

Parton fragmentation functions

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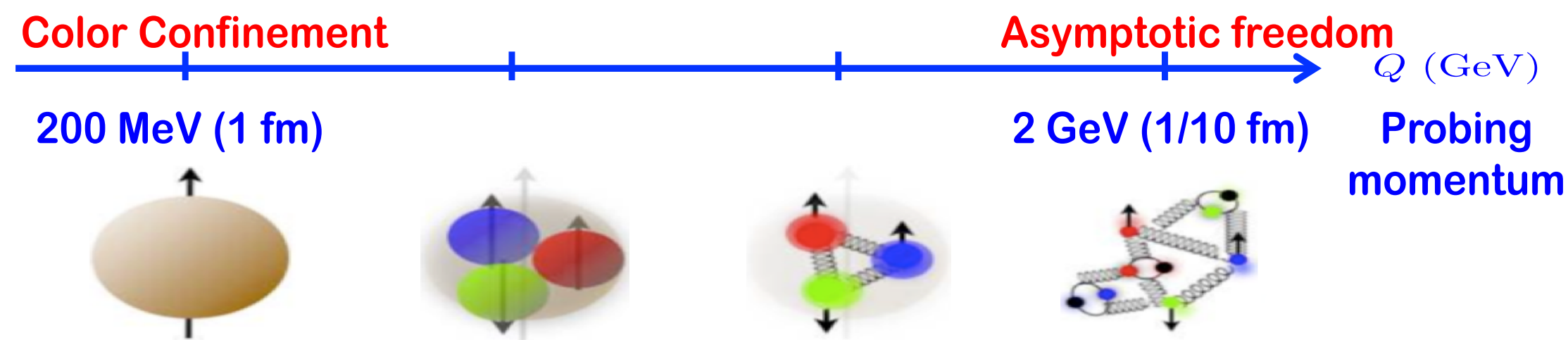


A fundamental property of QCD - color confinement

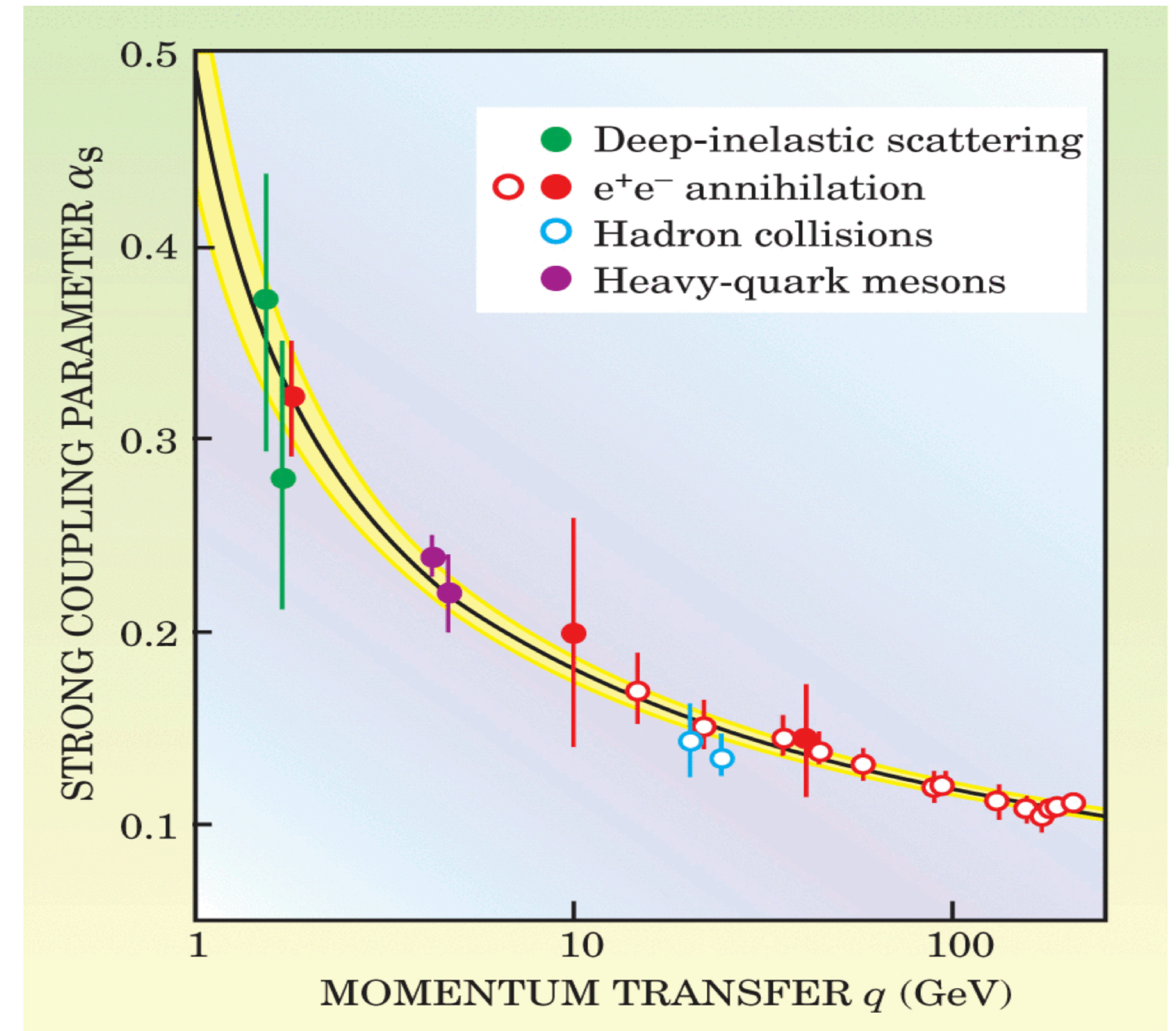
◆ QCD as the fundamental theory of strong interaction

$$\mathcal{L} = \bar{\Psi}_c(i\gamma^\mu\partial_\mu - m)\Psi_c + g\bar{\Psi}_c\gamma^\mu T_a\Psi_c G_\mu^a - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu}$$

$$G_{\mu\nu}^a \equiv \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - gf_{abc}G_\mu^b G_\nu^c$$

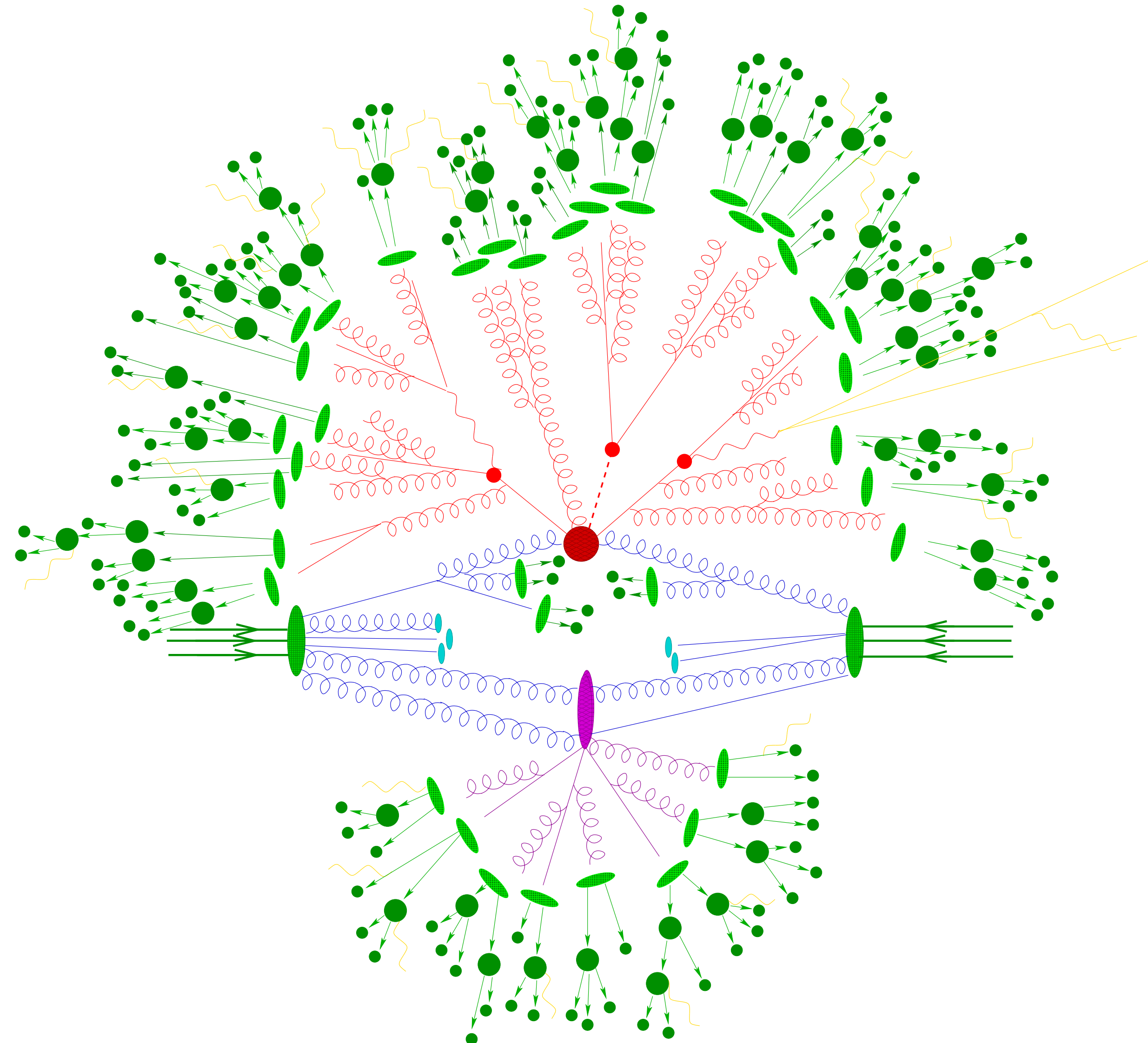


Color confinement in low energy/long distance scale



High energy scattering event

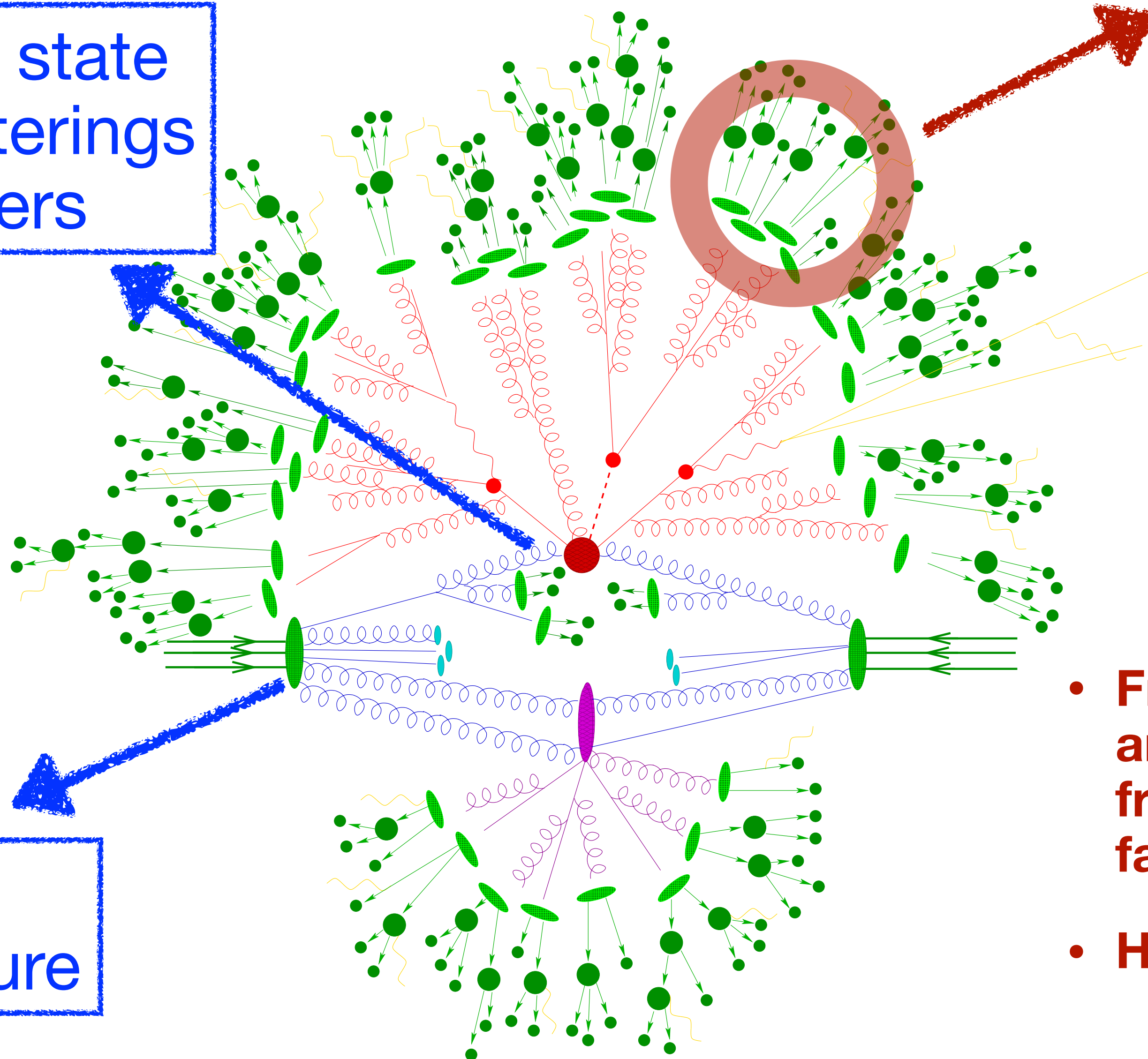
◆ Proton-proton collision



Hadronization is everywhere

◆ Proton-proton collision

Intermediate state
partonic scatterings
and showers



Initial state
hadron structure

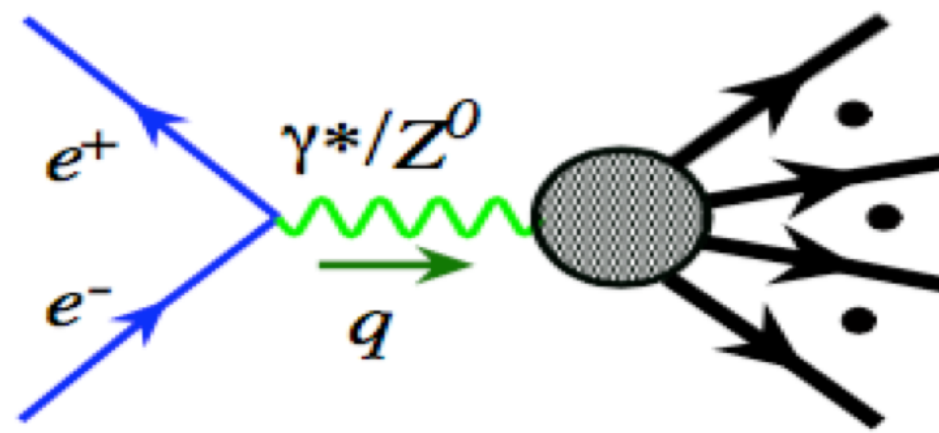
Final state
hadronization

- String fragmentation in PYTHIA
- Cluster hadronization in HERWIG and SHERPA
- Coalescence in AMPT
- **Fragmentation Functions (FFs) are defined within the framework of QCD factorization!**
- **Hadronization \neq FFs**

Multiple channels to explore parton hadronization

◆ Indispensable joint efforts from **experiments** and QCD theory

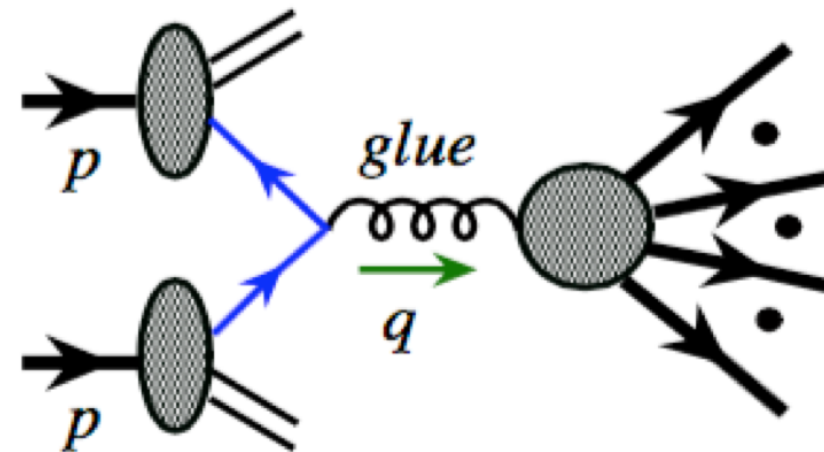
Lepton-lepton colliders



BEPC, SuperKEKB

- ▶ No hadron in the initial-state
- ▶ Hadrons are emerged from energy
- ▶ Not ideal for studying hadron structure, **but ideal for FFs**

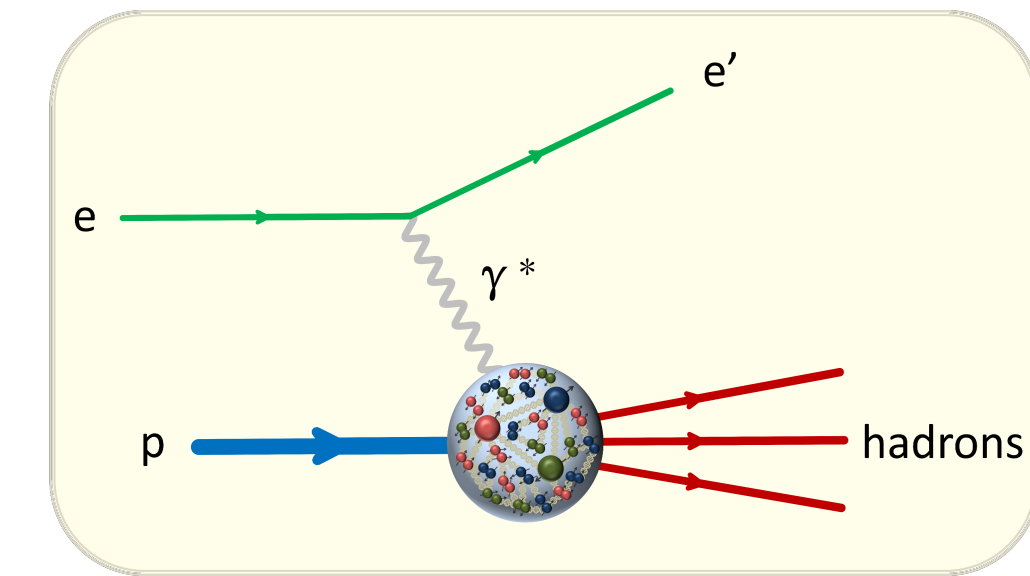
Hadron-hadron colliders



RHIC, LHC

- ▶ Hadrons in the initial-state
- ▶ Hadrons are emerged from energy
- ▶ Currently used for studying hadron structure and FFs

lepton-hadron colliders

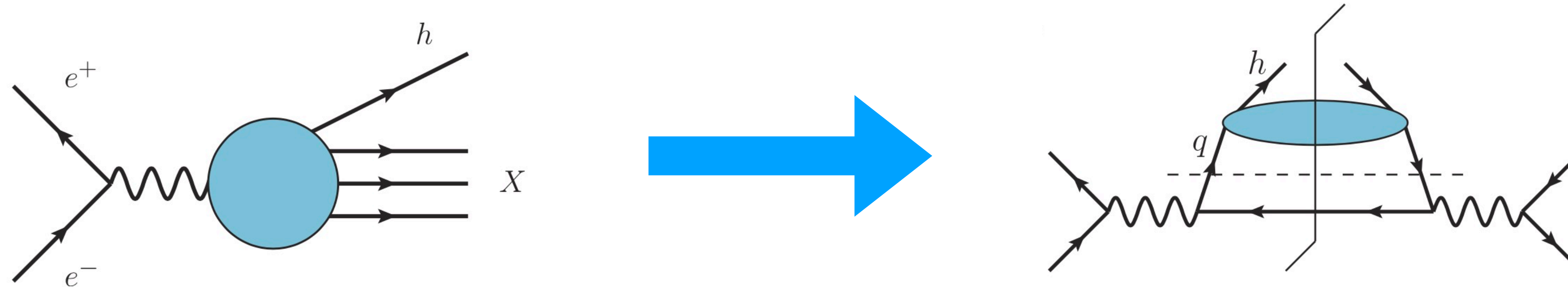


HERA, JLab

- ▶ Hadrons in the initial-state
- ▶ Hadrons are emerged from energy
- ▶ Ideal for studying hadron structure, can also involve FFs

A clean access to fragmentation functions

◆ QCD factorization in electron-positron annihilation



- Leading power/twist collinear factorization

$$\sigma^{e^+e^- \rightarrow hX} = \hat{\sigma}_{e^+e^- \rightarrow i} \otimes D_{i \rightarrow h}$$

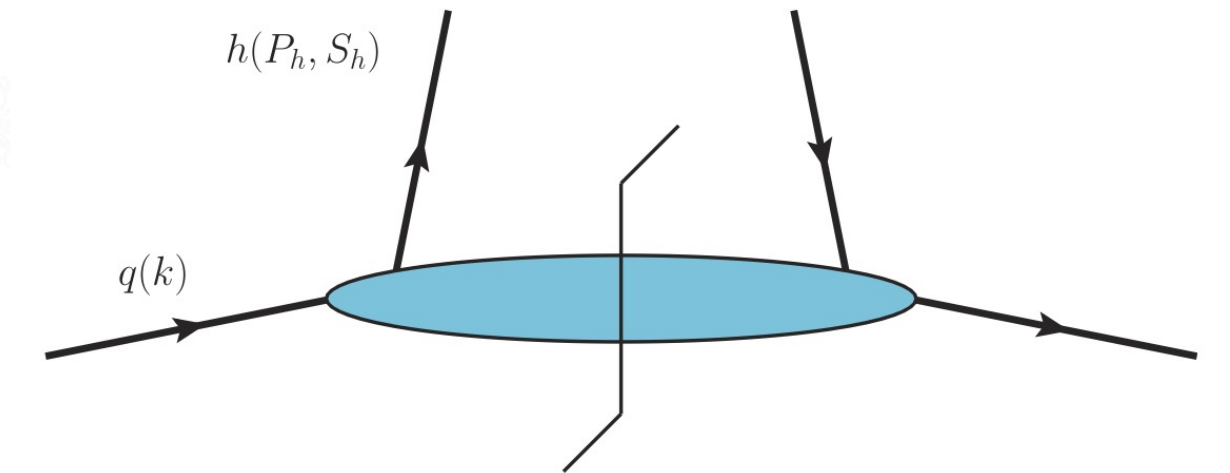
- Large momentum transfer $Q \gg \Lambda_{QCD}$
- High precision control of $\hat{\sigma}$
- D : fragmentation function, also called parton decay function, encodes the information on how partons produced in hard scattering hadronize into the detected color singlet hadronic bound state.

Fragmentation Functions

◆ Leading twist unpolarized fragmentation functions

- Operator definition

$$D_1^{h/q}(z) = \frac{z}{4} \sum_X \int \frac{d\xi^+}{2\pi} e^{ik^-\xi^+} \text{Tr} \left[\langle 0 | \mathcal{W}(\infty^+, \xi^+) \psi_q(\xi^+, 0^-, \vec{0}_T) | P_h, S_h; X \rangle \right. \\ \left. \times \langle P_h, S_h; X | \bar{\psi}_q(0^+, 0^-, \vec{0}_T) \mathcal{W}(0^+, \infty^+) | 0 \rangle \gamma^- \right].$$



- Probability densities for finding color-neutral particles inside partons

- Momentum sum rule
$$\sum_h \sum_{S_h} \int_0^1 dz z D_1^{h/q}(z) = 1$$

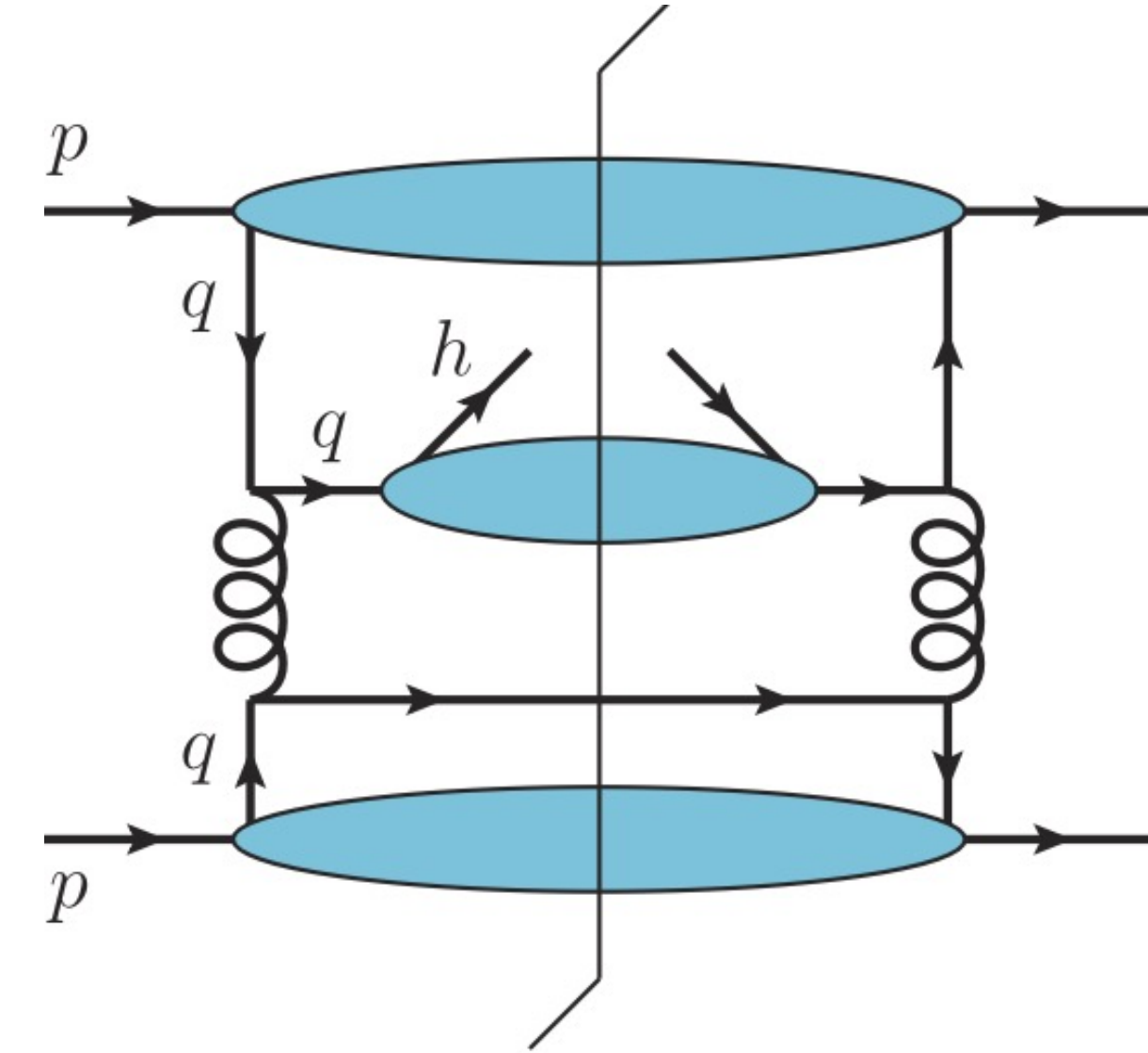
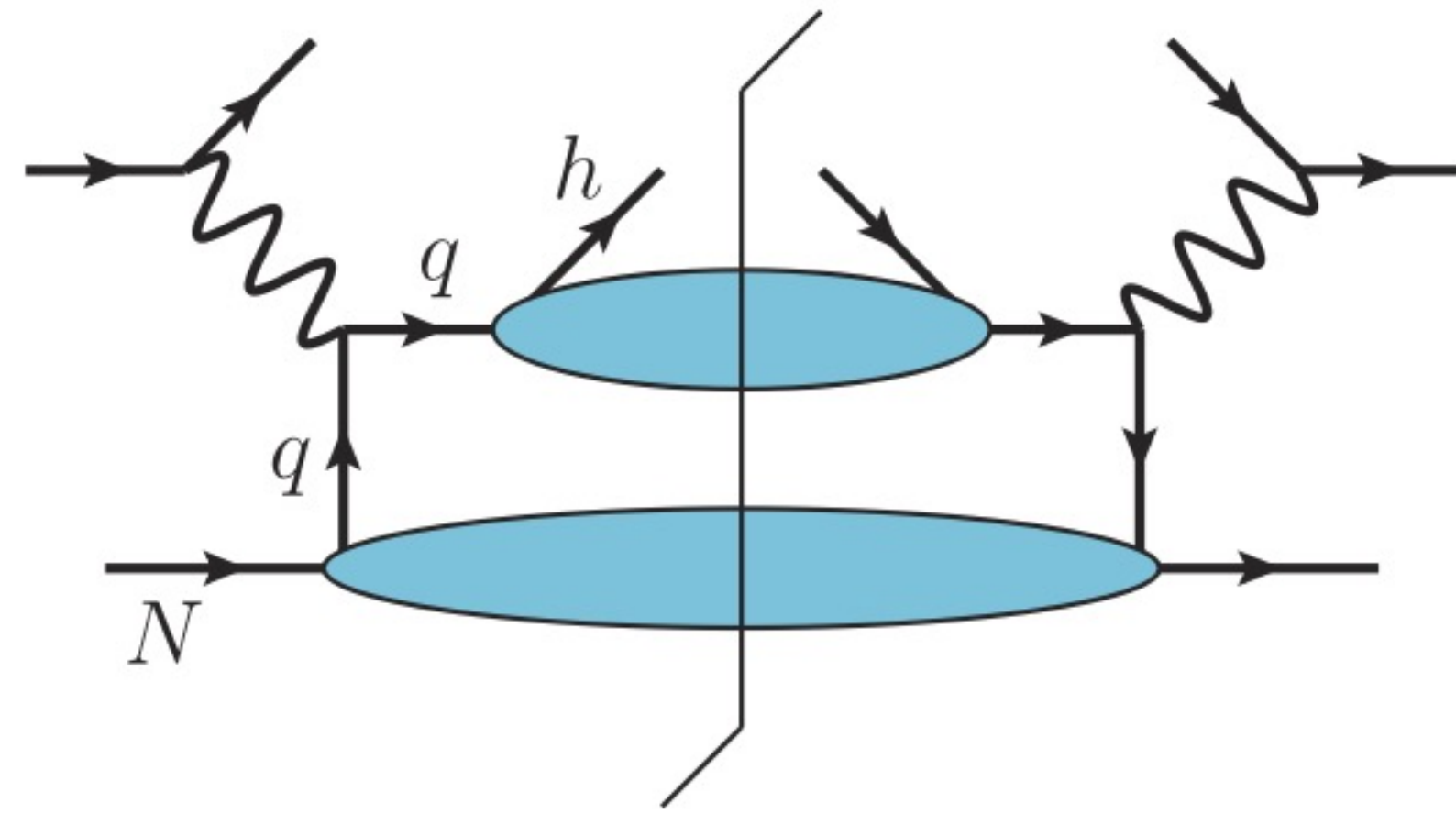
- Time-like DGLAP QCD evolution

$$\frac{d}{d \ln \mu^2} D_1^{h/i}(z, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \sum_j \int_z^1 \frac{du}{u} P_{ji}(u, \alpha_s(\mu^2)) D_1^{h/j}\left(\frac{z}{u}, \mu^2\right)$$

Perturbative splitting function:
$$P_{ji}(u, \alpha_s(\mu^2)) = P_{ji}^{(0)}(u) + \frac{\alpha_s(\mu^2)}{2\pi} P_{ji}^{(1)}(u) + \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2 P_{ji}^{(2)}(u) + \dots$$

Fragmentation Functions

◆ Access to FFs in ep and pp collisions: universality of FFs



- Factorization in semi-inclusive deep inelastic scattering

$$\sigma^{lp \rightarrow l'hX} = f_{i/p} \otimes \hat{\sigma}_{li \rightarrow j} \otimes D_{j \rightarrow h}$$

- Factorization in single inclusive hadron production in proton-proton collisions

$$\sigma^{pp \rightarrow hX} = f_{i/p} \otimes f_{j/p} \otimes \hat{\sigma}_{ij \rightarrow k} \otimes D_{k \rightarrow h}$$

Methodology for global extraction of FFs

Fitting Framework

Construction of χ_{global}^2 from χ_n^2

χ_n^2 Construction

Generation of Theory Data

FFs Evolution

Coefficient functions

Experimental Data

FF Parametrization

Minimization of χ_{global}^2

Hessian Matrix

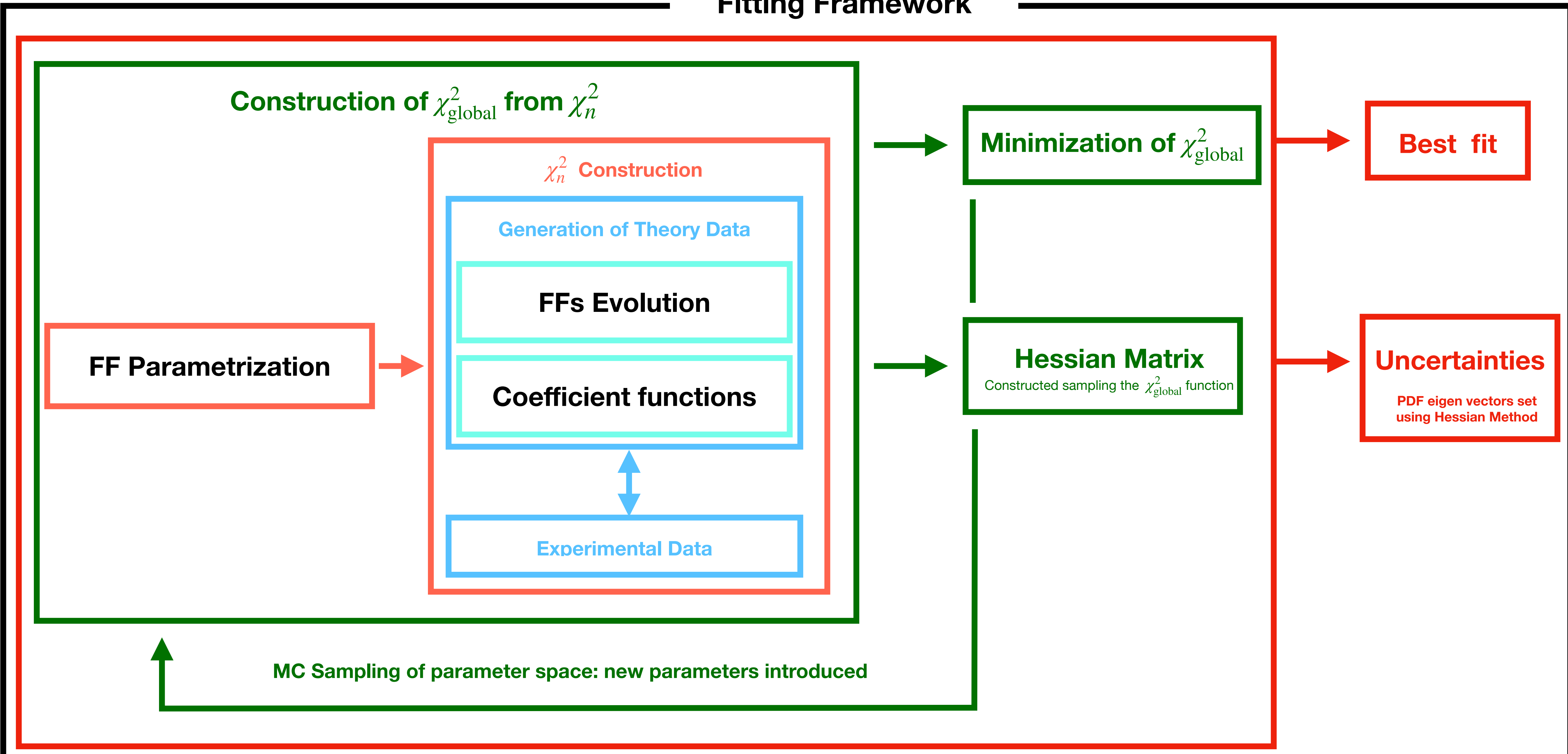
Constructed sampling the χ_{global}^2 function

Best fit

Uncertainties

PDF eigen vectors set
using Hessian Method

MC Sampling of parameter space: new parameters introduced



FF global fitting panorama

◆ Joint efforts from experiments and theory in extracting FFs

Table courtesy of E.R.Nocera

	DHESS	HKNS	JAM	NNFF1.0/1.1h
SIA	✓	✓	✓	✓
SIDIS	✓	✗	✗	✗
PP	✓	✗	✗	✓
statistical treatment	Iterative Hessian 68% - 90%	Hessian $\Delta\chi^2 = 15.94$	Monte Carlo	Monte Carlo
parametrisation	standard	standard	standard	neural network
HF scheme	ZM-VFN	ZM-VFN	ZM-VFN	ZM-VFN
hadron species	$\pi^\pm, K^\pm, p/\bar{p}, h^\pm$	$\pi^\pm, K^\pm, p/\bar{p}$	π^\pm, K^\pm	$\pi^\pm, K^\pm, p/\bar{p}, h^\pm$
latest update	PRD 91 (2015) 014035 PRD 95 (2017) 094019	PTEP 2016 (2016) 113B04	PRD 94 (2016) 114004	Eur.Phys.J.C 77 (2017) 8, 516 Eur.Phys.J.C 78 (2018) 8, 651
perturbative order	LO/NLO	LO/NLO	LO/NLO	LO/NLO/NNLO

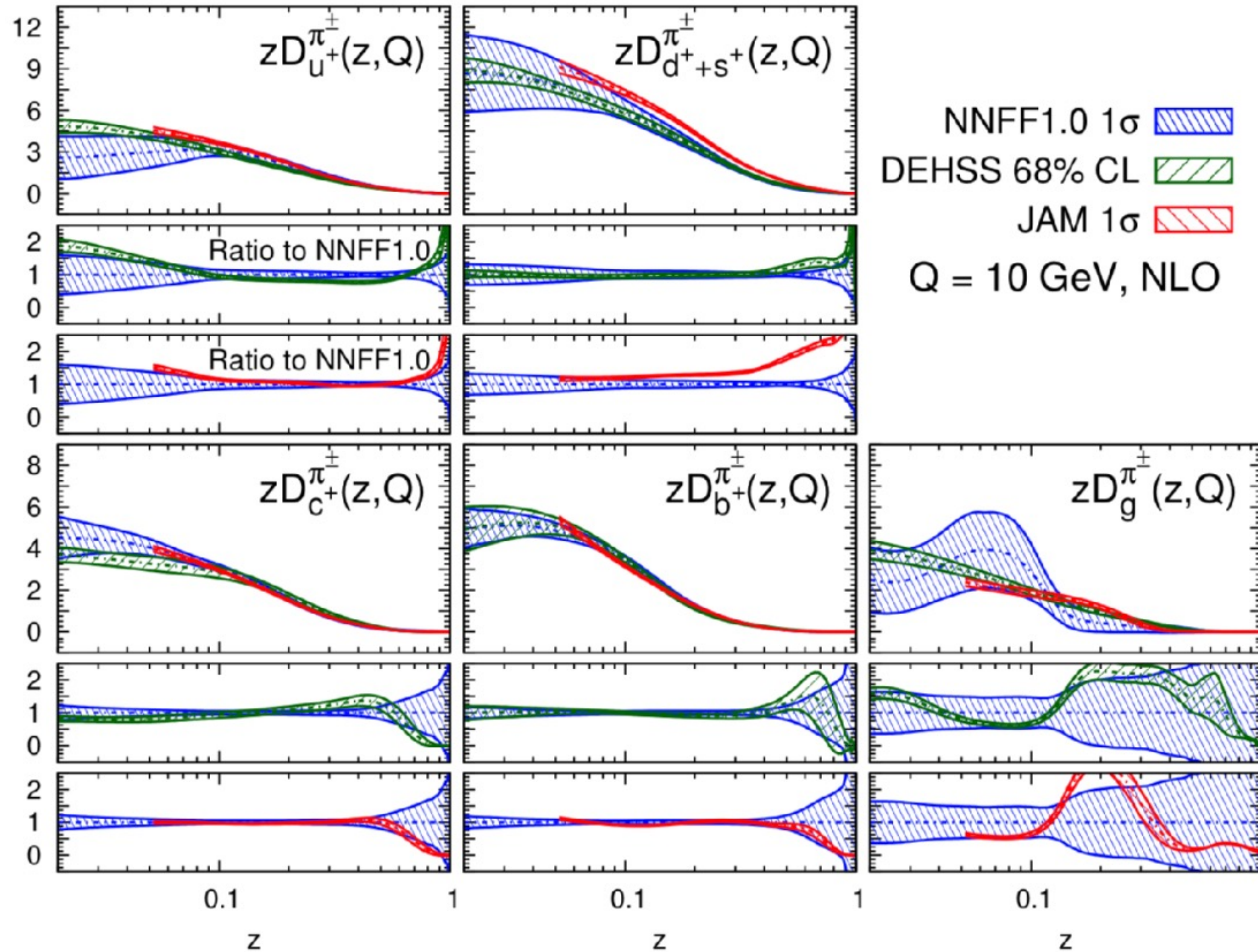
FF global fitting panorama

◆ Joint efforts from experiments and theory in extracting FFs

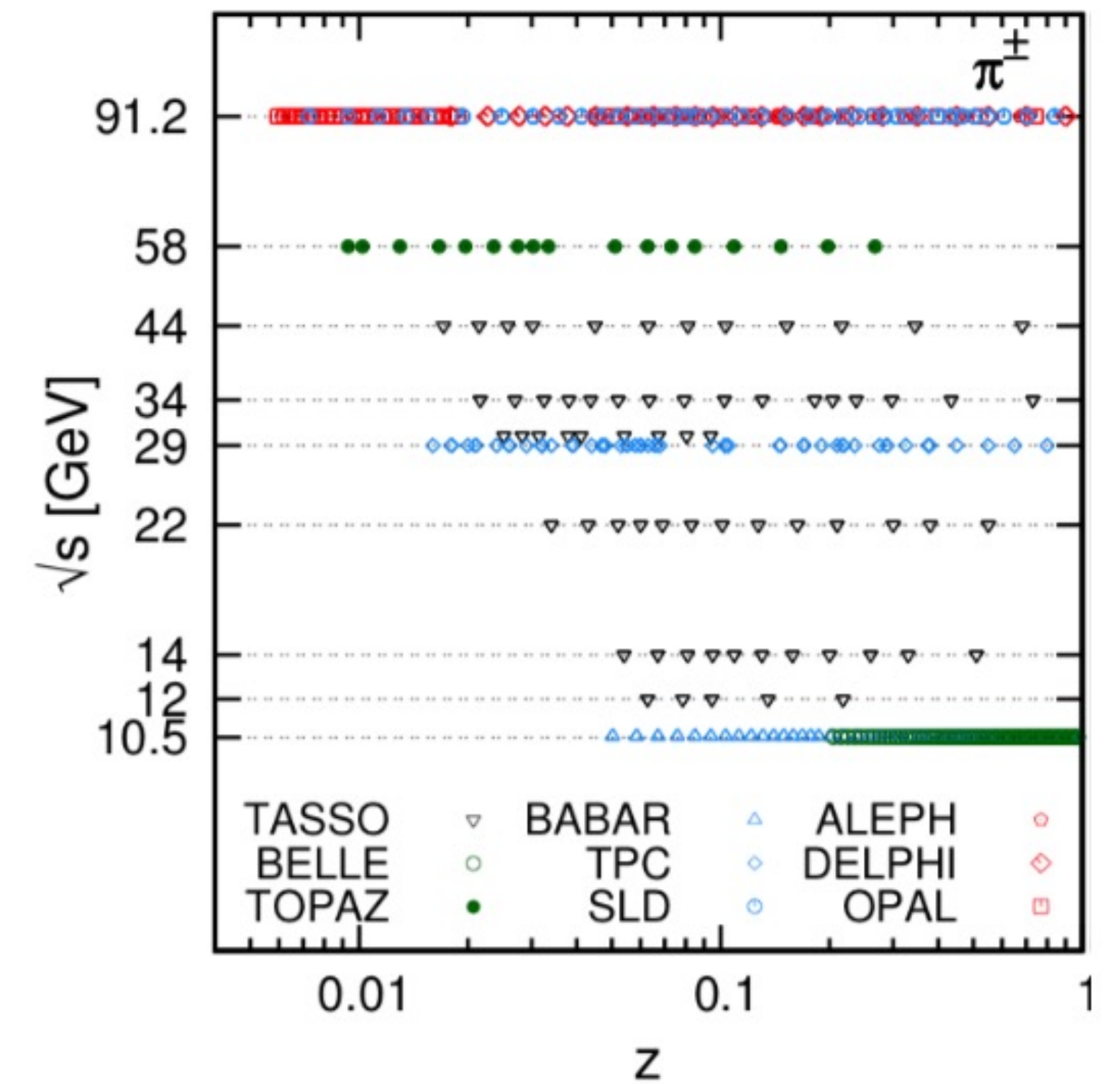
	MAPFF	BSFSV	ARS	AKRS
SIA SIDIS PP	✓ ✓ (Approximate NLL+NLP) ✗	✓ ✓ (Approximate NLL+NLP) ✗	✓ ✗ ✗	✓ ✗ ✗
Statistical treatment	Monte Carlo	NO	NO	NO
Parametrization	Neural network	Standard	Standard	Standard
HF Scheme	ZM-VFNS	ZM-VFNS	ZM-VFNS	ZM-VFNS
Hadron spices	π^\pm, K^\pm	π^\pm	π^\pm	π^\pm
Latest update	PRD 104 (2021) 3, 03 4007 PLB 834 (2022) 1374 56	PRL 129 (2022) 1, 01 2002	PRD 92 (2015) 11, 11 4017	PRD 95 (2017) 5, 054 003
perturbative order	LO/NLO/NNLO	NLO/NNLO	LO/NLO/NNLO	LO/NLO/NNLO

FFs panorama

◆ The best known FFs - π

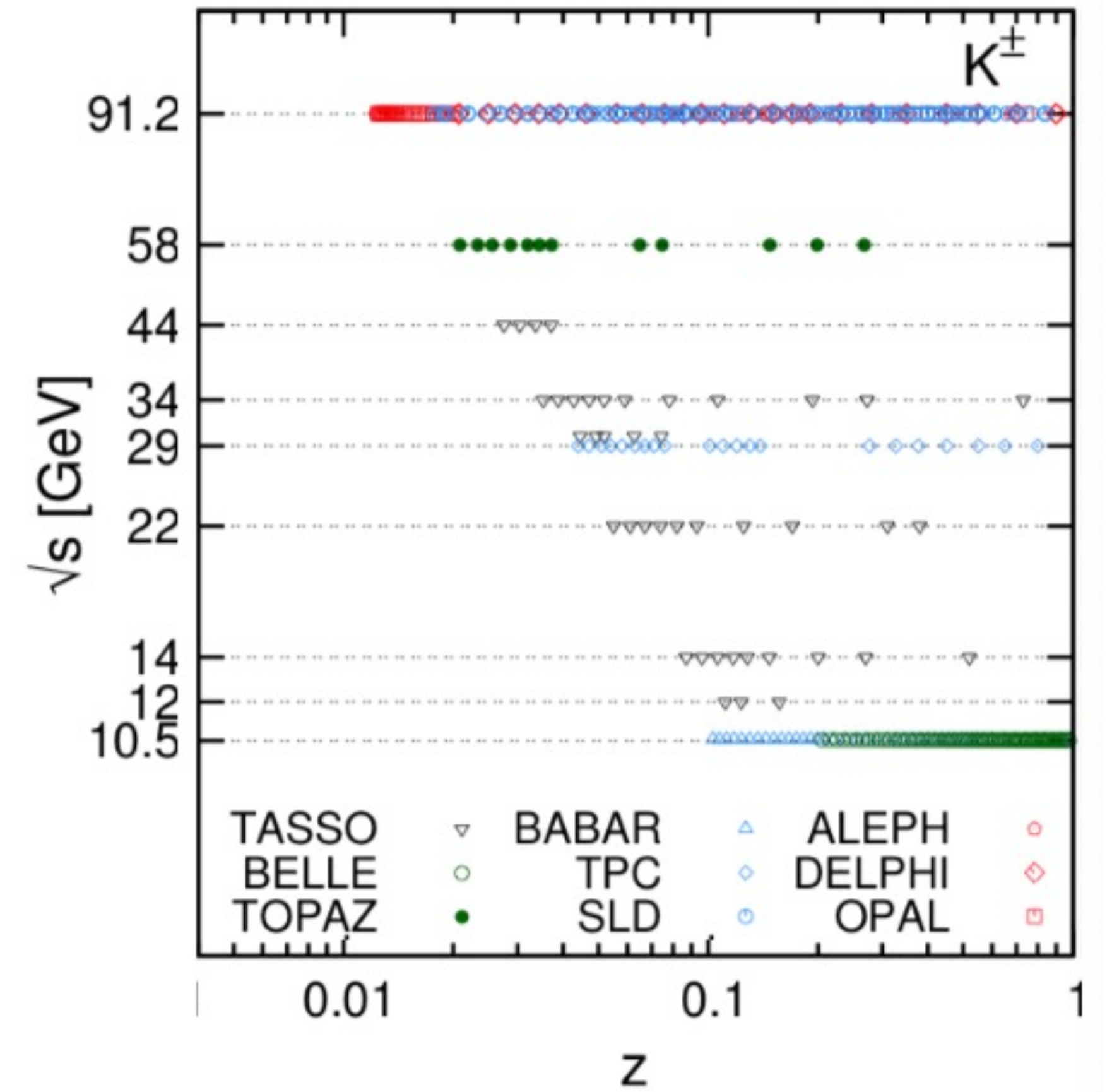
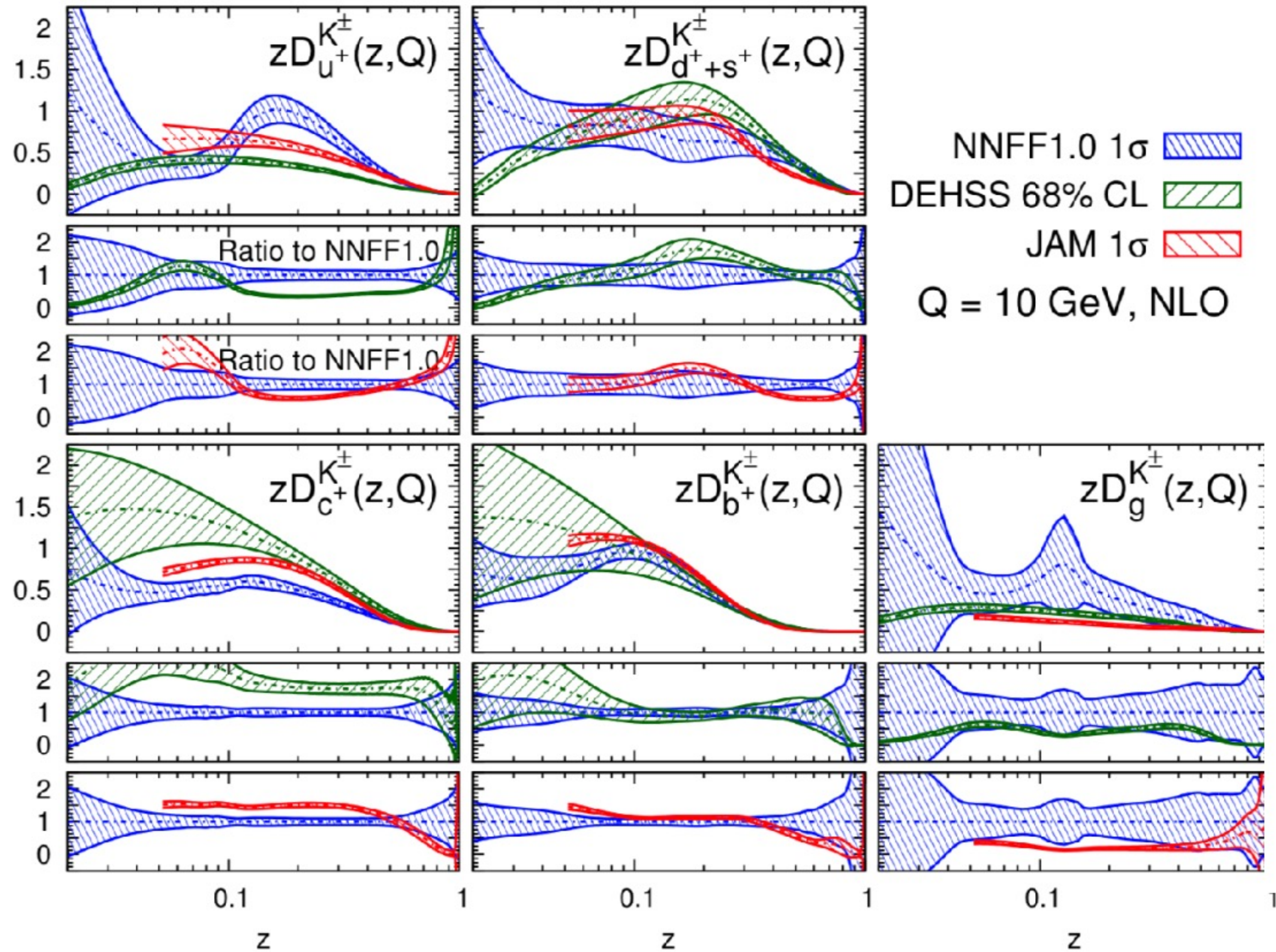


V. Bertone et al. [NNPDF collaboration] *Eur. Phys. J. C* 77 (2017) 8, 516
 D. de Florian et al., *Phys. Rev. D* 91 (2015), 4035, *D* 95 (2017), 094019
 N. Sato et al. [JAM Collaboration] *Physical Review D*, 94 (2016) 11, 114004



FFs panorama

◆ kaon FFs

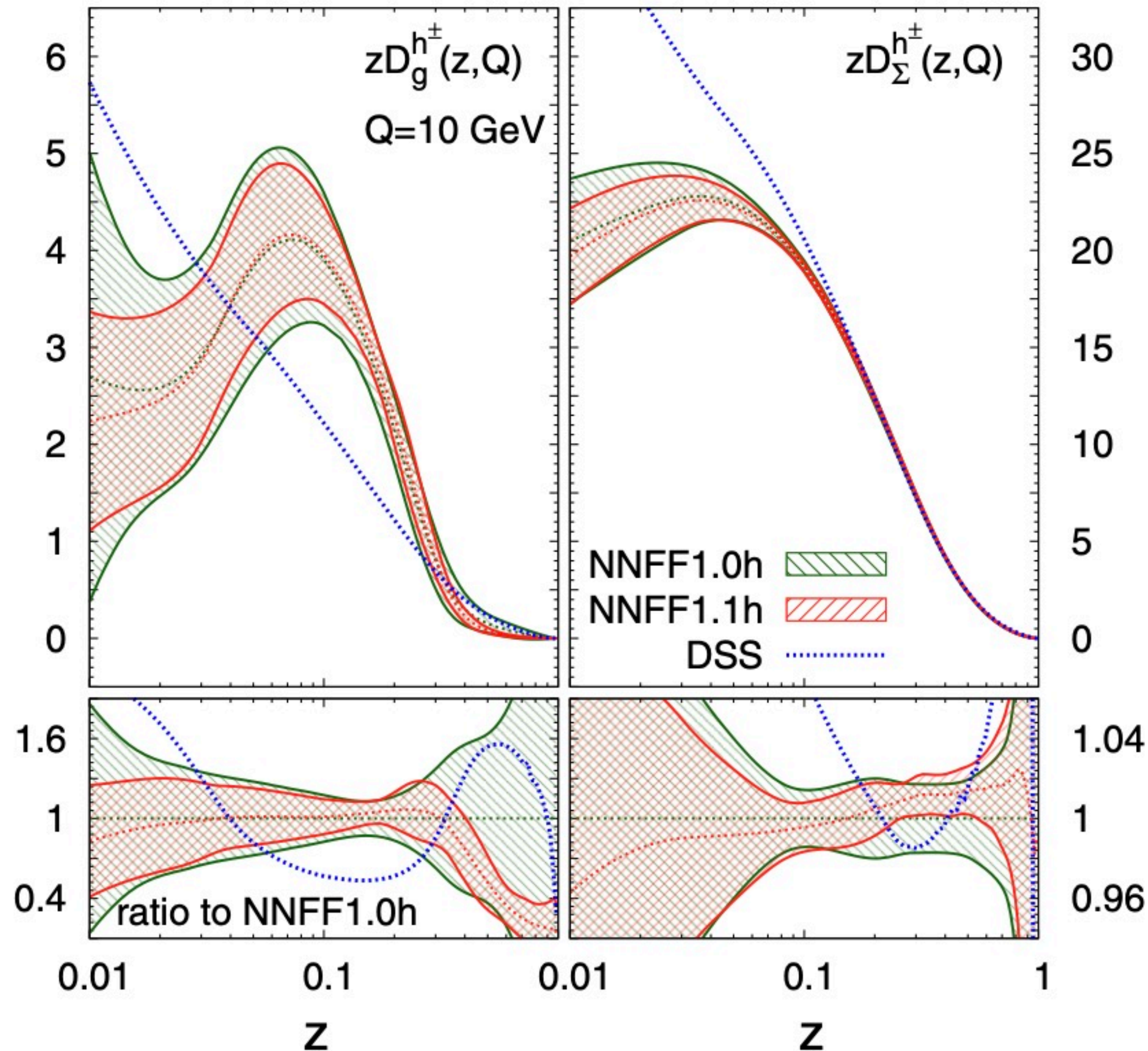


Fragmentation Functions

◆ Charged hadron FFs

NNPDF, EPJC 2018

DSS, PRD 2007



It is proved that FFs are universal, why they look different?

- ▶ Different selections of experimental data (kinematic cut)
- ▶ Different parametrization for FFs at initial scale, NNFF unbiased? DSS biased?
- ▶ Everything else is the same

More measurements are needed to further constrain the FFs, SIA will play very important role!

New opportunities in probing FFs

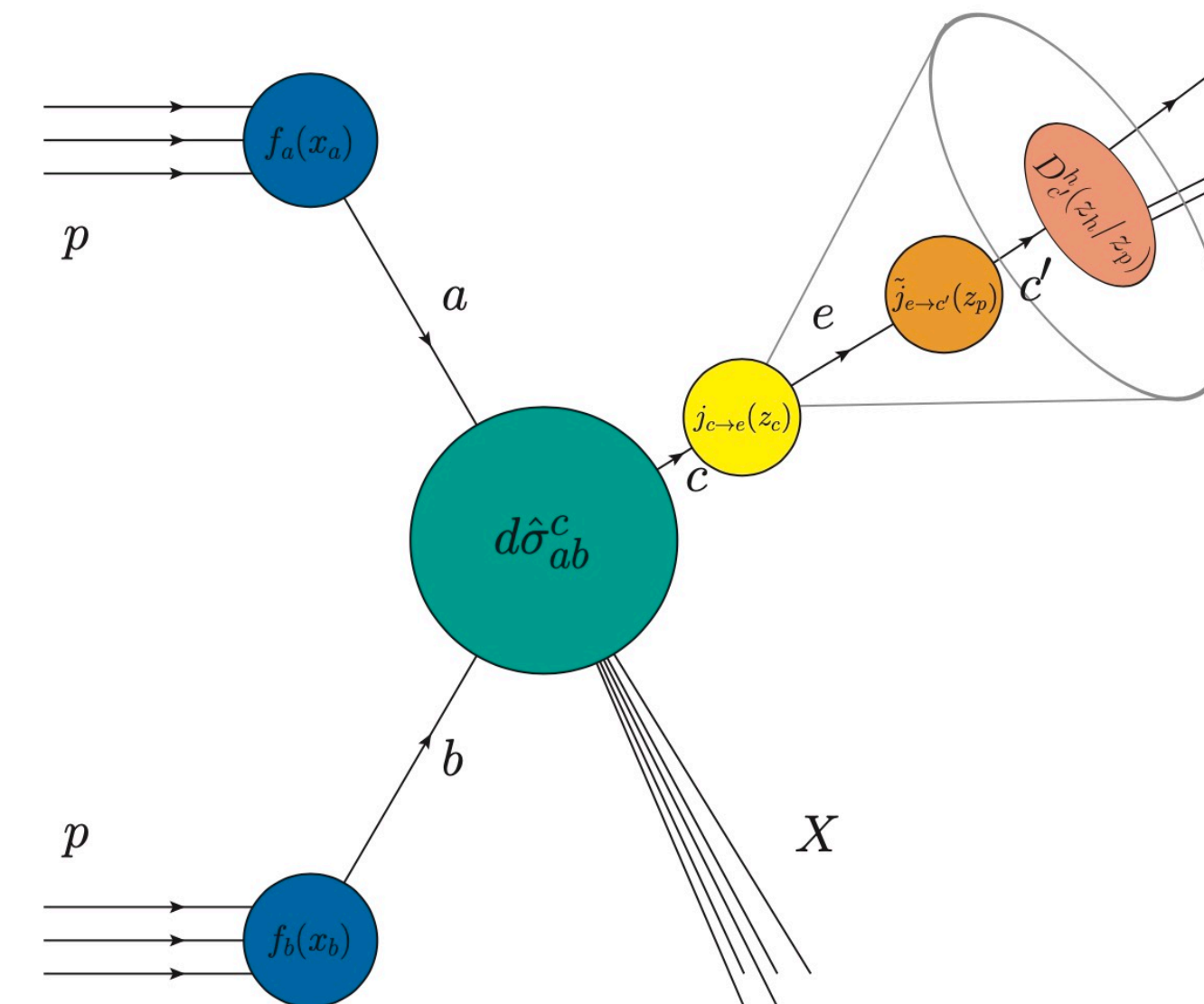
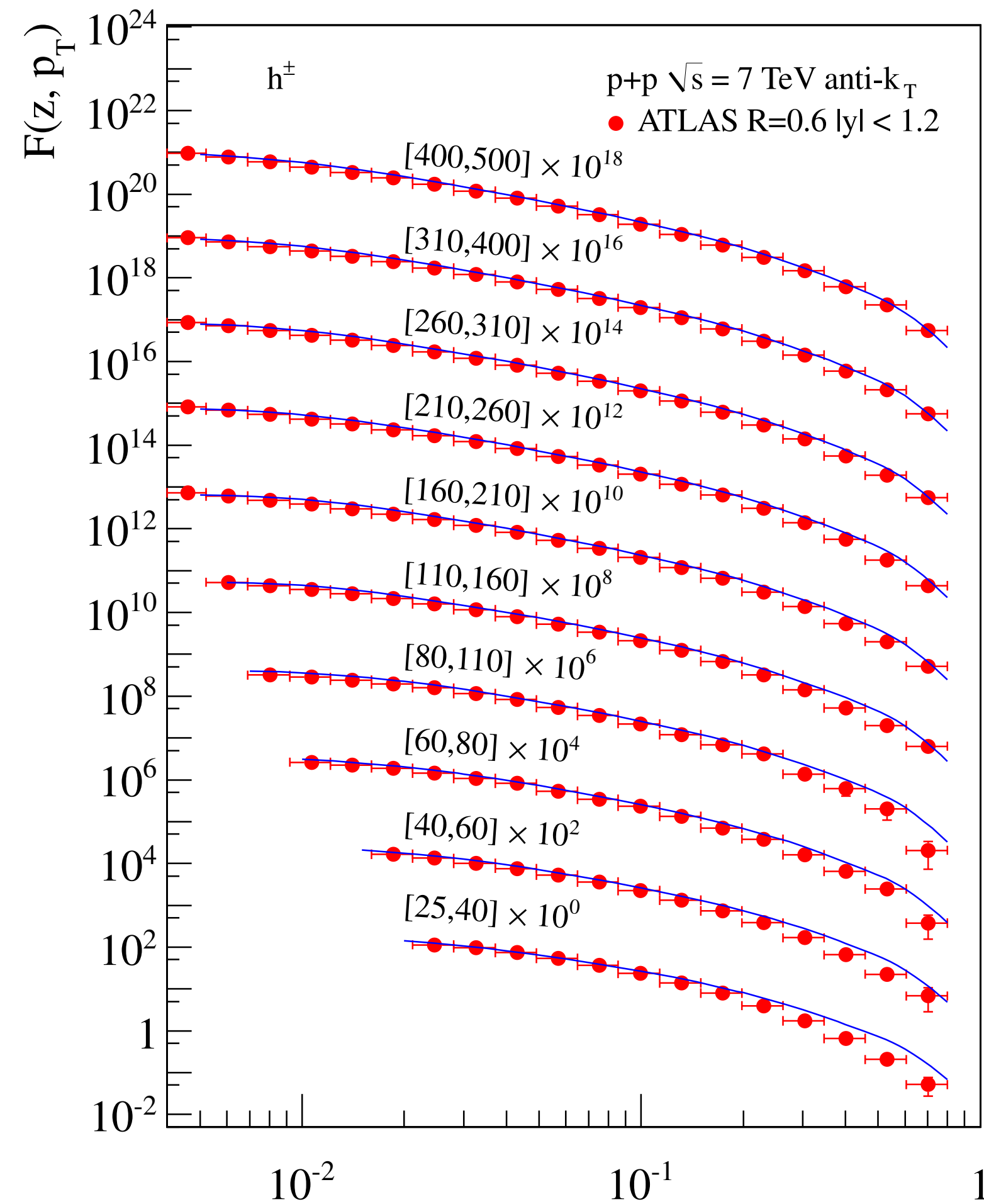
◆ Jet fragmentation function

Chien, Kang, Ringer, Vitev, **HX**, JHEP (2016)

$$\sigma^{pp \rightarrow J(h)X} = f_{i/p} \otimes f_{j/p} \otimes \hat{\sigma}_{ij \rightarrow k} \otimes \mathcal{G}_{k \rightarrow J(h)}$$

$$\mathcal{G}_{i \rightarrow J(h)} = \mathcal{F}_{ij} \otimes D_{j \rightarrow h}$$

$$F(z_h, p_T) = \frac{d\sigma^{J(h)}}{dp_T d\eta dz_h} \bigg/ \frac{d\sigma}{dp_T d\eta}$$



$$z_h = \frac{p_T^h}{p_T}$$

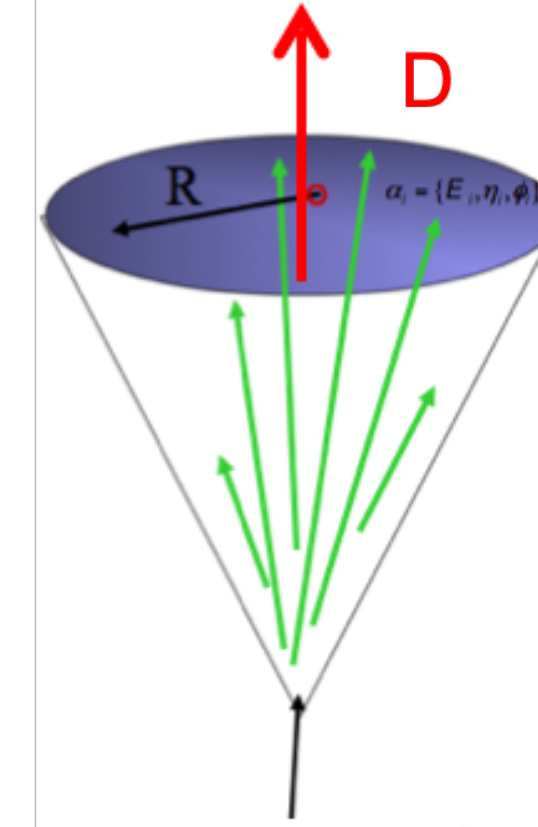
Light hadrons work very well

Heavy flavor in jet

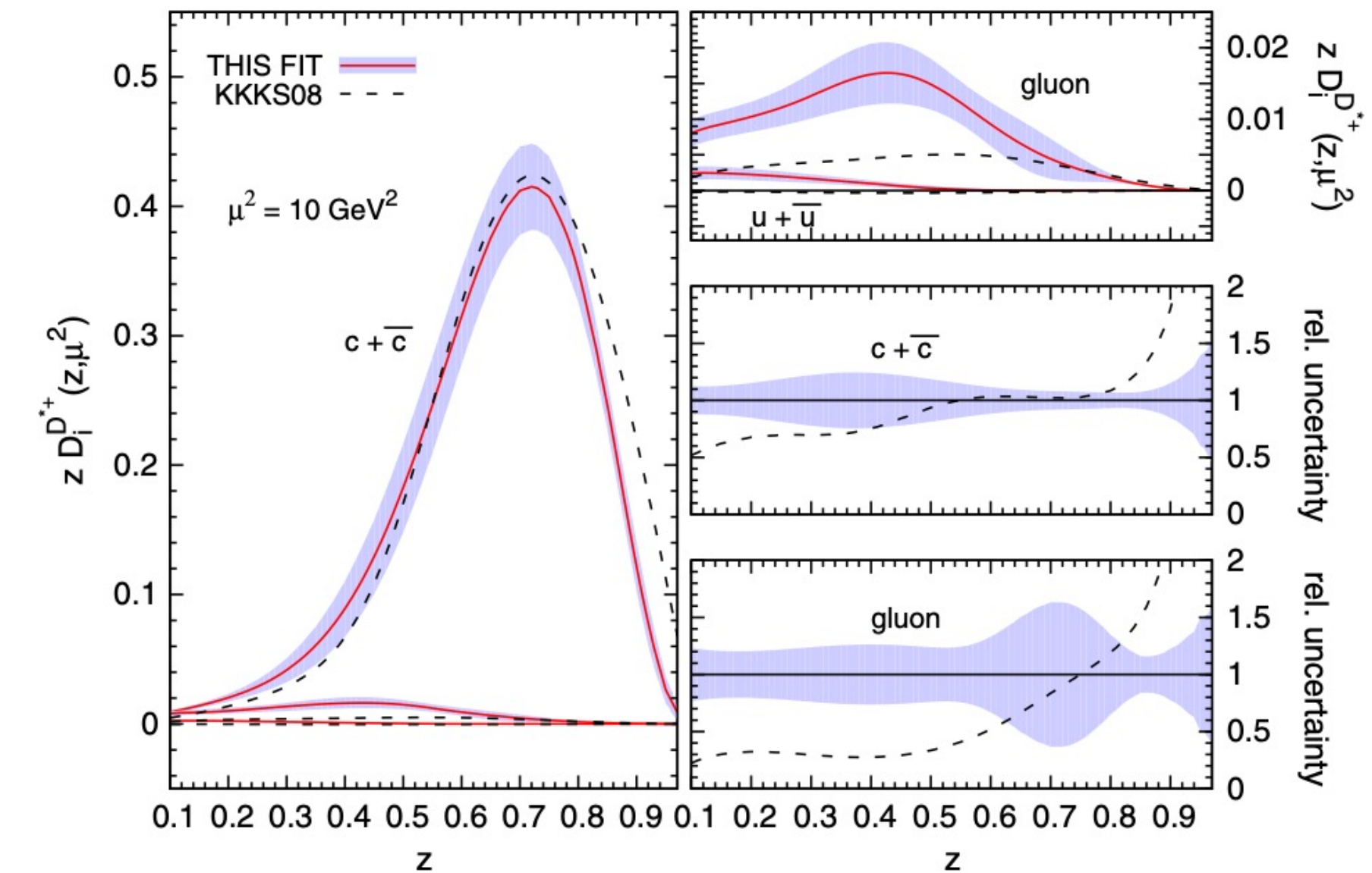
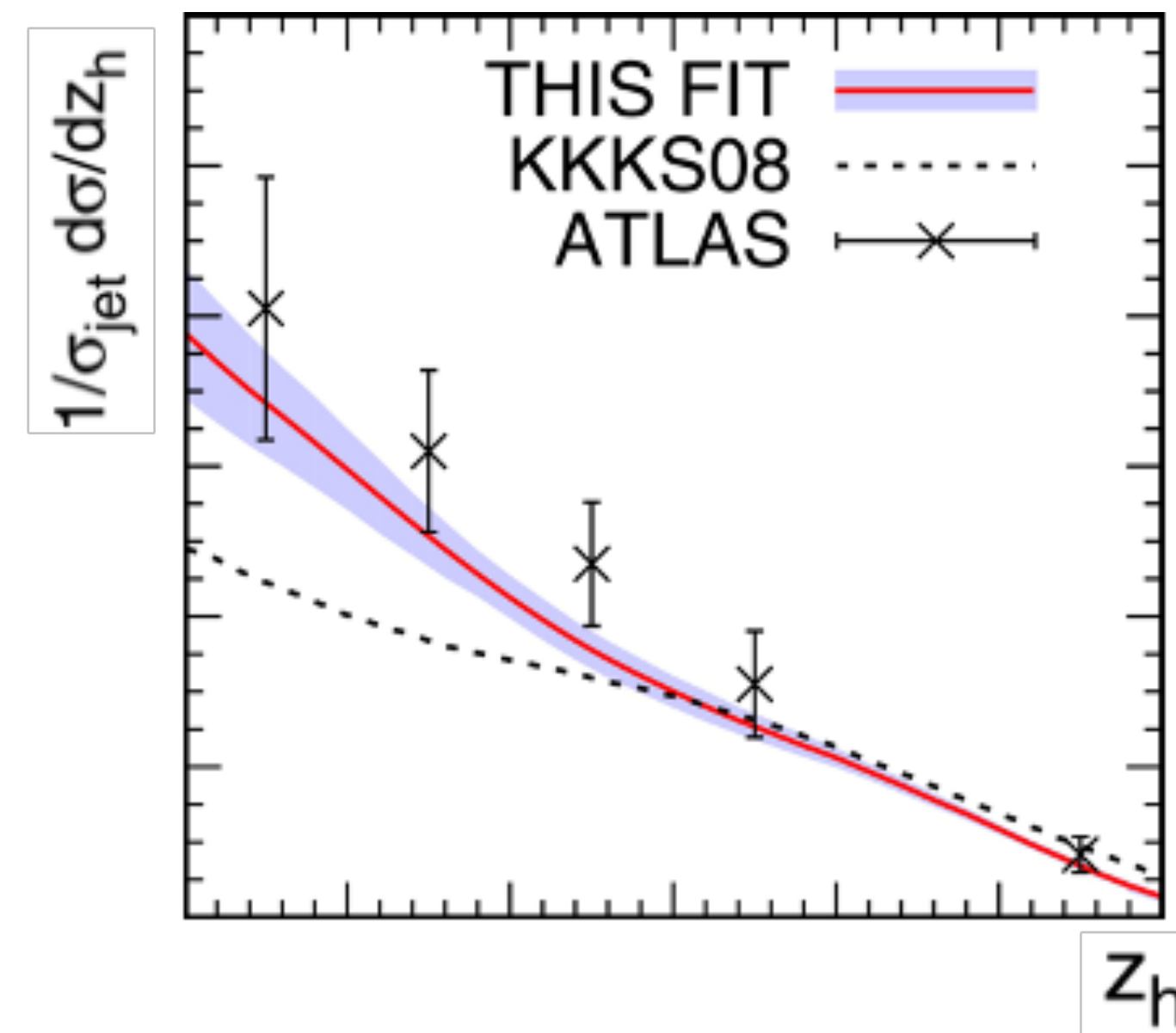
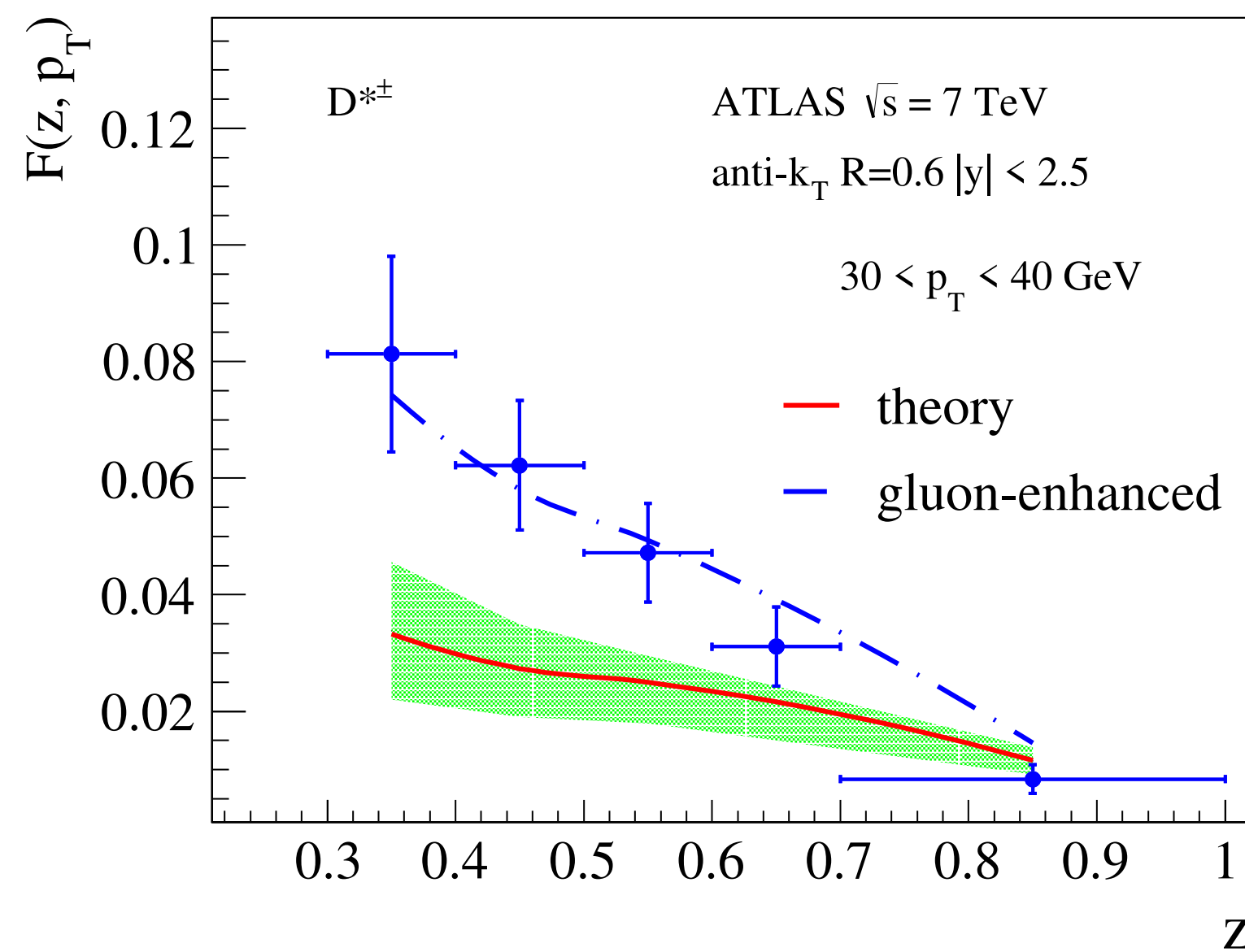
◆ Jet fragmentation function for D meson

$$F(z_h, p_T) = \frac{d\sigma^{J(h)}}{dp_T d\eta dz_h} \bigg/ \frac{d\sigma}{dp_T d\eta}$$

Chien, Kang, Ringer, Vitev, **HX**, JHEP (2016)



AKSRV, PRD (2017)



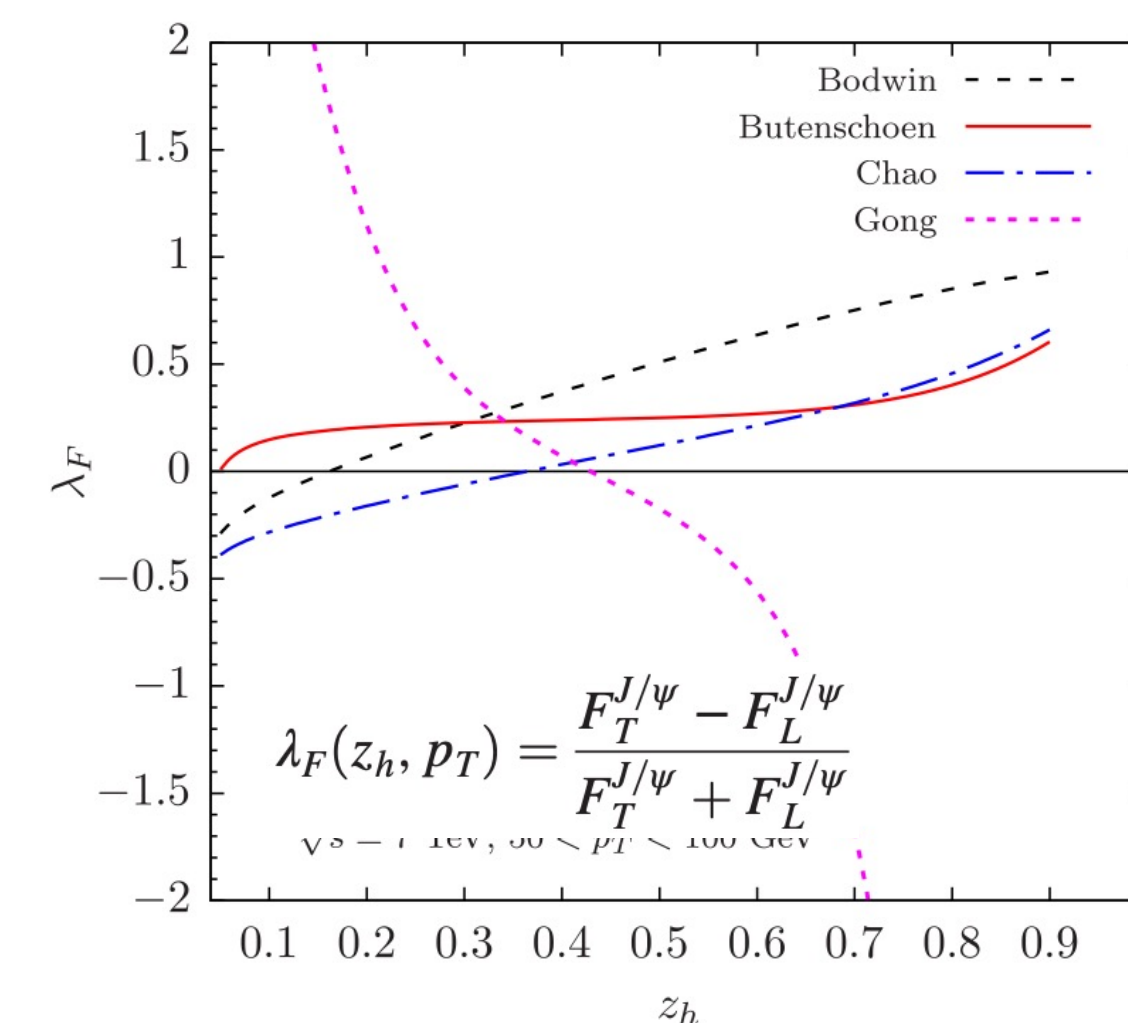
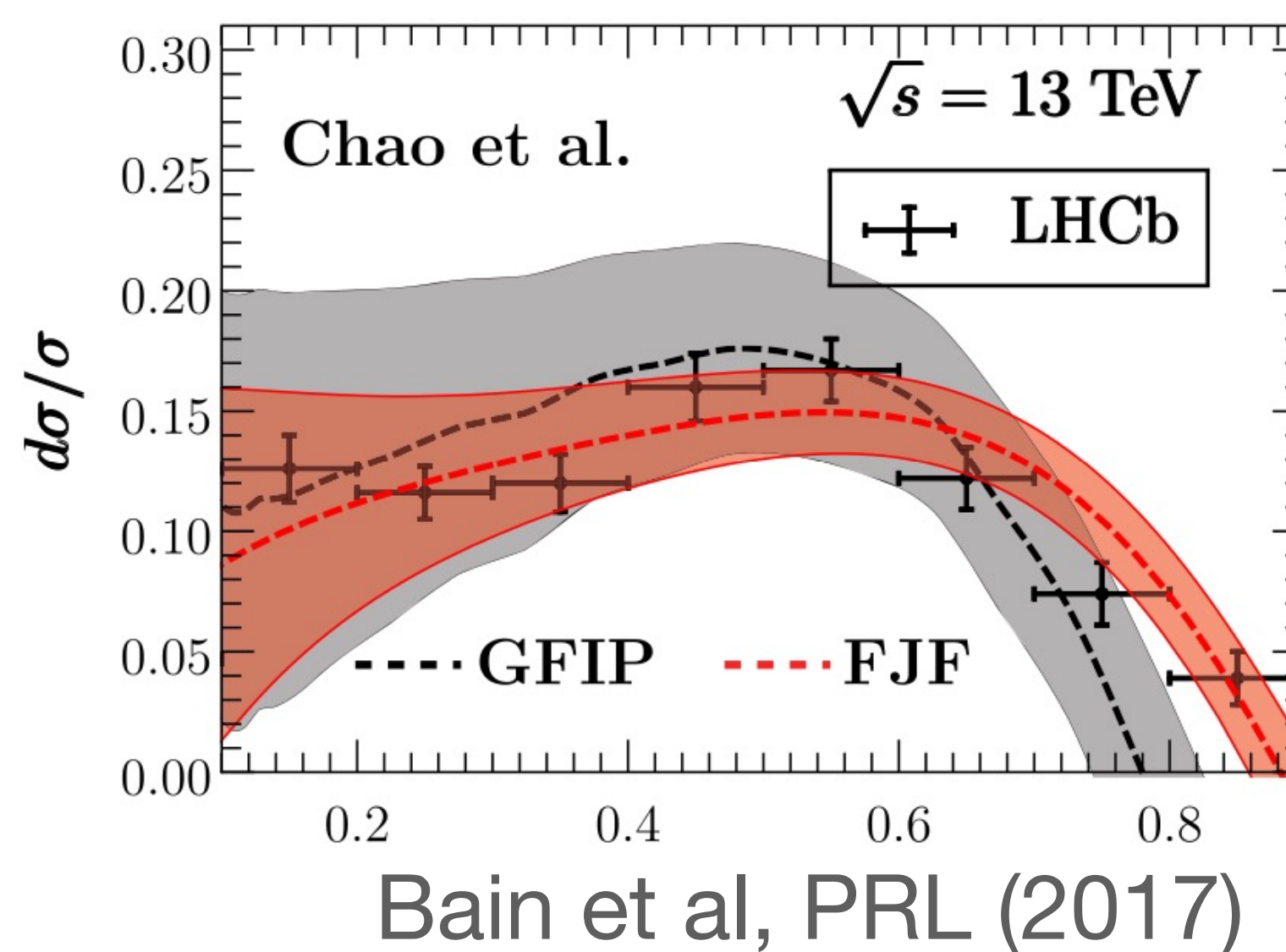
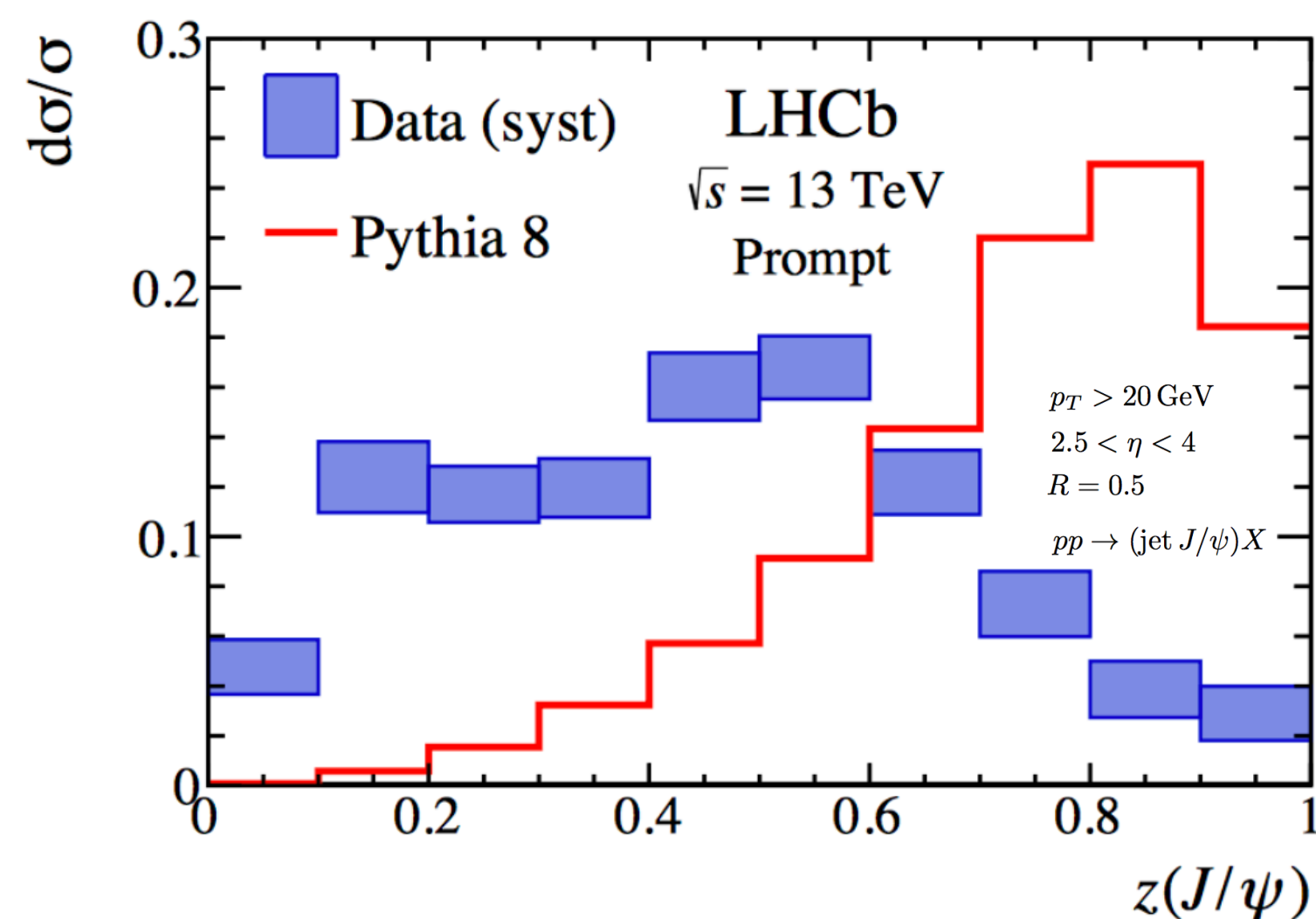
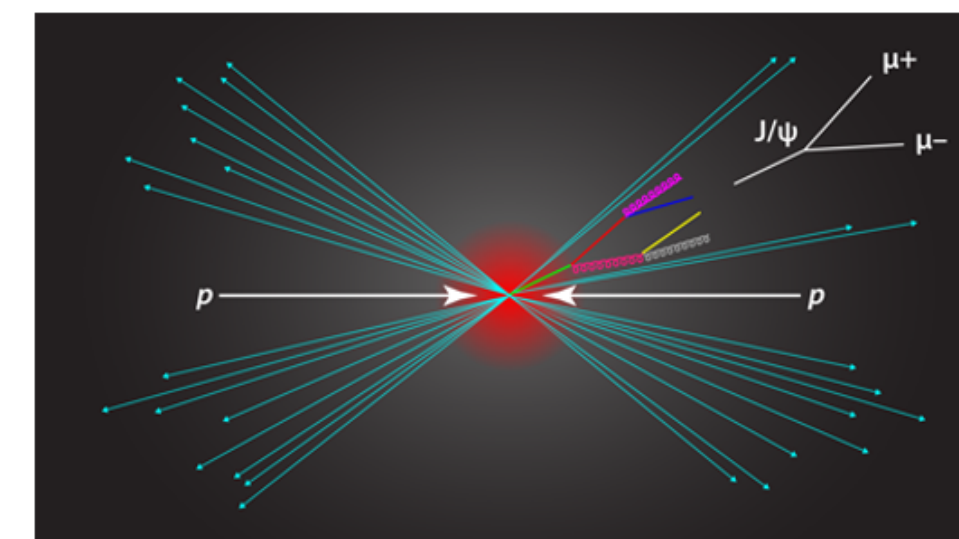
- Failed to describe D meson production in jet using KKK08 FFs
- Leads to new constrain of heavy flavor FFs using measurement of D in jet

Heavy flavor in jet

◆ Jet fragmentation function for J/ψ

$$\frac{d\sigma^{J/\psi}}{dp_T d\eta dz_h} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes \mathcal{G}_c^{J/\psi}$$

$$\mathcal{G}_i^{J/\psi}(z, z_h, p_{\text{jet}}^+, R, \mu) = \sum_j \int_{z_h}^1 \frac{dz'_h}{z'_h} \mathcal{J}_{ij}(z, z_h/z'_h, p_{\text{jet}}^+, R, \mu) \times D_j^{J/\psi}(z'_h, \mu) + \mathcal{O}(m_{J/\psi}^2 / (p_{\text{jet}}^+ R)^2)$$

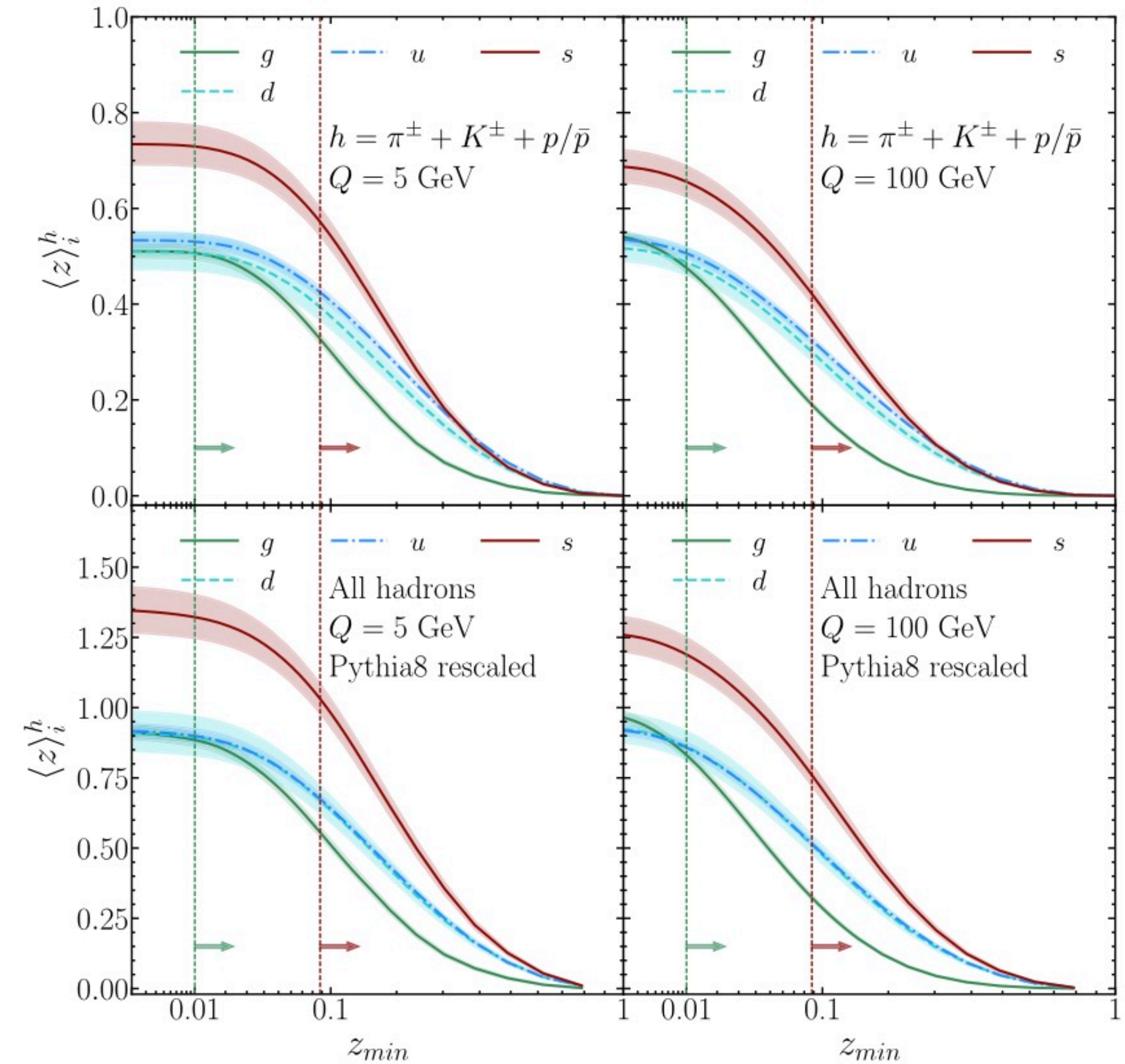
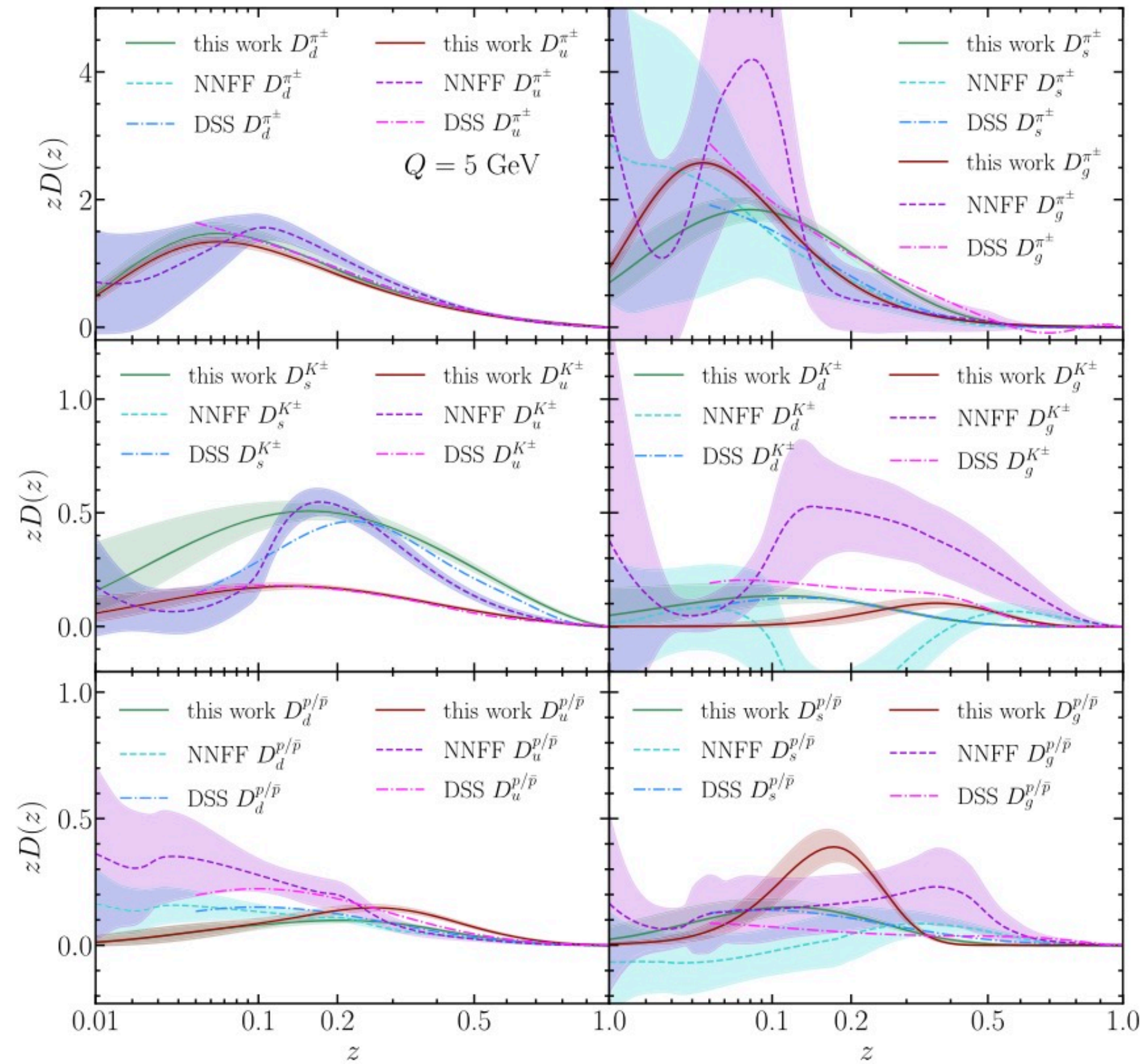


- Disagreement between default Pythia and data

- New insight into the shower mechanism for J/ψ production, and new constrain of LDMEs

New efforts from NPC

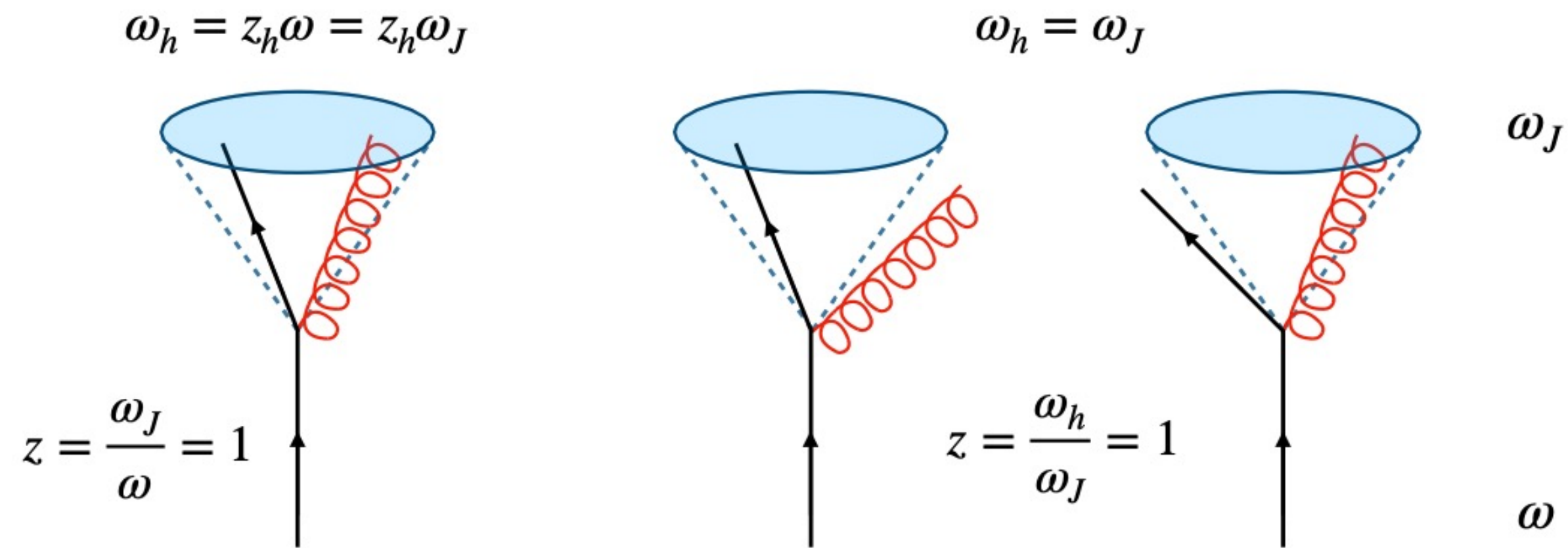
◆ **Joint analysis from NPC (SJTU+SCNU+IMP)** Gao, Liu, Shen, **HX**, Zhao, PRL, 2024



- Higher precision determination of FFs for charged hadron
- Hint for violation of momentum sum rule?

Parton to hadron fragmentation in jet

◆ A comprehensive analysis for jet fragmentation functions



		Quark polarization		
		U	L	T
Hadron polarization	U	$\mathcal{D}_1 =$		$\mathcal{H}_1^\perp =$
	L		$\mathcal{G}_{1L} =$	$\mathcal{H}_{1L}^\perp =$
	T	$\mathcal{D}_{1T}^\perp =$	$\mathcal{G}_{1T} =$	$\mathcal{H}_1 =$
				$\mathcal{H}_{1T}^\perp =$

Kang, **HX**, Zhao, Zhou, JHEP, 2024

- Collinear fragmenting jet function in semi-inclusive jet production

$$\Delta_{(T)} \mathcal{G}_i^h(z, z_h, \omega_J, \mu) = \sum_j \int_{z_h}^1 \frac{dz'_h}{z'_h} \Delta_{(T)} \mathcal{J}_{ij}(z, z'_h, \omega_J, \mu) \Delta_{(T)} D_j^h\left(\frac{z_h}{z'_h}, \mu\right)$$

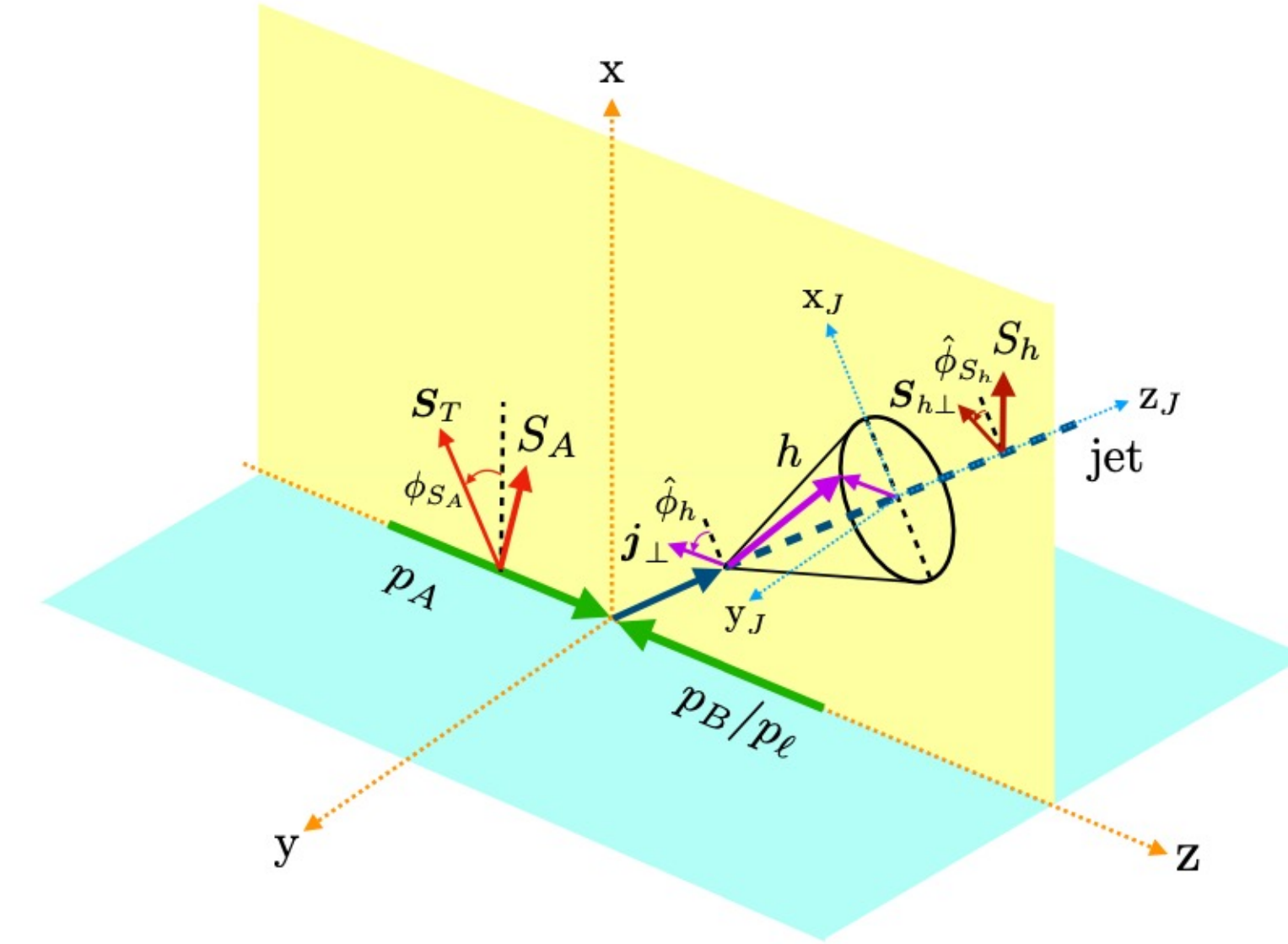
- An alternative way to explore different types of FFs
- Similar FJFs can be defined in exclusive jet production

Parton to hadron fragmentation in jet

◆ single inclusive jet production in hadronic collisions

Kang, HX, Zhao, Zhou, 2024

$$\begin{aligned}
 \frac{d\sigma^{p(S_A)+p \rightarrow \text{jet}h(S_h)} X}{d\eta d^2\mathbf{p}_T dz_h d^2\mathbf{j}_\perp} &= F_{UU,U} + |\mathbf{S}_T| \sin(\phi_{S_T} - \phi_h) F_{TU,U}^{\sin(\phi_{S_T} - \phi_h)} \\
 &+ \Lambda_h \left[\lambda F_{LU,L} + |\mathbf{S}_T| \cos(\phi_{S_T} - \phi_h) F_{TU,L}^{\cos(\phi_{S_T} - \phi_h)} \right] \\
 &+ |\mathbf{S}_{h_T}| \left[\sin(\phi_h - \phi_{S_h}) F_{UU,T}^{\sin(\phi_h - \phi_{S_h})} + \lambda \cos(\phi_h - \phi_{S_h}) F_{LU,T}^{\cos(\phi_h - \phi_{S_h})} \right. \\
 &\quad \left. + |\mathbf{S}_T| \left(\cos(\phi_{S_T} - \phi_{S_h}) F_{TU,T}^{\cos(\phi_{S_T} - \phi_{S_h})} \right. \right. \\
 &\quad \left. \left. + \cos(2\phi_h - \phi_{S_T} - \phi_{S_h}) F_{TU,T}^{\cos(2\phi_h - \phi_{S_T} - \phi_{S_h})} \right) \right], \quad (4.11)
 \end{aligned}$$



• Unpolarized case as an example

$$\begin{aligned}
 F_{UU,U}(z_h, j_\perp) &= \frac{\alpha_s^2}{s} \sum_{a,b,c} \int_{x_1^{\min}}^1 \frac{dx_1}{x_1} f_1^{a/A}(x_1, \mu) \int_{x_2^{\min}}^1 \frac{dx_2}{x_2} f_2^{b/B}(x_2, \mu) \\
 &\quad \times \int_{z^{\min}}^1 \frac{dz}{z^2} \hat{H}_{ab}^c(\hat{s}, \hat{p}_T, \hat{\eta}, \mu) \mathcal{D}_1^{h/c}(z, z_h, j_\perp^2, Q) \\
 &\equiv \mathcal{C}[ff\mathcal{D}_1\hat{H}],
 \end{aligned}$$

$$\mathcal{D}_1^{h/c}(z, z_h, \omega_J R, \mathbf{j}_\perp, \mu) = \hat{\mathcal{C}}_{c \rightarrow i}^U(z, \omega_J R, \mu) \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{j}_\perp \cdot \mathbf{b}/z_h} \tilde{\mathcal{D}}_1^{h/i}(z_h, \mathbf{b}, \mu_J, \nu) \tilde{\mathcal{S}}_i(\mathbf{b}, \mu_J, \nu R)$$

$$F_{TU,U}^{\sin(\phi_S - \phi_h)}(z_h, j_\perp) = \mathcal{C} \left[\frac{j_\perp}{z_h M_h} h_1 f_1 \mathcal{H}_1^\perp \Delta_T \hat{H} \right],$$

$$F_{LU,L}(z_h, j_\perp) = \mathcal{C} \left[g_{1L} f_1 \mathcal{G}_{1L} \Delta_L \hat{H} \right],$$

$$F_{TU,L}^{\cos(\phi_S - \phi_h)}(z_h, j_\perp) = -\mathcal{C} \left[\frac{j_\perp}{z_h M_h} h_1 f_1 \mathcal{H}_{1L}^\perp \Delta_T \hat{H} \right],$$

$$F_{UU,T}^{\sin(\phi_h - \phi_{S_h})}(z_h, j_\perp) = -\mathcal{C} \left[\frac{j_\perp}{z_h M_h} f_1 f_1 \mathcal{D}_{1T}^\perp \hat{H} \right],$$

$$F_{LU,T}^{\cos(\phi_h - \phi_{S_h})}(z_h, j_\perp) = -\mathcal{C} \left[\frac{j_\perp}{z_h M_h} g_{1L} f_1 \mathcal{G}_{1T} \Delta_L \hat{H} \right],$$

$$F_{TU,T}^{\cos(\phi_S - \phi_{S_h})}(z_h, j_\perp) = \mathcal{C} \left[h_1 f_1 \mathcal{H}_1 \Delta_T \hat{H} \right],$$

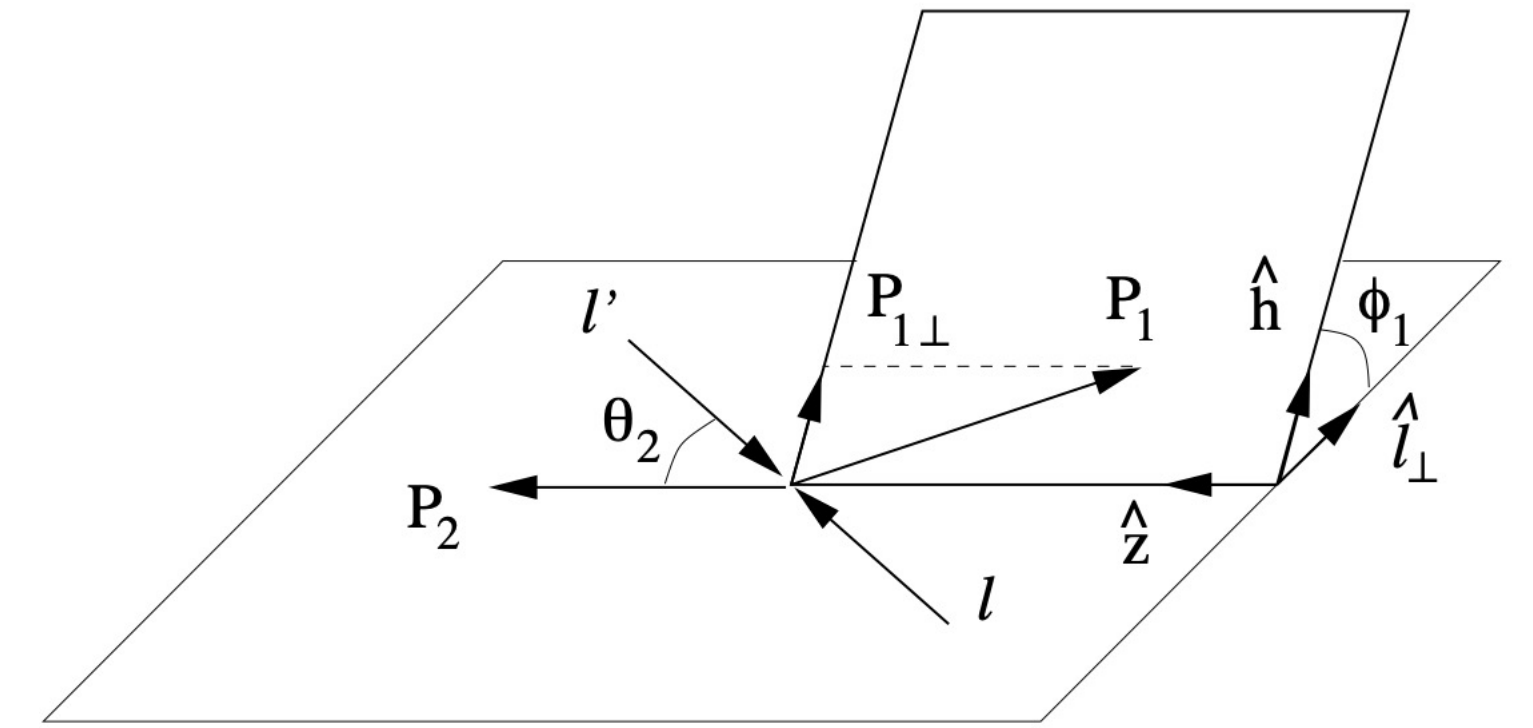
$$F_{TU,T}^{\cos(2\phi_h - \phi_S - \phi_{S_h})}(z_h, j_\perp) = -\mathcal{C} \left[\frac{j_\perp^2}{2z_h^2 M_h^2} h_1 f_1 \mathcal{H}_{1T}^\perp \Delta_T \hat{H} \right].$$

Opportunities to probe various FFs at BES/STCF

- Probe collinear FFs

$$\frac{d\sigma^O(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2} = \frac{3\alpha^2}{Q^2} \sum_{a,\bar{a}} e_a^2 \left\{ A(y) \left(D_1 \bar{D}_1 - \lambda_1 \lambda_2 G_1 \bar{G}_1 \right) + B(y) |\mathbf{S}_{1T}| |\mathbf{S}_{2T}| \cos(\phi_{S_1} + \phi_{S_2}) \left(H_1 \bar{H}_1 \right) \right\}$$

$$\frac{d\sigma^L(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2} = \frac{3\alpha^2}{Q^2} \lambda_e \sum_{a,\bar{a}} e_a^2 \left\{ \frac{C(y)}{2} \left(\lambda_2 D_1 \bar{G}_1 - \lambda_1 G_1 \bar{D}_1 \right) \right\}$$



- Back to back unpolarized hadron production in unpolarized SIA - TMDFFs

$$\frac{d\sigma^O(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2\mathbf{q}_T} = \frac{3\alpha^2}{Q^2} z_1^2 z_2^2 \left\{ A(y) \mathcal{F} [D_1 \bar{D}_1] + B(y) \cos(2\phi_1) \mathcal{F} \left[\left(2 \hat{\mathbf{h}} \cdot \mathbf{k}_T \hat{\mathbf{h}} \cdot \mathbf{p}_T - \mathbf{k}_T \cdot \mathbf{p}_T \right) \frac{H_1^\perp \bar{H}_1^\perp}{M_1 M_2} \right] \right\}$$

- One hadron polarized and another one unpolarized

$$\frac{d\sigma^O(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2\mathbf{q}_T} = \frac{3\alpha^2}{Q^2} z_1^2 z_2^2 \left\{ \dots + B(y) \lambda_1 \sin(2\phi_1) \mathcal{F} \left[\left(2 \hat{\mathbf{h}} \cdot \mathbf{k}_T \hat{\mathbf{h}} \cdot \mathbf{p}_T - \mathbf{k}_T \cdot \mathbf{p}_T \right) \frac{H_{1L}^\perp \bar{H}_1^\perp}{M_1 M_2} \right] - A(y) |\mathbf{S}_{1T}| \sin(\phi_1 - \phi_{S_1}) \mathcal{F} \left[\hat{\mathbf{h}} \cdot \mathbf{k}_T \frac{D_{1T}^\perp \bar{D}_1}{M_1} \right] + B(y) |\mathbf{S}_{1T}| \sin(\phi_1 + \phi_{S_1}) \mathcal{F} \left[\hat{\mathbf{h}} \cdot \mathbf{p}_T \frac{H_1 \bar{H}_1^\perp}{M_2} \right] + B(y) |\mathbf{S}_{1T}| \sin(3\phi_1 - \phi_{S_1}) \mathcal{F} \left[\left(4 \hat{\mathbf{h}} \cdot \mathbf{p}_T (\hat{\mathbf{h}} \cdot \mathbf{k}_T)^2 - 2 \hat{\mathbf{h}} \cdot \mathbf{k}_T \mathbf{k}_T \cdot \mathbf{p}_T - \hat{\mathbf{h}} \cdot \mathbf{p}_T k_T^2 \right) \frac{H_{1T}^\perp \bar{H}_1^\perp}{2M_1^2 M_2} \right] \right\},$$

A complete analysis can be found in [hep-ph/9702281](https://arxiv.org/abs/hep-ph/9702281)

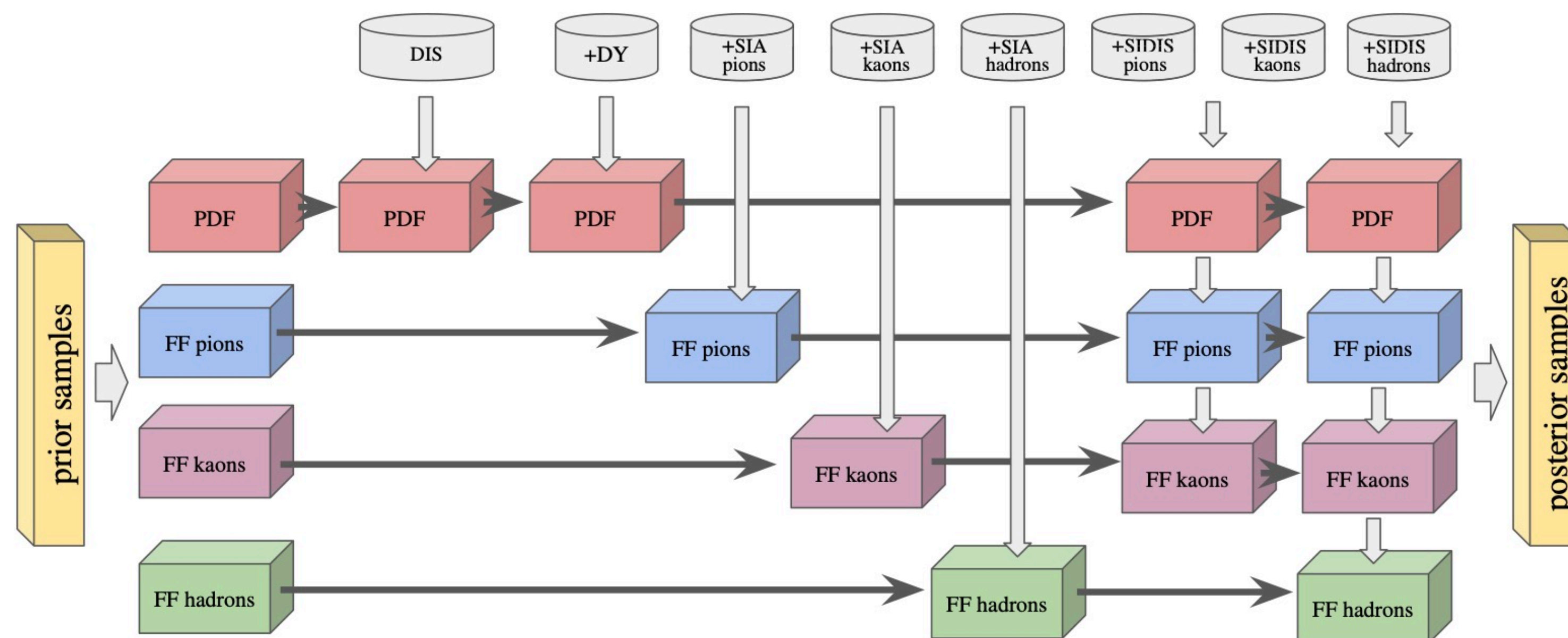
Leading Quark TMDFFs ○ → Hadron Spin ⊙ → Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		$D_1 = \odot$ Unpolarized		$H_1^\perp = \odot - \ominus$ Collins
	L		$G_1 = \odot \rightarrow - \ominus \rightarrow$ Helicity	$H_{1L}^\perp = \odot \rightarrow - \ominus \rightarrow$
Polarized Hadrons	T	$D_{1T}^\perp = \odot - \ominus$ Polarizing FF	$G_{1T}^\perp = \odot - \ominus$	$H_1 = \odot - \ominus$ Transversity $H_{1T}^\perp = \odot - \ominus$

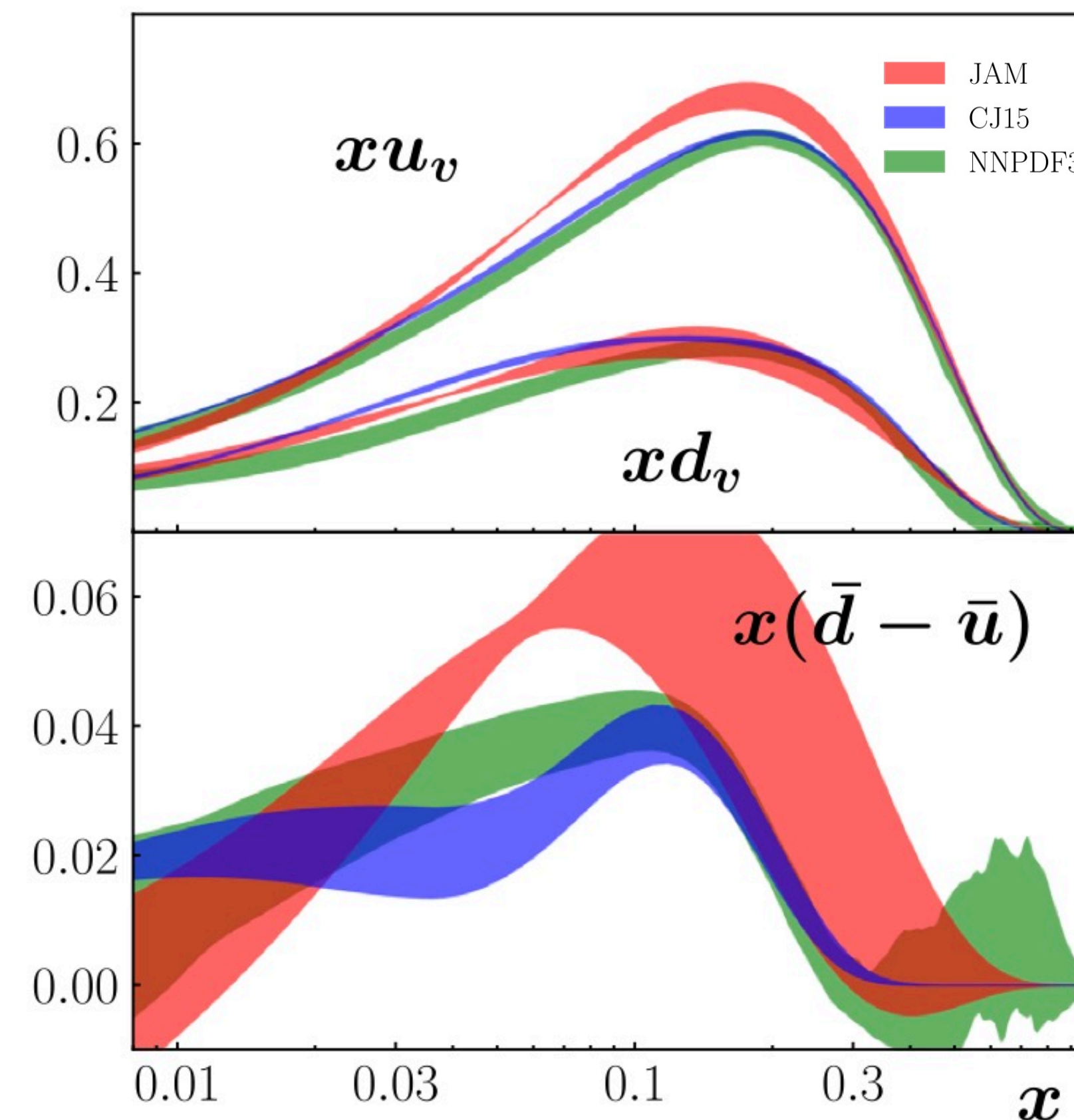
FFs as a tool to probe nucleon structure

◆ Probe the nucleon structure

JAM, PRD 2021



$$\sigma^{lp \rightarrow l'hX} = f_{i/p} \otimes \hat{\sigma}_{li \rightarrow j} \otimes D_{j \rightarrow h}$$

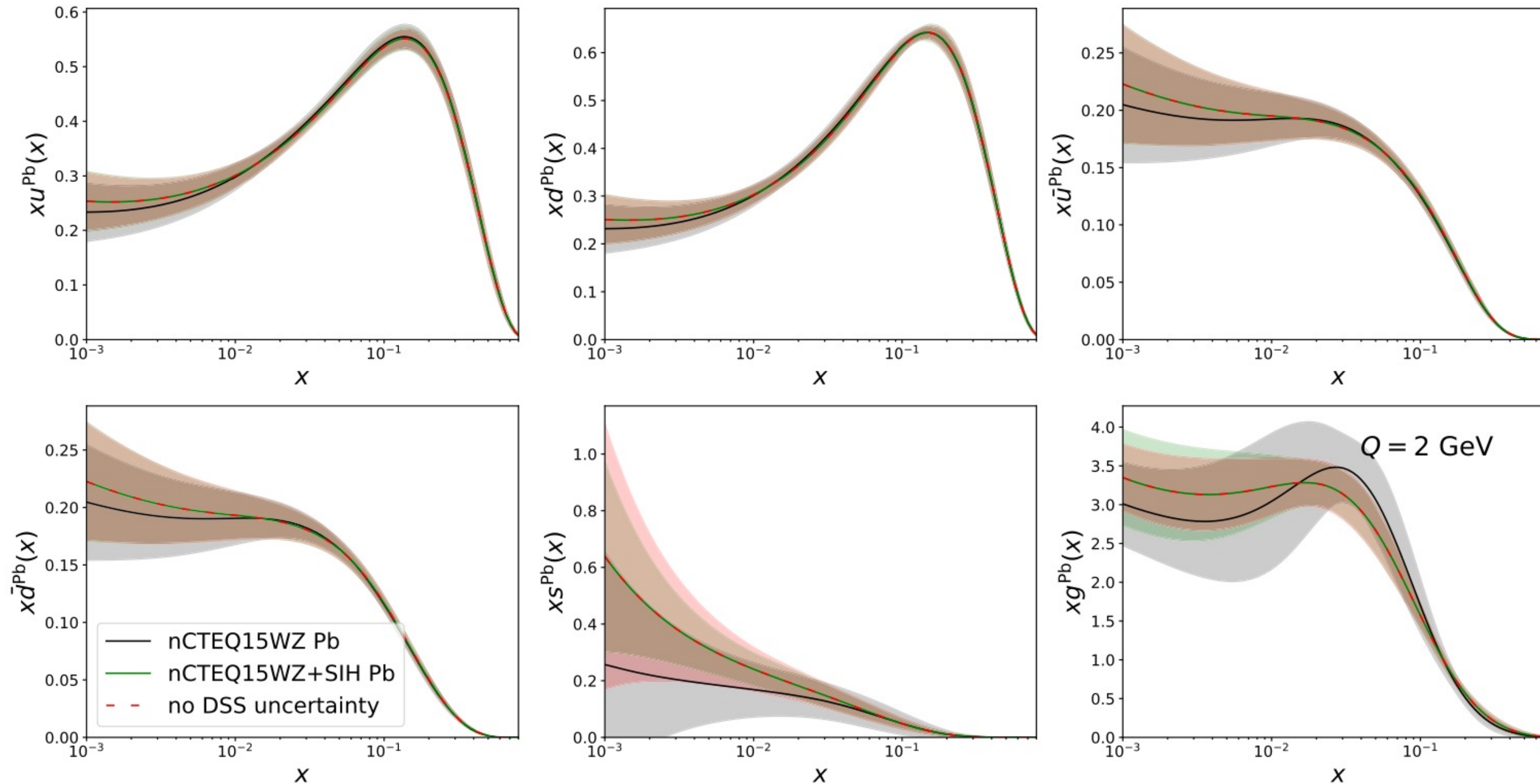


A simultaneous fit of PDF and FF can provide further constrain for flavor separation

FFs as a tool to probe nuclear PDFs

◆ Inclusive hadron production in p+Pb collisions

Duwentaster et al, PRD 2021

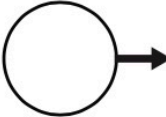
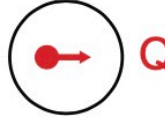


$$\sigma^{pPb \rightarrow hX} = f_{i/p} \otimes f_{j/Pb} \otimes \hat{\sigma}_{ij \rightarrow k} \otimes D_{k \rightarrow h}$$



Precise information of FF is helpful for nuclear PDF determination

Transverse momentum dependent FFs

◆ FFs: 8 transverse momentum dependent FFs at leading twist

Leading Quark TMDFFs  Hadron Spin  Quark Spin

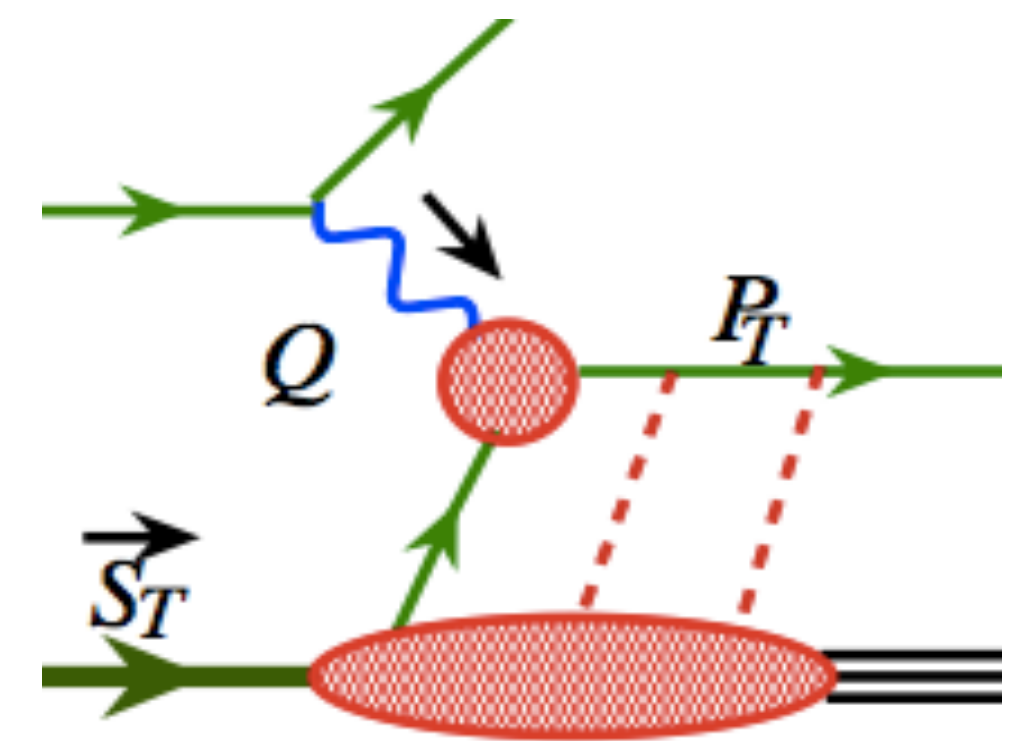
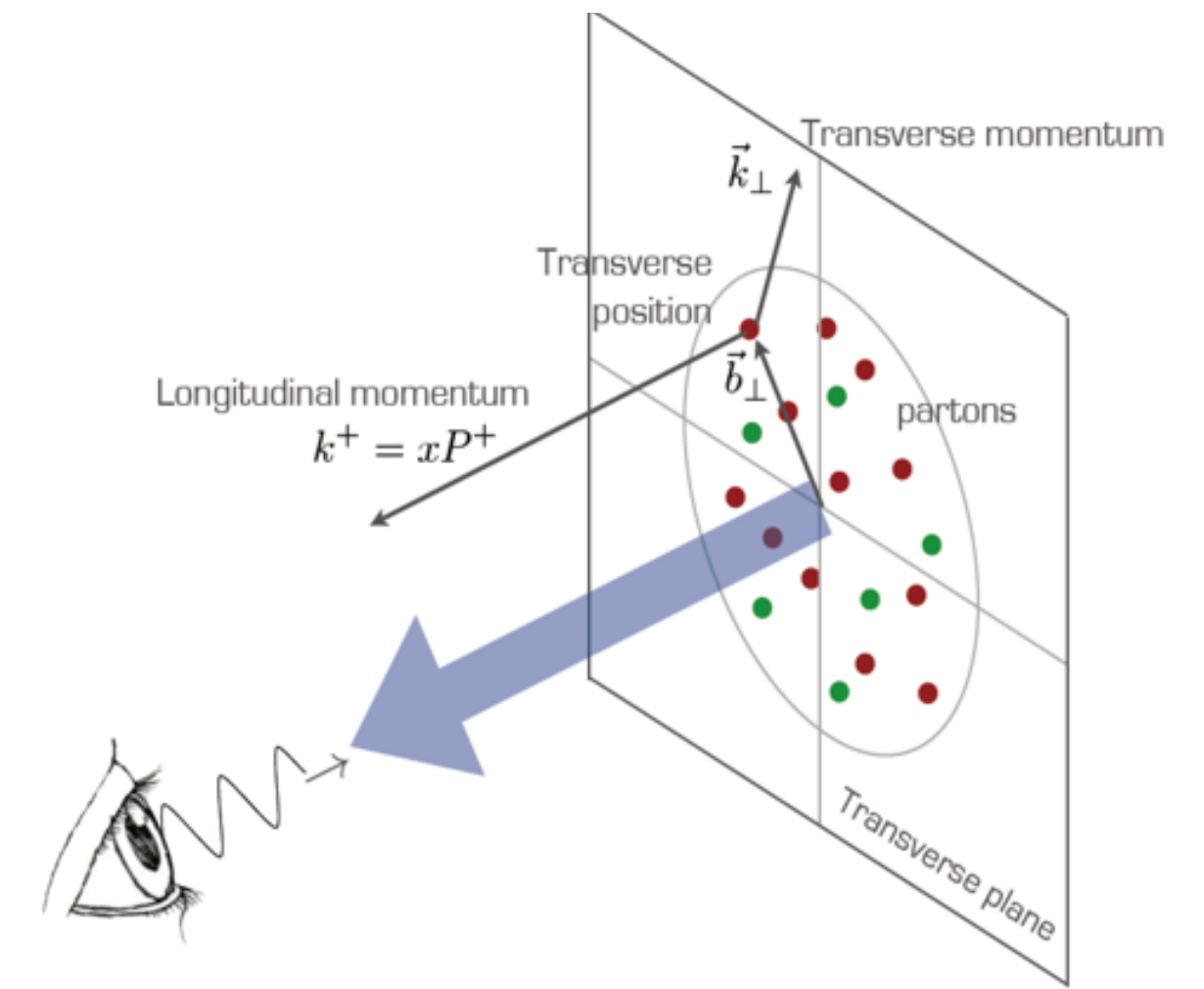
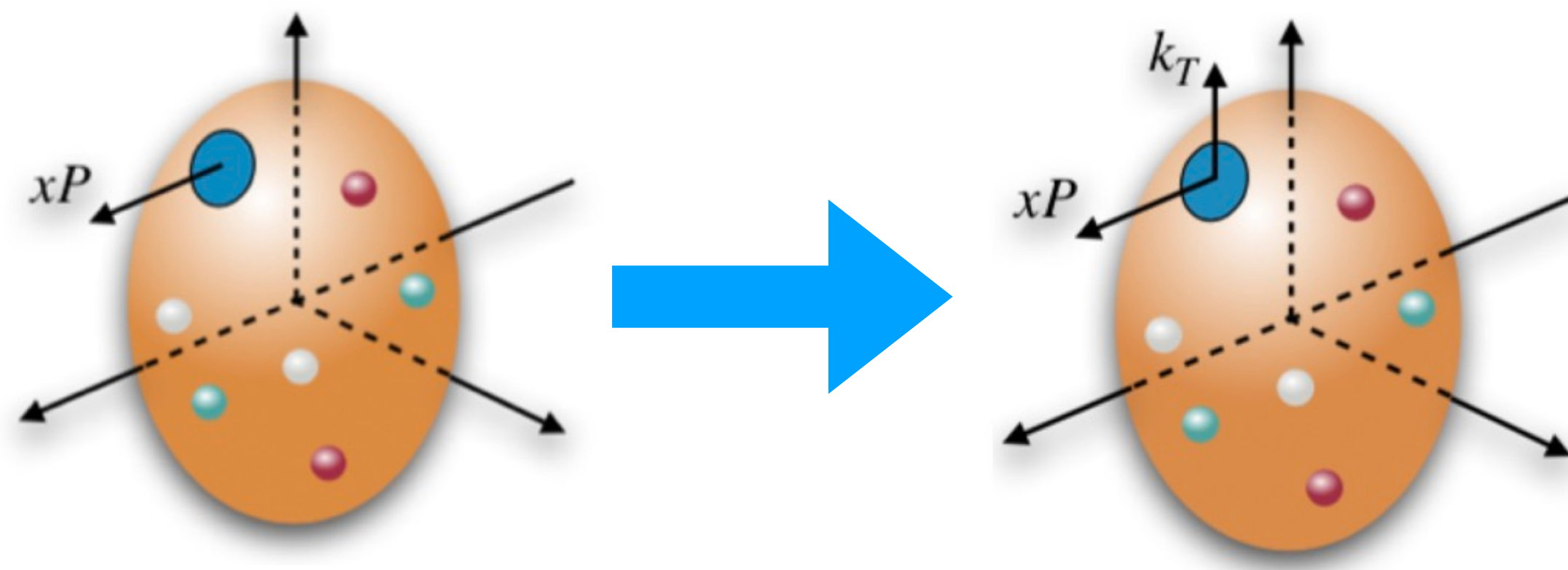
		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Polarized Hadrons	L		$G_1 = \text{Hadron Spin} \rightarrow - \text{Hadron Spin} \rightarrow$ Helicity	$H_{1L}^\perp = \text{Quark Spin} \rightarrow - \text{Quark Spin} \rightarrow$
	T	$D_{1T}^\perp = \text{Hadron Spin} \uparrow - \text{Hadron Spin} \downarrow$ Polarizing FF	$G_{1T}^\perp = \text{Hadron Spin} \uparrow - \text{Hadron Spin} \uparrow$	$H_{1T} = \text{Quark Spin} \uparrow - \text{Quark Spin} \uparrow$ Transversity $H_{1T}^\perp = \text{Quark Spin} \uparrow - \text{Quark Spin} \rightarrow$
Unpolarized (or Spin 0) Hadrons		$D_1 = \text{Hadron Spin} \odot$ Unpolarized		$H_1^\perp = \text{Quark Spin} \rightarrow - \text{Quark Spin} \rightarrow$ Collins

Leading Gluon TMDFFs  Hadron Spin  Gluon Operator Helicities

		Gluon Operator Polarization		
		Un-Polarized	Helicity 0 antisymmetric	Helicity 2
Polarized Hadrons	L		$G_{1L}^g = \text{Gluon Operator Helicities} \rightarrow - \text{Gluon Operator Helicities} \rightarrow$ Helicity	$H_{1L}^{\perp g} = \text{Gluon Operator Helicities} \rightarrow + \text{Gluon Operator Helicities} \rightarrow$
	T	$D_{1T}^{\perp g} = \text{Hadron Spin} \uparrow - \text{Hadron Spin} \downarrow$	$G_{1T}^{\perp g} = \text{Gluon Operator Helicities} \uparrow - \text{Gluon Operator Helicities} \uparrow$	$H_{1T}^g = \text{Gluon Operator Helicities} \uparrow + \text{Gluon Operator Helicities} \uparrow$ Transversity $H_{1T}^{\perp g} = \text{Gluon Operator Helicities} \uparrow + \text{Gluon Operator Helicities} \rightarrow$
Unpolarized (or Spin 0) Hadrons		$D_1^g = \text{Hadron Spin} \odot$ Unpolarized		$H_1^{\perp g} = \text{Gluon Operator Helicities} \uparrow + \text{Gluon Operator Helicities} \downarrow$ Linearly Polarized

How to probe transverse momentum dependent FFs

◆ Transverse momentum dependent PDFs (TMDs)



SIDIS: $Q \gg P_T$

- Probing nucleon 3D structure requires two momentum scales
- Hard scale $Q_1 \gg 1/fm$ localizes the probes (particle nature of quarks/gluons)
- Soft scale $Q_2 \sim 1/fm$ accesses the transverse motion of quarks/gluons

◆ Same requirement on two scales also apply to TMDFFs

Nuclear modified transverse momentum dependent FFs

◆ TMD factorization for SIDIS

Alrashed, Anderle, Kang, Terry, **HX**, PRL 2022

$$\frac{d\sigma^A}{dx dQ^2 dz d^2P_{h\perp}} = \sigma_0 H(Q) \sum_q e_q^2 \int_0^\infty \frac{b db}{2\pi} J_0\left(\frac{bP_{h\perp}}{z}\right) f_{q/n}^A(x, b; Q) D_{h/q}^A(z, b; Q)$$

◆ TMDs

$$f_{q/n}^A(x, b; Q) = \left[C_{q\leftarrow i} \otimes f_{i/n}^A \right] (x, \mu_{b_*}) \exp \left\{ -S_{\text{pert}}(\mu_{b_*}, Q) - S_{\text{NP}}^f(b, Q, A) \right\}$$

$$D_{h/q}^A(z, b; Q) = \frac{1}{z^2} \left[\hat{C}_{i\leftarrow q} \otimes D_{h/i}^A \right] (z, \mu_{b_*}) \exp \left\{ -S_{\text{pert}}(\mu_{b_*}, Q) - S_{\text{NP}}^D(b, z, Q, A) \right\}$$

Our assumptions

- Perturbative information is left unchanged by the nuclear medium.

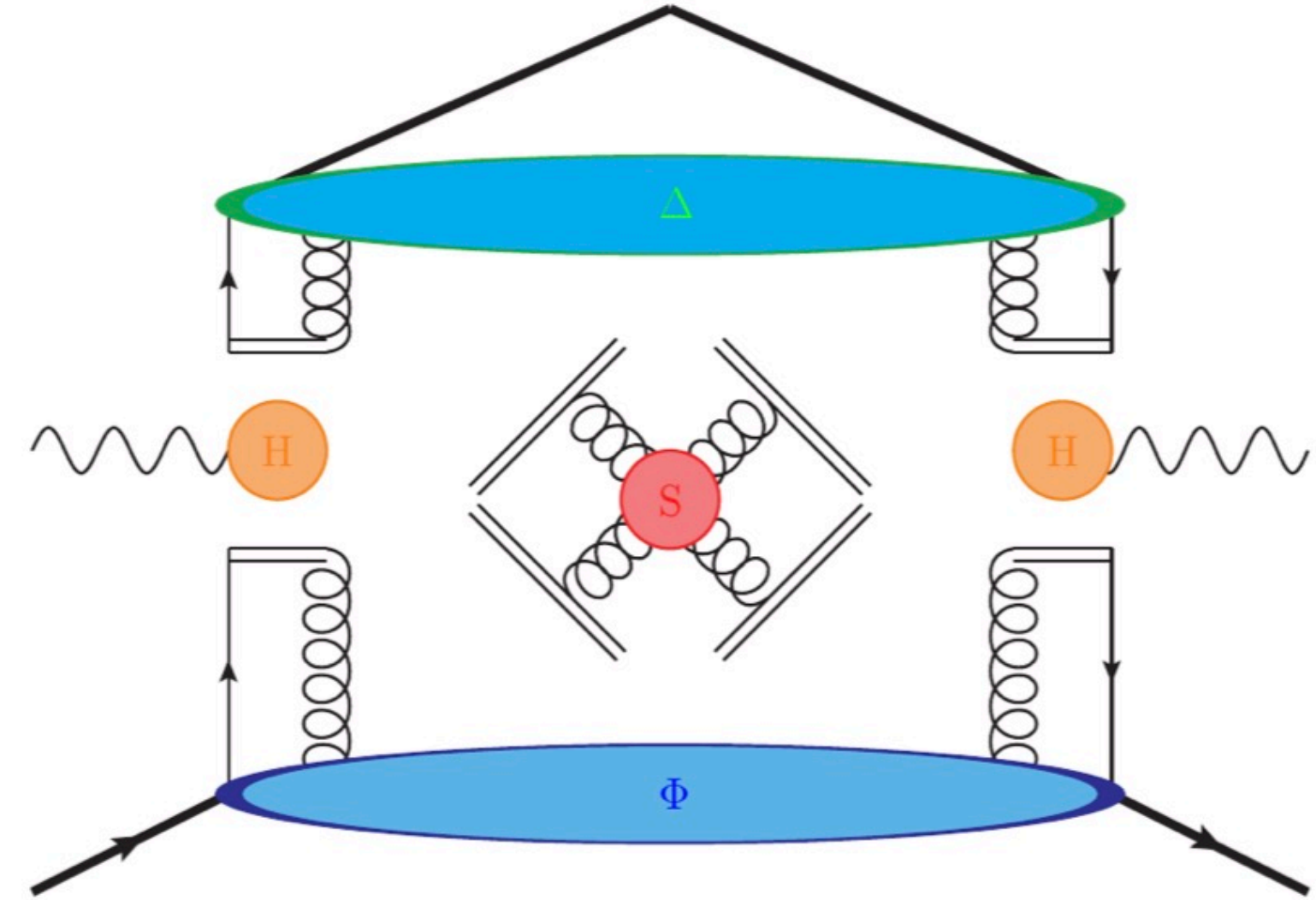
$C_{q\leftarrow i}$, $\hat{C}_{i\leftarrow q}$, and S_{pert} are unchanged.

- Non-perturbative information is modified.

$f_{i/n}^A$, $D_{h/i}^A$, S_{NP}^D , and S_{NP}^f are altered.

$$S_{\text{NP}}^D(z, b, Q, A) = S_{\text{NP}}^D(z, b, Q) + b_N \left(A^{1/3} - 1 \right) \frac{b^2}{z^2}$$

$$S_{\text{NP}}^f(z, b, Q, A) = S_{\text{NP}}^f(z, b, Q) + b_N \left(A^{1/3} - 1 \right) \frac{b^2}{z^2}$$



Nuclear modified transverse momentum dependent FFs

◆ Data selections

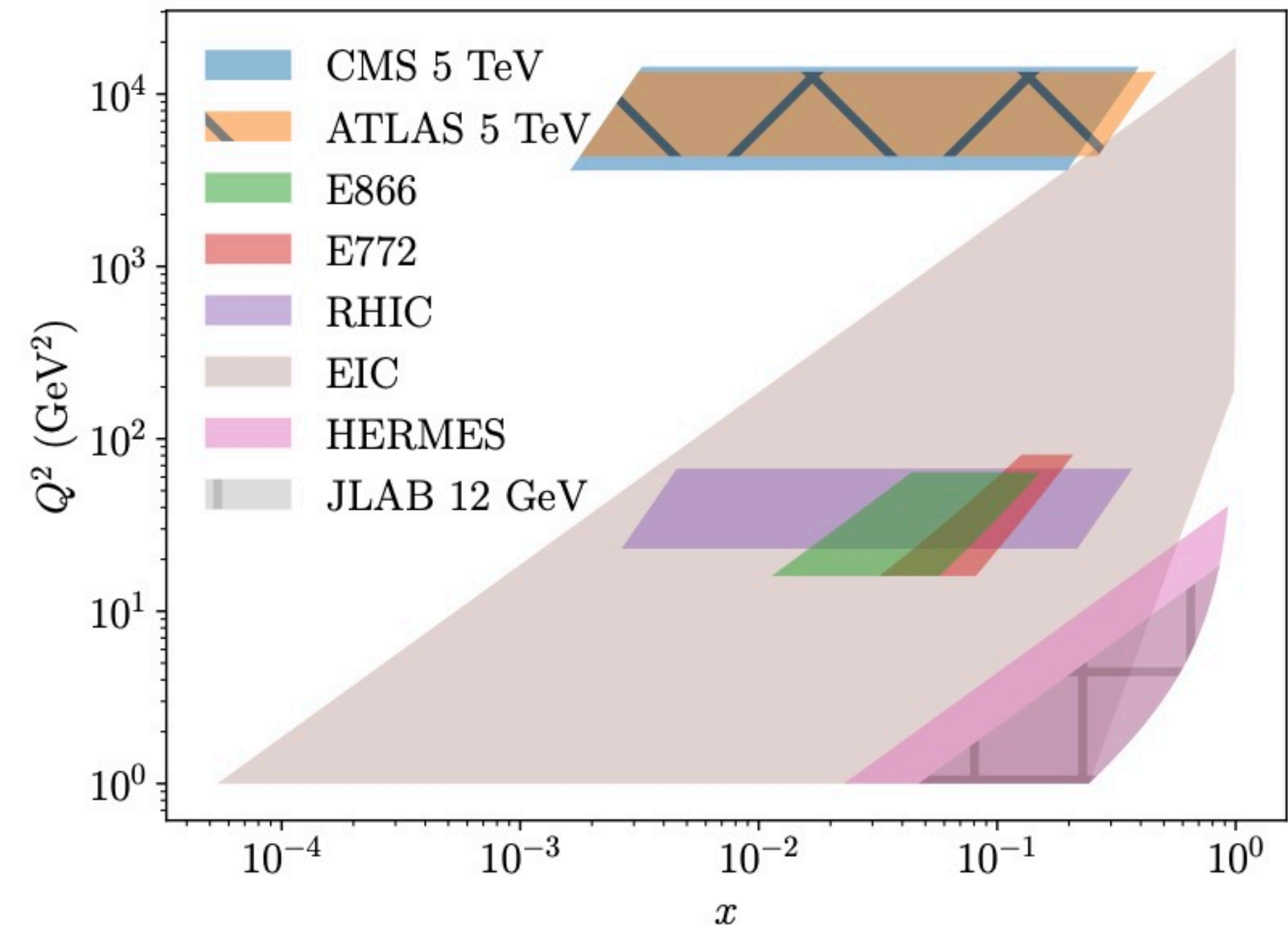
Drell-Yan Measurements

- $R_{AB} = \frac{d\sigma_A}{dq_{\perp}} / \frac{d\sigma_B}{dq_{\perp}}$
- E866
- E772
- Prelim. RHIC
- $d\sigma/dq_{\perp}$ (p Pb)
- ATLAS
- CMS

SIDIS Measurements

- Multiplicity ratio $R_h^A = M_h^A / M_h^D$.
- HERMES 2007
- Prelim. JLab
- Planned JLab
- Possible EIC.

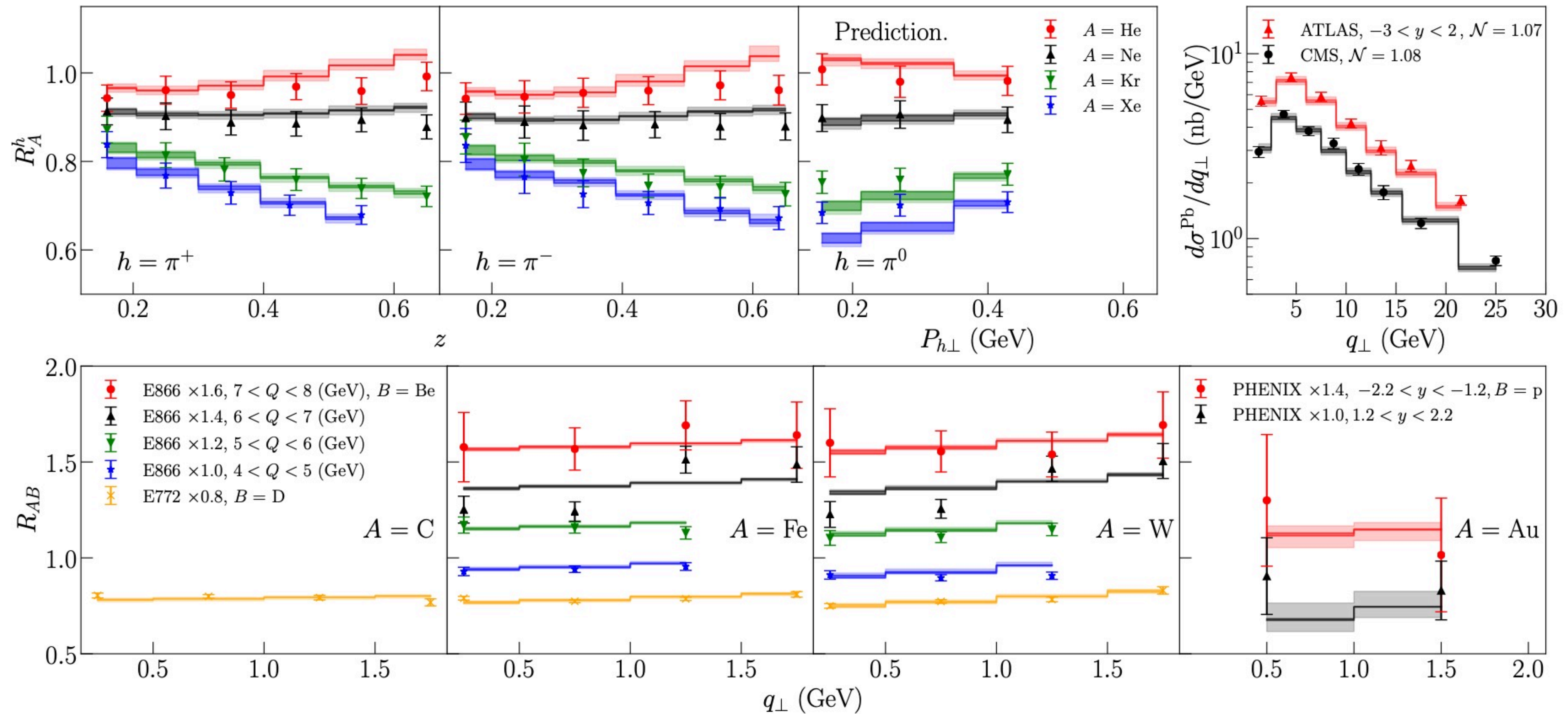
Collaboration	Process	Baseline	Nuclei	N_{dat}	χ^2
HERMES [36]	SIDIS (π)	D	Ne, Kr, Xe	27	16.3
RHIC [44]	DY	p	Au	4	2.0
E772 [42]	DY	D	C, Fe, W	16	20.1
E866 [43]	DY	Be	Fe, W	28	43.3
CMS [45]	γ^*/Z	NA	Pb	8	9.7
ATLAS [46]	γ^*/Z	NA	Pb	7	13.1
Total				90	105.2



Nuclear imaging in 3D

◆ Global fitting of nuclear TMDs

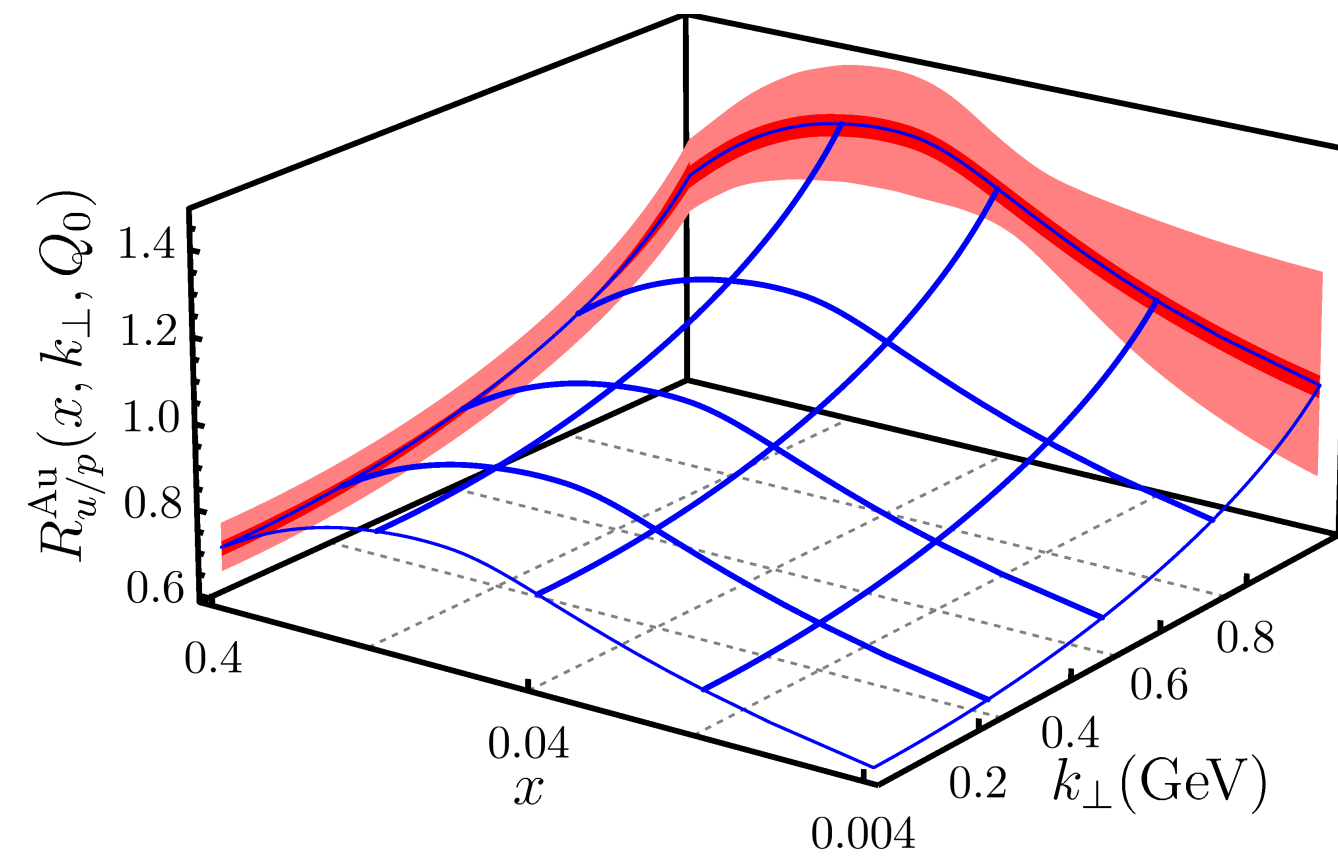
Alrashed, Anderle, Kang, Terry, **HX**, PRL 2022



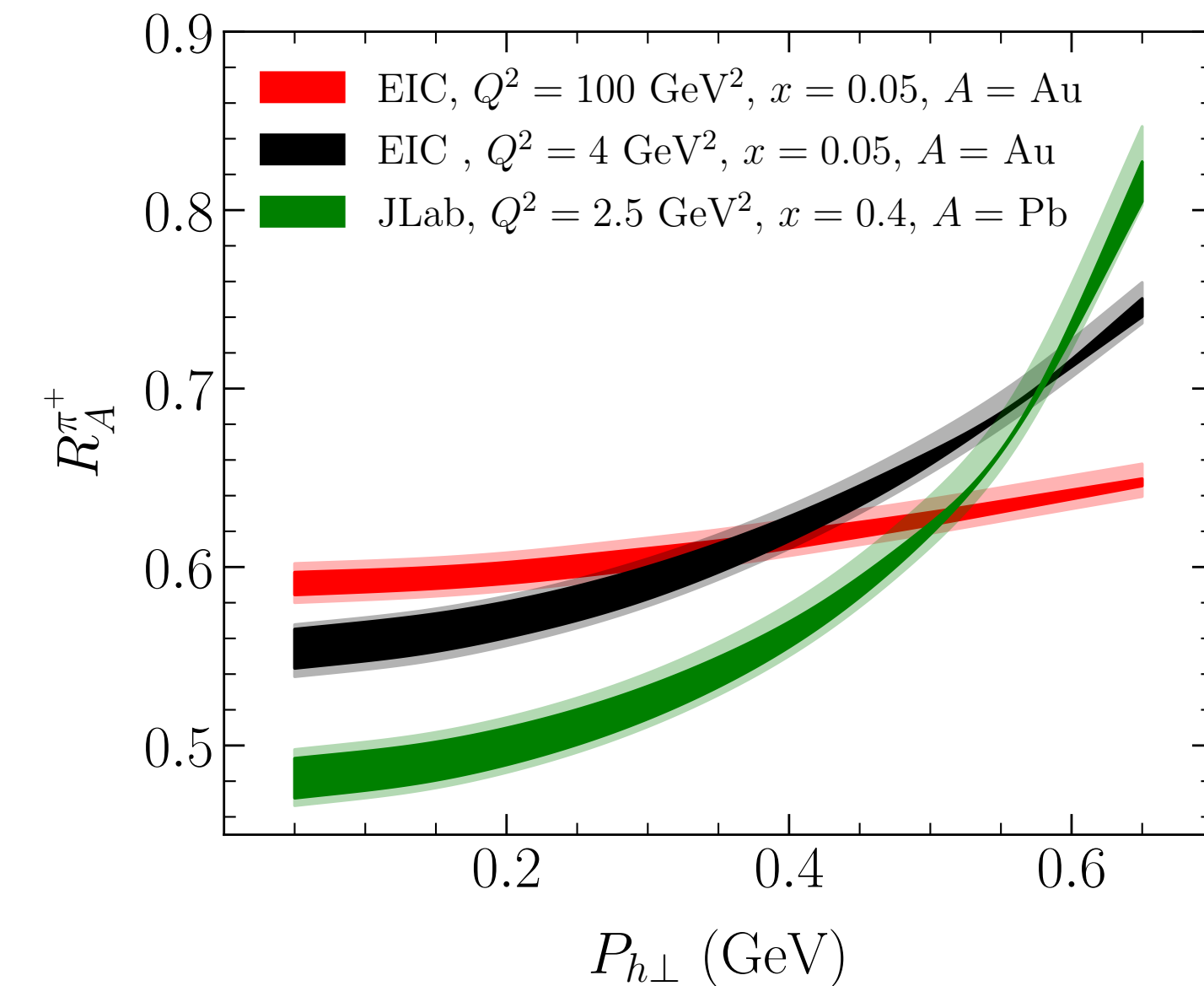
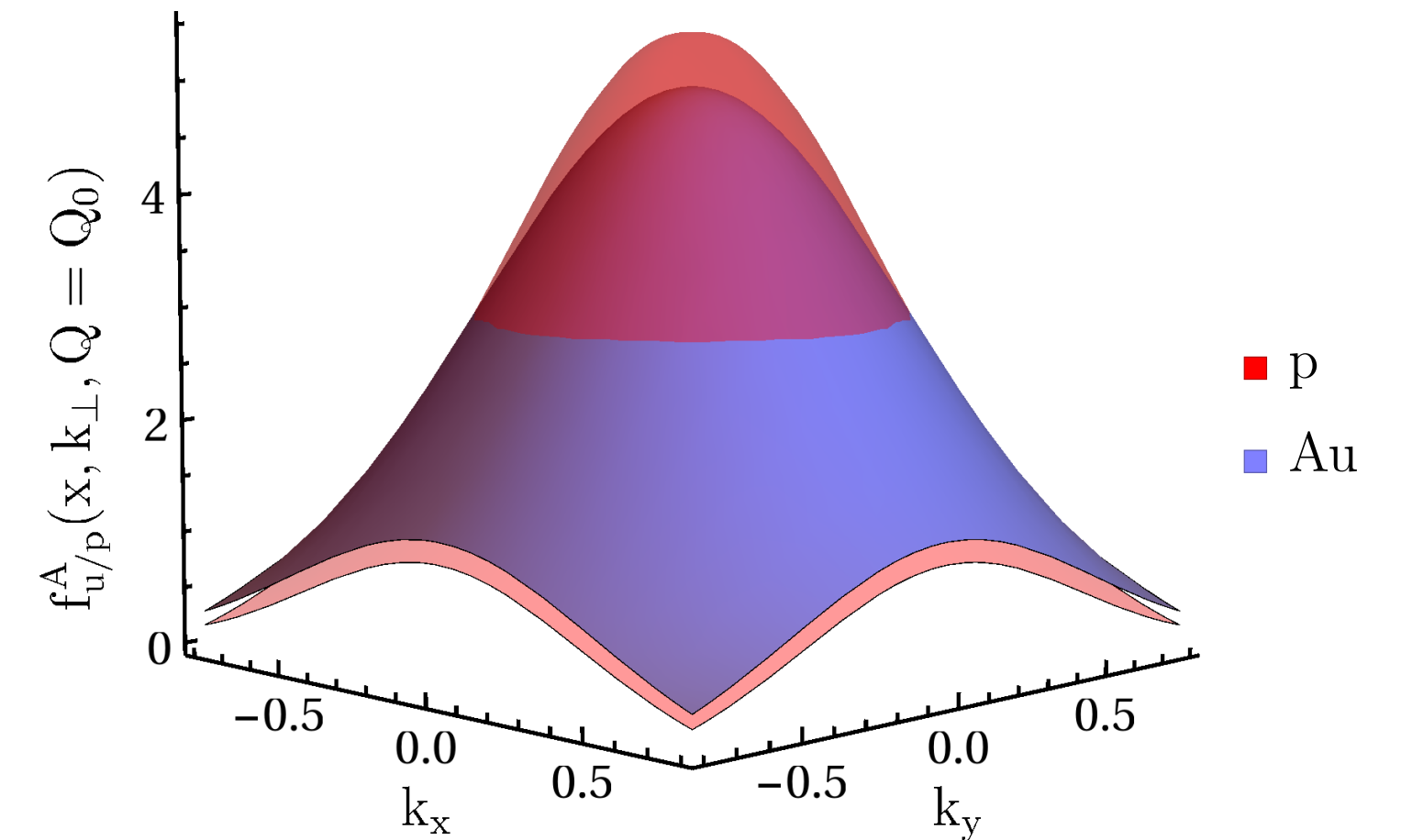
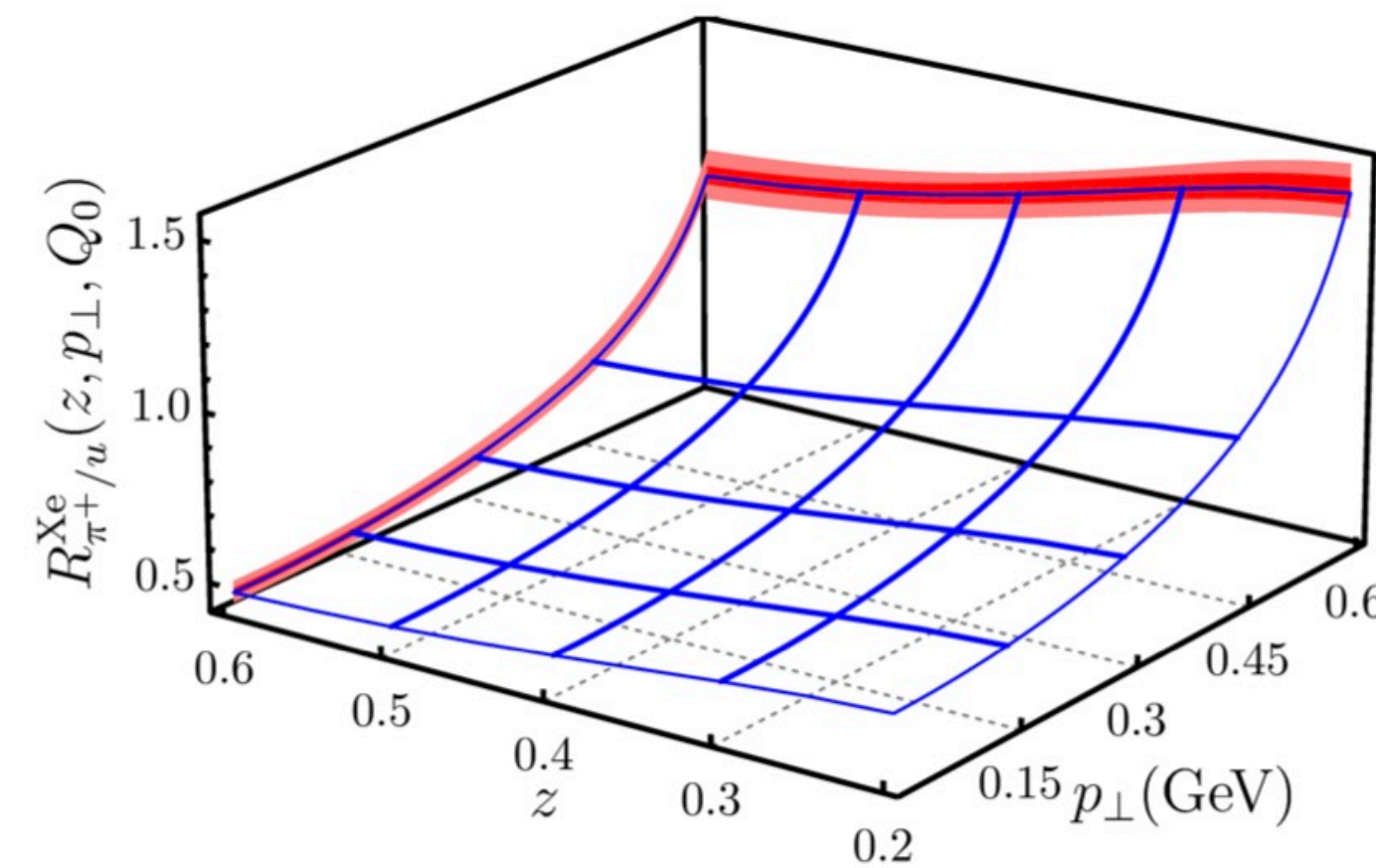
Reasonable good overall description on world data from HERMES, FNAL, RHIC, LHC

Nuclear imaging in 3D

$$R_{u/p}^{\text{Au}}(x, k_{\perp}, Q_0) = \frac{f_{u/p}^{\text{Au}}(x, k_{\perp}, Q_0)}{f_{u/p}(x, k_{\perp}, Q_0)}$$



$$\mathcal{R}_{\pi^+/u}^{\text{Xe}}(z, p_{\perp}, Q_0) = \frac{D_{\pi^+/u}^{\text{Xe}}(z, p_{\perp}, Q_0)}{D_{\pi^+/u}(z, p_{\perp}, Q_0)}$$





- First time quantitative determination of nuclear TMDs
- Identification of transverse momentum broadening in nuclei

Testing leading power QCD factorization at BES/STCF

- ◆ What's the boundary for Q^2 to ensure the validity of leading twist QCD factorization?
- ◆ Generalized factorization theorem

$$\begin{aligned} \sigma_{phys}^h = & \left[\alpha_s^0 C_2^{(0)} + \alpha_s^1 C_2^{(1)} + \alpha_s^2 C_2^{(2)} + \dots \right] \otimes T_2(x) \\ & + \frac{1}{Q} \left[\alpha_s^0 C_3^{(0)} + \alpha_s^1 C_3^{(1)} + \alpha_s^2 C_3^{(2)} + \dots \right] \otimes T_3(x) \\ & + \frac{1}{Q^2} \left[\alpha_s^0 C_4^{(0)} + \alpha_s^1 C_4^{(1)} + \alpha_s^2 C_4^{(2)} + \dots \right] \otimes T_4(x) \\ & + \dots \end{aligned}$$

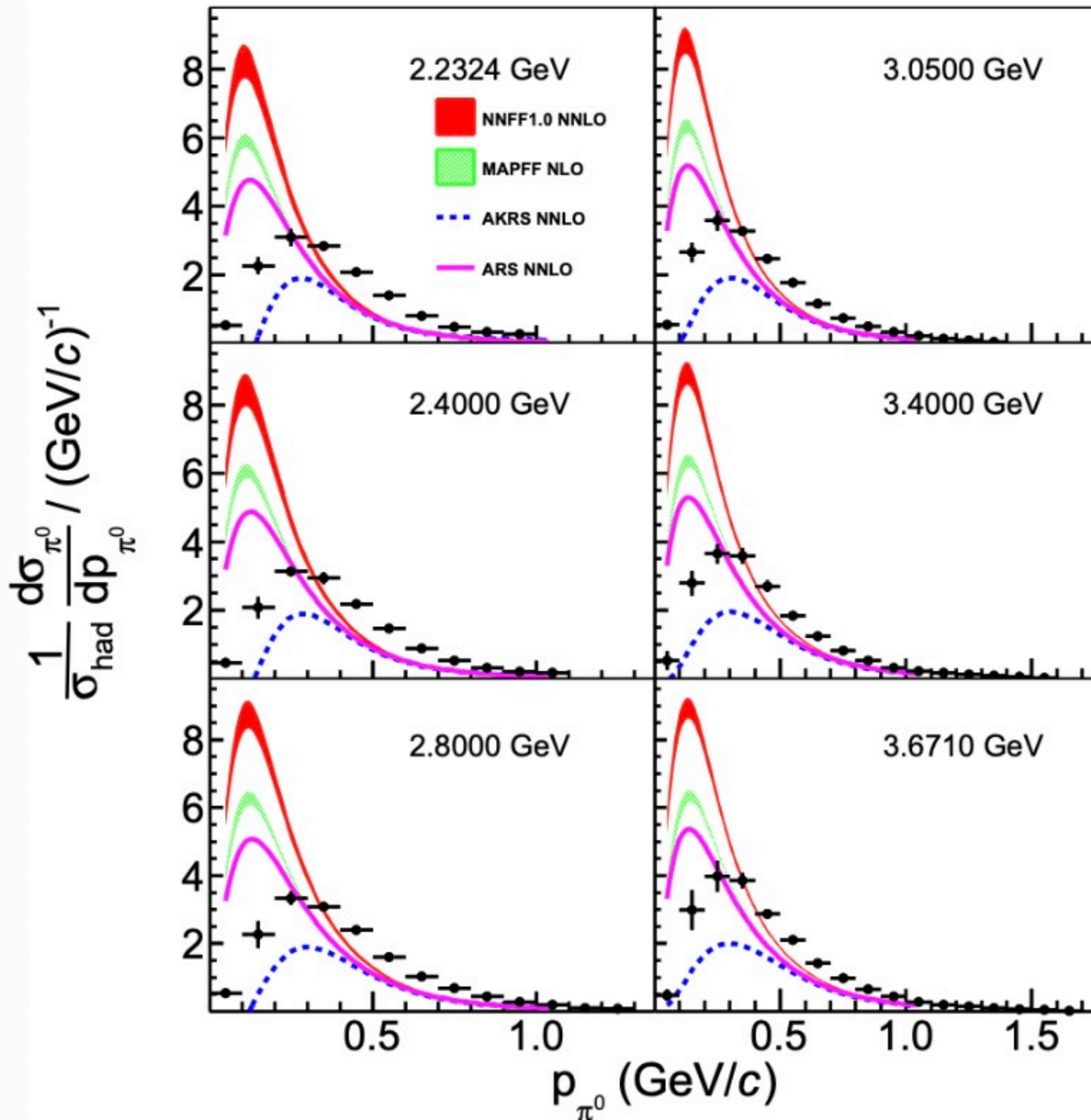
perturbative expansion 

Multiple scattering expansion 

Test leading power QCD factorization at BES/STCF

Predictions on low- z and low- Q^2 do not agree with data and depend on chosen FFs:

BESIII Collaboration • M. Ablikim et al. e-Print: 2211.11253



Data input, initial evolution scale μ_0 and kinematical cuts are different for the FFs.

NNLO (SIA data only)

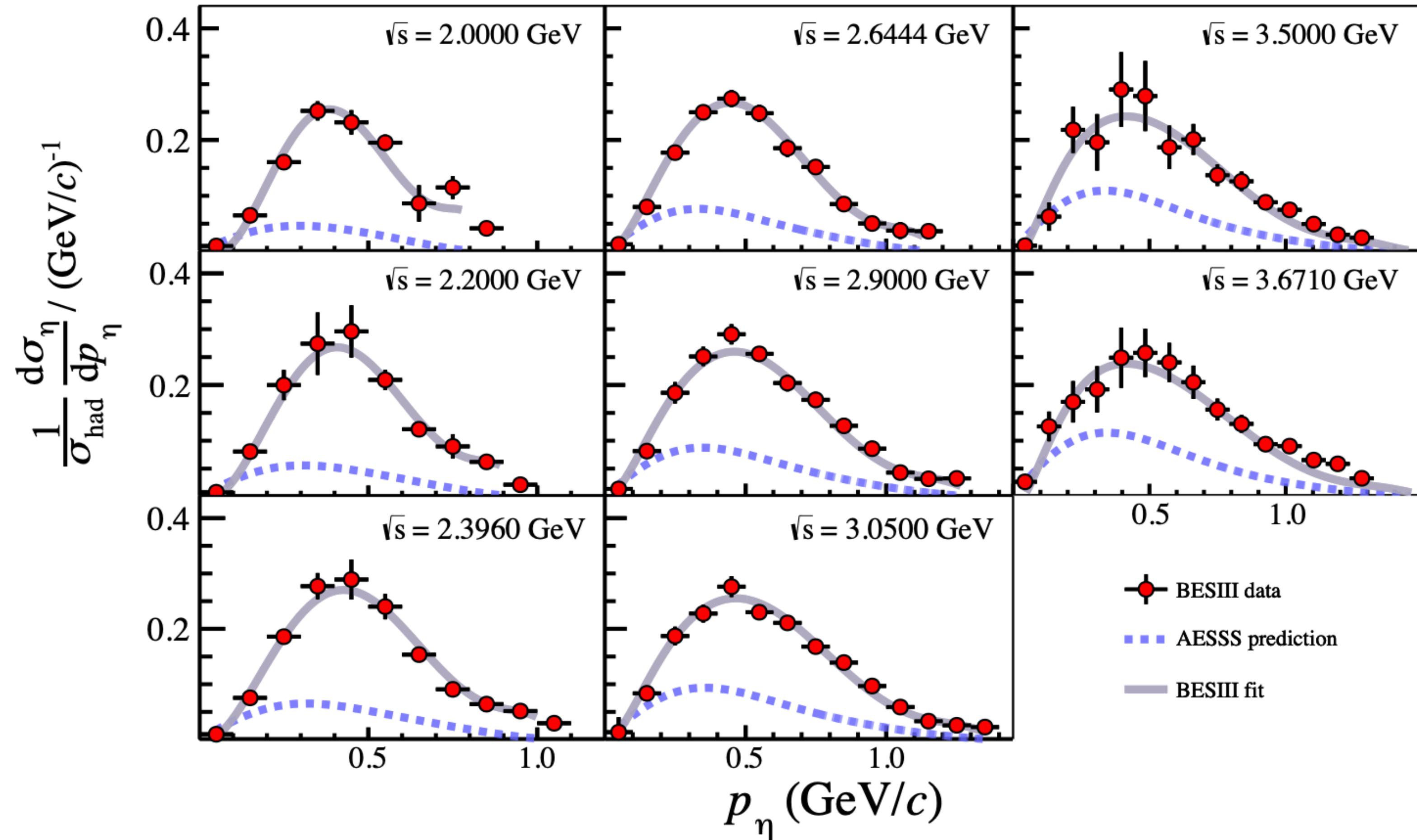
- ARS → Fixed order calculation
- AKRS → Includes small- z resum.
- NNFF1.0 → Includes hadron mass effects

NLO

- MAPFF → Includes low energy SIDIS data

Testing leading power QCD factorization at BES/STCF

◆ A test from data driving analysis of high twist contribution



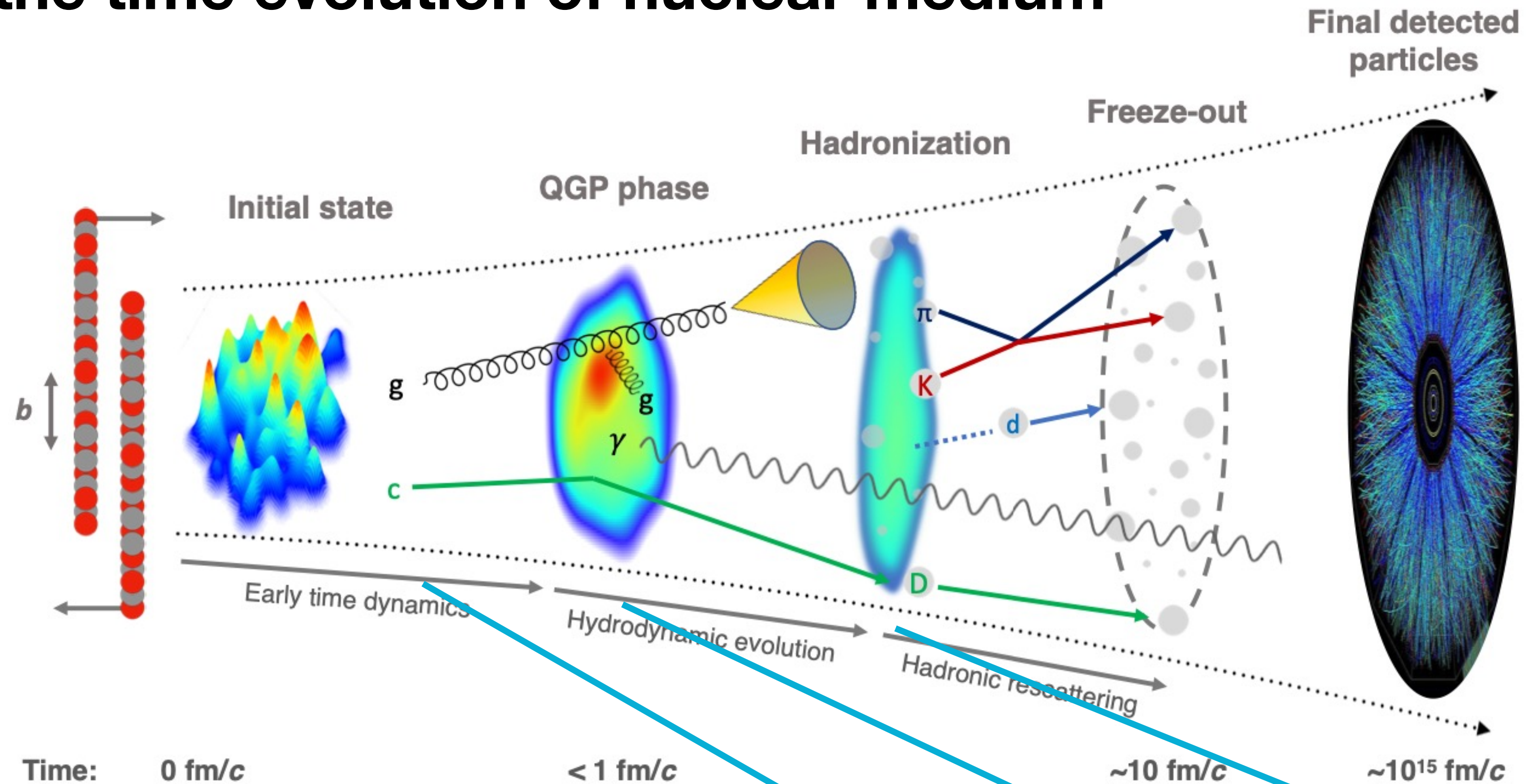
$$\sigma \approx \sigma^{LT} \left[1 + \sum_i N_i \frac{x^{a_i} (1-x)^{b_i}}{Q^{2i}} \right]$$

- Hint of leading twist factorization breaking?
- BES/STCF kinematic coverage is unique to answer this question!

BESIII + Li, Anderle, **HX**, PRL, 2024
 Li, Anderle, **HX**, Zhao, 2024

FFs as a tool to probe hot dense medium

◆ Track the time evolution of nuclear medium



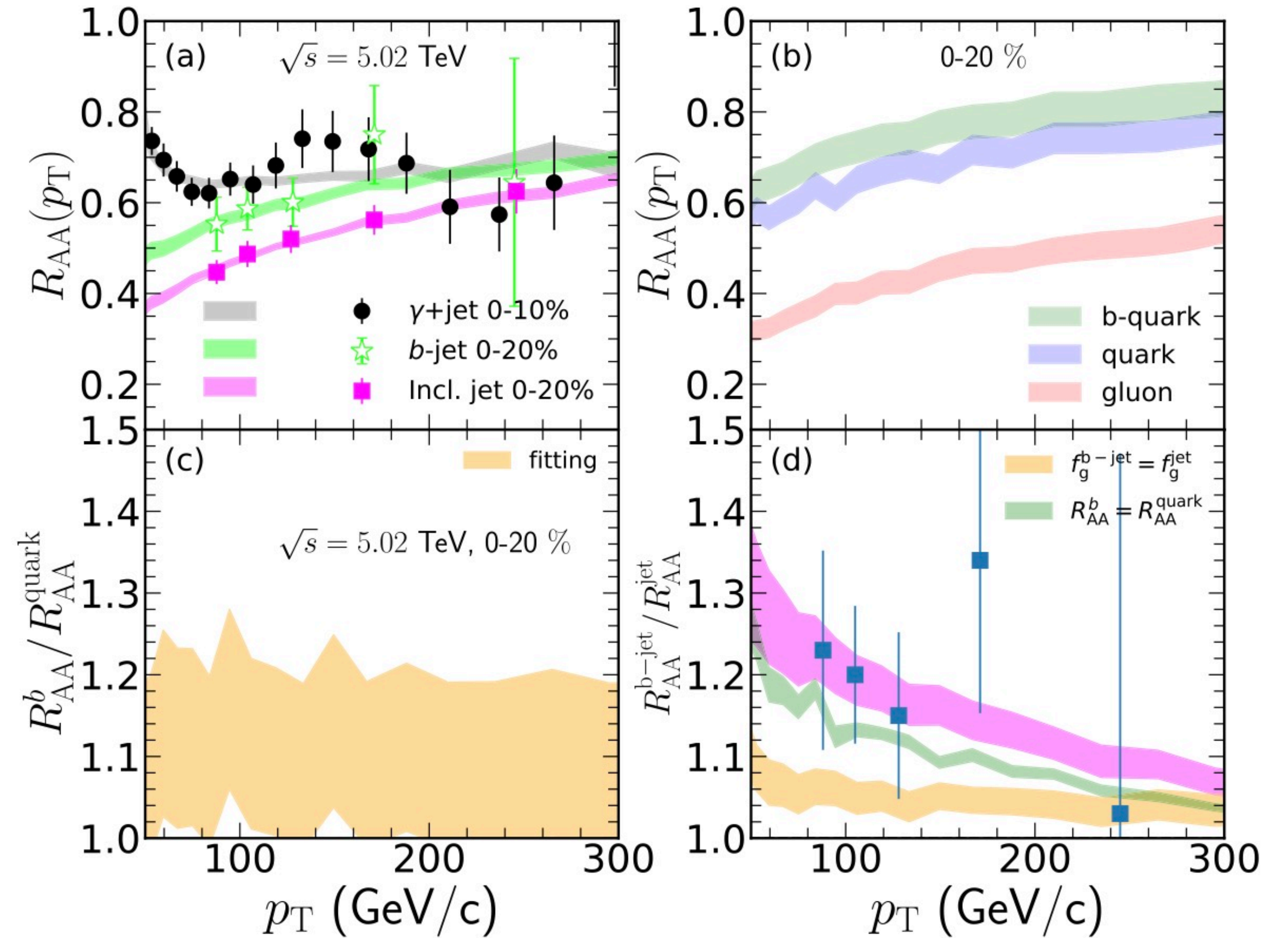
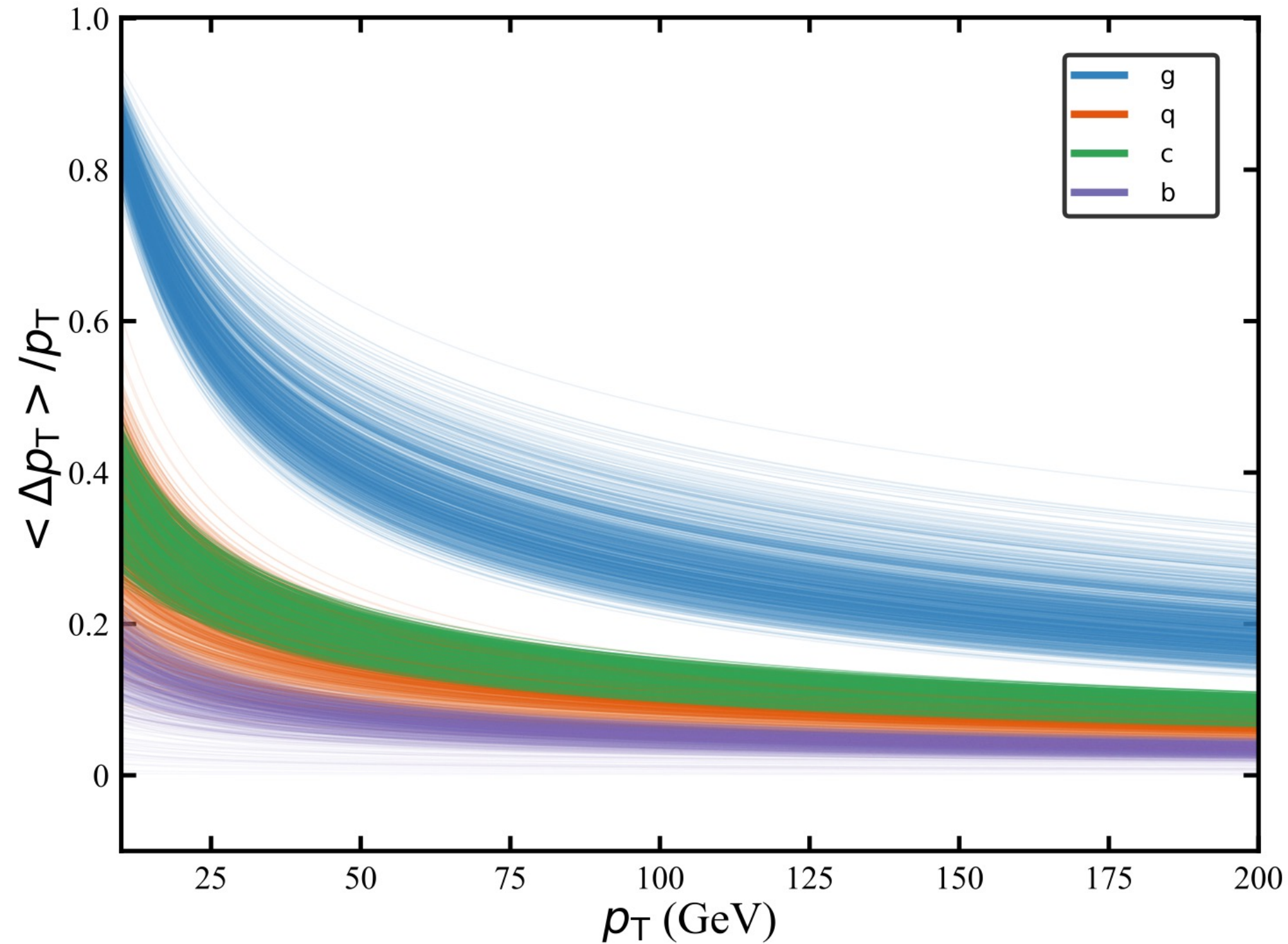
$$\sigma^{AA \rightarrow hX} = f_{i/A} \otimes f_{j/A} \otimes \tilde{\sigma}_{ij \rightarrow k} \otimes D_{k \rightarrow h}$$

- Observables involving FFs: single inclusive hadron, di-hadron, photon/Z tagged hadron, jet fragmentation function

FFs as a tool to probe hot dense medium

◆ Extract the medium property

Xing, Cao, Qin, 2023



Zhang, Wang, HX, Zhang, PLB, 2024

- Verify the flavor hierarchy of parton energy loss in medium
- Extract the jet transport parameter of quark-gluon plasma

Summary

- ◆ Introduction of collinear and transverse momentum dependent fragmentation functions
- ◆ Benefits of using FFs to probe nucleon/nuclear/hot dense medium property
- ◆ Unique opportunities to test QCD factorization for hadron production at BES/STCF

Thanks for your attention!