QCD临界点附近的动力学研究

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STAR区域研讨会

◆会议时间◆

2024年10月10日–10月15日

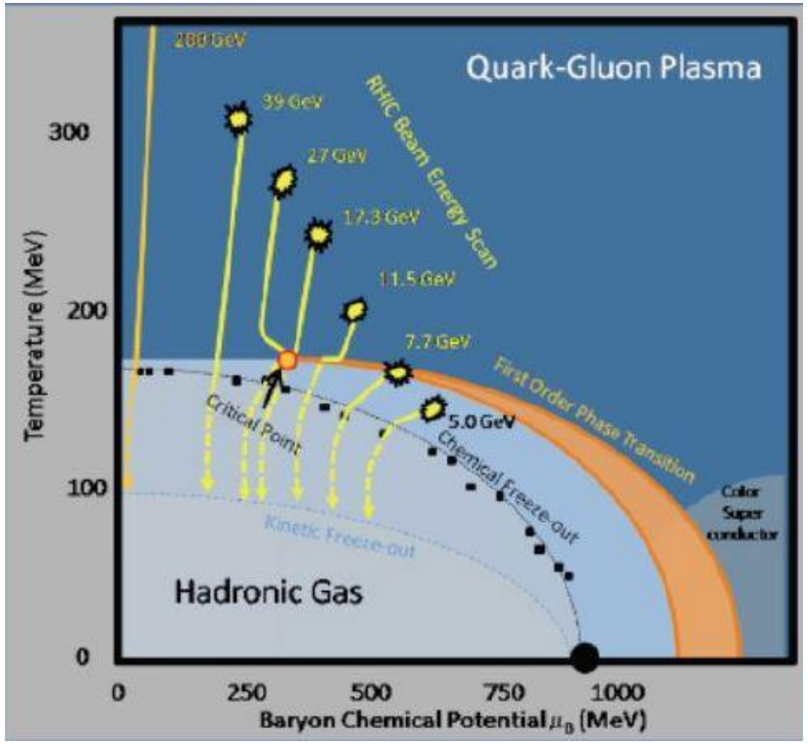
QCD phase diagram

- **Lattice QCD** (small μ_B finite T):
  - Crossover
- **Effective models** (large μ_B)
  - 1st order phase transition

→ Critical point

- Lattice QCD: sign problem at large μ_B
- Effective models: parameters dependent

→ **Heavy-ion collisions :**

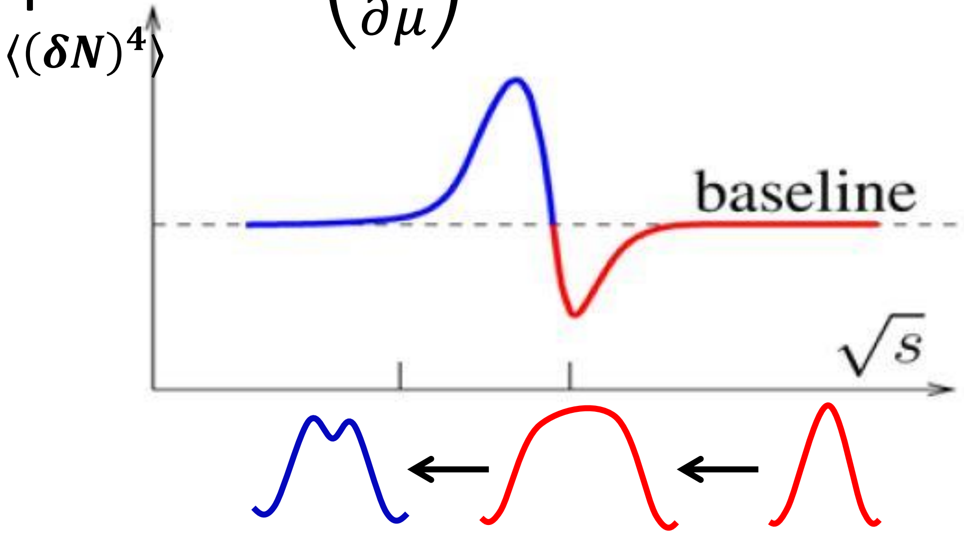


  - tuning $\sqrt{s_{NN}}$,mapping $T-\mu$ phase diagram: RHIC(BES),NICA,FAIR,J_PARC,HIAF….

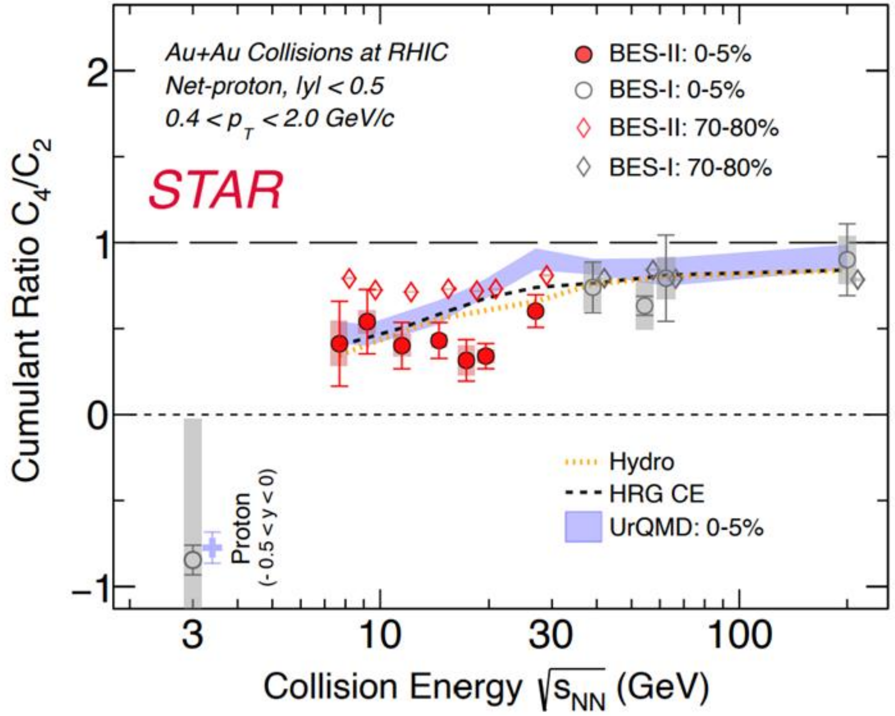
Net-proton fluctuations near critical point

- Characteristic feature of critical point:
  - long range correlation
  - large fluctuations
- **Non-monotonicity** of Net-Proton Cumulant

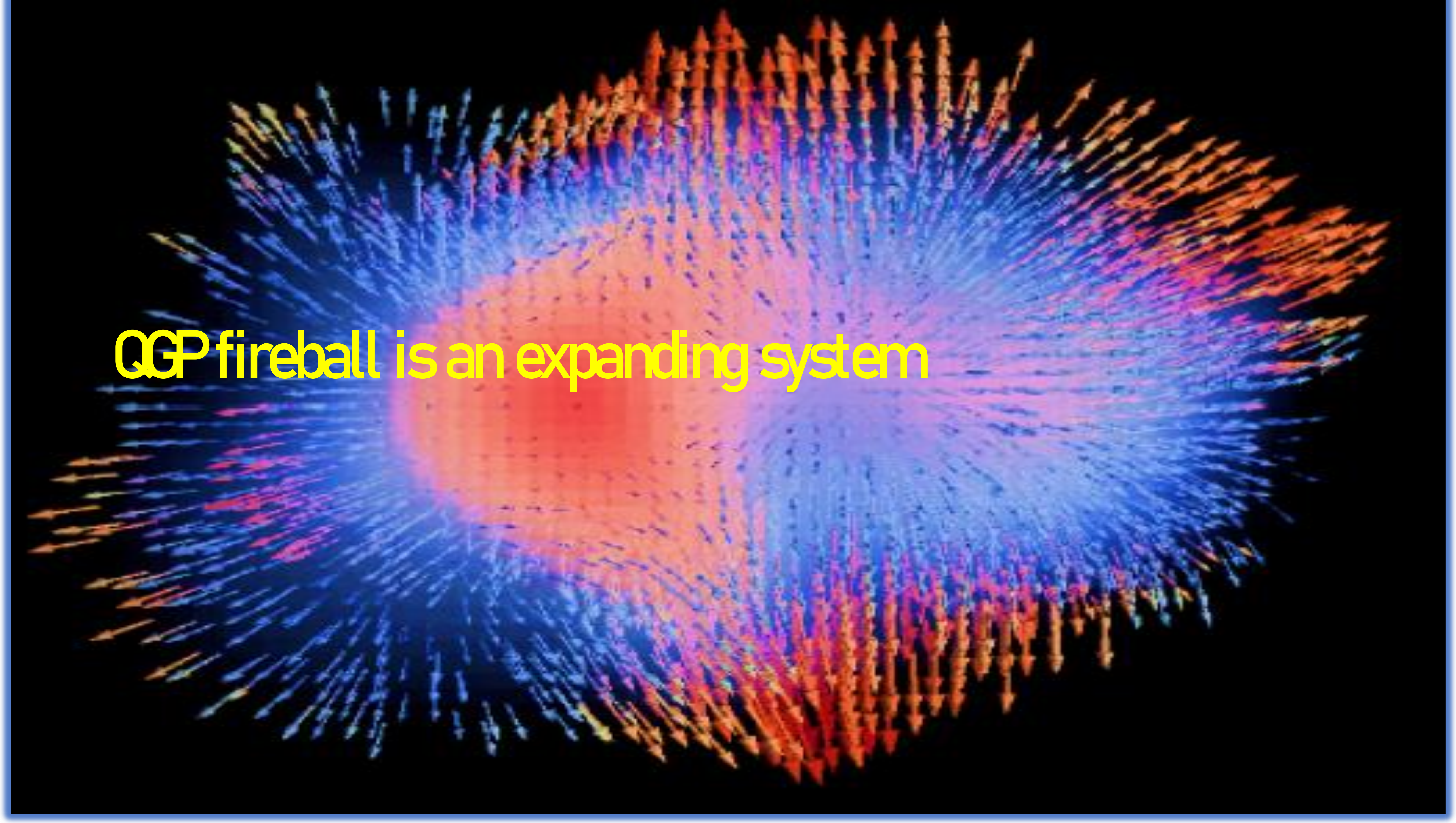
$\sim$ susceptibilities $\left(\frac{\partial}{\partial\mu}\right)^n \ln Z$



M.Stephanov, PRL 107,052301



STAR, CPOD 2024


QGP fireball is an expanding system

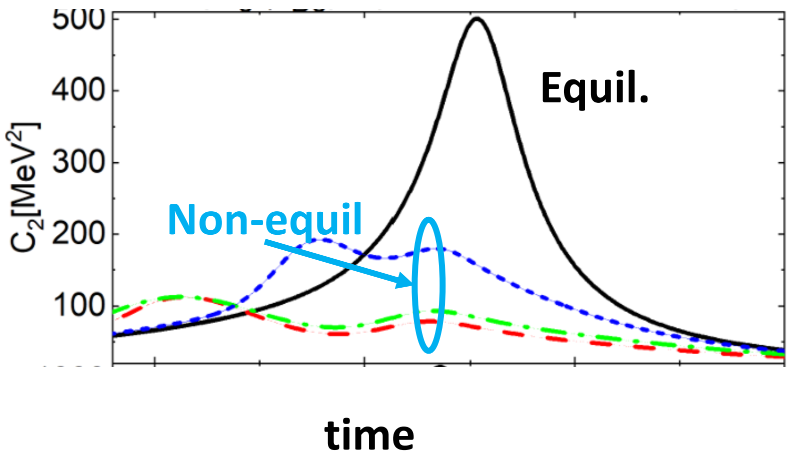
Dynamical effects modifies the critical fluctuations

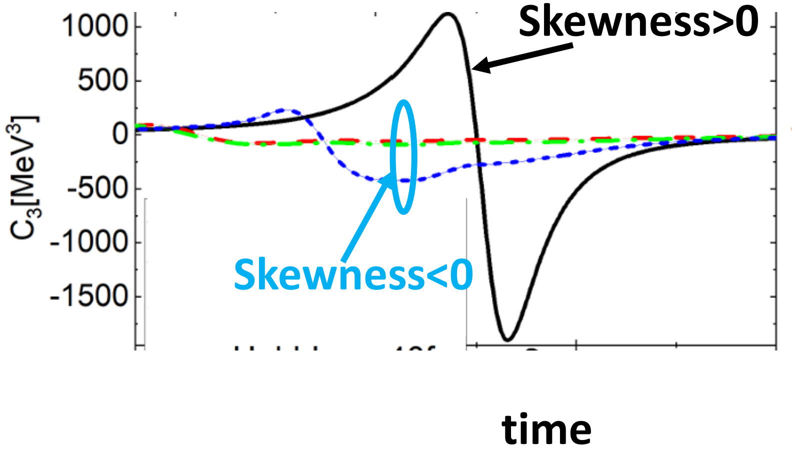
- Expanding QGP fireball forces the fluid cell swipes the critical regions with finite time τ_{expand};
- Long range correlation in the system near the critical point requires long time to correlate with each other $\tau_{relax} \sim \xi^{z}$;
- In heavy-ion collision, near the QCD critical point,

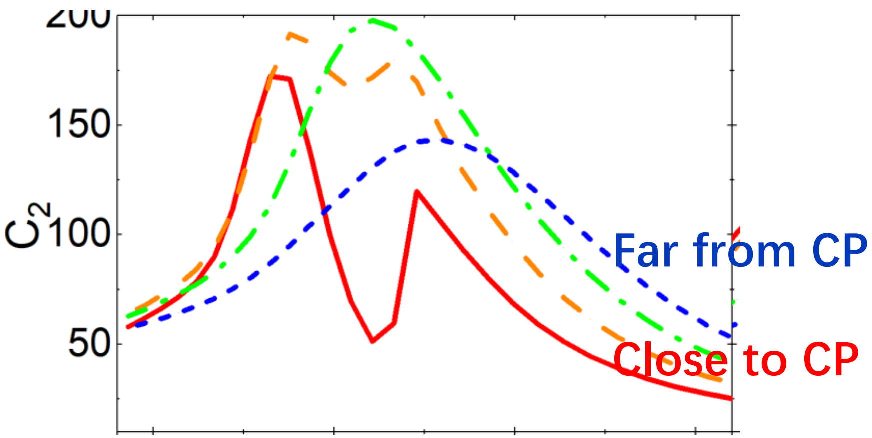
Shian Tang, Shanjin Wu, Huichao Song, PRC(202

$$\tau_{relax} \gg \tau_{expand}$$

- Expansion drives the system has no enough time to relax to equilibrium







Suppression of the fluctuations

Reverse the sign of skewness

Largest $C_2 \neq$ closet to CP

Dynamical models near QCD critical point

- **QCD critical point belongs to model H** (Symmetry and dynamical analysis)**:** D.T.Son,Stephanov(04)
- **Model H** (conserved baryon $\mathbf{n} \approx \boldsymbol{n} + \boldsymbol{\sigma}$ + transverse momentum density)**:**

$$\frac{\partial \boldsymbol{n}}{\partial \boldsymbol{t}} = \boldsymbol{\Gamma}\left[\mathbf{n}, \boldsymbol{\pi}^{\boldsymbol{T}}\right] + \mathbf{noise}, \qquad \frac{\partial \boldsymbol{\pi}^{\boldsymbol{T}}}{\partial \boldsymbol{t}} = \boldsymbol{\eta}\left[\boldsymbol{n}, \boldsymbol{\pi}^{\boldsymbol{T}}\right] + \boldsymbol{noise}$$

Numerical simulation is time consuming.

- **Simplified models:**
    - **Model A** (order parameter field) S.Mukherjee et al15’ 16’, L.Jiang et al17’, S.Wu et al 19’, S.Tang et al 23’,

$$\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{t}} = \boldsymbol{\Gamma}[\boldsymbol{\sigma}\ ] + \mathbf{noise}$$

    - **Model B** (conserved field) M.Sakaida et al 17’, S.Wu et al 19’, M.Nahrgang et al 19’, G.Pihan et al 22’…

$$\frac{\partial \boldsymbol{n}}{\partial \boldsymbol{t}} = \boldsymbol{\nabla}^{2}\boldsymbol{\Gamma}[\boldsymbol{n}\ ] + \mathbf{noise}$$

Dynamical models near QCD critical point

More efficient modeling in heavy-ion collisions: **Hydrodynamics + Critical fluctuations**

- **Non-equilibrium chiral hydrodynamics** M. Nahrgang et al 11'12'14'16'19'

$$\partial_\mu T^{\mu\nu}_{fluid} = \partial_\mu \mathrm{T}^{\mu\nu}_{\sigma}, \qquad \frac{\partial\sigma}{\partial t} = \Gamma[\sigma] + noise$$

- **Fluctuating hydrodynamics** J.Kapusta et al 12',12', K.Murase et al 13', X.An et al 19',21'…

$$\partial_\mu T^{\mu\nu}_{fluid} = noise \;, \partial_\mu N^\mu = noise$$

- **Hydro+, hydro++…** (hydro + slow modes) M. Stephanov et al 18'19'20', N. Abbasi et al 22', L. Du et al 20',……

$$\partial_\mu T^{\mu\nu}_{*} = 0, \partial_\mu N^\mu = 0, \qquad \frac{\partial\phi}{\partial t} = \Gamma[\phi]$$

- **Hydro-kinetics** D.Teaney et al 17'18'19'22'…

$$\partial_\mu T^{\mu\nu} = 0, \partial_\mu N^\mu = 0, \quad \partial_t(noise\ correlator) = \#$$

See reviews: e.g. Shanjiu Wu, et al.,2104.13250; Lipei Du et al. 2402.10183; Xin An et al., 2108.13867; Marcus Bluhm et al., 2001.08831; Adam Bzdak et al.,1906.00936;M.Asakawa et al.,1512.05308

Dynamical models near QCD critical point

- **QCD critical point belongs to model H** (Symmetry and dynamical analysis)**:** D.T.Son,Stephanov(04)
- **Model H** (conserved baryon $\mathbf{n} \approx \boldsymbol{n} + \boldsymbol{\sigma}$ + transverse momentum density)**:**

$$\frac{\partial n}{\partial t} = \Gamma[\mathrm{n}, \pi^T] + \mathrm{noise}, \qquad \frac{\partial \pi^T}{\partial t} = \eta[n, \pi^T] + noise$$

Numerical simulation is time consuming.

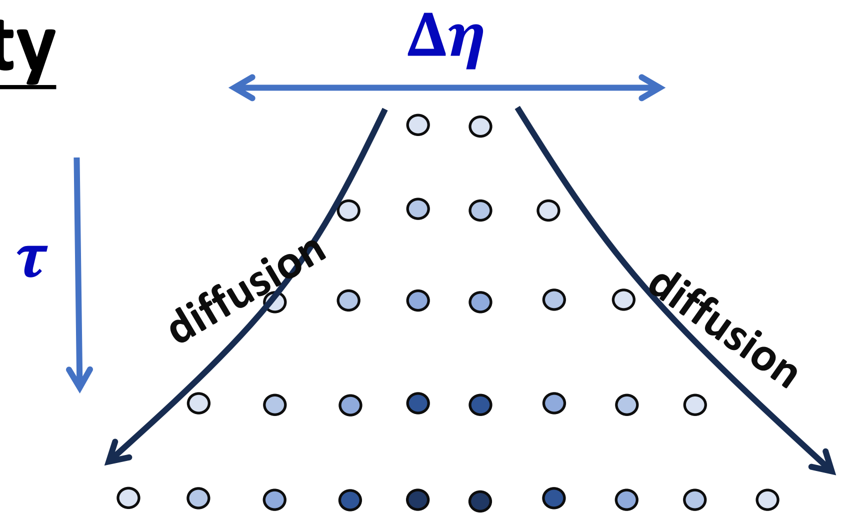
- **Simplified models:**
  - **Model A** (order parameter field) S.Mukherjee et al15’ 16’, L.Jiang et al17’, S.Wu et al 19’, S.Tang et al 23’,

$$\frac{\partial \sigma}{\partial t} = \Gamma[\sigma] + \mathrm{noise}$$

  - **Model B** (conserved field) M.Sakaida et al 17’, S.Wu et al 19’, M.Nahrgang et al 19’, G.Pihan et al 22’…

$$\frac{\partial n}{\partial t} = \nabla^2 \Gamma[n] + \mathrm{noise}$$

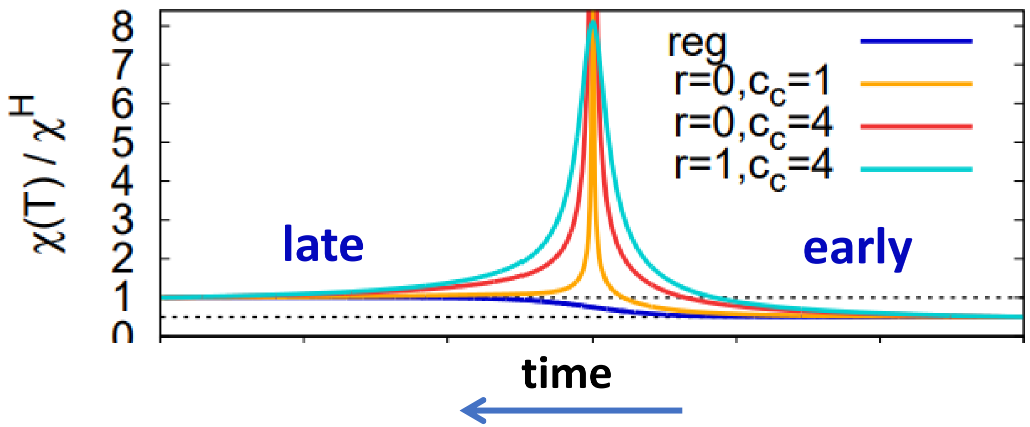
Dynamics of conserved net-baryon density



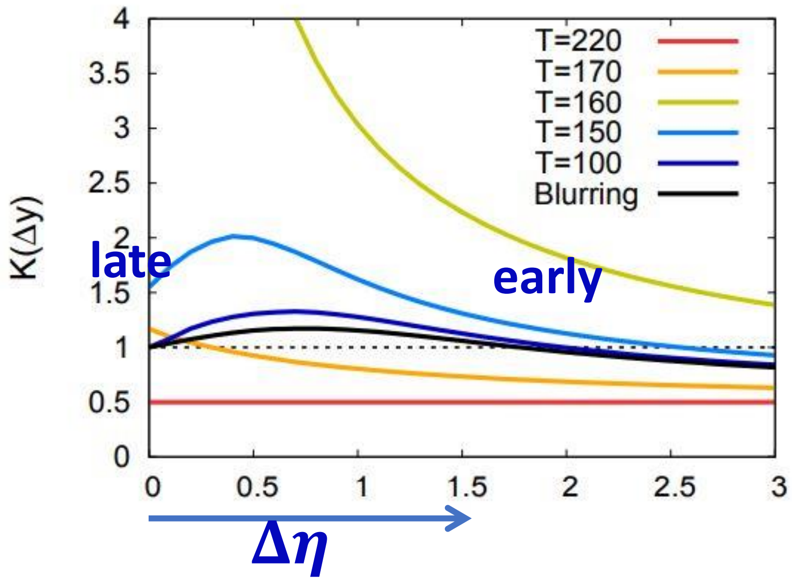
- Diffusion of conserved baryon near critical point:

$$\partial_\tau n = \nabla^2 n + \text{noise}$$

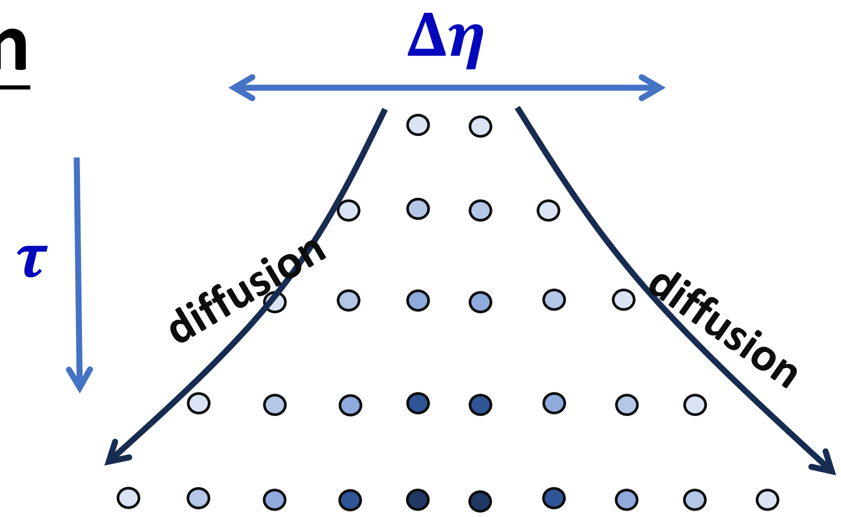
- The process of diffusion consumes time.
- Larger $\Delta\eta \sim$ the early evolution.



Sakaida et al, PRC.95.064905(2017)



Conserved net-baryon with non-Gaussian

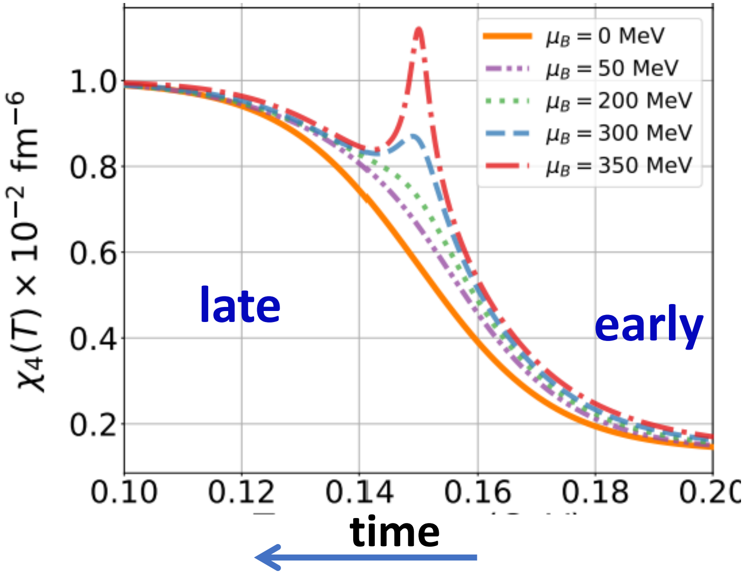


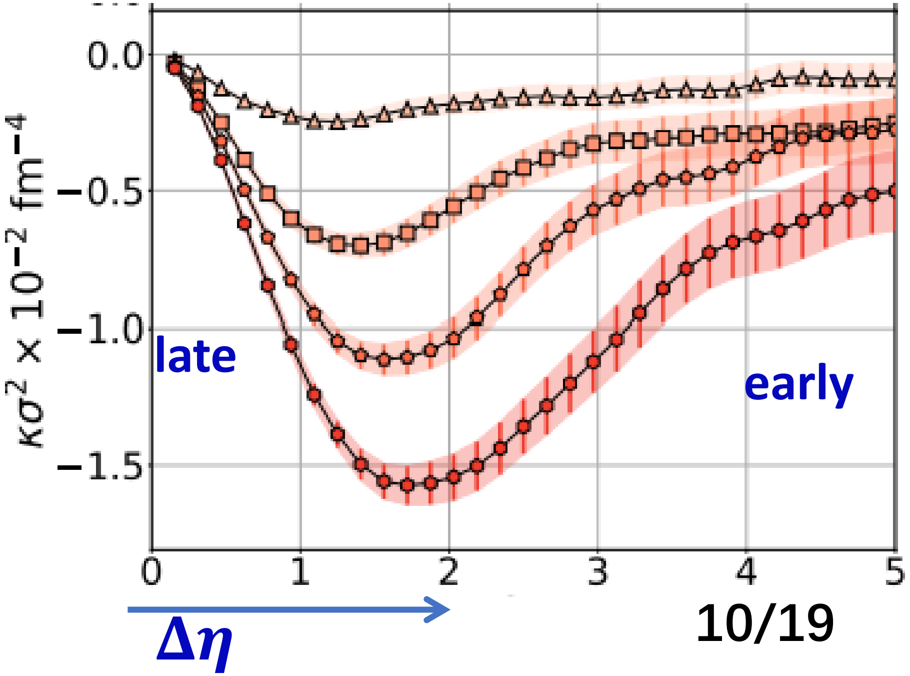
- Diffusion of conserved baryon near critical point:

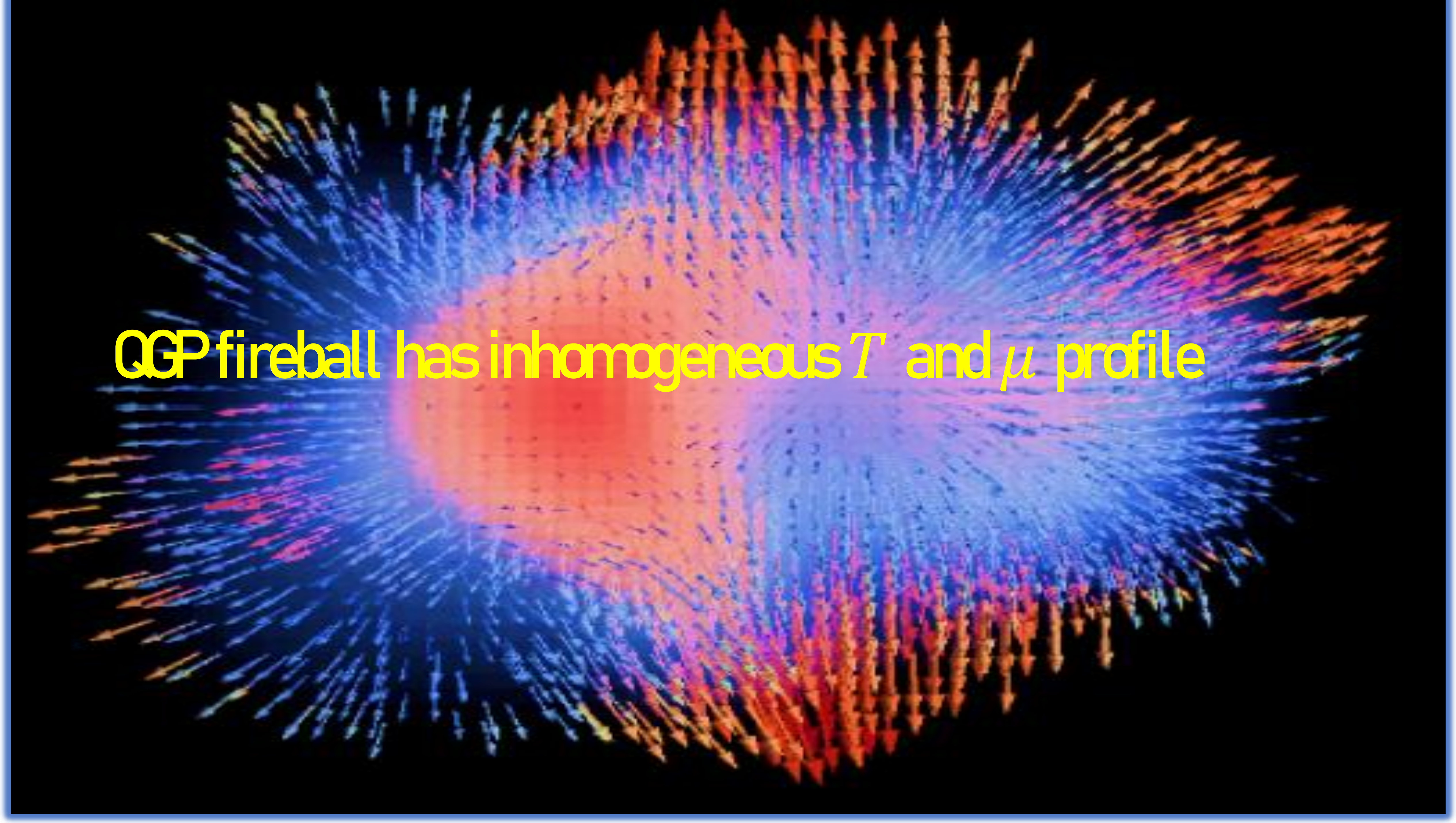
$$\partial_\tau n = \nabla^2(n + n^2 + n^3) + \text{noise}$$

- The process of diffusion consumes time.
- Larger $\Delta\eta \sim$ the early evolution.

G.Pihan et al., PRC.107.014908(2022)

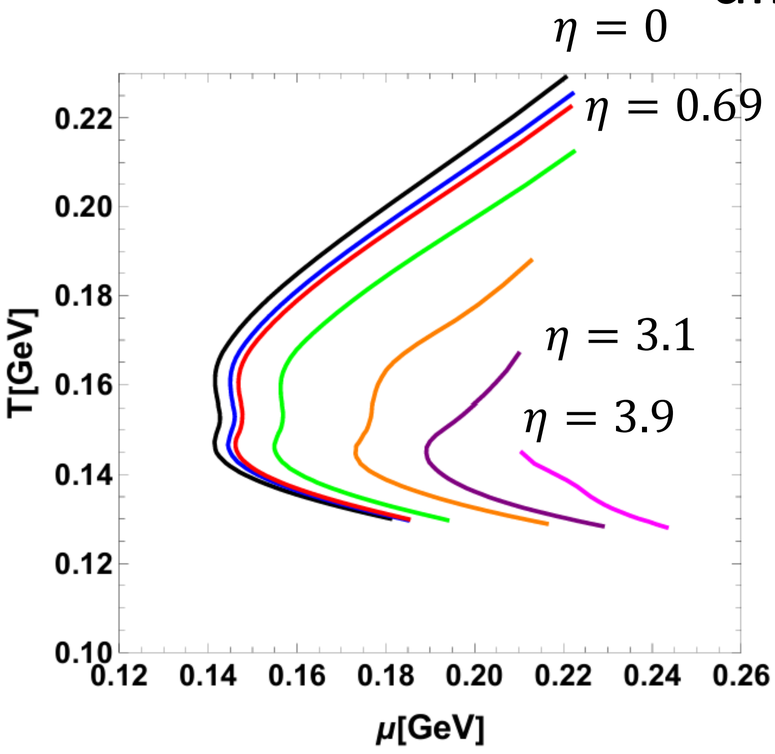




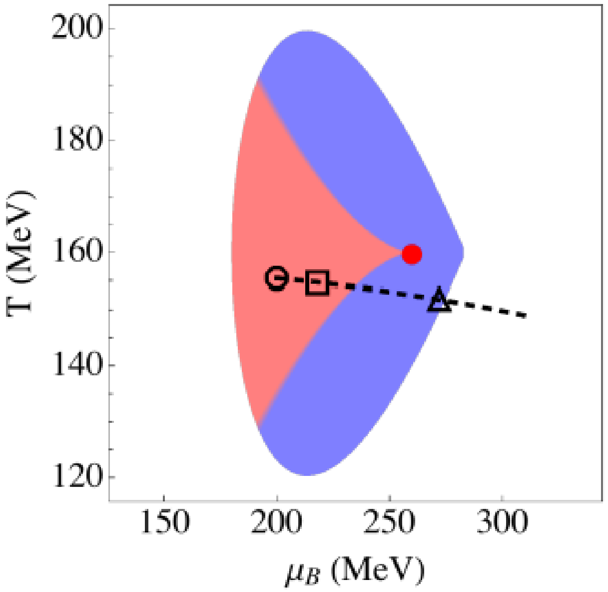

QGP fireball has inhomogeneous T and μ profile

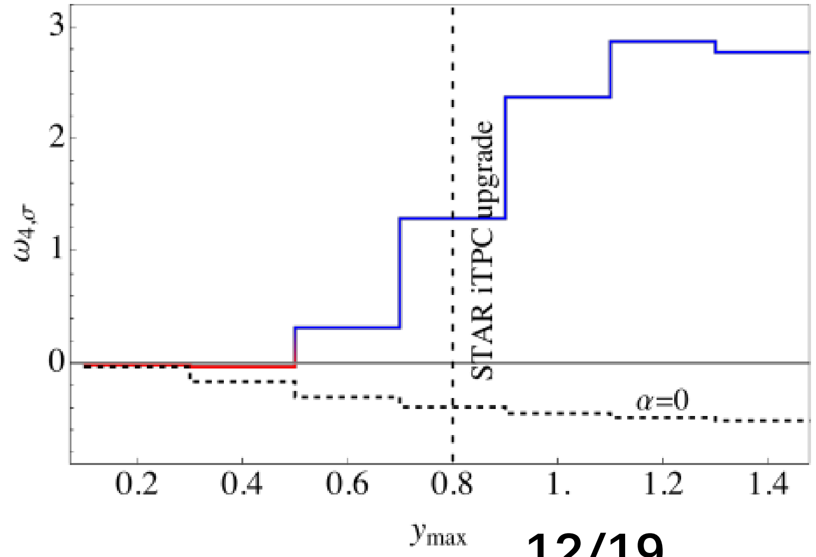
Inhomogeneous QGP profile

- Different rapidity ~ different trajectories
- ~ detect different region of critical region
- ~ different critical behavior



J.Brewer et al., PhysRevC.98.061901

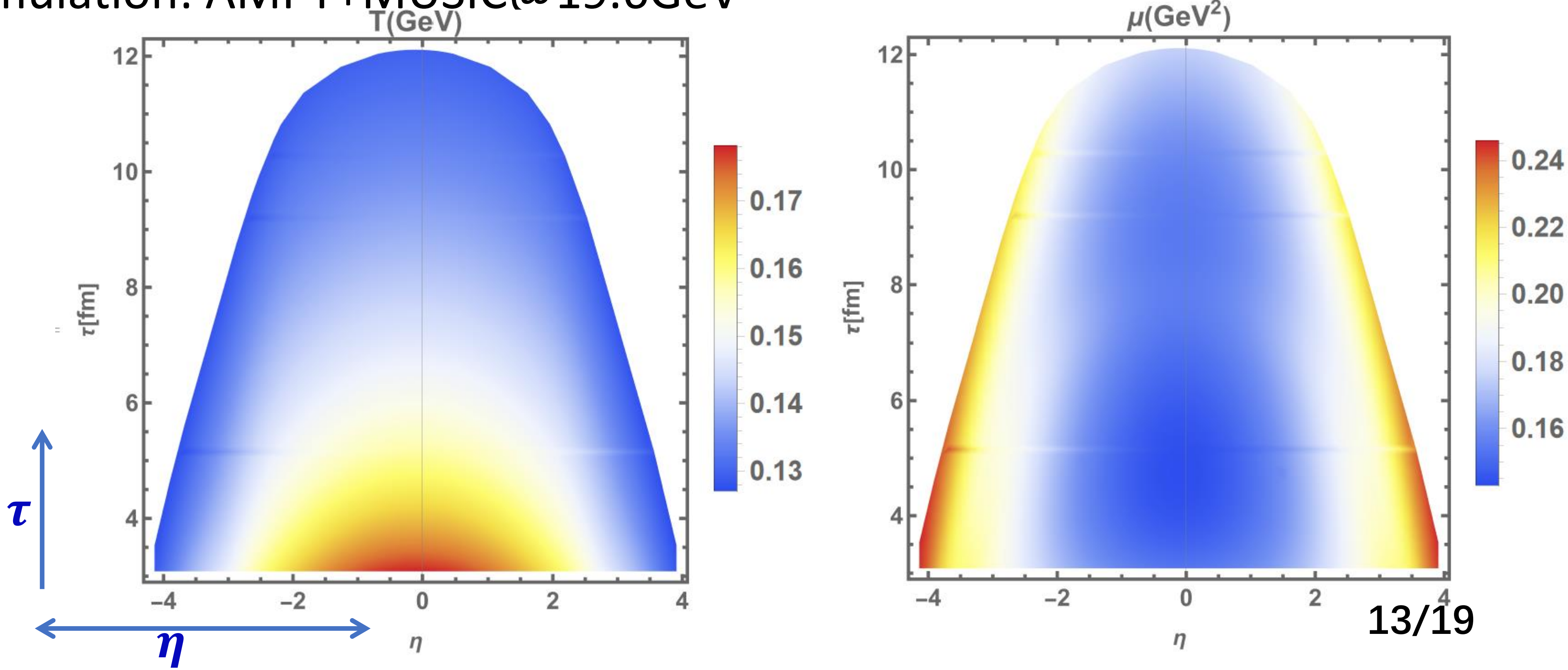




Inhomogeneous T and μ profile from hydro simulation

- This talk aims to study the inhomogeneous QGP profile effects on the diffusion of net-baryon
- Hydro simulation: AMPT+MUSIC@19.6GeV

Shanjin Wu 2406.12325

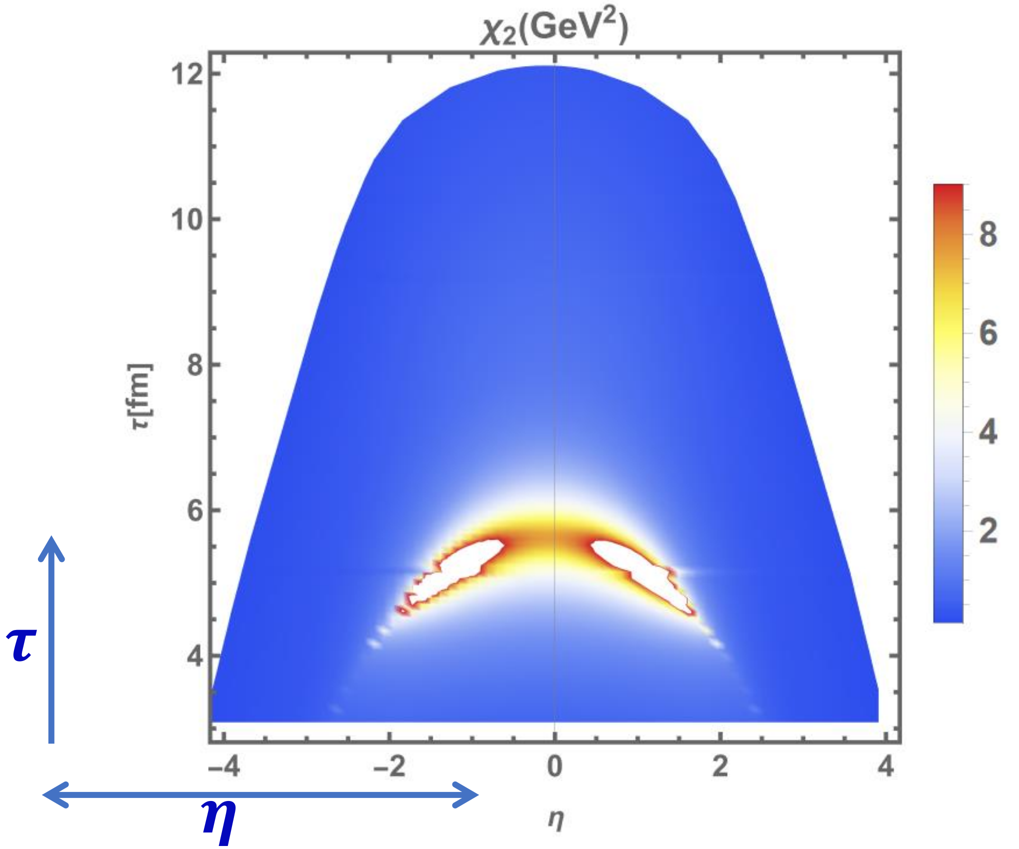


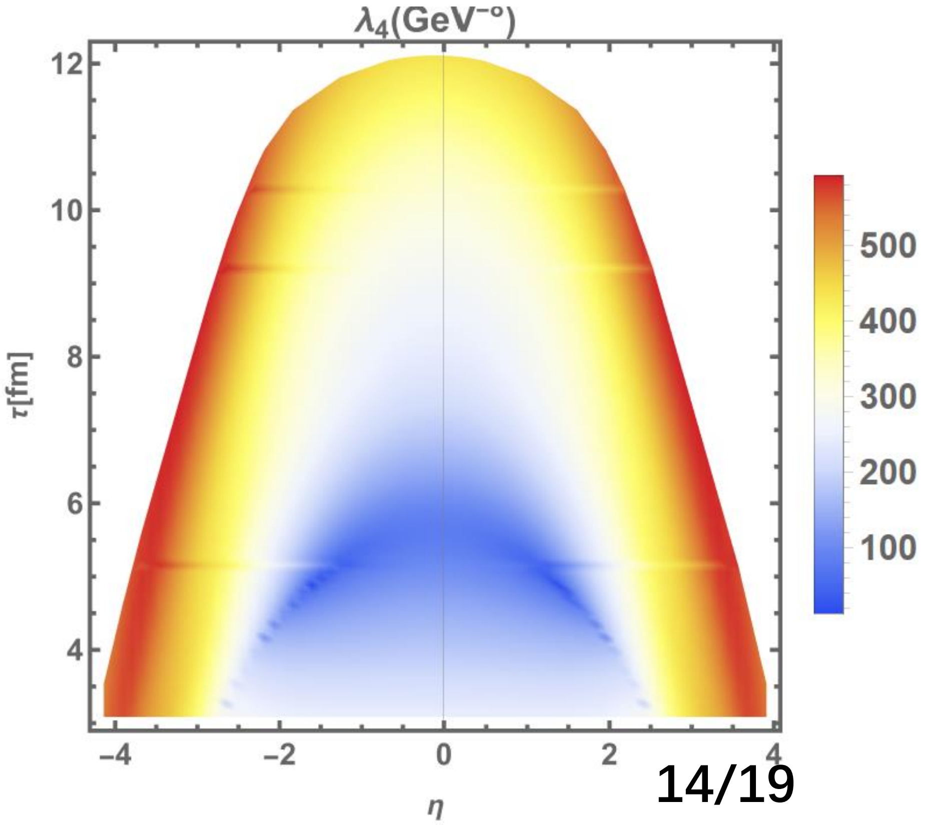
Conserved net-baryon with inhomogeneous T and μ profile

- Diffusion of conserved baryon near critical point: Shanjin Wu 2406.12325

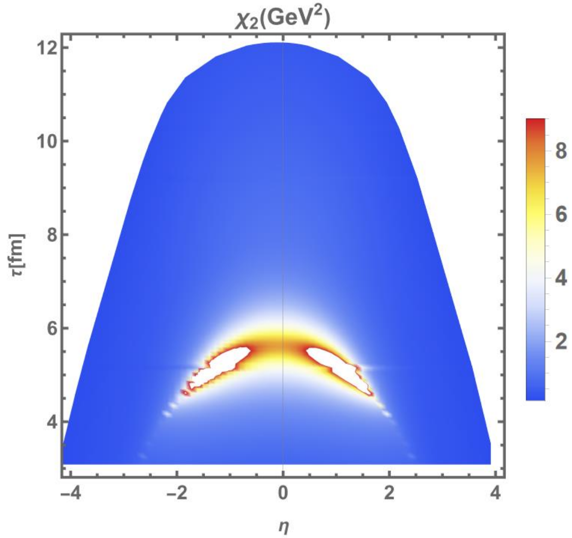
$$\partial_\tau n = \nabla^2\left(\frac{n}{\chi_2} + \lambda_3 n^2 + \lambda_4\, n^3\right) + \text{noise}$$

χ_2, λ_4 from Ising model





Conserved net-baryon fluctuations



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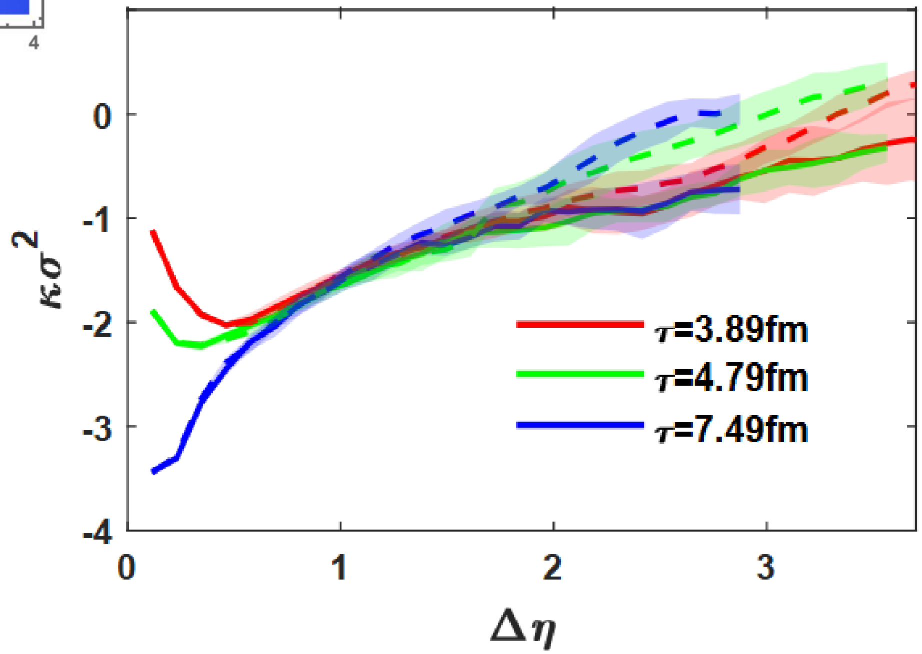
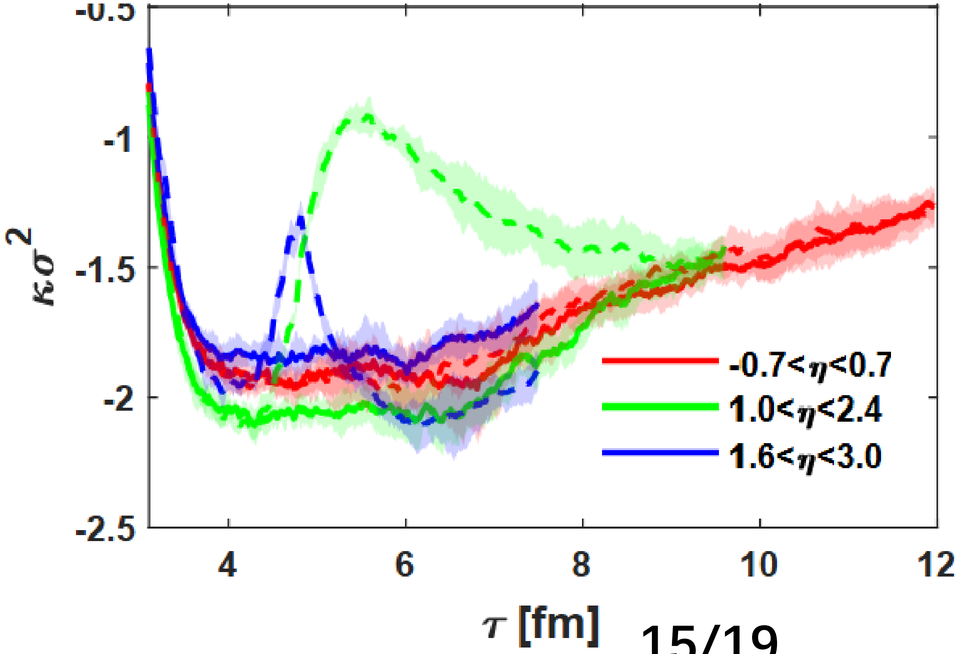
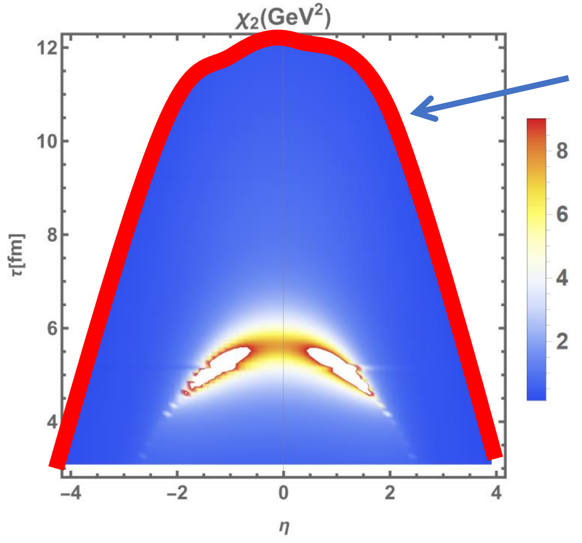
Inhomogeneous profile effects is significant at large rapidity

Solid: uniform profile; Dashed: inhomogeneous profile





Conserved net-baryon fluctuations at freeze-out surface



Freeze-out surface

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Net-baryon number at freeze-out surface

$$N_B = g \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^0} \int d\sigma_\mu p^\mu f(\boldsymbol{x}, \boldsymbol{p})$$

In Bjorken limit $$= \frac{2gA}{(2\pi)^2} \int_{-\Delta\eta}^{\Delta\eta} d\eta \tau_f \exp\left(\frac{\mu}{T}\right) T^3 \left(\frac{m}{T}\right)^2 K_2\left(\frac{m}{T}\right)$$

Net-baryon fluctuations at freeze-out surface

$$\delta N_B = \frac{2gAm^2}{(2\pi)^2} \int_{-\Delta\eta}^{\Delta\eta} d\eta \exp\left(\frac{\mu}{T}\right) \frac{\delta n_B(\tau_f, \eta)}{\chi} K_2\left(\frac{m}{T}\right)$$

B.Ling, T.Springer and M.Stephanov, PhysRevC.89.064901

Conserved net-baryon fluctuations at freeze-out surface

$$\delta N_B = \frac{2gAm^2}{(2\pi)^2}\int_{-\Delta\eta}^{\Delta\eta} d\eta \exp\left(\frac{\mu}{T}\right)\frac{\delta n_B(\tau_f,\eta)}{\chi}K_2\left(\frac{m}{T}\right)$$

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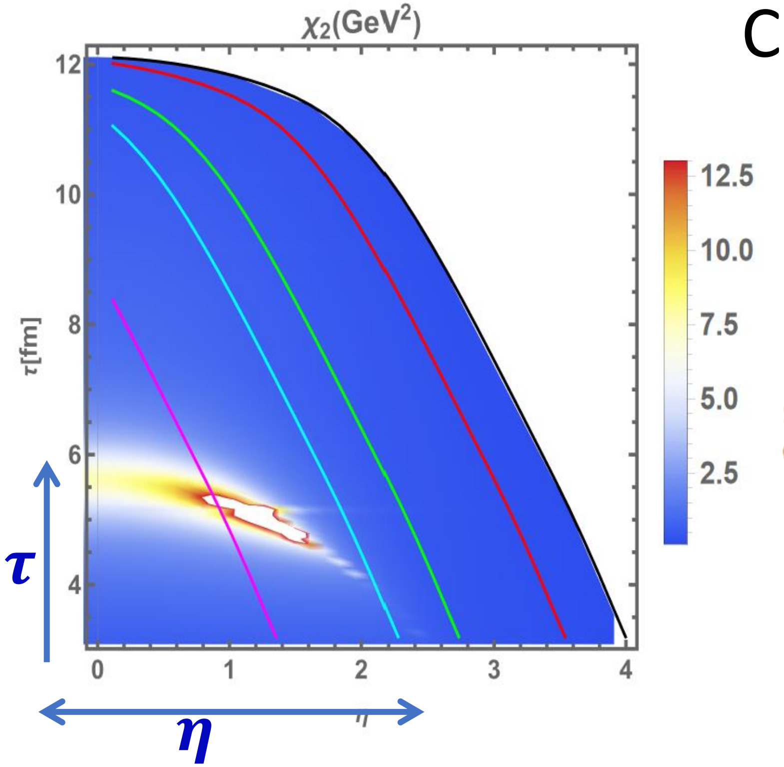
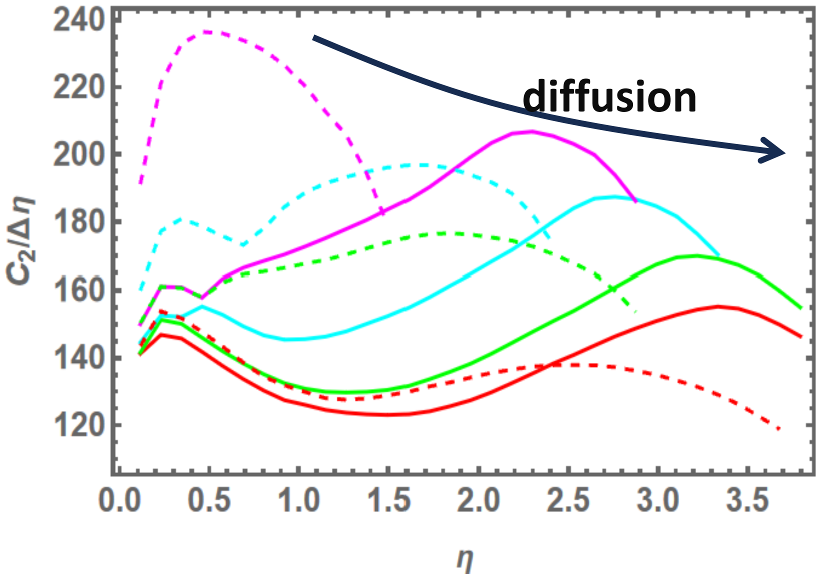
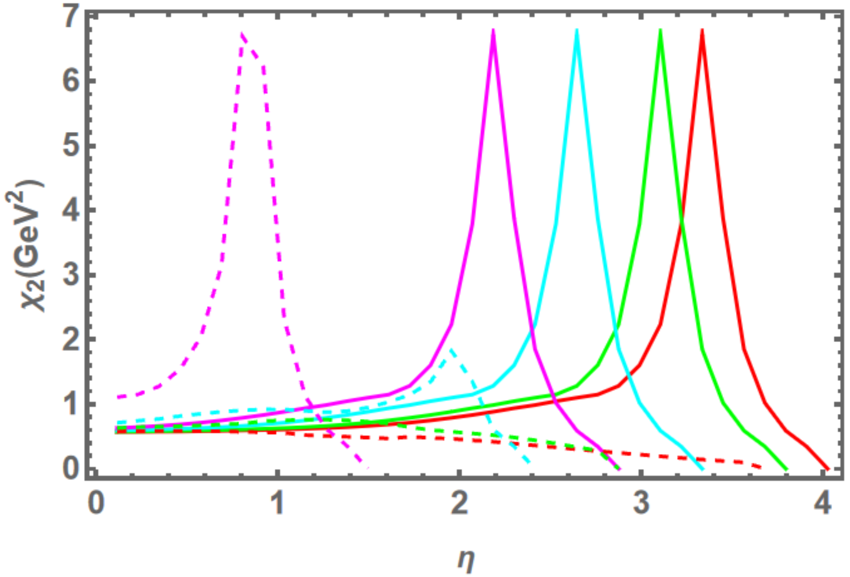


Critical slowing down effects of the fluctuations

$C_2 = \langle(\delta n_B)^2\rangle$

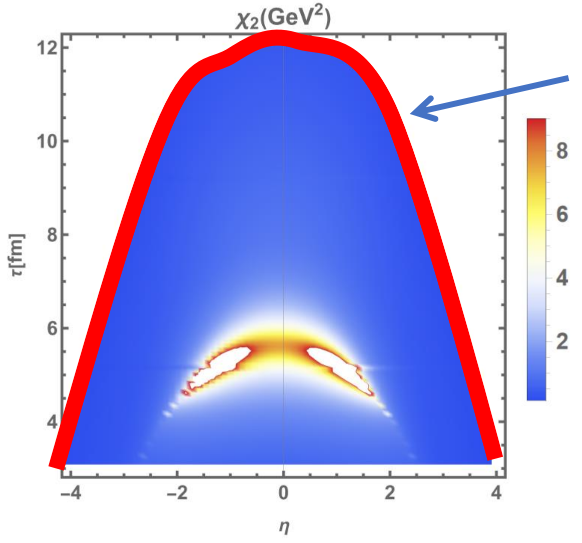


χ_2



Solid: uniform profile; Dashed: inhomogeneous profile

Conserved net-baryon fluctuations at freeze-out surface



Freeze-out surface

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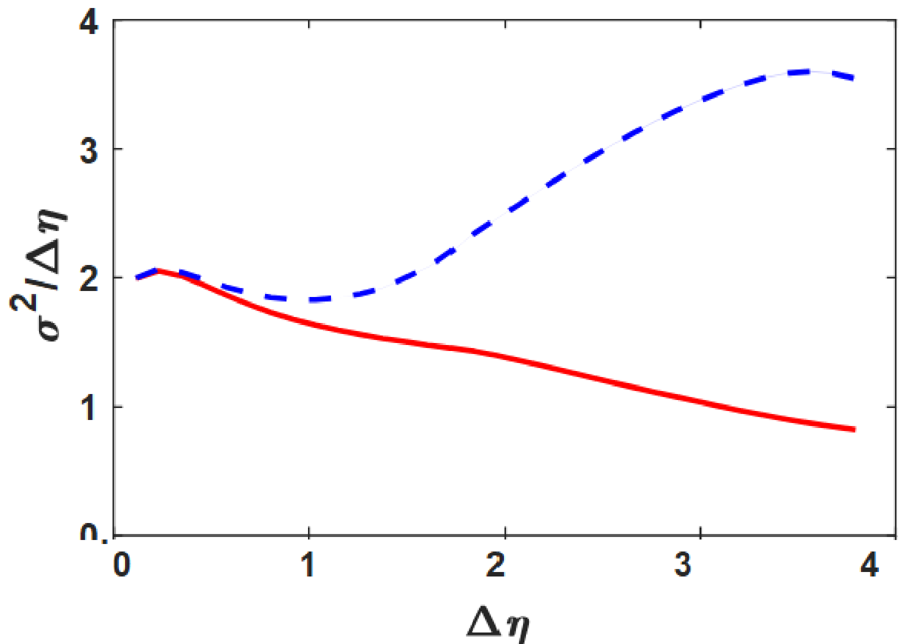
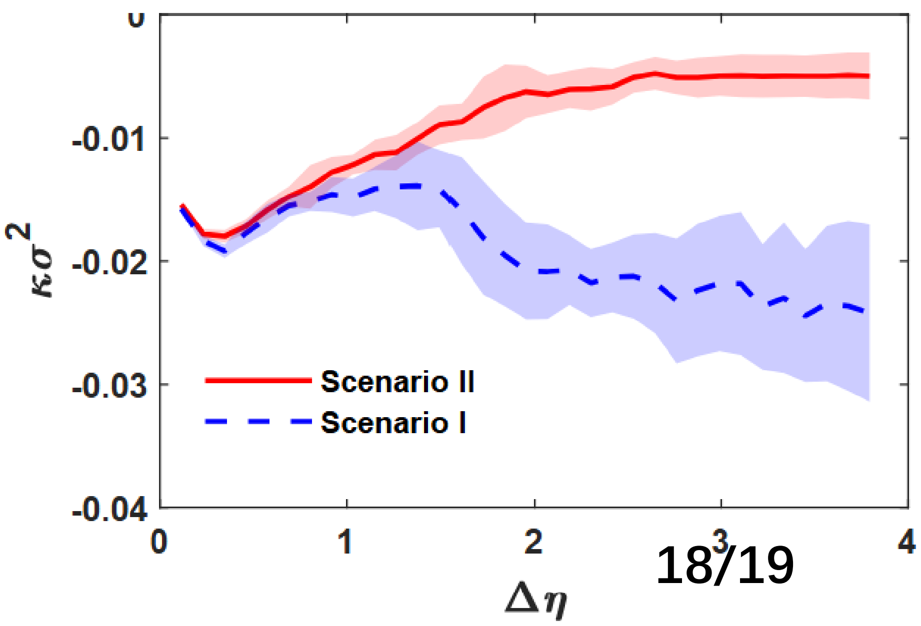
Critical slowing down => Significant inhomogeneous profile effects at large rapidity

Solid: uniform profile; Dashed: inhomogeneous profile





Summary

- Dynamical modeling the QGP evolution near the QCD critical point is essential for the study of fluctuations in heavy-ion experiments;
- The diffusion of conserved net-baryon density preserves the early evolution history and behaves non-monotonically with increasing rapidity;
- Considering the inhomogeneous T and μ profile has significant effects at large rapidity.
- Multiplicative noise

Thank you!