NNLO corrections to W-pair and H^+H^- production

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Based on [JHEP 12 (2024) 038, JHEP 12 (2024) 136, 2409.08879]

In collaboration with Ren-You Zhang, Wen-Jie He, Zhi-Xing Zhang, Shu-Xiang Li and Xiao-Feng Wang

Outline

I. Research background

II. W-pair production

III. H^+H^- production

Research background

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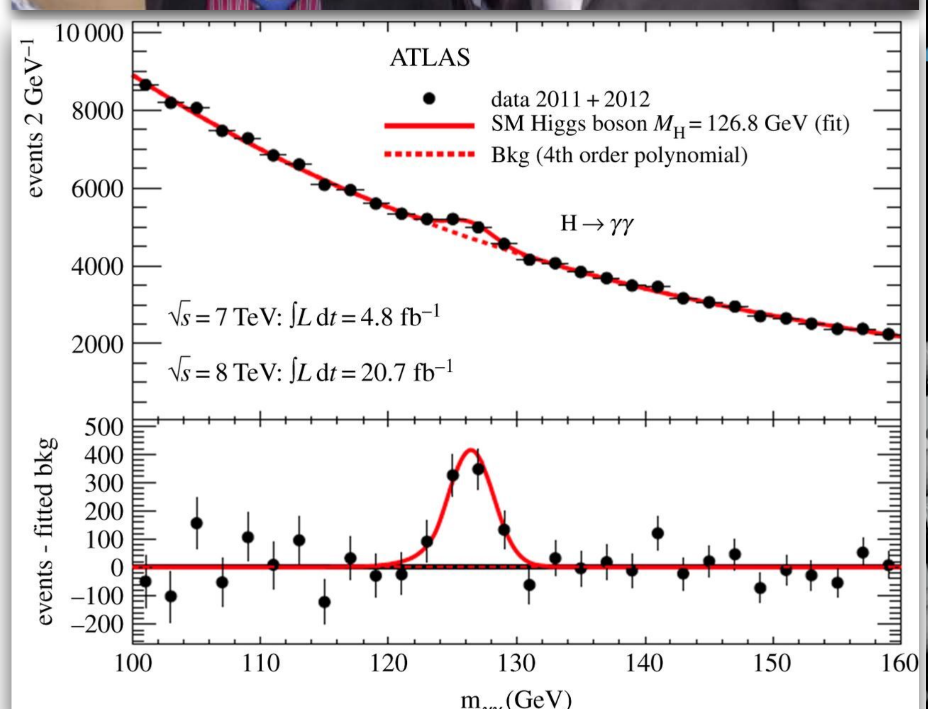
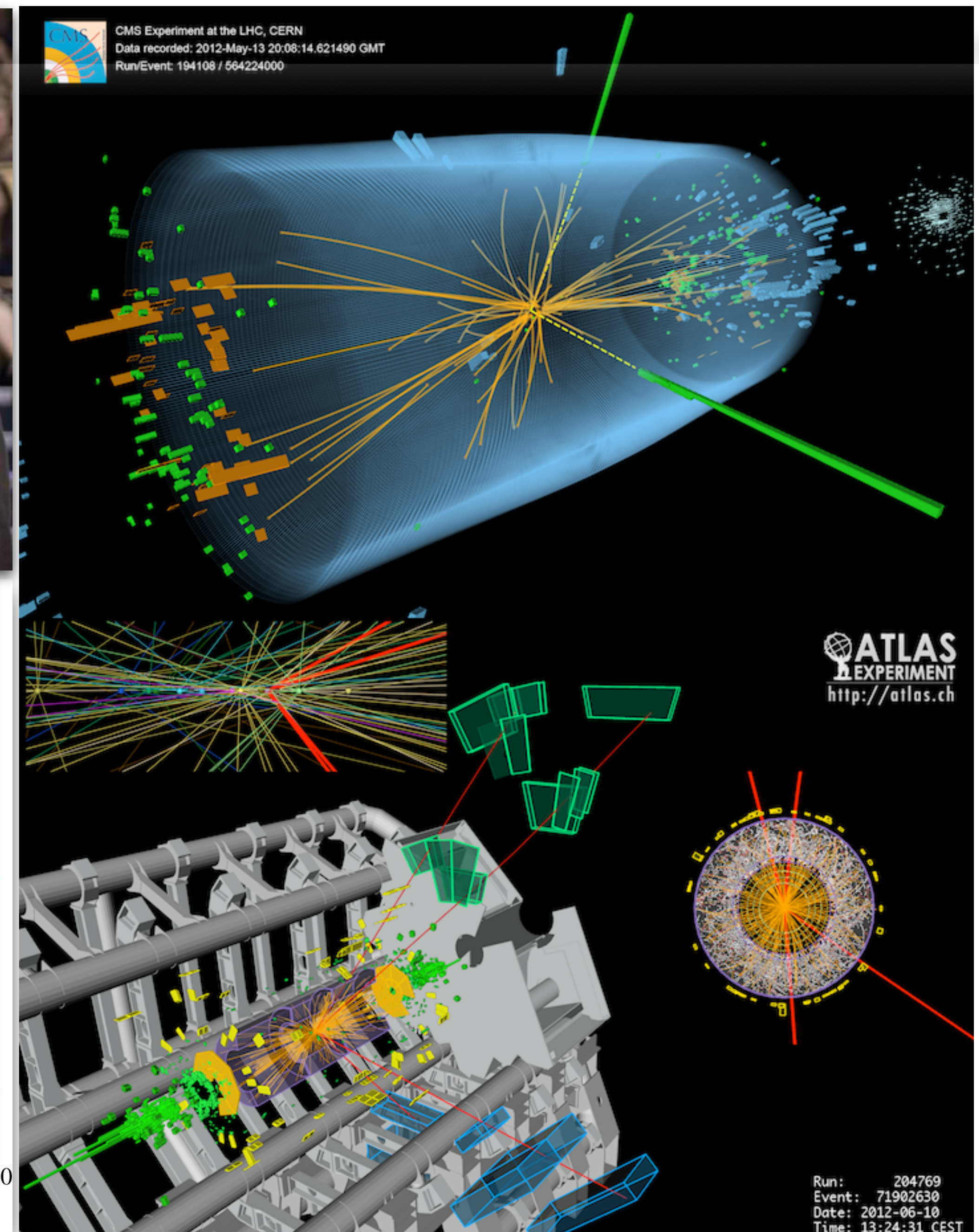
Higgs Discvover

😊 😊 😊 SM Win Win Win ✌️✌️✌️





[CMS, `12]

[ATLAS, `12]

Research background

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W-mass anomaly

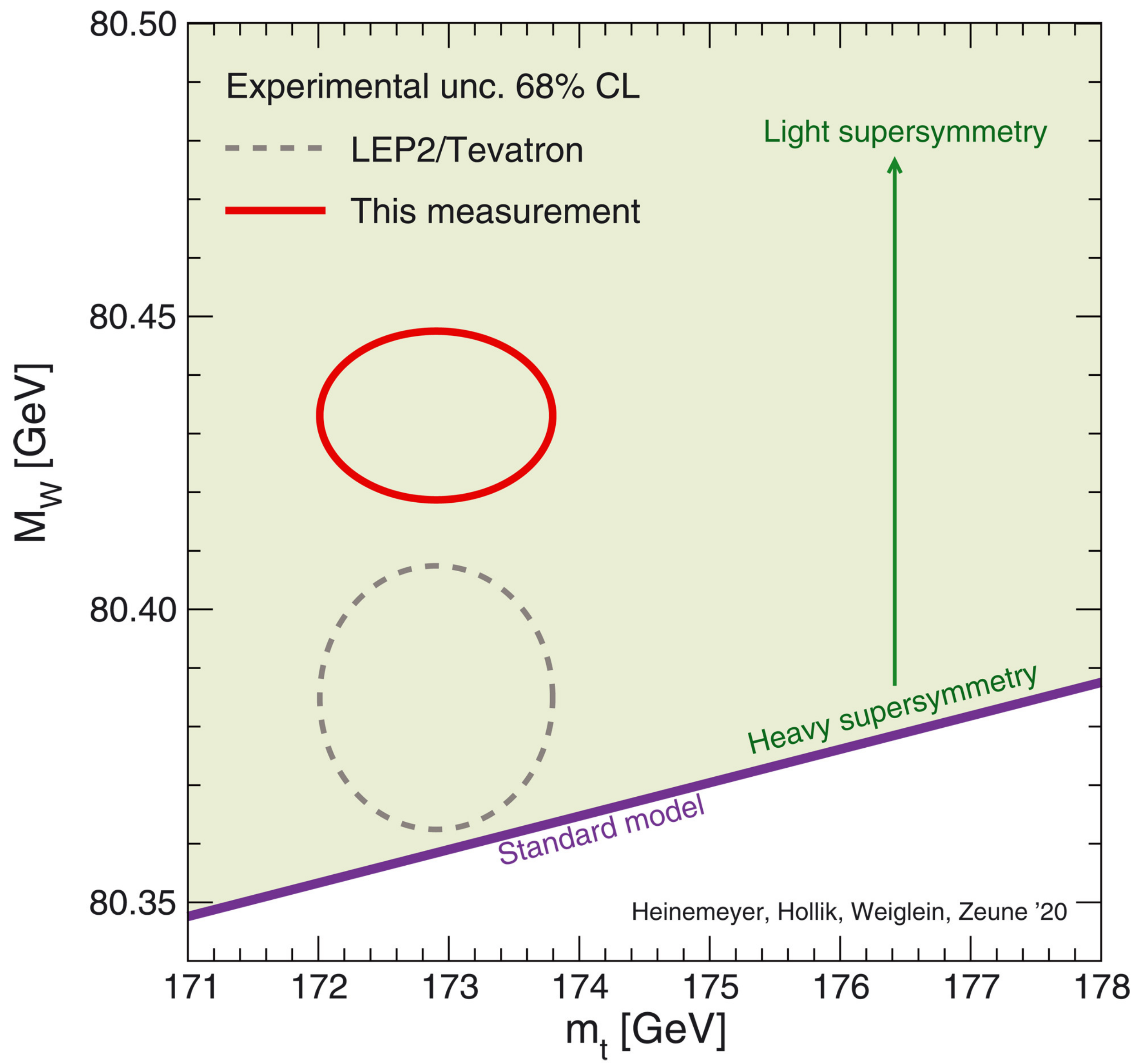
[CDF, `22]

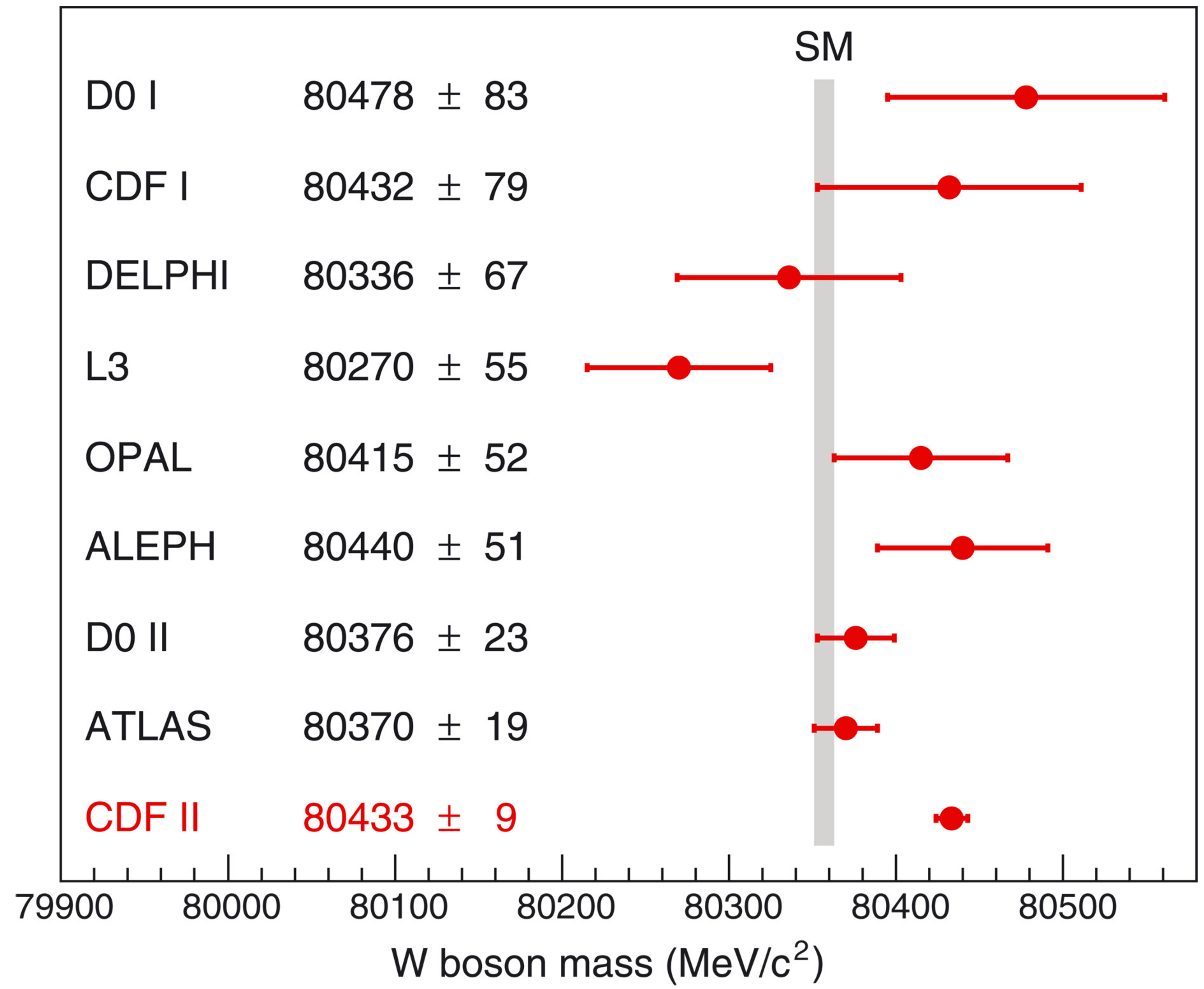
7 σ

CDF II ⟺ **SM**

$$m_W^{\rm CDF} = 80.4335 \pm 0.0094 \mathrm{GeV}$$

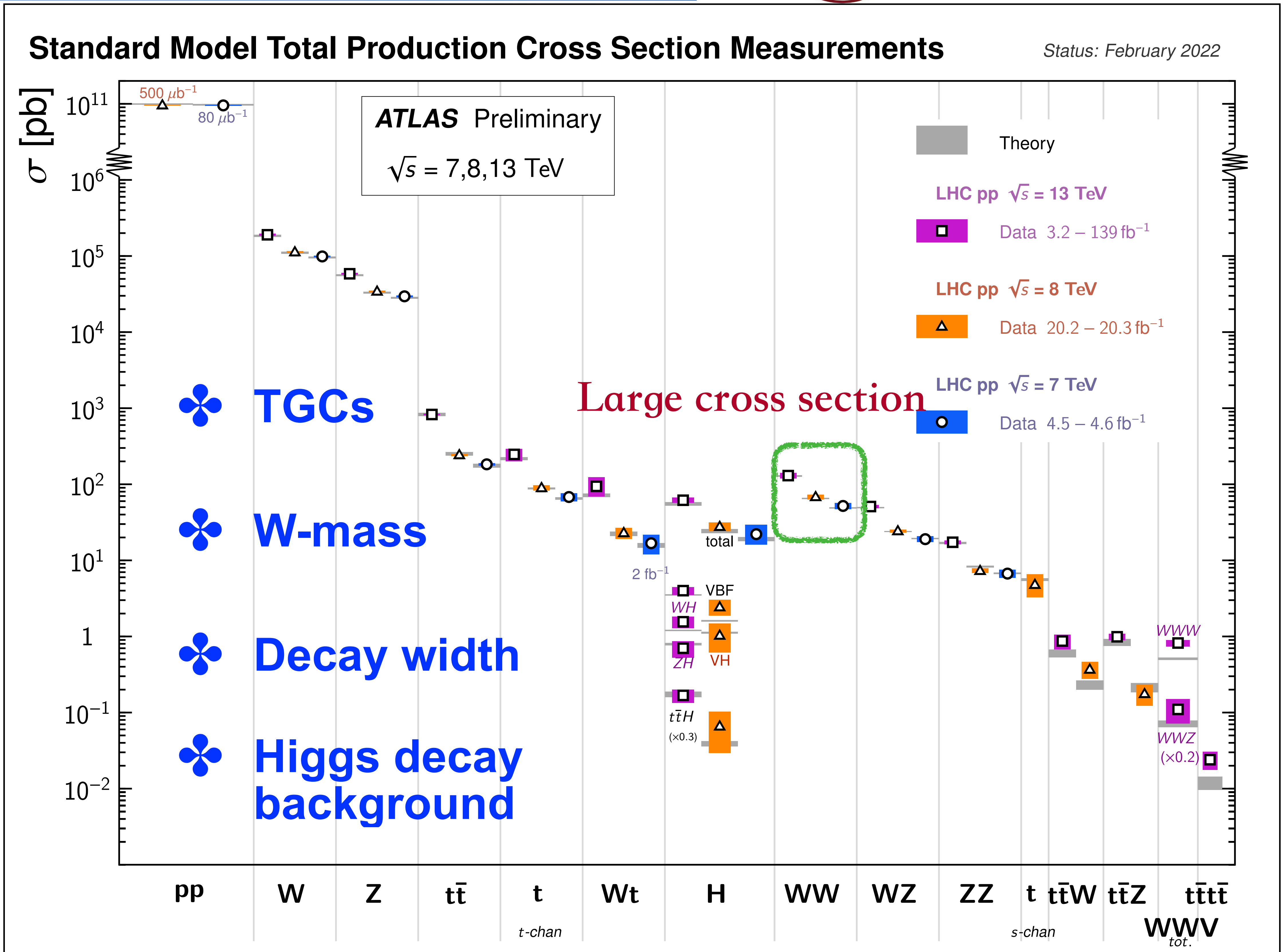
Requires a better understand of W physics!!!





Research background

W-pair



Large cross section

- **TGCs**
- **W-mass**
- **Decay width**
- **Higgs decay background**

Research background

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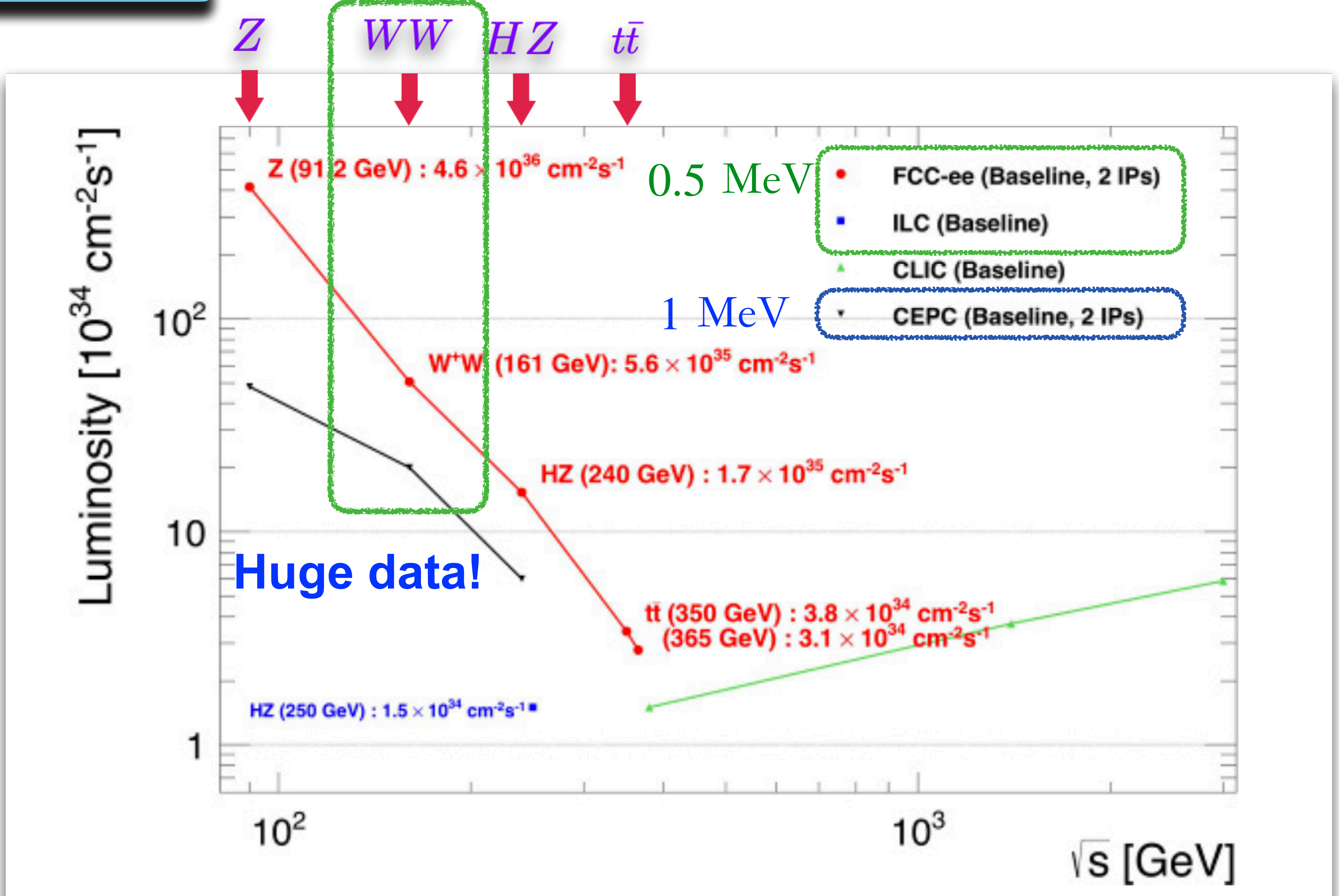
Lepton colliders

Improve theoretical prediction!!!



[ILC, `13]

[CEPC, `18]

[FCC, `19]

NNLO QCD-EW corrections to W-pair production at electron-positron colliders

Based on [JHEP 12 (2024) 038]

With Shu-Xiang Li and Xiao-Feng Wang, Ren-You Zhang, et. al

Calculation setup

- **Consider the following process**

$$e^+(p_1,\lambda_1) + e^-(p_2,\lambda_2) \to W^+(p_3,\lambda_3) + W^-(p_4,\lambda_4)\,,$$

- **Unpolarized differential cross sections**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\beta}{64\pi^2 s}\frac{1}{4}\sum_{\lambda_{1,\ldots,4}} \left|\mathcal{M}(s,t,\lambda_{1,\ldots,4})\right|^2$$

- **Perturbative expansion**

$$\sigma = \boxed{\sigma_{\mathrm{LO}} + \Delta\sigma^{\mathcal{O}(\alpha)}_{\mathrm{NLO}}} \boxed{+\Delta\sigma^{\mathcal{O}(\alpha\alpha_s)}_{\mathrm{NNLO}} + \Delta\sigma^{\mathcal{O}(\alpha^2)}_{\mathrm{NNLO}} + \cdots}$$

$\Downarrow$ **Well Studied** [M. Bohm `88] [A. Denner `93]

$\Downarrow$ **Still Unknown**

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Workflow of calculations

I. Generating Feynman amplitudes

- **FeynArts**
- QGRAF

II. Scattering amplitudes calculation

1. Dirac algebra,Lorentz algebra, color algebra evaluation
2. Tensor reduction
3. IBP reduction (Laporta algorithm)
4. ***MI calculation (CDEs)***

III. UV divergence

- **On-shell renormalization scheme**
- $\overline{\text{MS}}$ renormalization scheme

IV. IR divergence

- **Phase-space slicing method**
- Subtraction method

V. Phase-space integrals

- **Monte-Carlo**
- Reverse unitarity

$e^+e^- \to W^+W^-$@ QCD-EW

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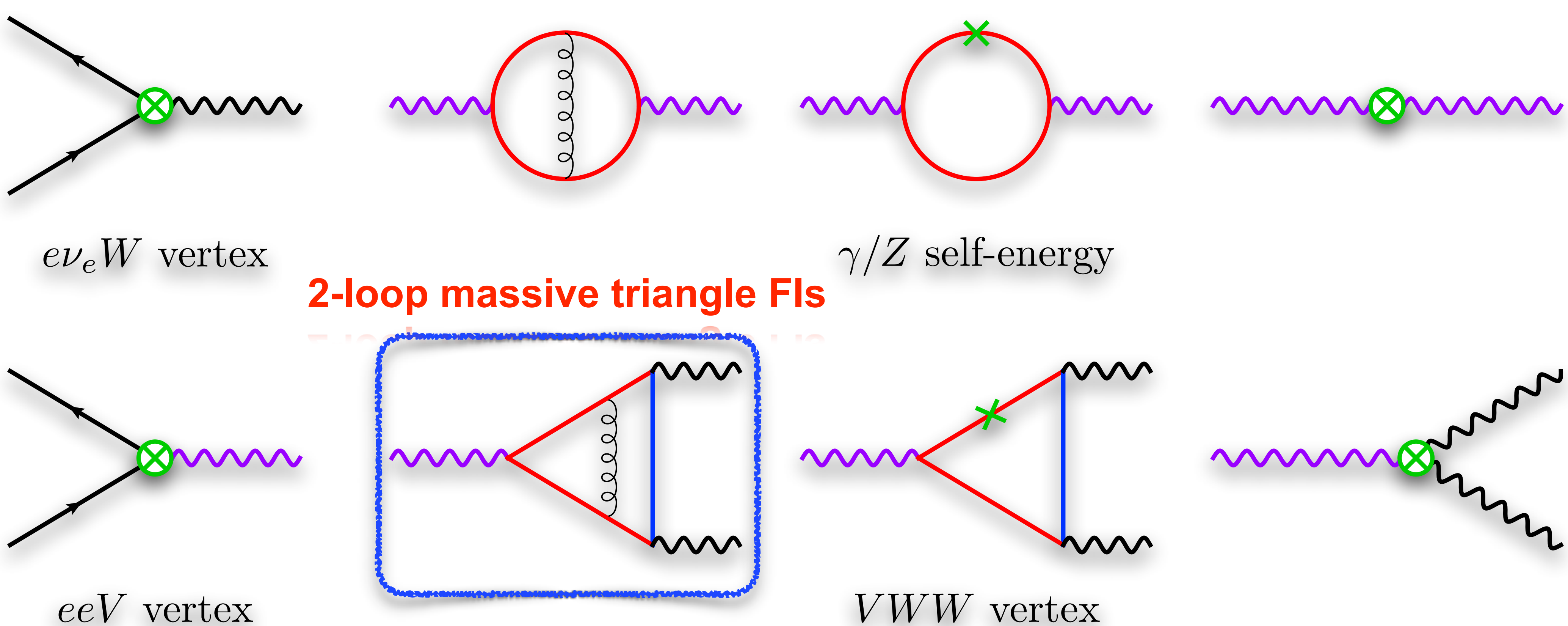
NNLO QCD⊗EW corrections

- $\mathcal{O}(\alpha\alpha_s)$ **corrections to amplitudes**

$$\delta\mathcal{M}^{\mathcal{O}(\alpha\alpha_s)} = \delta\mathcal{M}^{\mathcal{O}(\alpha\alpha_s)}_{e\nu_e W} + \delta\mathcal{M}^{\mathcal{O}(\alpha\alpha_s)}_{eeV} + \delta\mathcal{M}^{\mathcal{O}(\alpha\alpha_s)}_{\gamma/Z\text{-S.E.}} + \delta\mathcal{M}^{\mathcal{O}(\alpha\alpha_s)}_{VWW}$$

- **Feynman diagrams**

× $\mathcal{O}(\alpha_s)$ ⊗ $\mathcal{O}(\alpha\alpha_s)$



$e^+e^- \to W^+W^-$@ QCD-EW

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Integral family

- **Two-loop triangle integrals**

$$F(\alpha_1,\ldots,\alpha_7)=\int \mathcal{D}^d l_1 \mathcal{D}^d l_2 \frac{1}{D_1^{\alpha_1}\ldots D_7^{\alpha_7}}$$

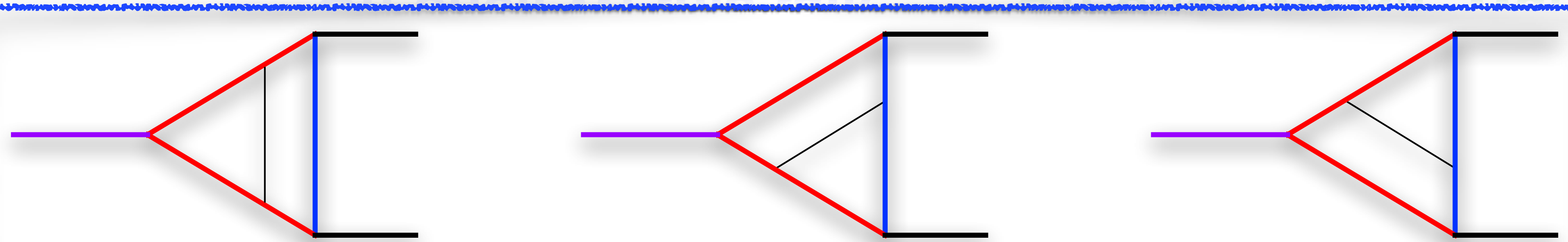
- **Propagators**

Full mass dependence!

$$D_1=l_1^2-m_t^2,\quad D_2=l_2^2-m_t^2,\quad D_3=(l_1-p_3)^2-m_b^2,\quad D_4=(l_2-p_3)^2-m_b^2$$

$$D_5=(l_1-p_3-p_4)^2-m_t^2,\quad D_6=(l_2-p_3-p_4)^2-m_t^2,\quad D_7=(l_1-l_2)^2\,.$$

- **Topologies** **On-shell W: this talk** **Off-shell W: X.F Wang's talk**



$$\mathcal{S}\equiv(1,1,1,0,1,1,1)_F\cup(1,0,1,1,1,1,1)_F\cup(1,1,1,1,1,0,1)_F$$

$e^+e^- \to W^+W^-$@ QCD-EW

Canonical basis

✤ **32 MIs in this family**

$$d\mathbf{F}(\vec{x},\epsilon) = d\tilde{\mathbb{A}}(\vec{x},\epsilon)\mathbf{F}(\vec{x},\epsilon)$$

$$\mathbf{I} = \mathbb{T}\mathbf{F}$$

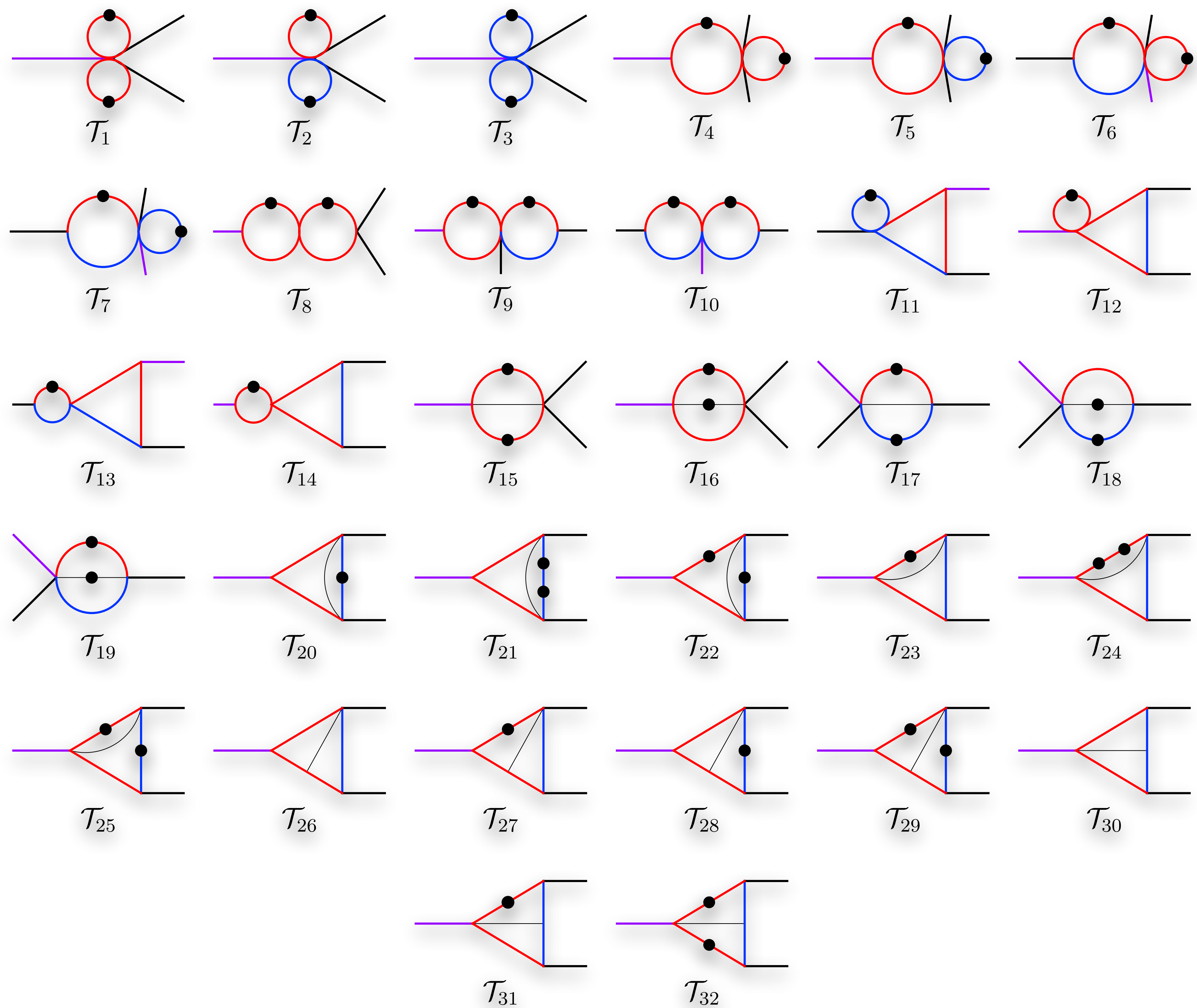
[J. Henn, `13]

$$d\mathbf{I}(\vec{x},\epsilon) = \epsilon d\mathbb{A}(\vec{x})\mathbf{I}(\vec{x},\epsilon)$$

Magnus series

[M. Argeri, `14]

ϵ factorization



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General solutions

See Prof. YZ's talk

- **Path-ordered iterated integral:**

$$\mathbf{I}(\vec{x},\epsilon) = \mathcal{P}\exp\left\{\epsilon\int_\gamma d\mathbb{A}\right\}\mathbf{I}\left(\vec{x}_0,\epsilon\right) \Longleftarrow \boxed{\text{Boundary}}$$

- **Order by order**

$$\mathbf{I}^{(n)}(\vec{x}) = \begin{cases} \mathbf{I}^{(0)}(\vec{x}_0) & n=0, \\ \mathbf{I}^{(n)}(\vec{x}_0) + \int_\gamma d\mathbb{A}(\vec{x})\mathbf{I}^{(n-1)}(\vec{x}) & n>0. \end{cases}$$

- **Iterated integrals**

$$K(t) = \mathrm{d}A/\mathrm{d}t$$

$$\int_\gamma \underbrace{\mathrm{d}A\cdots\mathrm{d}A}_{n\text{ times}} = \int_0^1 \mathrm{d}t_1 \boxed{K(t_1)} \int_0^{t_1}\mathrm{d}t_2\boxed{K(t_2)}\cdots\int_0^{t_{n-1}}\mathrm{d}t_n\boxed{K(t_n)}$$

[K. Chen, `77]

$e^+e^- \to W^+W^-$@ QCD-EW

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Rationalization

- **Three square roots**

$$\lambda_1 = \sqrt{s(s-4m_t^2)}\,, \qquad \lambda_2 = \sqrt{s(s-4m_W^2)}\,,$$

$$\lambda_3 = \sqrt{(m_W^2 - m_t^2 - m_b^2)^2 - 4\,m_t^2\,m_b^2}\,.$$

- **Change of variables**

$$\frac{s}{m_t^2} = -\frac{(1-x)^2}{x}, \quad \frac{m_W^2}{m_t^2} = -\frac{(1-x)^2 z}{x(1+z)^2}, \quad \frac{m_b^2}{m_t^2} = \left(1+\frac{y}{z+1}\right)\left(1-\frac{(1-x)^2 z}{xy(z+1)}\right).$$

- **Rationalized simultaneously**

$$\lambda_1' = \frac{(1-x)(1+x)}{x}\,, \quad \lambda_2' = \frac{(1-x)^2(1-z)}{x(1+z)}\,, \quad \lambda_3' = \frac{(1-x)^2 z + xy^2}{xy(1+z)}$$

Symbol letters

- **Dlog-form**

$$d\mathbf{I}(\vec{x},\epsilon) = \epsilon d\mathbb{A}(\vec{x})\mathbf{I}(\vec{x},\epsilon)\,, \quad d\mathbb{A}(\vec{x}) = \sum_i \mathbb{M}_i \mathrm{dlog}\eta_i$$

Constant matrix

letters

- **20 rational letters**

$$\begin{aligned}
\eta_1 &= x, & \eta_2 &= y \\
\eta_3 &= z, & \eta_4 &= 1-x \\
\eta_5 &= 1+x, & \eta_6 &= 1+y \\
\eta_7 &= 1-z, & \eta_8 &= 1+z \\
\eta_9 &= x+z, & \eta_{10} &= y+z \\
\eta_{11} &= 1+xz, & \eta_{12} &= 1-x+y \\
\eta_{13} &= -1+x+xy, & \eta_{14} &= 1+y+z \\
\eta_{15} &= y+z-xz, & \eta_{16} &= xy+xz-z \\
\eta_{17} &= xy-(1-x)^2, & \eta_{18} &= xy-(1-x)^2 z \\
\eta_{19} &= xy^2+(1-x)^2 z, & \eta_{20} &= xy(1+z)-(1-x)^2 z
\end{aligned}$$

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Multiple polylogarithms

See Prof. YZ's talk

$$\mathbf{I}^{(n)}(\vec{x}) = \mathbf{I}^{(n)}(\vec{x}_0) + \int_\gamma d\mathbb{A}(\vec{x})\mathbf{I}^{(n-1)}(\vec{x}) \qquad d\mathbb{A}(\vec{x}) = \sum_i \mathbb{M}_i d\log\eta_i$$

rational

- **MPLs**

$$G(a_1, \ldots, a_n; z) = \int_0^z \frac{1}{t-a_1} G(a_2, \ldots, a_n; t)\, \mathrm{d}t\,,$$

with

$$G(a_1; z) = \int_0^z \frac{1}{t-a_1} \mathrm{d}t \quad a_1 \neq 0\,, \qquad G(\underbrace{0, \ldots, 0}_{n \text{ times}}; z) = \frac{\log^n(z)}{n!}$$

[Goncharov, \`98,\`01]

- **eMPLs**

Unrationalizable square root *See X.F Wang's talk*

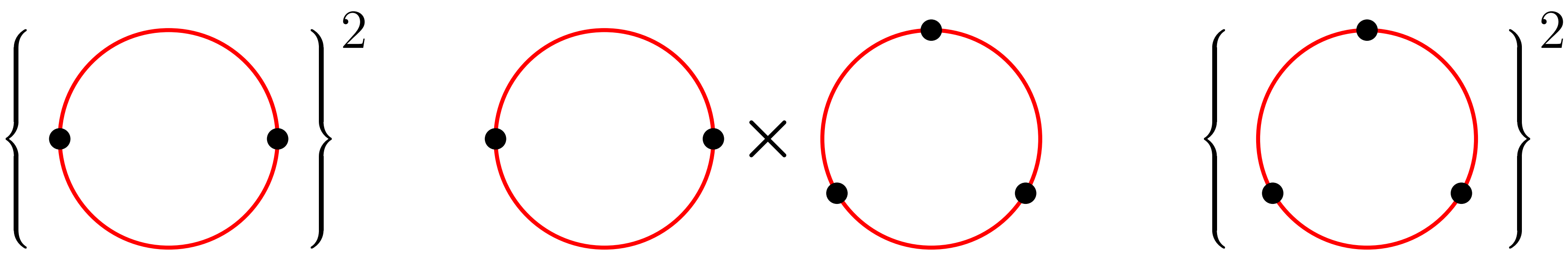
$e^+e^- \to W^+W^-$@ QCD-EW

Boundary conditions

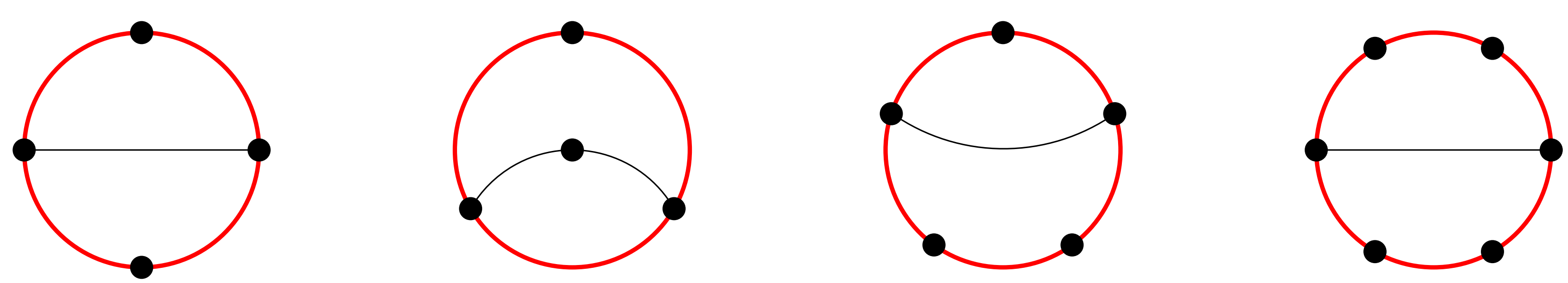
- Vanishing inputs ($s = m_W^2 = 0, m_b^2 = m_t^2$)

$$\mathrm{I}_i(\vec{x}_0, \epsilon) = \begin{cases} 1, & i = 1, 2, 3, \\ 0, & i \neq 1, 2, 3. \end{cases}$$

- Vacuums



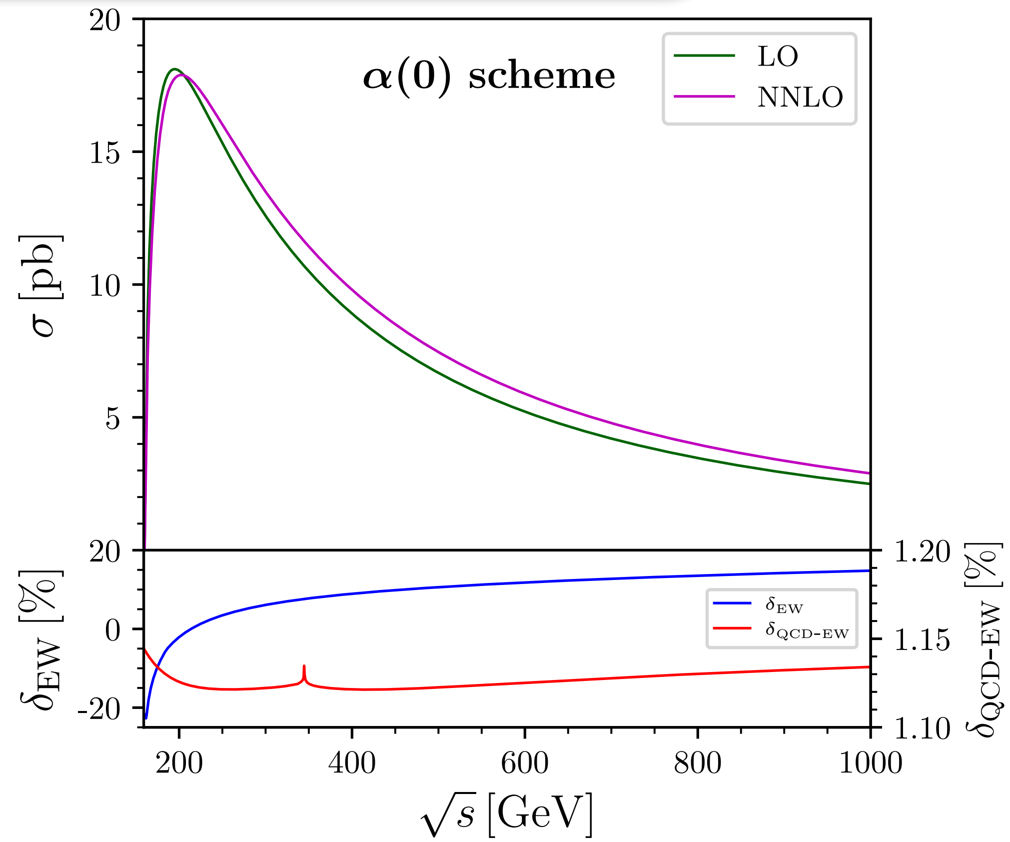
$\mathcal{T}_{1,2,3}$ $\mathcal{T}_{4,5,6,7,11,12}$ $\mathcal{T}_{8,9,10,13,14}$

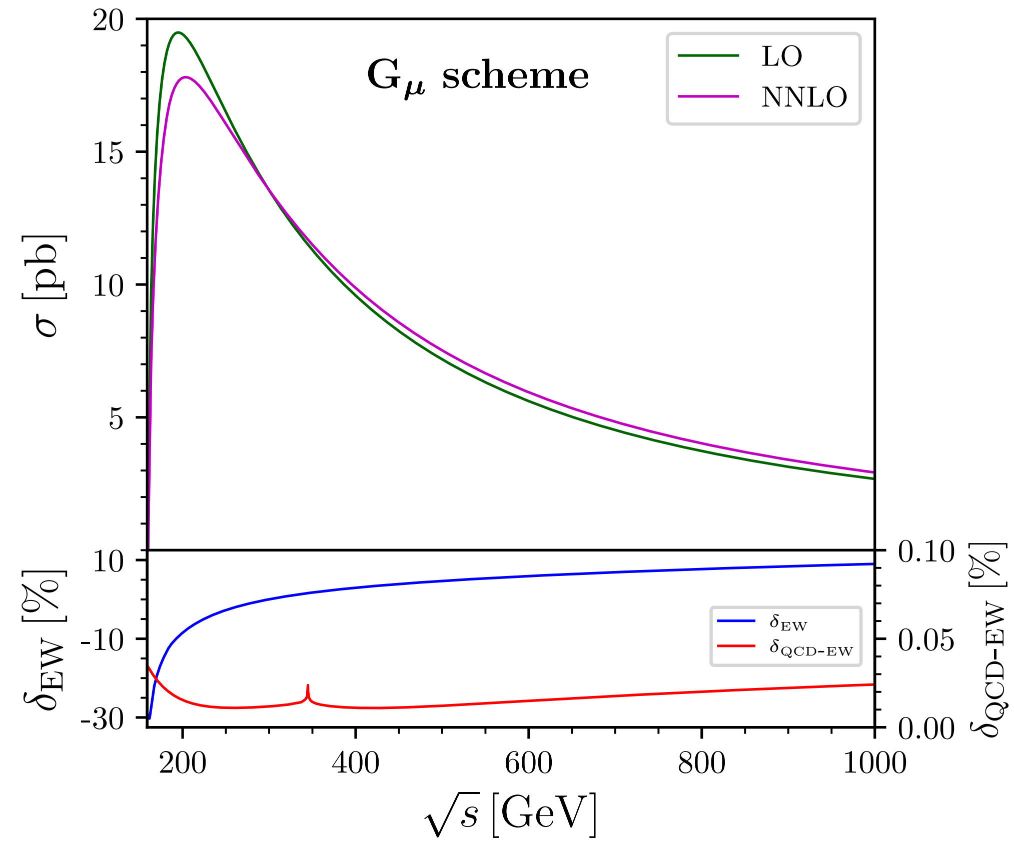


$\mathcal{T}_{15,17,20,23,26,30}$ $\mathcal{T}_{16,18,19}$ $\mathcal{T}_{21,22,24,25,27,28,31}$ $\mathcal{T}_{29,32}$

$e^+e^- \rightarrow W^+W^-$@ QCD-EW

Total cross section





$\sqrt{s}$ [GeV]	Scheme	$\sigma_{\rm LO}$ [pb]	$\sigma_{\rm NLO}$ [pb]	$\delta_{\rm EW}$ [%]	$\sigma_{\rm NNLO}$ [pb]	$\delta_{\rm QCD\text{-}EW}$ [%]
240	$\alpha(0)$	15.93577	16.3653	2.6952	16.5440	1.1216
	G_μ	17.14888	16.5484	−3.5015	16.5503	0.0113
500	$\alpha(0)$	6.673608	7.38135	10.6051	7.45626	1.1225
	G_μ	7.181637	7.51822	4.6867	7.51909	0.0122

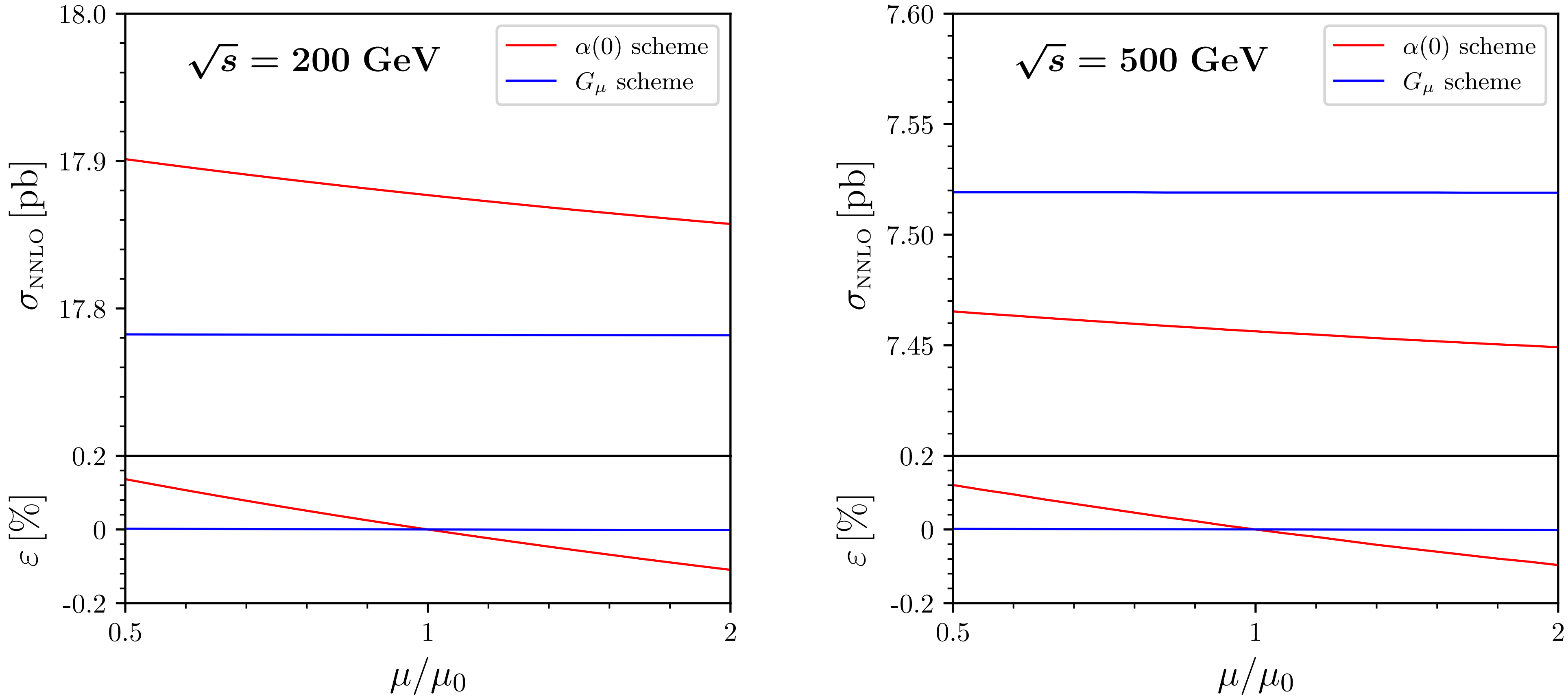
QCD-EW corrections

$\sim 1\%$

Suppressed by G_μ scheme

$e^+e^- \to W^+W^-$@ QCD-EW

Scale dependence



$\sqrt{s}$ [GeV]	Scheme	$\sigma(\mu_0/2)$ [pb]	$\sigma(\mu_0)$ [pb]	$\sigma(2\mu_0)$ [pb]	$\varepsilon_{\mathrm{scale}}$ [%]
200	$\alpha(0)$	17.9012	17.8768	17.8572	0.25
	G_μ	17.7823	17.7820	17.7817	0.003
500	$\alpha(0)$	7.46526	7.45626	7.44904	0.22
	G_μ	7.51920	7.51909	7.51901	0.003

Scale uncertainties

0.2-0.3%

$e^+e^- \to W^+W^-$@ QCD-EW

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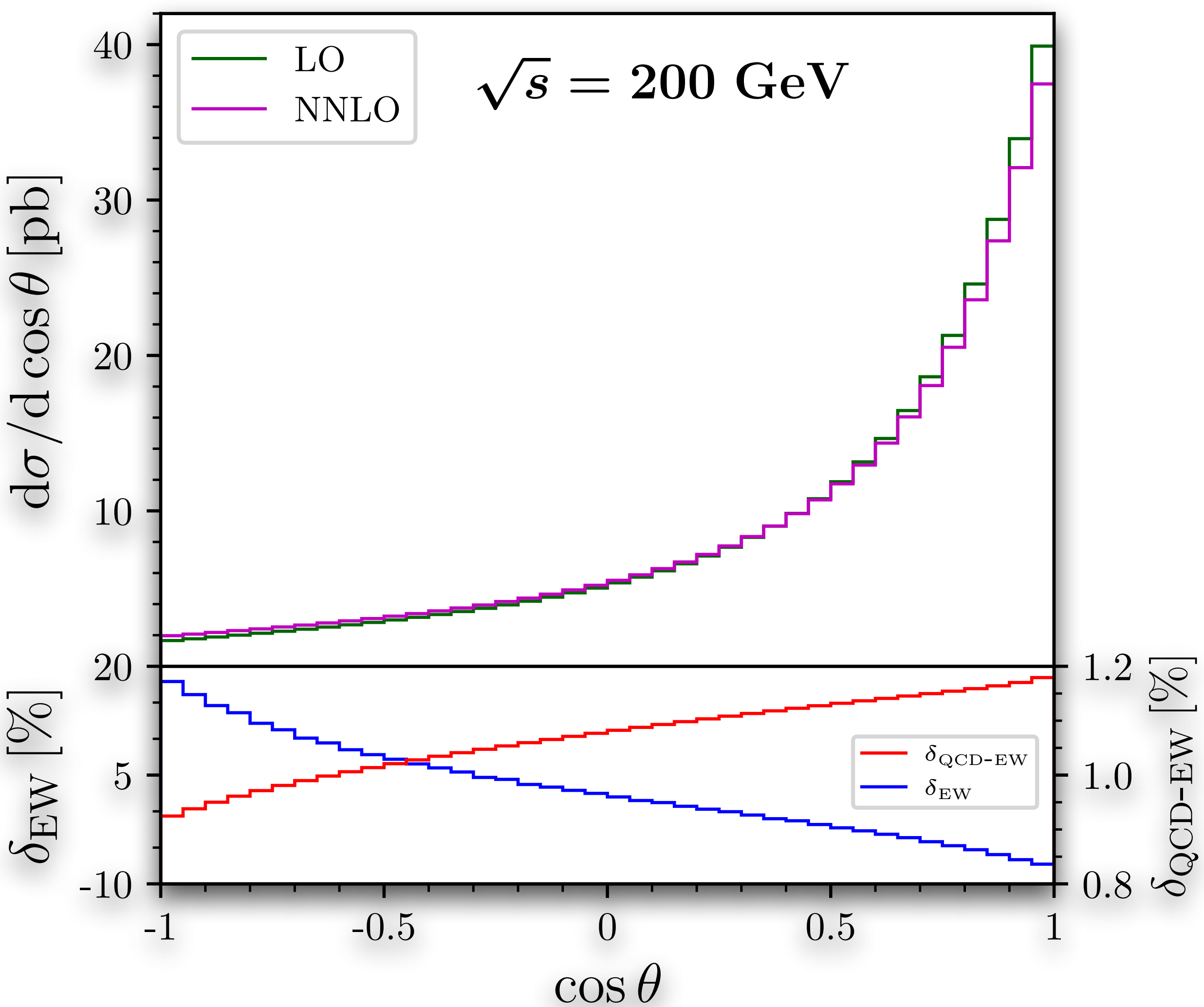
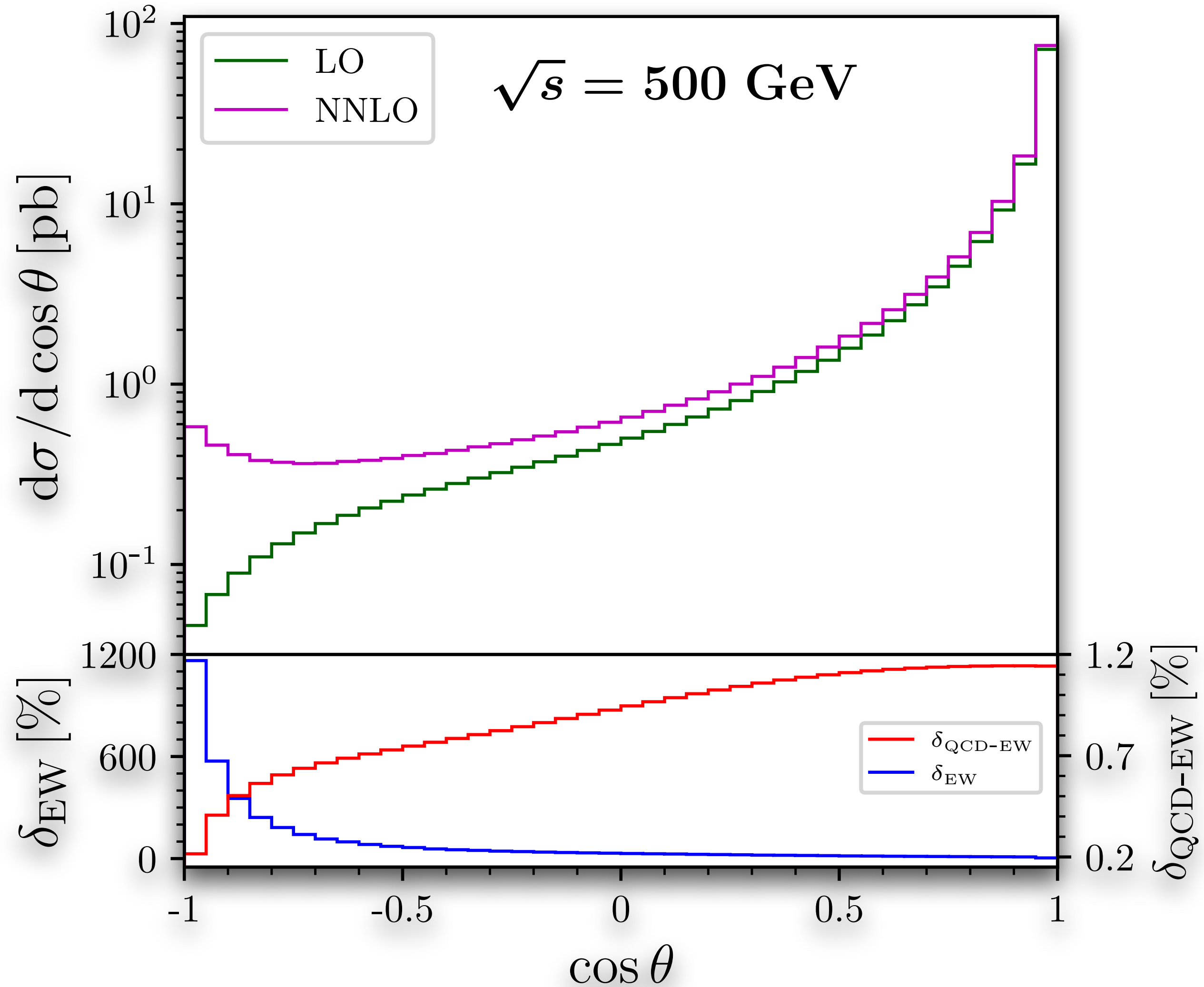
Scattering angle distributions





- **Symmetry relation**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_{W^-}} = \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_{W^+}}\Big|_{\theta\to\pi-\theta},$$

Strongly peak in the forward direction!!!

Two-loop planar master integrals for NNLO QCD corrections to W-pair production at hadron colliders

Based on [JHEP 12 (2024) 136]

**With Wen-Jie He, Shu-Xiang Li,
Xiao-Feng Wang, et. al**

Two-loop MIs for W-pair

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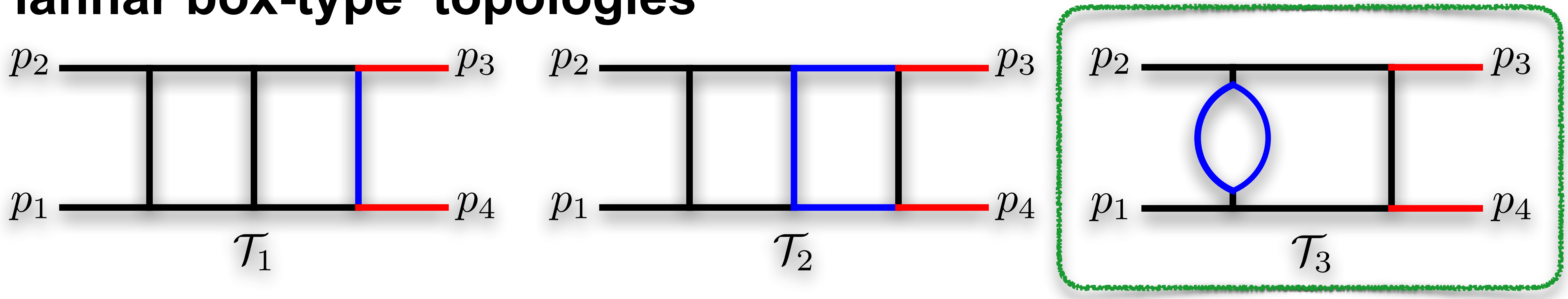
Integral family

$$q(p_1) + \bar{q}(p_2) \to W^+(p_3) + W^-(p_4)$$

- **Plannar box-type topologies**



- **Two-loop master integrals**

$\mathcal{S}_3 \equiv (1, 1, 1, 1, 0, 0, 1, 1, 0)_F$

$$F(n_1, \ldots, n_9) = \int \mathcal{D}^d l_1 \mathcal{D}^d l_2 \frac{1}{D_1^{n_1} \ldots D_9^{n_9}}$$

- **Propagators**

$$D_1 = l_1^2, \quad D_2 = (l_1 + p_1)^2, \quad D_3 = (l_1 - p_2)^2,$$
$$D_4 = l_2^2 - m_t^2, \quad D_5 = (l_2 + p_1)^2 - m_t^2, \quad D_6 = (l_2 - p_2)^2 - m_t^2,$$
$$D_7 = (l_1 - l_2)^2 - m_t^2, \quad D_8 = (l_1 - p_2 - p_3)^2, \quad D_9 = (l_2 - p_2 - p_3)^2.$$

Two-loop MIs for W-pair

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✤ Elliptic sector

$$\boldsymbol{s}_{\text{Elliptic}} = (0,\ 1,\ 1,\ 1,\ 0,\ 0,\ 1,\ 1,\ 0).$$

✤ Maximul cut

$$\text{Maxcut}(f_{13}) = \frac{1}{4\,\pi^3\sqrt{x\,(x-4\,z)}}\int\frac{d\xi}{\sqrt{(\xi-\xi_1)\,(\xi-\xi_2)\,(\xi-\xi_3)\,(\xi-\xi_4)}} + \mathcal{O}(\epsilon)$$

✤ Elliptic curve

$$\vartheta(\xi)^2 = (\xi-\xi_1)\,(\xi-\xi_2)\,(\xi-\xi_3)\,(\xi-\xi_4)$$

with

$$\xi_1 = -\frac{y\,(x-2\,z)+2\,z^2+2\,z\sqrt{x\,y+(y-z)^2}}{x-4\,z},$$

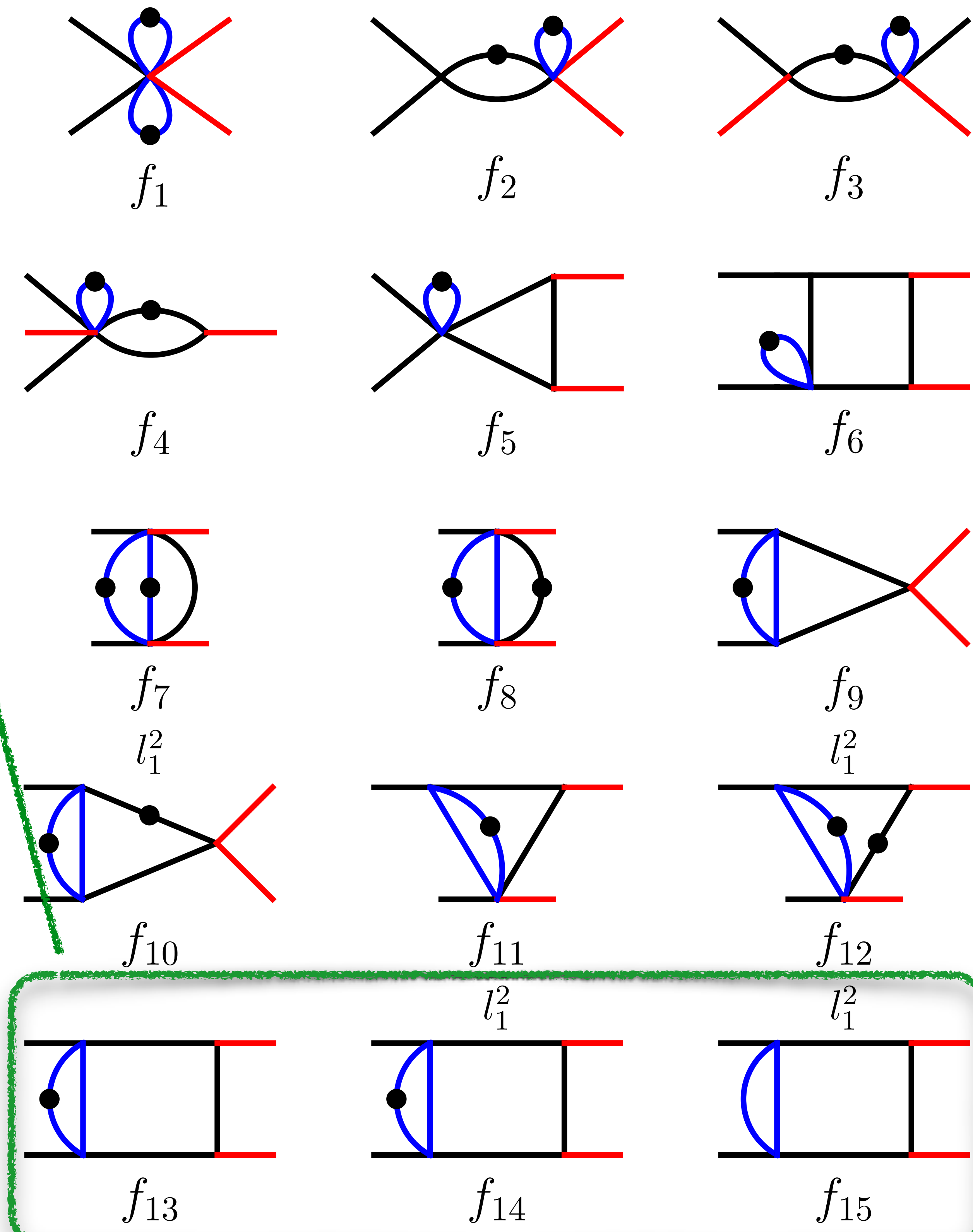
$$\xi_2 = -\frac{y\,(x-2\,z)+2\,z^2-2\,z\sqrt{x\,y+(y-z)^2}}{x-4\,z},$$

$$\xi_3 = 0, \qquad \xi_4 = 4.$$

✤ Periods

$$K(x) = \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-x^2t^2)}}$$

$$\Psi_1 = \frac{4\,K(k)}{U_3^{1/2}}, \qquad \Psi_2 = \frac{4\,i\,K(1-k)}{U_3^{1/2}},$$



Two-loop MIs for W-pair

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Linear basis

$$g_1 = \epsilon^2 f_1$$
$$g_2 = \epsilon^2 f_2\, x$$
$$g_3 = \epsilon^2 f_3\, y$$
$$g_4 = \epsilon^2 f_4\, z$$
$$g_5 = \epsilon^3 f_5\, r_3'$$
$$g_6 = \epsilon^3 f_6\, x\, y$$
$$g_7 = \epsilon^2 f_7\, y$$
$$g_8 = \epsilon^2 f_8\, r_2' + 1/2\, f_7\, r_2'$$
$$g_9 = \epsilon^3 f_9\, x$$
$$g_{10} = \epsilon^2 f_{10}\, r_1' - \epsilon^3 f_9\, r_1'$$
$$g_{11} = \epsilon^3 f_{11}\,(y-z)$$
$$g_{12} = \left[\epsilon^2 f_{12}\, z + \epsilon^3 f_{11}\,(y-z) - \epsilon^2 f_8\, y\right] r_4'/(y-z)$$
$$g_{13} = \epsilon^3 \frac{\pi\, r_3'}{\Psi_1} f_{13}$$
$$g_{14} = \epsilon^3 f_{14}\, r_3'$$
$$g_{15} = \frac{1}{\epsilon}\, \frac{\Psi_1^2}{2\,\pi\, i\, W_y}\, \frac{\partial g_{13}}{\partial y}$$

- Wronskian

$$\boxed{W_y = \Psi_1 \frac{\partial \Psi_2}{\partial y} - \Psi_2 \frac{\partial \Psi_1}{\partial y}\,,}$$

- Square roots

$$r_1'^2 = x\,(x-4)\,, \quad r_2'^2 = y\,(y+4)\,,$$
$$r_3'^2 = x\,(x-4\,z)\,, \; r_4'^2 = (z-y)\,(y-z+4)\,.$$

Linear form DEs

Lower triangular ⇒ Iterated integral

[L. Adams, `18]

$$\mathrm{d}\mathbf{g}(\vec{x},\epsilon) = \left[\boxed{A^{(0)}(\vec{x})} + \epsilon\, \mathrm{d}A^{(1)}(\vec{x})\right] \mathbf{g}(\vec{x},\epsilon).$$

Two-loop MIs for W-pair

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Validation

Good agreements!!!

Branch	G	Results (Analytic / AMFlow)	
$\mathcal{T}_{3F}$	g_{13}	$3.2548275606982903057\,\epsilon^3$	(Analytic)
		$3.2548275606982903051\,\epsilon^3$	(AMFlow)
	g_{14}	$-2.3375605560182768 7\,\epsilon^3$	(Analytic)
		$-2.3375605560182769 1\,\epsilon^3$	(AMFlow)
	g_{15}	$-0.094132461909778403737363\,\epsilon^2 - 0.071565531400688 7\,\epsilon^3$	(Analytic)
		$-0.094132461909778403737361\,\epsilon^2 - 0.071565531400688 5\,\epsilon^3$	(AMFlow)

Mixed QCD-EW corrections to THDM charged Higgs-pair production at lepton colliders

Based on [2312.17207]

With Zhi-Xing Zhang, Shu-Xiang Li, Ren-You Zhang, et. al

$e^+e^- \to H^+H^-$ @ QCD-EW

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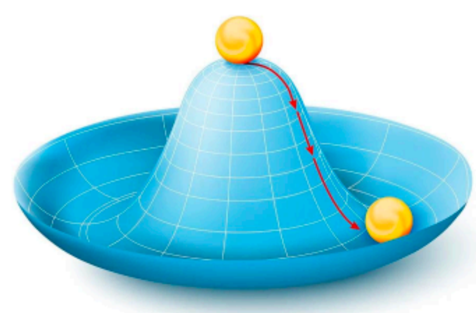
THDM background

- **Simplest extensions of SM Higgs sector**
- **Scalar sector of many other models (SUSY)**
- **Rich phenomenology**
  - Charged Higgs bosons
  - CP-violation
  - Dark matter candidates
  - BSM signals
  - …

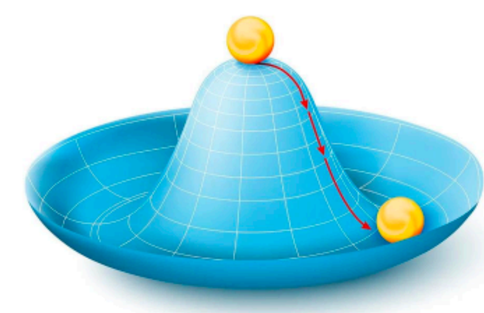
SM



$\times 1$

$\mathbf{Z,\ W^{\pm},\ h}$

THDM



$\times 2$

$\mathbf{Z,\ W^{\pm},\ h}$
$\mathbf{A,\ H^{\pm},\ H}$

$e^+e^- \to H^+H^-$ @ QCD-EW

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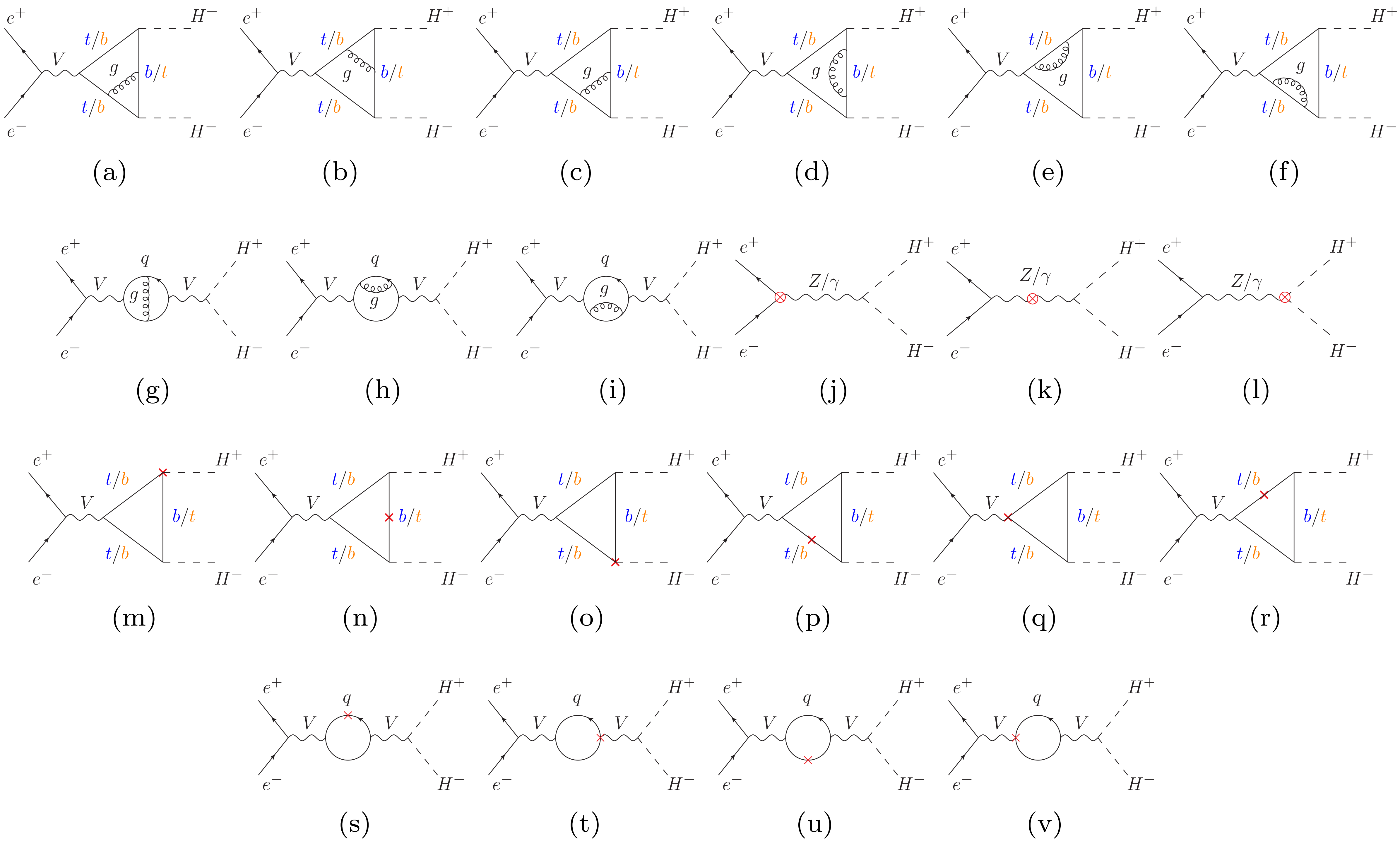
Feynman Diagrams

Two-loop MIs: same as W-pair

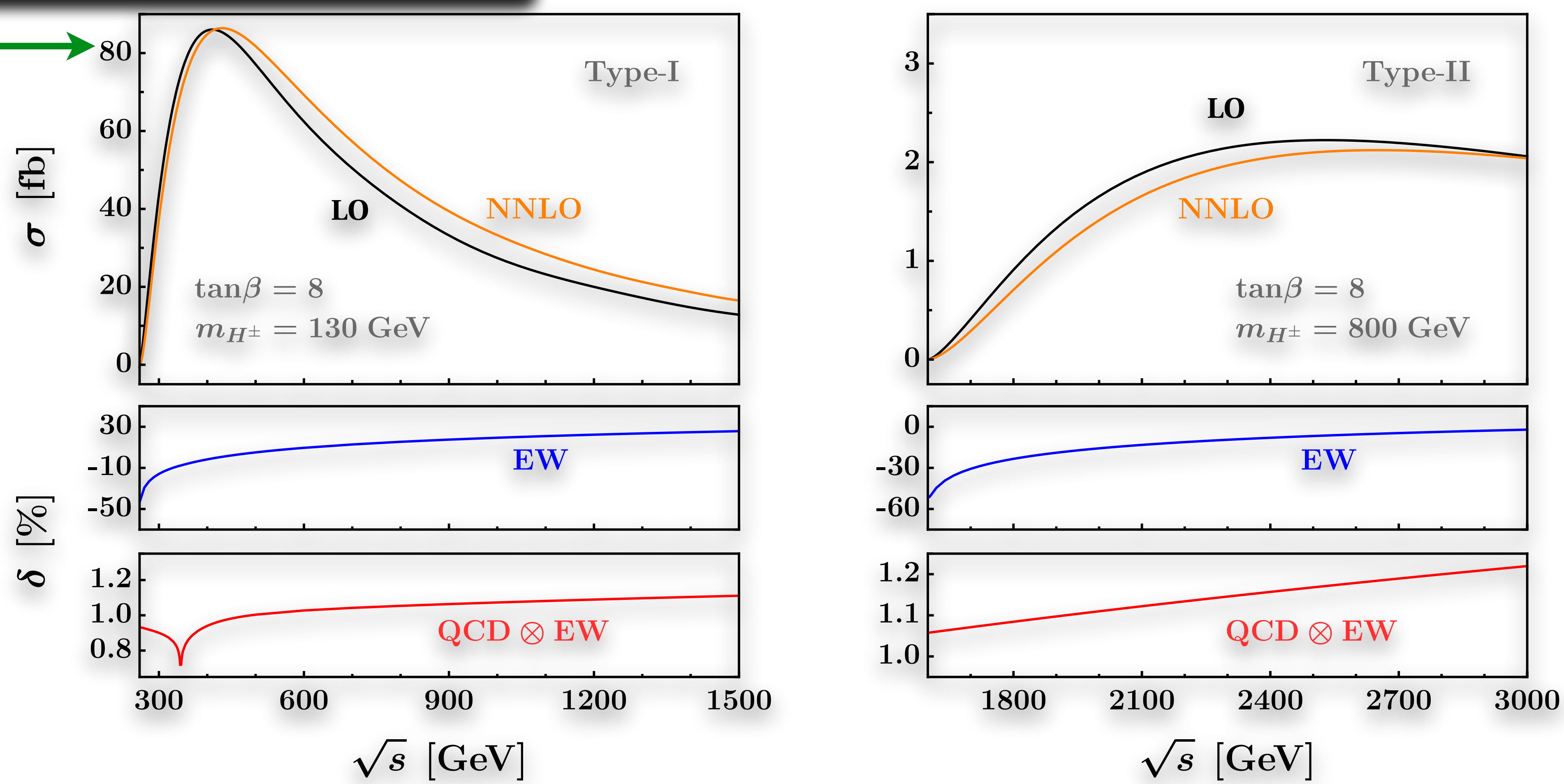


$e^+e^- \to H^+H^-$ @ QCD-EW

Total cross sections

Potential to be detected!!!

80fb



QCD-EW corrections

~1%

Type	scheme	σ_0 [fb]	δ_{EW} [%]	σ_{NLO} [fb]	$\delta_{\mathrm{QCD}\otimes\mathrm{EW}}$ [%]	σ_{NNLO} [fb]
I	$\alpha(m_Z)$	13.01	25.72	16.35	1.11	16.50
	$\alpha(0)$	11.52	12.03	12.90	1.04	13.02
II	$\alpha(m_Z)$	2.06	−2.11	2.02	1.22	2.04
	$\alpha(0)$	1.82	−10.91	1.62	1.15	1.64

$e^+e^- \to H^+H^-$ @ QCD-EW

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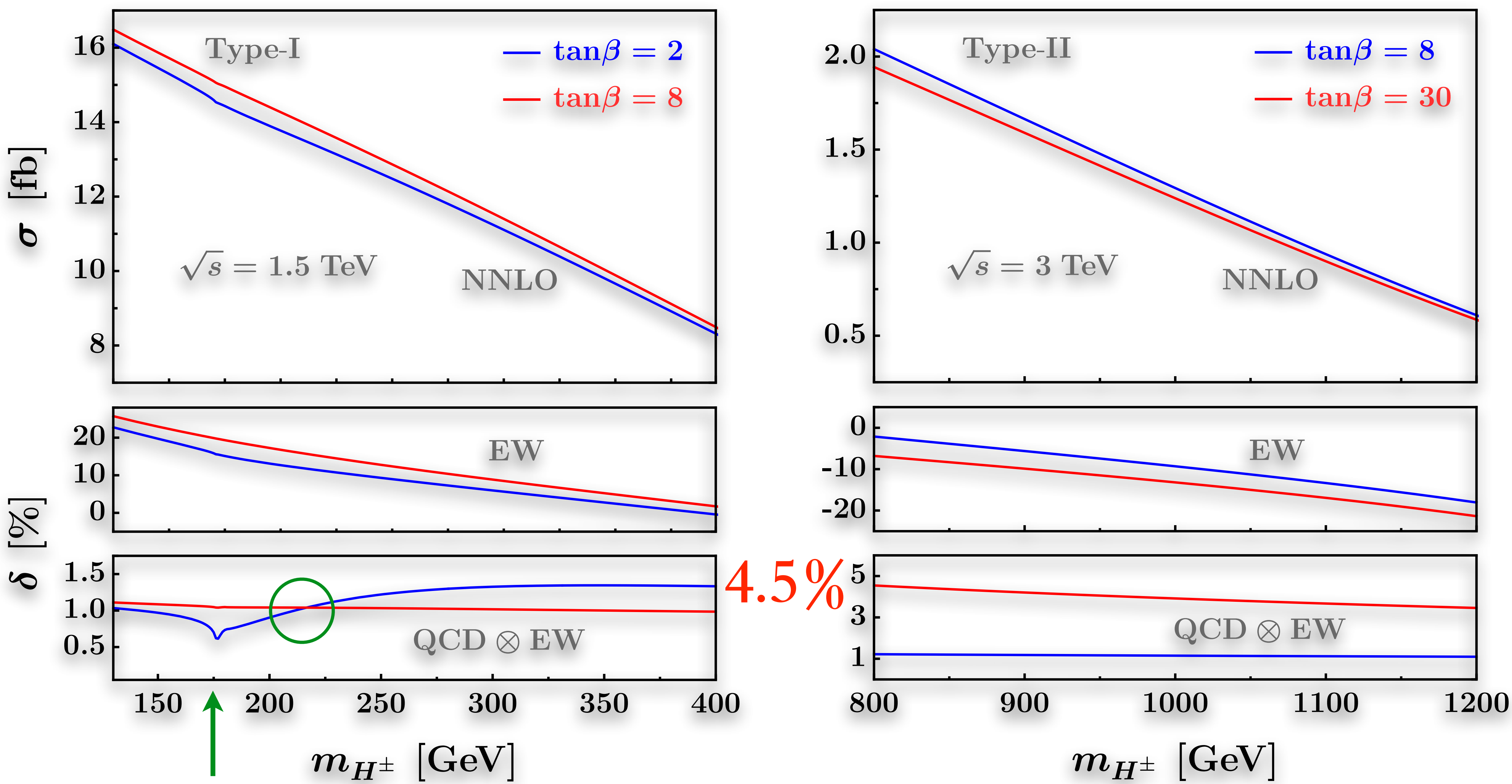
m_{H^+} scan plots



4.5%

$m_t + m_b$

$e^+e^- \to H^+H^-$ @ QCD-EW

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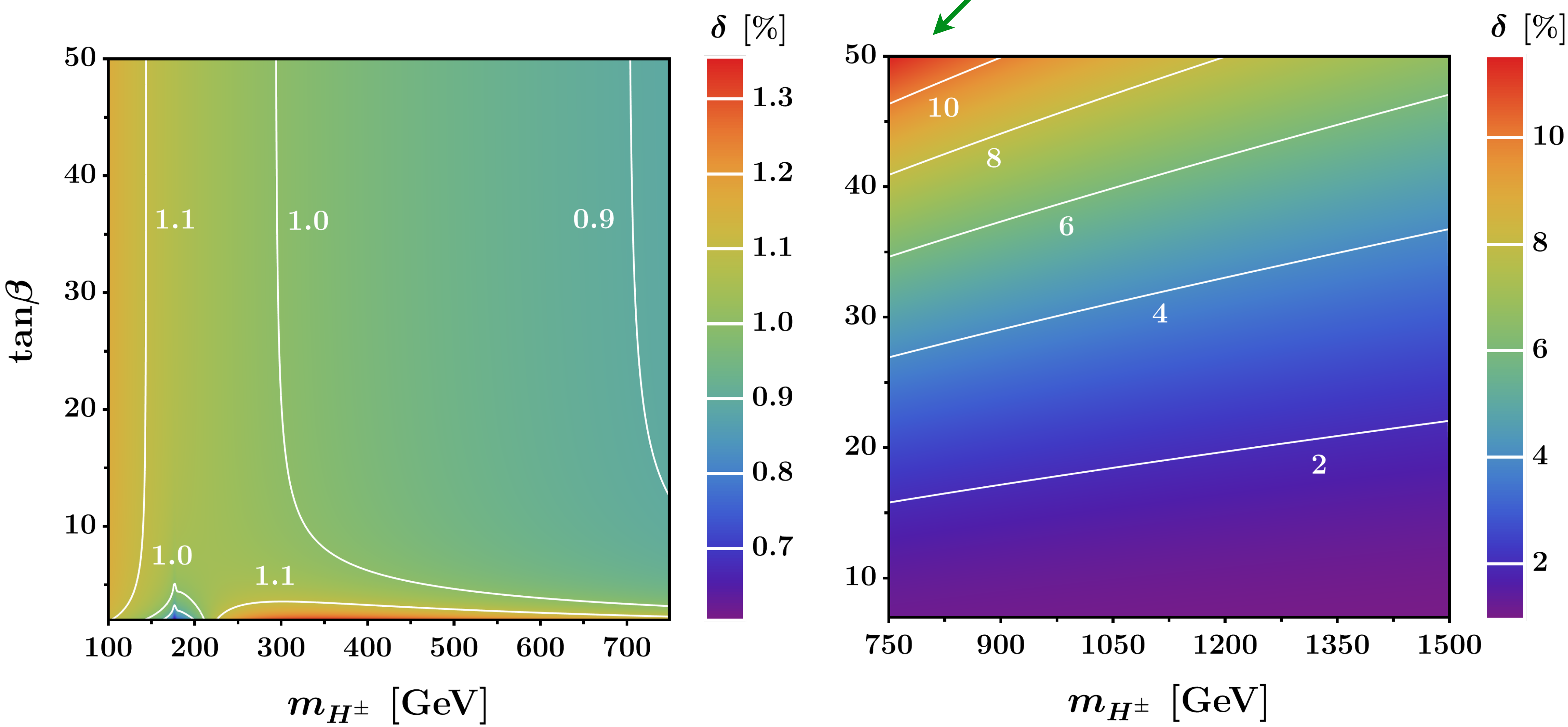
$\mathcal{O}(\alpha\alpha_s)$ corrections density plots

>10% Non-negligible!!!



Summary

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- NNLO QCD-EW corrections to W-pair production at lepton colliders (~1%)

- Planar MIs for NNLO QCD corrections to W-pair production at hadron colliders (elliptic integrals)

- NNLO QCD-EW corrections to charged Higgs-pair production at lepton colliders (~1%)

Thanks

Backup

$e^+e^- \to W^+W^-$@ QCD-EW

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NLO EW corrections

- **EW corrections**

$$\Delta\sigma_{\rm EW} = \sigma_{\rm virtual} + \sigma_{\rm real} + \sigma_{\rm h.o.ISR}$$

- **Virtual corrections**

$$\frac{d\sigma_{\rm virtual}}{d\Omega} = \frac{\beta}{64\pi^2 s}\frac{1}{4}\sum_{\lambda_{1,\ldots,4}} 2\,{\rm Re}\,\mathcal{M}_0^*\,\delta\mathcal{M}^{\mathcal{O}(\alpha)}$$

On-shell

[A.Denner `97]

- **Real photon radiation**

$$\sigma_{\rm real} = \sigma_{\rm S}(\delta_s) + \sigma_{\rm HC}(\delta_s, \delta_c) + \sigma_{\rm H\bar{C}}(\delta_s, \delta_c)$$

Slicing method

[B. W. Harris, `02]

- **Initial-state radiation**

$$\sigma_{\rm h.o.ISR} = \sigma_{\rm ISR-LL} - \sigma_{\rm LL,sub}$$

Structure functions

[W. Beenakker, `96]

$e^+e^- \to W^+W^-$ (MIs)

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Analytic continuation

Euclidean region
$s < 0 \wedge m_W^2 < 0$

$\xrightarrow[m_t^2 - i0^+ ,\, m_b^2 - i0^+]{s + i0^+ ,\, m_W^2 + i0^+}$

Physical region
$s > 4\, m_W^2 > 0$

- **Inverse transfrom**

$$x = \frac{\sqrt{4\,m_t^2 - s} - \sqrt{-\,s}}{\sqrt{4\,m_t^2 - s} + \sqrt{-\,s}}\,, \qquad z = \frac{\sqrt{-\,s} - \sqrt{4\,m_W^2 - s}}{\sqrt{-\,s} + \sqrt{4\,m_W^2 - s}}\,,$$

$$y = (1+z)\,\frac{1}{2\,m_t^2}\left[m_b^2 - m_t^2 - m_W^2 + \sqrt{\lambda(m_W^2,\, m_t^2,\, m_b^2)}\,\right].$$

Family	Kinematic configuration	x	z	y
$\mathcal{F}$	$2\,m_W < \sqrt{s} < 2\,m_t$	$e^{i\vartheta}$	z	$y - i\,0^+$
	$2\,m_t < \sqrt{s}$	$x + i\,0^+$	z	$y - i\,0^+$
$\mathcal{F}^*$	$2\,m_W < \sqrt{s}$	$x + i\,0^+$	z	$y - i\,0^+$

$e^+e^- \to W^+W^-$@ QCD-EW

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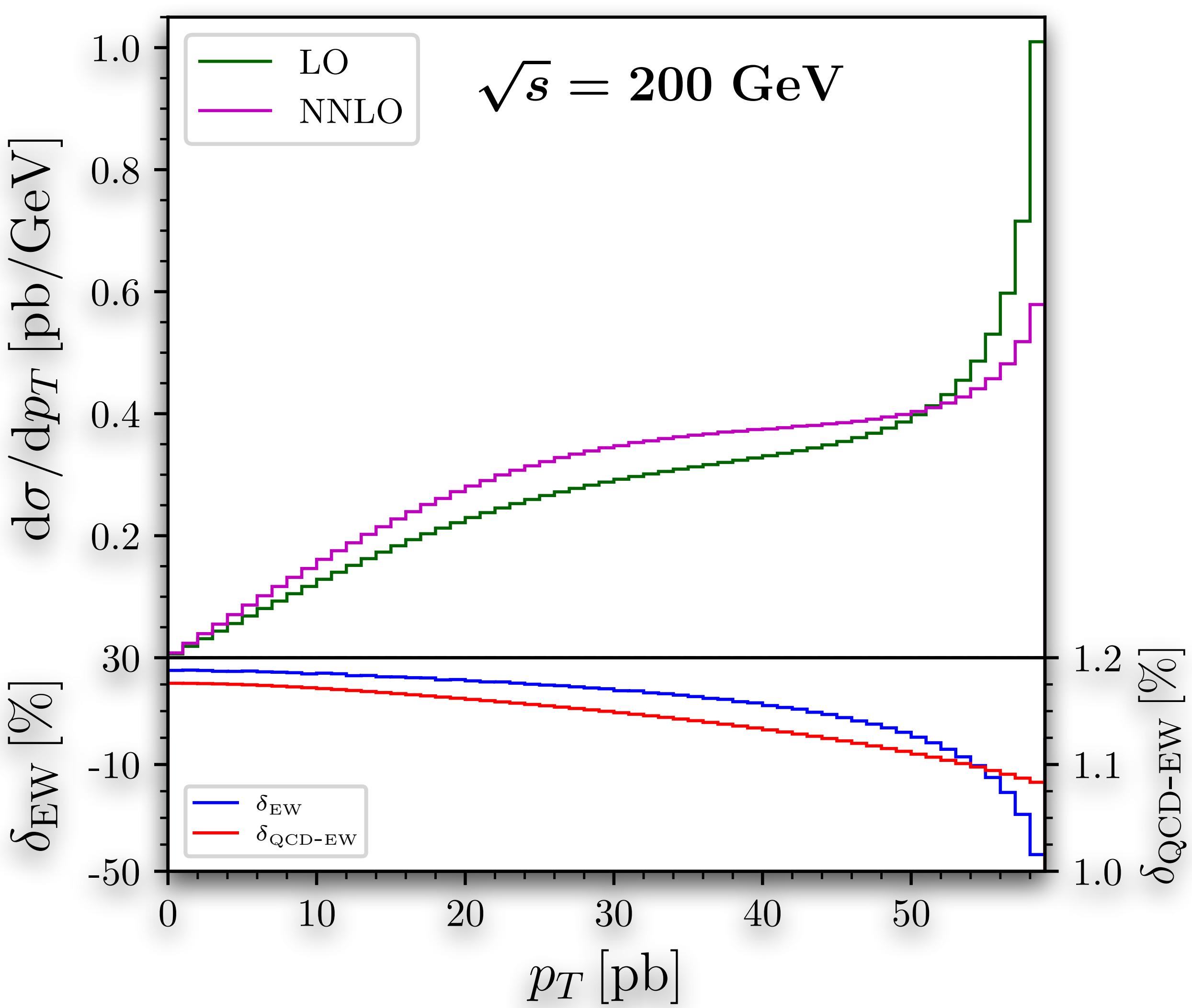
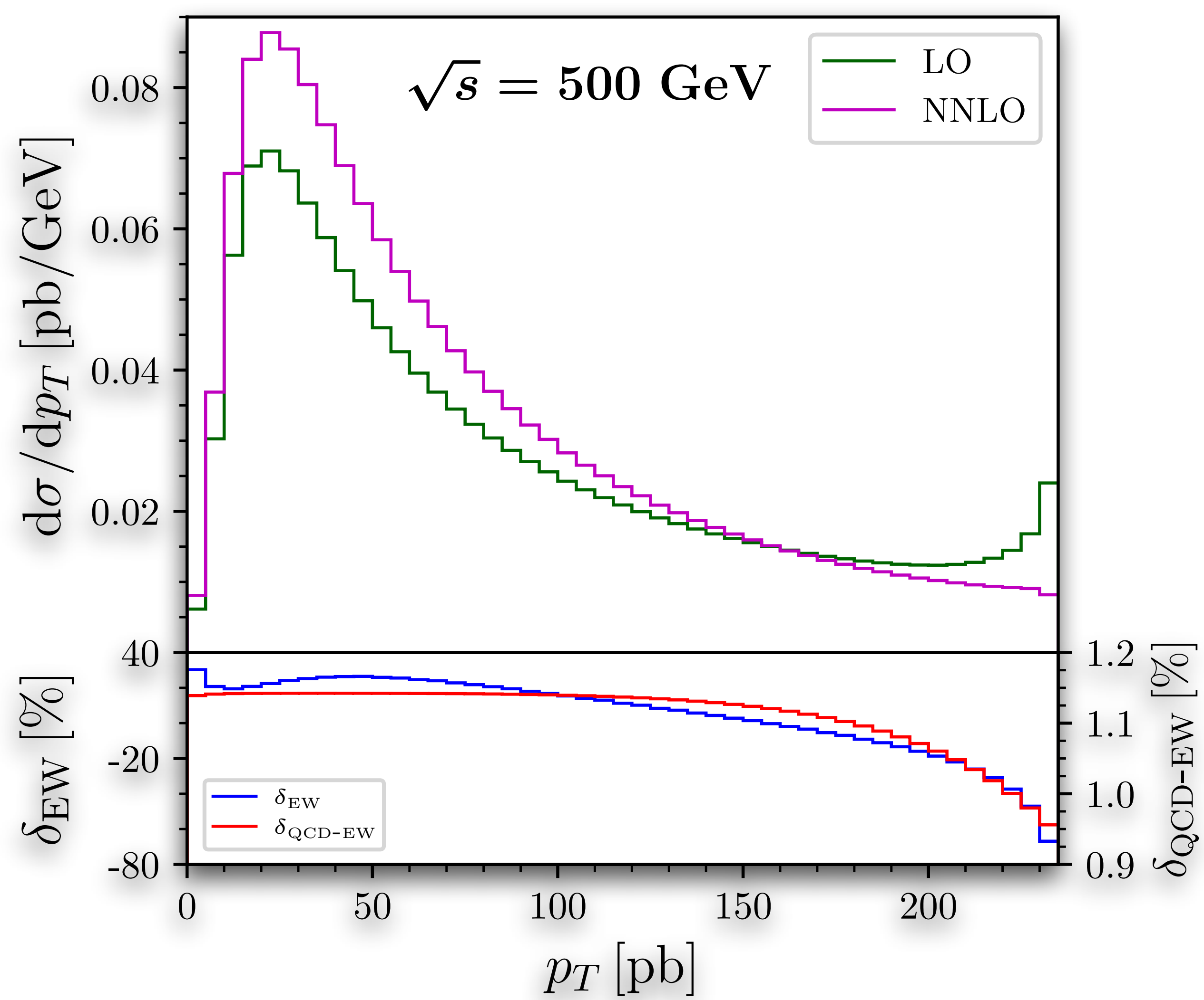
Kinematic distributions





- **Symmetry relation**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_{T,W^-}} = \frac{\mathrm{d}\sigma}{\mathrm{d}p_{T,W^+}}$$

Two-loop MIs for W-pair

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Boundary conditions

- **Expansion @ $\vec{x}_0 = (x, 1, 0)$**

$$A_x^{(0,1)}(\vec{x}) = A_{x,(0,0)}^{(0,1)}(x), \quad A_y^{(0,1)}(\vec{x}) = \frac{A_{y,(-1,0)}^{(0,1)}}{y - y_0}, \quad A_z^{(0,1)}(\vec{x}) = \frac{A_{z,(0,-1)}^{(0,1)}}{z - z_0}$$

- **Lowest-order approximation**

$$\mathrm{d}\tilde{\mathbf{g}}(\vec{x},\epsilon) = \epsilon \left[A_{x,(0,0)}^{(1)}(x)\ \mathrm{d}x + \frac{1}{y} A_{y,(-1,0)}^{(1)} \right] \tilde{\mathbf{g}}(\vec{x},\epsilon)$$

$$\tilde{g}(\vec{x}) = g(\vec{x}_0)$$

- **Degeneration**

$$r_1' = r_3' = \sqrt{x\,(x-4)}\,, \qquad r_2' = 0\,, \qquad r_4' = \sqrt{3}\,,$$

$$\Psi_1 = \frac{\pi}{2}\sqrt{x-4}\,, \qquad \partial_y \Psi_1 = \frac{\pi}{8}\,(2-x)\,\sqrt{x-4}\,.$$

- **Boundary**
  - Known integrals: $g_1, \cdots, g_8$ [S. Di Vita, `17]
  - Symmetry relation: $g_{11} = -\,g_9\,, \quad g_{12} = i g_{10} \quad @ \quad (x, y, z) = (1,0,1)$
  - Regularity condition:

✦ $x \to 0:\ g_9$ ✦ $x \to 3:\ g_{13}$ ✦ $x \to 4:\ g_{14}$

$e^+e^- \to H^+H^-$ @ QCD-EW

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THDM types

THDM	ϕ_1	ϕ_2	q_L	l_L	u_R	d_R	e_R
Type-I	+	−	+	+	−	−	−
Type-II	+	−	+	+	−	+	+
Type-X	+	−	+	+	−	−	+
Type-Y	+	−	+	+	−	+	−

Same quark Yukawa

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Constraints on THDM parameters

✤ Experiment constraints:

- Direct searches for BSM Higgs bosons [M. Aiko, `10, G. Abbiendi, `13]
- Flavor constraints: $B \to X_s\gamma$ [M. Misiak, `20, V. Khachatryan,`14, R. Aaij, `17]
- Anomalous magnetic moment of the muon [J. Haller, `18]
- SM Higgs boson coupling measurements [G. Aad, `15]
- Oblique parameters [W. Grimus, `15, W. Grimus, `08, R. Workma, `22]

✤ Theoretical constraints:

- Vacuum stability [S. Nie, `99, A. Barroso, `13, V. Branchina, `18,]
- Perturbative unitarity [S. Kanemura, `15]

$e^+e^- \to H^+H^-$ @ QCD-EW

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$\tan\beta$ scan plots