

# 《核与粒子物理实验方法》第1次作业 \*

1-2:

答:  $\frac{dN_b}{dt} = \lambda_a N_a - \lambda_b N_b$   
 $\frac{dN_a}{dt} = -\lambda_a N_a$   
 $N_b(t) = \frac{\lambda_a}{\lambda_b - \lambda_a} N_a(0)(e^{-\lambda_a t} - e^{-\lambda_b t})$

1-3:

答: 1)  $A(0) = \lambda N(0) = \ln 2 / T_{1/2} N(0) = 2.2 \times 10^8$   
2) 平衡时,  $\lambda_n N_n(t) = \lambda_1 N_1(t) \Rightarrow N_n(t) = \frac{\lambda_1}{\lambda_n} N_1(0) e^{-\lambda_1 t}$  将平衡时, 由 1-2,  $N_n(t) = \frac{\lambda_1}{\lambda_n} N_1(0)(e^{-\lambda_1 t} - e^{-\lambda_n t})$   
 $0.97 \frac{\lambda_1}{\lambda_n} N_1(0) e^{-\lambda_1 t} = \frac{\lambda_1}{\lambda_n} N_1(0)(e^{-\lambda_1 t} - e^{-\lambda_n t})$   $t = \ln 0.03 / (\lambda_1 - \lambda_n) = 101a$

1-4:

答:  $N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(0)(e^{-\lambda_1 t} - e^{-\lambda_2 t})$  当  $dN_2(t)/dt = 0$ ,  $t = \frac{1}{\lambda_1 - \lambda_2} \ln \frac{\lambda_1}{\lambda_2} = 501s$

1-7:

答:

Cs-137

	Energy (keV)	Probability (%)	Nature	$\log ft$
$\beta_{0,2}^-$	513,97 (17)	94,57 (26)	Forbidden 1 <sup>st</sup> unique	9,7
$\beta_{0,1}^-$	892,17 (18)	0,0006 (1)	Forbidden 2 <sup>nd</sup> unique	16,6
$\beta_{0,0}^-$	1175,63 (17)	5,43 (26)	Forbidden 2 <sup>nd</sup> non-unique	12,8

$$A/m = \lambda N / (\frac{N}{N_A} * 60g/mol) \Rightarrow m = 8.8mg$$

1-8

答: 1) 禁止, 反应前质量小于反应后 2) 允许 3) 禁止, 电荷不守恒 4) 允许

1-9

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答:  $Q = \Delta U - \Delta La - \Delta Br = 177.5 MeV$

### 1-10

答: 1)  $V = e^2/(4\pi\epsilon r) = 1.027 MeV, T = V/2, \frac{3}{2}kt = T, T = 3.88 \times 10^9 K$

2)  $Q = 2\Delta(^2_1H) - \Delta(^3_1H) = 4.02 MeV$

### 1-11

答: 1)  $R = r_0 A^{1/3}, r_0 \simeq 1.2 fm, V = \frac{4}{3}\pi R^3, \rho = M/V = (A/N_A)\frac{1}{V} = 2.3 \times 10^{17} kg \cdot m^{-3}$

2)  $R \simeq 10^4 m$

### 1-12

答: 书 1.22 公式,  $\beta^-$  和  $\beta^+$  衰变均可发生

### 1-13

答:  $\lambda - \lambda_0 = \frac{h}{mc}(1 - \cos\theta)$

动能差为  $\Delta E_k = h\Delta\frac{c}{\lambda} = -hc\frac{\Delta\lambda}{\lambda^2} = 2E_\gamma^2/(mc^2)$

$E_\gamma = (\frac{1}{2}mc^2 E_k)^{1/2}$

反冲动能 5.7MeV, 光子能量 52MeV

反冲动能 1.4MeV, 光子能量 96MeV

### 1-14

答:  $\frac{d\sigma}{d\Omega} = \frac{1}{L} \frac{dN}{d\Omega}, L = FN_A at \rho A^{-1}$

通量 F, 面积 a, 厚度 t, 密度  $\rho$

散射到立体角  $\Delta\Omega$  概率:

$\eta = dN/N_{in} = \frac{dN}{d\Omega} N_A at \rho A^{-1} \Delta\Omega$

$\Delta\Omega = S/r^2$

$\frac{d\sigma}{d\Omega} = \frac{a^2}{16\sin^4\frac{\theta}{2}}, a = \frac{e^2 Z_1 Z_2}{4\pi\epsilon_0 E}$

当  $\eta = 150$  时最小, 代入得到  $\eta_{min} = 3.75 \times 10^{-7}, \eta_{min} N_{in} > 10/s, N_{in} > 2.66 \times 10^7$

### 1-15

答: 衍射极小时,  $F(q) \sim 0$ , 求  $y = \tan x$  和  $y = x$  的,  $x$  大于 0 且有上限

$x_{max} = q_{max}R/\hbar, q = 2psin\frac{\theta}{2}, q_{max} = 2E/c$

代入  $x_{max} = 11.7$ , 在  $0 < x < 11.7$  里有三个解

### 1-16

答:

铅临界能量

$E_c = \frac{A}{Z+1.24}$

带电粒子辐射损失能量:  $E = E_0 e^{-X/X_0}$

代入  $\Delta E = E_0 - E = 41.42 \text{ MeV}$

曲率半径  $r = p/300ZB$ , p 单位是  $\text{MeV}$

$r_0 = 14 \text{ cm}, r_1 = 4.8 \text{ cm}, \Delta r/r_0 = 65.7\%$

### 1-19

答：书 1.46 相对论卢瑟福散射公式，代入数据得到卢瑟福散射截面  $64 \text{ mb}$ , 书 1.48 可以得到 Mott 散射截面  $63 \text{ mb}$

$$\eta = dN/N_{in} = \frac{dN}{d\Omega} N_A at \rho A^{-1} \Delta\Omega$$

$$\Delta\Omega = S/r^2$$

散射角 15 度，概率  $\eta = 2.37 \times 10^{-6}$

$N_{in} = I/e = 6.25 \times 10^{13}/\text{s}$  电子计数率为  $N_{in}\eta = 1.48 \times 10^8/\text{s}$

### 1-20

答：

$$p = p' \cos\theta + P' \cos\phi$$

$$p' \sin\theta = P' \sin\phi$$

$$E + M = E' + E'_P$$

$$\rightarrow E' = \frac{E}{1 + \frac{E}{M}(1 - \cos\theta)}$$

### 1-22

答： 2)  $\lambda = \frac{\hbar c}{E} \simeq 200 \text{ MeV fm}/E, E = 20 \text{ MeV}$

3) 探针最好是带电轻子，中子光子不带电，中子有结构

### 1-23

答：

动量转移大小 (书 1.45):  $q = 2q \sin\frac{\theta}{2} = 87.16 \text{ MeV}$

Mott 散射，书 1.46 和 1.48, 可得截面  $188.1 \text{ mb}$

均匀分布微分截面，形状因子  $F(q^2) = 3(\sin x - x \cos x)x^3, x = |q|R, r = r_0 A^{1/3}$

可得  $F(q^2) = 45.08$

由 1.49 可得截面  $8.48 \text{ b}$

### 1-25 答：

- 1) 轻子数不守恒
- 2) 电荷不守恒
- 3) 重子数不守恒
- 4) 能动量不守恒
- 5) 轻子数不守恒
- 7) 轻子数不守恒

### 1-27

答: 在末态粒子动量中心系,  $P_i^\mu = (E_1 + m_2, P_1)$ ,  $P_f^\mu = (\Sigma E_{M_i}, 0)$

$$(E_{ath} + m_2)^2 - p_1^2 = (\Sigma M_i)^2$$

$$E_{ath}^2 - p_1^2 = m_1^2$$

$$E_{ath} = ma + T_{ath}$$

$$T_{ath} = \frac{(\Sigma M_i)^2 - (m_1 + m_2)^2}{2m_2}$$

### 1-28

答:

- a) 奇异数不守恒
- b) 重子数不守恒; 电荷不守恒
- c) 重子数不守恒;
- d) 光子, 电磁相互作用; 奇异数不守恒
- e) 两个电荷不守恒

### 1-30

答: 1)  $y = \frac{1}{2} \ln\left(\frac{1+p/E}{1-p/E}\right) = \frac{1}{2} \ln\left(\frac{1+\beta}{1-\beta}\right)$

2)  $\beta = 0, y_{min} = 0, \beta = \beta_p, y_{max} = 9.5367$

3)  $y = \frac{1}{2} \ln\left(\frac{1+\cos\theta}{1-\cos\theta}\right) = -\ln\tan\frac{\theta}{2} = \eta$

4)  $\theta = \pi/2, \eta = 0, y = 0$

$\theta = 1^\circ, \eta = 4.74, y = 4.74$

### 1-31

答:

- 1) 夸克自旋  $1/2$ , 3 个夸克不能组成自旋为整的重子
- 2) 反夸克带电为  $+1/3$  或  $-2/3$
- 3) s 夸克奇异数-1, 带电  $-1/3$ , 没有  $4/3$  电荷的反夸克