Which Higgs Potential? & Which Standard Model Gauge Group?

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Mainly based on hep-ph/2404.04229 in collaboration with **Hao-Lin Li**
& hep-ph/1907.02078 with **Pankaj Agrawal, Debashis Saha, Jiang-Hao Yu, C.-P. Yuan**

Which Higgs Potential?
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Which Standard Model Gauge Group?

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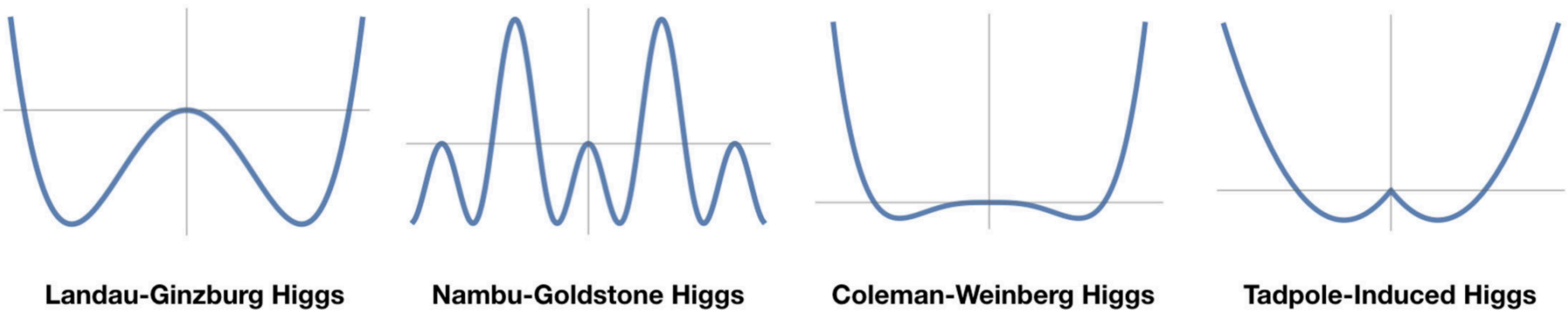
We don't know

Which Higgs potential?

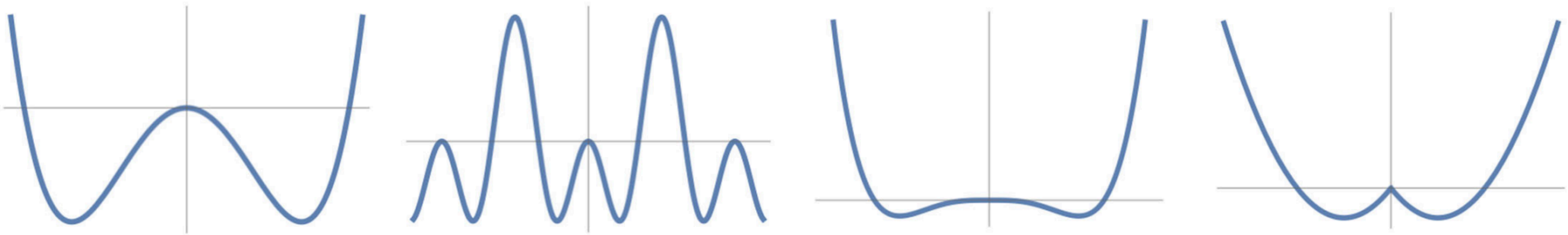
- Higgs potential can be different in various BSM scenarios



$$V(H) \simeq \begin{cases} -m^2 H^\dagger H + \lambda (H^\dagger H)^2 + \frac{c_6 \lambda}{\Lambda^2}(H^\dagger H)^3, & \text{Elementary Higgs} \\ -a \sin^2(\sqrt{H^\dagger H}/f) + b \sin^4(\sqrt{H^\dagger H}/f), & \text{Nambu-Goldstone Higgs} \\ \lambda (H^\dagger H)^2 + \epsilon (H^\dagger H)^2 \log \frac{H^\dagger H}{\mu^2}, & \text{Coleman-Weinberg Higgs} \\ -\kappa^3 \sqrt{H^\dagger H} + m^2 H^\dagger H, & \text{Tadpole-induced Higgs} \end{cases}$$

- Experimentally, we need to measure the Higgs self coupling to distinguish them

Which Higgs potential?



Landau-Ginzburg Higgs | Nambu-Goldstone Higgs | Coleman-Weinberg Higgs | Tadpole-Induced Higgs

If new physics has **decoupling** nature, such as the Landau-Ginzburg or Nambu-Goldstone Higgs boson, the self coupling is close to the SM value.

Otherwise, if new physics is **non-decoupling**, the deviation from SM is significant, the ratio of self coupling w.r.t. the SM value is close to 5/3 for Coleman-Weinberg and nearly zero for tadpole Higgs boson.

Which SM gauge group?

Generalized global symmetries

Generalized Global Symmetries

Davide Gaiotto (Perimeter Inst. Theor. Phys.), Anton Kapustin (Stony Brook U.), Nathan Seiberg (Princeton, Inst. Advanced Study), Brian Willett (Princeton, Inst. Advanced Study)
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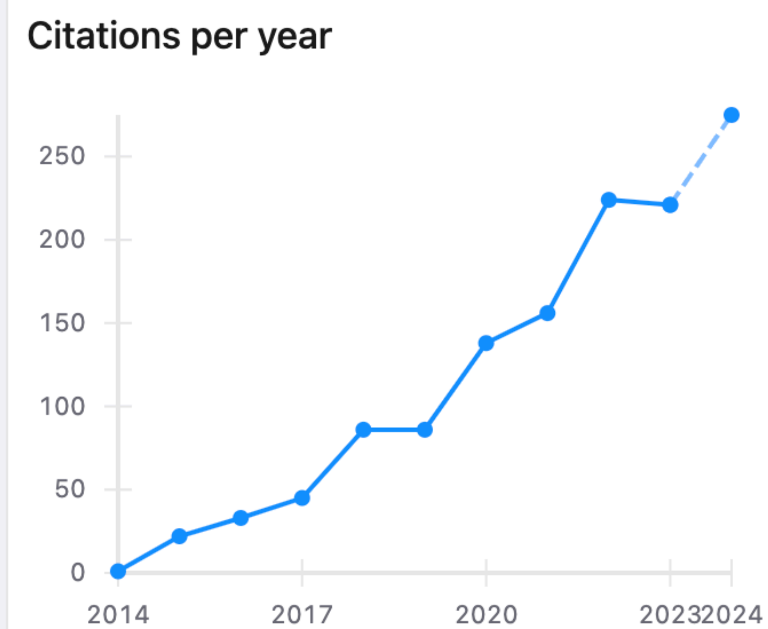
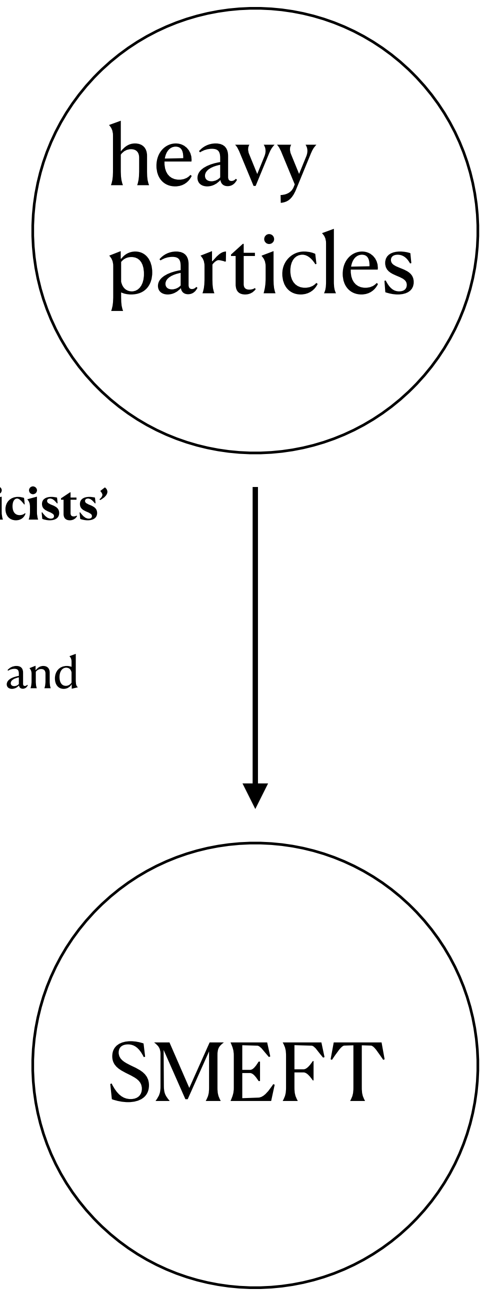


- The global symmetries in QFT are defined as topological operators.
- In this view, people found many generalizations and new applications of global symmetries, mainly in hep-th and condensed matter communities.
- How do we use it in particle physics?

Unification of two perspectives



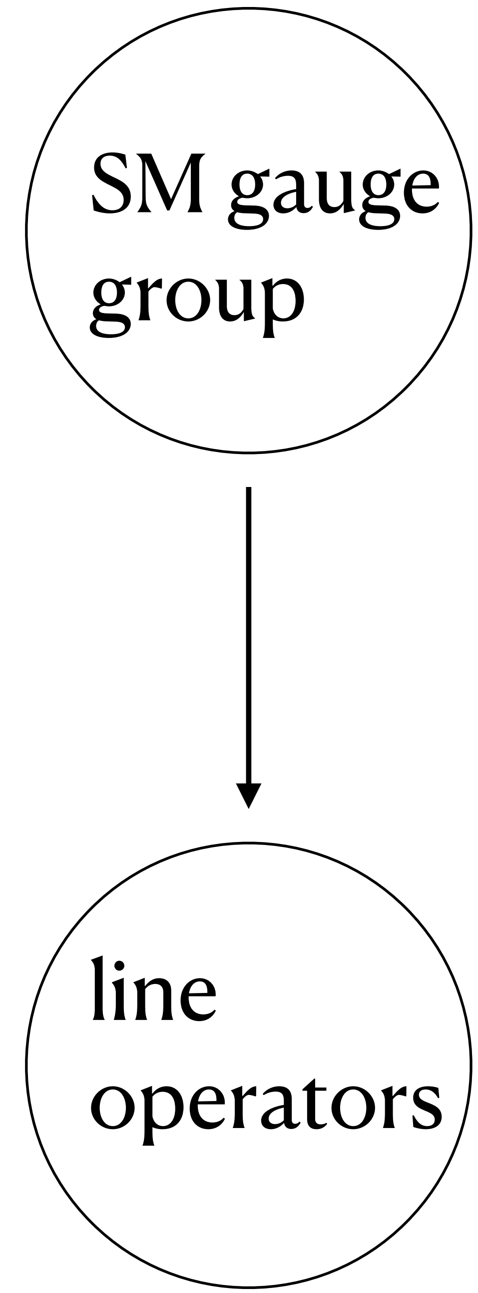
Particle physicists' viewpoint:

SM as an EFT and reductionism

Viewpoint of generalized symmetry:

Given the Lie algebra, there can be different Lie groups.

Gauge theories based on those Lie groups are considered as different theories, since they are distinguished by **one-form symmetries** acting on line operators.



heavy particles (with infinite masses) = Wilson/'t Hooft line operators

Toy Model

Example: SU(2) versus SO(3) groups

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- The consequence of the $\mathbb{Z}_2$ quotient:
  $SO(3)$ only has integer spin representations,
  $SU(2)$ can have both half-integer and integer spin representations
- In general, one can define $G \sim \dfrac{\tilde{G}}{H}$, where H is a subgroup of the center
  and all the allowed reps. are invariant under the H group

[Aharony, Seiberg, Tachikawa, 13]

Example: SU(2) versus SO(3) gauge theories

- Consider a low-energy theory with all the matter fields (including gauge bosons and Dirac fermions) in the adjoint representation of $SU(2)$. Suppose this is what has been discovered experimentally.

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- Instead, the gauge group can be either $SU(2)$ or $SO(3)$

- In fancier language, the gauge group $G = \frac{SU(2)}{\Gamma}$, where $\Gamma = 1,\ \mathbb{Z}_2$
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- Coming back to low-energy, heavy particle can be described by high dim. operators in EFT (if there is decoupling limit).

The Standard Model

SM particle content

- The matter content (+ gauge fields in the adjoints)

Table 29.1 Charges of Standard Model fields. □ indicates that the field transforms in the fundamental representation, and – indicates that a field is uncharged.

Field	$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	e_R	ν_R	$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	u_R	d_R	H
SU(3)	–	–	–	□	□	□	–
SU(2)	□	–	–	□	–	–	□
$U(1)_Y$	$-\frac{1}{2}$	-1	0	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

[M. Schwartz QFT & SM textbook]

- The $\tilde{G} = SU(3)_c \times SU(2)_L \times U(1)_Y$ appears to be the gauge group, naively

- Nonetheless, much like the $SU(2)$ in the toy model, we are not sure this is the genuine gauge group. To find the genuine gauge group, we need to take a quotient to remove the trivial group elements.

Which SM gauge group?

- The ambiguity comes from the following $\mathbb{Z}_6$ group acting trivially on all SM fields. (This is analogous to the $\mathbb{Z}_2$ center in the toy model.)

[… O'Raifeartaigh, 86; … Tong, 17; …]

$$\mathbb{Z}_6 = \{\alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6 = 1\}$$

$$\alpha = \left(e^{\frac{2\pi i}{3}}\mathbb{1}_{3\times 3},\ e^{\pi i}\mathbb{1}_{2\times 2},\ e^{\frac{2\pi i}{6}}\right)$$

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- The generator α act on a rep. (R_3, R_2, Q_Y) as

$$U_\alpha(R_3, R_2, Q_Y) = e^{\frac{2\pi i}{3}\mathcal{N}(R_3) + i\pi\mathcal{N}(R_2) + \frac{2\pi i}{6}(6Q_Y)} = e^{2\pi i\left(\frac{\mathcal{N}(R_3)}{3} + \frac{\mathcal{N}(R_2)}{2} + Q_Y\right)}$$

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- Hence the condition for the $\mathbb{Z}_6$ group acting trivially, i.e. $U_\alpha = 1$, is

$$\mathcal{N}(R_3) = 6Q_Y \bmod 3 \qquad \& \qquad \mathcal{N}(R_2) = 6Q_Y \bmod 2$$

- All SM fields are invariant under the $\mathbb{Z}_6$ group (check it!)

Which SM gauge group?

- There are ***four choices***, differing by the global structure of the gauge group (or one-form global symmetry):

$$G = \frac{\tilde{G}}{\Gamma} = \frac{SU(3)_c \times SU(2)_L \times U(1)_Y}{\Gamma} \qquad \Gamma = \mathbb{Z}_6,\ \mathbb{Z}_3,\ \mathbb{Z}_2,\ 1$$

- To pin down which one is realized in nature, we need to discover new particles which are not invariant under $\mathbb{Z}_6$ (which we call "$\mathbb{Z}_6$ exotic particles") in experiments.

- Q: What is the SM gauge group?
- A: We need to discover new heavy particles. There are four scenarios as follows:
  - All particles are invariant under $\mathbb{Z}_6$, Γ remains undetermined as in the SM. However, if this is the case it might be better to write $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y / \mathbb{Z}_6$.
  - At least one heavy particle is not invariant under $\mathbb{Z}_3$ but invariant under $\mathbb{Z}_2$ (hence not invariant under $\mathbb{Z}_6$), Γ can be either $\mathbb{Z}_2$ or 1.
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  - At least one heavy particle is invariant under neither $\mathbb{Z}_2$ nor $\mathbb{Z}_3$ (hence not invariant under $\mathbb{Z}_6$), Γ is uniquely determined to be 1.

[See in the backup slides and hep-ph/2404.04229 for concrete examples]

Conclusions

- We don't know about either the microscopic nature of EWSB (hence the shape of the Higgs potential) or the SM gauge group
- Future collider experiments are highly desirable to understand these questions better

— We must measure the Higgs self coupling

— If we are lucky, we might discover new particles not invariant under the $\mathbb{Z}_6$ group

(In the paper we discuss from the SMEFT perspective, so indirect signatures can also be interesting)

Backups

Examples of heavy particles & SM gauge group

- (R_3, R_2, Q_Y) = (fundamental, fundamental, 0) is allowed when $\Gamma = 1$ but forbidden when $\Gamma = \mathbb{Z}_{2,3,6}$
- (R_3, R_2, Q_Y) = (fundamental, fundamental, 2/3) is allowed when $\Gamma = 1$ or $\mathbb{Z}_3$, but forbidden when $\Gamma = \mathbb{Z}_2$ or $\mathbb{Z}_6$
- (R_3, R_2, Q_Y) = (fundamental, fundamental, 1/2) is allowed when $\Gamma = 1$ or $\mathbb{Z}_2$, but forbidden when $\Gamma = \mathbb{Z}_3$ or $\mathbb{Z}_6$

Decoupling heavy $\mathbb{Z}_6$ exotics

- One can use SMEFT
- We showed they never appear in tree-level ultraviolet completions, see hep-ph/2404.04229
- One-loop dictionary between heavy particles and SMEFT operators becomes highly desirable.
- Interesting phenomenological implications, in particular an upper bound for reheating temperature.
- Systematic studies on future colliders warranted

Non-decoupling scalars for EWSB

- Q: What about the scalars that can trigger EWSB?
- A: They don't decouple and they are not $\mathbb{Z}_6$ exotics.
- Proof: 1) Since color is unbroken, the scalars must be neutral under $SU(3)_c$ (i.e. singlet rep. has N-ality zero). 2) In the notation of (j, Q_Y) the quantum numbers are subject to the following constraints to accommodate a electric neutral component:

$$-j \leq Q_Y \leq j \qquad \text{and} \qquad j + Q_Y \in \mathbb{Z}$$

- Q_Y is either integer or half-integer since j is, hence

$$0 = 6Q_Y \mod 3 \longrightarrow \text{invariant under } \mathbb{Z}_3$$

- Furthermore, let's compute $2j - 6Q_Y$

$$2(j - Q_Y) - 4Q_Y = 0 \mod 2 \longrightarrow \text{invariant under } \mathbb{Z}_2$$

- Invariance under both $\mathbb{Z}_{2,3}$ implies invariance under $\mathbb{Z}_6$

Higher-form symmetries

- Free Maxwell theory with no matter:
  the Gauss law is understood as electric $U(1)$ 1-form symmetry

- Pure $SU(N)$ gauge theory with no matter:
  the center of the gauge group measures the N-ality of a Wilson line, which is understood as electric Z_N 1-form symmetry

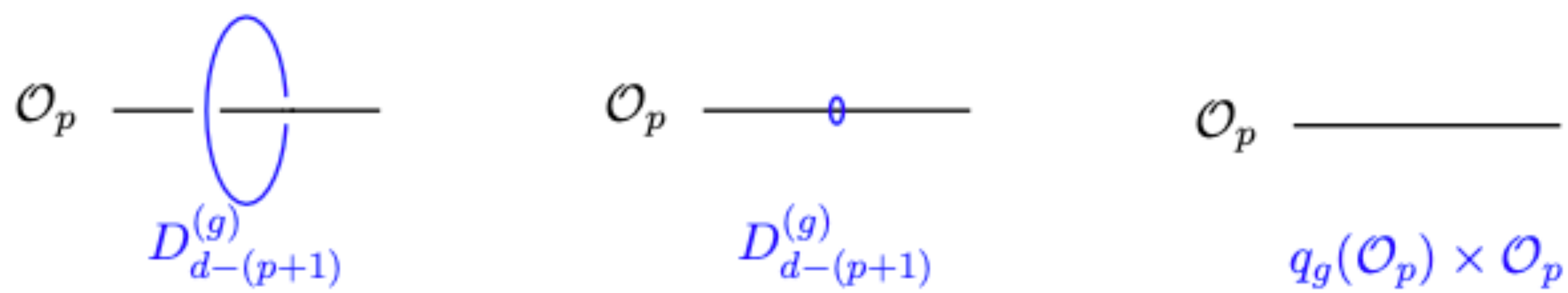
- Adding matter fields breaks the electric 1-form symmetry explicitly, i.e. Wilson lines can be screened/trivialized by particles.

- Nevertheless, the notions of electric 1-form symmetry and Wilson lines are still valid below the mass scale of the heavy particles that screen the Wilson lines. As such, the 1-form symmetry is viewed as accidental at low energy.

One-form symmetries and Line Operators

[ICTP lectures by Schafer-Nameki, 2023]

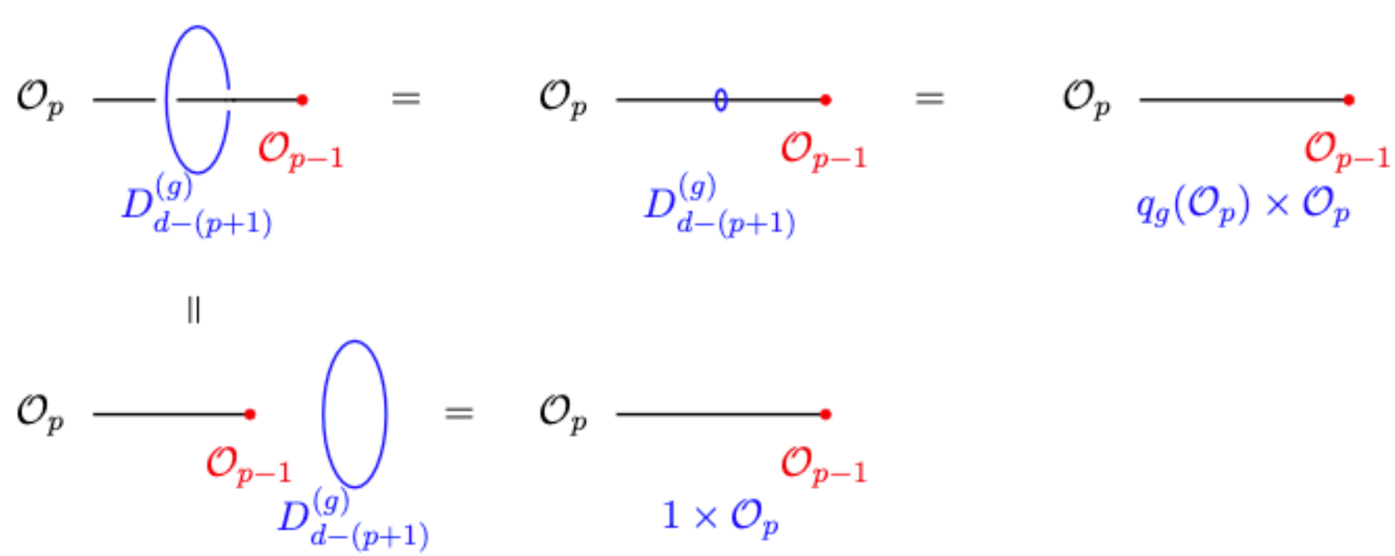
- A p-form global symmetry is generated by a dimension $(d-p-1)$ dimensional topological operator D_{d-p-1} acting on a p dimensional charged operator $\mathcal{O}_p$ as in the following:



(2.13)

- Higher form symmetries (i.e. $p > 1$) are abelian.

- Screening the charge: p-form symmetry can be screened (trivialized) by $p-1$ dimensional operators $\mathcal{O}_{p-1}$ which live at the end of $\mathcal{O}_p$



One-form symmetries and Line Operators

[ICTP lectures by Schafer-Nameki, 2023]

- One useful perspective is to think in terms of the equivalence relations between charged operators $\mathcal{O}_p$

$$\mathcal{O}_p^{(1)} \sim \mathcal{O}_p^{(2)} \quad \Leftrightarrow \quad \exists\ O_{p-1} \text{ at the junction between } \mathcal{O}_p^{(1)} \text{ and } \mathcal{O}_p^{(2)} . \qquad (2.28)$$

- Example: in a pure Yang-Mill theory with simply-connected gauge group G, Wilson lines of all possible charges under the center $\mathbb{Z}_G$ are allowed. Since the only local operators are in the adjoint which is not charged under center, all these $\mathbb{Z}_G$ charged Wilson lines are inequivalent and so the 1-form symmetry is the center. Also it's obvious that adding additional matter can trivialize some of the Wilson lines, hence breaking the 1-form symmetry to a subgroup.

- Taking the quotient Γ restricts the allowed Wilson lines, but it allows for more 't Hooft lines. There are different ways of adding the lines (called choices of "polarizations").

Centers for simply-connected groups

G	Z_G	$q(\boldsymbol{F})$
$SU(N)$	$\mathbb{Z}_N$	$1 \mod N$
$\mathrm{Spin}(4N)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(1,1) \mod (2,2)$
$\mathrm{Spin}(4N+2)$	$\mathbb{Z}_4$	$2 \mod 4$
$\mathrm{Spin}(2N+1)$	$\mathbb{Z}_2$	$1 \mod 2$
E_6	$\mathbb{Z}_3$	$1 \mod 3$
E_7	$\mathbb{Z}_2$	$1 \mod 2$
E_8	$\mathbb{Z}_1$	$1 \mod 1$

Table 1: Simply-connected Lie groups G and their centers Z_G, as well as the charge of the fundamental representation $\boldsymbol{F}$ under the generator(s) of the center.

$$Z_G = \mathrm{Center}(G) = \{g \in G : \; gh = hg \text{ for all } h \in G\} \,. \tag{2.18}$$

[ICTP lectures by Schafer-Nameki, 2023]