The Enigma of QCD from Spectroscopy

Si-Xue Qin

（秦思学）

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Introduction: Many-body problem

THE CONDENSED MATTER PHYSICS OF QCD

KRISHNA RAJAGOPAL AND FRANK WILCZEK
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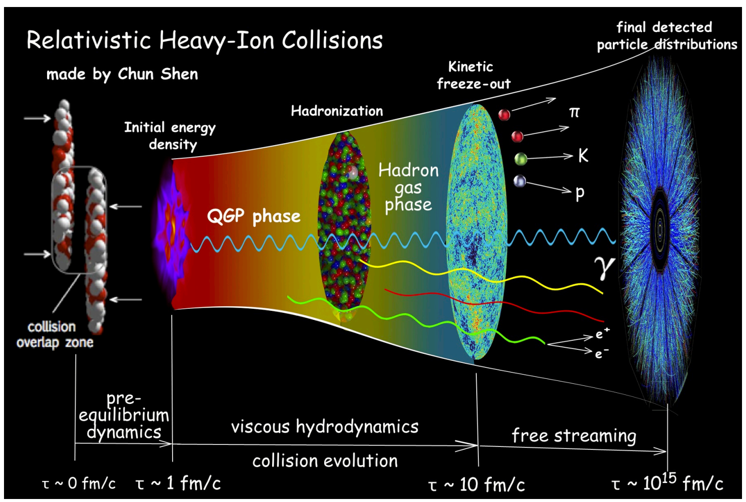
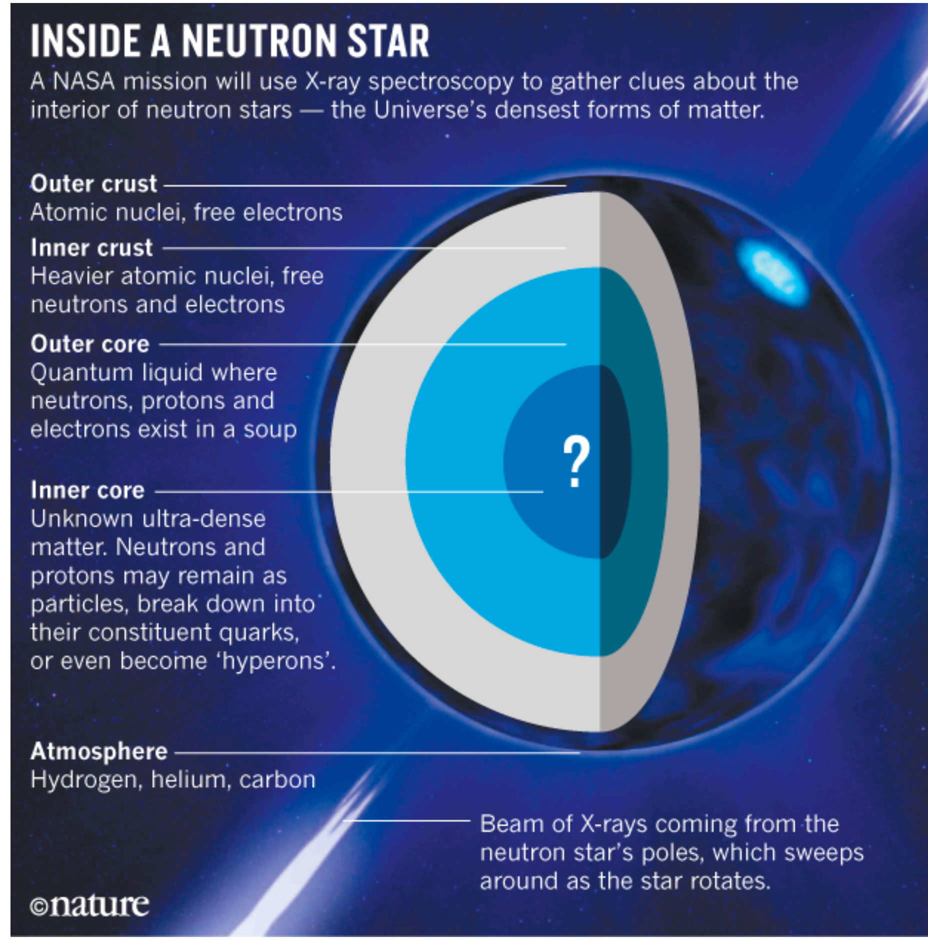
Important progress in understanding the behavior of hadronic matter at high density has been achieved recently, by adapting the techniques of condensed matter theory. At asymptotic densities, the combination of asymptotic freedom and BCS theory make a rigorous analysis possible. New phases of matter with remarkable properties are predicted. They provide a theoretical laboratory within which chiral symmetry breaking and confinement can be studied at weak coupling. They may also play a role in the description of neutron star interiors. We discuss the phase diagram of QCD as a function of temperature and density, and close with a look at possible astrophysical signatures.





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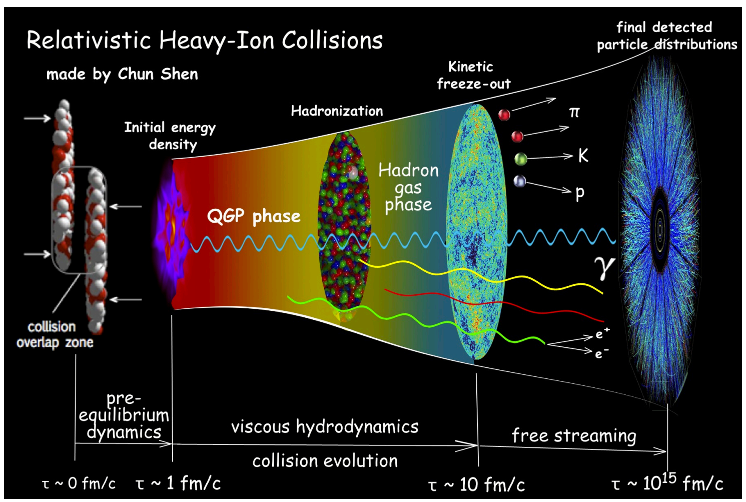

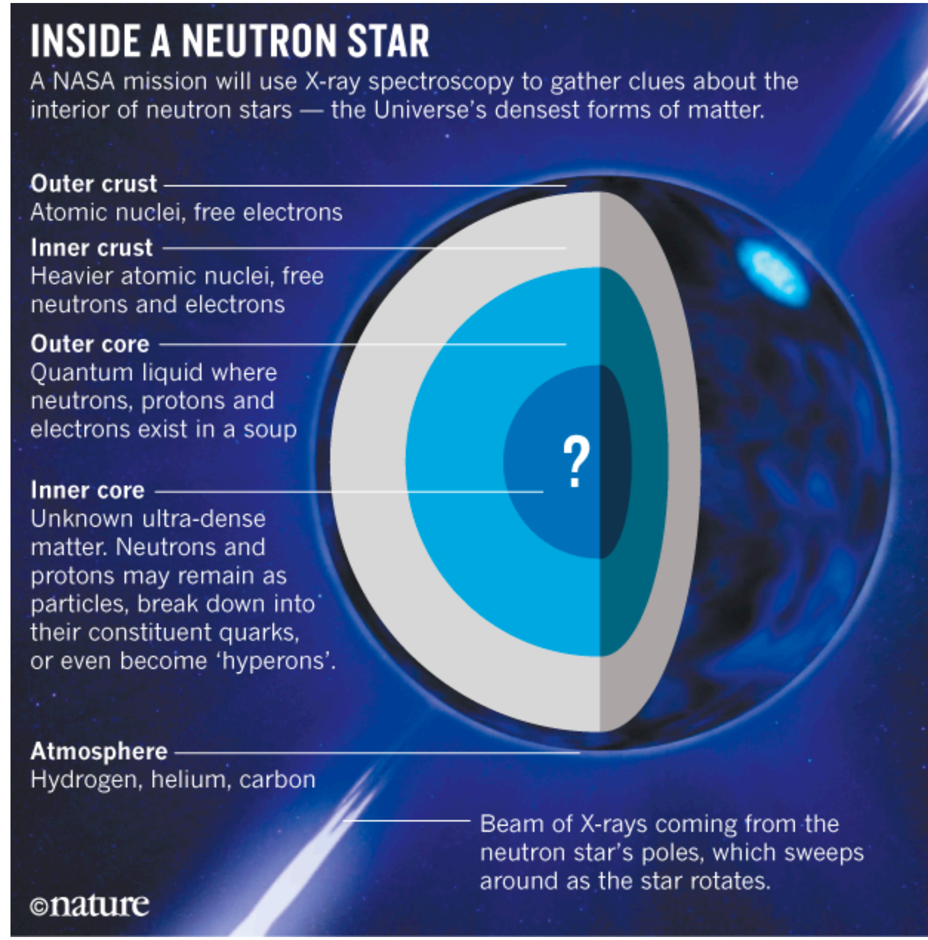

"More Is Different"

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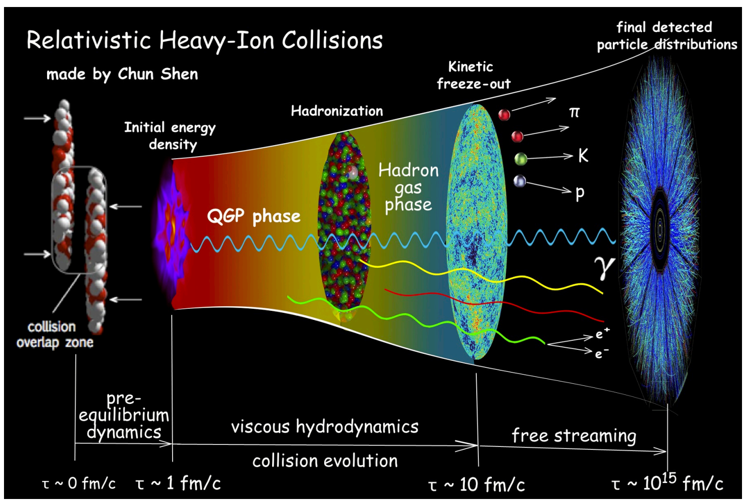
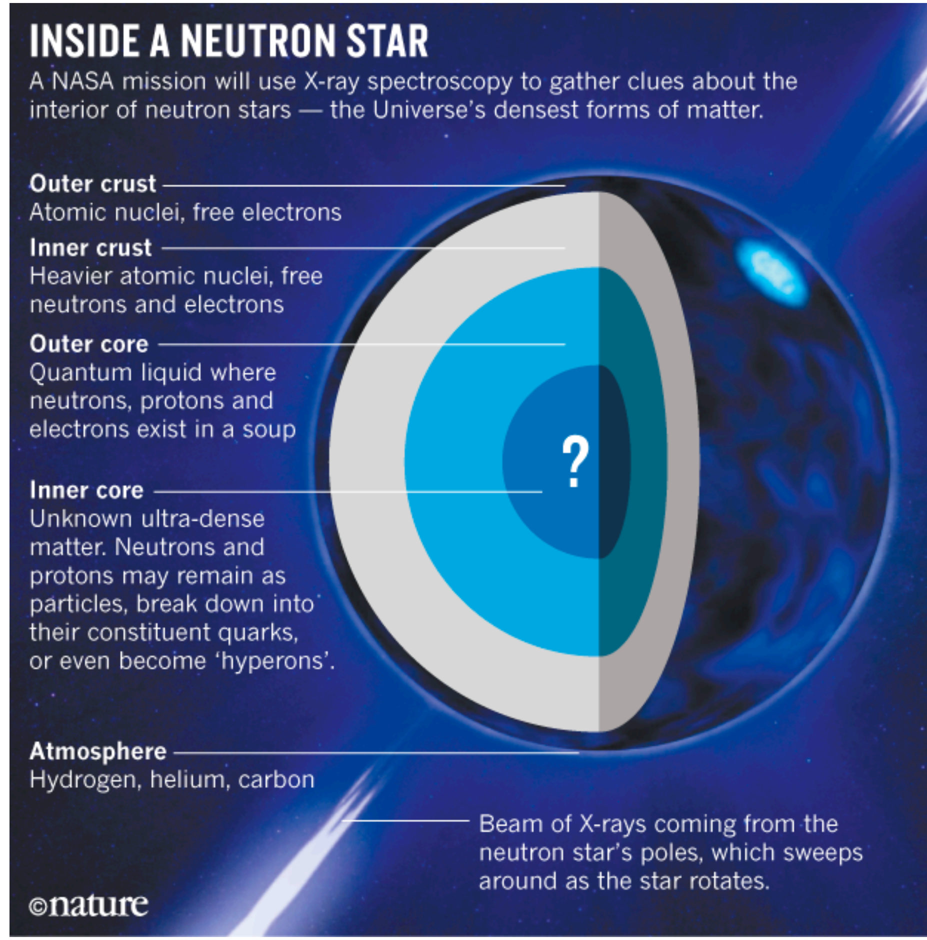
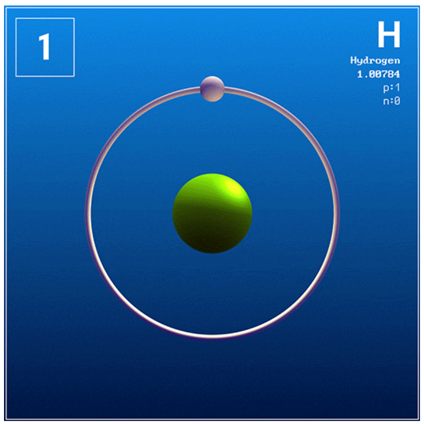




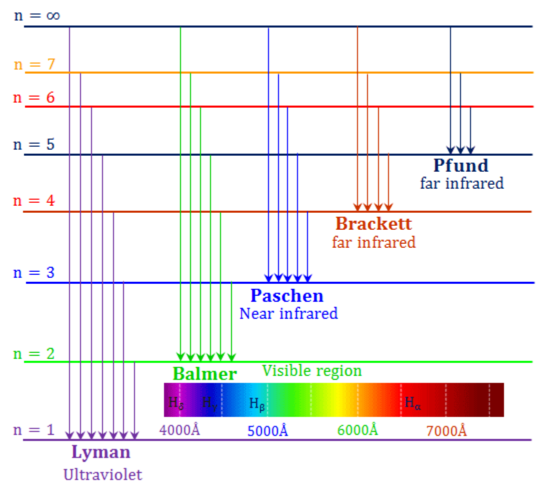
"More Is Different" From What?

Introduction: Few-body problem



$$H|\psi_n\rangle = E_n|\psi_n\rangle$$

$$H = H_{\text{kinetic}} + H_{\text{Coulomb}}$$



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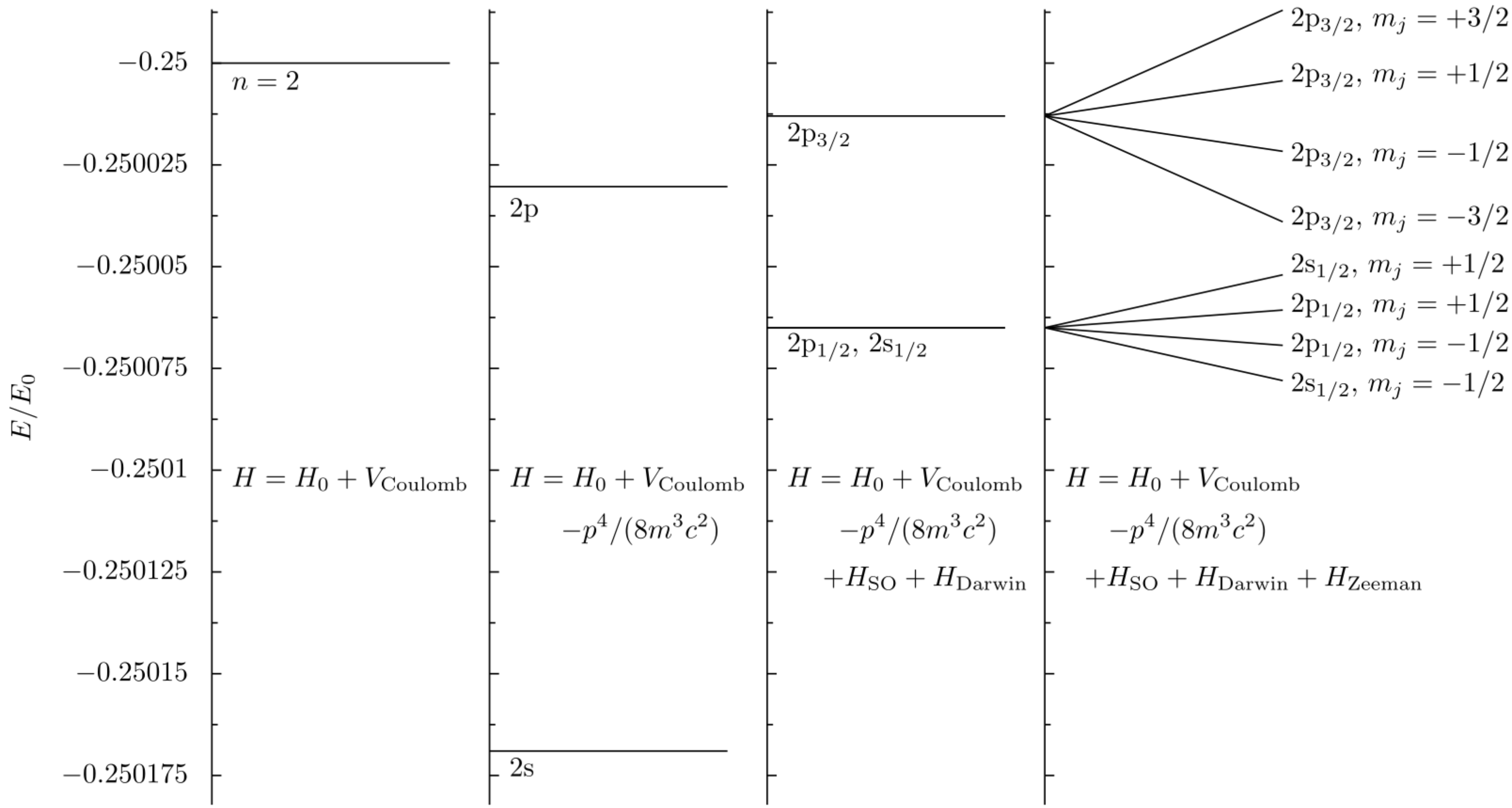

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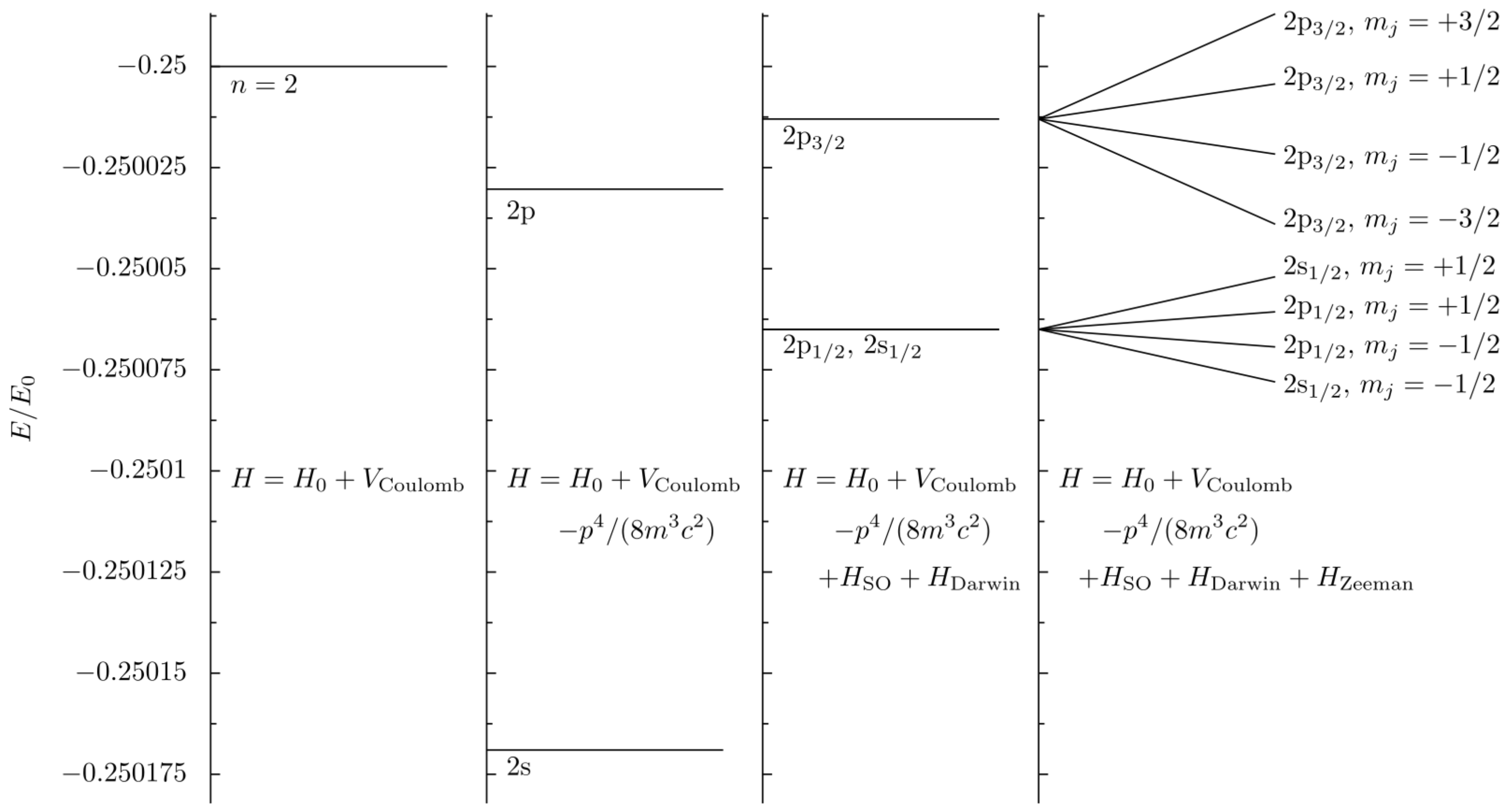
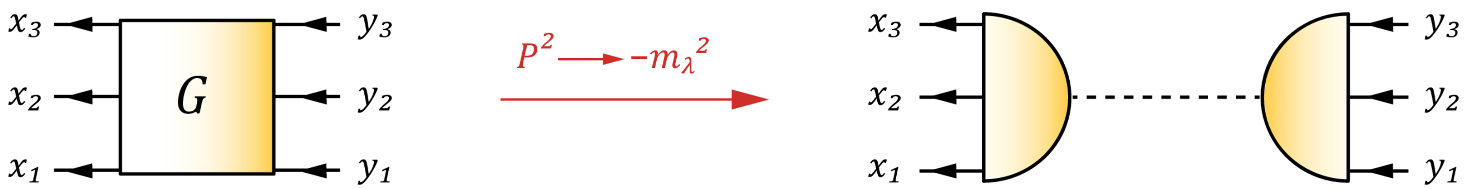


$$H = H_{\mathrm{kinetic}} + H_{\mathrm{Coulomb}} + H_{\mathrm{spin-orbit}} + H_{\mathrm{relativistic}} + H_{\mathrm{QED}}$$

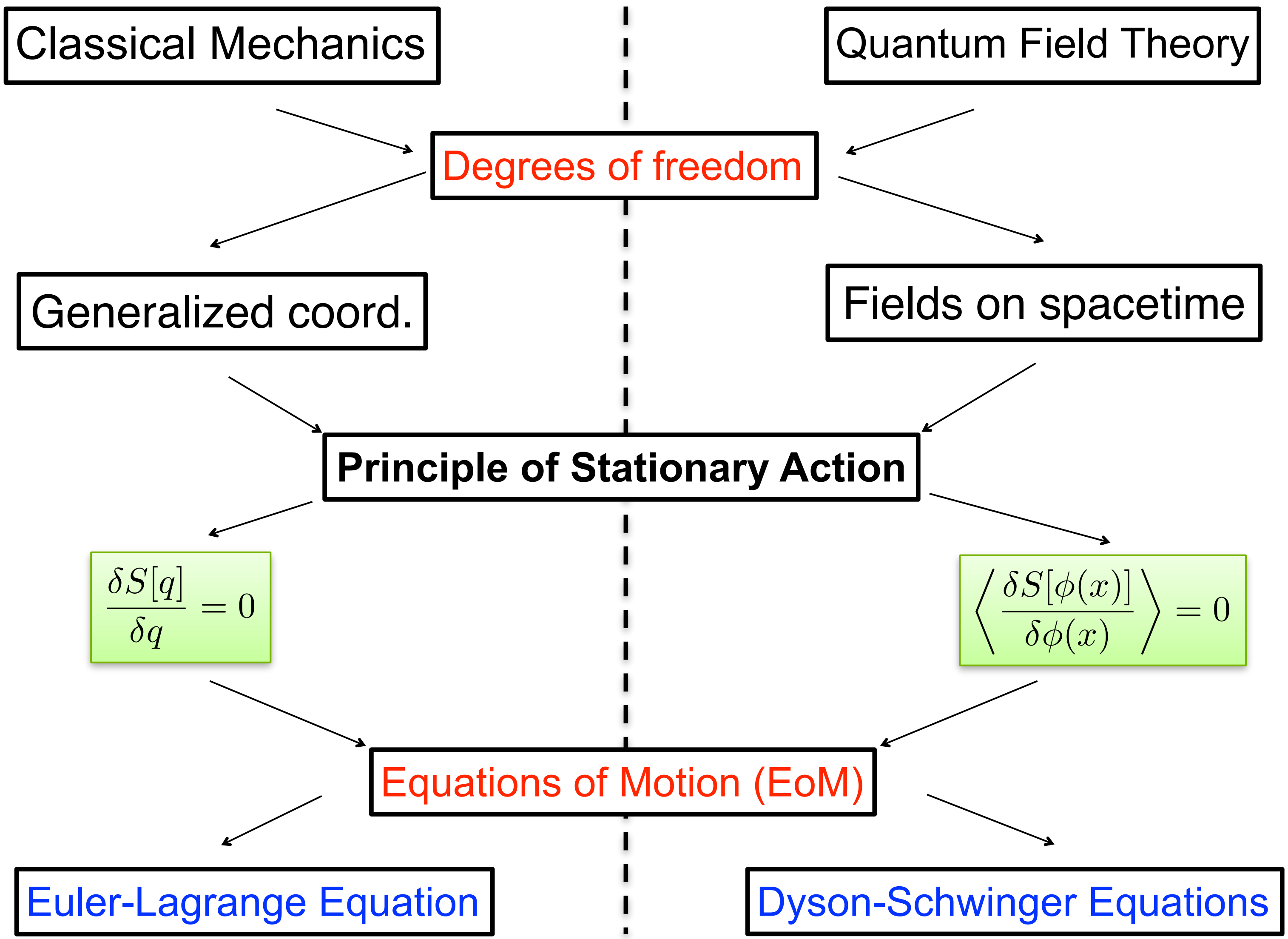
Introduction: Nonperturbative QCD Framework

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left[i(\gamma^\mu D_\mu)_{ij} - m\delta_{ij} \right] \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$



$$G^{(6)}(x_1, x_2, x_3, y_1, y_2, y_3) = \langle \Omega | q(x_1) q(x_2) q(x_3) q(y_1) q(y_2) q(y_3) | \Omega \rangle$$

Introduction: Nonperturbative QCD Framework



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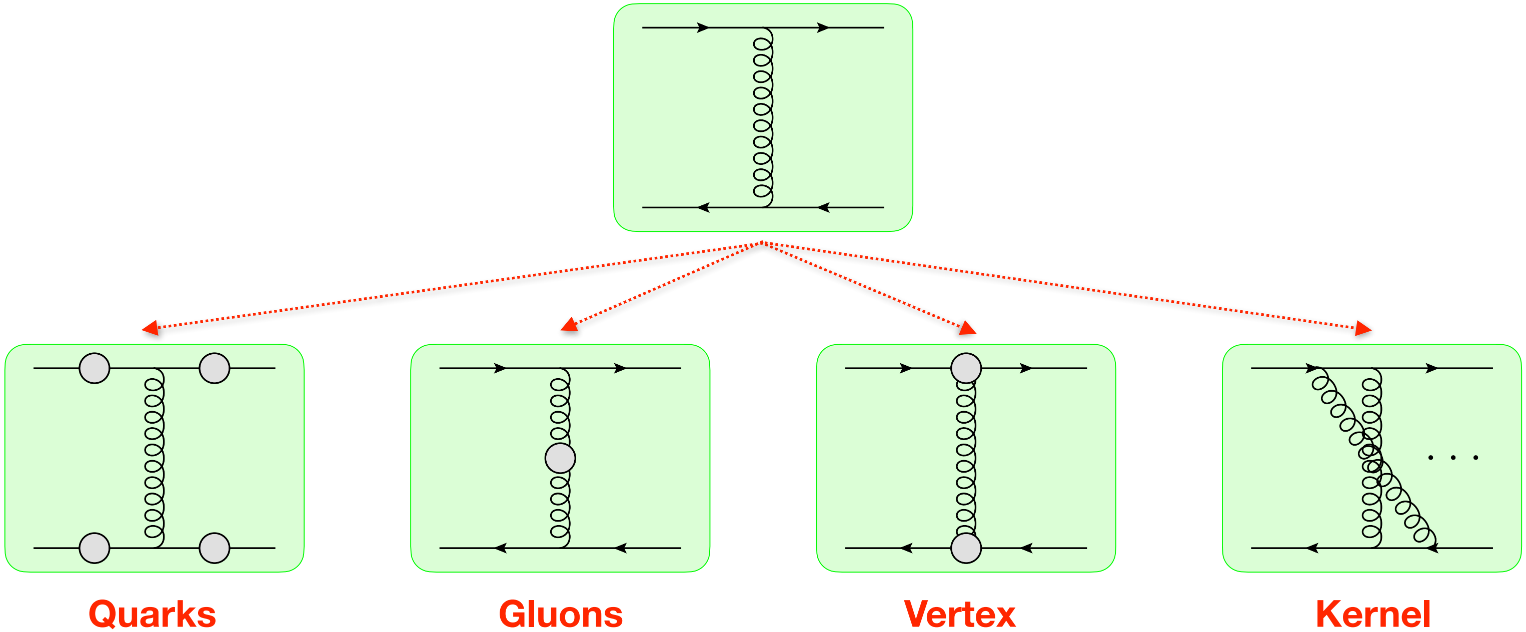

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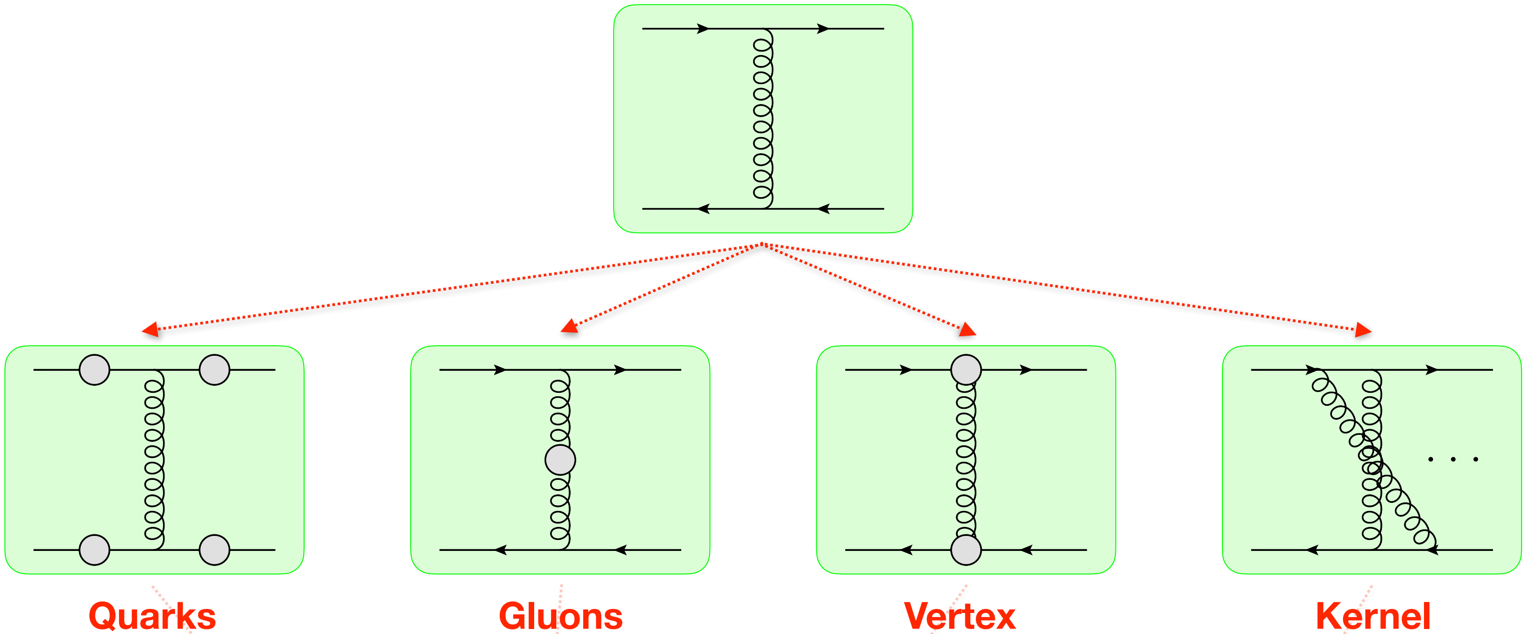

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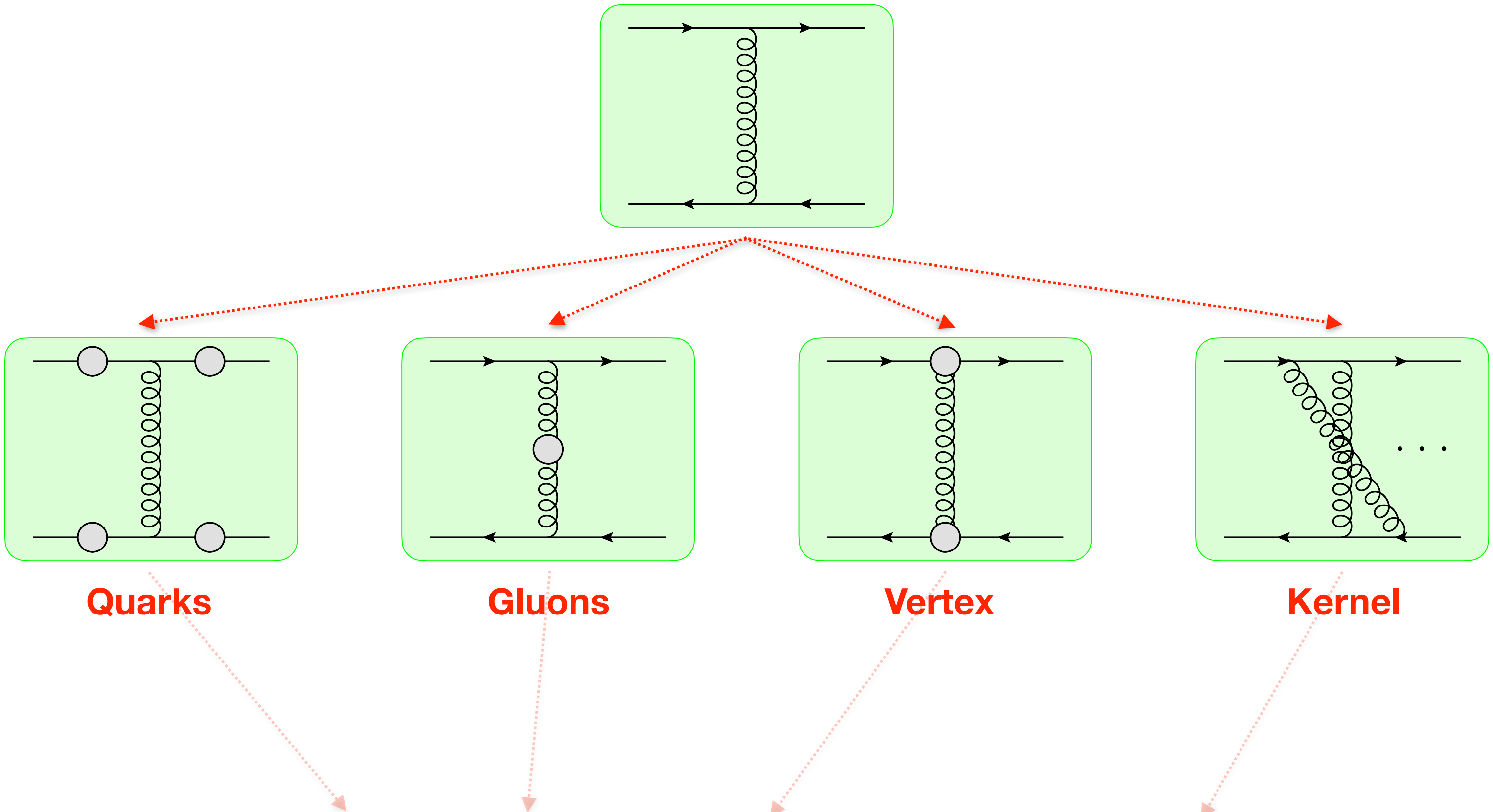

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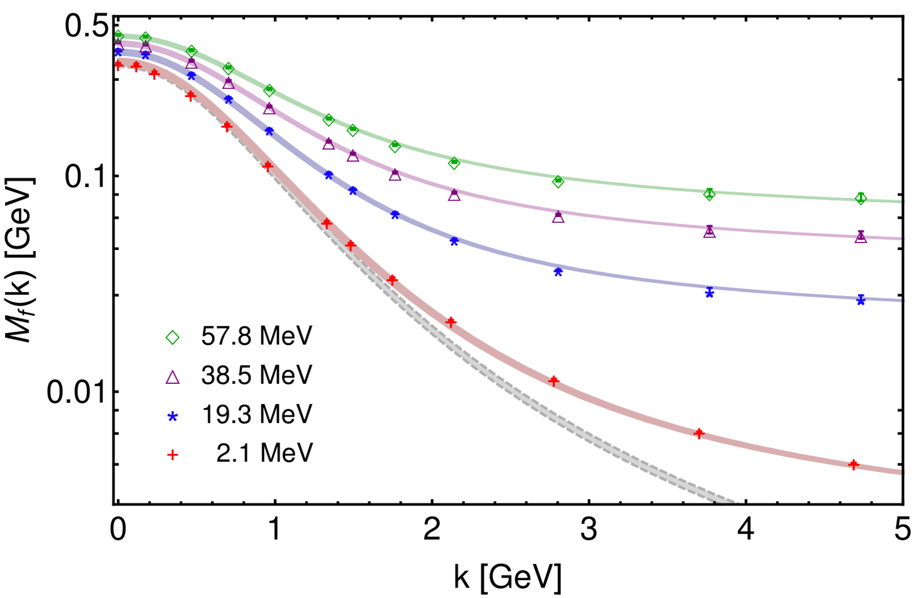
Basics

Basics: Quarks are dispersive quasi-particles

$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

Chang, Yang, et. al., PRD 104, 094509 (2021)



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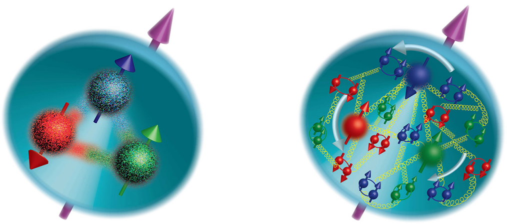

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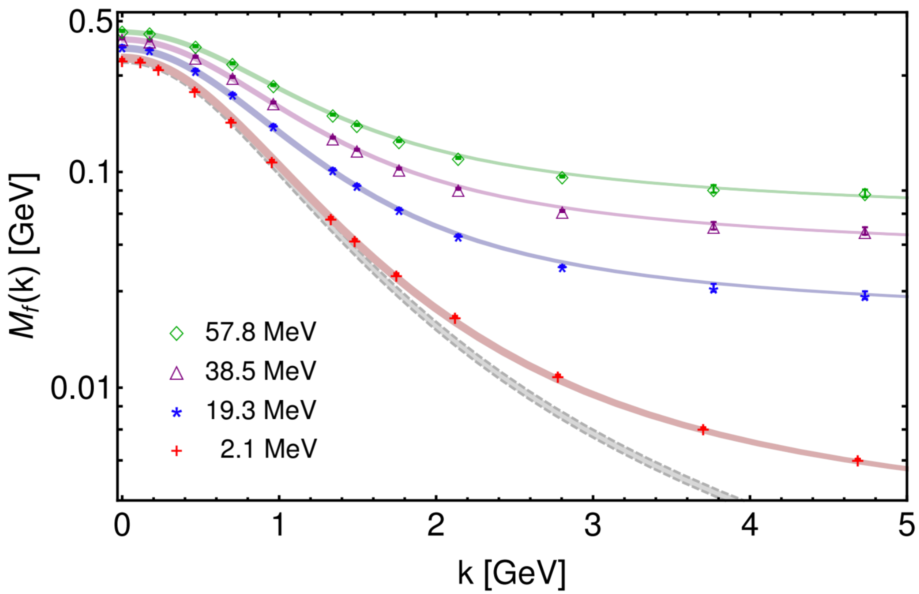

1. The quark's **effective mass** runs with its momentum.
2. The most **constituent mass** of a light quark comes from a cloud of gluons.
3. The mass has a **fast transition** between non-pert. and pert. at about 1GeV.

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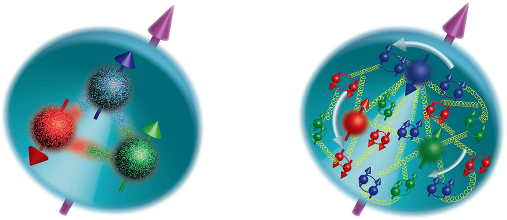

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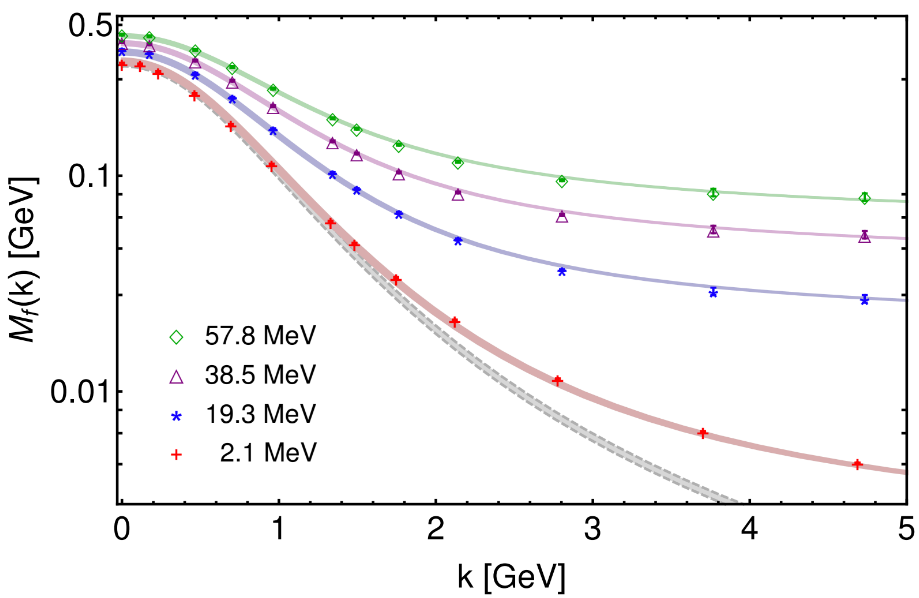

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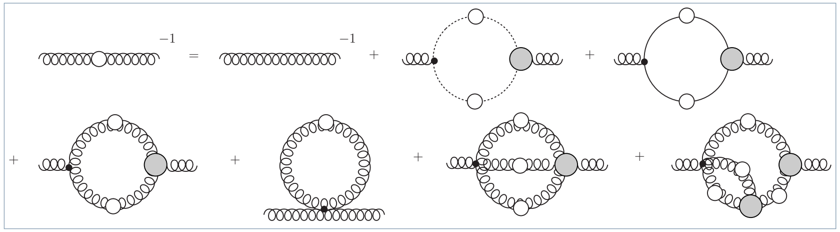
Vacuum — invisible highly dispersive medium

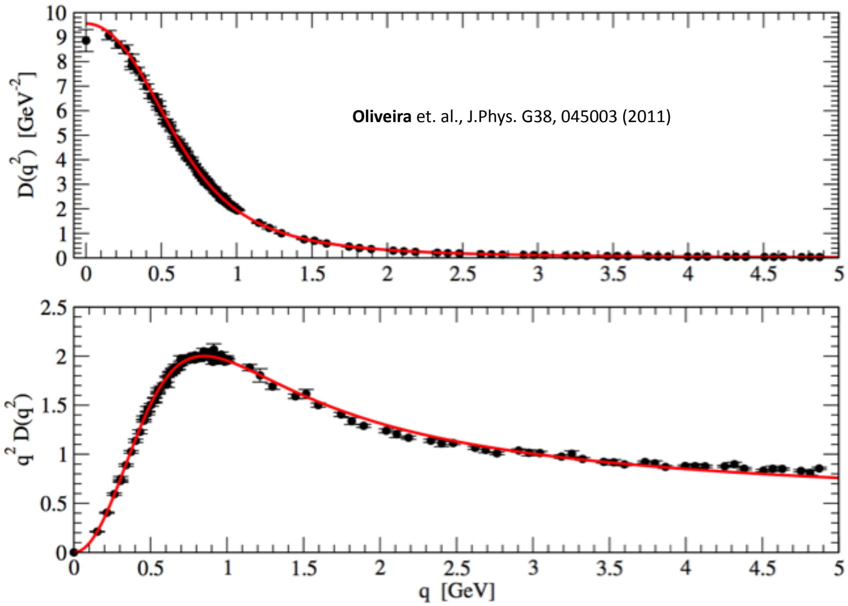
Basics: Gluons are massive quasi-particles

Gluon gap equation:

Aguilar, Binosi, Papavassiliou and Rodriguez-Quintero



Lattice QCD simulations:



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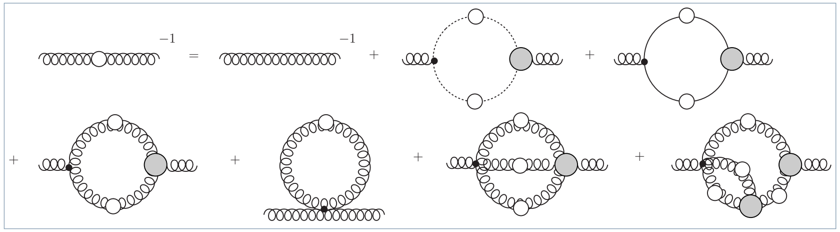

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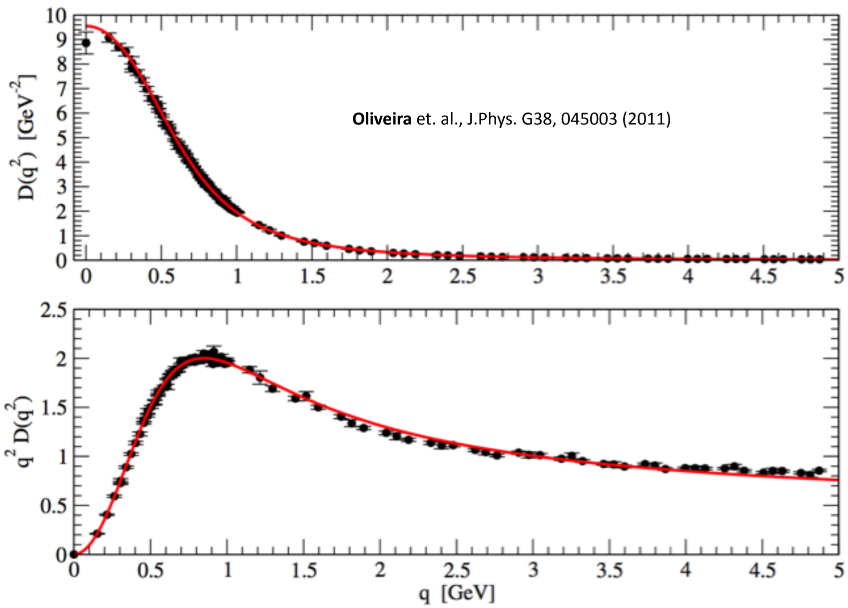


- The interaction can be decomposed: ***gluon running mass*** + ***effective running coupling***

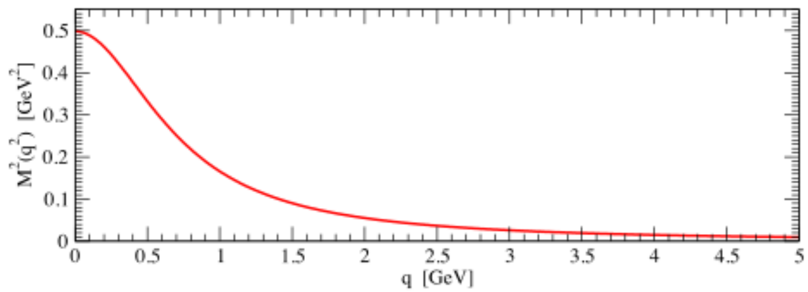
$$g^2 D_{\mu\nu}(k) = \mathcal{G}(k^2)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_{RL}(k^2)}{k^2 + m_g^2(k^2)}$$

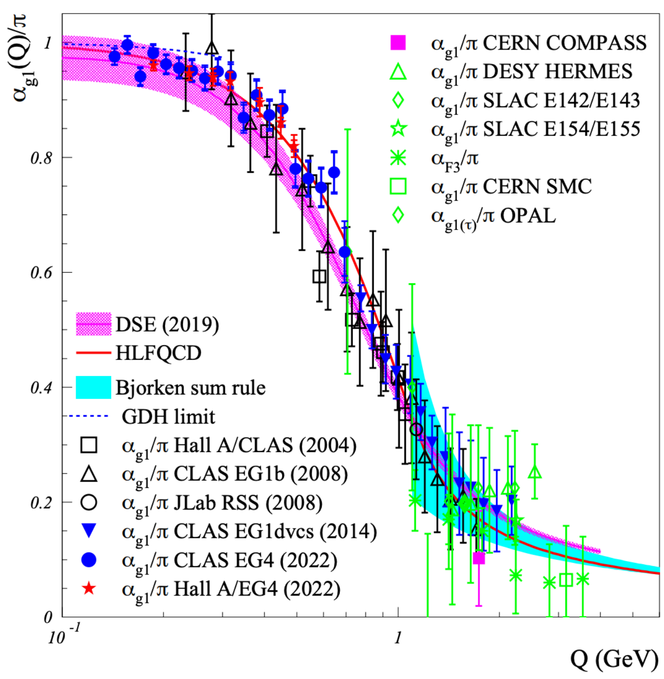
- In QCD: Gluons are ***cannibals*** — a particle species whose members become **massive** by eating each other — **quasi-particles**!

Basics: Gluons are massive quasi-particles

Gluon mass function: **Oliveira** et. al., J.Phys. G38, 045003 (2011)



Running coupling: **Deur, Brodsky, Roberts,** PPNP, 104081 (2024)



See, e.g., PRC 84, 042202(R) (2011)

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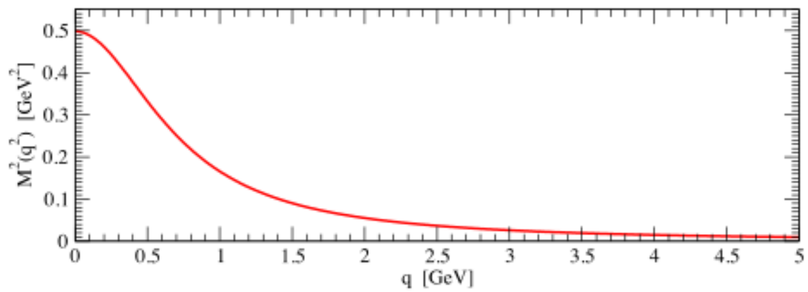

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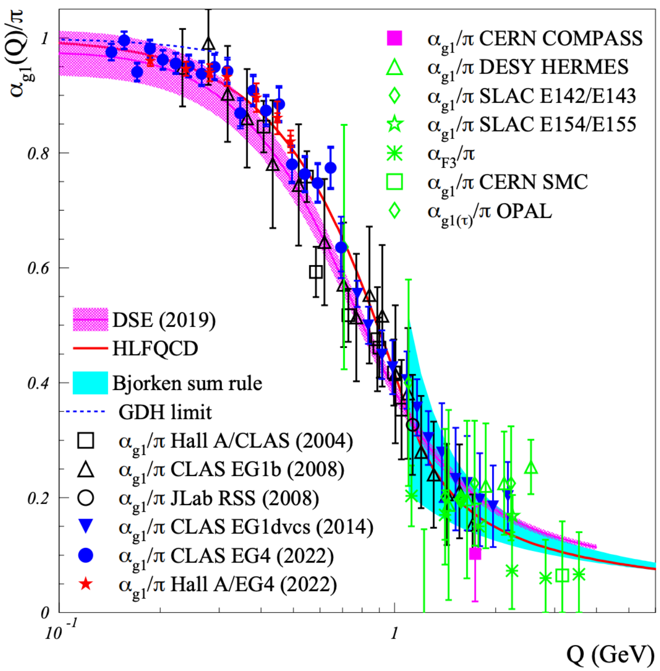


1. The dressed gluon can be well parameterized by a **mass scale**

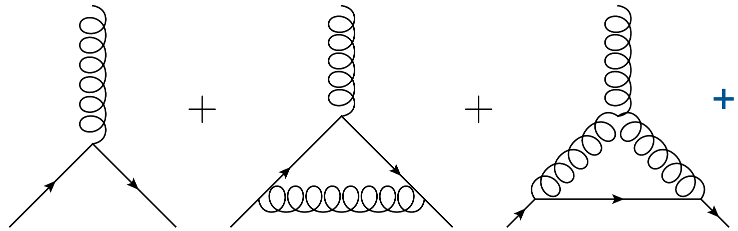
$$m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2}$$

$$M_g \sim 700 \text{ MeV}$$

2. The effective running coupling **saturates** in the infrared limit.

- converge to: $\alpha_s(0) \sim \pi$
- transition at: $Q \sim 1$ GeV

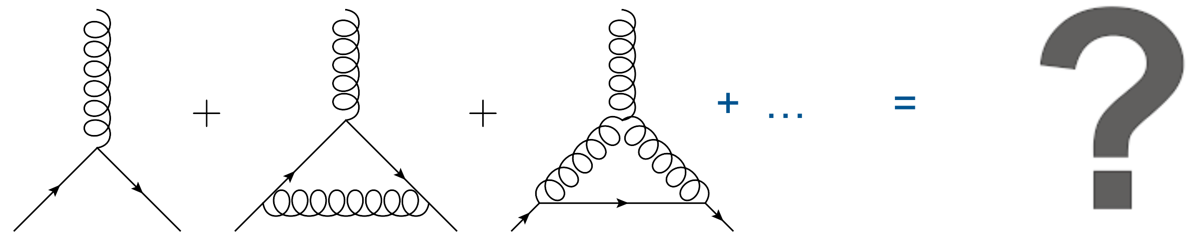
See, e.g., PRC 84, 042202(R) (2011)

Quark-gluon vertex:  + … = ?

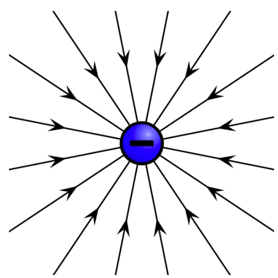
See, e.g., PLB722, 384 (2013)

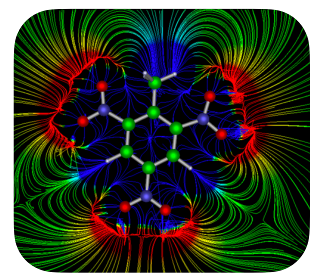
Basics: Vertex has DCSB-rendered appearance

Quark-gluon vertex:



point charge





distributed charges

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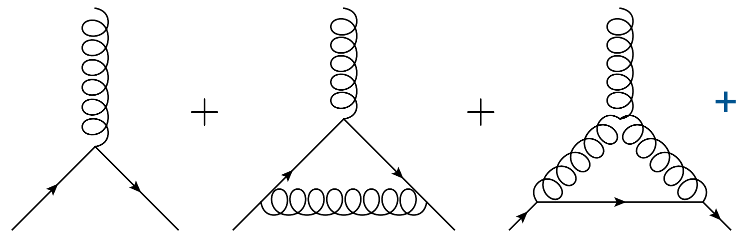

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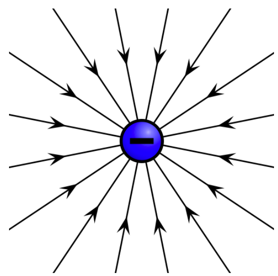

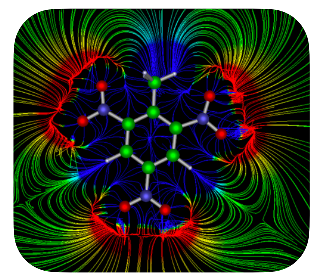

distributed charges

✦ The **Dirac** and **Pauli** terms: for an on-shell fermion, the vertex can be decomposed by two form factors:

$$\Gamma^\mu(P', P) = \gamma^\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M_f} Q^\nu F_2(Q^2)$$

12 terms

✦ The form factors express (color-)**charge** and (color-)**magnetization** densities. And the so-called **anomalous moment** is proportional to the **Pauli** term.

See, e.g., PLB722, 384 (2013)

Basics: Vertex has DCSB-rendered appearance

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Equation of Motion (DSEs)

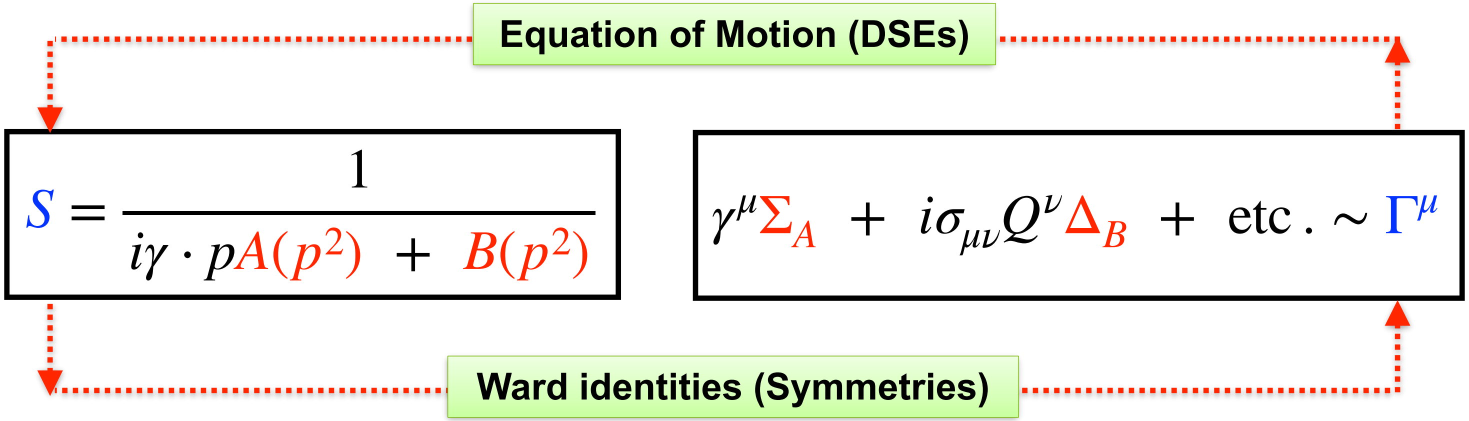
$$S = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

$$\gamma^\mu \Sigma_A + i\sigma_{\mu\nu} Q^\nu \Delta_B + \text{etc.} \sim \Gamma^\mu$$

Ward identities (Symmetries)

See, e.g., PLB722, 384 (2013)

Basics: Vertex has DCSB-rendered appearance



1. There is a dynamic chiral symmetry breaking (**DCSB**) **feedback**. **DCSB** is closely related to the **Pauli term**:

$$F_2 \sim \text{DCSB}$$

See, e.g., PLB722, 384 (2013)

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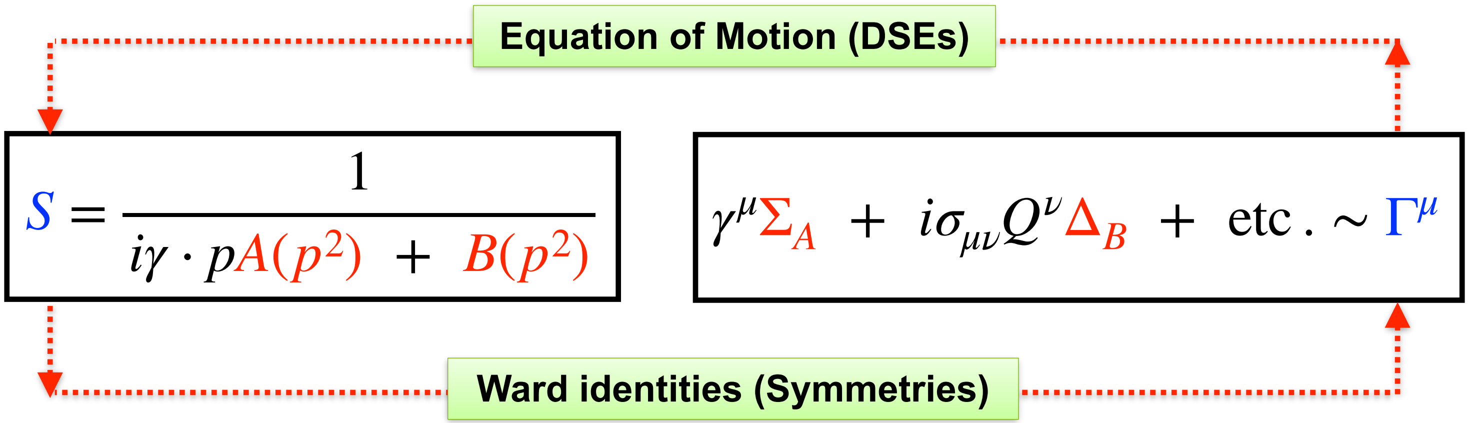

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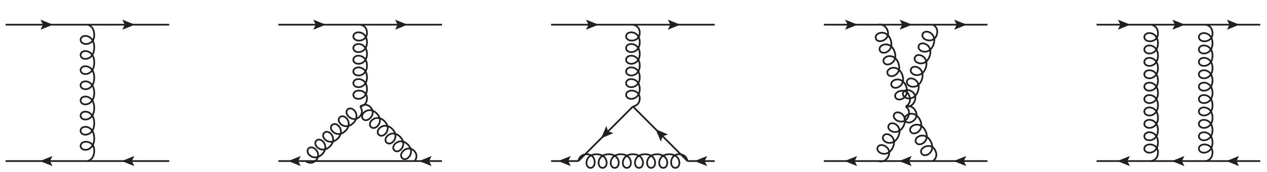
$$F_2 \sim \text{DCSB}$$

2. The **appearance** of the vertex is dramatically modified by the **dynamics**. The vertex can be phenomenologically expressed as:

$$\Gamma^\mu \sim \gamma^\mu + i\eta\sigma_{\mu\nu} Q^\nu \Delta_B$$

See, e.g., PLB722, 384 (2013)

Basics: Kernel has the Dirac and Pauli terms



+ … = ?

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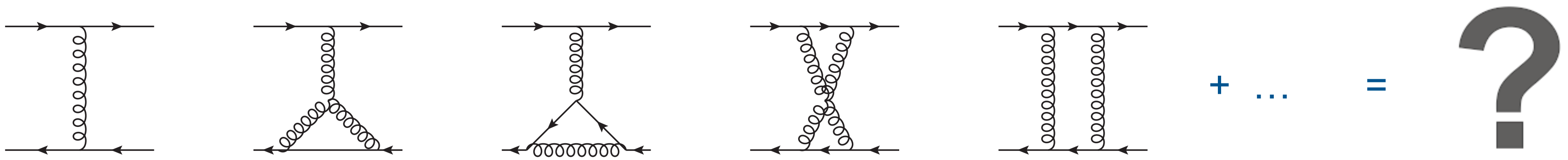
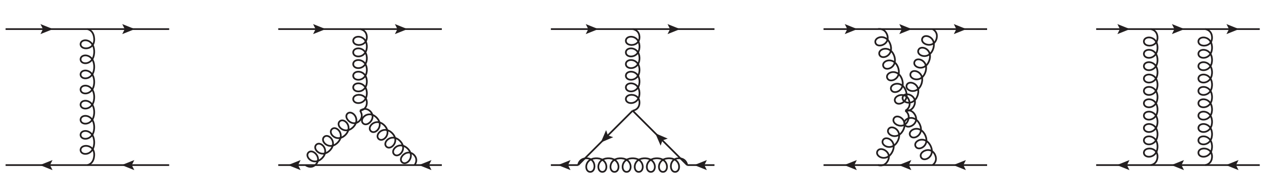


✦ The **discrete** and **continuous** **symmetries** strongly constrain the kernel:

Poincaré symmetry	Gauge symmetry
C-, P-, T-symmetry	Chiral symmetry

Basics: Kernel has the Dirac and Pauli terms



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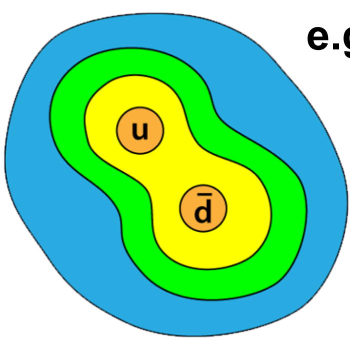
e.g., pion



1. **Bound state** of quark and anti-quark, but abnormally light:

$$M_\pi \ll M_u + M_{\bar{d}}$$

2. **Goldstone's theorem:** If a generic continuous symmetry is spontaneously broken, then new massless scalar particles appear in the spectrum of possible excitations.

Basics: Kernel has the Dirac and Pauli terms

✦ Proper decomposition:

$$K^{(2)} = \left[K_{L0}^{(+)} \otimes K_{R0}^{(-)}\right] + \left[K_{L0}^{(-)} \otimes K_{R0}^{(+)}\right] + \left[K_{L1}^{(-)} \otimes_{+} K_{R1}^{(-)}\right] + \left[K_{L1}^{(+)} \otimes_{+} K_{R1}^{(+)}\right] + \left[K_{L2}^{(-)} \otimes_{-} K_{R2}^{(-)}\right] + \left[K_{L2}^{(+)} \otimes_{-} K_{R2}^{(+)}\right]$$

$$\text{with} \quad \gamma_5 K^{(\pm)} \gamma_5 = \pm K^{(\pm)}, \quad \otimes_{\pm} := \tfrac{1}{2}(\otimes \pm \gamma_5 \otimes \gamma_5)$$

discrete

✦ Deformed WTIs:

$$\begin{aligned}
\Sigma_B(k_+) &= \int_{dq} \left\{K_{L0}^{(+)}[\Delta_{\sigma_A}^{\pm}]K_{R0}^{(-)} - K_{L1}^{(-)}[\sigma_B(q_+)]K_{R1}^{(-)} + K_{L1}^{(+)}[\sigma_B(q_-)]K_{R1}^{(+)}\right\} \\
0 &= \int_{dq} \left\{K_{L0}^{(+)}[\sigma_B(q_-)]K_{R0}^{(-)} - K_{L0}^{(-)}[\sigma_B(q_+)]K_{R0}^{(+)} + K_{L2}^{(+)}[\Delta_{\sigma_A}^{\pm}]K_{R2}^{(+)}\right\} \\
[\Sigma_A(k_+) - \Sigma_A(k_-)] &= \int_{dq} \left\{K_{L0}^{(+)}[-\sigma_B(q_+)]K_{R0}^{(-)} + K_{L0}^{(-)}[\sigma_B(q_-)]K_{R0}^{(+)} + K_{L2}^{(-)}[\Delta_{\sigma_A}^{\pm}]K_{R2}^{(-)}\right\} \\
-\Sigma_B(k_-) &= \int_{dq} \left\{K_{L0}^{(-)}[\Delta_{\sigma_A}^{\pm}]K_{R0}^{(+)} + K_{L1}^{(-)}[\sigma_B(q_-)]K_{R1}^{(-)} + K_{L1}^{(+)}[-\sigma_B(q_+)]K_{R1}^{(+)}\right\}
\end{aligned}$$

continuous

See, e.g., CPL 38 (2021) 7, 071201

Basics: Kernel has the Dirac and Pauli terms

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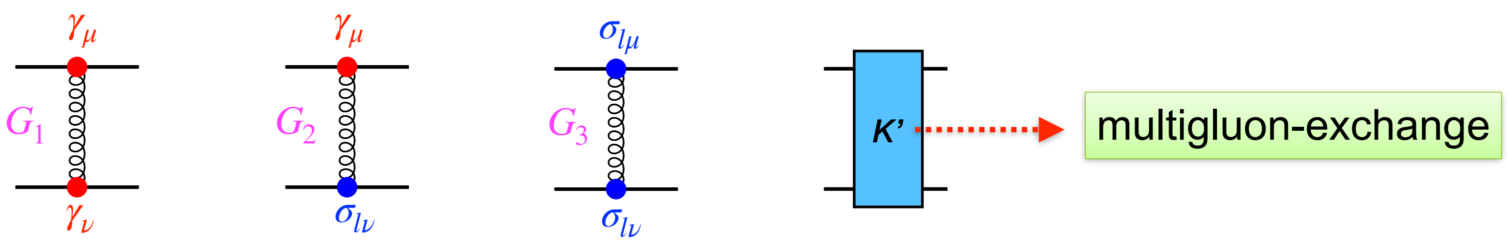
[\Sigma_A(k_+) - \Sigma_A(k_-)] &= \int_{dq} \left\{K_{L0}^{(+)}[-\sigma_B(q_+)]K_{R0}^{(-)} + K_{L0}^{(-)}[\sigma_B(q_-)]K_{R0}^{(+)} + K_{L2}^{(-)}[\Delta_{\sigma_A}^{\pm}]K_{R2}^{(-)}\right\} \\
-\Sigma_B(k_-) &= \int_{dq} \left\{K_{L0}^{(-)}[\Delta_{\sigma_A}^{\pm}]K_{R0}^{(+)} + K_{L1}^{(-)}[\sigma_B(q_-)]K_{R1}^{(-)} + K_{L1}^{(+)}[-\sigma_B(q_+)]K_{R1}^{(+)}\right\}
\end{aligned}$$

continuous

1. A realistic kernel must involves the Dirac and Pauli structures:



See, e.g., CPL 38 (2021) 7, 071201

Basics: Kernel has the Dirac and Pauli terms

✦ Proper decomposition:

$$K^{(2)} = \left[K_{L0}^{(+)} \otimes K_{R0}^{(-)}\right] + \left[K_{L0}^{(-)} \otimes K_{R0}^{(+)}\right] + \left[K_{L1}^{(-)} \otimes_+ K_{R1}^{(-)}\right] + \left[K_{L1}^{(+)} \otimes_+ K_{R1}^{(+)}\right] + \left[K_{L2}^{(-)} \otimes_- K_{R2}^{(-)}\right] + \left[K_{L2}^{(+)} \otimes_- K_{R2}^{(+)}\right]$$

$$\text{with} \quad \gamma_5 K^{(\pm)} \gamma_5 = \pm K^{(\pm)}, \quad \otimes_\pm := \tfrac{1}{2}(\otimes \pm \gamma_5 \otimes \gamma_5)$$

discrete

✦ Deformed WTIs:

$$\begin{aligned}
\Sigma_B(k_+) &= \int_{dq} \left\{ K_{L0}^{(+)}[\Delta_{\sigma_A}^{\pm}]K_{R0}^{(-)} - K_{L1}^{(-)}[\sigma_B(q_+)]K_{R1}^{(-)} + K_{L1}^{(+)}[\sigma_B(q_-)]K_{R1}^{(+)} \right\} \\
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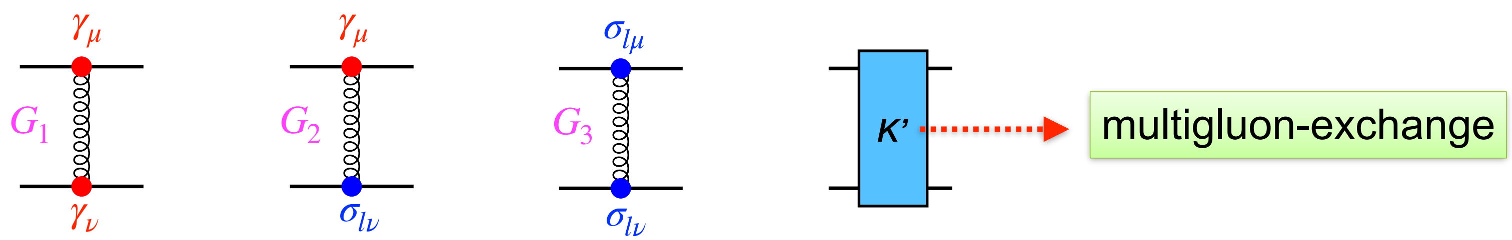


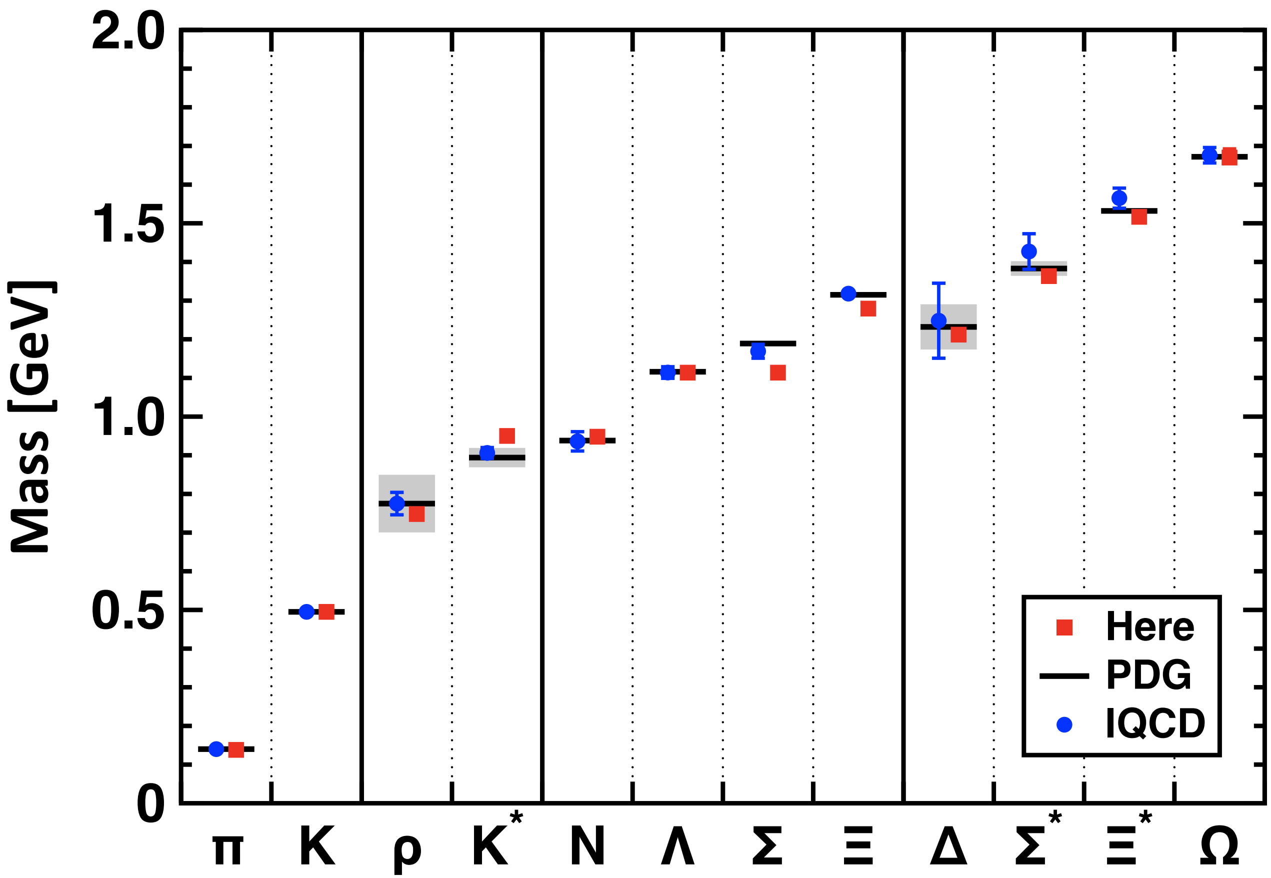
2. G_2 and G_3 are proportional to the Pauli term in the vertex, and thus to **DCSB**:

$$G_2, G_3 \sim \mathrm{DCSB}$$

See, e.g., CPL 38 (2021) 7, 071201

Ground states

Ground states: Light & Strange flavor spectra



The interaction strength and current quark masses are fixed by properties of pseudo-scalar mesons, e.g., pion, kaon, and etc.

See, e.g., Few-Body Syst 60, 26 (2019)

Ground states: Light & Strange flavor spectra

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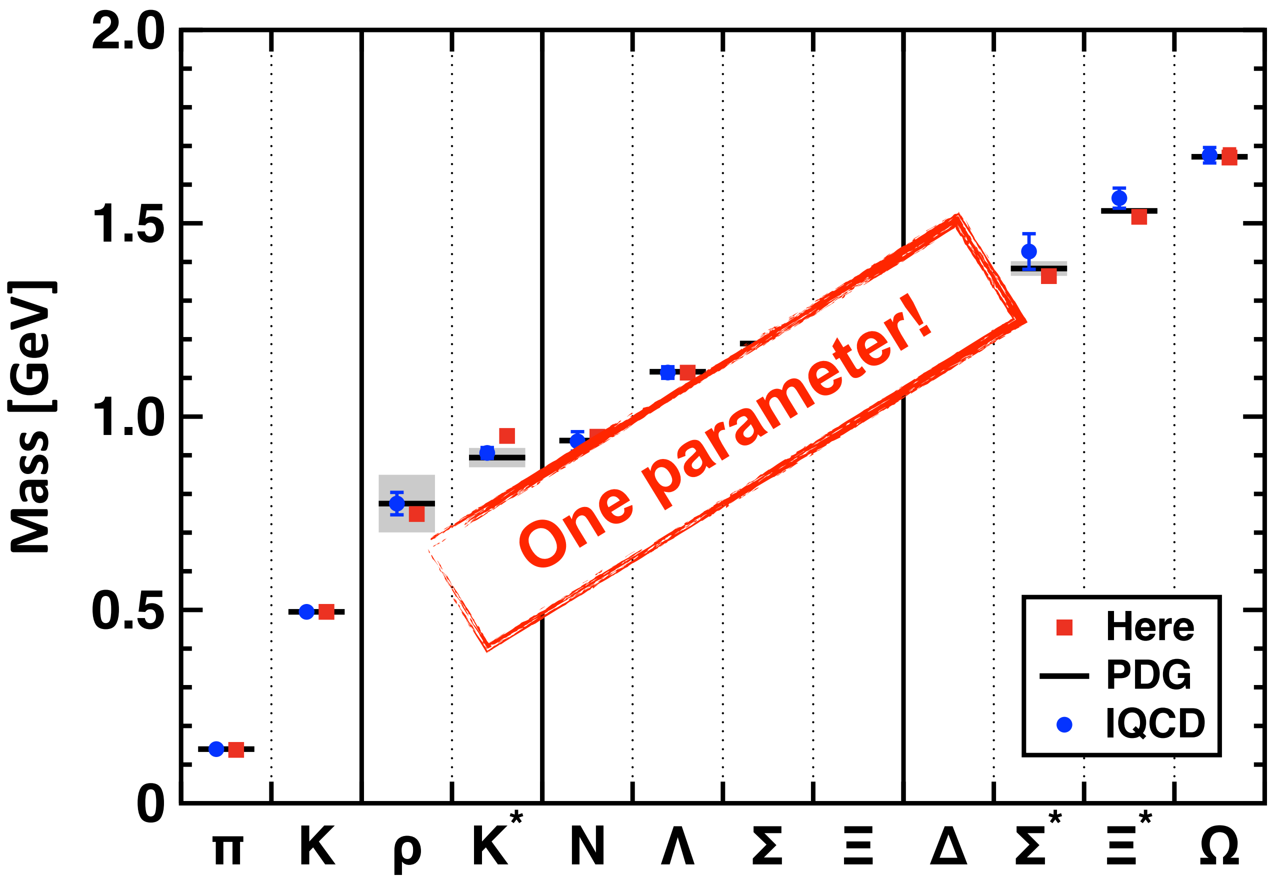

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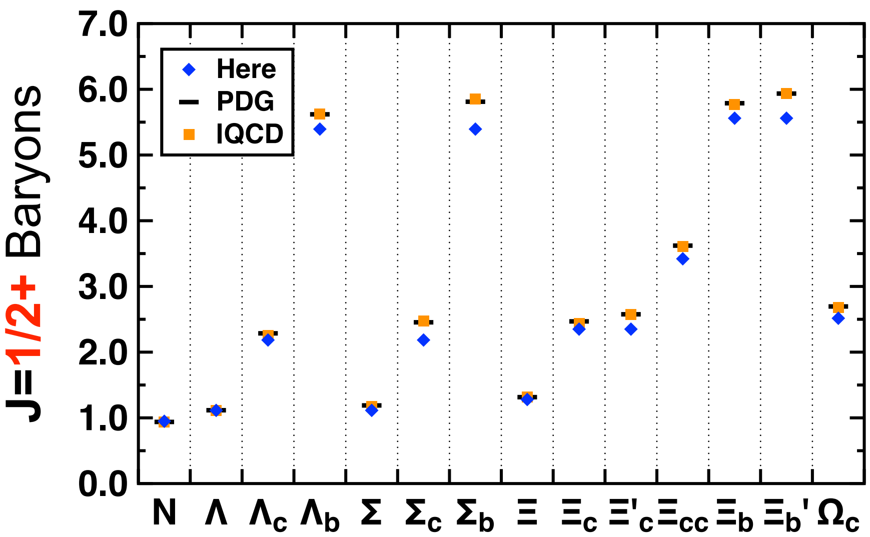
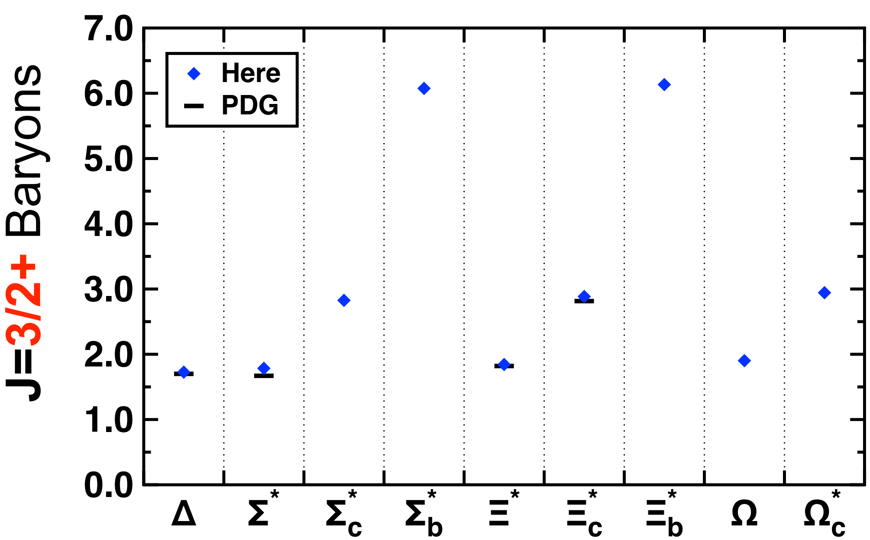
Ground states: Charm & Bottom flavor spectra





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Ground states: Charm & Bottom flavor spectra

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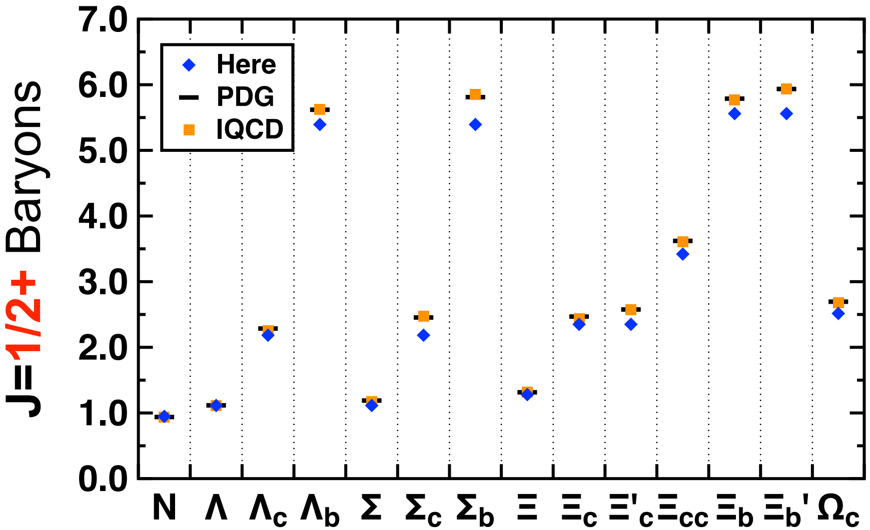
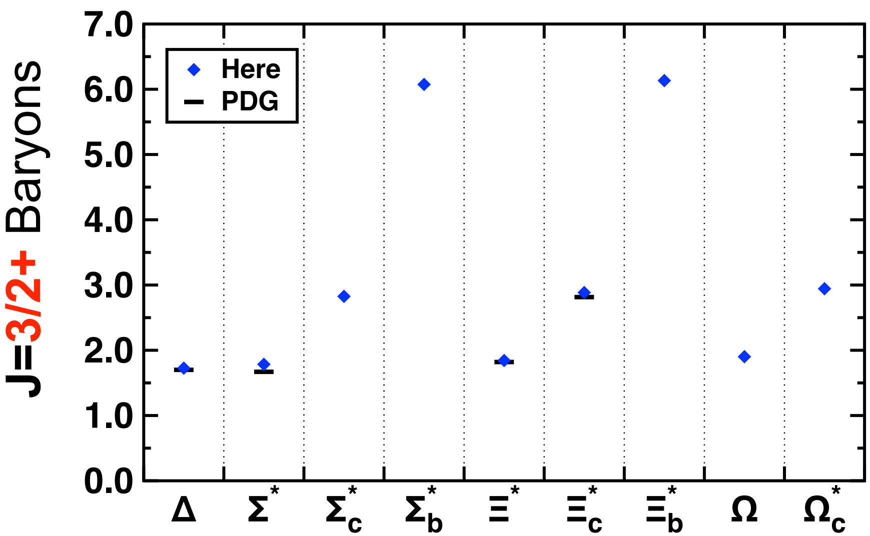
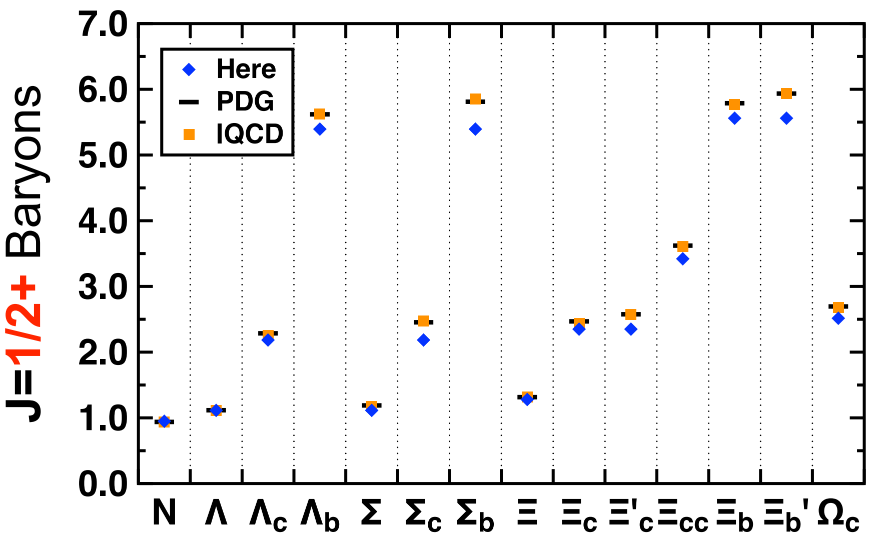


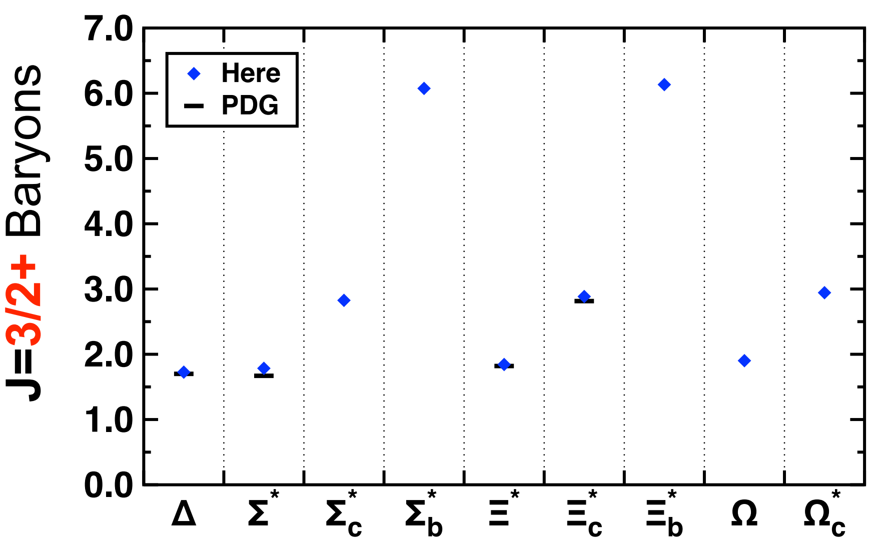


- The mean-absolute-relative-difference between the calculated values for the ground-states and the known empirical masses is about 5%.

See, e.g., Few-Body Syst 60, 26 (2019)

Ground states: Charm & Bottom flavor spectra



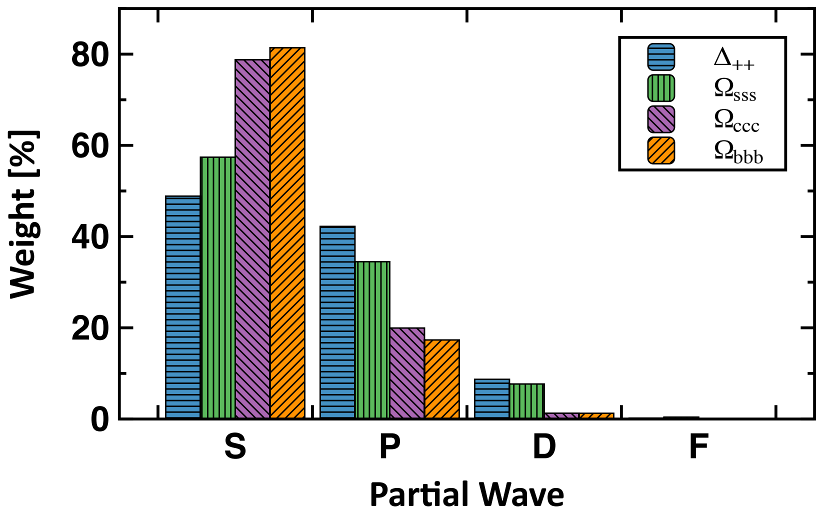


- ✦ The mean-absolute-relative-difference between the calculated values for the ground-states and the known empirical masses is about 5%.
- ✦ The ground spectra is NOT sensitive to the structures beyond the leading terms in the vertex and the kernel.

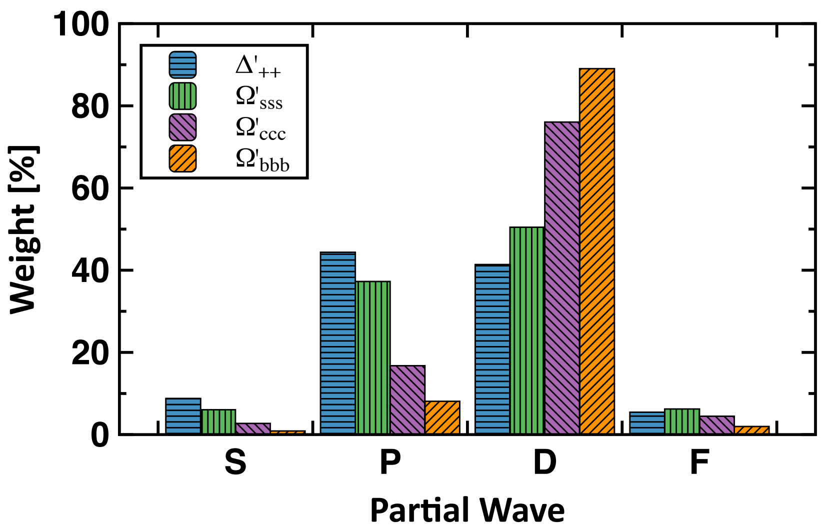
See, e.g., Few-Body Syst 60, 26 (2019)

Excited states

Excited states: Multiple partial waves



✓ S-waves dominate for ground states, but P-waves grow for light baryons.



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See, e.g., PRD 97, 114017 (2018)

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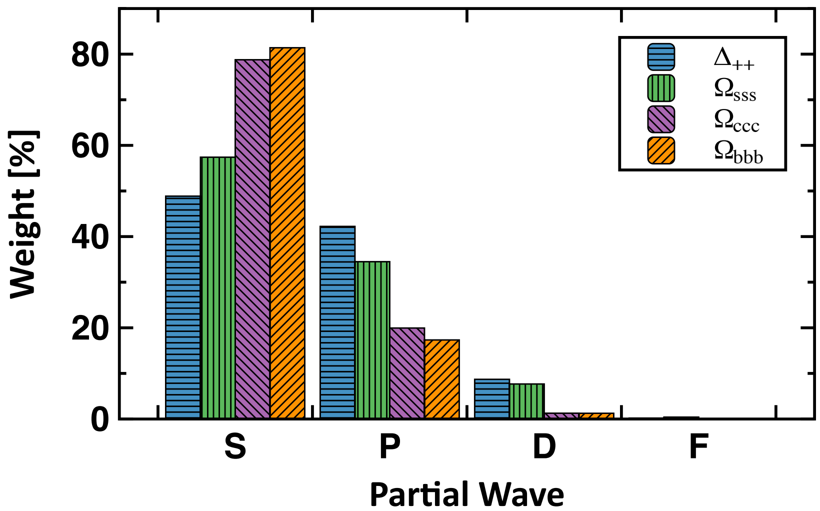

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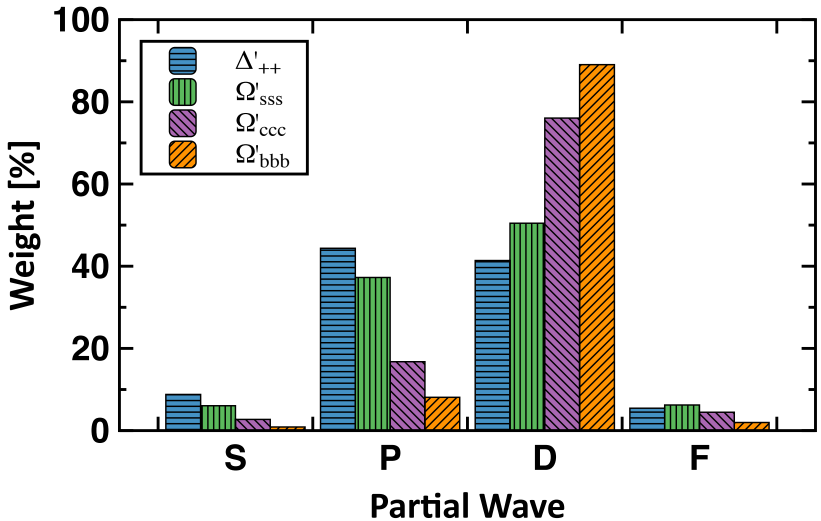
Why NR potential models work



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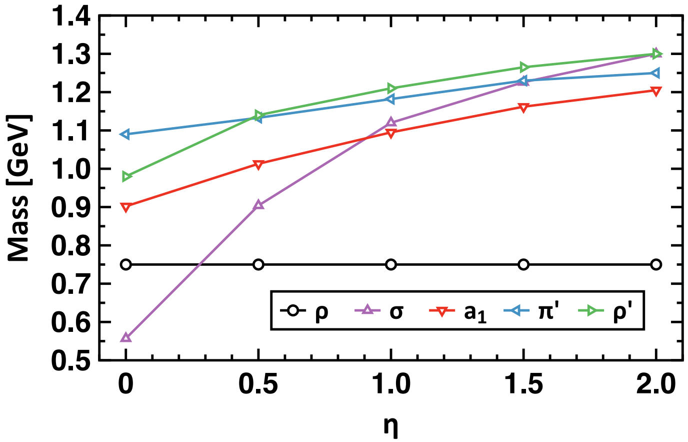
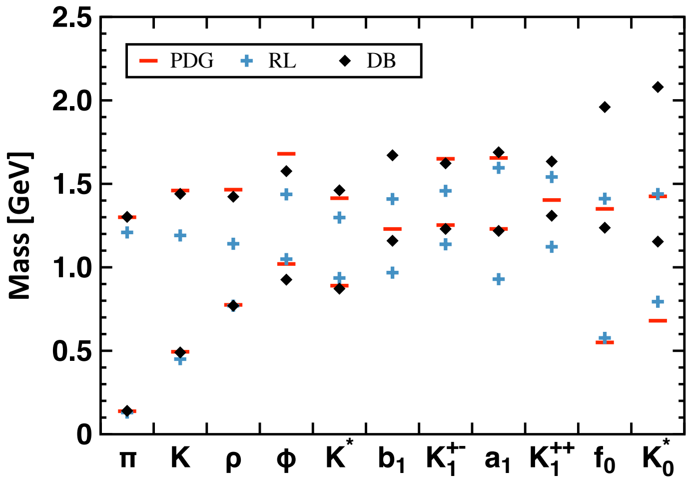
Excited states: Spin-orbit interaction

➡ Impact of the Pauli term (**a**nomalous **m**oment):



➡ Light & strange meson spectrum:



See, e.g., CPL 38, 071201 (2021) & EPJA 59, 39 (2023)

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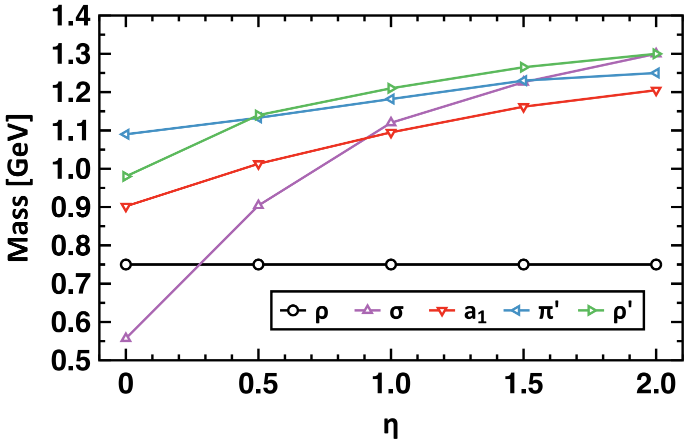
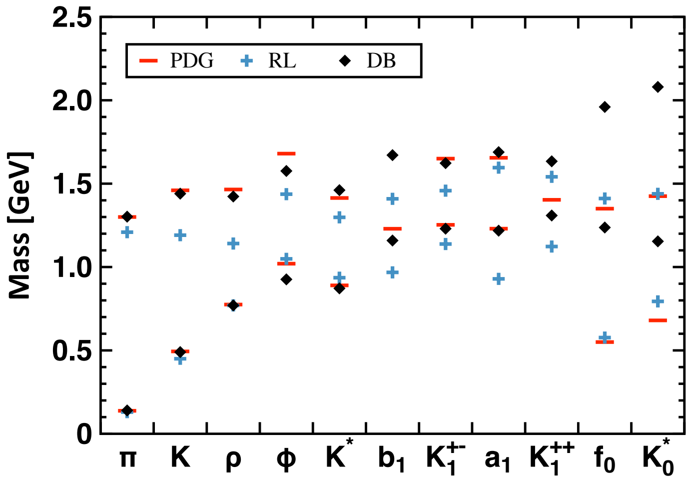


✦ With increasing the AM strength, the $a_1-\rho$ mass-splitting rises very rapidly. From a quark model perspective, the DCSB-enhanced kernel increases spin-orbit repulsion.

➡ Light & strange meson spectrum:



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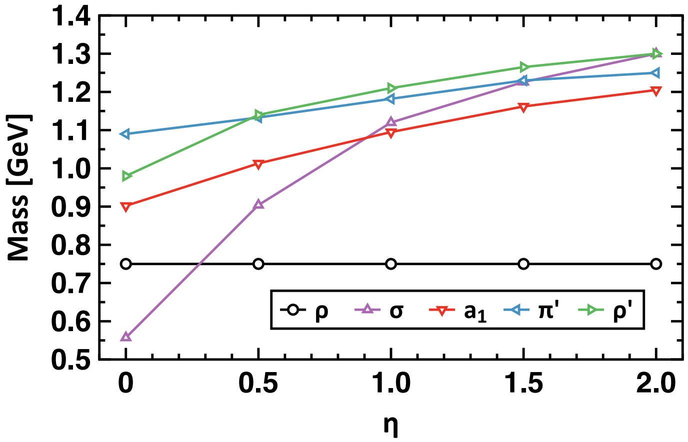

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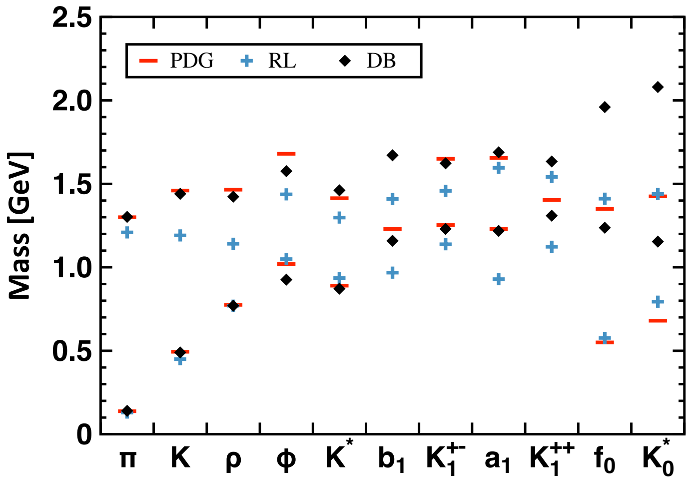
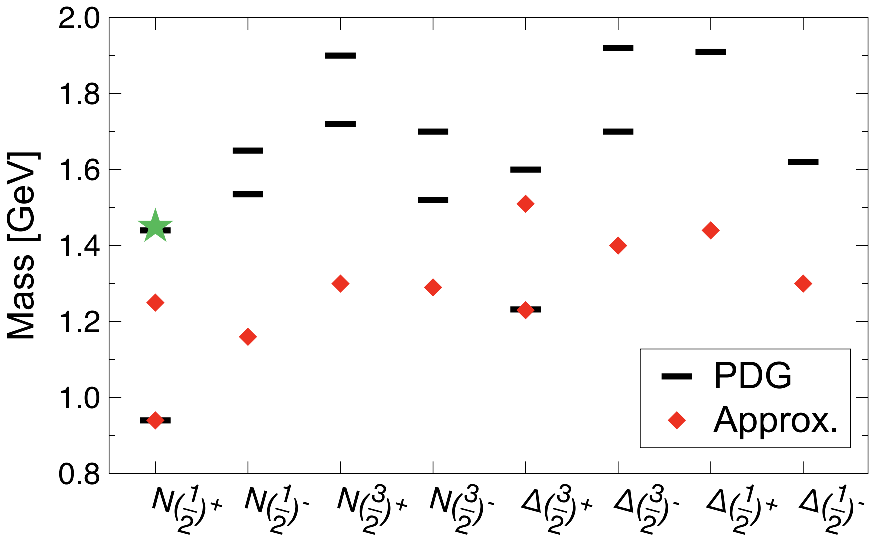
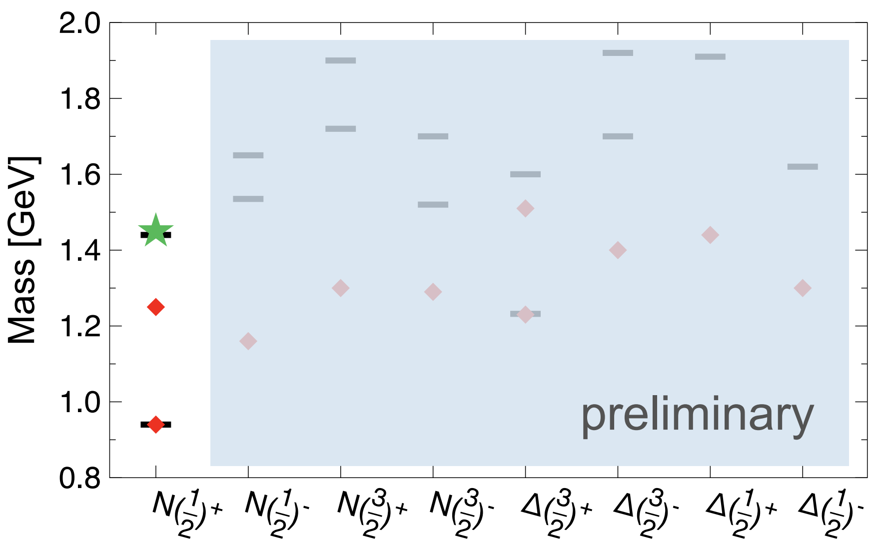


✦ The magnitude and ordering of all excitation states can be fixed with the DCSB-enhanced kernel.

See, e.g., CPL 38, 071201 (2021) & EPJA 59, 39 (2023)

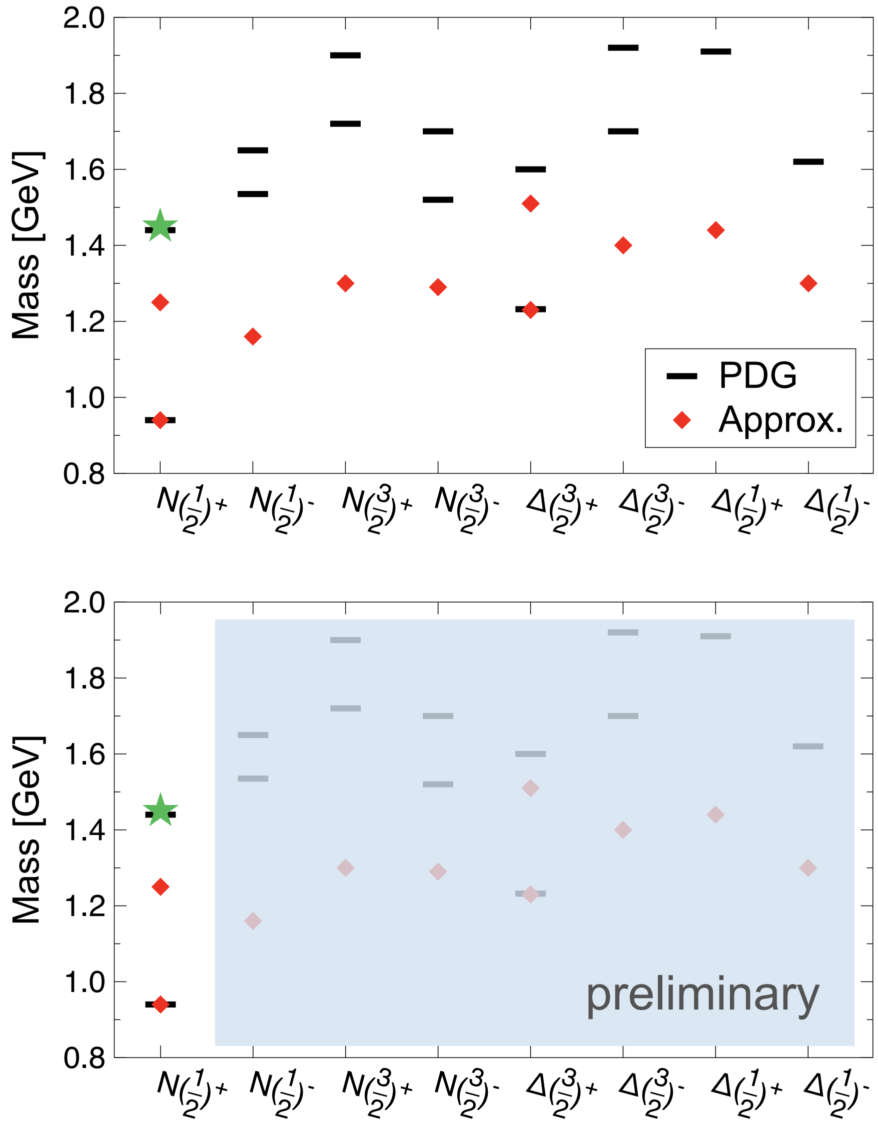
Excited states: DCSB-rendered spectra





In progress

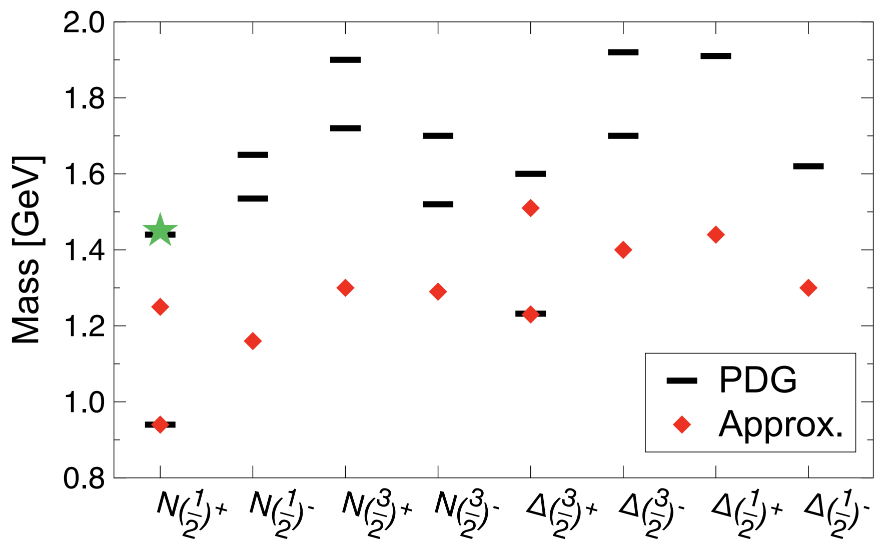
Excited states: DCSB-rendered spectra

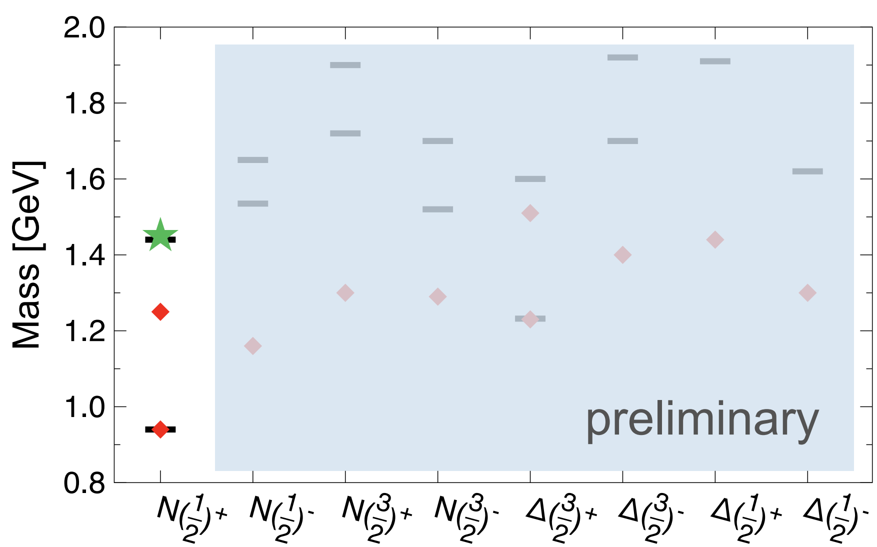


✦ The magnitude and ordering of radial or angular excitation states are WRONG in the approximation lacking of DCSB effect.

In progress

Excited states: DCSB-rendered spectra





- The magnitude and ordering of radial or angular excitation states are WRONG in the approximation lacking of DCSB effect.

- The DCSB-enhanced kernel boost up 1st excitation nucleon, and can potentially fix the full spectra.

In progress

Summary

✦ The framework of few-body bound-state equations, which describes hadrons in continuum QCD, and its basics (e.g., quark, gluon, vertex, kernel) are introduced.

✦ Baryon properties are studied: **a**) ground states — full mass spectrum of J=0, 1/2, 1, 3/2; **b**) excited states — partial waves, spin-orbit interaction, DCSB-rendered spectra.

Summary

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- The framework of few-body bound-state equations, which describes hadrons in continuum QCD, and its basics (e.g., quark, gluon, vertex, kernel) are introduced.

- Baryon properties are studied: **a**) ground states — full mass spectrum of J=0, 1/2, 1, 3/2; **b**) excited states — partial waves, spin-orbit interaction, DCSB-rendered spectra.

Outlook

- Use the three-body Faddeev equation to a wider range of applications in baryon problems of QCD: transition form factors, parton distribution functions, and etc.

- Hopefully, iterating with future high precision experiments on **light** and **heavy** hadrons, from spectroscopy to structures, we may provide a **faithful path** to understand **QCD**.