粲强子非轻衰变过程中轻标量介子 $a_0(1710/1817)$的理论研究

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PRD105(2022)116010, 107(2023)034001, 108(2023)114004,

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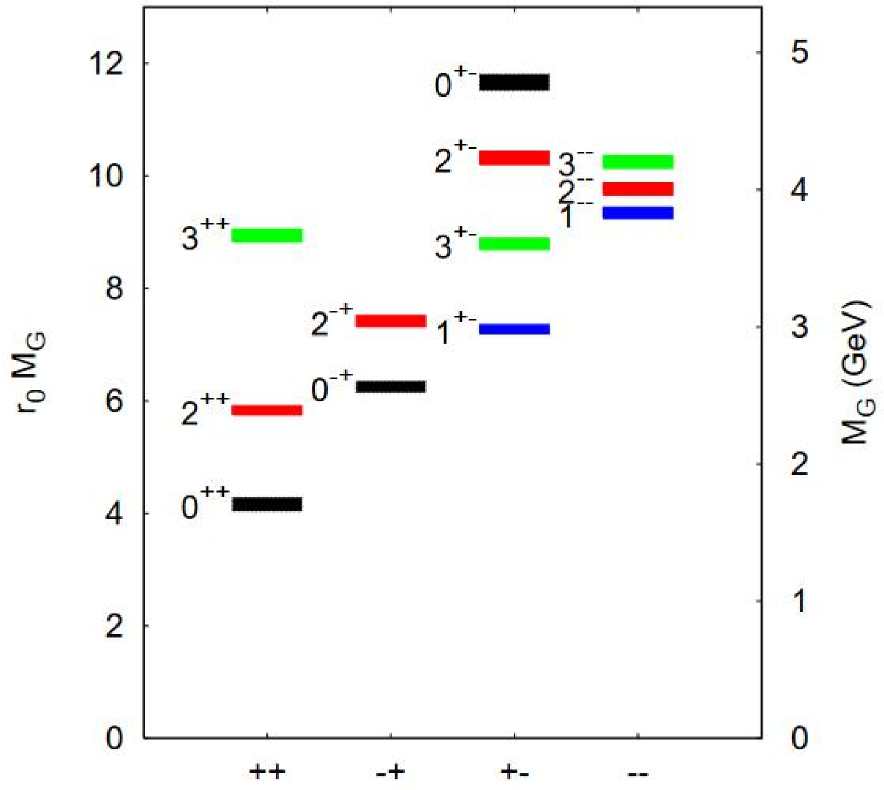
Motivation-Scalar mesons

- **Masses puzzle:** $a_0(980)/f_0(980)$, $K(700)$, $a_0(500)$
- $a_0(980)/f_0(980)$: $q\bar{q}$, **tetraquark, hadronic molecules**
- $f_0(1370)$, $f_0(1500)$, $f_0(1710)$
- $f_0(1710)$: **glueball,** $s\bar{s}$, $K^*\bar{K}^*$

Y. Chen, PRD73(2006)014516



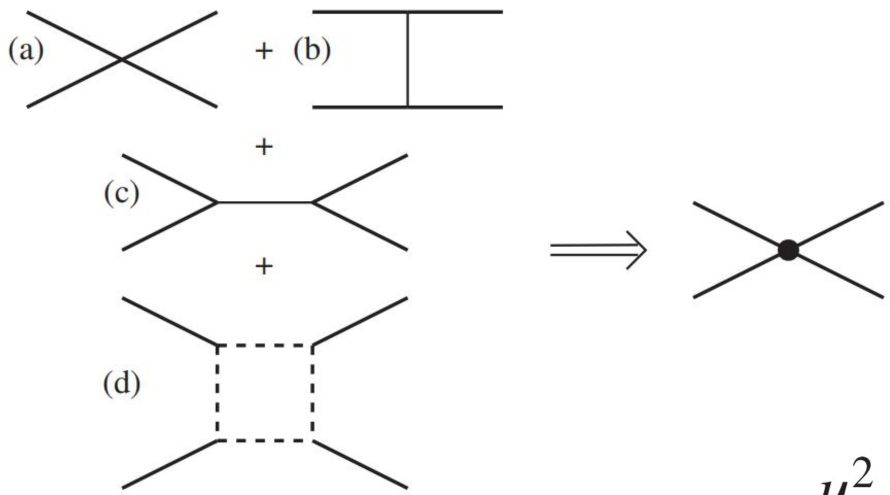
PDG: Tentative classification

	Γ [MeV]	isospin i	structure
$a_0(980)$	~ 50	1	$K\bar{K}, qq\bar{q}\bar{q}$
$f_0(980)$	~ 50	0	$K\bar{K}, qq\bar{q}\bar{q}$
$f_0(500)$	~ 800	0	$\pi\pi, qq\bar{q}\bar{q}$
$K_0^*(700)$	~ 600	$\frac{1}{2}$	$K\pi, qq\bar{q}\bar{q}$
$a_0(1450)$	265	1	$u\bar{d}, d\bar{u}, d\bar{d} - u\bar{u}$
$f_0(1370)$	~ 400	0	$d\bar{d} + u\bar{u}$
$f_0(1710)$	125	0	$s\bar{s}$
$K_0^*(1430)$	294	$\frac{1}{2}$	$u\bar{s}, d\bar{s}, s\bar{u}, s\bar{d}$

VV interaction from hidden-gauge lagrangians

□The hidden-gauge Lagrangian Geng-Oset, PRD79(2009)074009

$$g = \frac{M_V}{2f},$$



$$\mathcal{L} = -\frac{1}{4}\langle \bar{V}_{\mu\nu}\bar{V}^{\mu\nu}\rangle + \frac{1}{2}M_v^2\langle [V_\mu - (i/g)\Gamma_\mu]^2\rangle,$$

$$\bar{V}_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu],$$

$$\Gamma_\mu = \frac{1}{2}\{u^\dagger[\partial_\mu - i(v_\mu + a_\mu)]u + u[\partial_\mu - i(v_\mu - a_\mu)]u^\dagger\},$$

$$\mathcal{L}_{VVVV} = \frac{1}{2}g^2\langle [V_\mu, V_\nu]V^\mu V^\nu\rangle,$$

$$u^2 = U = \exp\left(\frac{i\sqrt{2}\Phi}{f}\right)$$

$$\mathcal{L}_{VVV} = ig\langle (\partial_\mu V_\nu - \partial_\nu V_\mu)V^\mu V^\nu\rangle$$
$$= ig\langle V^\mu \partial_\nu V_\mu V^\nu - \partial_\nu V_\mu V^\mu V^\nu\rangle$$
$$= ig\langle (V^\mu \partial_\nu V_\mu - \partial_\nu V_\mu V^\mu)V^\nu\rangle.$$

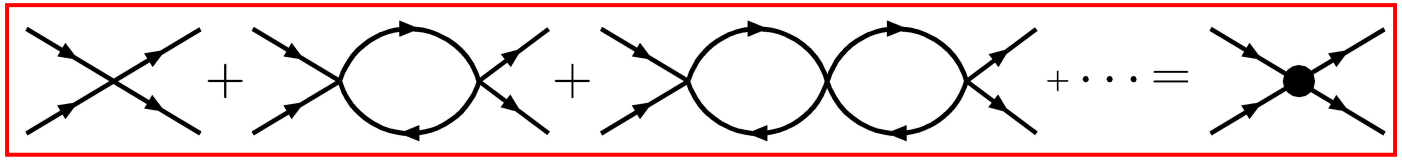
$$V_\mu = \begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_\mu, \quad \Phi = \begin{pmatrix} \frac{\eta}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}.$$

$$\mathcal{L}_{V\Phi\Phi} = -ig\langle V_\mu[\Phi, \partial^\mu\Phi]\rangle.$$

Bethe-Salpeter equation

□Transition amplitude T

V + VGV + VGVGV + ··· = T



□On-shell factorization

$$\begin{aligned} VGV &= \int d^4q V(q^2) \frac{i}{q^2 - m_1^2 + i\epsilon} \frac{1}{(P-q)^2 - m_2^2 + i\epsilon} V(q^2) \\ &= V(m^2) G V(m^2) \end{aligned}$$

$$V(q^2) = V(m^2) + \frac{\partial V}{q^2}\Big|_{q^2=m^2}(q^2 - m^2) \qquad G = i\int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - M_{V1}^2} \frac{1}{q^2 - M_{V2}^2}$$

$$T = (1 - VG)^{-1} V$$

Free parameter

□Loop function

$$G = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 - M_{V1}^2}\frac{1}{q^2 - M_{V2}^2}$$

□Subtraction constants in dimensional regularization (DR)

$$\begin{aligned} G &= \frac{1}{16\pi^2}\left(\alpha + Log\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s}Log\frac{m_2^2}{m_1^2}\right. \\ &+ \left.\frac{p}{\sqrt{s}}\left(Log\frac{s - m_2^2 + m_1^2 + 2p\sqrt{s}}{-s + m_2^2 - m_1^2 + 2p\sqrt{s}} + Log\frac{s + m_2^2 - m_1^2 + 2p\sqrt{s}}{-s - m_2^2 + m_1^2 + 2p\sqrt{s}}\right)\right) \end{aligned}$$

□cutoff values in cutoff method

$$G = \int_0^{q_{max}} \frac{q^2dq}{(2\pi)^2}\frac{\omega_1 + \omega_2}{\omega_1\omega_2[(P^0)^2 - (\omega_1 + \omega_2)^2 + i\epsilon]}$$

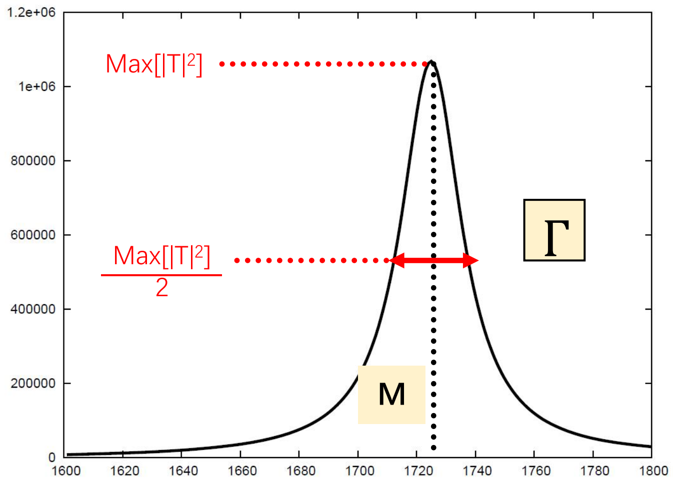
Since we work with hadronic phenomena, one often uses a cutoff value of about 1 GeV to guide calculations in DR method.

Dynamically generated state

- To identify resonances, one goes to the complex plane to look for poles.
- Around the pole position, the amplitude can be approximated by

$$T_{ij} = \frac{g_i g_j}{s - s_{\text{pole}}},$$

$$M = \text{Re}\sqrt{s}_{\text{pole}} \quad \text{and} \quad \Gamma = 2 \times \text{Im}\sqrt{s}_{\text{pole}}$$



Dynamically generated states

- **strangeness=0, isospin=0, and spin=2**

	pole position (no two p's)	real axis (with two p's)
$f_2(1270)$	**(1275, 2)**	**(1276, 97)**
$f'_2(1525)$	**(1525, 6)**	**(1525, 45)**

Two subtraction constants slightly tuned to reproduce exactly the masses!

- **strangeness=0, isospin=0, and spin=0**

	pole position (no two p's)	real axis (two p's)
$f_0(1370)$	**(1512, 51)**	**(1523, 257)**
$f_0(1710)$	**(1726, 28)**	**(1721, 133)**

Table: Branching ratios of the $f_0(1710)$ in comparison with data.

	Our model	PDG
$\Gamma(\pi\pi)/\Gamma(K\bar{K})$	$< 1\%$	$< 11\%$ at 95% C.L.
$\Gamma(\eta\eta)/\Gamma(K\bar{K})$	$\sim 50\%$	$(48 \pm 15)\%$

No tuning of the parameters, their positions are fixed by those of the f_2 states!

Dynamically generated states

Notation: (mass, width) in MeV

$I^G(J^{PC})$	Theory			PDG data		
	pole position	real axis		name	mass	width
		$\Lambda_b = 1.4$ GeV	$\Lambda_b = 1.5$ GeV			
$0^+(0^{++})$	(1512,51)	(1523,257)	(1517,396)	$f_0(1370)$	1200∼1500	200∼500
$0^+(0^{++})$	(1726,28)	(1721,133)	(1717,151)	$f_0(1710)$	1724 ± 7	137 ± 8
$0^+(1^{++})$	(1802,78)	(1802,49)		f_1		
$0^+(2^{++})$	(1275,2)	(1276,97)	(1275,111)	$f_2(1270)$	1275.1 ± 1.2	$185.0^{+2.9}_{-2.4}$
$0^+(2^{++})$	(1525,6)	(1525,45)	(1525,51)	$f_2'(1525)$	1525 ± 5	73^{+6}_{-5}
$1^-(0^{++})$	(1780,133)	(1777,148)	(1777,172)	a_0		
$1^+(1^{+-})$	(1679,235)	(1703,188)		b_1		
$1^-(2^{++})$	(1569,32)	(1567,47)	(1566,51)	$a_2(1700)$??		
$1/2(0^+)$	(1643,47)	(1639,139)	(1637,162)	K		
$1/2(1^+)$	(1737,165)	(1743,126)		$K_1(1650)$?		
$1/2(2^+)$	(1431,1)	(1431,56)	(1431,63)	$K_2^*(1430)$	1429 ± 1.4	104 ± 4

Effects of PP coupled channels

PHYSICAL REVIEW D **104,** 114001 (2021)

Further study of $f_0(1710)$ with the coupled-channel approach and the hadron molecular picture

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The $f_0(1710)$ was previously proposed to be a dynamically generated state with interactions between vector mesons. We extend the study of $f_0(1710)$ by including its coupling to channels of pseudoscalar mesons within the coupled-channel approach. The channels involved are $K^*\bar{K}^*, \rho\rho, \omega\omega, \phi\phi, \omega\phi, \pi\pi, K\bar{K}, \eta\eta$. We show that the pole assigned to $f_0(1710)$ does not change much. Then we calculate the partial decay widths of $f_0(1710) \to K^*\bar{K}^* \to \pi\pi, K\bar{K}, \eta\eta$ as the coupled channel dynamically generated state as well as assuming it to be a pure $K^*\bar{K}^*$ molecule. In both cases the ratios of partial decay widths agree fairly with that in PDG.

Other predictions for $a_0(1710)$

PHYSICAL REVIEW D **83,** 016007 (2011)

Low-lying even-parity meson resonances and spin-flavor symmetry

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(Received 6 May 2010; published 20 January 2011)

$\sqrt{s_R}$	$\eta\pi$	$\bar{K}K$	$\omega\rho$	$\phi\rho$	$\bar{K}^*K^*$
(991, −46)	2906	3831	775	4185	5541
(1442, −5)	907	285	10898	677	3117
(1760, −12)	790	1241	667	5962	5753

PHYSICAL REVIEW D **97,** 034030 (2018)

Strong decays of the higher isovector scalar mesons

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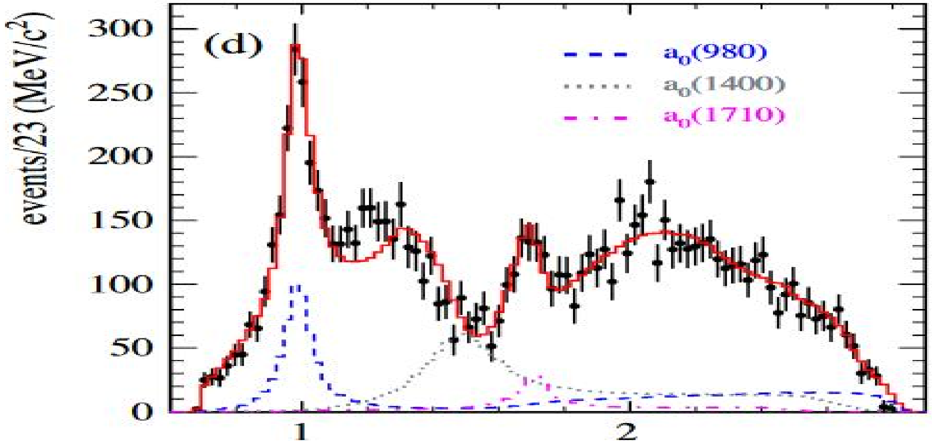
TABLE IV. Decay widths of $a_0(2^3P_0)$ (in MeV). The initial state mass is set to be 1744 MeV.

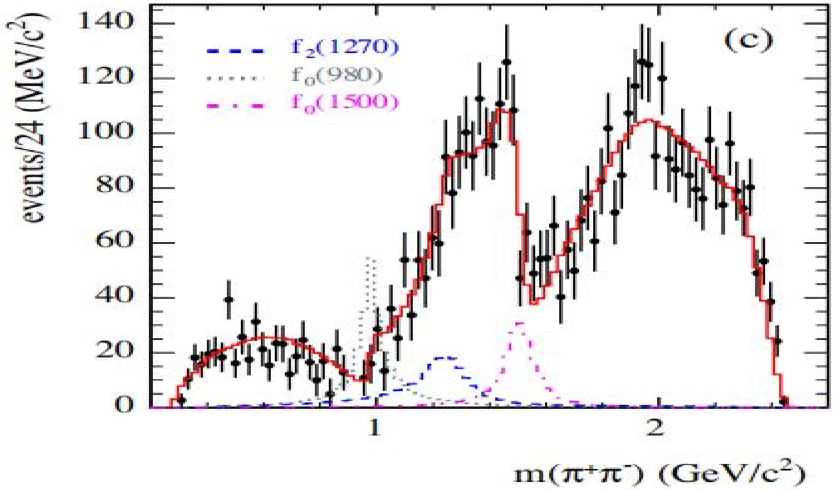
Channel	Mode	$\Gamma_i(2^3P_0)$
$0^+ \to 0^-0^-$	$\pi\eta$	7.91
	$\pi\eta'$	2.46
	$\pi\eta(1475)$	19.25
	$\pi\eta(1295)$	20.86
	$K\bar{K}$	1.07
$0^+ \to 0^-1^+$	$\pi b_1(1235)$	213.08
	$\pi f_1(1285)$	38.54
	$\pi f_1(1420)$	1.02
$0^+ \to 1^-1^-$	$\rho\omega$	59.96
Total width		364.12

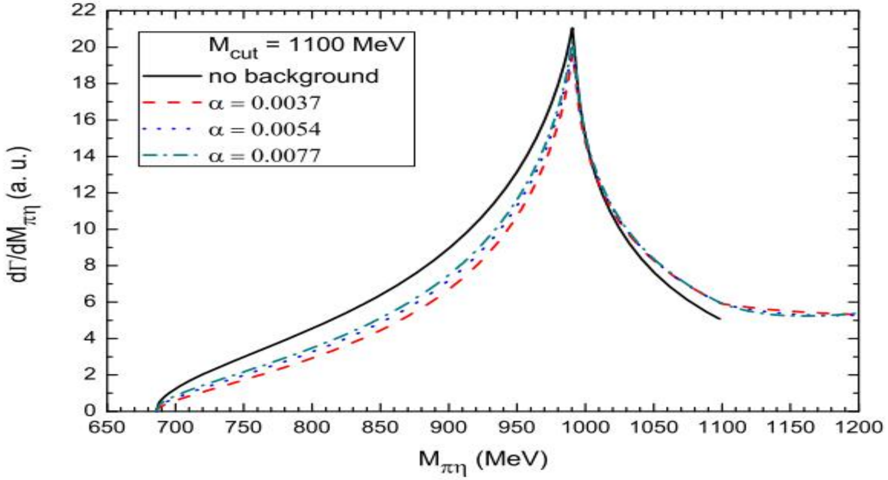
$a_0(1710)$

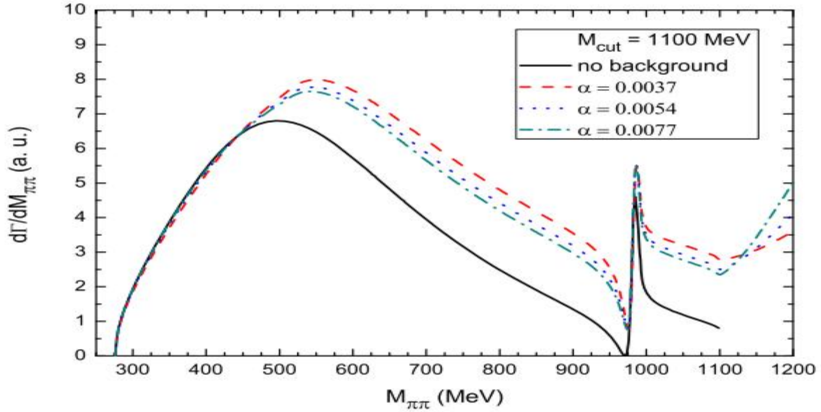
□BaBar: $\eta_c \to \eta\pi^+\pi^-$ BaBar: PRD104(2021)072002

$$m(a_0(1700)) = 1704 \pm 5_{\text{stat}} \pm 2_{\text{sys}}\ \text{MeV}/c^2,$$
$$\Gamma(a_0(1700)) = 110 \pm 15_{\text{stat}} \pm 11_{\text{sys}}\ \text{MeV}/c^2.$$







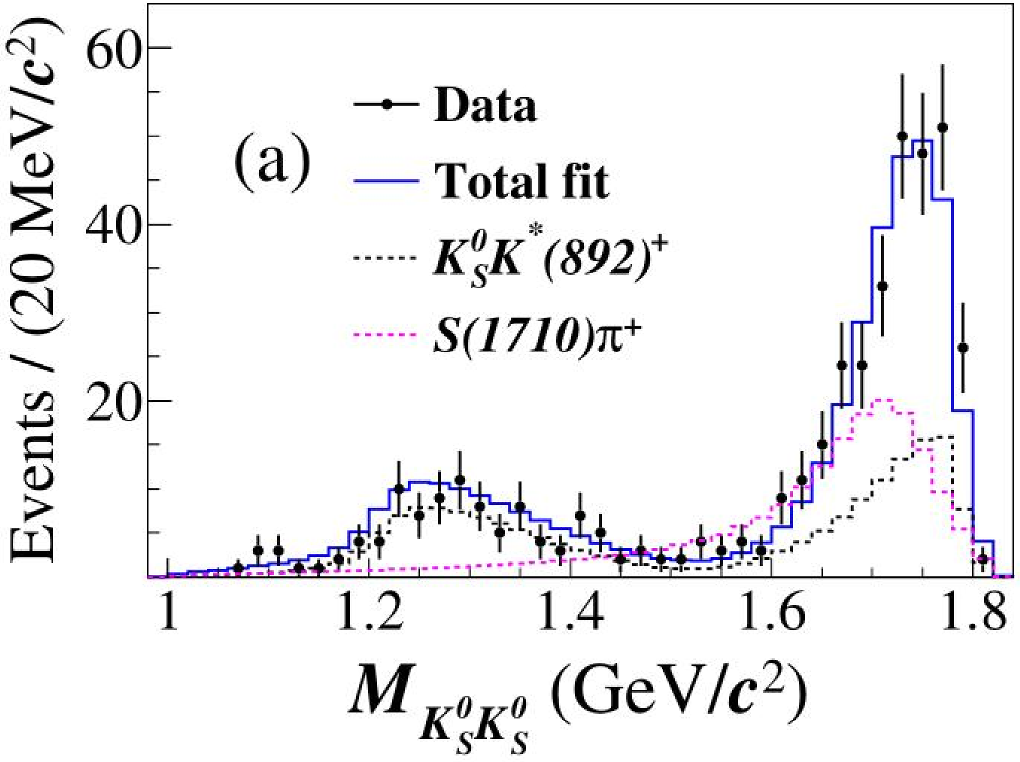


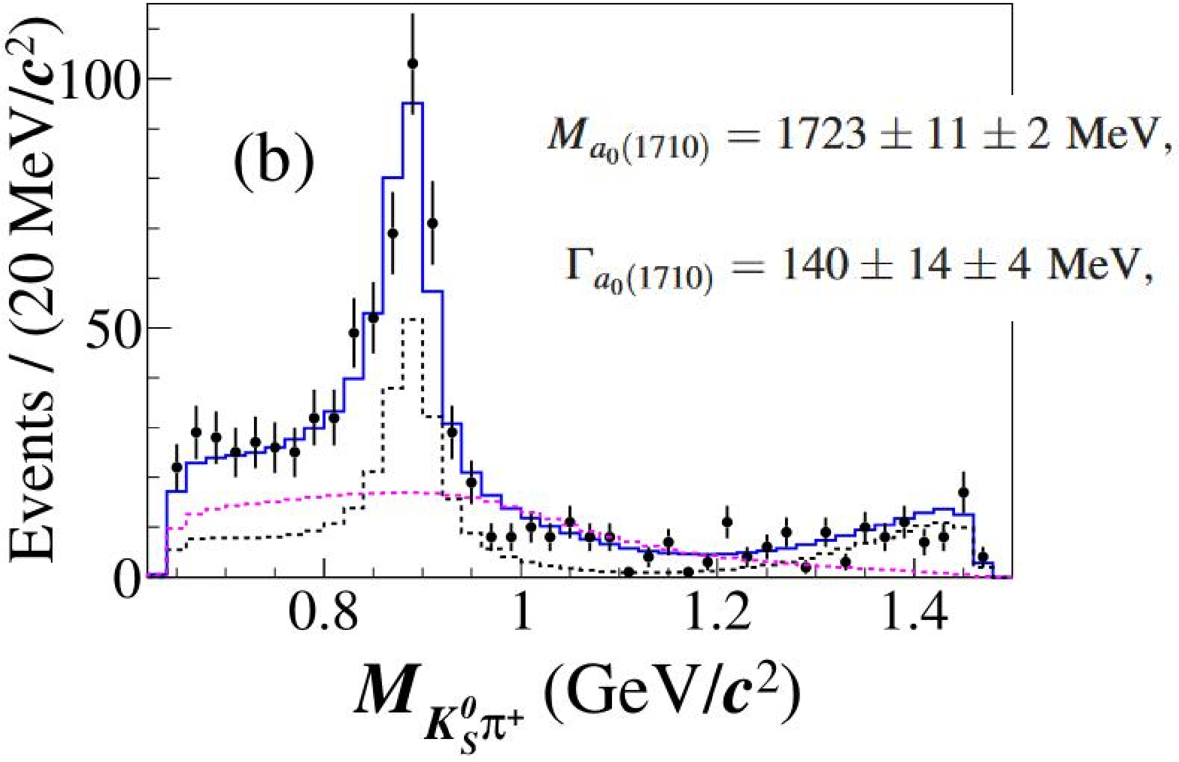
Liang-Xie-Oset PLB766 (2017) 59

BESIII measurements

□ $D_s^+ \to K_s^0 K_s^0 \pi^+$

BESIII: PRD105 (2022) 5, L051103

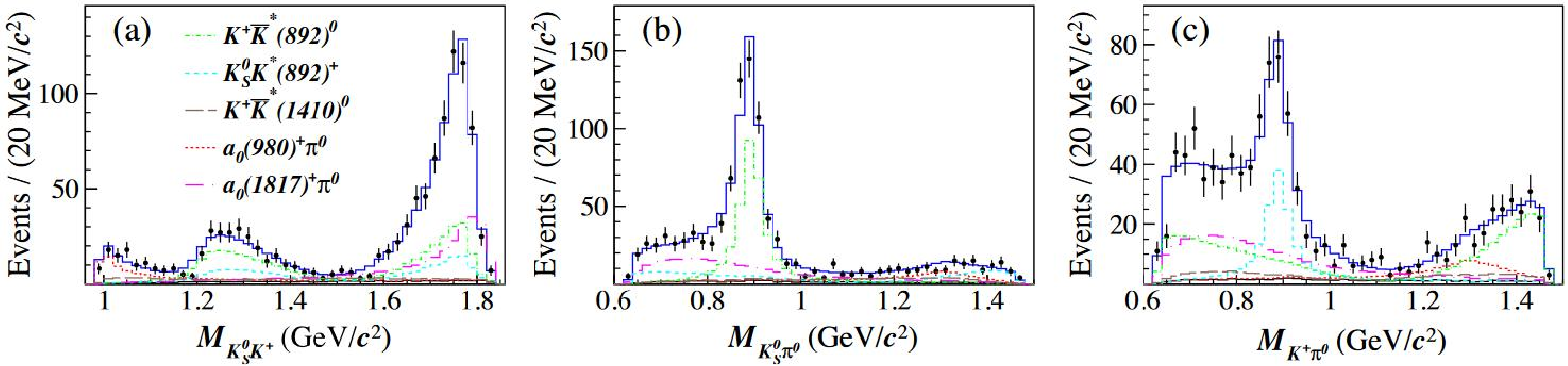




BESIII measurements

□ $D_s^+ \to K_S^0 K^+ \pi^0$

BESIII: PRL129, 182001



$$
\begin{aligned}
M_{a_0(1710)} &= 1817 \pm 8 \pm 20\ \text{MeV}, \\
\Gamma_{a_0(1710)} &= 97 \pm 22 \pm 15\ \text{MeV}.
\end{aligned}
$$

Too close to the boundary region!!!

Collaboration	$M_{a_0(1710)}$	$\Gamma_{a_0(1710)}$	Ref.
BABAR	$1704 \pm 5 \pm 2$	$110 \pm 15 \pm 11$	[8]
BESIII	$1723 \pm 11 \pm 2$	$140 \pm 14 \pm 4$	[9]
BESIII	$1817 \pm 8 \pm 20$	$97 \pm 22 \pm 15$	[10]

VV interactions

Eur. Phys. J. C (2022) 82:509
https://doi.org/10.1140/epjc/s10052-022-10460-4

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Regular Article - Theoretical Physics

Two dynamical generated a_0 resonances by interactions between vector mesons

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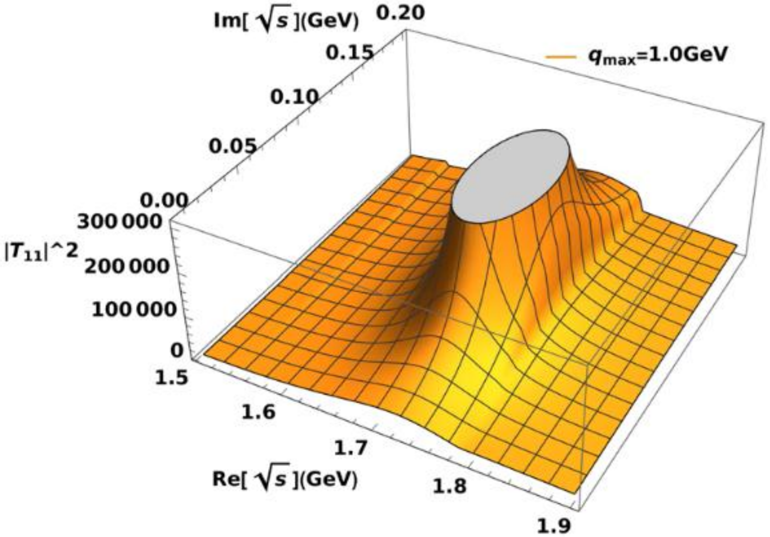


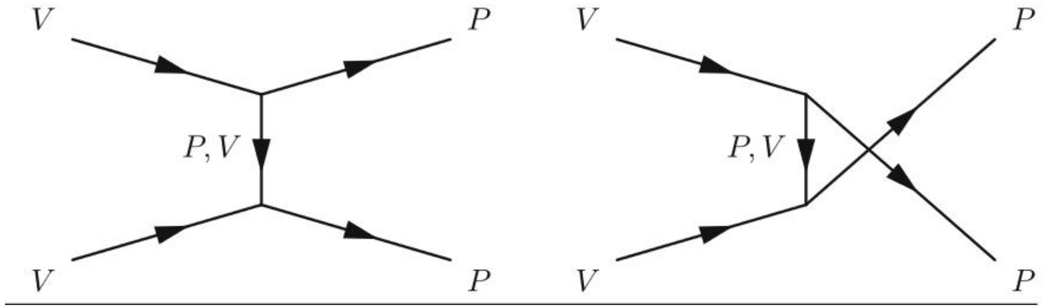


Table 3 The resonance pole for different cutoffs

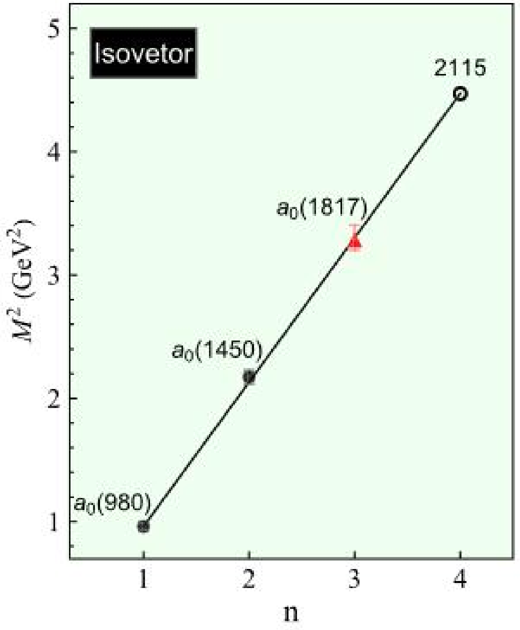
q_{max} (GeV)	0.9	1.0	1.1
Pole (GeV)	$1.76 - 0.09i$	$1.72 - 0.10i$	$1.69 - 0.11i$

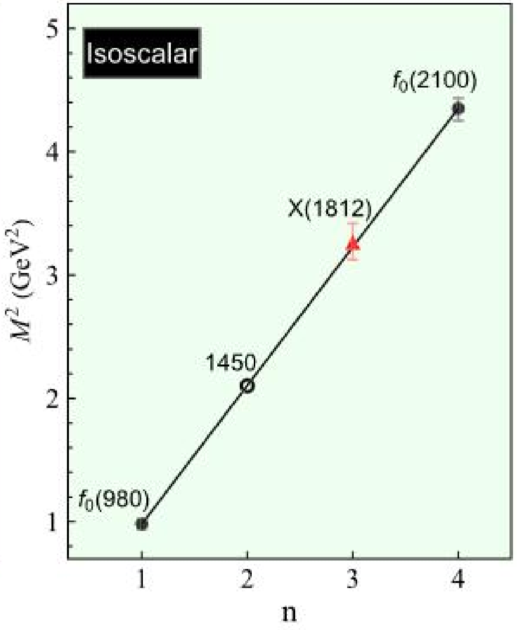
$a_0(1817)$

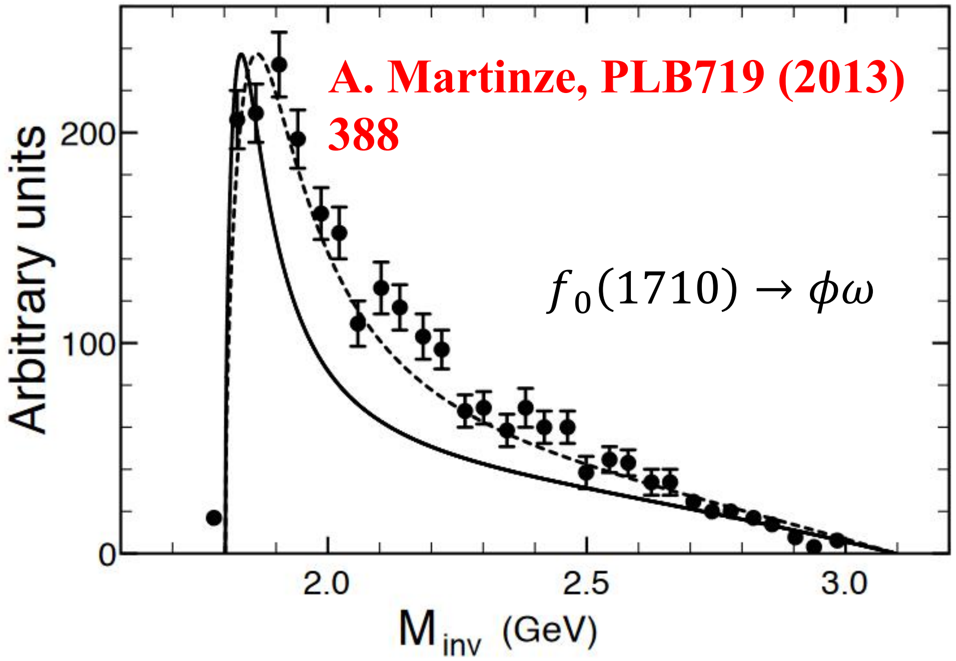
$a_0(1817)$ & $X(1812)(J/\psi \to \gamma\phi\omega)$

D.Guo, PRD105(2022)114014

BESIII: Phys.Rev.Lett. 96 (2006) 162002, PRD87(2013)032008







A. Martinze, PLB719 (2013) 388

$f_0(1710) \to \phi\omega$

Other studies

◻Molecule

- Z.Y. Wang, Y.W. Peng, J.Y Yi, W.C. Luo, C. W. Xiao, PRD107 (2023) 1116018
- Oset-Dai-Geng, EPJC82 (2022) 3, 225,Sci.Bull. 68 (2023) 243

◻Four quark in MIT model

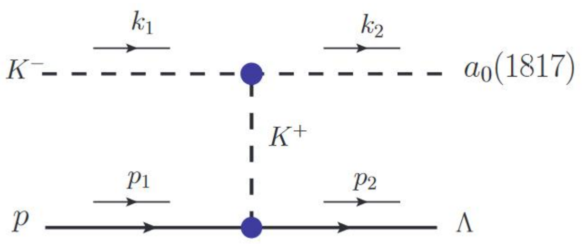
- N.N. Achasov, 2306.04478

◻Search for a0(1710)

- Abreu-Wang-Oset, Eur.Phys.J.C 83 (2023) 3, 243
- Ewang-JJXie-LSGeng, PRD108(2023)114004
- Xiao-Yun Wang,Hui-Fang Zhou, Xiang Liu, 2306.12815

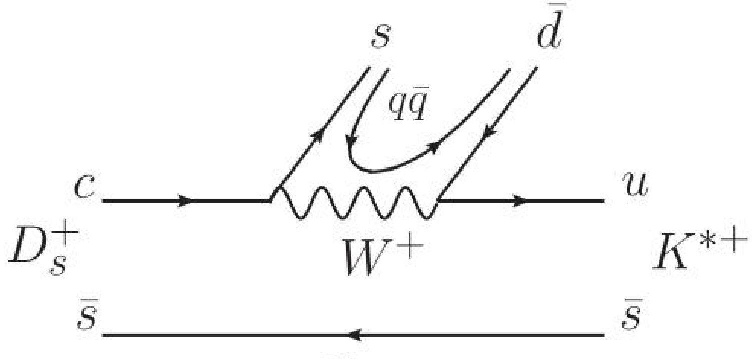
$$J/\psi \to \phi K^+K^-(K^0\bar{K}^0)$$

$$\eta_c \to \bar{K}^0K^+\pi^-$$

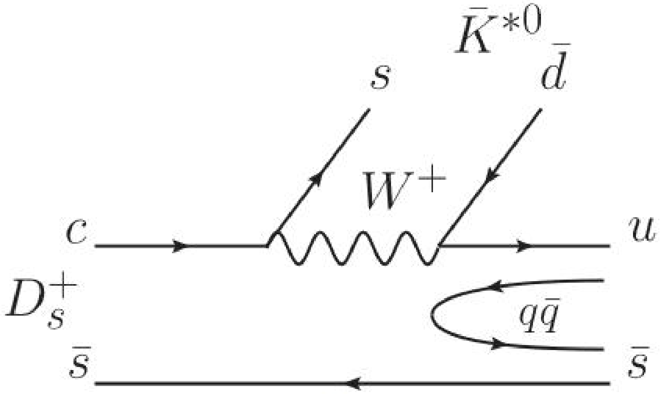


Mechanism of the $D_s^+ \to K_S^0 K_S^0 \pi^+$

□Weakly decay



(a)



(b)

$$D_s^+ \to V_1[s\bar{d} \to s(\bar{u}u + \bar{d}d + \bar{s}s)\bar{d}](u\bar{s} \to K^{*+}),$$

$$D_s^+ \to V_2[u\bar{s} \to u(\bar{u}u + \bar{d}d + \bar{s}s)\bar{s}](\bar{d}s \to \bar{K}^{*0}),$$

$$\sum_{i=u,d,s} s\bar{q}_i q_i \bar{d} = M_{3i}M_{i2} = (M^2)_{32},$$

$$\sum_{i=u,d,s} u\bar{q}_i q_i \bar{s} = M_{1i}M_{i3} = (M^2)_{13},$$

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$

Mechanism of the $D_s^+ \to K_S^0 K_S^0 \pi^+$

□Weakly decay+Hadronization

$$P = \begin{pmatrix} \frac{\eta}{\sqrt{3}} + \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{3}} - \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \frac{\sqrt{6}\eta'}{3} \end{pmatrix},$$

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}.$$

$$(M^2)_{32} \to (V \cdot P)_{32} = \pi^+ K^{*-} - \frac{1}{\sqrt{2}} \pi^0 \bar{K}^{*0},$$

$$(M^2)_{13} \to (P \cdot V)_{13} = \pi^+ K^{*0} + \frac{1}{\sqrt{2}} \pi^0 K^{*+},$$

$$D_s^+ \to V_1 \left(\pi^+ K^{*-} K^{*+} - \frac{1}{\sqrt{2}} \pi^0 \bar{K}^{*0} K^{*+} \right),$$

$$D_s^+ \to V_2 \left(\pi^+ K^{*0} \bar{K}^{*0} + \frac{1}{\sqrt{2}} \pi^0 K^{*+} \bar{K}^{*0} \right).$$

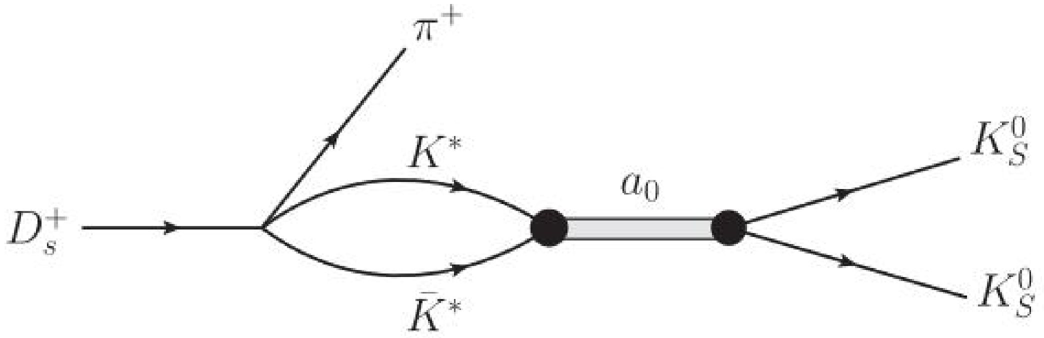
Mechanism of the $D_s^+ \to K_S^0 K_S^0 \pi^+$

☐Final state interaction

$$|K^{*0}\bar{K}^{*0}\rangle = \frac{1}{\sqrt{2}}(|K^*\bar{K}^*, I=1\rangle - |K^*\bar{K}^*, I=0\rangle),$$

$$|K^{*+}K^{*-}\rangle = -\frac{1}{\sqrt{2}}(|K^*\bar{K}^*, I=1\rangle + |K^*\overline{K^*}, I=0\rangle).$$

$$\begin{aligned} &V_1 K^{*+}K^{*-} + V_2 K^{*0}\bar{K}^{*0} \\ &= -\frac{V_1}{\sqrt{2}}(|K^*\bar{K}^*, I=1\rangle + |K^*\bar{K}^*, I=0\rangle) \\ &\quad + \frac{V_2}{\sqrt{2}}(|K^*\bar{K}^*, I=1\rangle - |K^*\bar{K}^*, I=0\rangle) \\ &= \frac{V_2 - V_1}{\sqrt{2}}|K^*\bar{K}^*, I=1\rangle - \frac{V_2 + V_1}{\sqrt{2}}|K^*\bar{K}^*, I=0\rangle. \end{aligned}$$



$$\mathcal{M}_a = \frac{V_2 - V_1}{4}\tilde{G}_{K^*\bar{K}^*}(M_{K_S^0K_S^0}) \times \frac{g_{K^*\bar{K}^*}g_{K\bar{K}}}{M^2_{K_S^0K_S^0} - M^2_{a_0(1710)} + iM_{a_0(1710)}\Gamma_{a_0(1710)}},$$

Set	$M_{a_0(1710)}$	$\Gamma_{a_0(1710)}$	$g_{K^*\bar{K}^*}$	$\Gamma_{K\bar{K}}$
I (Refs. [13,15])	1777	148	$(7525, -i1529)$	36
II (Ref. [28])	1720	200	$(8731, -i2200)$	74

Mechanism of the $D_s^+ \to K_S^0 K_S^0 \pi^+$

□G function

$$\omega(\tilde{m}_1^2) = \frac{1}{N}\mathrm{Im}\left(\frac{1}{\tilde{m}_1^2 - m_{K^*}^2 + i\Gamma(\tilde{m}_1^2)\tilde{m}_1}\right)$$

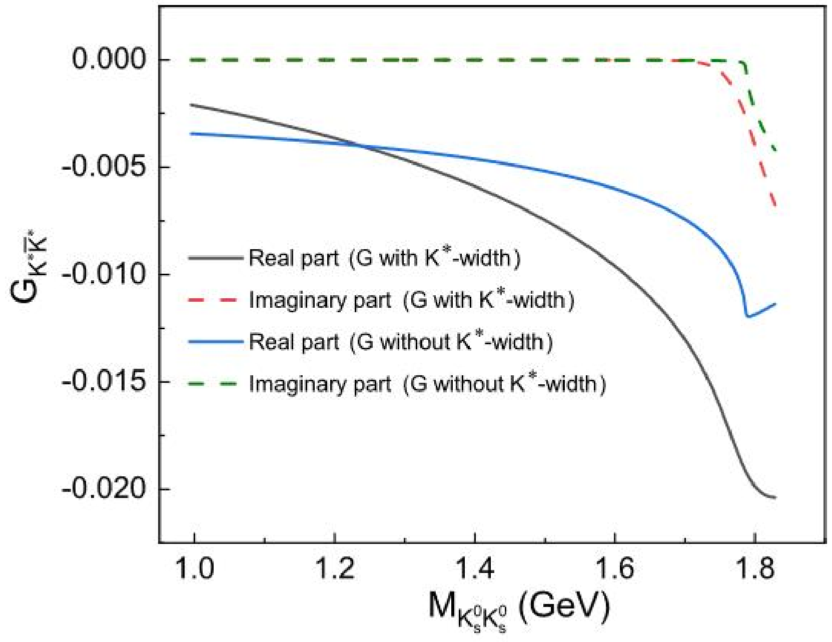
$$G_{K^*\bar{K}^*}(M_{K_S^0K_S^0}) = \int_{m_-^2}^{m_+^2}\int_{m_-^2}^{m_+^2} d\tilde{m}_1{}^2 d\tilde{m}_2^2 \times \omega(\tilde{m}_1^2)\omega(\tilde{m}_2^2)\tilde{G}(M_{K_S^0K_S^0}, \tilde{m}_1^2, \tilde{m}_2^2),$$

$$N = \int_{\tilde{m}_-^2}^{\tilde{m}_+^2} d\tilde{m}_1^2 \mathrm{Im}\left(\frac{1}{\tilde{m}_1^2 - m_{K^*}^2 + i\Gamma(\tilde{m}_1^2)\tilde{m}_1}\right),$$

$$\tilde{G} = \frac{1}{16\pi^2}\left\{a_\mu + \mathrm{In}\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s}\mathrm{In}\frac{m_2^2}{m_1^2} \times \frac{p}{\sqrt{s}}[\mathrm{In}(s - (m_2^2 - m_1^2) + 2p\sqrt{s}) + \mathrm{In}(s + (m_2^2 - m_1^2) + 2p\sqrt{s}) - \mathrm{In}(-s + (m_2^2 - m_1^2) + 2p\sqrt{s}) - \mathrm{In}(-s - (m_2^2 - m_1^2) + 2p\sqrt{s})]\right\}$$

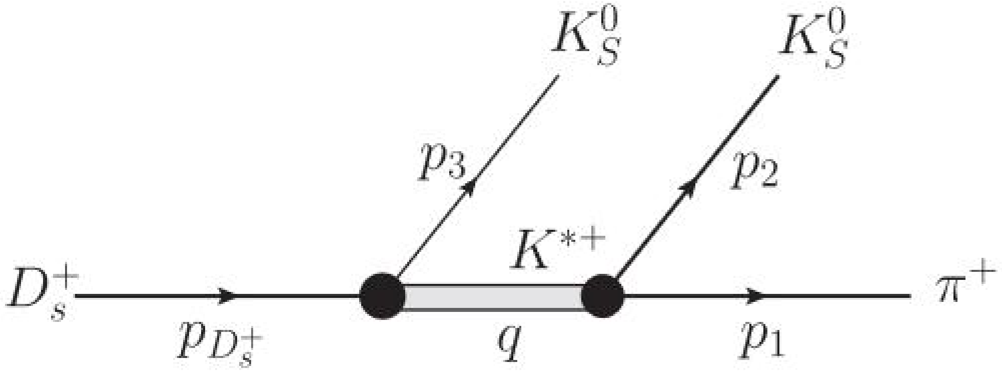
$$\Gamma(\tilde{m}_1^2) = \Gamma_{K^*}\frac{\tilde{k}^3}{k^3},$$

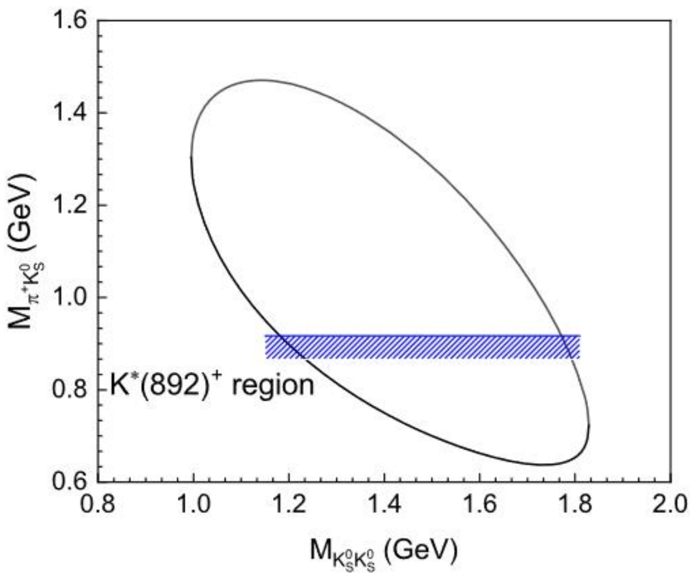
$$\tilde{k} = \frac{\lambda(\tilde{m}_1^2, m_\pi^2, m_K^2)}{2\tilde{m}_1},$$



Mechanism of the $D_s^+ \to K_S^0 K_S^0 \pi^+$

□The contribution of K^*





$$
\begin{aligned}
\mathcal{M}_b = {} & \frac{g_{D_s \bar{K} K^*} g_{K^* K \pi}}{2} \frac{1}{q^2 - m_{K^{*+}}^2 + i m_{K^{*+}} \Gamma_{K^{*+}}} \\
& \times \left[(m_{K_S^0}^2 - m_{\pi^+}^2) \left(1 - \frac{q^2}{m_{K^{*+}}^2} \right) \right. \\
& + 2 p_1 \cdot p_3 \frac{m_{\pi^+}^2 - m_{K_S^0}^2 - m_{K^{*+}}^2}{m_{K^{*+}}^2} \\
& \left. + 2 p_2 \cdot p_3 \frac{m_{\pi^+}^2 - m_{K_S^0}^2 + m_{K^{*+}}^2}{m_{K^{*+}}^2} \right] \\
& + (\text{exchange term with } p_2 \leftrightarrow p_3),
\end{aligned}
$$

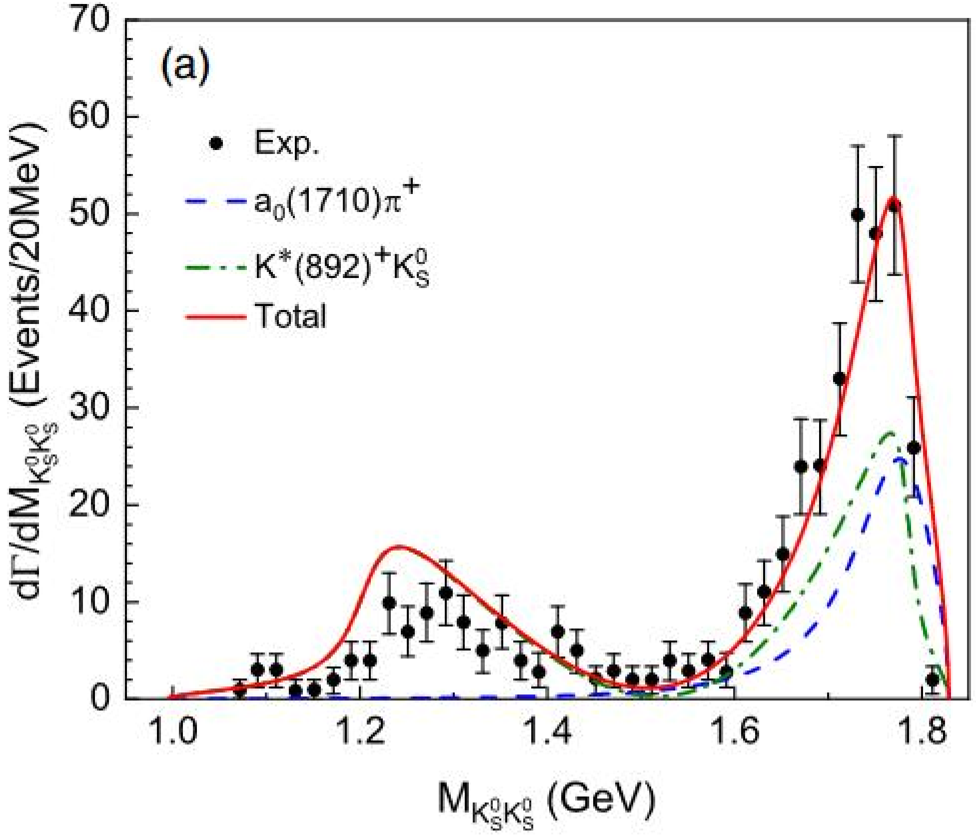
$$
\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b,
$$

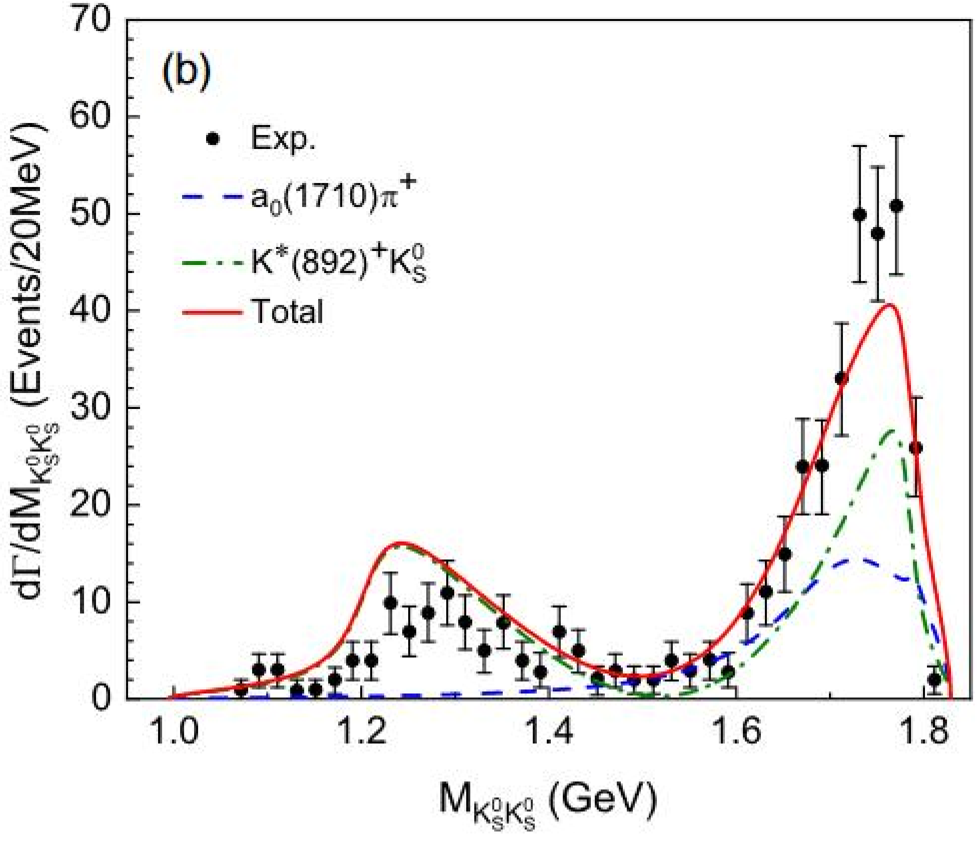
$$
\frac{d^2\Gamma}{dM_{K_S^0 K_S^0} dM_{\pi K_S^0}} = \frac{M_{K_S^0 K_S^0} M_{\pi K_S^0}}{128 \pi^3 m_{D_s^+}^3} (|\mathcal{M}_a|^2 + |\mathcal{M}_b|^2),
$$

Results

□ $D_s^+ \to K_S^0 K_S^0 \pi^+$

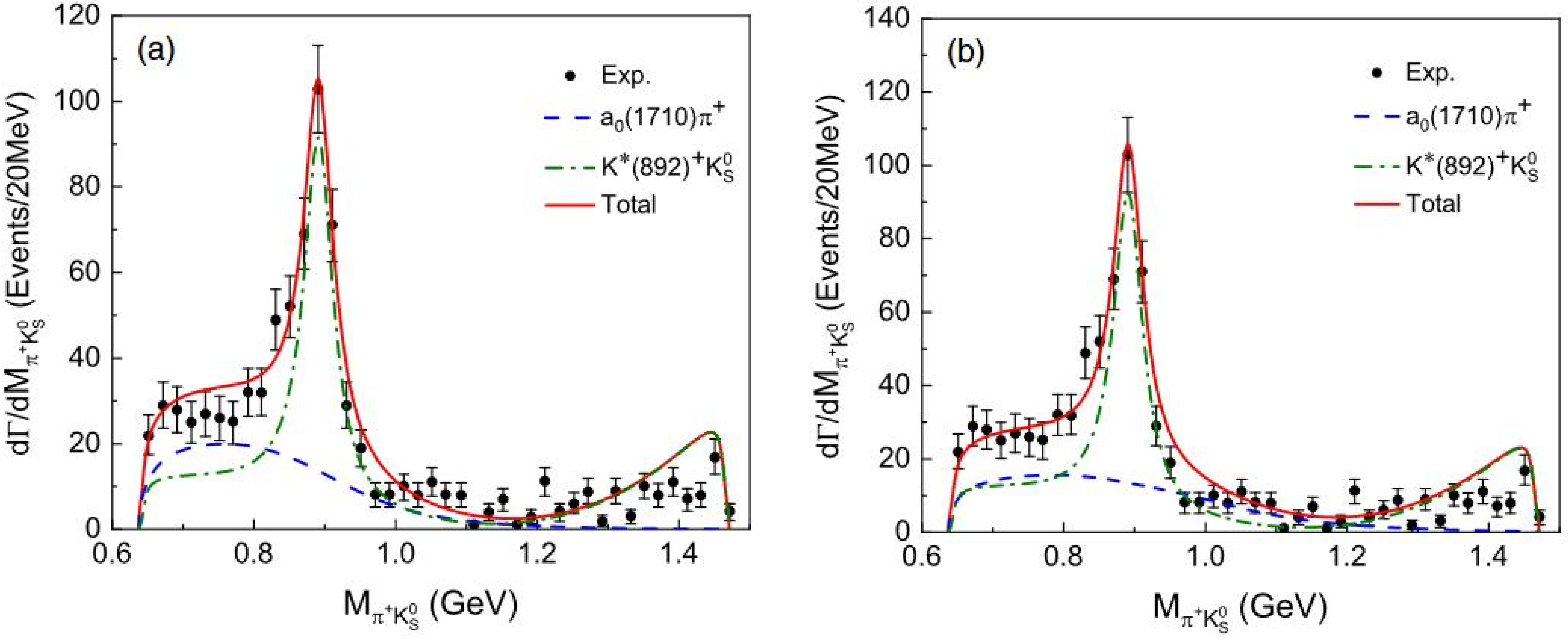
Set	$M_{a_0(1710)}$	$\Gamma_{a_0(1710)}$	$g_{K^*\bar{K}^*}$	$\Gamma_{K\bar{K}}$
I (Refs. [13,15])	1777	148	$(7525, -i1529)$	36
II (Ref. [28])	1720	200	$(8731, -i2200)$	74





Results

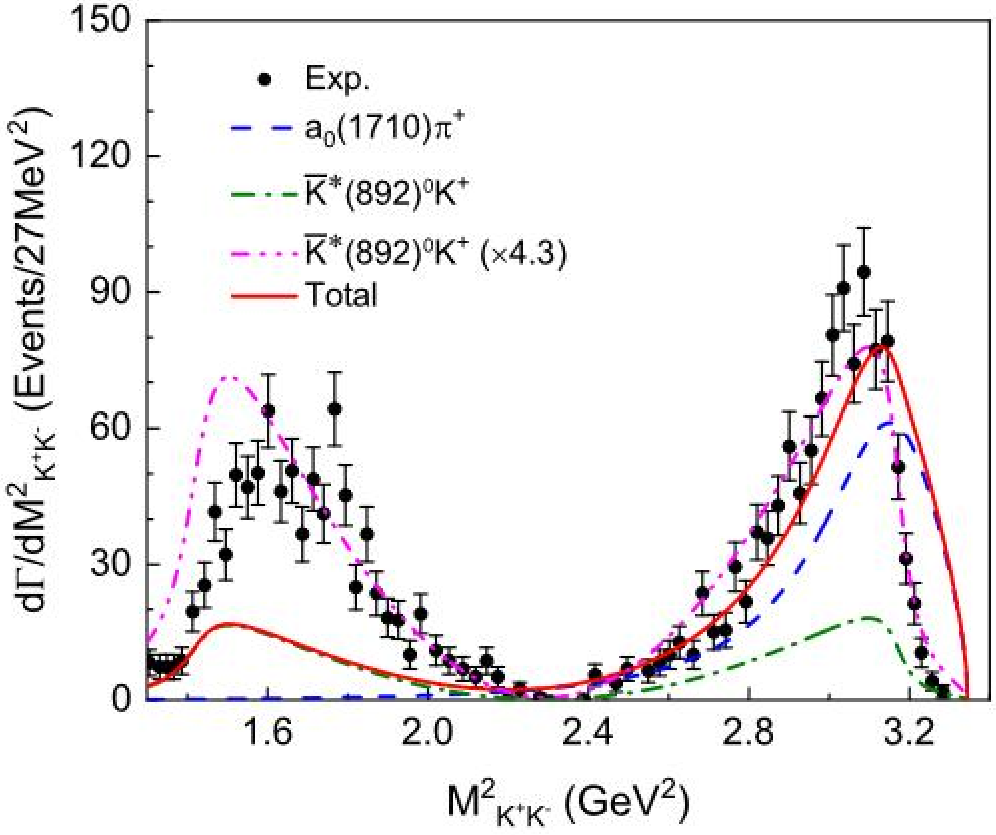
- $D_s^+ \to K_S^0 K_S^0 \pi^+$

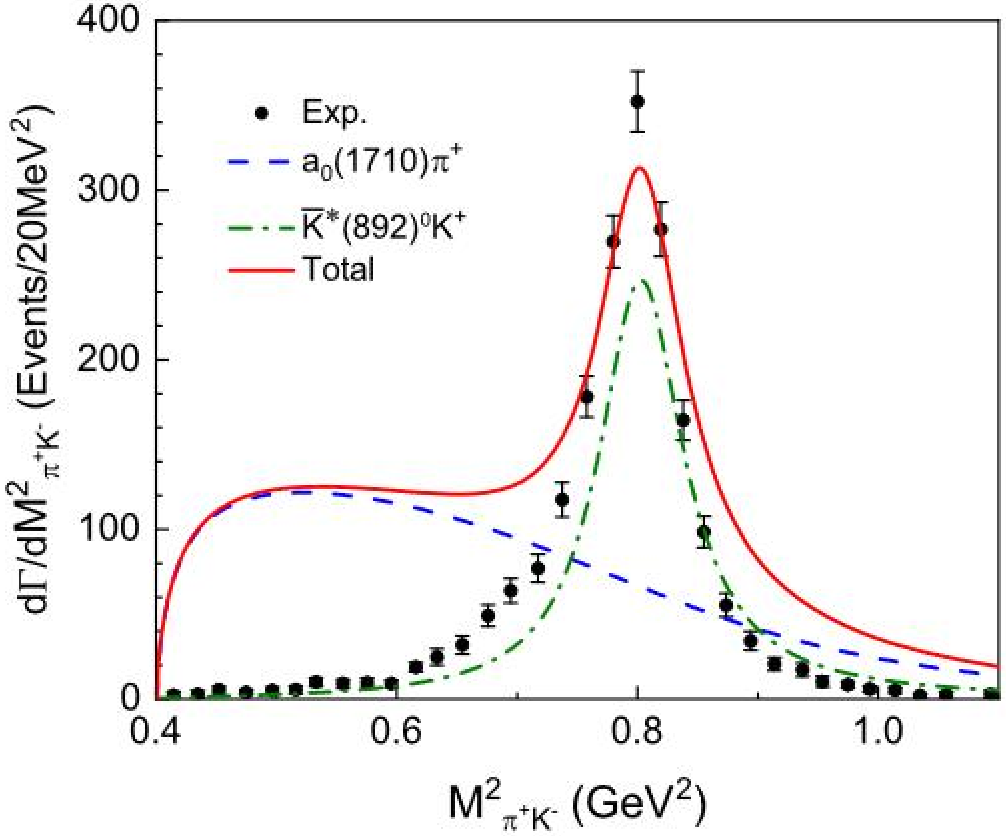


Results

□ $D_s^+ \to K^+K^-\pi^+$

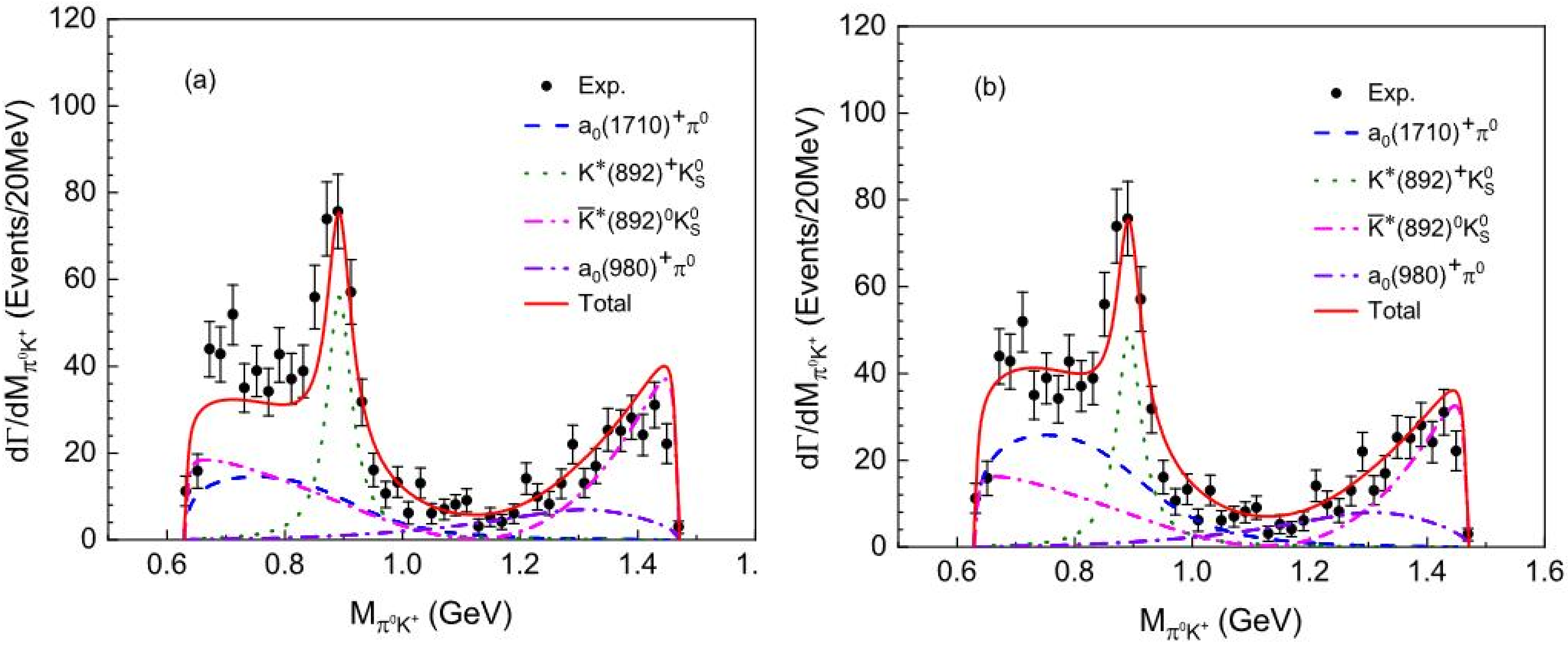
BESIII: PRD104(2021)012016





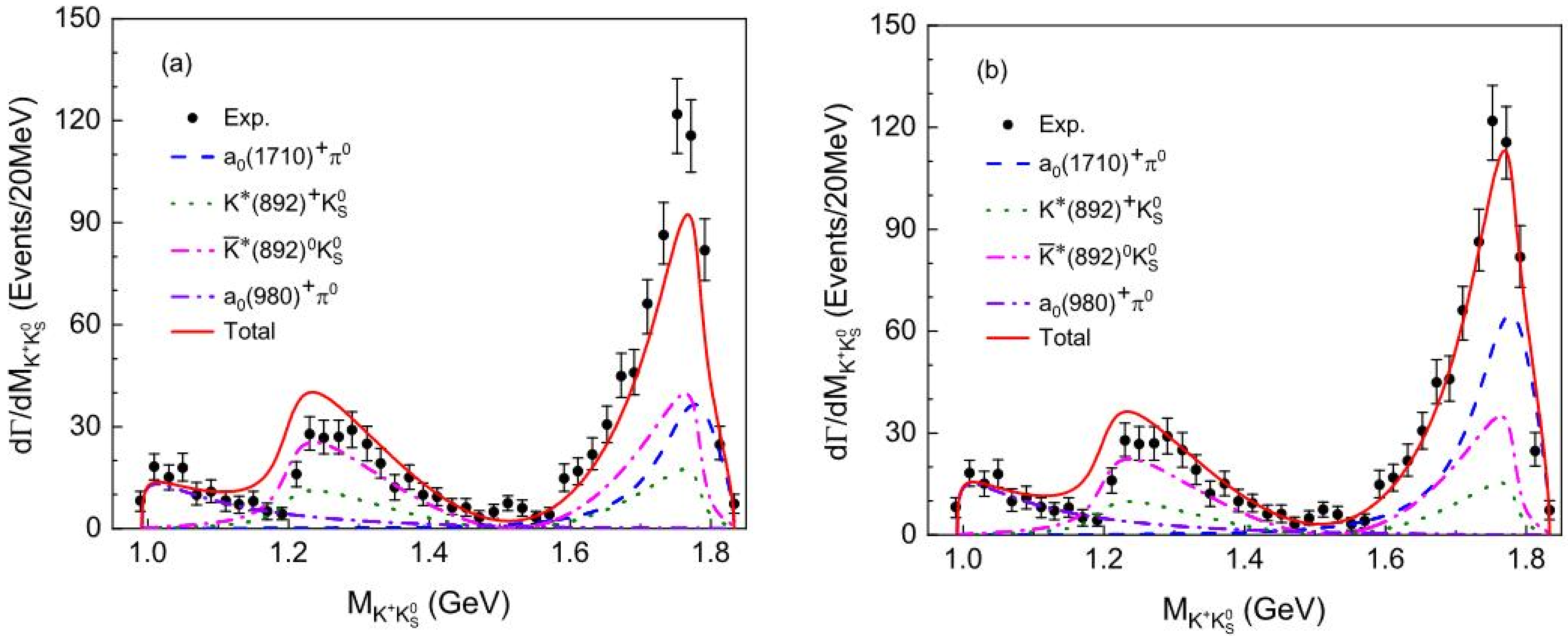
Results

- $D_s^+ \to K_S^0 K^+ \pi^0$



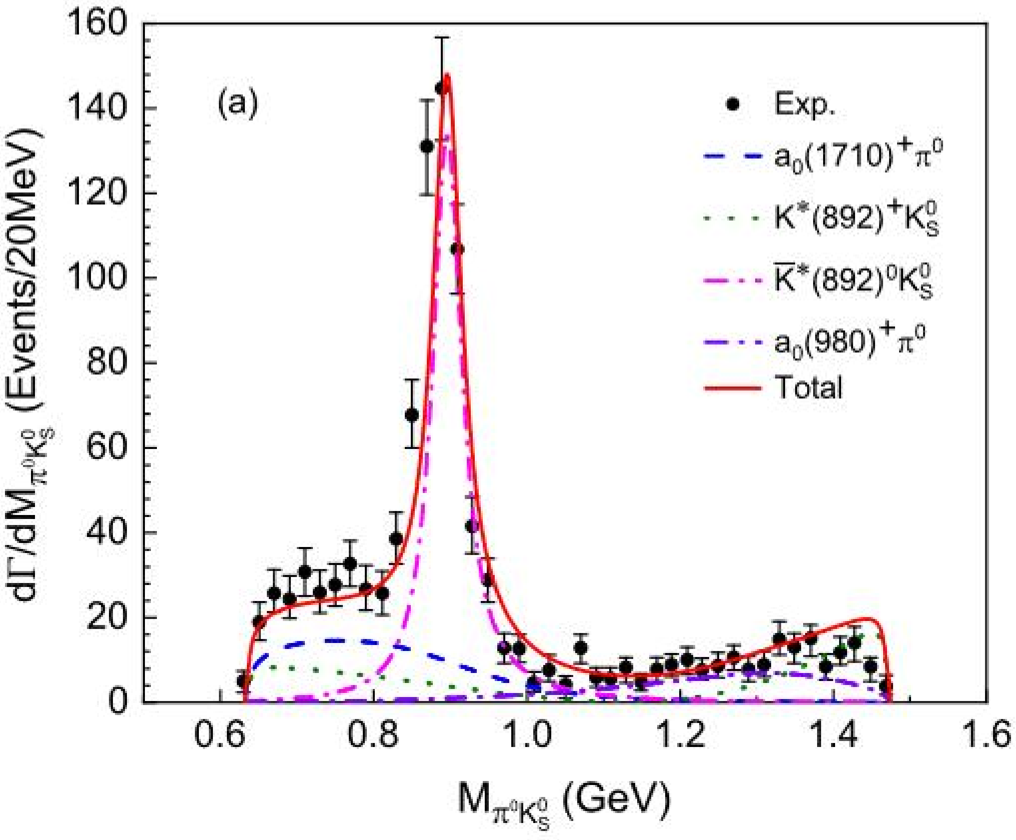
Results

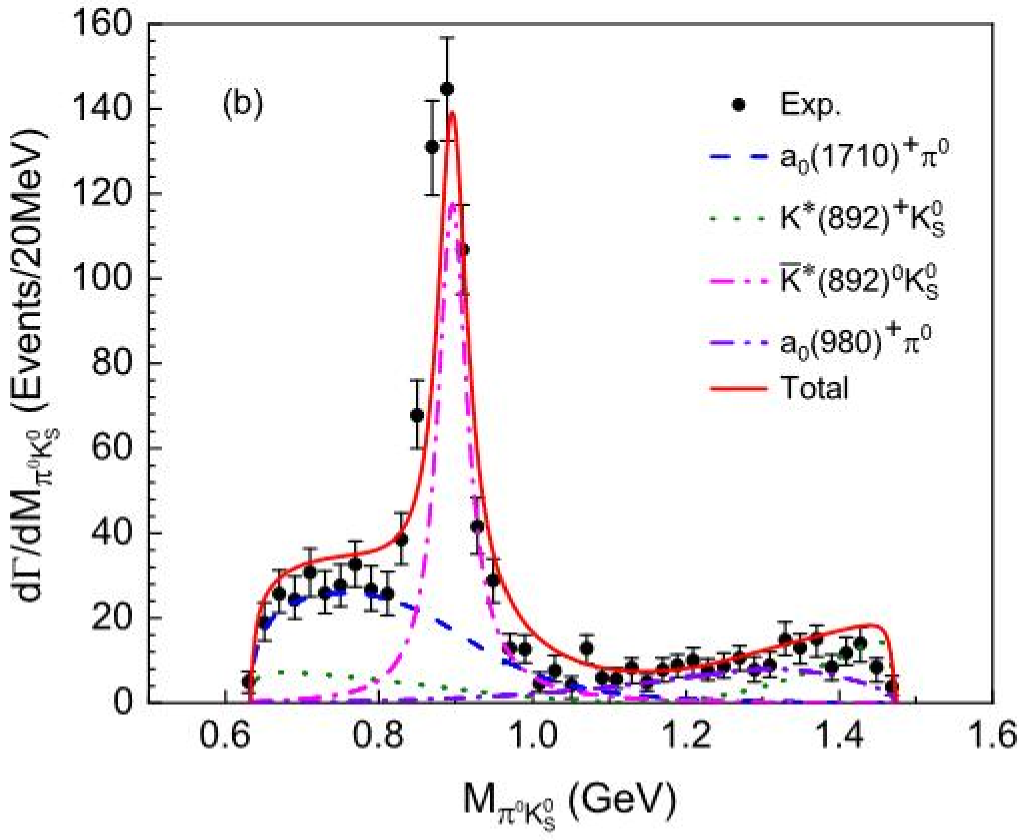
□ $D_s^+ \to K_S^0 K^+ \pi^0$



Results

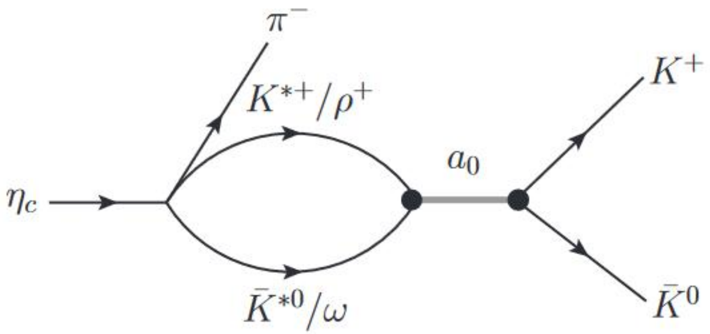
□ $D_s^+ \to K_S^0 K^+ \pi^0$





$a_0(1710)$ in $\eta_c \to KK\pi$

□ $\eta_c \to KK\pi$



$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix},$$

$$P = \begin{pmatrix} \frac{\eta}{\sqrt{3}} + \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{3}} - \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \frac{\sqrt{6}\eta'}{3} \end{pmatrix}$$

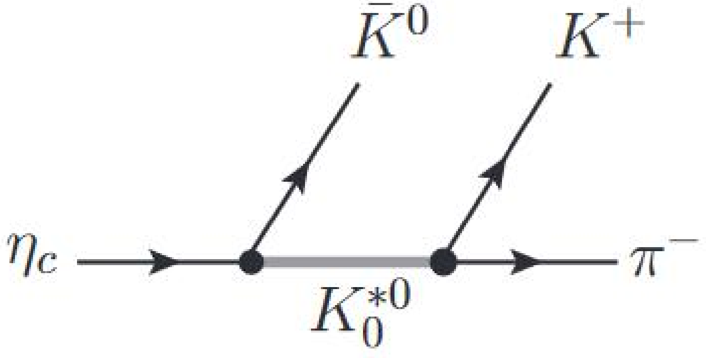
$$\begin{aligned} & <VVP> \\ = \ & (VV)_{12}P_{21} \\ = \ & \pi^- \sum_i V_{1i}V_{i2} \\ = \ & \pi^- \left[\rho^+\left(\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}}\right) + \left(-\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}}\right)\rho^+ + \bar{K}^{*0}K^{*+}\right] \\ = \ & \pi^- \left[\sqrt{2}\rho^+\omega + \bar{K}^{*0}K^{*+}\right]. \end{aligned} \quad (3)$$

$$\begin{aligned} \mathcal{M}_a \ = \ & V_p \times (G_{\bar{K}^{*0}K^{*+}} t_{\bar{K}^{*0}K^{*+}\to\bar{K}^0K^+} \\ & + \sqrt{2}G_{\omega\rho^+} t_{\omega\rho^+\to\bar{K}^0K^+}), \end{aligned}$$

$$\begin{aligned} G_i(M_{\bar{K}^0K^+}) \ = \ & \int_{m_{1-}^2}^{m_{1+}^2} \int_{m_{2-}^2}^{m_{2+}^2} d\tilde{m}_1^2 d\tilde{m}_2^2 \times \\ & \omega(\tilde{m}_1^2)\omega(\tilde{m}_2^2)\tilde{G}(M_{\bar{K}^0K^+}, \tilde{m}_1^2, \tilde{m}_2^2), \end{aligned}$$

$$\begin{aligned} \omega(\tilde{m}_i^2) \ = \ & \frac{1}{N}\mathrm{Im}\left[\frac{1}{\tilde{m}_i^2 - m_{V_i}^2 + i\Gamma(\tilde{m}_i^2)\tilde{m}_i}\right], \\ N \ = \ & \int_{\tilde{m}_{i-}^2}^{\tilde{m}_{i+}^2} d\tilde{m}_i^2 \mathrm{Im}\left[\frac{1}{\tilde{m}_i^2 - m_{V_i}^2 + i\Gamma(\tilde{m}_i^2)\tilde{m}_i}\right], \end{aligned}$$

□$K_0(1430)$



$$\mathcal{L} = g_{\eta_c K K_0^*} \eta_c K K_0^*$$
$$\mathcal{L} = g_{K_0^* K\pi} K_0^* K \pi.$$

$$\Gamma_{\eta_c \to K_0^* K} = \frac{g^2_{\eta_c K_0^* K}}{8\pi} \frac{|\mathbf{P}|}{m^2_{\eta_c}},$$
$$\Gamma_{K_0^* \to K\pi} = \frac{g^2_{K_0^* K\pi}}{8\pi} \frac{|\mathbf{P}|}{m^2_{K_0^*}},$$

$$\mathcal{M}_b = \frac{g_{\eta_c K^+ K_0^{*-}} g_{K_0^{*-} \bar{K}^0 \pi^-}}{M^2_{\bar{K}^0\pi^-} - M^2_{K_0^*} + i M_{K_0^*} \Gamma_{K_0^*}},$$

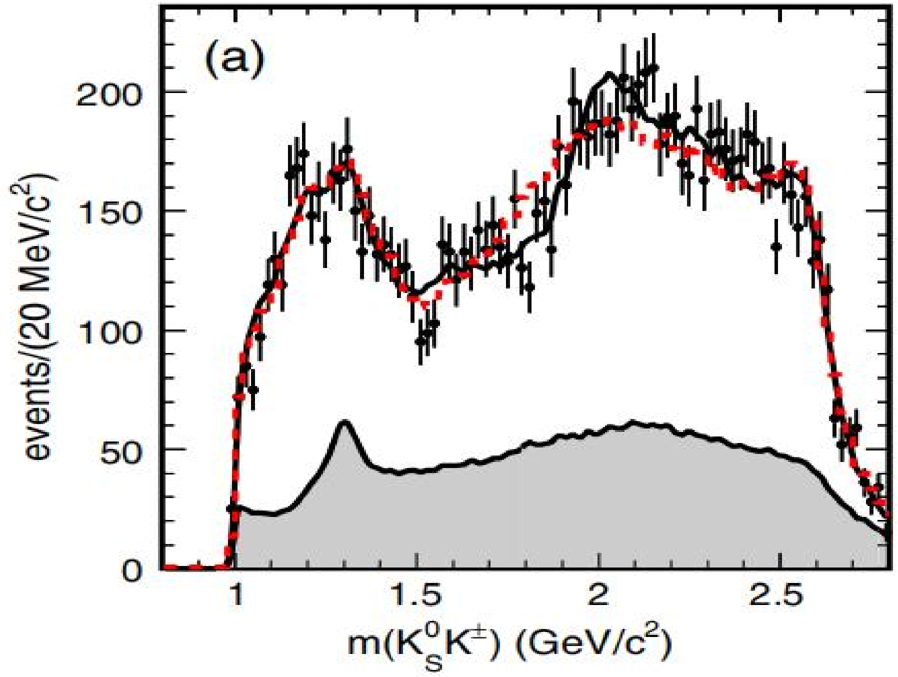
$$\mathcal{M}_c = \frac{g_{\eta_c \bar{K}^0 K_0^{*0}} g_{K_0^{*0} K^+ \pi^-}}{M^2_{K^+\pi^-} - M^2_{K_0^*} + i M_{K_0^*} \Gamma_{K_0^*}},$$

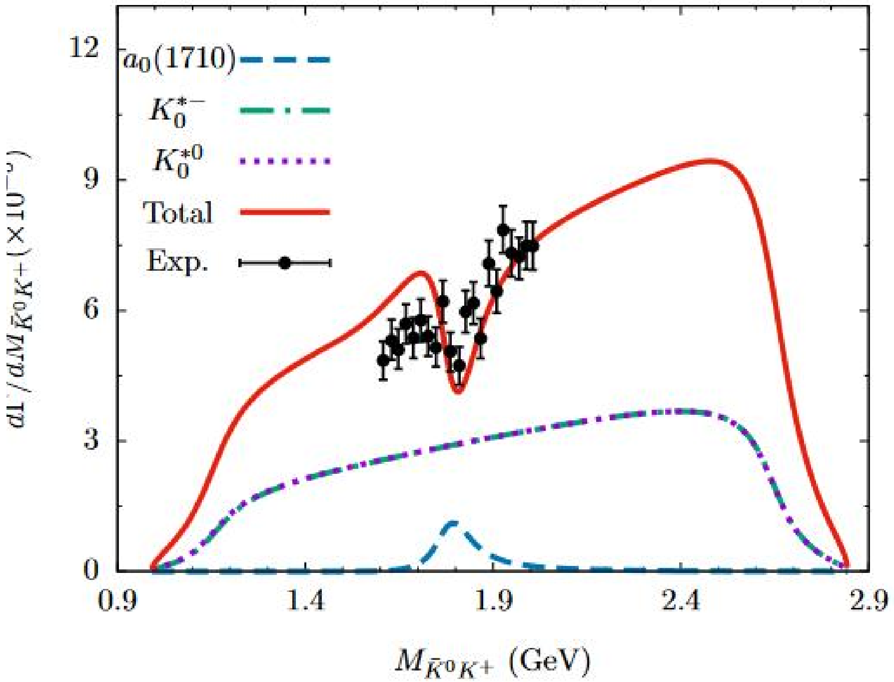
$$\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c,$$

$$\frac{d^2\Gamma}{dM_{\bar{K}^0K^+} dM_{K^+\pi^-}} = \frac{M_{\bar{K}^0K^+} M_{K^+\pi^-}}{128\pi^3 m^3_{\eta_c}} |\mathcal{M}|^2,$$
$$\frac{d^2\Gamma}{dM_{\bar{K}^0K^+} dM_{\bar{K}^0\pi^-}} = \frac{M_{\bar{K}^0K^+} M_{\bar{K}^0\pi^-}}{128\pi^3 m^3_{\eta_c}} |\mathcal{M}|^2.$$

Results

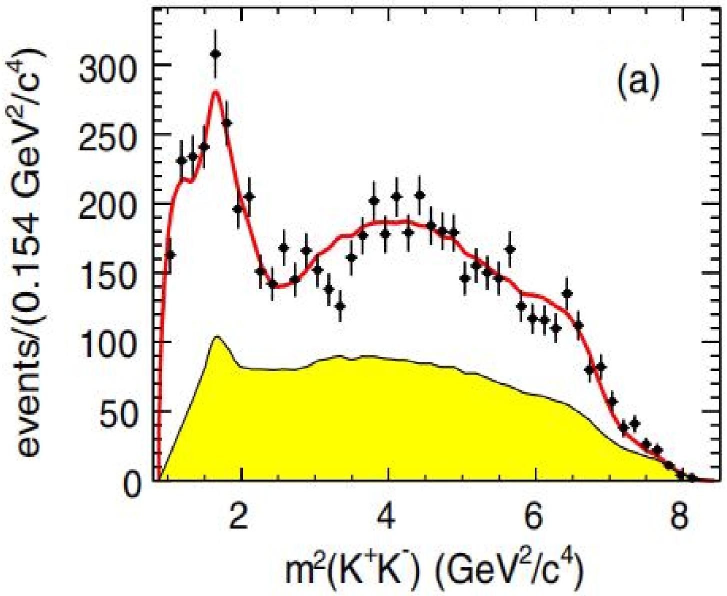
- $\overline{K}_s^0 K^{\pm}$ invariant mass distribution

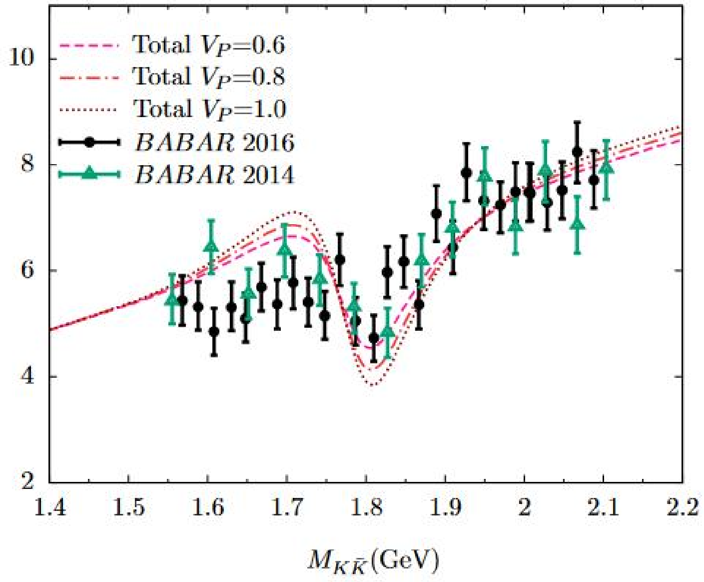


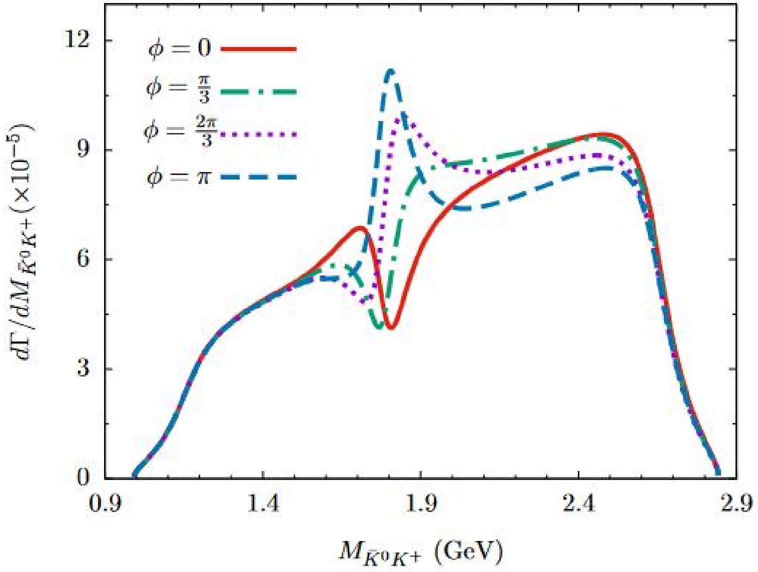


BABAR: PRD 93(2016)012005

- $\eta_c \rightarrow K^+K^-\pi$^0



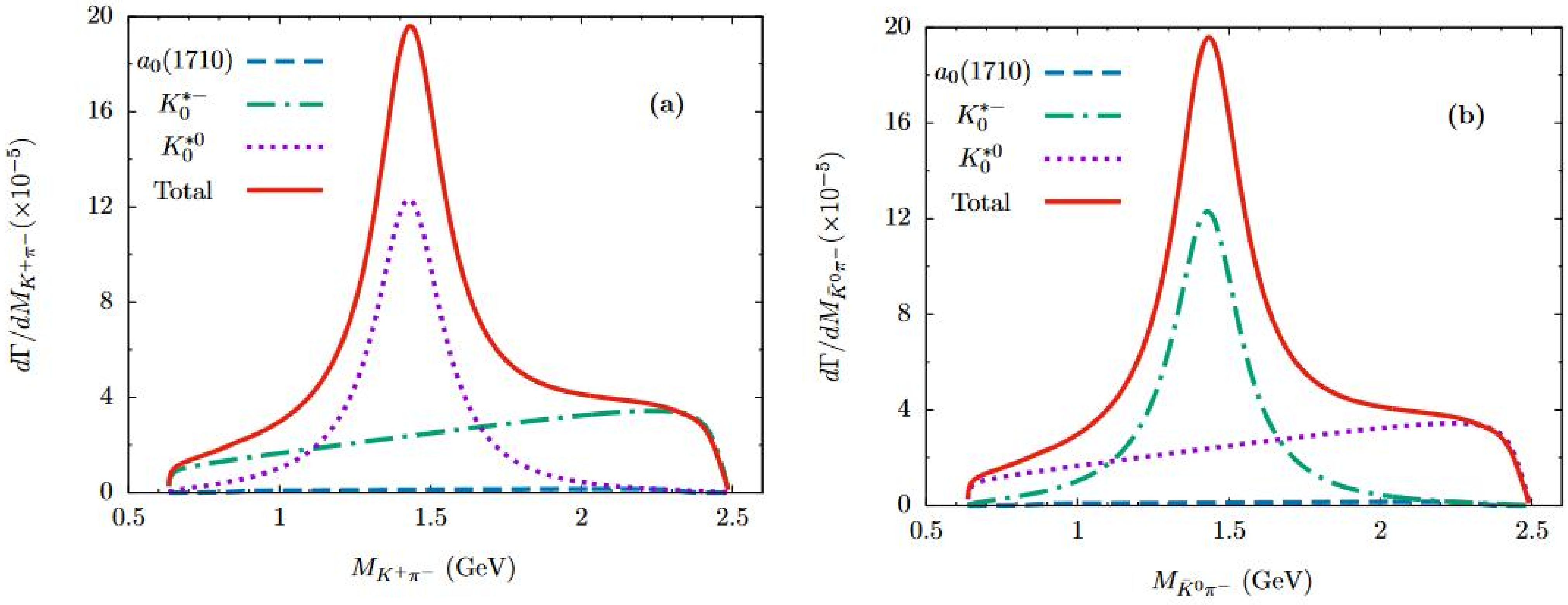




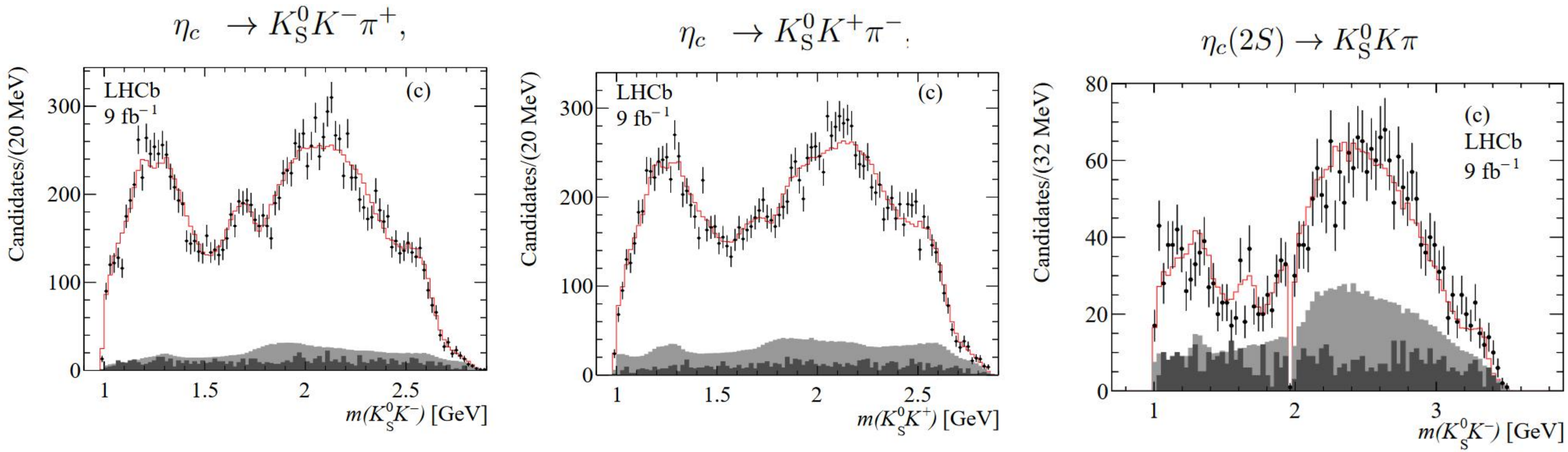
Belle: PRD89(2014)112004

Results

- $K^{\pm}\pi^{-}$ invariant mass distribution



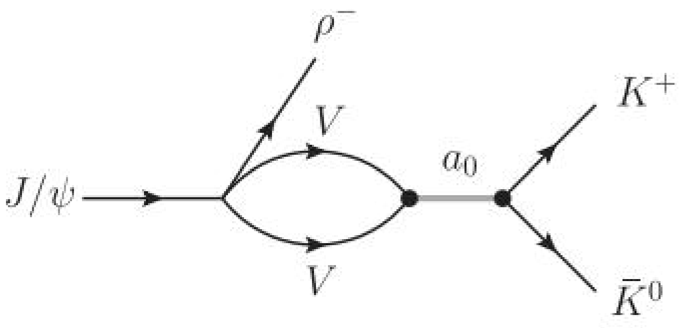
LHCb measurements 2304.14891

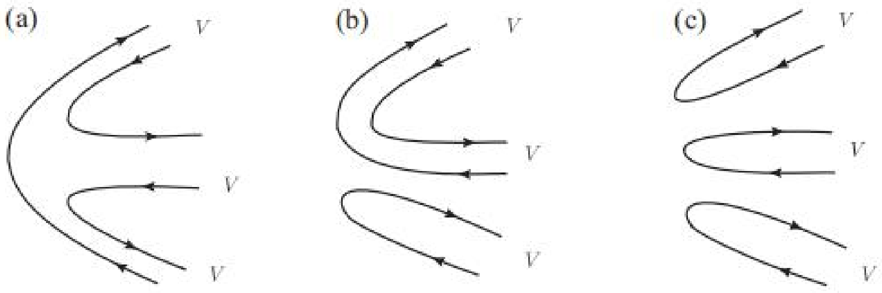


Resonance	Mass [MeV]	Γ [MeV]	Δ(2 log $\mathcal{L}$)	Significance
$K_0^*(1430)$	$1493 \pm 4 \pm 7$	$215 \pm 7 \pm 4$	-	-
$K_0^*(1950)$	$1980 \pm 14 \pm 19$	$229 \pm 26 \pm 16$	316	17.8σ
$a_0(1700)$	$1736 \pm 10 \pm 12$	$134 \pm 17 \pm 61$	161	12.7σ
$\kappa(2600)$	$2662 \pm 59 \pm 201$	$480 \pm 47 \pm 72$	1338	36.6σ

$a_0(1710)$ in $J/\psi \to \bar{K}^0 K^+ \rho^-$

- **Reaction mechanism**





$\langle VVV \rangle$, $\langle VV \rangle\langle V \rangle$, $\langle V \rangle\langle V \rangle\langle V \rangle$,

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}.$$

$$\langle VVV \rangle:\ \alpha \times \left[\frac{\rho^+\rho^0}{\sqrt{2}} + 3\sqrt{2}\omega\rho^+ + 3\bar{K}^{*0}K^{*+}\right]\rho^-, \qquad \beta/\alpha = 0.32$$

PRD79(2009)074009

$$\langle VV \rangle\langle V \rangle:\ \beta \times [2\sqrt{2}\omega\rho^+ + 2\phi\rho^+]\rho^-.$$

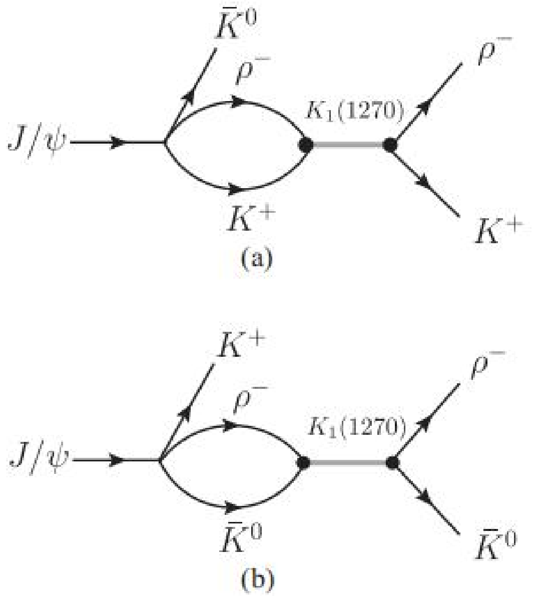
$$\begin{aligned} \mathcal{M}_a = V_p \times [&3\alpha G_{\bar{K}^{*0}K^{*+}} t_{\bar{K}^{*0}K^{*+} \to \bar{K}^0 K^+} \\ &+ (2\sqrt{2}\beta + 3\sqrt{2}\alpha) G_{\omega\rho^+} t_{\omega\rho^+ \to \bar{K}^0 K^+} \\ &+ 2\beta G_{\phi\rho^+} t_{\phi\rho^+ \to \bar{K}^0 K^+}], \end{aligned}$$

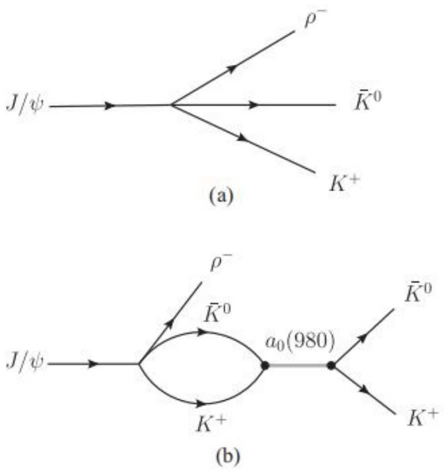
$$\begin{aligned} G_i(M_{\bar{K}^0 K^+}) = &\int_{m_{1-}^2}^{m_{1+}^2} \int_{m_{2-}^2}^{m_{2+}^2} d\tilde{m}_1^2 d\tilde{m}_2^2 \\ &\times \omega(\tilde{m}_1^2)\omega(\tilde{m}_2^2)\tilde{G}(M_{\bar{K}^0 K^+}, \tilde{m}_1^2, \tilde{m}_2^2), \end{aligned}$$

$$t_{i \to \bar{K}^0 K^+} = \frac{g_i \times g_{K\bar{K}}}{M_{\bar{K}^0 K^+}^2 - M_{a_0(1710)}^2 + i M_{a_0(1710)} \Gamma_{a_0(1710)}},$$

$a_0(1710)$ in $J/\psi \to \bar{K}^0 K^+ \rho^-$

- $\boldsymbol{K_1(1270)}$ & $\boldsymbol{a_0(980)}$





$$\mathcal{M}_b = V'_p \times G_{K^+\rho^-} t_{K^+\rho^- \to K^+\rho^-},$$

$$t_{K^+\rho^- \to K^+\rho^-} = \frac{g_{K^+\rho^-} g_{K^+\rho^-}}{M^2_{K^+\rho^-} - M^2_{K_1} + iM_{K_1}\Gamma_{K_1}},$$

PRD75(2007)014017

TABLE IV. Pole positions and coupling constants of the two poles of the $K_1(1270)$ [46]. All values are in units of MeV.

	First pole	Second pole
Pole position $\sqrt{s_0}$	$1195 - i123$	$1284 - i73$
$g_{K\rho}$	$-1671 + i1599$	$4804 + i395$

$$\mathcal{M}_d = V'_p[1 + G_{K\bar{K}} t_{\bar{K}^0K^+ \to \bar{K}^0K^+}],$$

$$T = [1 - VG]^{-1} V,$$

PLB821(2021)136617

$$V_{K\bar{K}\to K\bar{K}} = -\frac{1}{4f^2} s,$$

$$V_{K\bar{K}\to \pi\eta} = \frac{\sqrt{6}}{12f^2}\left(3s - \frac{8}{3}m_K^2 - \frac{1}{3}m_\pi^2 - m_\eta^2\right),$$

$$V_{\pi\eta\to K\bar{K}} = V_{K\bar{K}\to\pi\eta},$$

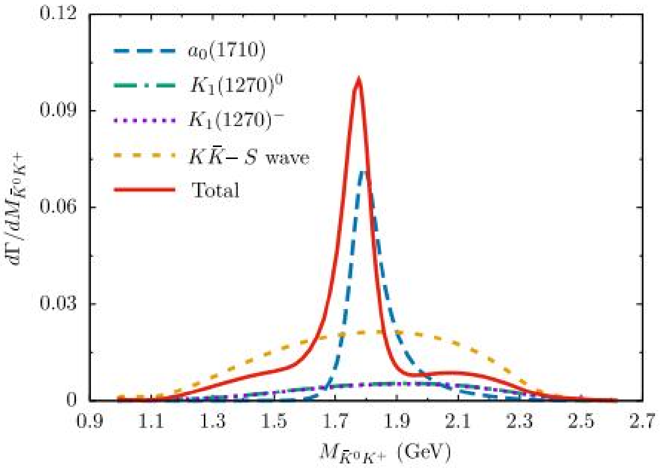
$$V_{\pi\eta\to\pi\eta} = -\frac{1}{3f^2} m_\pi^2,$$

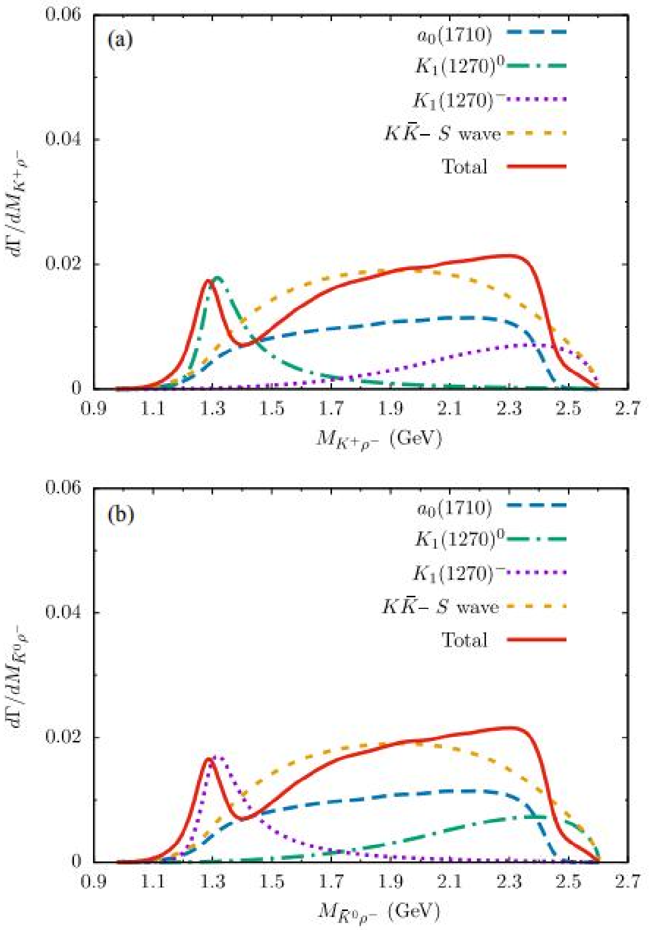
$$\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d,$$

$$\frac{d^2\Gamma}{dM_{\bar{K}^0K^+} dM_{K^+\rho^-}} = \frac{M_{\bar{K}^0K^+} M_{K^+\rho^-}}{128\pi^3 m^3_{J/\psi}} |\mathcal{M}|^2,$$

$$\frac{d^2\Gamma}{dM_{\bar{K}^0K^+} dM_{\bar{K}^0\rho^-}} = \frac{M_{\bar{K}^0K^+} M_{\bar{K}^0\rho^-}}{128\pi^3 m^3_{J/\psi}} |\mathcal{M}|^2.$$

Results





BABAR: PRD95(2017)072005

$$\mathcal{B}(J/\psi \to K_S^0 K^\pm \rho^\mp) =$$

$$(1.87 \pm 0.18 \pm 0.34) \times 10^{-3}$$

- BESIII

$$(10.09 \pm 0.04) \times 10^9 \ J/\psi$$

- STCF per year

$$3.4 \times 10^{12} \ J/\psi$$

Front. Phys. 19, 14701 (2024).

Summary

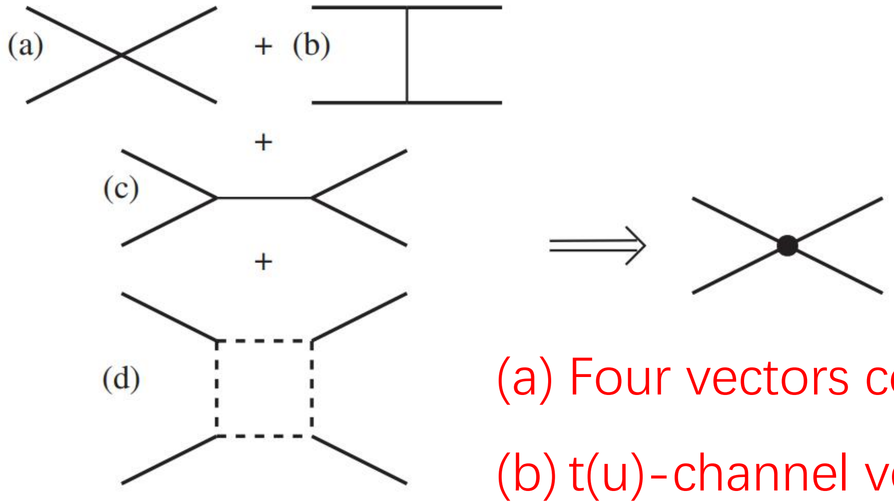
- Our results are in good agreement with the BESIII measurements, which supports the $K^*\bar{K}^*$ molecule of $a_0(1710)$.
- The $a_0(1710)$ mass and width are crucial to understand its internal structure.
- Precise experimental measurements are necessary.

致谢：
国家自然科学基金重大项目课题

Thank you very much!

VV interaction from hidden-gauge lagrangians

□Tree level transition amplitudes



(a) Four vectors contact

(b) t(u)-channel vector exchange

} Most important! Provide attraction!

(c) S-channel vector exchange Basically p-wave, neglected!

(d) Box diagram

- Provides decays to two pseudoscalars;
- Contributes only to spin 0 and 2;
- Real part small, only imaginary part considered!