Higgs Potential
From Standard Model to EFT and UV Models

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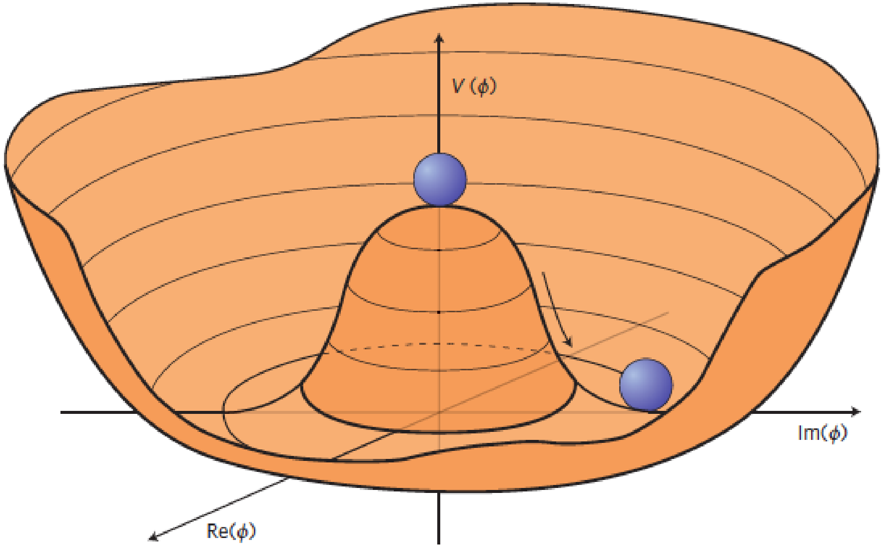
Higgs potential and Electroweack Symmetry breaking

$$V_{SM}(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$

$$\phi = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

$$V(h) = \frac{1}{2}m_h^2 h^2 + \frac{1}{3!}\lambda_3 h^3 + \frac{1}{4!}\lambda_4 h^4$$



$$\lambda_3^{\rm SM} = \frac{3m_h^2}{v} \simeq 190\,\mathrm{GeV} \quad \text{and} \quad \lambda_4^{\rm SM} = \frac{3m_h^2}{v^2} \simeq 0.77$$

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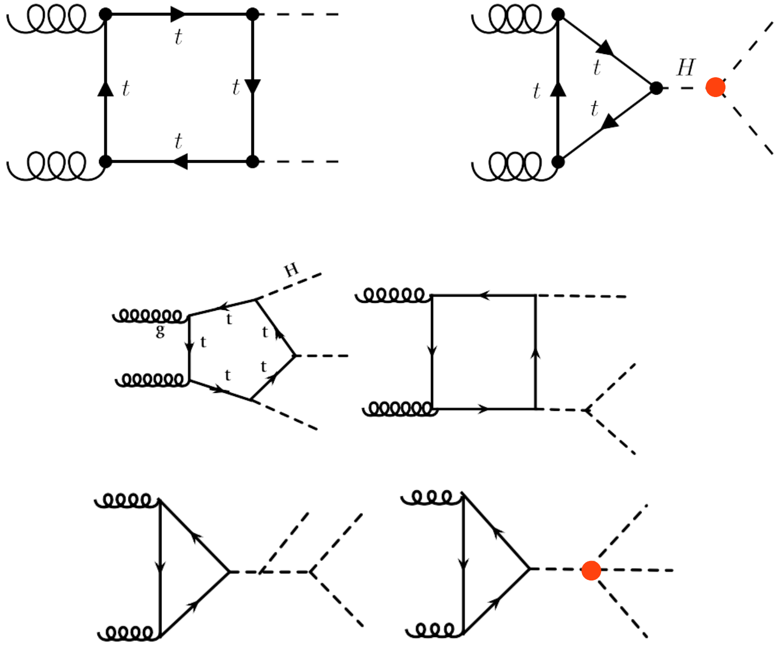


Outline

- Review of higher order correction for ggHH
- EFT framework for ggHH
- EFT UV completion – J-basis UV correspondence $|H|^6$
- Global fit constraints for EFT and UV models
- Higgs quartic couplings measurments
- Theoretical bounds on Higgs self couplings

$ggHH$ production in SM — QCD Correction

➢ Heavy Top Limit:

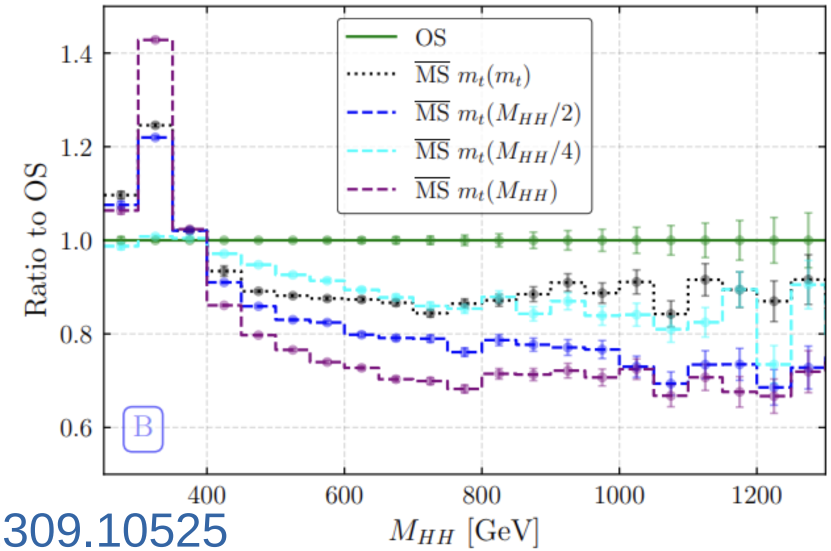
scale uncertainties ~ 3%

– N3LO+N3LL: Ajjath, Shao,2209.03914
– N3LO with Top mass effects: Chen, Li, Shao, Wang, 1912.13001
– N2LO: De Florian, Mazzitelli 1305.5206; Grigo, Melnikov, Steinhauser 1408.2422

➢ Full Top Mass:

– NLO: Baglio,et.al. 1811.05692; Davies, et.al. 1907.06408 ; Bagnaschi, et.al 2309.10525
– N2LO Approx: Grazzini, et.al, 1803.02463

top mass effects ~ 5%



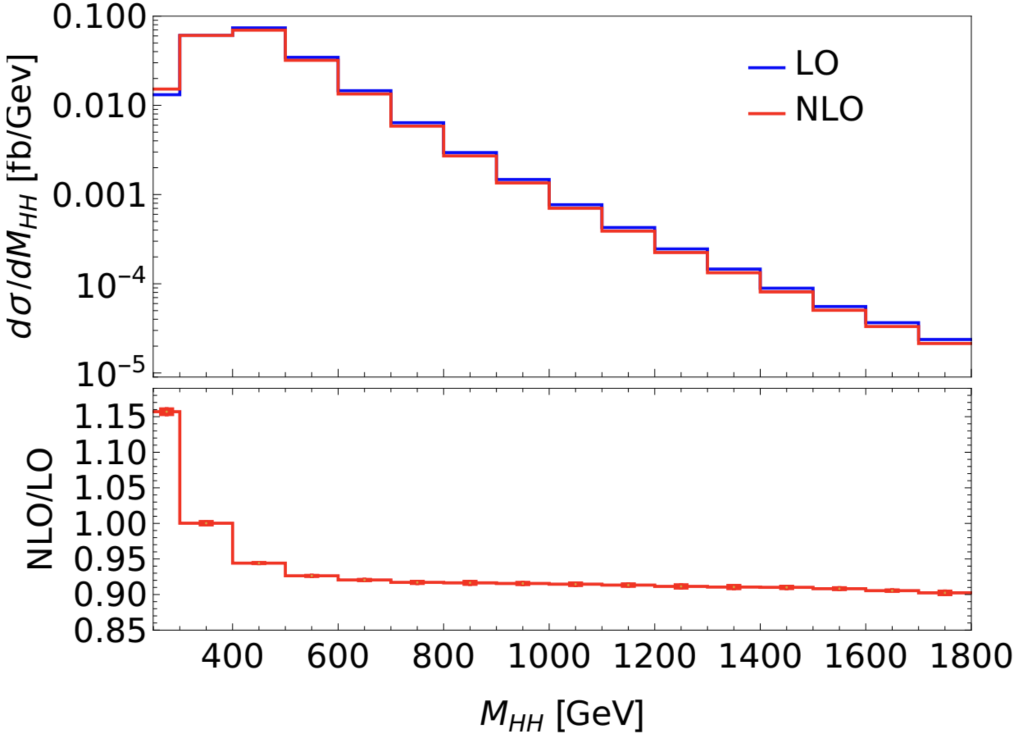
2309.10525

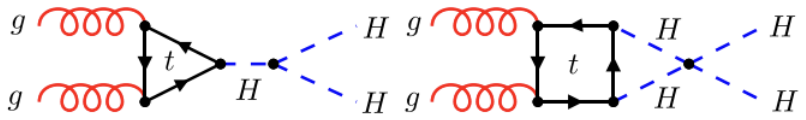
On-shell vs MSbar
top mass scheme ~ 20%

See Bi and Wang's talk for detail

$ggHH$ production in SM — EW Correction

Full NLO EW: Bi, Huang, Huang, Ma, Yu, 2311.16963





Sensitive to the quartic Higgs coupling

– Partial top Yukawa + Higgs self-couplings: Heinrich, et.al. 2407.04653
– Higgs self-couplings with modifier: Li, et.al. 2407.14716
– full EW in large-mt expansion: Davies, Schönwald, Steinhauser, Zhang 2308.01355

See Bi and Wang's talk for detail

$ggHH$ production in Effective field theory

- Standard Model Effective Field Theory (SMEFT)

Gradzkowsi, et. al. 1008.4884; **HLL**, Ren, Xiao, Yu, Zhen 2005.00008

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^2} O_i^{\text{dim6}} + \sum_i \frac{c_i}{\Lambda^4} O_i^{\text{dim8}} + \ldots$$

canonical mass dimension.

– Higgs as a component of SU(2) doublet. Linearly realized SU(2) symmetry.
– Suitable for weakly coupled UV theories.

- Higgs Effective Field Theory (HEFT)

Feruglio, 9301281; Grinstein, Trott, 0704.1505; Buchalla, et.al. 1307.5017; Sun, et.al 2206.07722, 2210.14939.

$$\mathcal{L}_{\text{HEFT}} = \mathcal{L}_{\chi_d=2} + \sum_{L,i} \left(\frac{1}{16\pi^2}\right)^L c_i^L O_i^L$$

chiral (loop) dimension.

– Higgs as singlet of SU(2), non-linearly realized SU(2) symmetry.
– Suitable for strongly coupled or non-decoupled UV theories.

$ggHH$ production in HEFT

$$\mathcal{L}_{\rm HEFT} = -m_t \left(c_t \frac{h}{v} + c_{tt} \frac{h^2}{v^2} \right) \bar{t}t - c_{hhh} \frac{m_h^2}{2v} h^3 + \frac{\alpha_s}{8\pi} \left(c_{ggh} \frac{h}{v} + c_{gghh} \frac{h^2}{v^2} \right) G^a_{\mu\nu} G^{a,\mu\nu} \; .$$

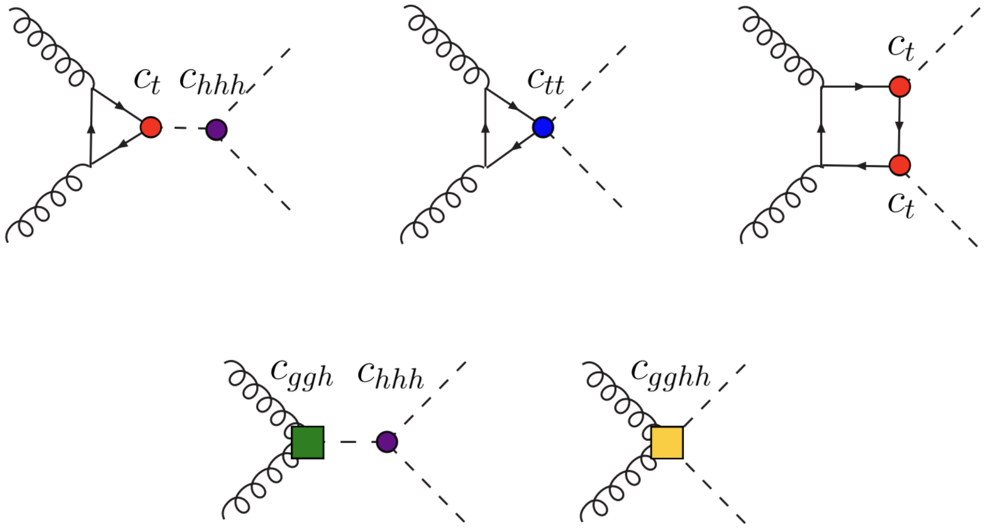


figure from 2304.01968

c_t, c_{ggh} More constraint by single Higgs production.

Latest result:

- ATLAS 2406.09971
combined ***ggHH*** and ***VBFHH***

$$-1.2 < c_{hhh} < 7.2$$

- CMS 2407.13554
combined **single** and **di-Higgs**

$$-1.2 < c_{hhh} < 7.5$$

Assuming all other coupling SM-like

- HL-LHC projection 1902.00134

$$0.1 < c_{hhh} < 2.3$$

$ggHH$ production in SMEFT

$$\Delta\mathcal{L}_{\text{Warsaw}} = \frac{C_{H,\Box}}{\Lambda^2}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + \frac{C_{HD}}{\Lambda^2}(\phi^\dagger D_\mu\phi)^*(\phi^\dagger D^\mu\phi) + \frac{C_H}{\Lambda^2}(\phi^\dagger\phi)^3$$
$$+ \left(\frac{C_{uH}}{\Lambda^2}\phi^\dagger\phi\bar{q}_L\tilde{\phi}\,t_R + h.c.\right) + \frac{C_{HG}}{\Lambda^2}\phi^\dagger\phi G^a_{\mu\nu}G^{\mu\nu,a}$$
$$+ \frac{C_{uG}}{\Lambda^2}(\bar{q}_L\sigma^{\mu\nu}T^aG^a_{\mu\nu}\tilde{\phi}\,t_R + \text{h.c.}).$$

Looply generated in a weakly coupled theory, effectively contributing 2-loop order in ggHH thus subleading

$$C_{H,\text{kin}} \equiv C_{H,\Box} - \frac{1}{4}C_{HD}$$

HEFT	Warsaw
c_{hhh}	$1 - 2\frac{v^2}{\Lambda^2}\frac{v^2}{m_h^2}C_H + 3\frac{v^2}{\Lambda^2}C_{H,\text{kin}}$
c_t	$1 + \frac{v^2}{\Lambda^2}C_{H,\text{kin}} - \frac{v^2}{\Lambda^2}\frac{v}{\sqrt{2}m_t}C_{uH}$
c_{tt}	$-\frac{v^2}{\Lambda^2}\frac{3v}{2\sqrt{2}m_t}C_{uH} + \frac{v^2}{\Lambda^2}C_{H,\text{kin}}$
c_{ggh}	$\frac{v^2}{\Lambda^2}\frac{8\pi}{\alpha_s}C_{HG}$
c_{gghh}	$\frac{v^2}{\Lambda^2}\frac{4\pi}{\alpha_s}C_{HG}$

A naive mapping exists between HEFT and SMEFT at leading order.

– HEFT is more general than SMEFT
– valid HEFT point may result in invalid SMEFT point (depend on Λ)
– NLO QCD correction 2204.13045

TriHiggs coupling – SMEFT UV completion

$\delta\lambda_3 \neq 0$ → $(\phi^\dagger\phi)^3$, $(\phi^\dagger\phi)^2 D^2$ → Tree-level UV origin →

- **– real singlet; – 2HDM; – real triplet;**
- **– complex triplet ($Y = 2Y_H$);**
- **– complex quadruplet ($Y = 3Y_H / Y_H$)**

Murphy, Dawson 1704.07851;
Corbett, Joglekar, **HLL**, Yu 1705.02551

J-Basis – UV correspondence

Fully automated with the package **`ABC4EFT`**

HLL, Ren, Xiao, Yu, Zheng
2201.04639

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```
TabulateUVs["H"^3 "H†"^3, 1]
```

$1 \to \{\{H \to \{3\},\ H^\dagger \to \{3\}\},\ H_i H_j H_k H^{\dagger i} H^{\dagger j} H^{\dagger k}\}$

S1(**1**,**1**,0)	(H,H†,S1) (H,H†,S1,S1) (H,H†,S1) (H,H†,S1) (S1,S1,S1)
S7(**1**,**3**,0)	(H,H†,S7) (H,H†,S7,S7) (H,H†,S7)
S4(**1**,**2**,1/2)	(H,H†,H†,S4)
S9(**1**,**4**,1/2)	(H,H†,H†,S9)
S8(**1**,**3**,1)	(H†,H†,S8) (H,H†,S8†,S8) (H,H,S8†)
S10(**1**,**4**,3/2)	(H†,H†,H†,S10)

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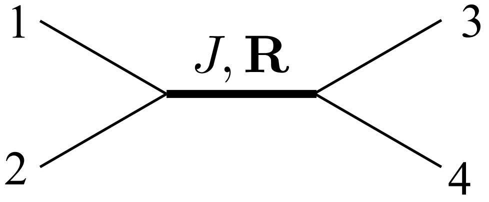
New Fields Lists	New Vertices Lists
S1(**1**,**1**,0)	(H,H†,S1) (H,H†,S1,S1) (H,H†,S1) (H,H†,S1) (S1,S1,S1)
S7(**1**,**3**,0)	(H,H†,S7) (H,H†,S7,S7) (H,H†,S7)
S4(**1**,**2**,1/2)	(H,H†,H†,S4)
S9(**1**,**4**,1/2)	(H,H†,H†,S9)
S8(**1**,**3**,1)	(H†,H†,S8) (H,H†,S8†,S8) (H,H,S8†)
S10(**1**,**4**,3/2)	(H†,H†,H†,S10)

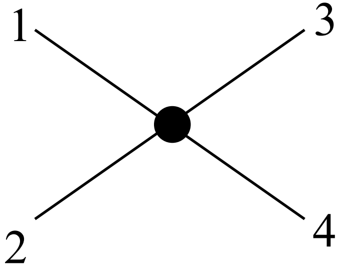
Short introduction to J-basis UV correspondence

- Intuition – diagrammatic matching

UV Resonance Φ — spin J, gauge rep: $\mathbf{R}$ $\qquad$ $\psi_1\psi_2\Phi, \psi_3\psi_4\Phi$



$$\mathcal{A}_{\text{UV}}^{J_{12}=J;\mathbf{R}_{12}=\mathbf{R}}$$



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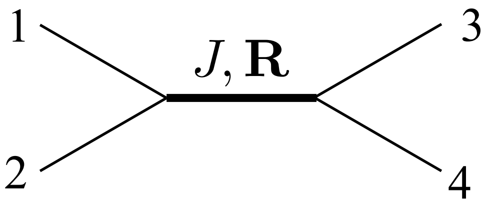
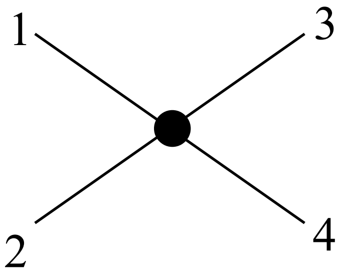


$$\mathcal{A}_{\text{UV}}^{J_{12}=J;\mathbf{R}_{12}=\mathbf{R}}$$

Expanding the propagator



$$\mathcal{A}_{\text{EFT}}^{J_{12}=J;\mathbf{R}_{12}=\mathbf{R}}$$

→ J-basis amplitude:
An extension of partial wave amplitude

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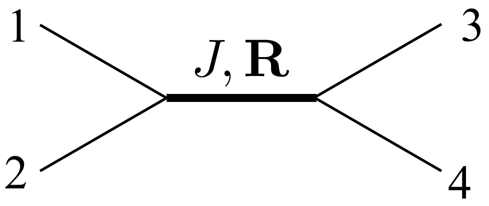

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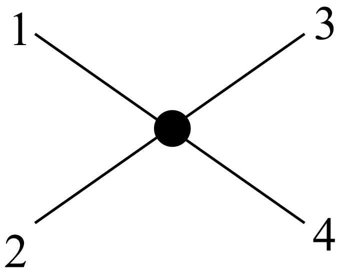

$$\mathcal{A}_{\mathrm{EFT}}^{J_{12}=J;\mathbf{R}_{12}=\mathbf{R}}$$

J-basis amplitude:
An extension of partial wave amplitude

$$\mathbb{C}_{12}\mathcal{A}_{\mathrm{EFT}}^{\mathbf{R}_{12}=\mathbf{R}} = C(\mathbf{R})\mathcal{A}_{\mathrm{EFT}}^{\mathbf{R}_{12}=\mathbf{R}}$$
$$\mathcal{W}_{12}^2\mathcal{A}_{\mathrm{EFT}}^{J_{12}=J} = sJ(J+1)\mathcal{A}_{\mathrm{EFT}}^{J_{12}=J}$$

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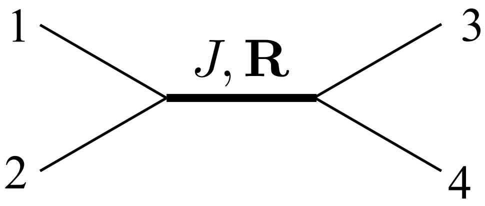

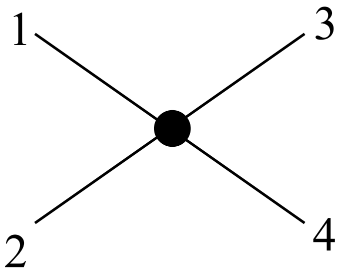

$$\mathcal{A}_{\text{UV}}^{J_{12}=J;\mathbf{R}_{12}=\mathbf{R}}$$

↓ Expanding the propagator

$$\mathcal{A}_{\text{EFT}}^{J_{12}=J;\mathbf{R}_{12}=\mathbf{R}}$$ → J-basis amplitude: An extension of partial wave amplitude

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↓ Amplitude-Operator Correspondence

$$\mathcal{O}^{J_{12}=J;\mathbf{R}_{12}=\mathbf{R}}$$ → J-basis operator:

Short introduction to J-basis UV correspondence

- Intuition – diagrammatic matching

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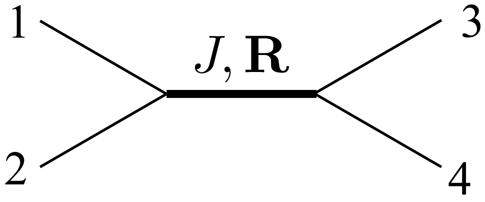

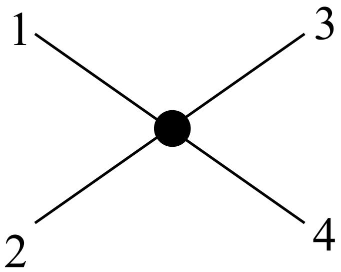

$\mathcal{A}_{\text{UV}}^{J_{12}=J;\mathbf{R}_{12}=\mathbf{R}}$

↓ Expanding the propagator

$\mathcal{A}_{\text{EFT}}^{J_{12}=J;\mathbf{R}_{12}=\mathbf{R}}$ → J-basis amplitude: An extension of partial wave amplitude

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↓ Amplitude-Operator Correspondence

$\mathcal{O}^{J_{12}=J;\mathbf{R}_{12}=\mathbf{R}}$ → J-basis operator:

$$\mathcal{O}^{J,\mathbf{R}}|J',\mathbf{R}'\rangle_{12} \sim \delta^{JJ'}\delta_{\mathbf{R}\mathbf{R}'}$$

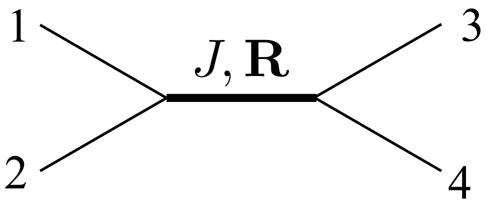
↓ $\mathcal{K}^{jp}$

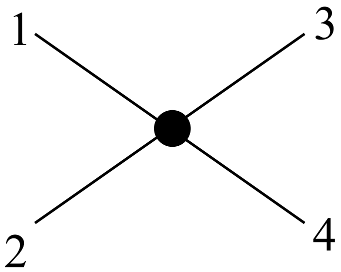
Physical Basis: $\{\mathcal{O}^p\}$

Short introduction to J-basis UV correspondence

- Intuition – diagrammatic matching

UV Resonance Φ — spin J, gauge rep: $\mathbf{R}$ $\quad\psi_1\psi_2\Phi, \psi_3\psi_4\Phi$





Reorganize into j-basis according to certain scattering channel

$$\mathcal{A}_{\text{UV}}^{J_{12}=J;\mathbf{R}_{12}=\mathbf{R}}$$

Expanding the propagator

$$\mathcal{A}_{\text{EFT}}^{J_{12}=J;\mathbf{R}_{12}=\mathbf{R}}$$

→ J-basis amplitude: An extension of partial wave amplitude

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Amplitude-Operator Correspondence

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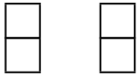
$\mathcal{K}^{jp}$

Physical Basis: $\{\mathcal{O}^p\}$

Short introduction to J-basis UV correspondence

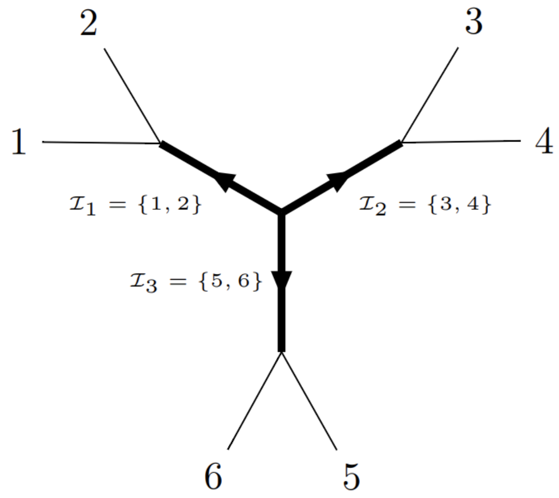
- A quick example: SMEFT dim-6: H^4D^2 , for $H_1^\dagger H_2 \to H_3^\dagger H_4$

J	$\mathbf{R}$	J-basis	$\mathcal{K}^{\text{jp}}$				P-basis	$\text{Sym}_{H,H^\dagger}$
0	$\mathbf{1}$	$(H_1^\dagger H_2)D^2(H_3^\dagger H_4)$	1	0	1	0	$Q_{\varphi\Box}$	
	$\mathbf{3}$	$(H_1^\dagger \tau^I H_2)D^2(H_3^\dagger \tau^I H_4)$	-1	4	-1	4	$Q_{\varphi D}$	
1	$\mathbf{1}$	$(H_1^\dagger i\overleftrightarrow{D}_\mu H_2)(H_3^\dagger i\overleftrightarrow{D}^\mu H_4)$	-1	-4	-1	4	$Q'_{\varphi\Box}$	
	$\mathbf{3}$	$(H_1^\dagger i\tau^I\overleftrightarrow{D}_\mu H_2)(H_3^\dagger i\tau^I\overleftrightarrow{D}^\mu H_4)$	-3	0	5	-8	$Q'_{\varphi D}$	

$$Q_{H\Box} \sim S(\mathbf{1}), S(\mathbf{3}), V(\mathbf{1}), V(\mathbf{3}) \qquad Q_{HD} \sim S(\mathbf{3}), V(\mathbf{1})$$

Short introduction to J-basis UV correspondence

Directly extend to multi-partition for operators with more than 4 fields



$$[W^2_{\mathcal{I}_1}, W^2_{\mathcal{I}_2}] = [W^2_{\mathcal{I}_1}, W^2_{\mathcal{I}_3}] = [W^2_{\mathcal{I}_3}, W^2_{\mathcal{I}_2}] = 0$$

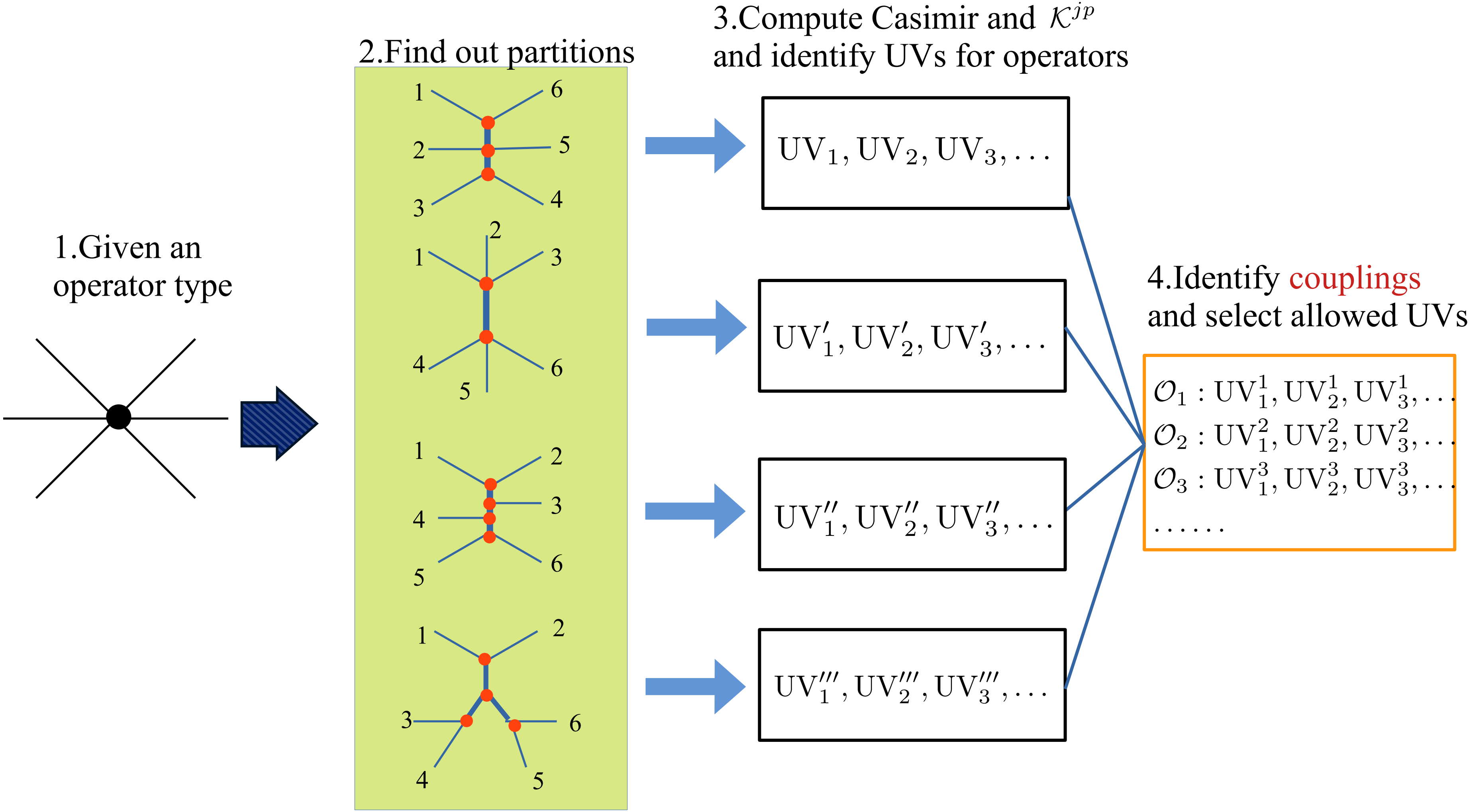
$$[\mathbb{C}_{\mathcal{I}_1}, \mathbb{C}_{\mathcal{I}_2}] = [\mathbb{C}_{\mathcal{I}_1}, \mathbb{C}_{\mathcal{I}_3}] = [\mathbb{C}_{\mathcal{I}_3}, \mathbb{C}_{\mathcal{I}_2}] = 0$$

Can Find simultanous eigenbasis for each Casimir operator

$$\mathbf{W}^2_{\mathcal{I}_i} \mathcal{A}^{\{J_i\},\{\mathbf{R}_i\}} = -s_{\mathcal{I}_i} J_i \left(J_i + 1\right) \mathcal{A}^{\{J_i\},\{\mathbf{R}_i\}}$$

$$\mathbb{C}_{\mathcal{I}_i} \mathcal{A}^{\{J_i\},\{\mathbf{R}_i\}} = C\left(\mathbf{R}_i\right) \mathcal{A}^{\{J_i\},\{\mathbf{R}_i\}}$$

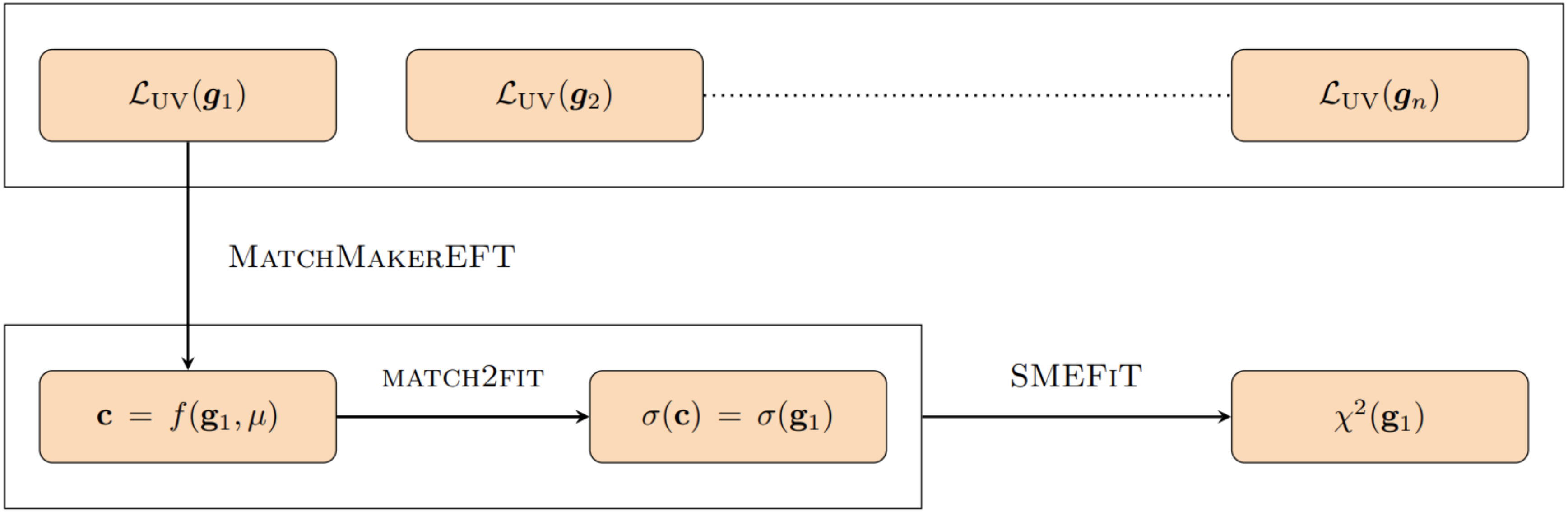
Short introduction to J-basis UV correspondence



Global fit constraints from SMEFT to UV

- Rapid developments of EFT Matching Tools enable the systematic matching results to 1-loop level. [DsixTools 2010.16341]; [CoDeX 1808.04403]; [MatchmakerEFT, 2112.10787]; [Matchete,2212.04510].
- Translate SMEFT Global Fit to the Global Fit of UV models

Hoeve, et.al, 2309.04523



Identify independent combination of UV parameters from the expression of Wilson coefficients

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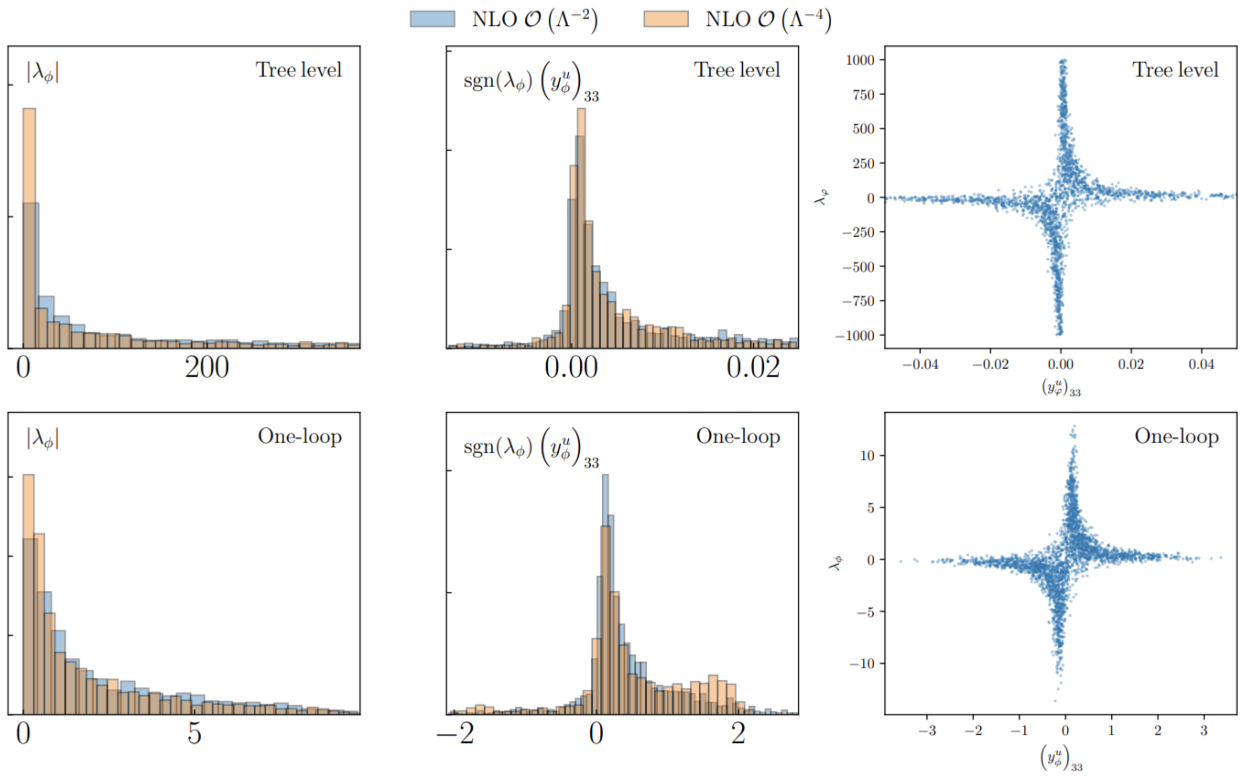


Toy 2HDM:

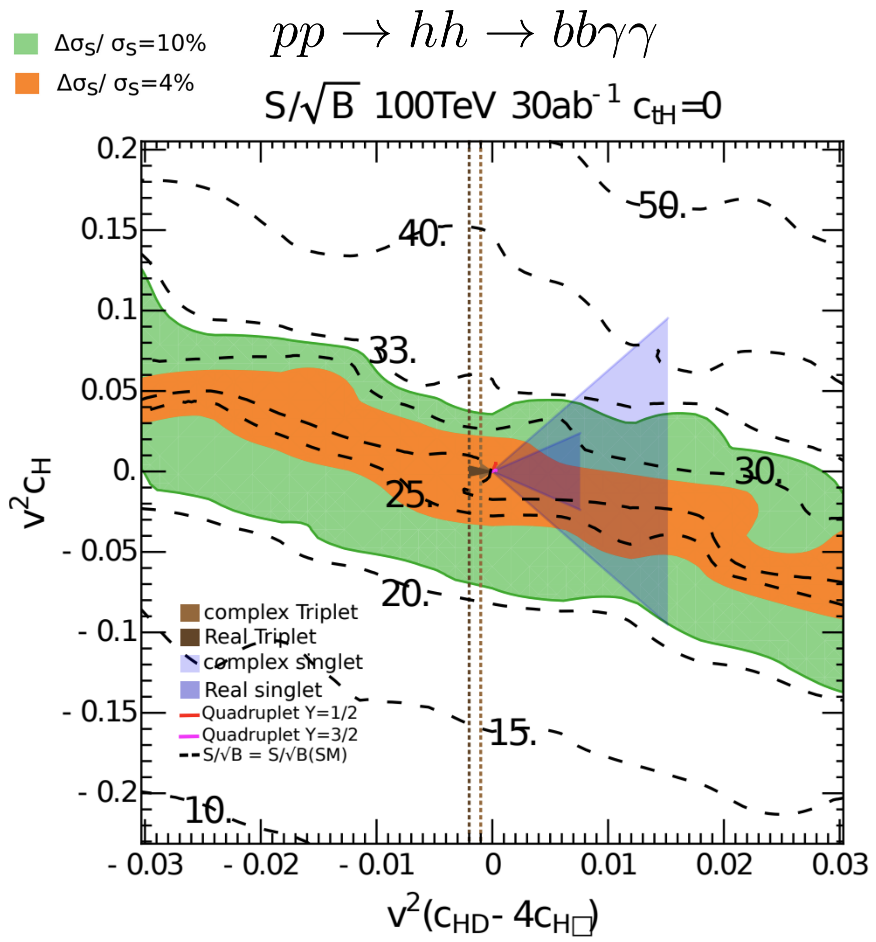
$$+(y^u_\phi)_{ij}\,\phi^\dagger i\sigma_2 \bar{q}^{T,i}_L u^j_R + \lambda_\phi\,\phi^\dagger\varphi|\varphi|^2$$

Posterior distributions from a Global fit with *Higgs+Top+diboson+EWPO*

Hoeve, et.al, 2309.04523

Global fit constraints from SMEFT to UV

- Constraints from Future Colliders



Custodial symmetry Breaking

Tree-level Matching results:

Theory:	c_H	$c_{H\Box}$	c_{HD}
$\mathbb{R}$ Triplet $(Y=0)$	$-\frac{g^2}{M^4}\left(\frac{\lambda_{H\Phi}}{8}-\lambda\right)$	$\frac{g^2}{8M^4}$	$-\frac{g^2}{2M^4}$
$\mathbb{C}$ Triplet $(Y=-1)$	$-\frac{\|g\|^2}{M^4}\left(\frac{\lambda_{H\Phi}}{4}+\frac{\lambda'}{8}-2\lambda\right)$	$\frac{\|g\|^2}{2M^4}$	$\frac{\|g\|^2}{M^4}$
$\mathbb{C}$ Quadruplet $(Y=1/2)$	$\frac{\|\lambda_{H3\Phi}\|^2}{M^2}$	-	$\frac{2\|\lambda_{H3\Phi}\|^2 v^2}{2M^4}$
$\mathbb{C}$ Quadruplet $(Y=3/2)$	$\frac{\|\lambda_{H3\Phi}\|^2}{M^2}$	-	$\frac{6\|\lambda_{H3\Phi}\|^2 v^2}{2M^4}$

Correlation

– Models that violate Custodial symmetry strongly constraint by EWPO,

– The correlation between $\mathbf{c_H}$ $\mathbf{c_{H\Box}}$ and $\mathbf{c_{HD}}$ leads to exceedingly small correction on $\boldsymbol{\lambda_3}$

Corbett, Joglekar, **HLL**, Yu 1705.02551

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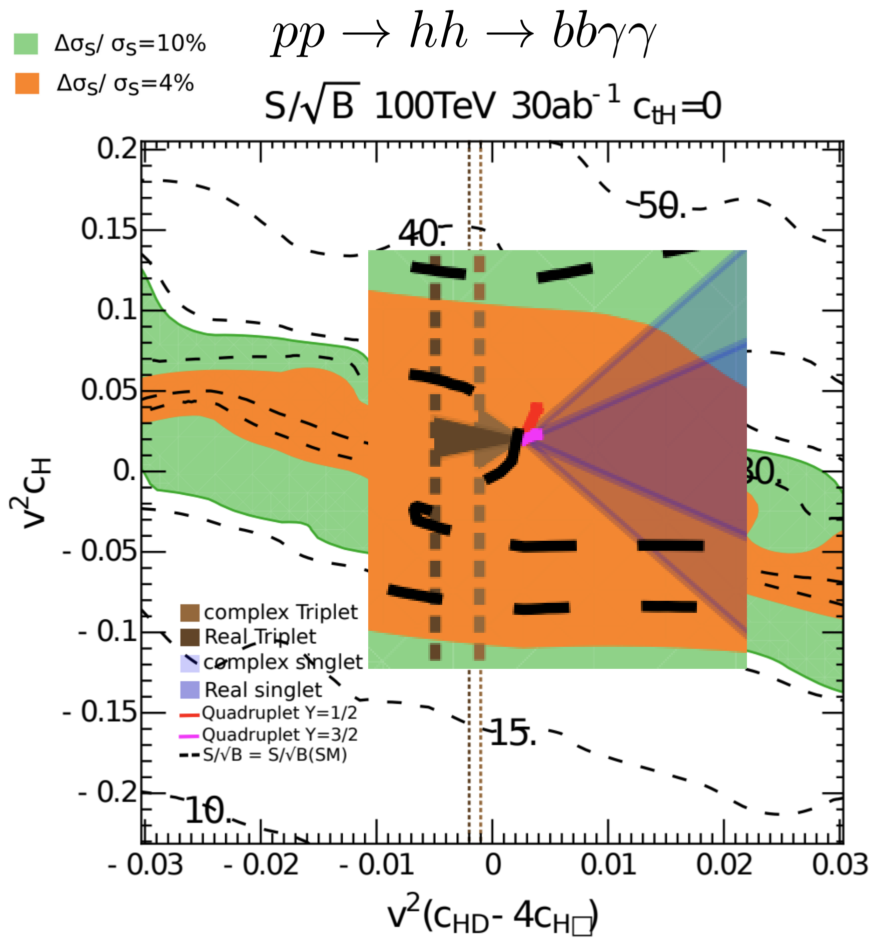

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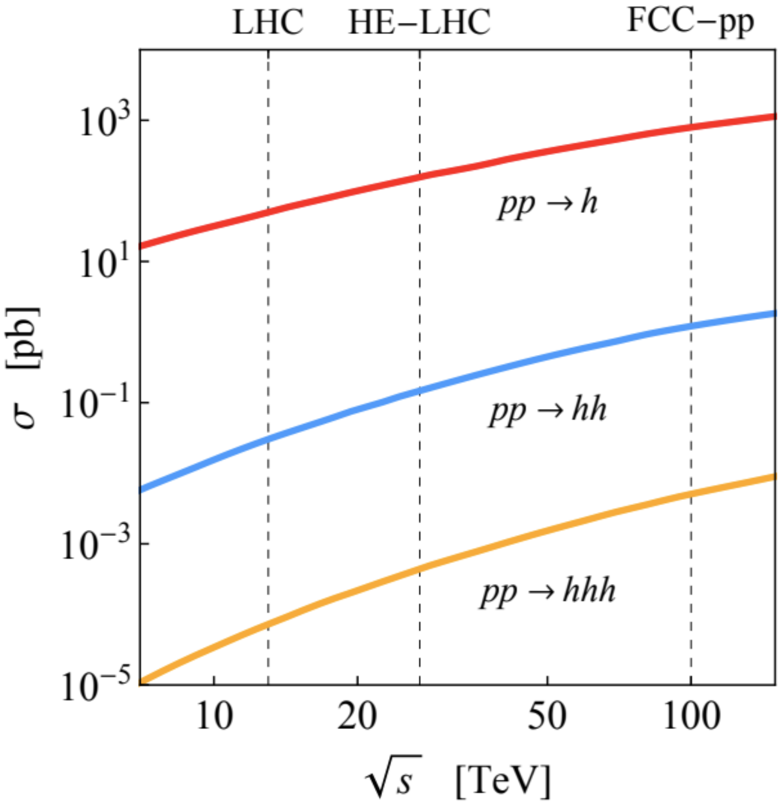
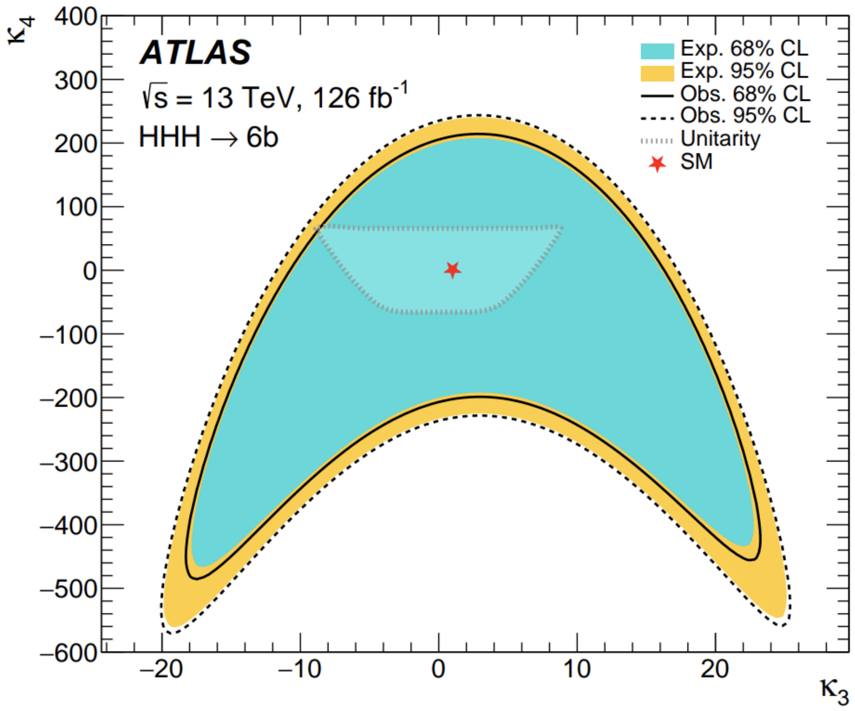
Higgs quartic coupling search

Direct measurement is very challenging:



Bizon, et.al., 1810.04665

First direct measurement by HHH → 6b



ATLAS: 2411.02040

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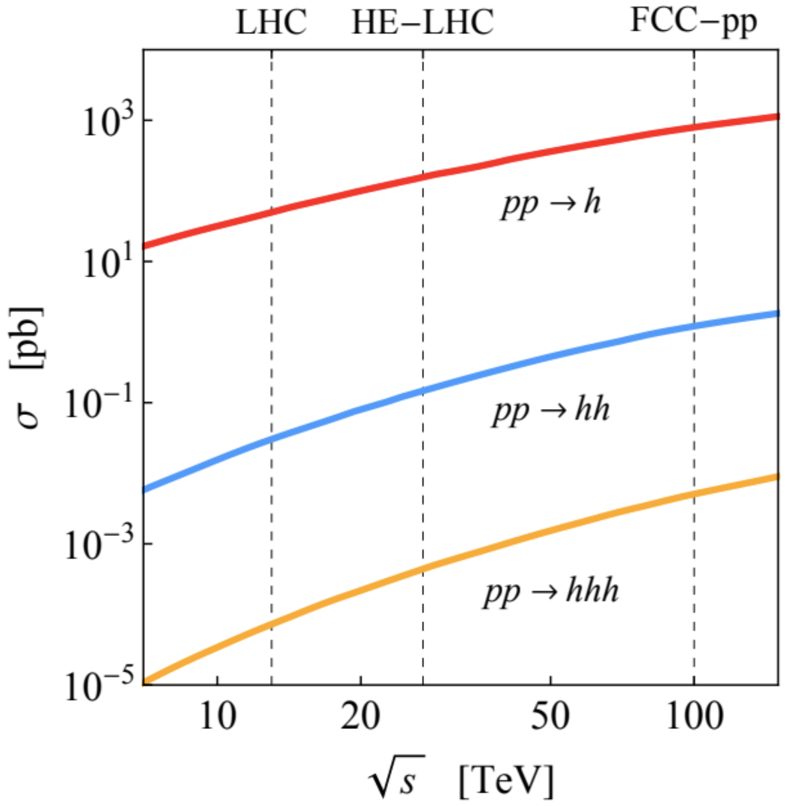
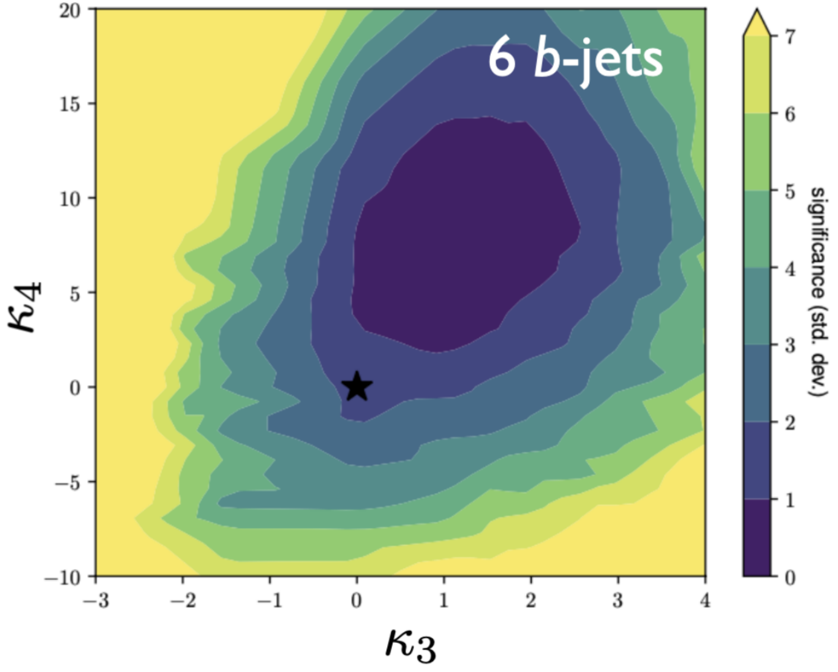


Bizon, et.al., 1810.04665

100 TeV projection can be found in :
HHH Whitepaper 2407.03015



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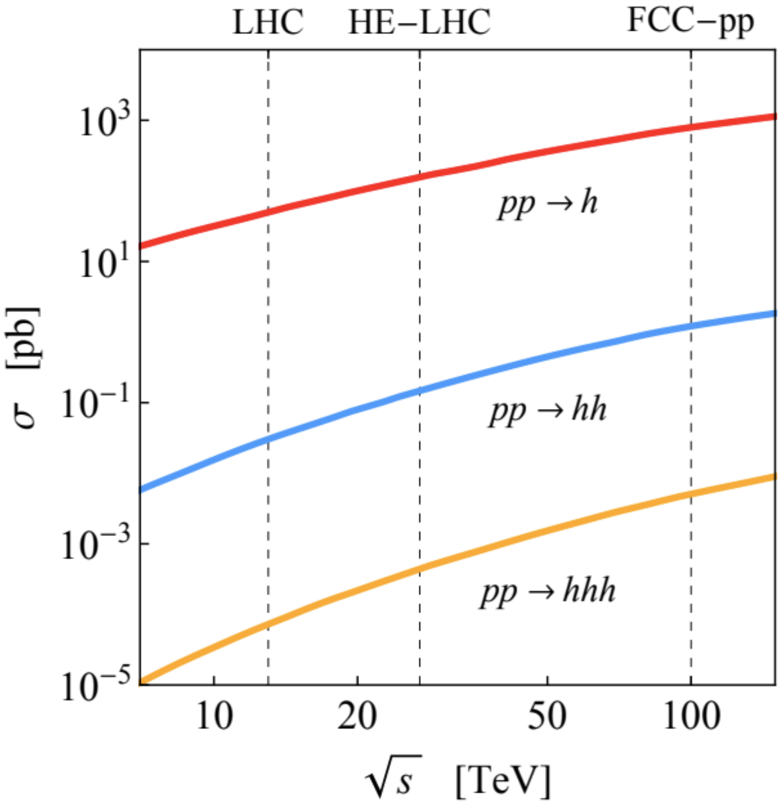

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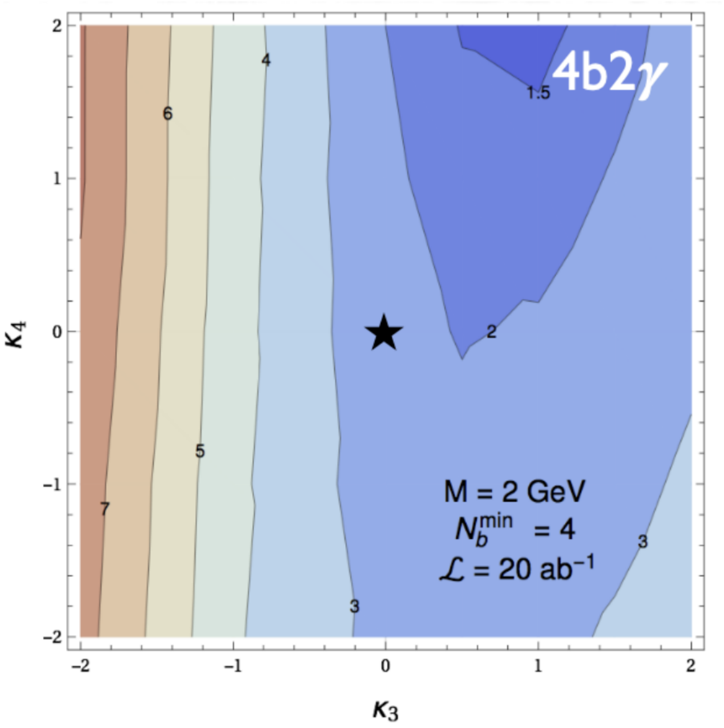
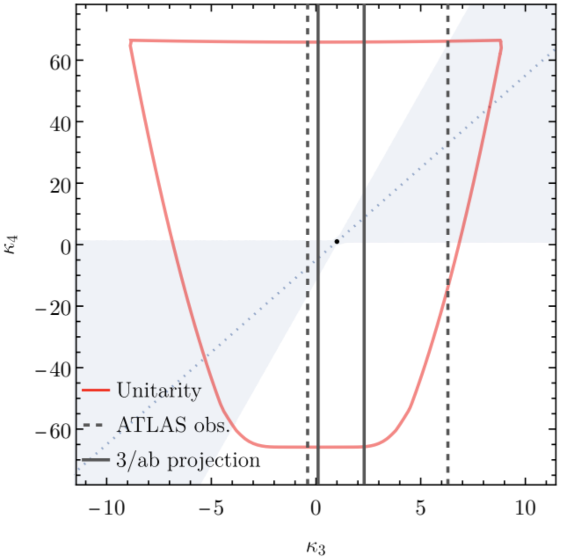


Other Constraints on Higgs Self-Couplings

- Perturbative Unitarity:



Partial wave for HH→HH scattering

$$a^0 = \frac{3M_H^2\sqrt{s^2-4M_H^2 s}}{32\pi s(s-M_H^2)v^2}\left[\kappa_4(s-M_H^2) - 3\kappa_3^2 M_H^2 + \frac{6\kappa_3^2 M_H^2(s-M_H^2)}{s-4M_H^2}\log\left(\frac{s}{M_H^2}-3\right)\right]$$

κ4 from high energy limit
κ3 from lower energy regiem

HHH Whitepaper 2407.03015
Liu, et.al 1803.04359

- Vaccum stability:

Hard to obtain general results without specifying UV   Luzio, Gröber, Spannowsky 1704.02311

– SMEFT upto dim-8 $|H|^8$ Falkowski, Rattazzi 1902.05936

– more on MCHM, CTHM see

Agrawal, et.al. 1907.02078

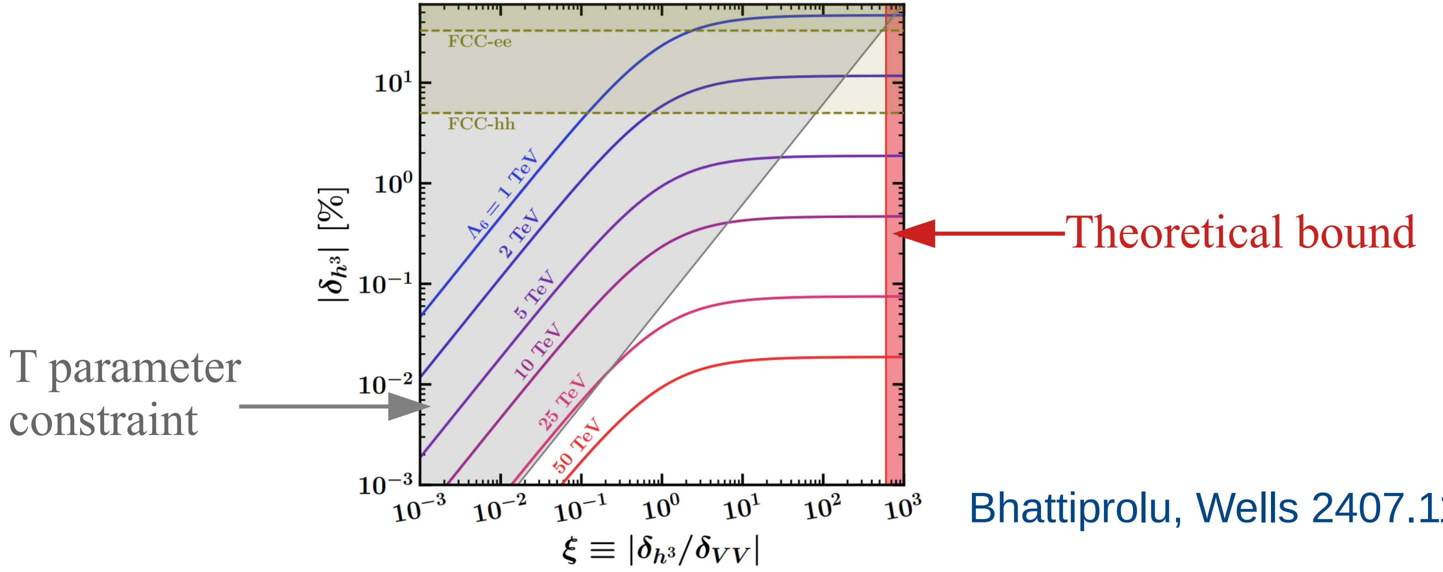
$$-\delta_{h^3} \leq \frac{2\sqrt{2c_8}\,v^3}{M^2 m_h} \approx \frac{\sqrt{c_8}}{(4\pi)^3}\left(\frac{26\,\text{TeV}}{M}\right)^2$$

Other Constraints on Higgs Self-Couplings

- Relation between δ_{h3} and δ_{VV} Durieux, McCullough, Salvioni 2209.00666
  Based on the argument of hbar counting
  Assuming no fine-tuning in the UV model

$$\left|\frac{\delta_{h^3}}{\delta_{VV}}\right| \lesssim \min\left[\left(\frac{4\pi v}{m_h}\right)^2, \left(\frac{M}{m_h}\right)^2\right] \sim 600 \text{ for } M > 3\text{TeV}$$

- Combining with the current EWPO indicates a maximum δ_{h3}~40%



Bhattiprolu, Wells 2407.11847

Summary

- Top mass scheme dominates the theory uncertainties of Di-Higgs cross-section
- Full Electroweak NLO correction of Di-Higgs significantly affects low mHH bin
- SMEFT and HEFT are framework for study the deviation from SM
- Tree-level UV completion for $|H|^6$ are all scalar extensions
- Direct measurement on quartic couplings are challenging even in 100TeV collider
- Deviations in hVV coupling and h^3 coupling can be correlated, improving hVV measurment set upper bound on h^3 couplings

Back Up

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4}g^2\phi^4$$

$$\frac{i}{\hbar}S = \frac{i}{\hbar}\int d^4x\, \mathscr{L}$$

$$\phi = \sqrt{\hbar}\hat{\phi}$$

$$\frac{i}{\hbar}S = i\int d^4x \left[\frac{1}{2}(\partial_\mu \hat{\phi})^2 - \frac{1}{2}m^2\hat{\phi}^2 - \frac{1}{4}(g^2\hbar)\hat{\phi}^4\right]$$

$$\frac{i}{\hbar}g^{n-2}\phi_1 \ldots \phi_n \quad \rightarrow \quad i(g^2\hbar)^{n/2-1}\hat{\phi}_1 \ldots \hat{\phi}_n$$

$(n-2)$ **Rule**:

an operator in a generic $\mathscr{L}$ containing n fields (irrespective of their spin) should carry $n-2$ powers of couplings.

L-loop n-pt → *n-2+2L* coupling order

$$c_6 \sim \kappa \quad , \qquad c_{H,R,T} \sim \frac{\kappa}{16\pi^2} \quad , \qquad c_{H_8,R_8,T_8} \sim \kappa$$

$$\delta_{h^3} \sim \kappa \frac{v^4}{M^2 m_h^2} \quad , \qquad \delta_{VV} \sim \kappa \frac{v^2}{M^2} \max\left[\frac{1}{16\pi^2}, \frac{v^2}{M^2}\right]$$

$$\Delta\widehat{T} \sim \kappa \frac{v^2}{M^2} \max\left[\frac{1}{16\pi^2}, \frac{v^2}{M^2}\right]$$

Short introduction to J-basis UV correspondence

- Once a amplitude basis is obtained, then one can find the possible resonance with the Casimir operator for certain channel

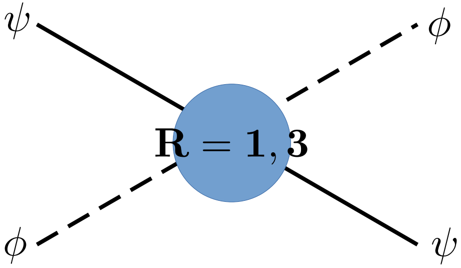
  Taking $L_1 L_2 H_3 H_4 D^2$ as an example

$$\mathcal{T}^{y}_{LLHH} = \begin{pmatrix} \epsilon^{ik}\epsilon^{jl} \\ \epsilon^{ij}\epsilon^{kl} \end{pmatrix}, \quad \underset{\{13\}}{\mathbb{C}_2} \circ \mathcal{T}^y = (\underset{\{13\}}{C_2})^{\mathrm{T}}.\mathcal{T}^y = \begin{pmatrix} 0 & 0 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} \epsilon^{ik}\epsilon^{jl} \\ \epsilon^{ij}\epsilon^{kl} \end{pmatrix}.$$

$C_2(\mathbf{1})$ $C_2(\mathbf{3})$

$$\mathcal{K}^{jy}_{G}.(\underset{\{13\}}{C_2})^{\mathrm{T}}(\mathcal{K}^{jy}_{G})^{-1} = \begin{pmatrix} 0 & 0 \\ 0 & 6 \end{pmatrix} \text{ with } \mathcal{K}^{jy}_{G} = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}$$

$$\Rightarrow \mathcal{T}^{j} = \mathcal{K}^{jy}_{G}\mathcal{T}^{y} = \begin{cases} \epsilon^{ik}\epsilon^{jl} & \mathbf{R} = \mathbf{1} \\ \epsilon^{ik}\epsilon^{jl} - 2\epsilon^{ij}\epsilon^{kl} & \mathbf{R} = \mathbf{3} \end{cases}$$



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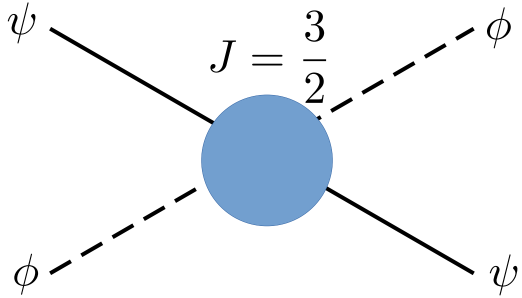
$$\mathcal{W}$$

$$\mathcal{M}^{y}_{\psi^2\phi^2 D^2} = \begin{pmatrix} s_{34}\langle 12\rangle \\ [34]\langle 13\rangle\langle 24\rangle \end{pmatrix}, \quad \mathbf{W}^2_{\{13\}}\mathcal{M}^y = s_{13}\begin{pmatrix} -\frac{15}{4} & 2 \\ 0 & -\frac{3}{4} \end{pmatrix}\mathcal{M}^y, \quad \mathcal{K}^{jy} = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \mathcal{M}^j = \mathcal{K}^{jy}\mathcal{M}^y = \begin{cases} 3s_{34}\langle 12\rangle + 2[34]\langle 13\rangle\langle 24\rangle & J = \frac{3}{2} \\ \langle 13\rangle\langle 24\rangle & J = \frac{1}{2} \end{cases}$$



Representation on the space of spinor functions: [M.-Y. Jiang et.al. 2001.04481]

$$\mathbf{J}^{\mathcal{I}}_{\mu\nu} = i \sum_{i\in\mathcal{I}} \left[\sigma^{\alpha\beta}_{\mu\nu} \left(\lambda_{i\alpha} \frac{\partial}{\partial \lambda^{\beta}_i} + \lambda_{i\beta} \frac{\partial}{\partial \lambda^{\alpha}_i} \right) + \bar{\sigma}^{\dot{\alpha}\dot{\beta}}_{\mu\nu} \left(\tilde{\lambda}_{i\dot{\alpha}} \frac{\partial}{\partial \tilde{\lambda}^{\dot{\beta}}_i} + \tilde{\lambda}_{i\dot{\beta}} \frac{\partial}{\partial \tilde{\lambda}^{\dot{\alpha}}_i} \right) \right]$$

$$\mathbf{W}^2_{\mathcal{I}} \equiv \left(\frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \mathbf{P}^{\mathcal{I}}_{\nu} \mathbf{J}^{\mathcal{I}}_{\rho\sigma} \right)^2 = \frac{1}{2} \mathbf{P}^2 \mathbf{J}^2 + \mathbf{P}_{\mu} \mathbf{J}^{\mu\nu} \mathbf{J}_{\nu\rho} \mathbf{P}^{\rho}$$

– For an operator type, the complete and independent amplitude basis $\mathcal{M}^y$ given by Young Tensor Method: [**HLL**, et.al. 2005.00008, 2201.04639]

– With the help of our amplitude reduction algorithm, one can obtain the representation matirx of the Casimir operator

$$W^2_{\mathcal{I}} \mathcal{M}^y_i = -s_{\mathcal{I}} \mathcal{W}_i{}^j \mathcal{M}^y_j$$

One can do the same thing for the gauge amplitude

$$\mathbb{C}_2 = \mathbb{T}^a\mathbb{T}^a, \text{ for both } SU(2) \text{ and } SU(3),$$
$$\mathbb{C}_3 = d^{abc}\mathbb{T}^a\mathbb{T}^b\mathbb{T}^c, \text{ for } SU(3) \text{ only},$$

$$\mathbb{T}^A_{\mathcal{I}} \circ \mathcal{T}_{I_1 I_2 \ldots I_N} = \sum_{i\in\mathcal{I}}^{N} (T^A_{r_i})^Z_{I_i} \mathcal{T}_{I_1 \ldots I_{i-1} Z I_{i+1} I_N}$$

Generator of Rep of the *i*-th particle

$$\mathbb{C}_{\mathcal{I}} \circ \mathcal{T}_i = \sum_j \mathcal{T}_j C_{ji}$$

A complete set of invariant tensors

Representation matrix of Casimir