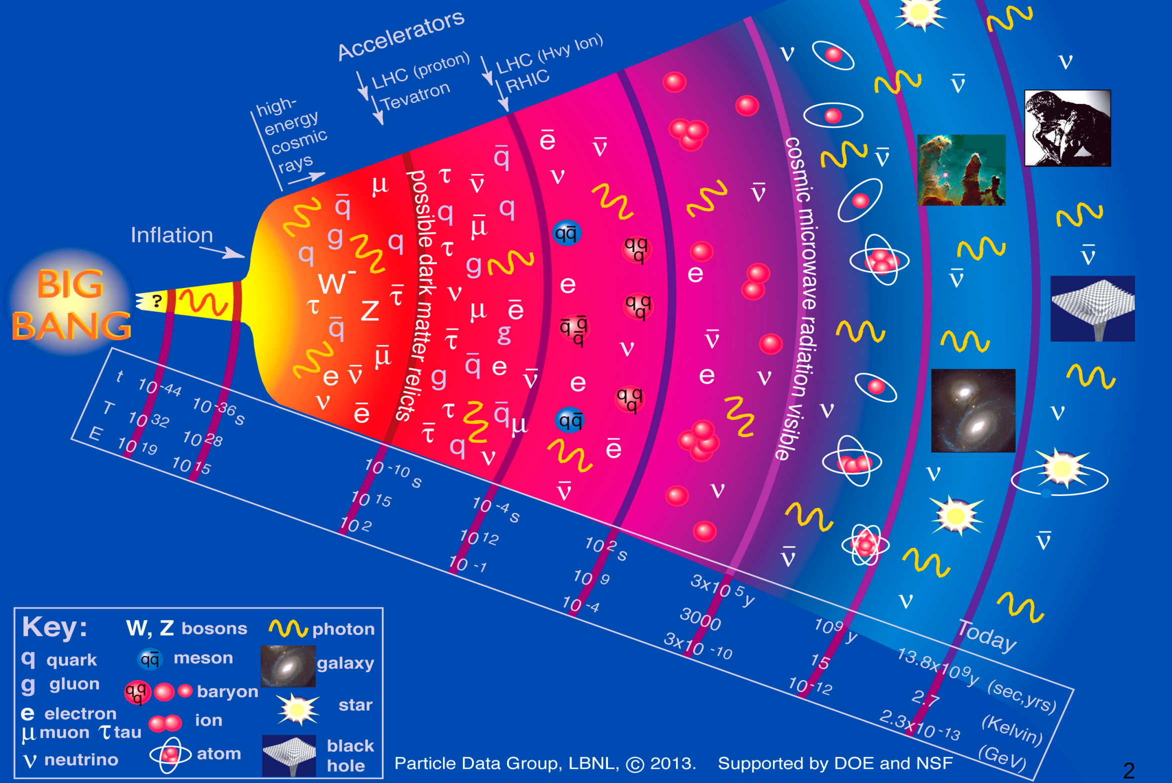
Neutrino cosmology

Special thanks to smart USTC students:
Xuheng Luo
Siyu Zhou

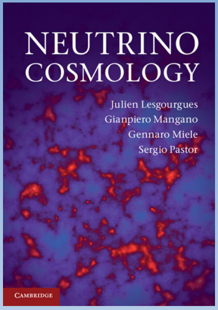
History of the Universe



Particle Data Group, LBNL, © 2013. Supported by DOE and NSF

Neutrinos play a crucial role in modern cosmology

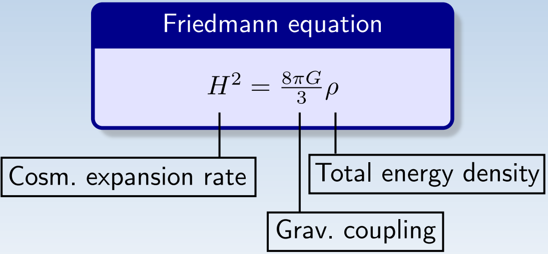
Recent years ...
both interest and importance ↑ ...


NEUTRINO COSMOLOGY
Julien Lesgourgues
Gianpiero Mangano
Gennaro Miele
Sergio Pastor
CAMBRIDGE

Recommend a textbook:
Julien Lesgourgues, *et al*,
Neutrino cosmology

also useful:
A.D. Dolgov, *Neutrino in cosmology*,
Physics Reports 370 (2002) 333-535

- contribute to the radiation in the early universe
  - roughly as much as photons (CMB)
  - thus affect the expansion of the universe
- participate in BBN reactions
- wash out small-scale structures (free streaming)
- ...


Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho$$

$\rho = \rho_\gamma + \rho_\nu + \cdots$
where $\cdots$ are subdominant
in the subMeV-eV universe.

The story of $N_{\text{eff}} = 3.045$

- How many species of neutrinos are there?
  - SM tells you three: ν_e, ν_μ, ν_τ
  - but this doesn't stop theorists from inventing more:
    - 3+1 $(\cdots + \nu_s)$? 3+3 (Dirac neutrinos)?
- How can you test these hypothesis?
  - Colliders, $Z \rightarrow$invisible, $N_\nu \approx 3$ (require couplings to Z)
  - Interestingly, the universe can also tell you "3"

The SM prediction

$$N_{\text{eff}} = 3.045$$

Cosmological observations

Current (Planck 2018): $N_{\text{eff}} = 2.99 \pm 0.17$;
Future (CMB-S4, CMB-HD, ...): $1\sigma = 0.03 \sim 0.014$

Why 3.045, not 3?

A brief outline of the next few slides

- $g_\star = 106.75$
- Energy conservation, entropy conservation, ...
- $T_\nu = 1.9$ K $< T_\gamma = 2.7$ K
- $N_{\rm eff} = 3.045$

$g_\star = 106.75$

The early universe is a hot thermal bath. The earlier, the hotter.

Very early → all SM fields thermalize

fermions/bosons obey Fermi-Dirac/Bose-Einstein distributions:

internal d.o.f

$$f = \frac{g_i}{e^{E/T} \pm 1}$$

$$n = \int \frac{d^3p}{(2\pi)^3} f \, , \quad \rho = \int \frac{d^3p}{(2\pi)^3} E f \, , s = \int \frac{d^3p}{(2\pi)^3} \left[\cdots \right] f$$

T very high → all particles are relativistic → $\int \boxed{\cdots}$ can be analytically computed.

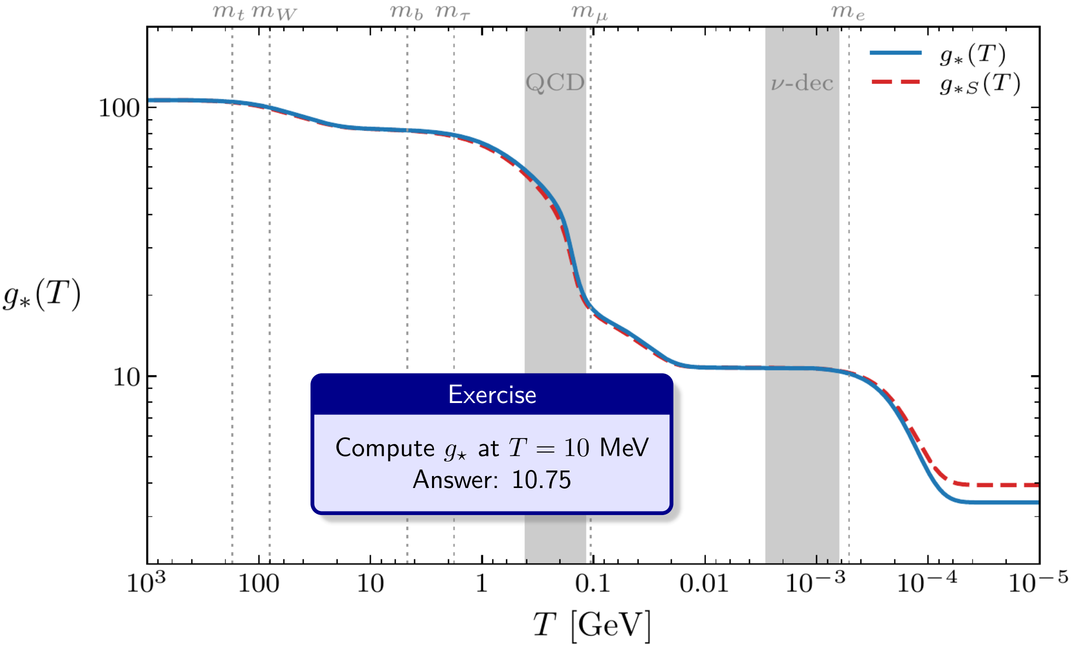
$$\rho = \frac{\pi^2}{30} g_i T^4 \times \begin{cases} 1 & \text{(BE)} \\ 7/8 & \text{(FD)} \end{cases} \qquad s = \frac{2\pi^2}{45} g_i T^3 \times \begin{cases} 1 & \text{(BE)} \\ 7/8 & \text{(FD)} \end{cases} \qquad n = \frac{\zeta(3)}{\pi^2} g_i T^3 \times \begin{cases} 1 & \text{(BE)} \\ 3/4 & \text{(FD)} \end{cases}$$

Define $g_\star$ by $\quad \rho_{\text{SM}} = \frac{\pi^2}{30} g_\star T^4$

Each boson: $g_\star \to g_\star + 1$; each fermion: $g_\star \to g_\star + 7/8$.

$$\boxed{g_\star = (3_c \times 3_f \times 2_{ud} \times 2_{LR} + 3_f \times 2_{\nu\ell} + 3) \times 2 \times \tfrac{7}{8} + 12 \times 2 + 4_H = 106.75}$$

How the SM $g_\star$ decreases as the universe cools down



Exercise

Compute $g_\star$ at $T = 10$ MeV
Answer: 10.75

From B. Wallisch's PhD thesis, 1810.02800

Energy and entropy conservation

$T\downarrow\ \Rightarrow$ more and more species annihilate or decay $\Rightarrow g_\star \downarrow$

release their energies to light species

Is the total energy conserved?

No! Think about just a single species (e.g., γ), how does ρ_γ evolve?

$$\rho_\gamma \propto a^{-4}$$

scale factor in the FRW metric: $(ds^2 = dt^2 - a^2 dx^2)$

Energy conservation:

Definition: $\rho a^4 =$ constant
Condition: negligible mass effect

Entropy conservation:

Definition: $s a^3 =$ constant
Condition: reversible

None of them always hold! Examples that break them:

(1) e^+e^- annihilation at $T \lesssim 1$ MeV; (2) out-of-equilibrium decay (3) freeze-in

More about entropy conservation

Entropy conservation:

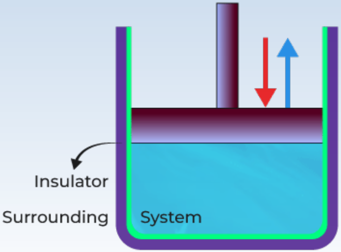
Definition: $sa^3 =$ constant
Condition: reversible

Often not well clarified in the literature/textbooks ...

Very often, students ask why and when ...

The key to understand it:

- reversibility
  - if the universe shrinks back, the same thermodynamic status can be recovered
- the 2nd law of thermodynamics
- adiabaticity
  - remember how to a balloon expands adiabatically?
  - what would happen if you lift the piston too fast?



$T_\gamma = 1.9$ K $< T_\gamma = 2.7$ K

Why today's ν (CNB) is cooler than γ (CMB)?

Because of neutrino decoupling and e^+e^- annihilation

- At $T \gg 1$ MeV, $\nu + \overline{\nu} \leftrightarrow e^+ + e^-$; conversion rate: $\Gamma \sim G_F^2 T^5$;
- Hubble expansion $H \sim T^2/m_{\rm pl}$;
- $\Gamma < H$ if $T \lesssim$ 2MeV;
- Below ~ 2 MeV, ν decouple from γ-$e^\pm$ plasma
- Below ~ 1 MeV, $e^\pm$ abundantly annihilate to γ
- ... causing today's $T_\nu < T_\gamma$

i.e. the probability of a ν finding a partner to achieve the conversion per unit time

How to quantitatively compute it?

Method 1:

solve Boltzmann equation
(fundamental and accurate)

Method 2:

count d.o.f $\Rightarrow T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$
(simple, less accurate)

$T_\gamma = 1.9$ K $< T_\gamma = 2.7$ K

Method 2:

count d.o.f $\Rightarrow T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$
(simple, less accurate)

Two assumptions:

- assume entropy conservation
- assume $2\ \mathrm{MeV} \gg 1$ MeV

(i) $T_\gamma \gg 2$ MeV: SM plasma $\ni (\nu, e^\pm, \gamma)$

$$T_\nu = T_\gamma = T_e$$

$$\frac{d}{dt}\left(s_\nu a^3 + s_\gamma a^3 + s_e a^3\right) = 0$$

2 MeV

(ii) 1 MeV $\ll T_\gamma \ll 2$ MeV: SM plasma $\ni (\nu, e^\pm, \gamma)$

$$T_\nu = T_\gamma = T_e$$

$$\frac{d}{dt}\left(s_\nu a^3\right) = \frac{d}{dt}\left(s_\gamma a^3 + s_e a^3\right) = 0$$

similar to (i) ... but ν decoupled

1 MeV

(iii) $T_\gamma \ll 1$ MeV: SM plasma $\ni (\nu, \gamma)$

$T_\nu \neq T_\gamma$ due to $e^\pm \to \gamma$

... entropy conserv. on each side

$$\left(s_\nu a^3\right)_{\text{iii}} = \left(s_\nu a^3\right)_{\text{ii}}, \ \left(s_\gamma a^3\right)_{\text{iii}} = \left(s_\gamma a^3 + s_e a^3\right)_{\text{ii}} \longrightarrow \boxed{\left(\frac{s_\nu}{s_\gamma}\right)_{\text{iii}} = \left(\frac{s_\nu}{s_\gamma + s_e}\right)_{\text{ii}}}$$

1 keV

$T_\gamma = 1.9$ K $< T_\gamma = 2.7$ K

… …

2 MeV

(ii) $1\ \text{MeV} \ll T_\gamma \ll 2\ \text{MeV}$: SM plasma $\ni (\nu,\, e^\pm,\, \gamma)$

$$T_\nu = T_\gamma = T_e$$

$$\tfrac{d}{dt}\left(s_\nu a^3\right) = \tfrac{d}{dt}\left(s_\gamma a^3 + s_e a^3\right) = 0$$

1 MeV

(iii) $T_\gamma \ll 1$ MeV: SM plasma $\ni (\nu,\, \gamma)$

$$T_\nu \neq T_\gamma \text{ due to } e^\pm \to \gamma$$

$$\left(s_\nu a^3\right)_{\text{iii}} = \left(s_\nu a^3\right)_{\text{ii}},\ \left(s_\gamma a^3\right)_{\text{iii}} = \left(s_\gamma a^3 + s_e a^3\right)_{\text{ii}}$$

$$\left(\frac{s_\nu}{s_\gamma}\right)_{\text{iii}} = \left(\frac{s_\nu}{s_\gamma + s_e}\right)_{\text{ii}}$$

1 keV

$$\left(\frac{g_\nu T_\nu^3}{g_\gamma T_\gamma^3}\right)_{\text{iii}} = \left(\frac{g_\nu T_\nu^3}{(g_\gamma + g_e)\, T_\gamma^3}\right)_{\text{ii}} \quad \longrightarrow \quad \left(\frac{\cancel{g_\nu} T_\nu^3}{g_\gamma T_\gamma^3}\right)_{\text{iii}} = \left(\frac{\cancel{g_\nu} \cancel{T_\nu^3}}{(g_\gamma + g_e)\, \cancel{T_\gamma^3}}\right)_{\text{ii}}$$

$$g_\nu = 3 \times 2 \times \frac{7}{8}$$

$$g_e = 2 \times 2 \times \frac{7}{8} = \frac{7}{2}$$

$$g_\gamma = 2$$

So today, we have

$$\boxed{T_\nu^3 = T_\gamma^3 \frac{g_\gamma}{g_\gamma + g_e} = \frac{4}{11} T_\gamma^3}$$

$N_{\rm eff} = 3.045$

$$\boxed{T_\nu^3 = T_\gamma^3 \frac{g_\gamma}{g_\gamma + g_e} = \frac{4}{11} T_\gamma^3}$$

How accurate is it? ...can be verified by Method 1!

Let's revisit our assumptions

- 2 MeV $\gg$ 1MeV:
  - ... obvious not very good approx.
- entropy conservation:
  - ... in fact, not exact

Moreover ...

- non-instantaneous decoupling
- flavor dependence
  - ν_e decouples later than $\nu_{\mu,\tau}$
- Finite-temperature QED correction
  - e.g. $m_\gamma \neq 0$ in plasma

... all lead to some corrections.

How to quantify the corrections?

$$N_{\rm eff} \equiv \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\left(\frac{\rho_{\rm rad} - \rho_\gamma}{\rho_\gamma}\right)$$

Now you can understand why there are $8/7$, $11/4$, and $4/3$

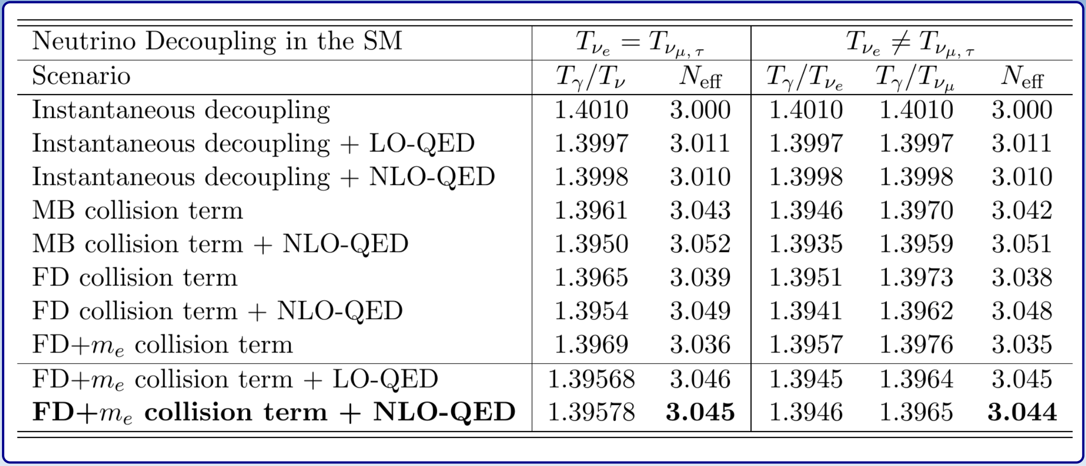
By this definition, if $T_\nu^3 = \frac{4}{11} T_\gamma^3$, it gives $N_{\rm eff} = 3$.

eventually

$$\boxed{N_{\rm eff} = 3.045}$$

$N_{\text{eff}} = 3.045$

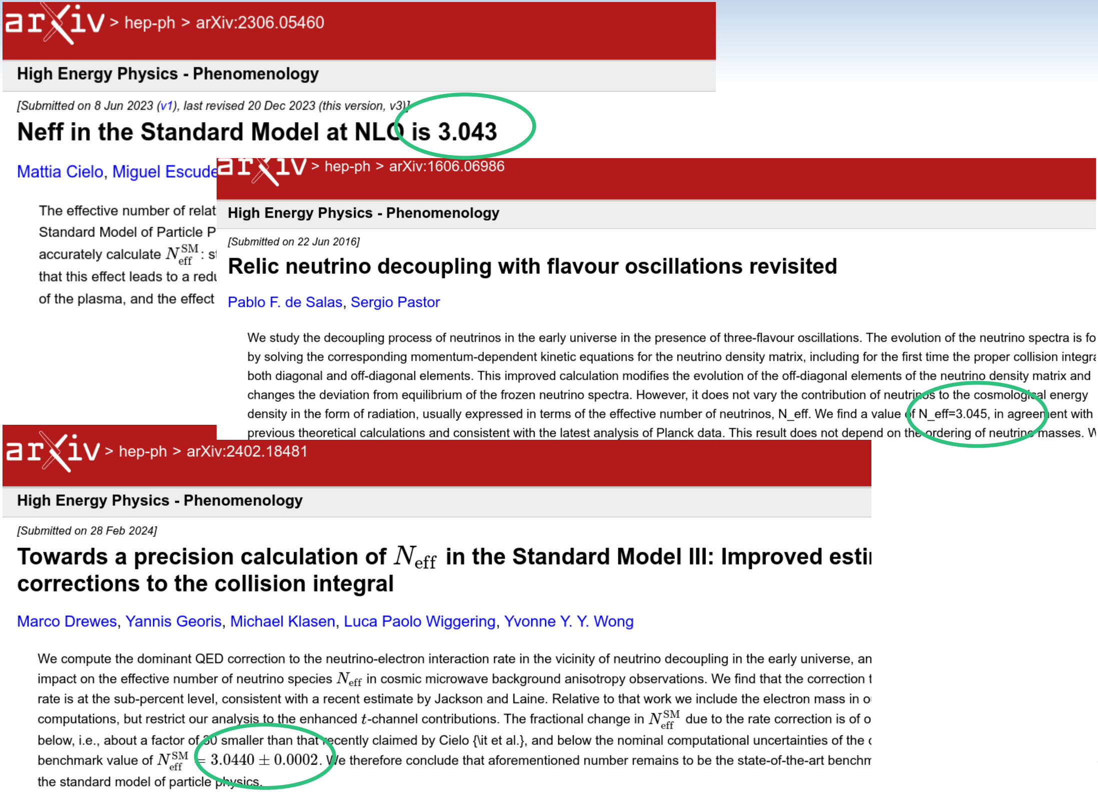
Including all known corrections → $N_{\text{eff}} = 3.045$



Neutrino Decoupling in the SM	$T_{\nu_e} = T_{\nu_{\mu,\tau}}$		$T_{\nu_e} \neq T_{\nu_{\mu,\tau}}$		
Scenario	T_γ/T_ν	N_{eff}	T_γ/T_{ν_e}	T_γ/T_{ν_μ}	N_{eff}
Instantaneous decoupling	1.4010	3.000	1.4010	1.4010	3.000
Instantaneous decoupling + LO-QED	1.3997	3.011	1.3997	1.3997	3.011
Instantaneous decoupling + NLO-QED	1.3998	3.010	1.3998	1.3998	3.010
MB collision term	1.3961	3.043	1.3946	1.3970	3.042
MB collision term + NLO-QED	1.3950	3.052	1.3935	1.3959	3.051
FD collision term	1.3965	3.039	1.3951	1.3973	3.038
FD collision term + NLO-QED	1.3954	3.049	1.3941	1.3962	3.048
FD+m_e collision term	1.3969	3.036	1.3957	1.3976	3.035
FD+m_e collision term + LO-QED	1.39568	3.046	1.3945	1.3964	3.045
FD+m_e collision term + NLO-QED	1.39578	**3.045**	1.3946	1.3965	**3.044**

From M. Escudero [2001.04466]

$N_{\rm eff} = 3.045?\ \ 3.044?\ \ 3.043?$



arXiv > hep-ph > arXiv:2306.05460

High Energy Physics - Phenomenology

[Submitted on 8 Jun 2023 (v1), last revised 20 Dec 2023 (this version, v3)]

Neff in the Standard Model at NLO is 3.043

Mattia Cielo, Miguel Escude

The effective number of relat
Standard Model of Particle P
accurately calculate $N_{\rm eff}^{\rm SM}$: s
that this effect leads to a redu
of the plasma, and the effect

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arXiv > hep-ph > arXiv:1606.06986

High Energy Physics - Phenomenology

[Submitted on 22 Jun 2016]

Relic neutrino decoupling with flavour oscillations revisited

Pablo F. de Salas, Sergio Pastor

We study the decoupling process of neutrinos in the early universe in the presence of three-flavour oscillations. The evolution of the neutrino spectra is fo
by solving the corresponding momentum-dependent kinetic equations for the neutrino density matrix, including for the first time the proper collision integr
both diagonal and off-diagonal elements. This improved calculation modifies the evolution of the off-diagonal elements of the neutrino density matrix and
changes the deviation from equilibrium of the frozen neutrino spectra. However, it does not vary the contribution of neutrinos to the cosmological energy
density in the form of radiation, usually expressed in terms of the effective number of neutrinos, N_eff. We find a value of N_eff=3.045, in agreement with
previous theoretical calculations and consistent with the latest analysis of Planck data. This result does not depend on the ordering of neutrino masses. W

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arXiv > hep-ph > arXiv:2402.18481

High Energy Physics - Phenomenology

[Submitted on 28 Feb 2024]

Towards a precision calculation of $N_{\rm eff}$ in the Standard Model III: Improved esti corrections to the collision integral

Marco Drewes, Yannis Georis, Michael Klasen, Luca Paolo Wiggering, Yvonne Y. Y. Wong

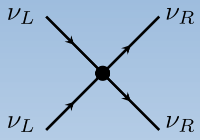
We compute the dominant QED correction to the neutrino-electron interaction rate in the vicinity of neutrino decoupling in the early universe, an
impact on the effective number of neutrino species $N_{\rm eff}$ in cosmic microwave background anisotropy observations. We find that the correction t
rate is at the sub-percent level, consistent with a recent estimate by Jackson and Laine. Relative to that work we include the electron mass in o
computations, but restrict our analysis to the enhanced t-channel contributions. The fractional change in $N_{\rm eff}^{\rm SM}$ due to the rate correction is of o
below, i.e., about a factor of 30 smaller than that recently claimed by Cielo {\it et al.}, and below the nominal computational uncertainties of the c
benchmark value of $N_{\rm eff}^{\rm SM} = 3.0440 \pm 0.0002$. We therefore conclude that aforementioned number remains to be the state-of-the-art benchm
the standard model of particle physics.

Going beyond $N_{\rm eff} = 3.045$

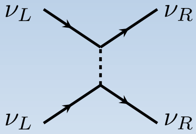
Many BSM physics might leads to $N_{\rm eff} \neq 3.045$.

For example, Dirac neutrinos with BSM interactions

Dirac neutrinos and $N_{\rm eff}$
X. Luo, W. Rodejohann, X. Xu, JCAP, 2005.01629

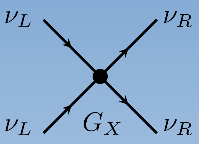


Dirac neutrinos and $N_{\rm eff}$: Part II. The freeze-in case
X. Luo, W. Rodejohann, X. Xu, JCAP, 2011.13059

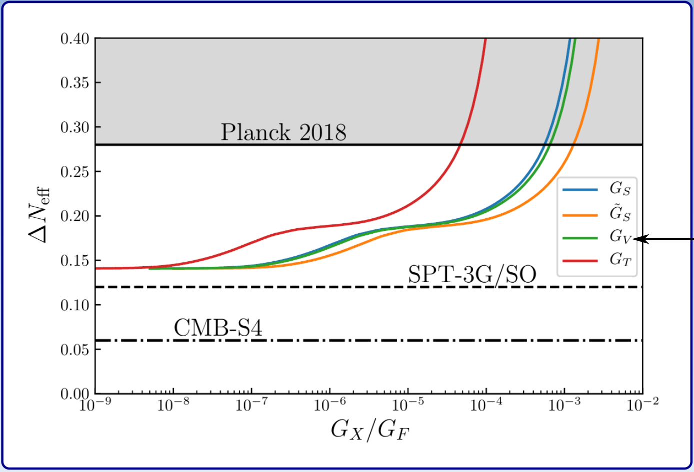


Going beyond $N_{\rm eff} = 3.045$

Dirac neutrinos and $N_{\rm eff}$
X. Luo, W. Rodejohann, X. Xu, JCAP, 2005.01629



... could be
Scalar (S),
Vector (V),
Tensor (T), ...

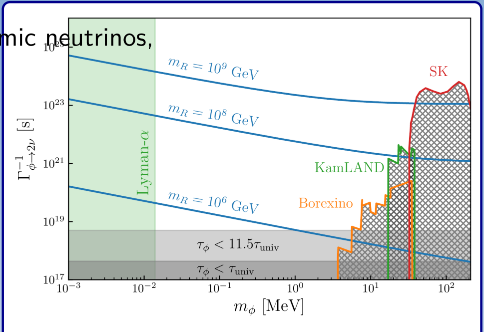


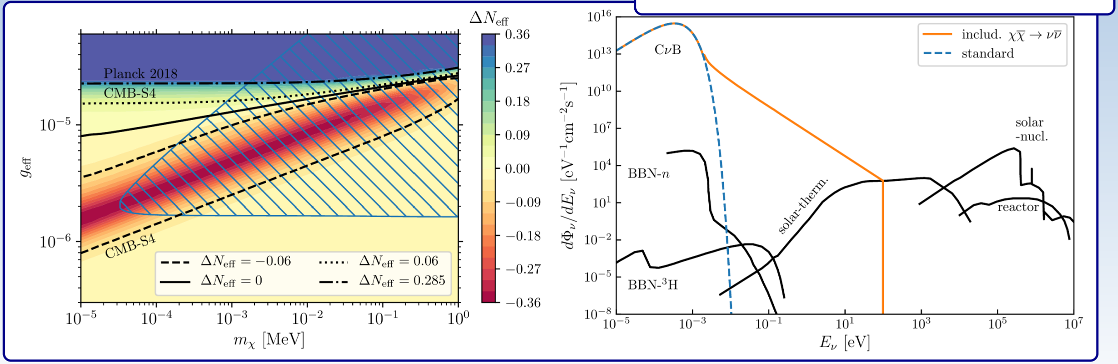
Current: $10^{-5} \sim 10^{-3} G_F$ excluded $\Rightarrow$ we're probing 50 TeV new physics!
Future: CMB-S4 $\Rightarrow \infty$ TeV ??? No! It starts to probe the freeze-in regime

Potential connections between Neutrino and DM

Some recent work from our group ...

[1] Imprints of light dark matter on the evolution of cosmic neutrinos, Isaac Wang, XJX, JCAP [2312.17151].
[2] The ν_R-philic scalar dark matter,
XJX, Siyu Zhou, Junyu Zhu, JCAP [2310.16346].
[3] Dark matter produced from right-handed neutrinos,
Shao-Ping Li, XJX, JCAP [2212.09109].
[4] Dark matter produced from neutrinos,
Marco Hufnagel, XJX, JCAP [2110.09883].





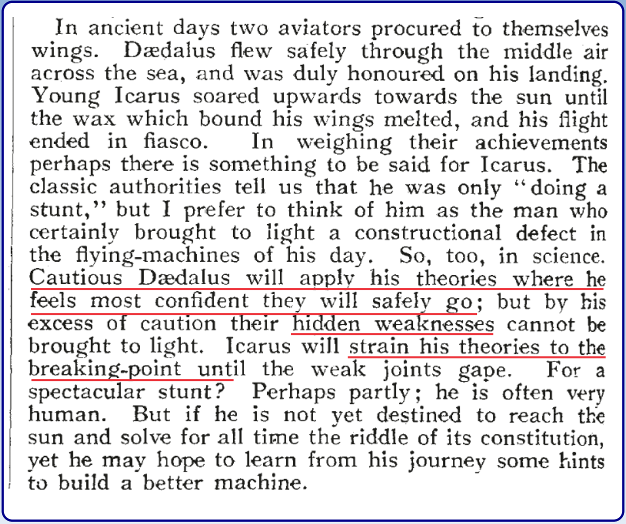
Overall, rich pheno. and excellent theoretical motivation ...

Summary

- The beginning of anomalies
  - solar neutrinos and ...
- The standard neutrino oscillation theory
  - in vacuum
  - in matter
    - why matter matters? the power of coherence
- Theories of neutrino masses
  - Seesaw, νMSM, ...
  - Majorana masses and $0\nu\beta\beta$
- Neutrino cosmology
  - $N_{\rm eff} = 3.045$
  - beyond $N_{\rm eff}$

Strain the theory to the breaking point

The last paragraph of Eddington's paper



In ancient days two aviators procured to themselves wings. Dædalus flew safely through the middle air across the sea, and was duly honoured on his landing. Young Icarus soared upwards towards the sun until the wax which bound his wings melted, and his flight ended in fiasco. In weighing their achievements perhaps there is something to be said for Icarus. The classic authorities tell us that he was only "doing a stunt," but I prefer to think of him as the man who certainly brought to light a constructional defect in the flying-machines of his day. So, too, in science. Cautious Dædalus will apply his theories where he feels most confident they will safely go; but by his excess of caution their hidden weaknesses cannot be brought to light. Icarus will strain his theories to the breaking-point until the weak joints gape. For a spectacular stunt? Perhaps partly; he is often very human. But if he is not yet destined to reach the sun and solve for all time the riddle of its constitution, yet he may hope to learn from his journey some hints to build a better machine.

"fly too close to the sun" 的希腊神话起源

D= 代达罗斯 I= 伊卡洛斯
D 安全飞回, 获得荣耀
I 越飞越高 → 粘羽毛的蜡融化 → 坠亡

The success of SM + the establishment of 3ν-mixing framework:

the beginning of end or the end of beginning?

Shall we be Daedalus or Icarus?