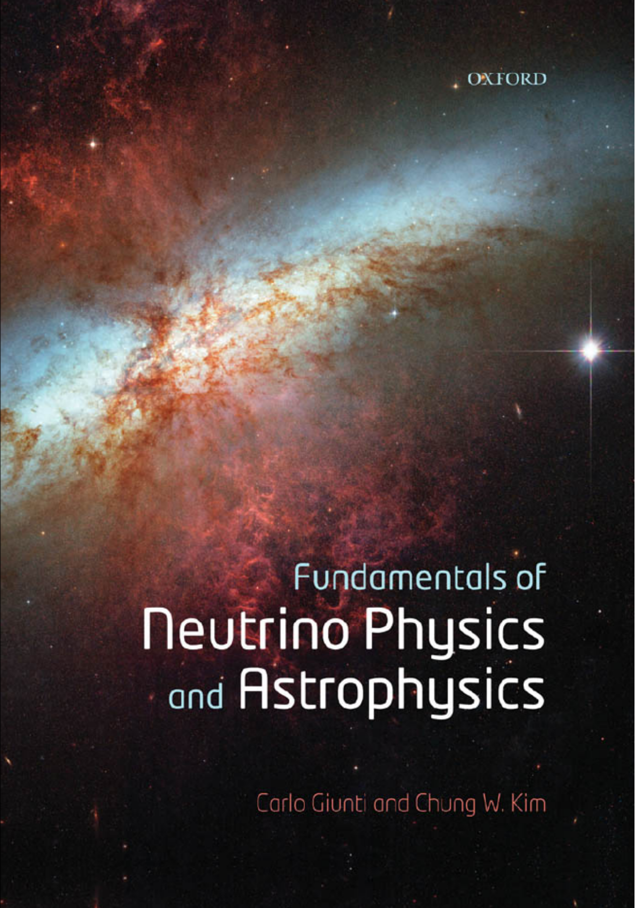
Neutrino oscillations


OXFORD
Fundamentals of
Neutrino Physics
and Astrophysics
Carlo Giunti and Chung W. Kim

Neutrino masses and mixing

flavor eigenstates $\neq$ mass eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \text{PMNS} \\ \text{matrix} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

PMNS=Pontecorvo-Maki-Nakagawa-Sakata

- If you're familiar with the CKM mixing in the quark sector, then it's fairly easy to understand this.
- Otherwise, here is the explanation:
  - ν is typically produced/detected via, e.g. $\nu + \boxed{\cdots} \to \boxed{\cdots} + \ell$ or $\boxed{\cdots} \to \nu + \bar{\ell} + \boxed{\cdots}$.
  - The accompanying charged lepton (ℓ) is experimentally much easier to be identified.
  - Hence we label the corresponding ν by ν_ℓ, which is a flavor eigenstate.
  - However, flavors eigenstates are not mass eigenstates, but linear combinations of them.
  - The linear (unitary) transformations between them is known as the PMNS matrix
  - Production/detection: use $(\nu_e,\ \nu_\mu,\ \nu_\tau)$; Propagation: use $(\nu_1,\ \nu_2,\ \nu_3)$.

Neutrino masses and mixing: relevant parameters and current measurements

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \text{PMNS} \\ \text{matrix} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

→ Three masses: m_1, m_2, m_3

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & c_{13}c_{23} \end{pmatrix}$$

where $s_{13} = \sin\theta_{13}$, $c_{13} = \cos\theta_{13}$, $s_{12} = \sin\theta_{12}$, ...

Relevant to ν-osc: 4 parameters in $U_{\rm PMNS}$ and 2 parameters in masses

→ $(\theta_{13},\ \theta_{12},\ \theta_{23},\ \delta_{\rm CP})$

→ $\Delta m^2_{21} \equiv m^2_2 - m^2_1$

$\Delta m^2_{31} \equiv m^2_3 - m^2_1$

Current measurements (not up-to-date on purpose):

$\theta_{13} \approx 8.5^\circ$, $\theta_{12} \approx 34^\circ$, $\theta_{23} \approx 45 \pm 6^\circ$; $\delta_{\rm CP} = \text{unknown}$;
$\Delta m^2_{21} \approx 7.4 \times 10^{-5}$ eV^2, $\Delta m^2_{31} \approx \pm 2.5 \times 10^{-3}$ eV^2.

→ sign not determined yet

www.nu-fit.org/?q=node/278

www.nu-fit.org

Menu

- Home
- Results
- Publications
- Members
- Login

Home

v5.3: Three-neutrino fit based on data available in March 2024

Menu

- Summary of data included
- Parameter ranges
- Leptonic mixing matrix
- Two-dimensional allowed regions
- One-dimensional χ^2 projections
- CP-violation: Jarlskog invariant
- CP-violation: unitarity triangle
- Tension between Solar and KamLAND data
- Synergies: atmospheric mass-squared splitting
- Synergies: disappearance data and θ_{23}
- Synergies: determination of θ_{23}
- Synergies: determination of δ_{CP}
- Synergies: determination of $\Delta m^2_{3\ell}$
- Correlation between δ_{CP} and other parameters

without SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
$\theta_{12}/^\circ$	$33.66^{+0.73}_{-0.70}$	$31.60 \to 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$
$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.407 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00058}$	$0.02029 \to 0.02391$	$0.02219^{+0.00059}_{-0.00057}$	$0.02047 \to 0.02396$
$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.90$
$\delta_{CP}/^\circ$	197^{+41}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.511^{+0.027}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.024}$	$-2.581 \to -2.409$

with SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 9.1$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
$\theta_{12}/^\circ$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$
$\sin^2\theta_{23}$	$0.454^{+0.019}_{-0.016}$	$0.411 \to 0.606$	$0.568^{+0.016}_{-0.021}$	$0.412 \to 0.611$
$\theta_{23}/^\circ$	$42.3^{+1.1}_{-0.9}$	$39.9 \to 51.1$	$48.9^{+0.9}_{-1.2}$	$39.9 \to 51.4$
$\sin^2\theta_{13}$	$0.02224^{+0.00056}_{-0.00057}$	$0.02047 \to 0.02397$	$0.02222^{+0.00069}_{-0.00057}$	$0.02049 \to 0.02420$
$\theta_{13}/^\circ$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.13}_{-0.11}$	$8.23 \to 8.95$
$\delta_{CP}/^\circ$	232^{+39}_{-25}	$139 \to 350$	273^{+24}_{-26}	$195 \to 342$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.505^{+0.024}_{-0.026}$	$+2.426 \to +2.586$	$-2.487^{+0.027}_{-0.024}$	$-2.566 \to -2.407$

The Schrödinger equation:

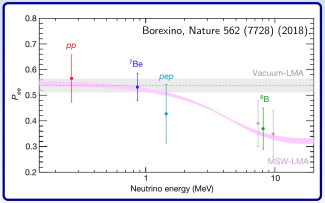
$$i\frac{d}{dL}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = H\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}, \tag{1}$$

$$H = \frac{1}{2E_\nu}U_{\rm PMNS}\begin{pmatrix}m_1^2 & & \\ & m_2^2 & \\ & & m_3^2\end{pmatrix}U_{\rm PMNS}^\dagger + \begin{pmatrix}V_e & & \\ & 0 & \\ & & 0\end{pmatrix}, \tag{2}$$

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & c_{13}c_{23} \end{pmatrix}$$

where $V_e \equiv \sqrt{2}G_F n_e$ is the MSW effective potential.

Solve Eq. (1) $\Rightarrow$



Why the evolution is governed by the Schrödinger equation?

Neutrinos are in general relativistic, while the Schrödinger equation only deals with nonrelativistic particles. So why can it be used here?

- It is a Schrödinger-like equation, not the Schrödinger equation
- The fundamental cause of ν-osc: dispersion relations
- It's misleading to say "ν-osc can be derived from QM, without QFT"
  - I would say "ν-osc can be derived even without QM"
- Smart students always want to rederive ν-osc from QFT
  - My suggestion: good exercise! But don't be too much on it.

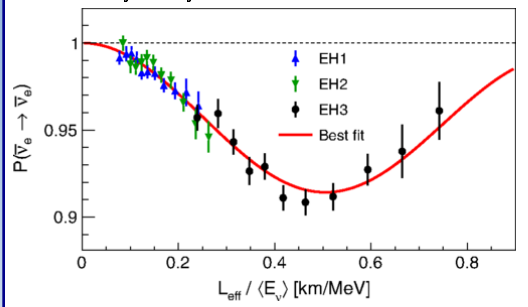
The probability of $\nu_e \to \nu_e$ after flying through a distance of L:

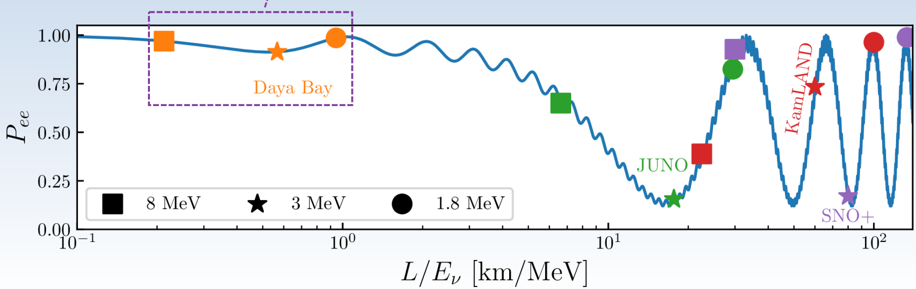
$$P_{ee} \approx 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m_{31}^2 L}{4E_\nu} + \mathcal{O}\left(\frac{\Delta m_{21}^2}{\Delta m_{31}^2}\right)$$

1-3 osc. first observed by Daya Bay
1-2 osc. first observed by KamLAND

Question:
Which one is more challenging?

Daya Bay collab. PRL. 115, 111802





A very well-known effect in the field of neutrino physics:

The matter effect, aka, the MSW effect

When ν propagates in matter, ν-osc is modified by matter.

...

Sounds obvious?

You may whisper: "... of course! Every particle propagating in a medium is affected by the medium."

When I was an undergraduate, I thought it was trivial too.

An estimate

ν can fly through the Sun/Earth so easily

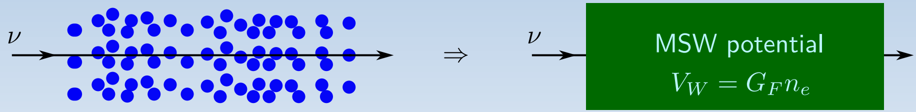
$$\sigma n L = 10^{-41}\text{cm}^2 \times 5\text{g}/\text{cm}^3/m_n \times 6400\text{km}$$
$$= 10^{-11}$$

only $1/10^{11}$ neutrinos are stopped by scattering

→

Question

Only 10^{-11}! Why matter matters?
Answer: coherency



The concept of "coherent forward scattering"

$\neq$ scattering with individual particles
$\equiv$ scattering with all particles simultaneously

All particles, together, form an effective potential

Neutrino oscillation in matter

$$i\frac{d}{dL}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = H\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}, \quad (1)$$

$$H = \frac{1}{2E_\nu} U_{\rm PMNS}\begin{pmatrix}m_1^2 & & \\ & m_2^2 & \\ & & m_3^2\end{pmatrix} U_{\rm PMNS}^\dagger + \begin{pmatrix}V_e & & \\ & 0 & \\ & & 0\end{pmatrix}, \quad (2)$$

The MSW effect is important in solar/atmospheric/accelerator neutrino oscillations

An outline of next few slides:

- For accelerator neutrinos
  - Freund's formula
- For solar neutrinos
  - Simple analytic formula for students
  - Adiabatic approximation
  - Earth matter effect (D/N asymmetry)

If you solve Eq. (1) by brute force, then you can ignore them all!

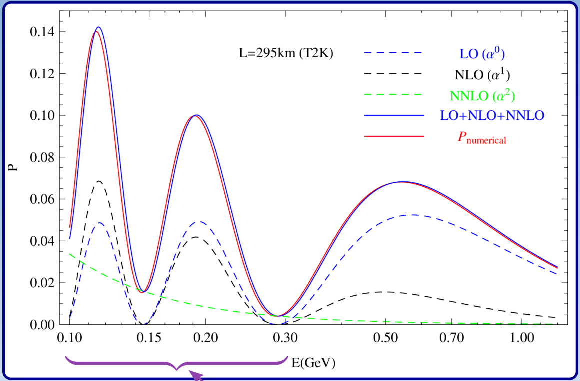
→ ... which I strongly discourage

Freund's formula

In long-baseline accelerator neutrino experiments (T2K, MINOS, NOvA, DUNE, etc.), there is a simple formula to include the MSW effect

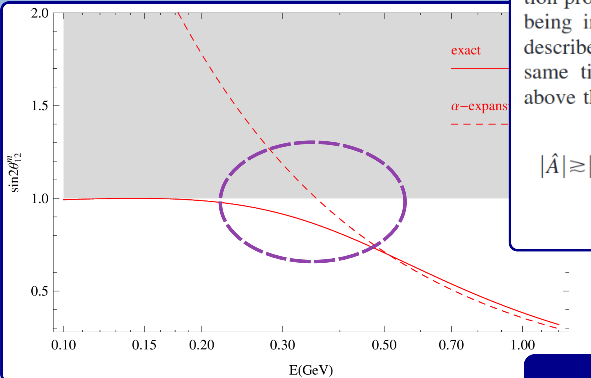
$$P(\nu_\mu \to \nu_e) = 4 s_{13}^2 c_{13}^2 s_{23}^2 \frac{\sin^2(1-A)\Delta}{(1-A)^2} + 8\alpha \frac{J_{CP}}{s_\delta} \cos(\Delta+\delta) \frac{\sin A\Delta}{A} \frac{\sin(1-A)\Delta}{1-A} + 4\alpha^2 s_{12}^2 c_{12}^2 c_{23}^2 \frac{\sin^2 A\Delta}{A^2}$$

$$A \equiv 2\sqrt{2}\frac{G_F N_e E_\nu}{\Delta m_{31}^2}, \quad \alpha \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \approx 0.03, \quad \Delta \equiv \frac{\Delta m_{31}^2 L}{4E_\nu}$$



- Originally derived by M. Freund [Phys.Rev.D 64 (2001) 053003]
- Very accurate in practical use
- obtained by series expansion in α
- The author expected it to be invalid at $E_\nu \lesssim 0.3\text{GeV}$

- obtained by series expansion in α
- expect ... invalid at $E_\nu \lesssim 0.3\text{GeV}$



VII. CONCLUSIONS

The purpose of this work was to find approximate analytic expressions for the neutrino mixing parameters and oscillation probabilities in the presence of matter. It was stated that being interested in approximate solutions it is difficult to describe both the solar and the atmospheric resonance at the same time. Therefore, this work is restricted to energies above the solar resonance according to

$$|\hat{A}| \gtrsim |\alpha| \;\Rightarrow\; E_\nu \gtrsim 0.45\ \text{GeV}\left(\frac{\Delta m^2_{21}}{10^{-4}\ \text{eV}^2}\right)\left(\frac{2.8\ \text{g/cm}^3}{\rho}\right). \quad (45)$$

- ... unexpectedly accurate below 0.3 GeV
  - 50% error $+50\%$ error $+ \ldots = \%1$ error

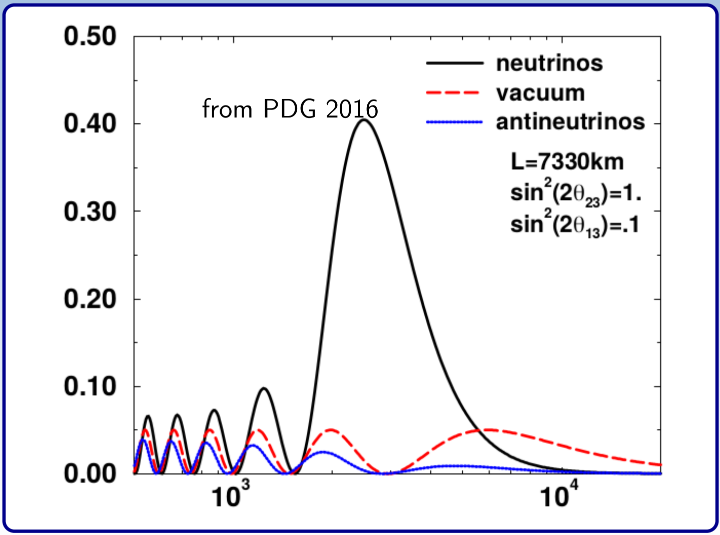
Take-home message:

For accelerator neutrino oscillations (GeV + Earth matter density), Freund's formula suffices.

- later studies reveal that there are hidden but guaranteed cancellations

A worth mentioning example:

- very long baseline, very high energy
  - huge enhancement by the MSW effect
- the so-called neutrino factory ... no longer interested after 2012



Simple analytic formula for students:

Maltoni and Smirnov [1507.05287]

$$P_{ee} = (c_{13} c_{13}^m)^2 \left(\frac{1}{2} + \frac{1}{2} \cos 2\theta_{12}^m \cos 2\theta_{12} \right) + (s_{13} s_{13}^m)^2 \,,$$

$$\left.\begin{aligned} \cos 2\theta_{12}^m &\approx \frac{\cos 2\theta_{12} - \beta_{12}}{\sqrt{(\cos 2\theta_{12} - \beta_{12})^2 + \sin^2 2\theta_{12}}} \,, \\ (s_{13}^m)^2 &\approx s_{13}^2 \, (1 + 2\beta_{13}) \,, \end{aligned}\right\} \text{ effective mixing angles}$$

$$\beta_{12} \equiv \frac{2 c_{13}^2 V_e^0 E_\nu}{\Delta m_{21}^2} \,,$$

$$\beta_{13} \equiv \frac{2 V_e^0 E_\nu}{\Delta m_{31}^2} \,.$$

If matter density $\to 0$ ($\beta_{12,13} \to 0$), $\cos 2\theta_{12}^m \to \cos 2\theta_{12}$; $s_{13}^m \to s_{13}$

- simple and fast, of practical use
  - especially for those students addicted to coding:-)
- sufficient accuracy
  - for current precision of measurement
- straightforward to see how P_{ee} varies with θ_{12}, θ_{13}, ...

If $\theta_{13} \to 0$, more simplified:

$$P_{ee}^{\odot} = \frac{1}{2} + \frac{1}{2} \cos 2\theta_{12}^m \cos 2\theta_{12}$$

$$\cos 2\theta_{12}^m \approx \frac{\cos 2\theta_{12} - \beta_{12}}{\sqrt{(\cos 2\theta_{12} - \beta_{12})^2 + \sin^2 2\theta_{12}}}\,, \qquad \theta_{12}^m \to \begin{cases} \theta_{12} & (\beta_{12} \to 0) \\ -1 & (\beta_{12} \to \infty) \end{cases}$$

$$\beta_{12} \equiv \frac{2V_e^0 E_\nu}{\Delta m_{21}^2}\,,$$

vacuum limit ($\beta_{12} \to 0$):

$$P_{ee} \approx c_{12}^4 + s_{12}^4 \approx 5/9$$

strong matter effect limit ($\beta_{12} \to \infty$):

$$P_{ee} \approx s_{12}^2 \approx \frac{1}{3}$$

vacuum limit ($\beta_{12} \to 0$):

$$P_{ee} \approx c_{12}^4 + s_{12}^4 \approx 5/9$$

strong matter effect limit ($\beta_{12} \to \infty$):

$$P_{ee} \approx s_{12}^2 \approx \frac{1}{3}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ \cdots & \cdots & c_{13}s_{23} \\ \cdots & \cdots & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\langle \nu_e | \nu_1 \rangle = c_{12}c_{13} \approx c_{12}$$

$$\langle \nu_e | \nu_2 \rangle = s_{12}c_{13} \approx s_{12}$$

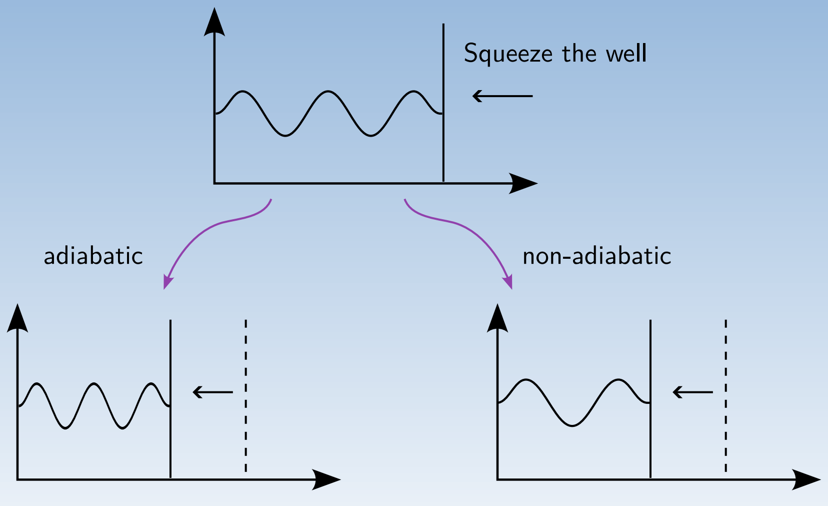
The result is easy to understand:

- When ν_e is produced, it consists of $c_{12}\nu_1 + s_{12}\nu_2$. Each mass eigenstate propagates to the Earth independently. Due to the long distance they lose coherence. At production, the probability of ν_e being ν_1 (ν_2) is c_{12}^2 (s_{12}^2); at detection, the probability of ν_1 (ν_2) being detected as ν_e is also c_{12}^2 (s_{12}^2). Hence the survival probability of ν_e at detection is given by $(c_{12}^2)^2 + (s_{12}^2)^2$.

- When ν_e is produced at the center with a high electron number density, it is almost pure ν_2^m due to the strong matter effect ($\theta_{12}^m \approx 90^\circ$). As the density slowly decreases to zero, the evolution of all mass eigenstates is adiabatic, which means ν_2^m will eventually come out to the surface as ν_2. Since the probability of ν_2 being detected as ν_e is s_{12}^2, the survival probability in the high-E_ν limit is simply s_{12}^2.

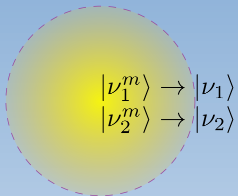
What does “adiabatic” mean?

Quantum mechanics:
Consider a wave function in a well …



The adiabatic approximation

What does “adiabatic” mean for solar neutrinos?



$$P_{ee} = \sum_i |U_{ei}^m|^2 |U_{ei}|^2$$

→ obtained by re-diagonalizing H:

$$H = \frac{1}{2E_\nu} U^m \text{diag}\left(\tilde{m}_1^2,\ \tilde{m}_2^2,\ \tilde{m}_3^2\right) U^{m\dagger}$$

How good is it?

$$\delta P \sim \frac{\gamma^2}{4} \lesssim 10^{-7}$$

More generally,

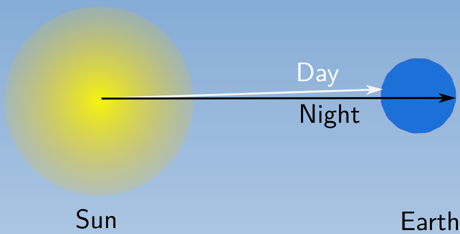
smirnov *et al* [hep-ph/0404042]

$$\gamma \equiv \frac{4\Delta m_{21}^2 E_\nu^2 \sin 2\theta_{12}}{\left(\Delta m_{21}^4 \sin^2 2\theta_{12} + (\Delta m_{21}^2 \cos 2\theta_{12} - 2E_\nu V_e)^2\right)^{3/2}} \frac{dV_e}{dr} \ll 1$$

most fragile at resonance: $\gamma_{\text{resonance}} = \frac{4E_\nu^2}{\Delta m_{21}^4 \sin^2 2\theta_{12}} \frac{dV_e}{dr}$

still good enough: 1 GeV $\to \gamma \approx 0.1$

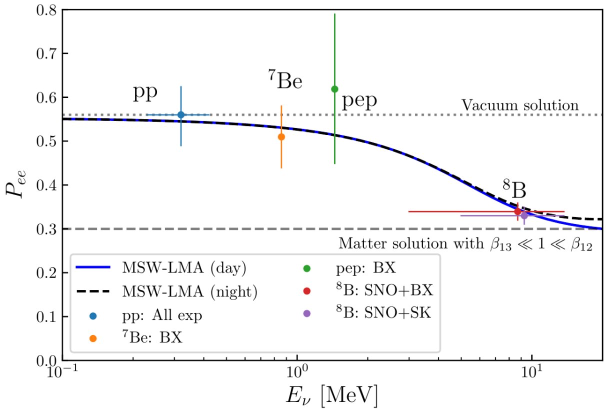
The Earth matter effect (aka Day-Night asymmetry)



$$\Delta P \equiv P_{ee}^{(\mathrm{day})} - P_{ee}^{(\mathrm{night})}$$

$$\approx \frac{1}{2} c_{13}^6 \frac{\cos 2\theta_{12}^m \sin^2 2\theta_{12} K V_\oplus}{K^2 - 2c_{13}^2 \cos 2\theta_{12} V_\oplus K + V_\oplus^2}$$

where $K = \Delta m_{21}^2/(2E_\nu)$,
$V_\oplus$ = MSW potential at Earth.



Neutrino 2022:
SK reported 3 σ significance:

$$A_{DN} \equiv 2\frac{R_D - R_N}{R_D + R_N} = (-2.9 \pm 0.9)\%$$