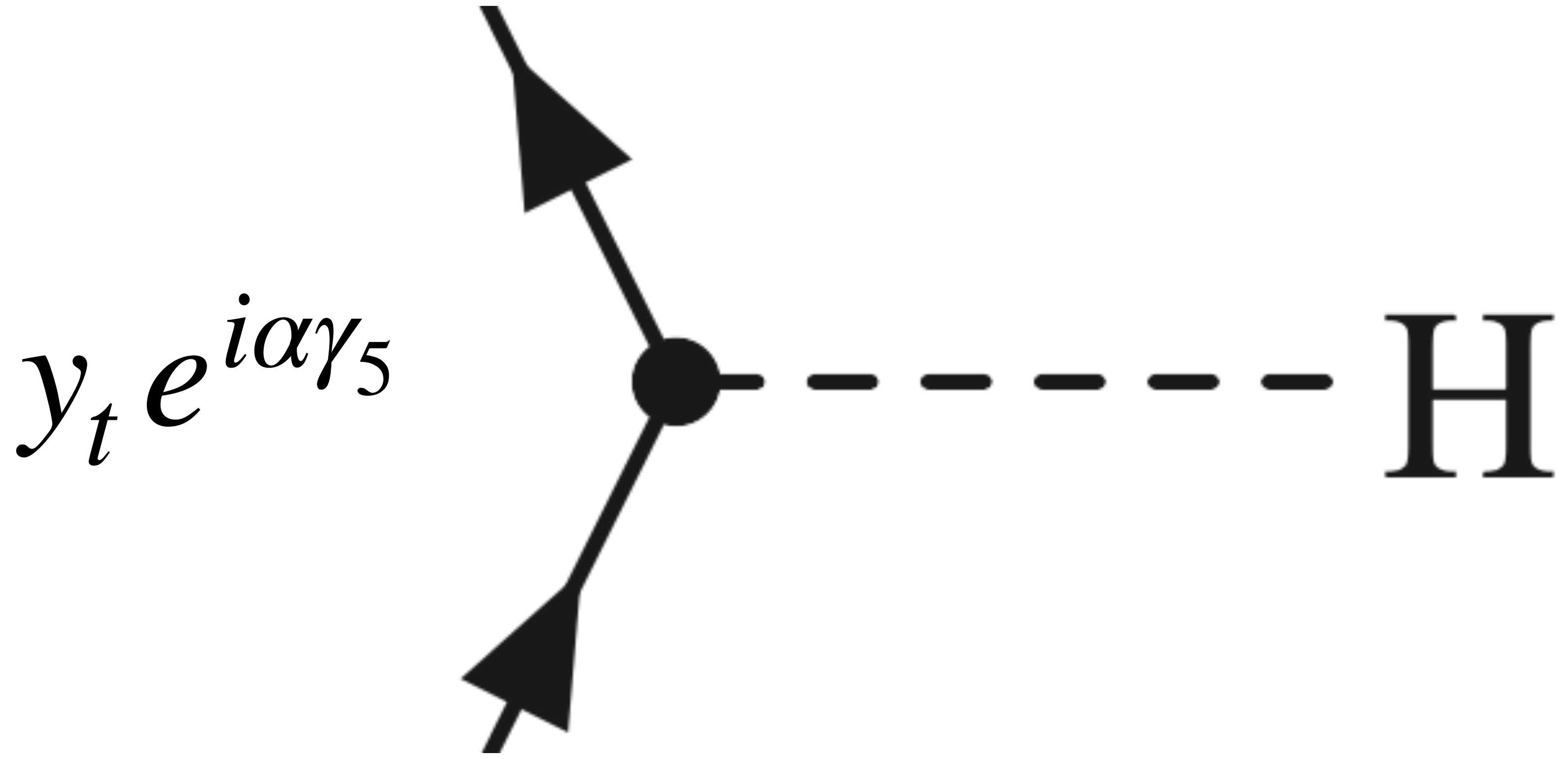
On the NNLO calculations for tTH production at hadron colliders

Li Lin Yang
Zhejiang University

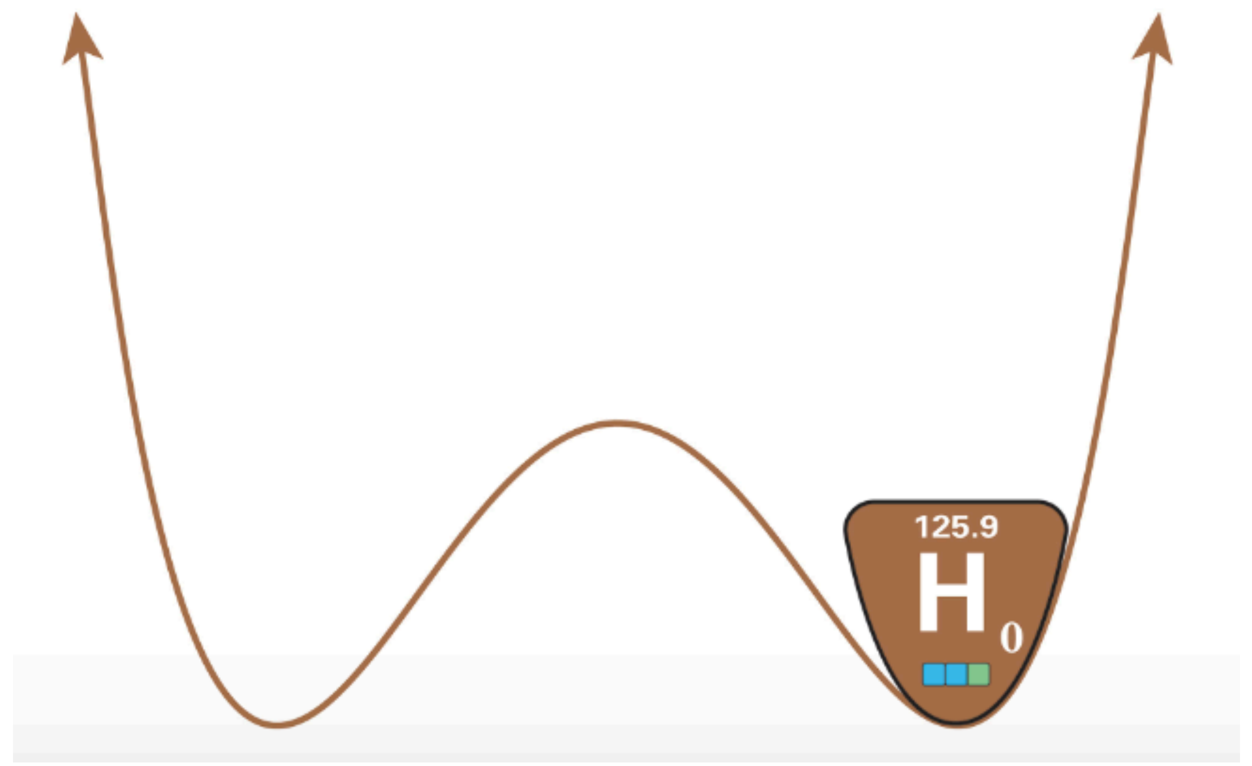
Higgs Potential 2024, 2024.12.19-23, Hefei

The top quark Yukawa coupling

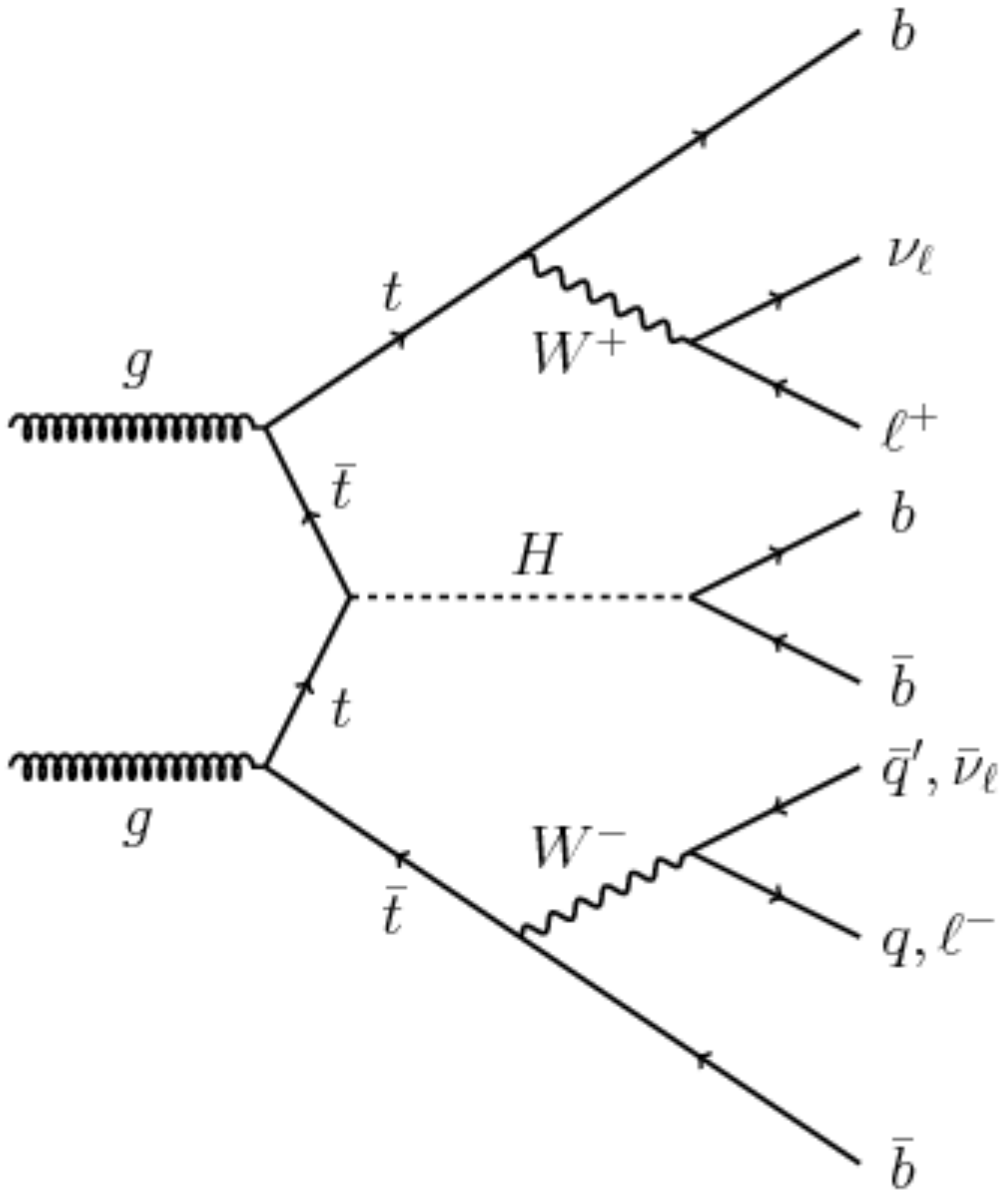


Relevant for

- Origin of masses of fundamental fermions
- Matter-anti-matter asymmetry (possible source of CP violation)
- Higgs effective potential (vacuum stability)

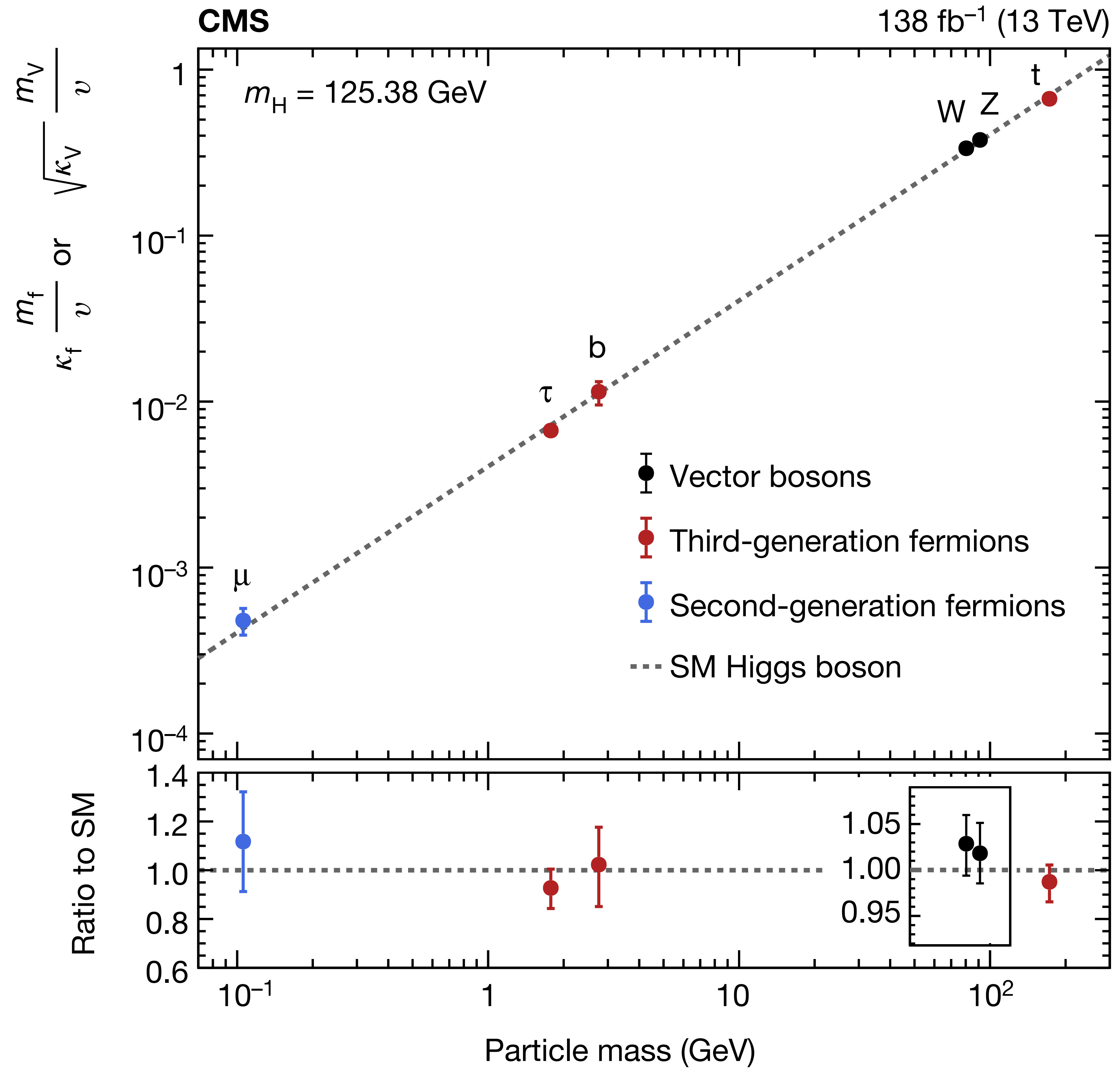


Associated tTH production

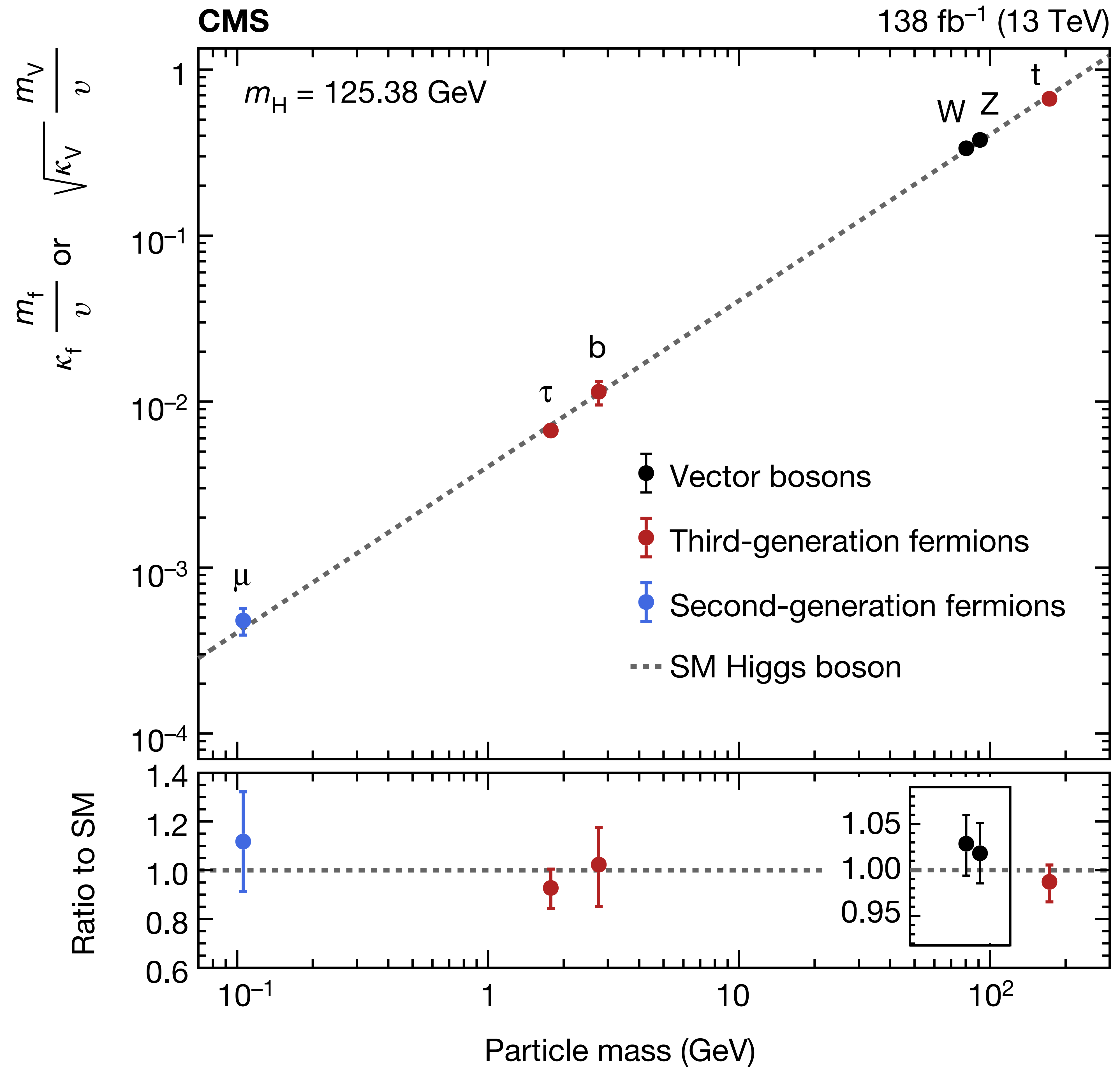


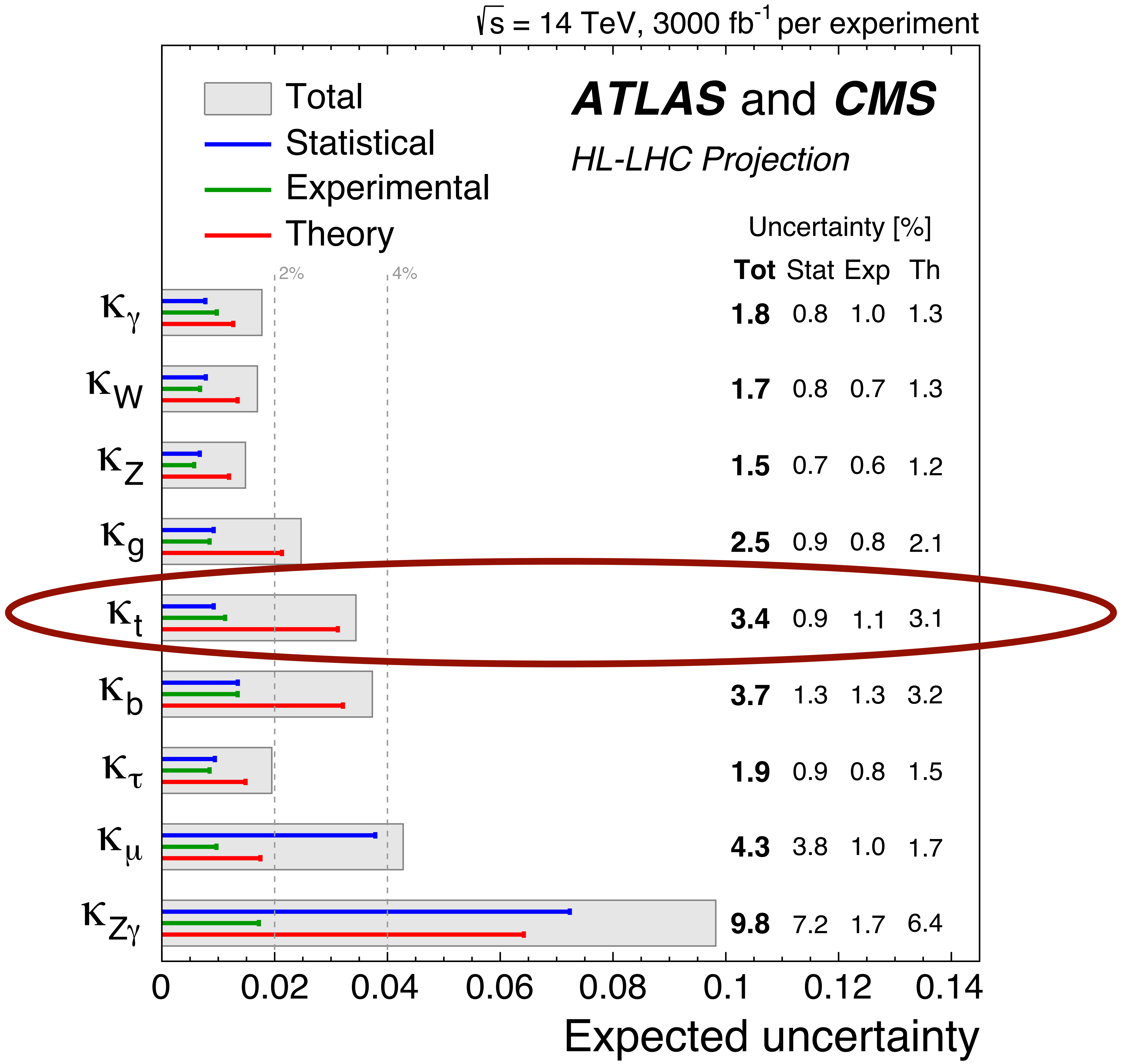
- Direct probe of top quark Yukawa coupling
- Observed in 2018 by ATLAS and CMS
- CP structure probed in 2020

The need for precision



The need for precision



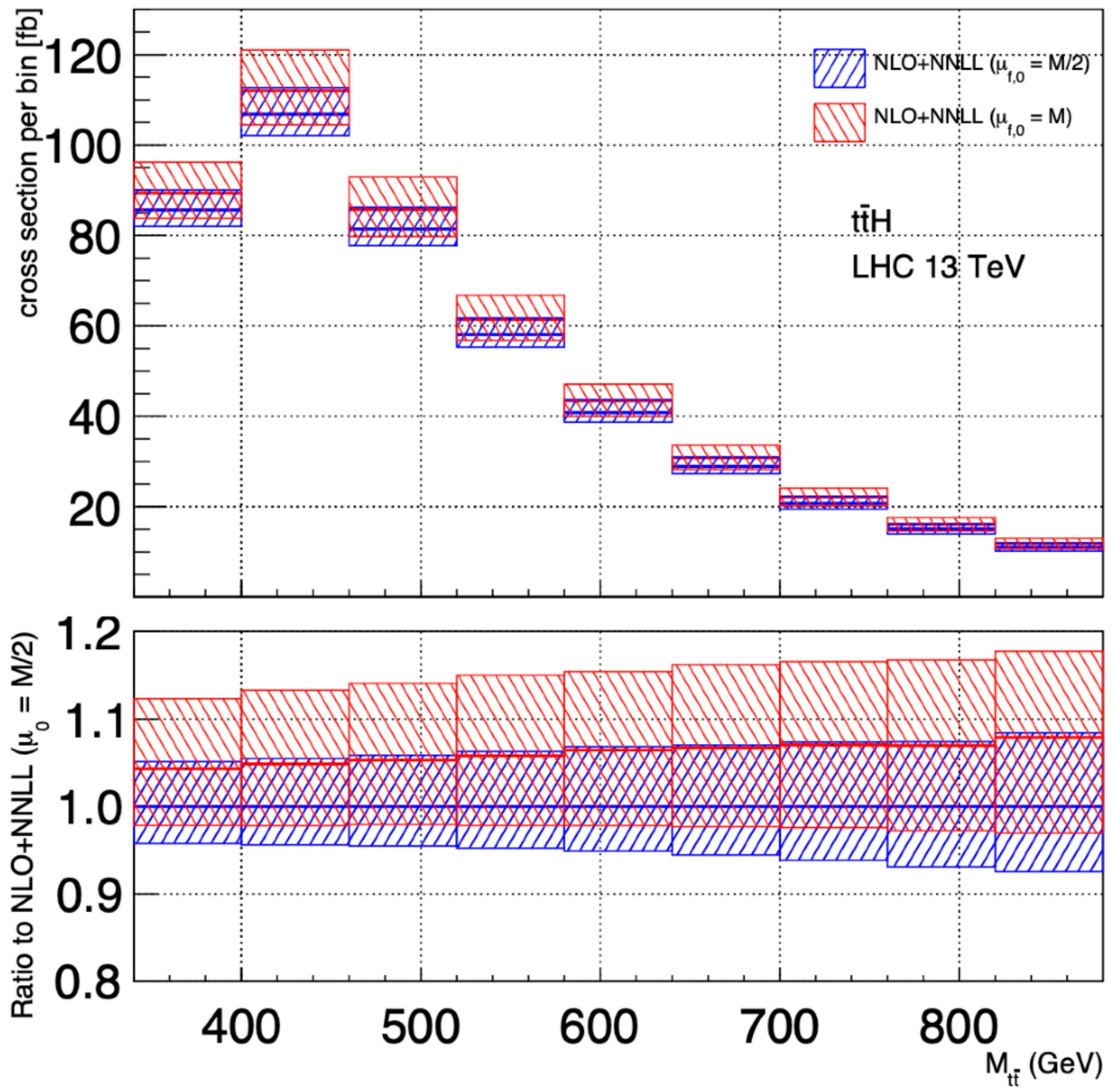


	Tot	Stat	Exp	Th
κ_γ	**1.8**	0.8	1.0	1.3
κ_W	**1.7**	0.8	0.7	1.3
κ_Z	**1.5**	0.7	0.6	1.2
κ_g	**2.5**	0.9	0.8	2.1
κ_t	**3.4**	0.9	1.1	3.1
κ_b	**3.7**	1.3	1.3	3.2
κ_τ	**1.9**	0.9	0.8	1.5
κ_μ	**4.3**	3.8	1.0	1.7
$\kappa_{Z\gamma}$	**9.8**	7.2	1.7	6.4

Theoretical status

- NLO + resummation   Broggio, Ferroglia, Pecjak, LLY: 1611.00049
- Coulomb corrections   Ju, LLY: 1904.08744

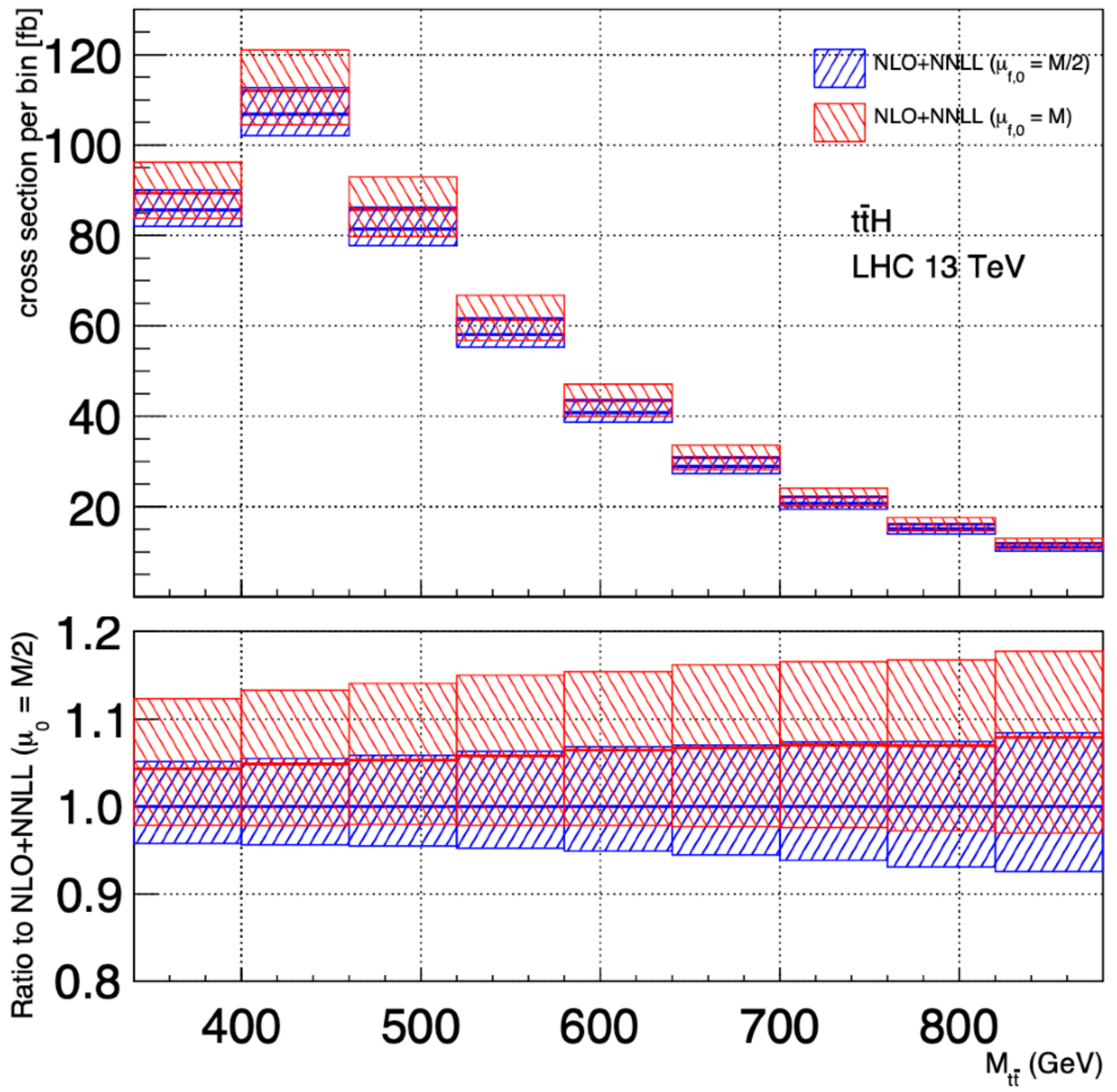
	13 TeV LHC (pb)	14 TeV LHC (pb)
NLO	$0.493^{+5.8\%}_{-9.2\%}$	$0.597^{+6.1\%}_{-9.2\%}$
NLL′+NLO	$0.521^{+1.9\%}_{-2.6\%}$	$0.630^{+2.3\%}_{-2.6\%}$
K-factor	1.06	1.06



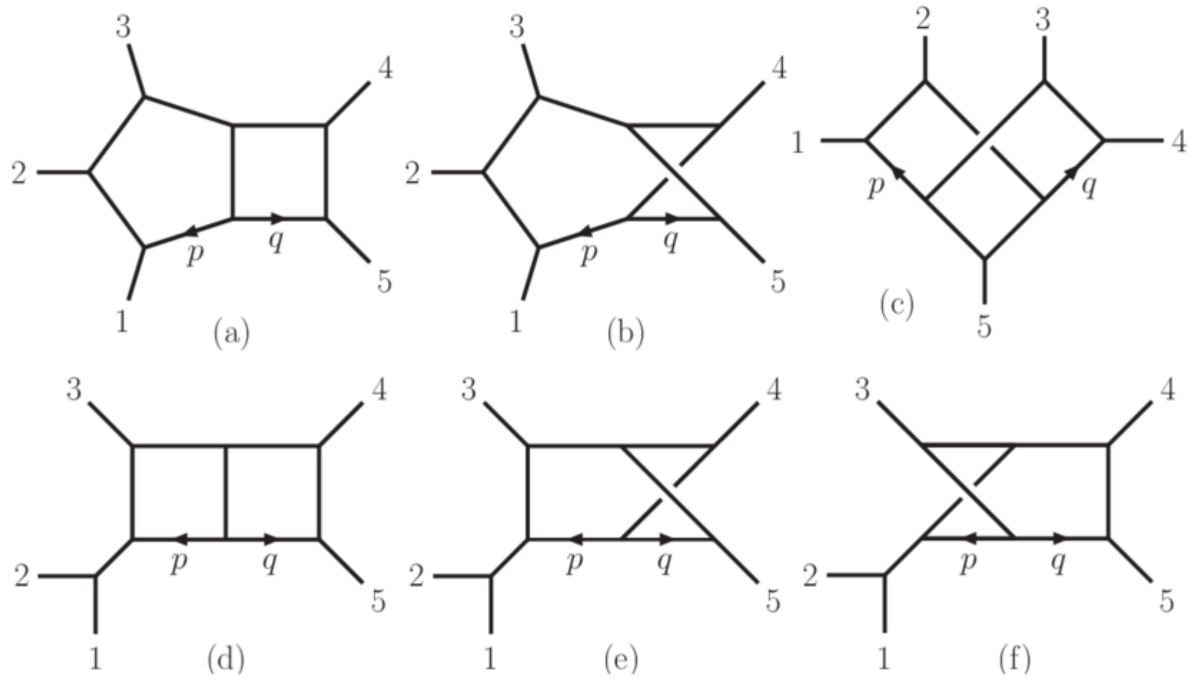
Theoretical status

- NLO + resummation Broggio, Ferroglia, Pecjak, LLY: 1611.00049
- Coulomb corrections Ju, LLY: 1904.08744

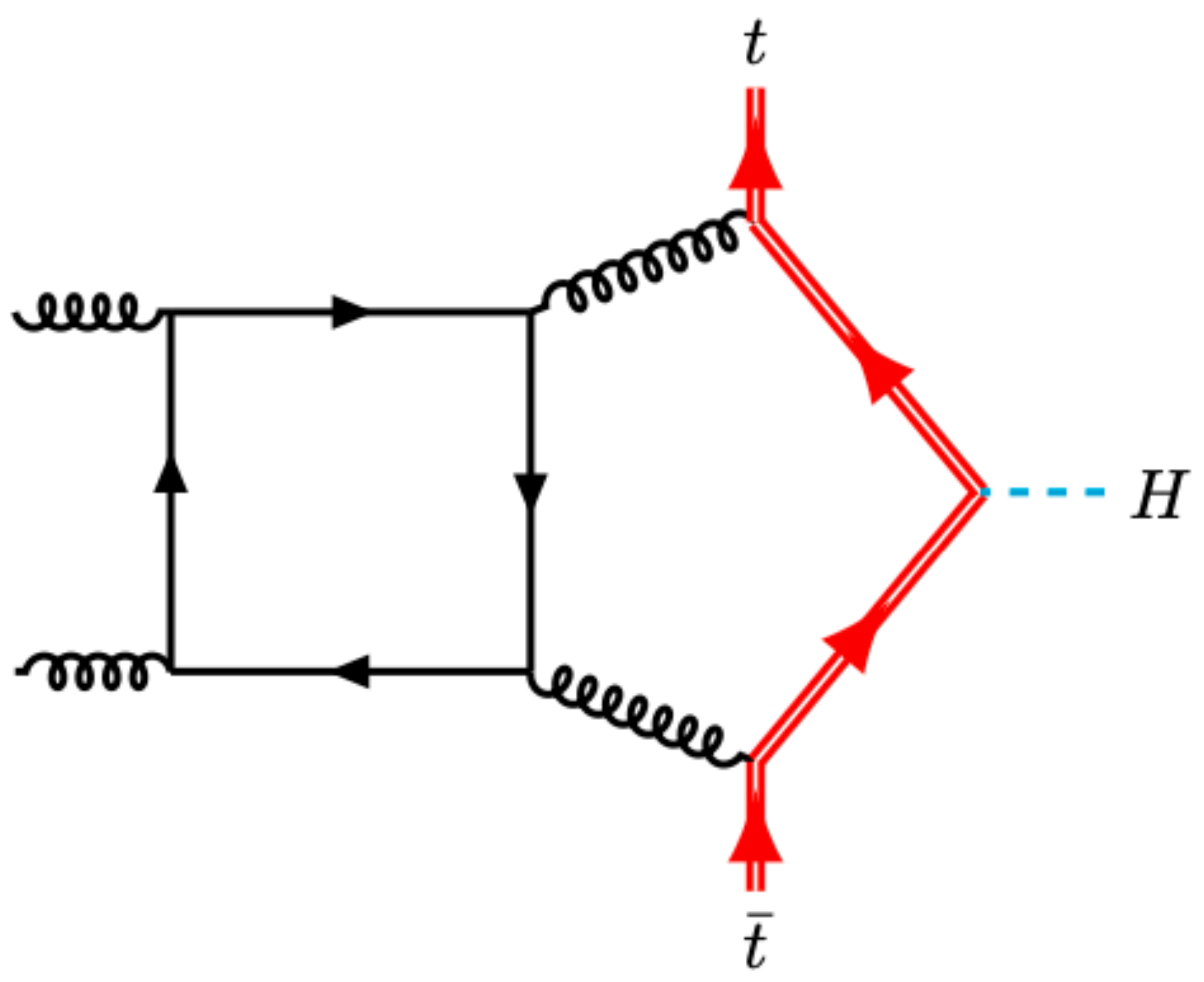
	13 TeV LHC (pb)	14 TeV LHC (pb)
NLO	$0.493^{+5.8\%}_{-9.2\%}$	$0.597^{+6.1\%}_{-9.2\%}$
NLL′+NLO	$0.521^{+1.9\%}_{-2.6\%}$	$0.630^{+2.3\%}_{-2.6\%}$
K-factor	1.06	1.06



- Bottlenecks towards NNLO
  - Two-loop amplitudes
  - IR subtraction



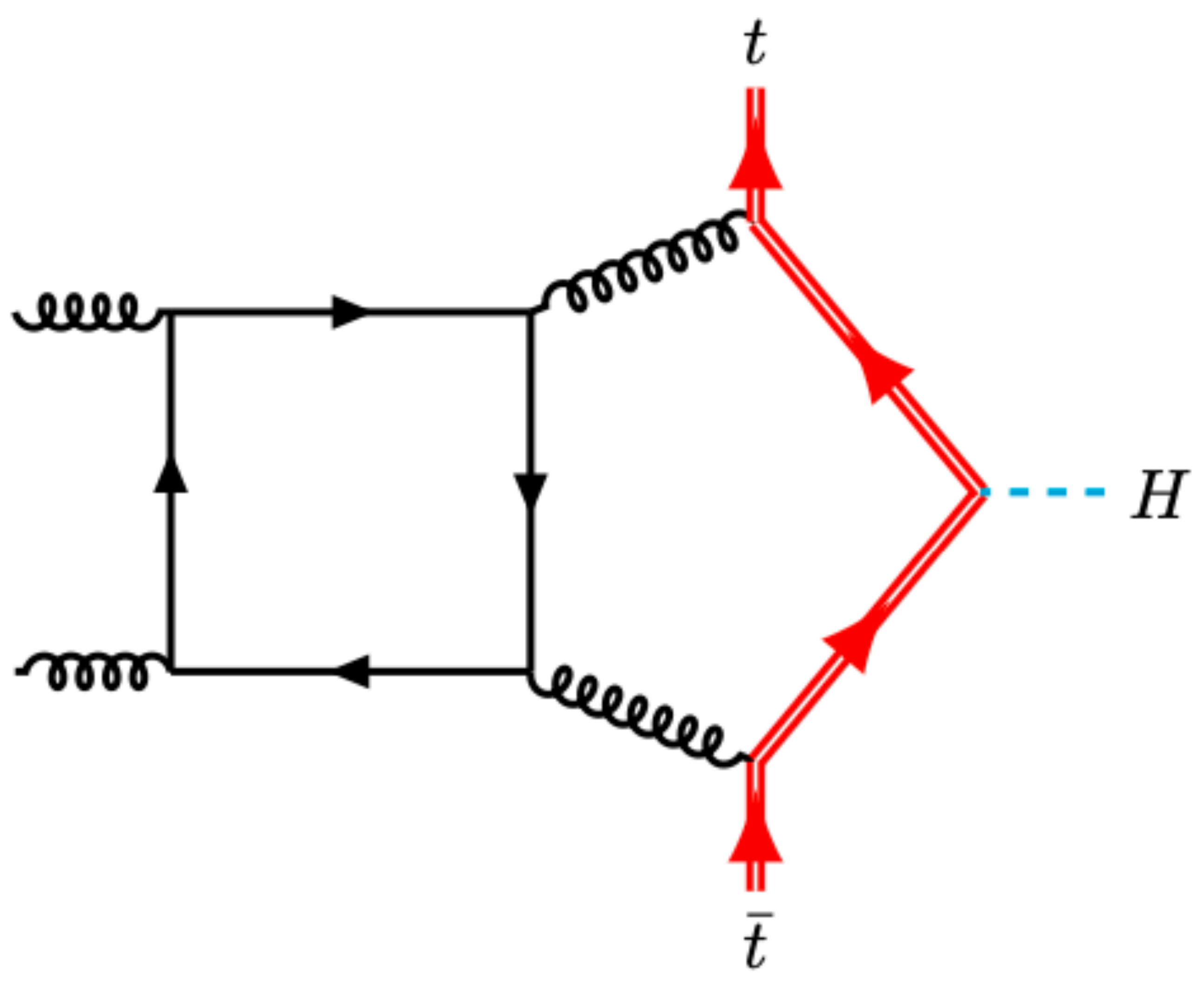
Two-loop amplitudes for $t\bar{t}H$



+ many more planar and non-planar families

- Two-loop five-point amplitudes with 7 scales
- Partial results for simpler families e.g.: 2312.08131, 2402.03301
- Full results require much more efforts (analytic + numeric methods)

Two-loop amplitudes for $t\bar{t}H$



+ many more planar and non-planar families

- Two-loop five-point amplitudes with 7 scales
- Partial results for simpler families e.g.: 2312.08131, 2402.03301
- Full results require much more efforts (analytic + numeric methods)

Many new developments not to be covered in this talk

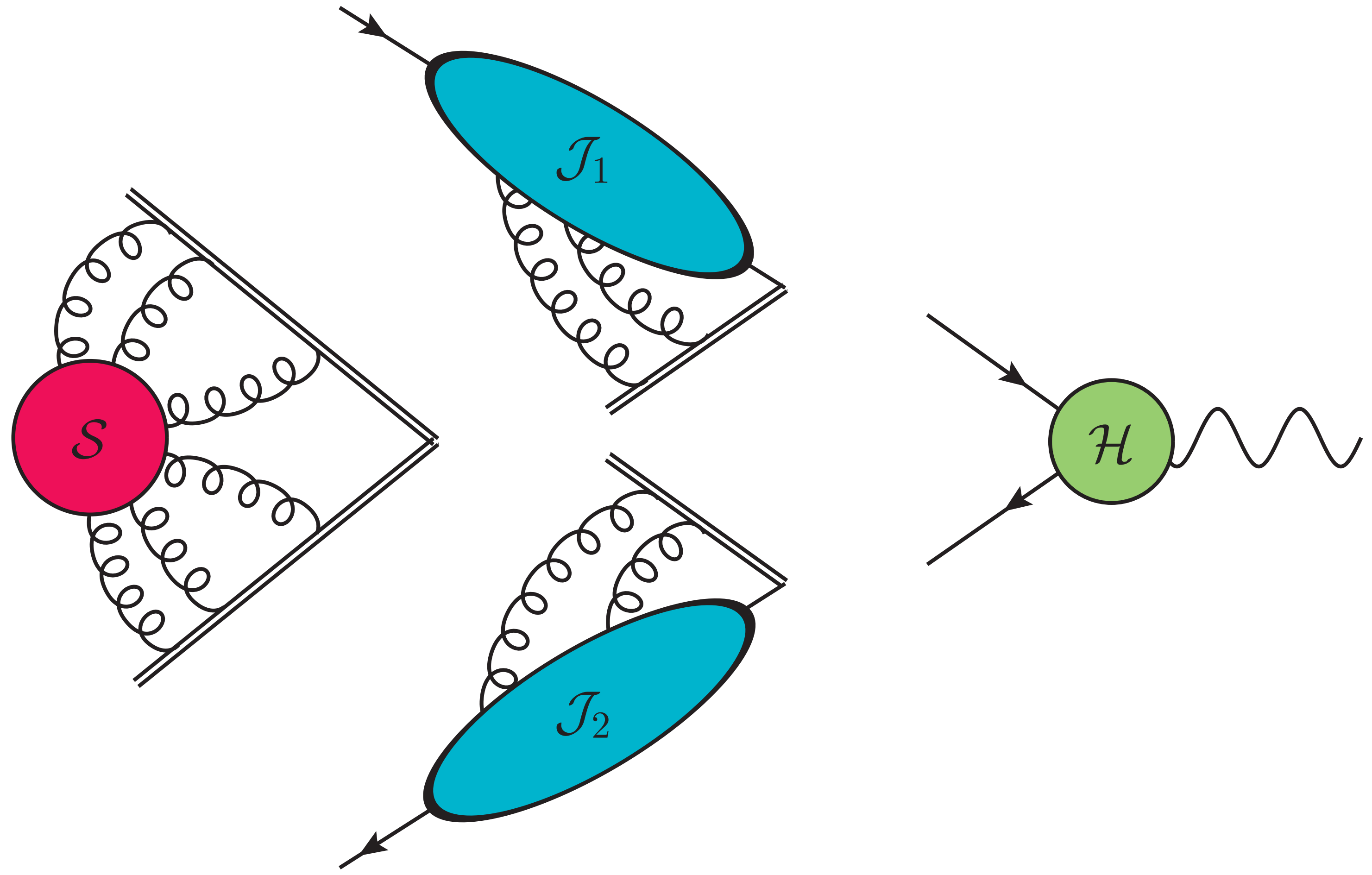
Two-loop IR singularities

Chen, Ma, Wang, LLY, Ye: 2202.02913

IR singularities of QCD amplitudes admit a universal structure due to soft/collinear factorization

$$Z^{-1}(\epsilon)\,\mathscr{M}^{\text{UV renormalized}}(\epsilon) = \mathcal{O}(\epsilon^0)$$

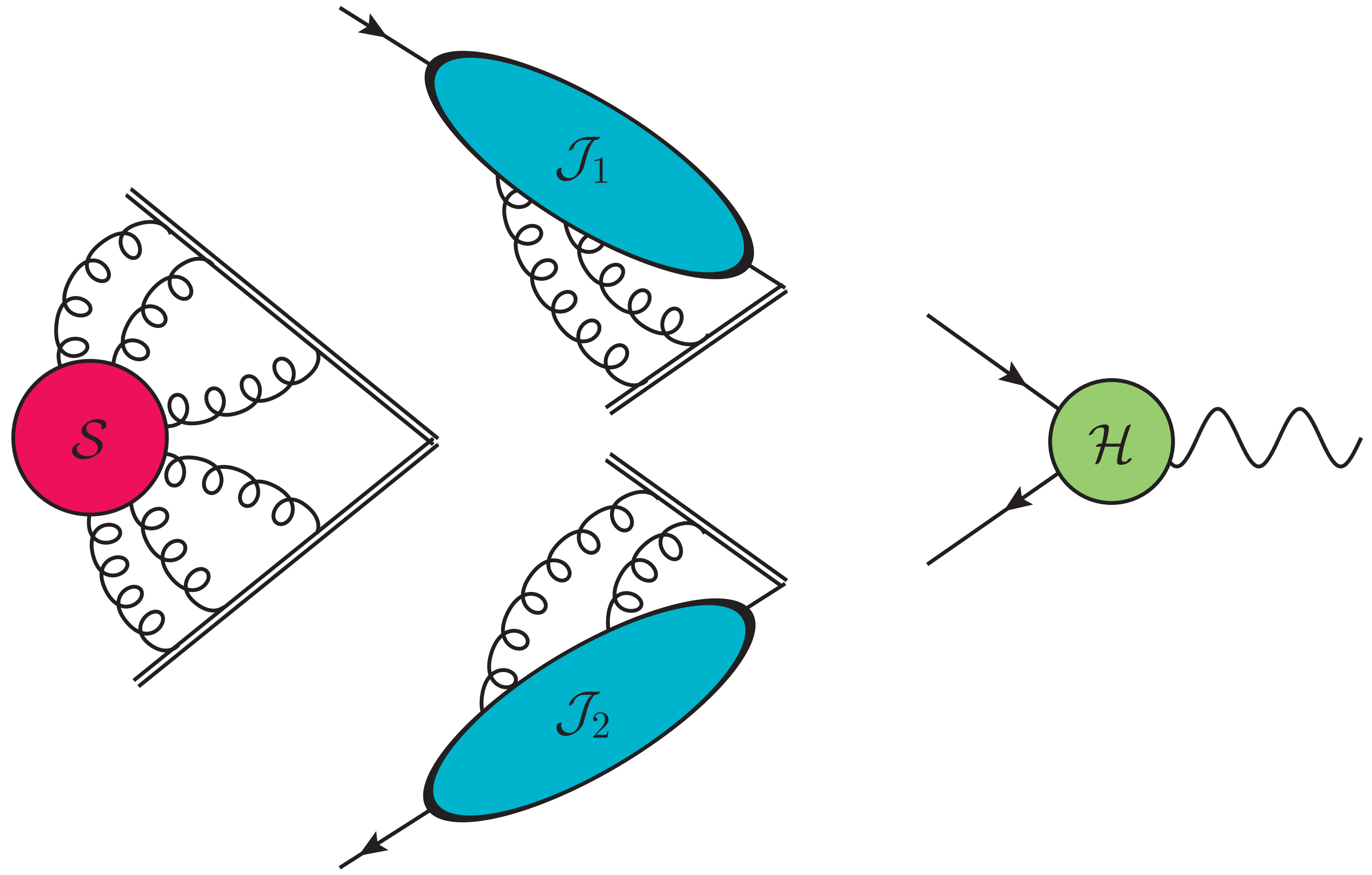
Two-loop poles = Two-loop Z-factor × One-loop amplitude to ϵ^1



Two-loop IR singularities

Chen, Ma, Wang, LLY, Ye: 2202.02913

IR singularities of QCD amplitudes admit a universal structure due to soft/collinear factorization



$$Z^{-1}(\epsilon)\,\mathscr{M}^{\text{UV renormalized}}(\epsilon) = \mathcal{O}(\epsilon^0)$$

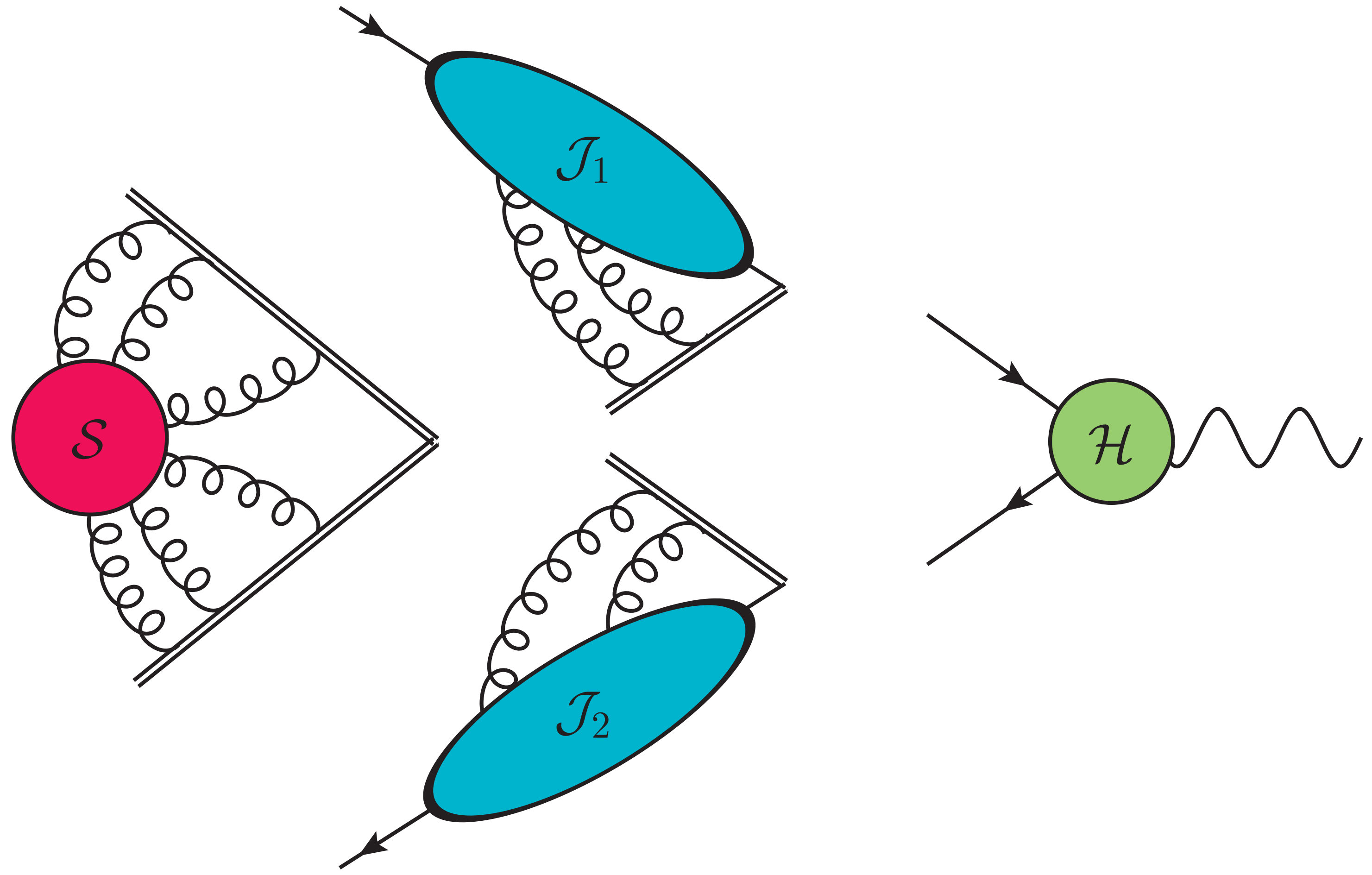
Two-loop poles = Two-loop Z-factor × One-loop amplitude to ϵ^1

Ferroglia, Neubert, Pecjak, LLY:
0907.4791, 0908.3676

Two-loop IR singularities

Chen, Ma, Wang, LLY, Ye: 2202.02913

IR singularities of QCD amplitudes admit a universal structure due to soft/collinear factorization



$$Z^{-1}(\epsilon)\,\mathcal{M}^{\text{UV renormalized}}(\epsilon) = \mathcal{O}(\epsilon^0)$$

Two-loop poles = Two-loop Z-factor × One-loop amplitude to ϵ^1

Ferroglia, Neubert, Pecjak, LLY: 0907.4791, 0908.3676

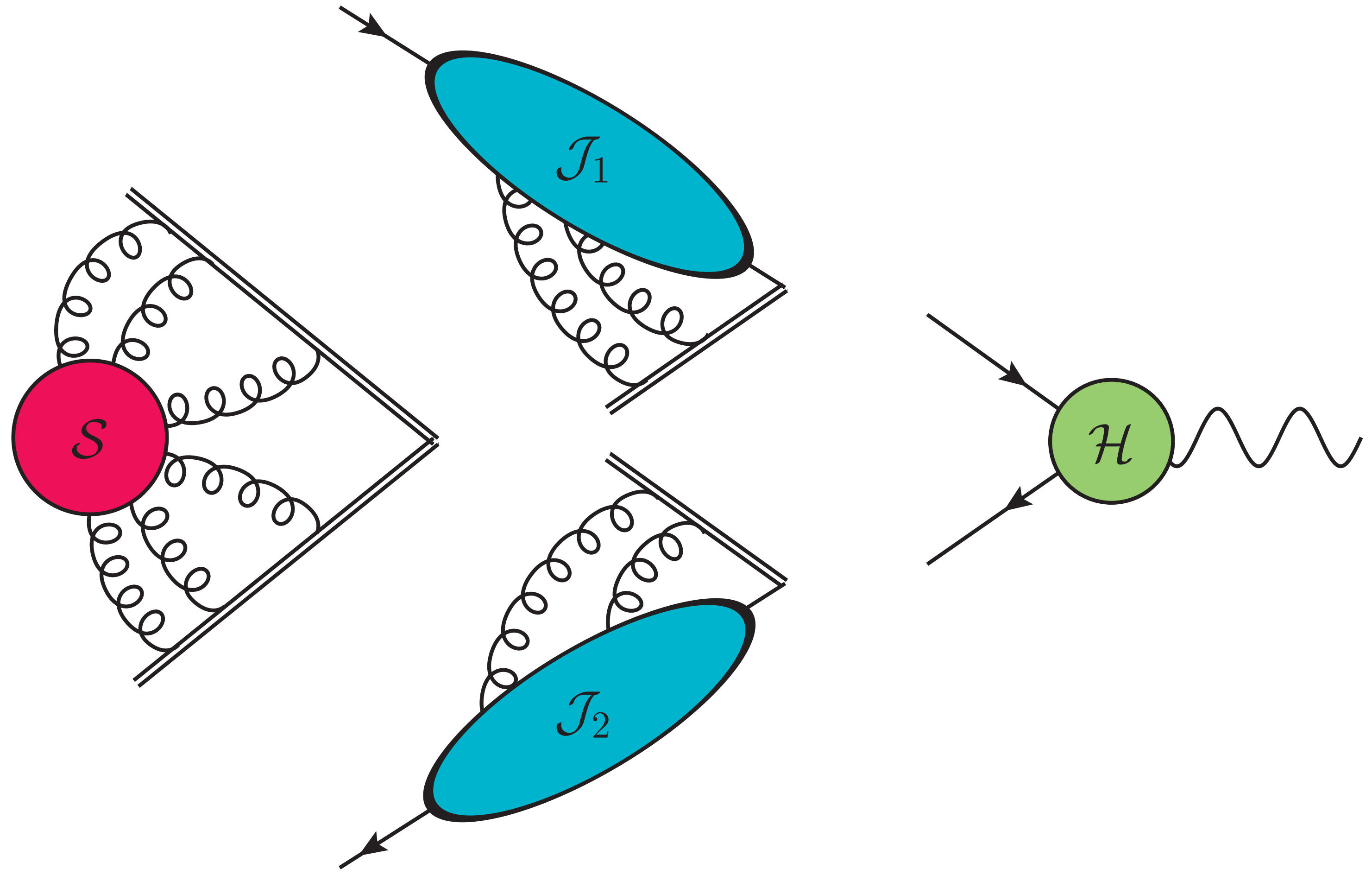
Generically known in terms of symbols

Chen, Ma, LLY: 2201.12998
Jiang, LLY: 2303.11657

Two-loop IR singularities

Chen, Ma, Wang, LLY, Ye: 2202.02913

IR singularities of QCD amplitudes admit a universal structure due to soft/collinear factorization



$$Z^{-1}(\epsilon)\,\mathcal{M}^{\text{UV renormalized}}(\epsilon) = \mathcal{O}(\epsilon^0)$$

Two-loop poles = Two-loop Z-factor × One-loop amplitude to ϵ^1

Ferroglia, Neubert, Pecjak, LLY: 0907.4791, 0908.3676

Generically known in terms of symbols

Chen, Ma, LLY: 2201.12998
Jiang, LLY: 2303.11657

- ➤ Predict two-loop IR poles for tTH
- ➤ Provide strong check on two-loop amplitudes
- ➤ Validate IR subtraction

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}
A^g	17.37022326	6.277797530	−162.1830217	559.8062598
B^g	−32.49510001	−34.75486260	−624.1343773	3901.332369
C^g		−9.463444735	−54.41556200	−497.5350517
D^g			143.6321997	−578.4857199
E^g_l		−20.26526047	46.54471184	−10.69967085
E^g_h			−24.23013938	79.68650479
F^g_l		37.91095001	−74.94866603	71.66904977
F^g_h			43.70151160	−132.3384924
G^g_l			4.731722368	85.25318119
G^g_h				6.363526190
H^g_l			3.860049613	−10.52987601
H^g_{lh}				8.076713126
H^g_h				
I^g_l			−7.221133335	19.49234494
I^g_{lh}				−14.56717053
I^g_h				
A^q	2.390051823	15.03938540	0.597121534	−34.95784899
B^q	−4.780103646	−22.69017086	49.54607207	106.0851578
C^q	2.390051823	7.650785464	−186.5751188	−21.39439443
D^q_l		−2.390051823	0.308675876	−6.605875838
D^q_h			6.244349191	4.860387981
E^q_l		2.390051823	1.610219156	77.52356965
E^q_h			−6.244349191	19.76269918
F^q_l				
F^q_{lh}				
F^q_h				

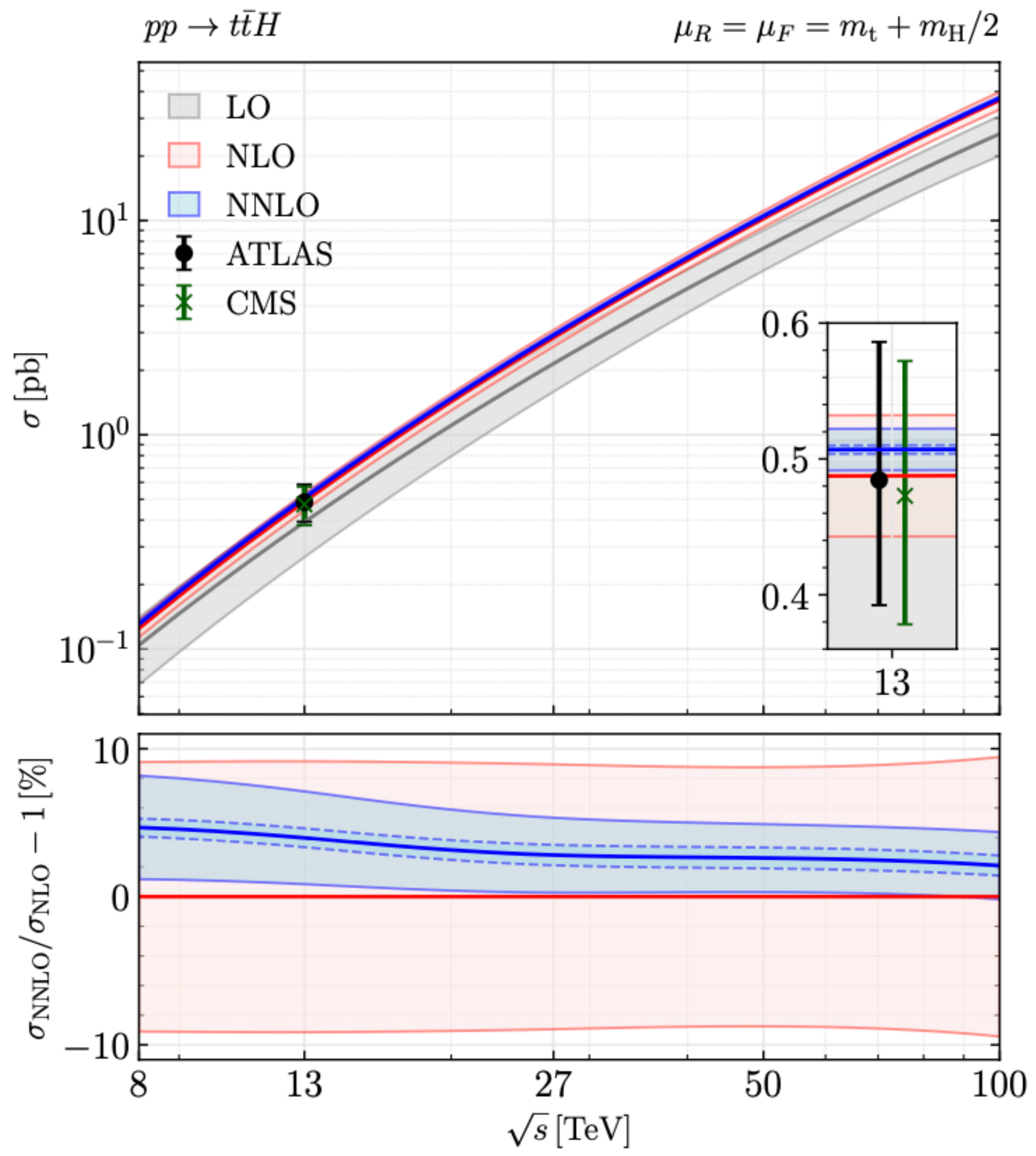
Table 1. IR poles decomposed as color coefficients for the phase-space point $x_{12} = 10$, $x_{13} = -1339/920$, $x_{14} = -2269/465$, $x_{23} = -1951/620$, $x_{24} = -1803/1810$ and $x_{34} = 5$.

Approximation with soft Higgs

Catani et al.: 2210.07846

Eikonal approximation: $2 \to 2$ kinematics

$$\mathcal{M}(\{p_i\},k) \simeq F(\alpha_S(\mu_R); \tfrac{m_t}{\mu_R}) \, \frac{m_t}{v} \sum_{i=3,4} \frac{m_t}{p_i \cdot k} \mathcal{M}(\{p_i\})$$



Approximation with soft Higgs

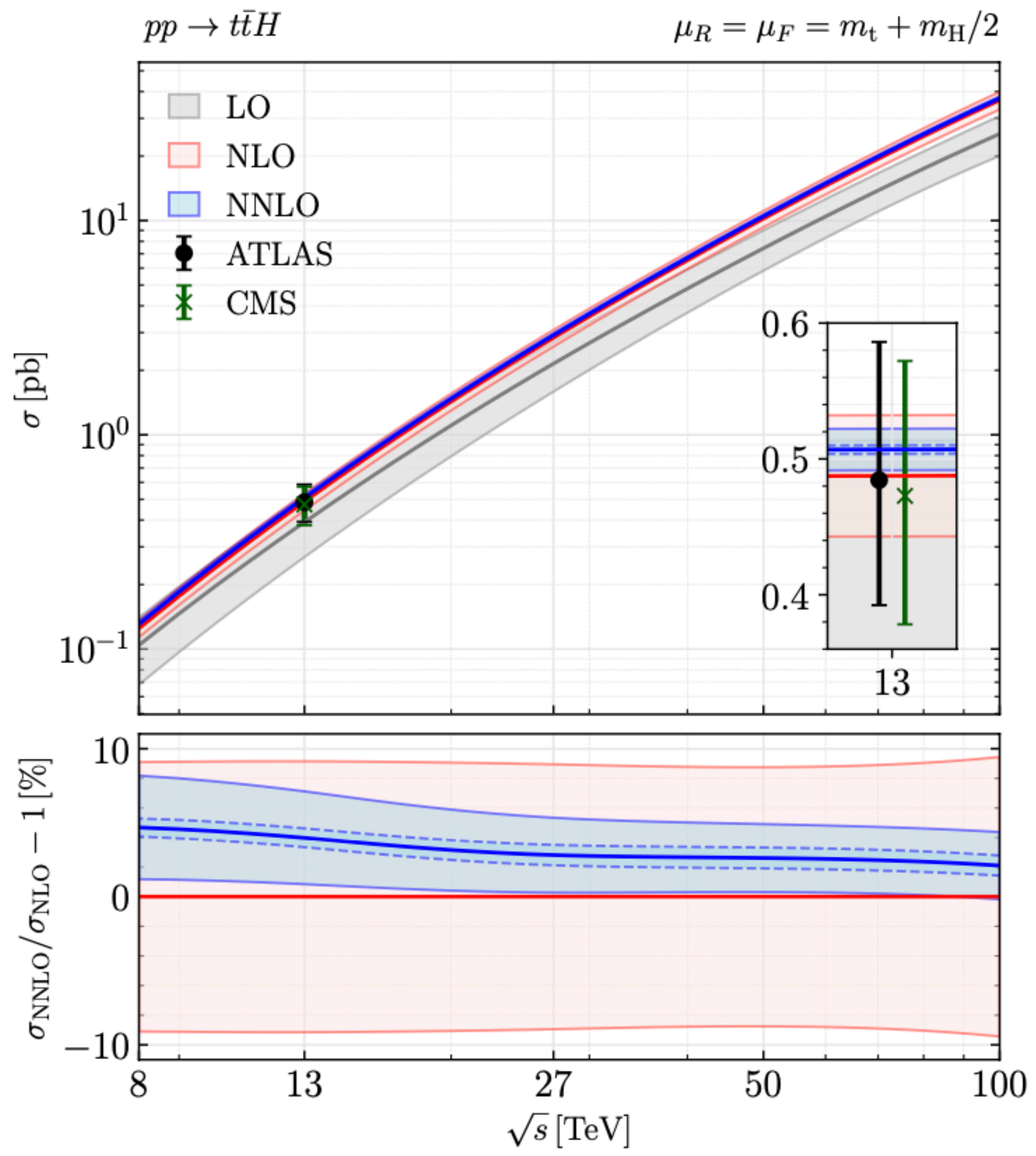
Catani et al.: 2210.07846

Eikonal approximation: 2 → 2 kinematics

$$\mathcal{M}(\{p_i\},k) \simeq F(\alpha_S(\mu_R); \tfrac{m_t}{\mu_R}) \frac{m_t}{v} \sum_{i=3,4} \frac{m_t}{p_i \cdot k} \mathcal{M}(\{p_i\})$$

Not a good approximation for two-loop amplitudes:

- One-loop already 30% error
- Two-loop estimated 100% error



Approximation with soft Higgs

Catani et al.: 2210.07846

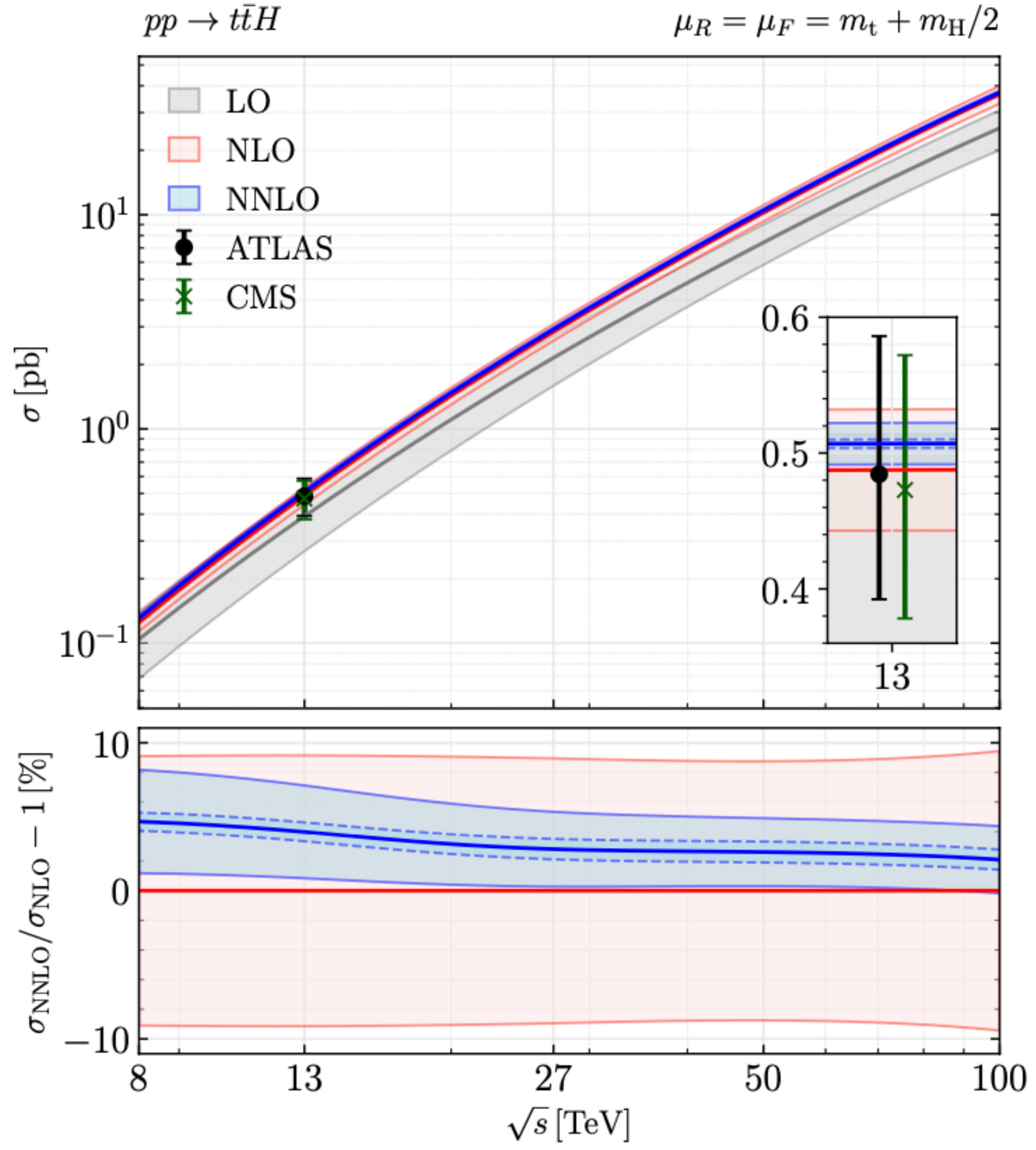
Eikonal approximation: $2 \to 2$ kinematics

$$\mathcal{M}(\{p_i\}, k) \simeq F(\alpha_S(\mu_R); \tfrac{m_t}{\mu_R}) \frac{m_t}{v} \sum_{i=3,4} \frac{m_t}{p_i \cdot k} \mathcal{M}(\{p_i\})$$

Not a good approximation for two-loop amplitudes:

- ➤ One-loop already 30% error
- ➤ Two-loop estimated 100% error

The argument was: two-loop amplitudes small for total cross section



Approximation with soft Higgs

Catani et al.: 2210.07846

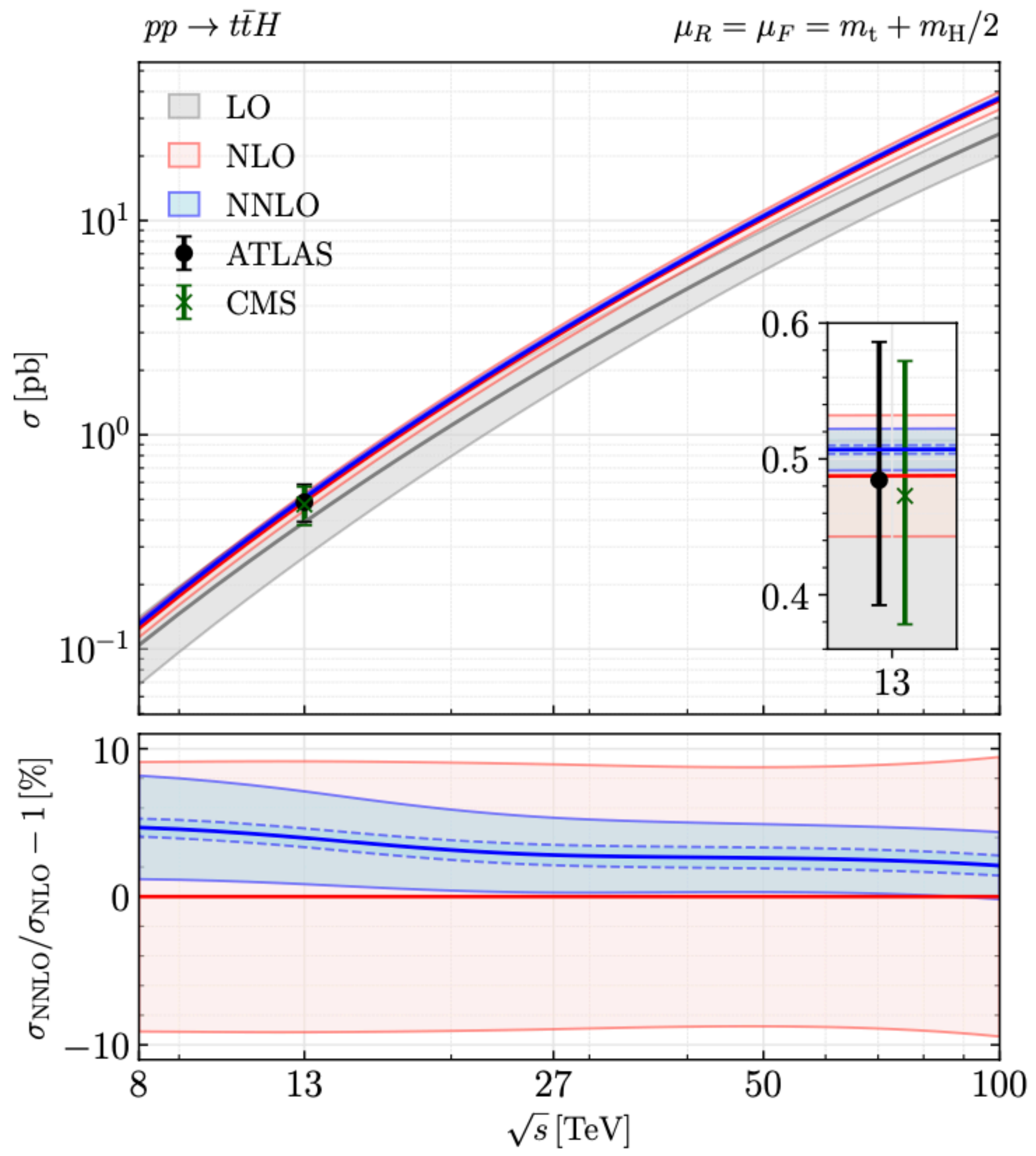
Eikonal approximation: $2 \to 2$ kinematics

$$\mathcal{M}(\{p_i\}, k) \simeq F(\alpha_S(\mu_R); \tfrac{m_t}{\mu_R}) \frac{m_t}{v} \sum_{i=3,4} \frac{m_t}{p_i \cdot k} \mathcal{M}(\{p_i\})$$

Not a good approximation for two-loop amplitudes:

- ➤ One-loop already 30% error
- ➤ Two-loop estimated 100% error

The argument was: two-loop amplitudes small for total cross section



What about differential cross sections?

Approximation in the high energy limit

It is known that a massive amplitude can be factorized into a massless amplitude and a collinear factor for each leg in the high-energy limit

Mitov, Moch: hep-ph/0612149

$$\mathcal{M}^{[\mathrm{p}],(m)}\left(\{k_i\},\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right) =$$

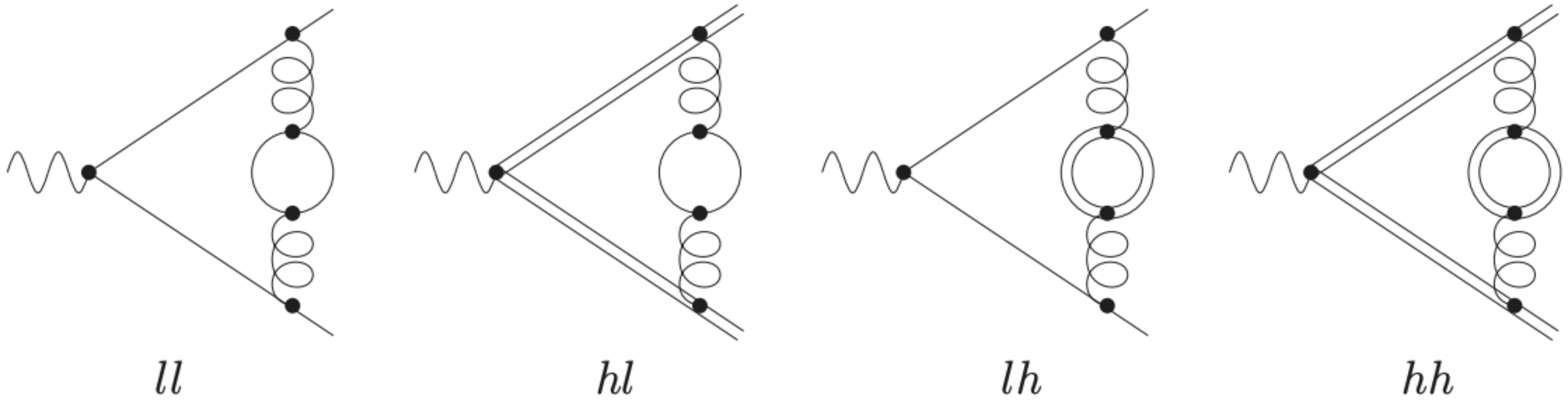
$$\prod_{i\in\,\{\text{all legs}\}}\left(Z^{(m|0)}_{[i]}\left(\frac{m^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right)\right)^{\frac{1}{2}} \times \mathcal{M}^{[\mathrm{p}],(m=0)}\left(\{k_i\},\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right)$$

Approximation in the high energy limit

It is known that a massive amplitude can be factorized into a massless amplitude and a collinear factor for each leg in the high-energy limit

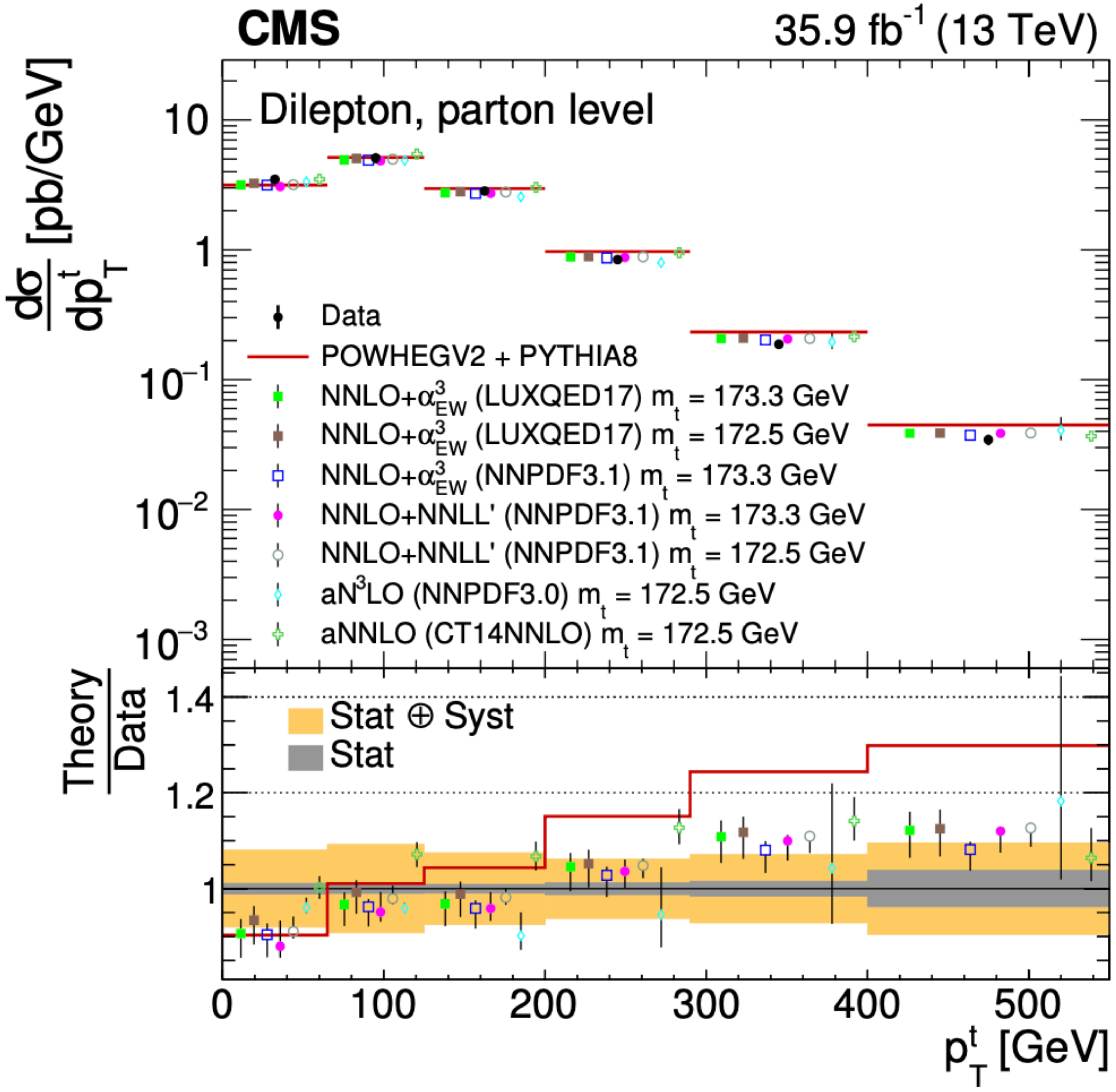
Mitov, Moch: hep-ph/0612149

$$\mathcal{M}^{[\mathrm{p}],(m)}\left(\{k_i\},\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right) = \prod_{i\in\,\{\mathrm{all\ legs}\}}\left(Z_{[i]}^{(m|0)}\left(\frac{m^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right)\right)^{\frac{1}{2}} \times \mathcal{M}^{[\mathrm{p}],(m=0)}\left(\{k_i\},\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right)$$



But the heavy-quark bubbles are not included!

Top quark pair production

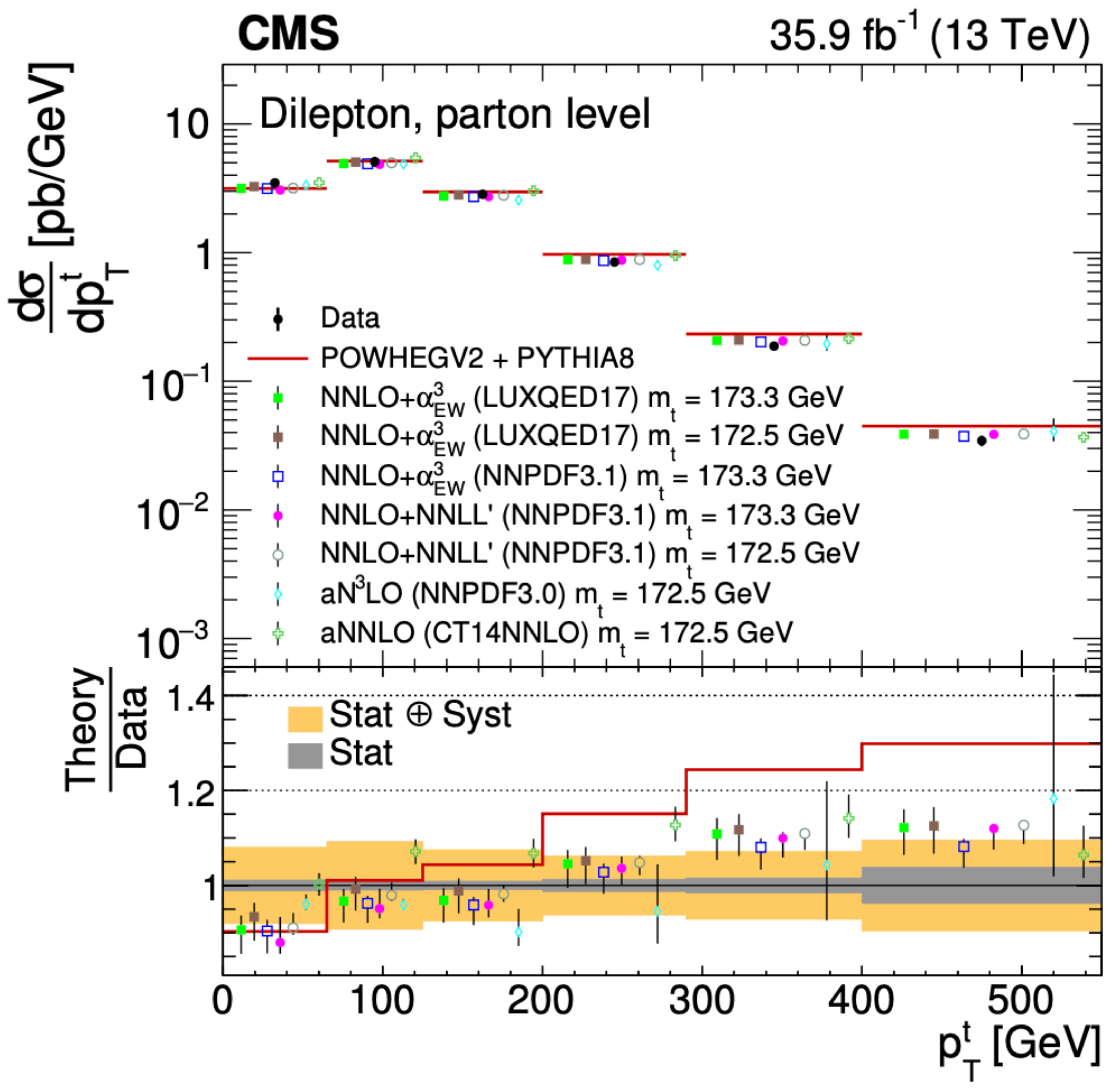


High energy factorization has been applied in the resummation for top quark pair production

1205.3662
1306.1537
1310.3836
1601.07020
1803.07623
1901.08281

Best precision:
NNLO+NNLL' in QCD + NLO in EW

Top quark pair production



High energy factorization has been applied in the resummation for top quark pair production

1205.3662
1306.1537
1310.3836
1601.07020
1803.07623
1901.08281

Best precision:
NNLO+NNLL' in QCD + NLO in EW

But the factorization of heavy quark bubbles was not understood…

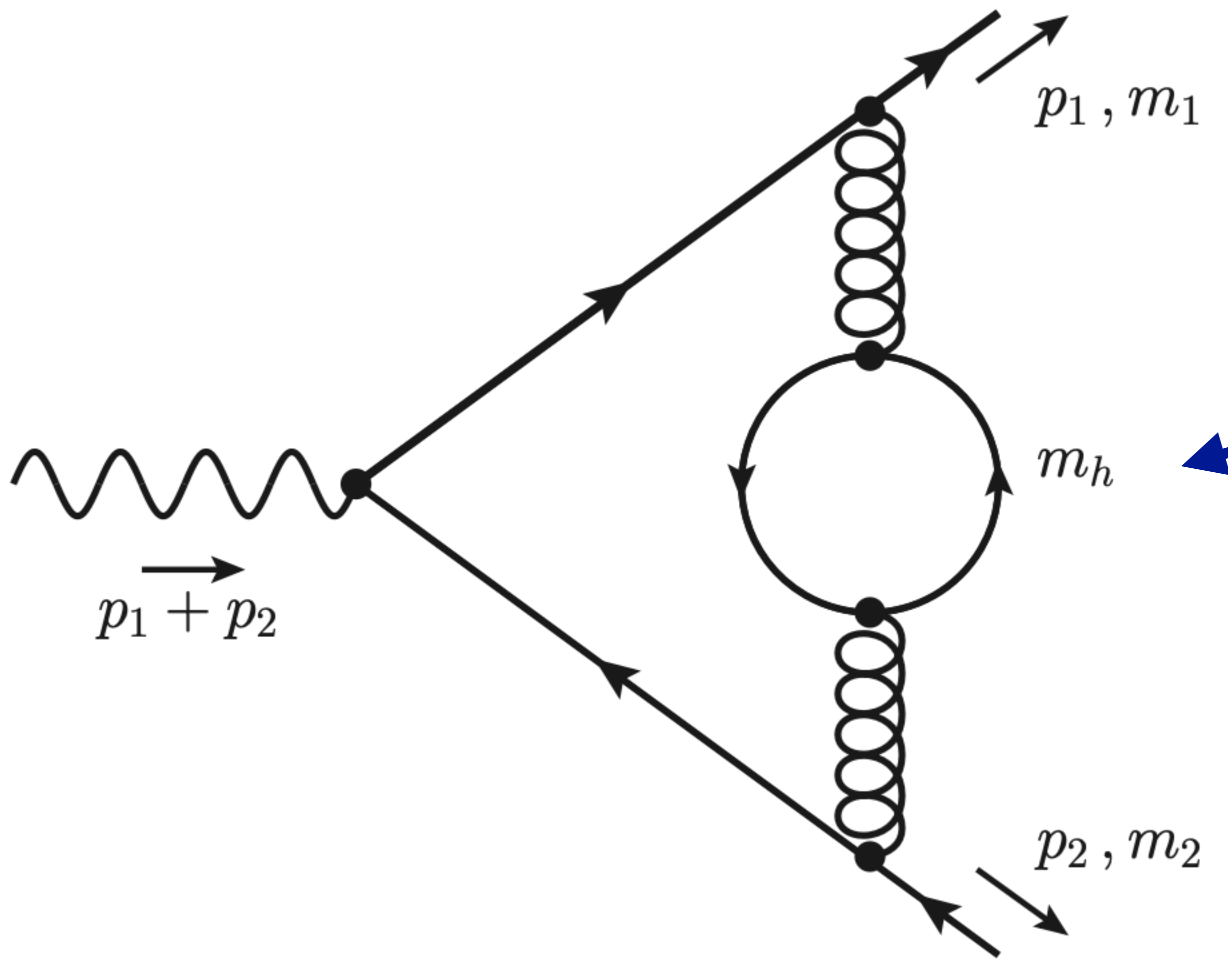
Heavy-quark bubbles

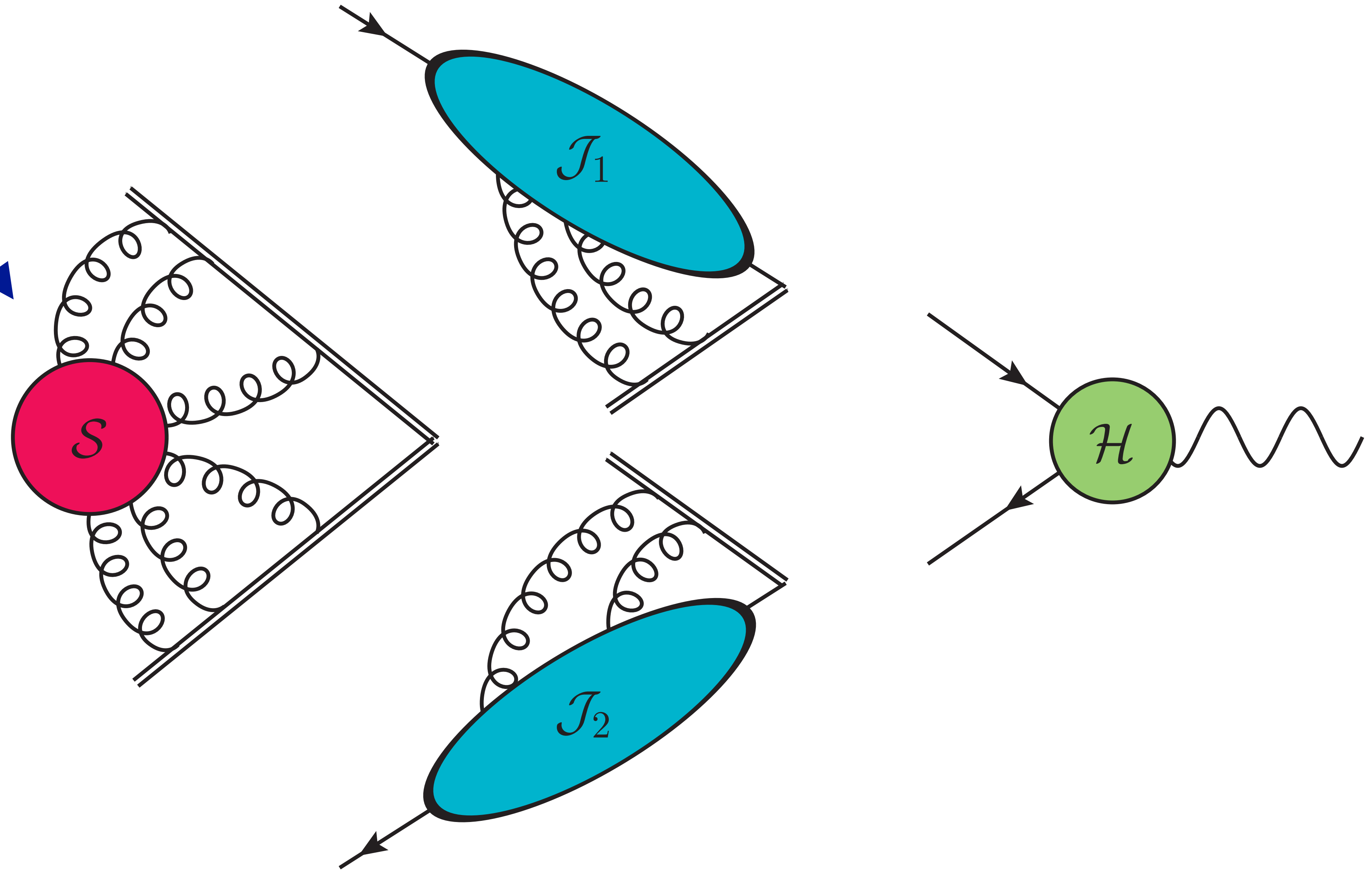
Wang, Xia, LLY, Ye: 2312.12242

A new factorization formula

$$\left|\mathcal{M}^{\text{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\{p\})\right\rangle$$

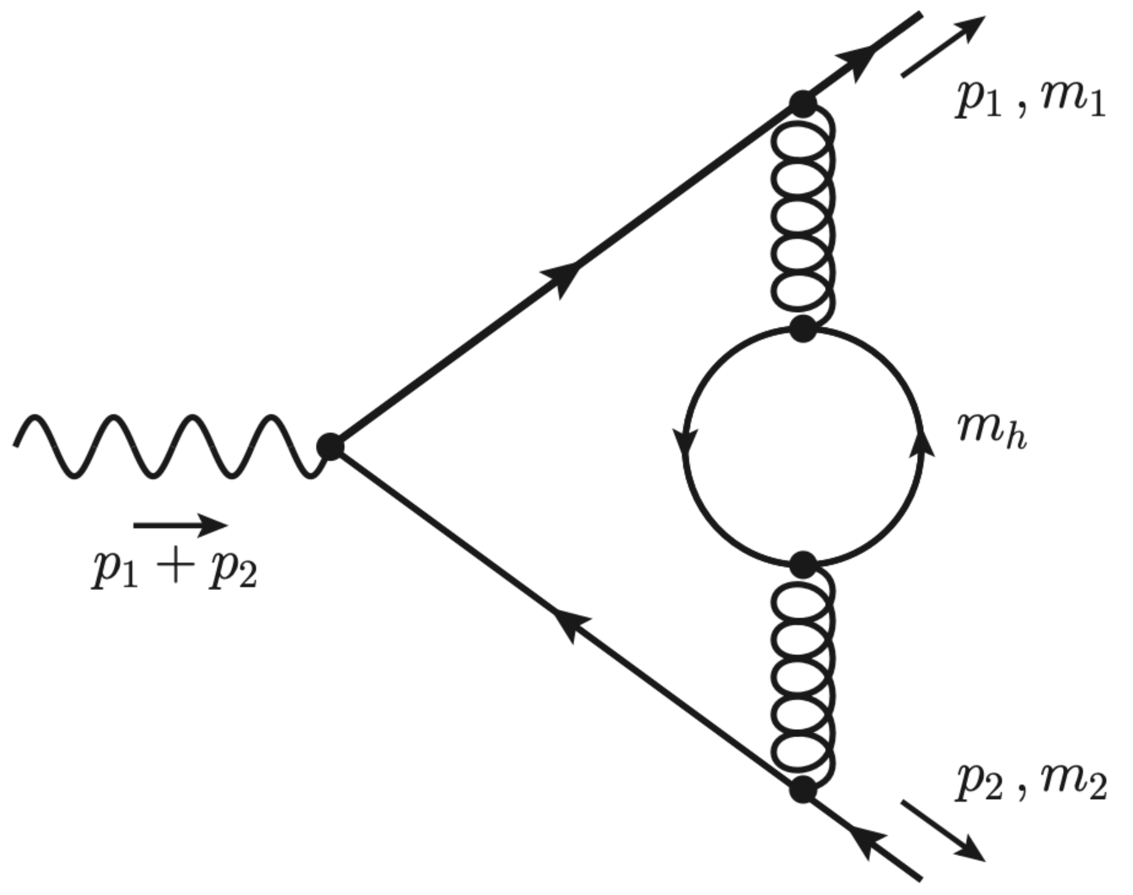
A new soft function





The new soft function

Wang, Xia, LLY, Ye: 2312.12242



$$\text{hard}: k^\mu \sim \sqrt{|s|}\,,$$
$$n_i\text{-collinear}: (n_i\cdot k,\ \bar{n}_i\cdot k,\ k_\perp) \sim \sqrt{|s|}\,(\lambda^2,\,1,\,\lambda)$$
$$\text{soft}: k^\mu \sim \sqrt{|s|}\,\lambda\,.$$

Rapidity divergence: analytic regulator

$$I_{\{a_i\}} \equiv \mu^{4\epsilon}\int \frac{dk_1}{(2\pi)^d}\frac{dk_2}{(2\pi)^d}\frac{1}{[k_1^2-m_h^2]^{a_1}}\frac{1}{[k_2^2-m_h^2]^{a_2}}\frac{1}{[(k_1+k_2)^2]^{a_3}}\frac{1}{[(k_1+k_2-p_1)^2-m_1^2]^{a_4}}$$
$$\times\frac{(-\tilde{\mu}^2)^\nu}{[(k_1+k_2+p_2)^2-m_2^2]^{a_5+\nu}}\frac{1}{[(k_1-p_1)^2]^{a_6}}\frac{1}{[(k_1+p_2)^2]^{a_7}}\,, \quad (3.4)$$

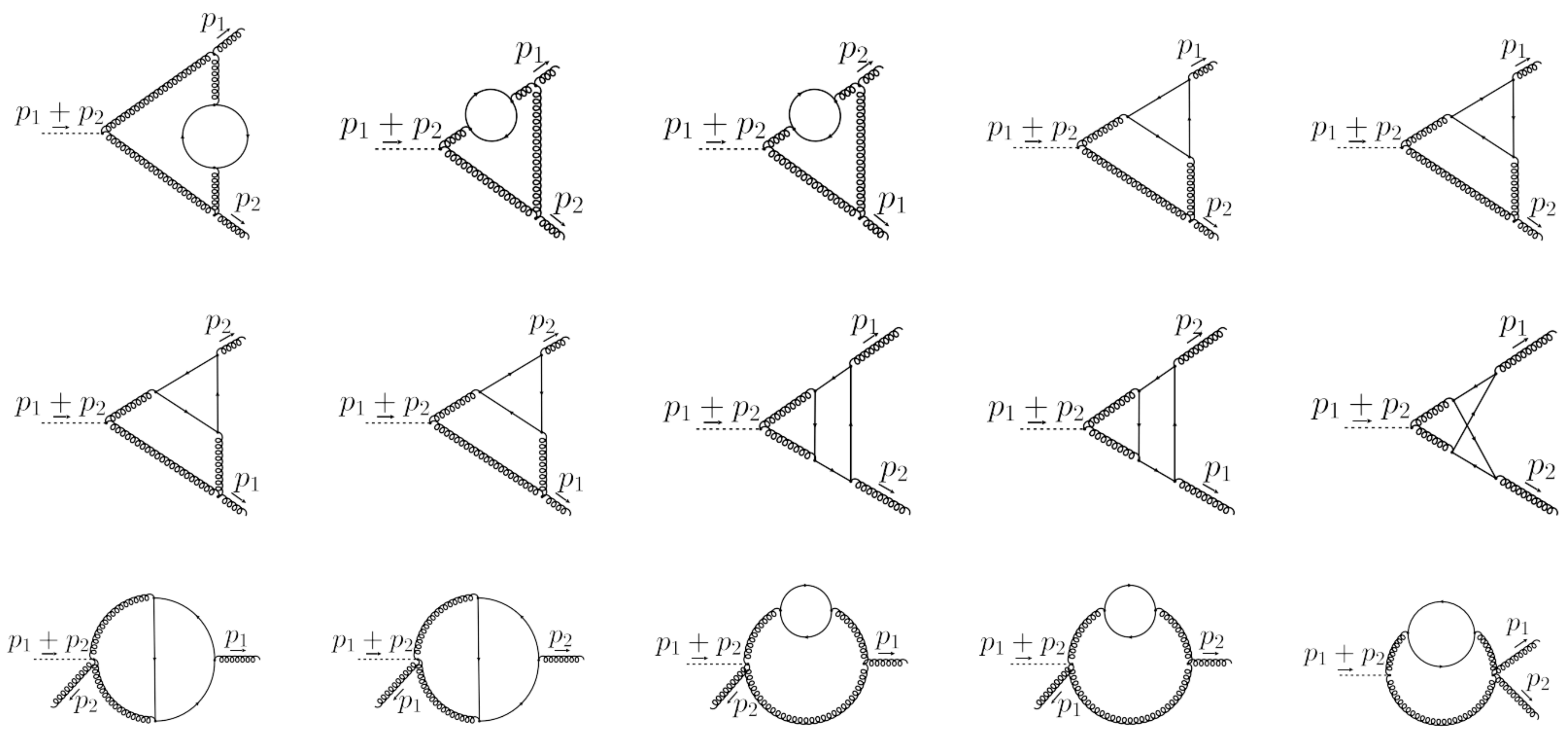
$$\boldsymbol{\mathcal{S}}(\{p\},\{m\}) = 1 + \left(\frac{\alpha_s}{4\pi}\right)^2 \sum_{\substack{i,j\\ i\neq j}} (-\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\sum_h \mathcal{S}^{(2)}(s_{ij},m_h^2) + \mathcal{O}(\alpha_s^3)$$

$$\mathcal{S}^{(2)}(s_{ij},m_h^2) = T_F\left(\frac{\mu^2}{m_h^2}\right)^{2\epsilon}\left(-\frac{4}{3\epsilon^2}+\frac{20}{9\epsilon}-\frac{112}{27}-\frac{4\zeta_2}{3}\right)\ln\frac{-s_{ij}}{m_h^2}$$

Validation of the new formula

Wang, Xia, LLY, Ye: 2312.12242

$$\left|\mathcal{M}^{\text{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\{p\})\right\rangle$$



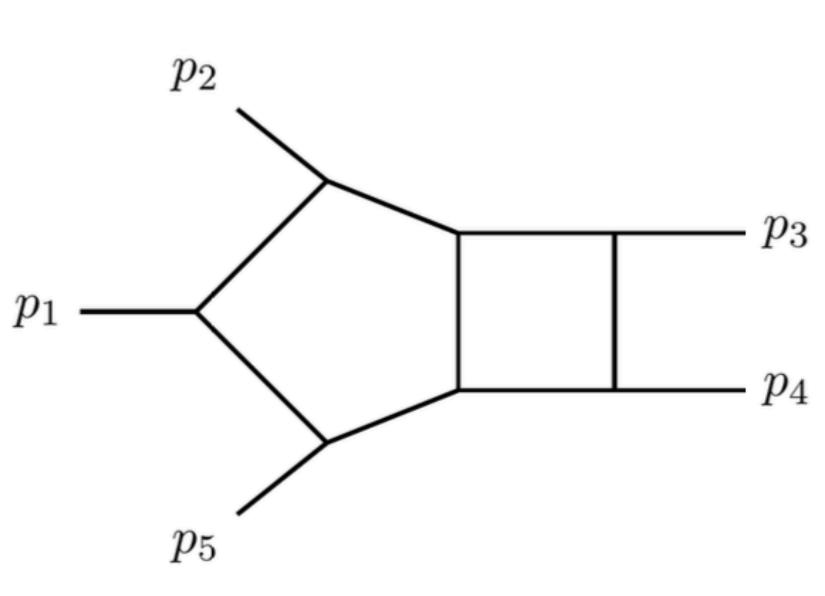
Checked in various situations:

- Quark form factors: heavy-heavy, heavy-light, light-light
- Gluon form factor
- Top quark pair amplitude

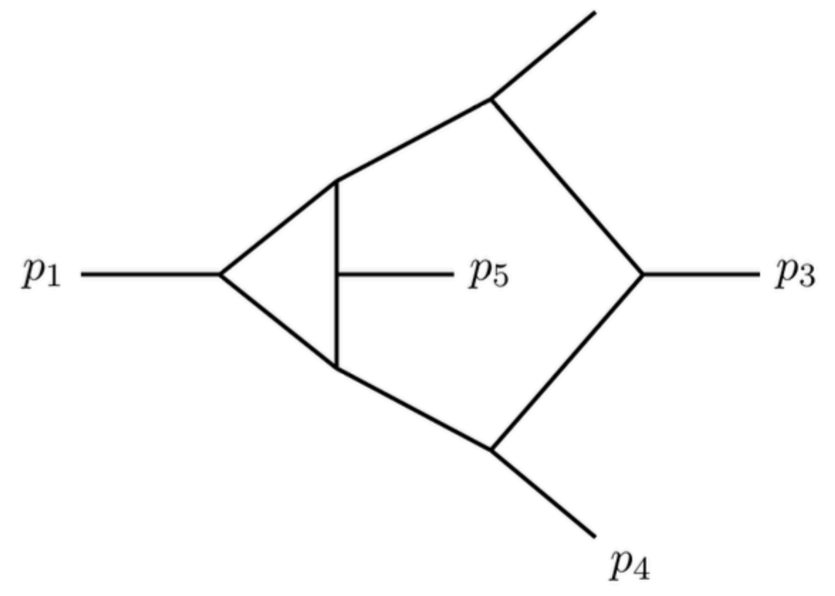
Two-loop amplitudes for tTH in the high-energy limit

Wang, Xia, LLY, Ye: 2402.00431

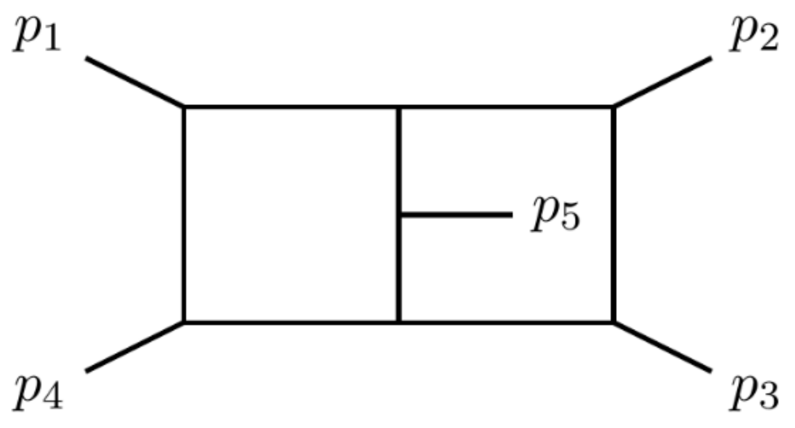
$$\left|\mathcal{M}^{\text{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\{p\})\right\rangle$$



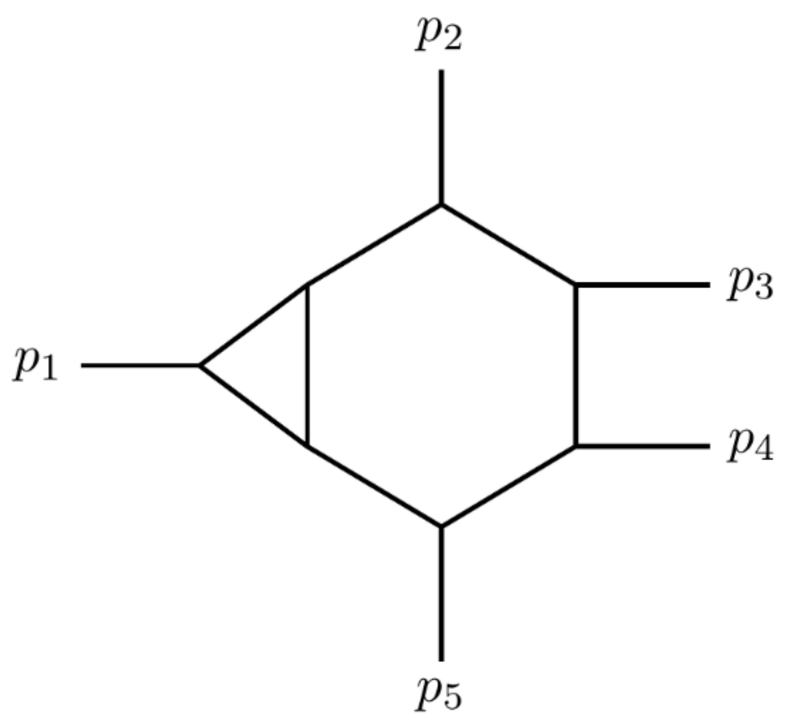
(a) planar pentagon-box (PB)



(b) non-planar hexagon-box (HB)



(c) non-planar double pentagon (DP)

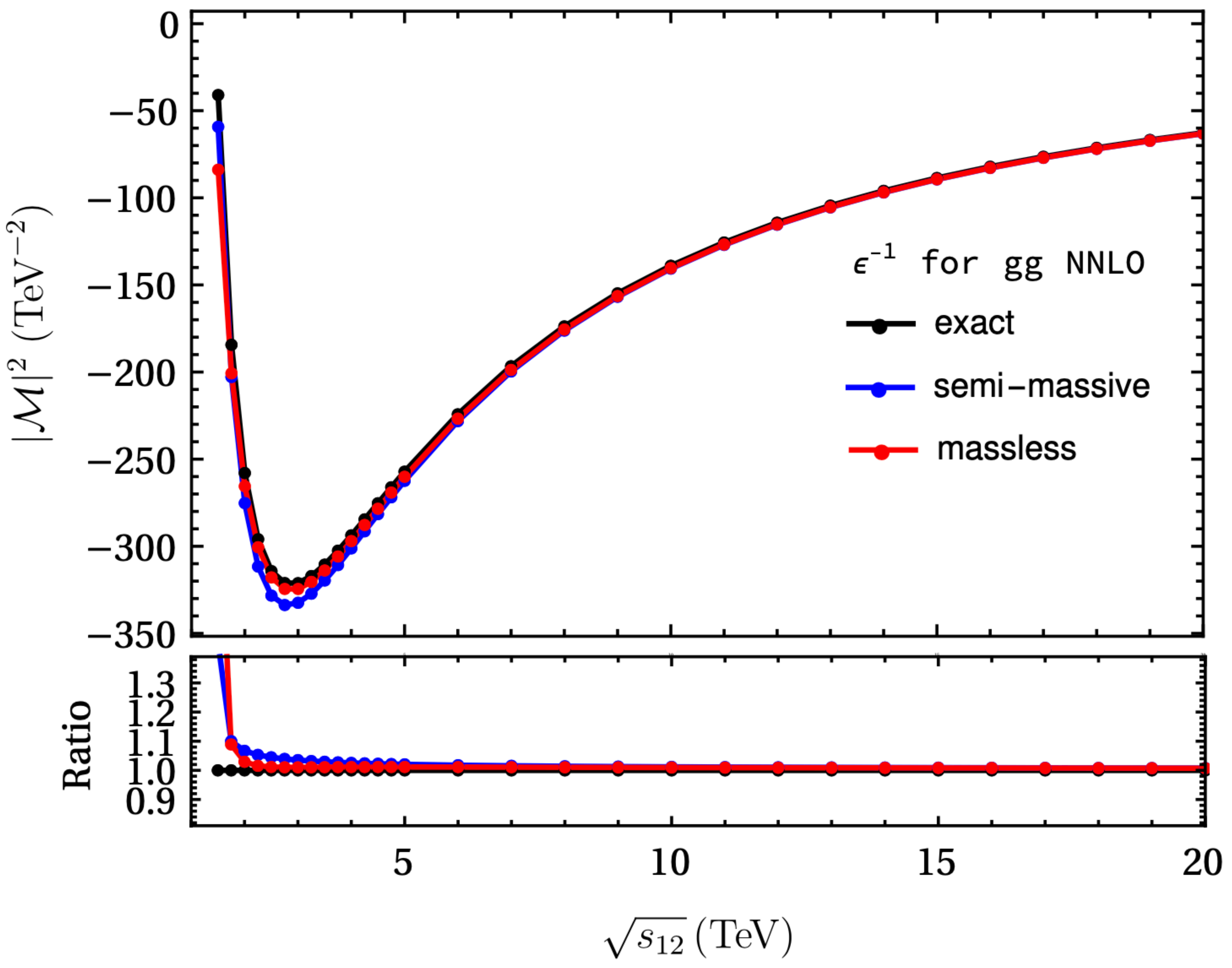


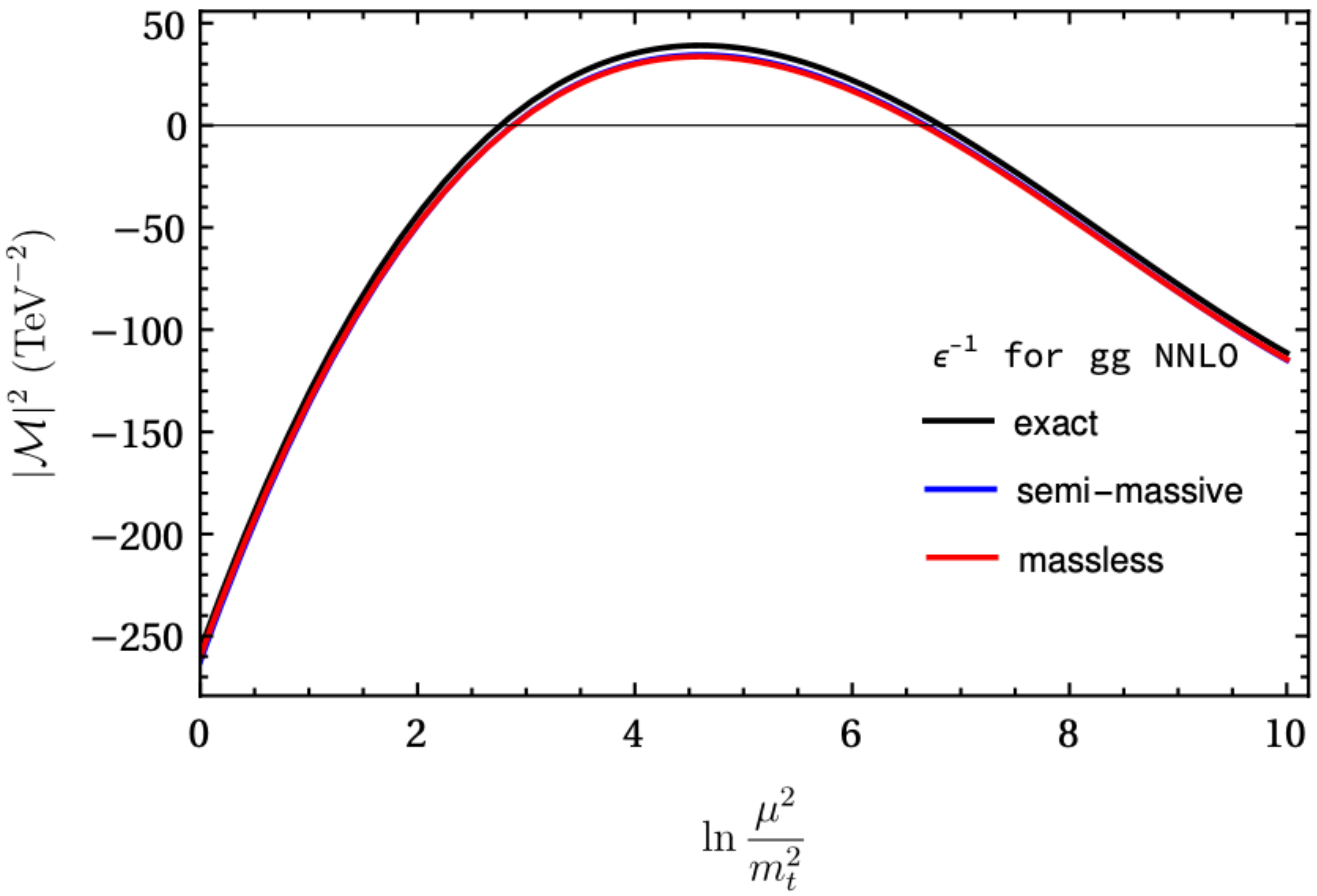
(d) planar hexagon-triangle (HT)

- Massless amplitudes computed using standard techniques
- Very large expressions, simplified using MultivariateApart
- Fast numeric evaluation with PentagonMI

Numerical results

Wang, Xia, LLY, Ye: 2402.00431



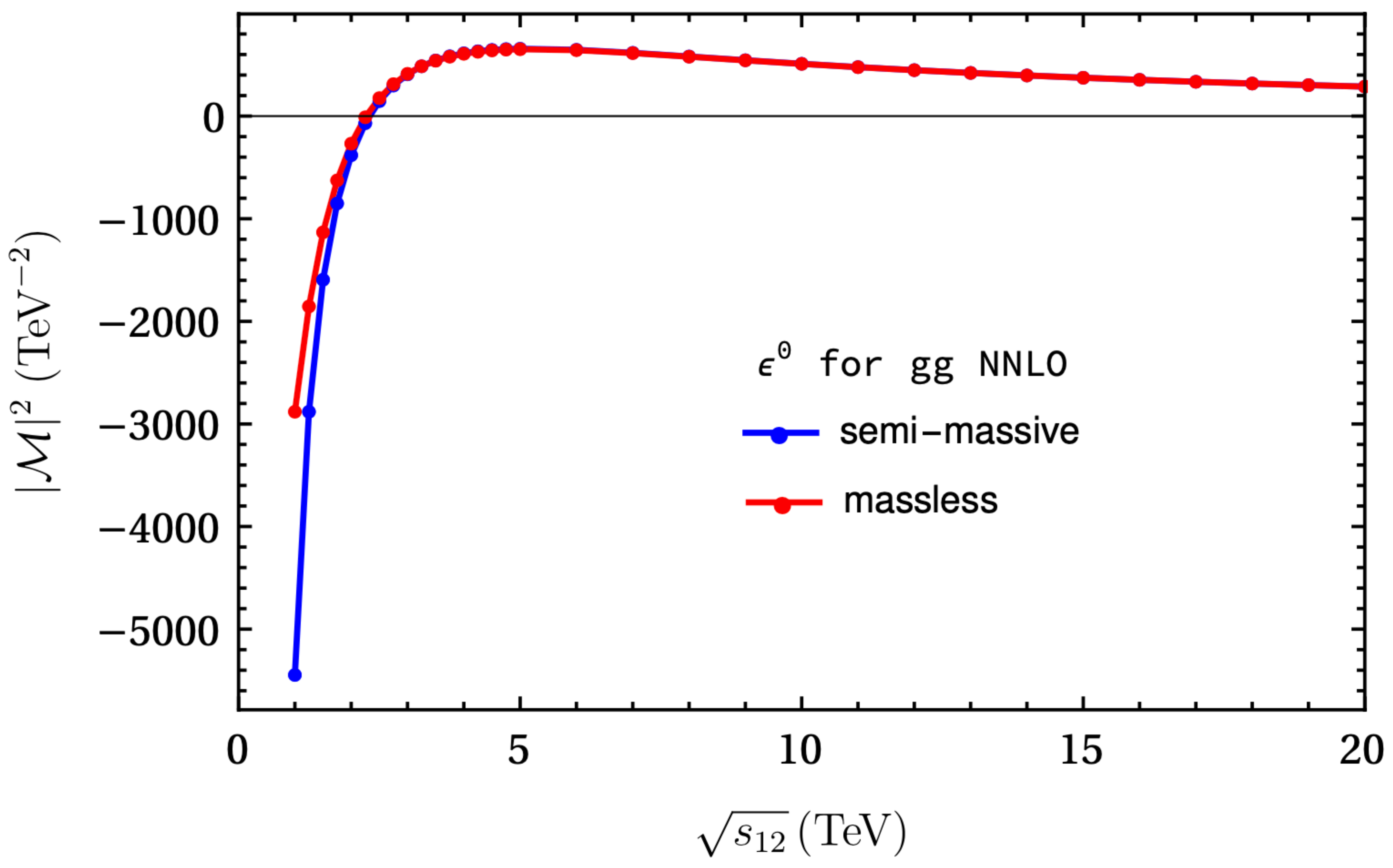


IR poles validated against exact results in Chen, Ma, Wang, LLY, Ye: 2202.02913

Note: without the heavy quark bubble, the scale-dependence would be wrong!

Numerical results

Wang, Xia, LLY, Ye: 2402.00431



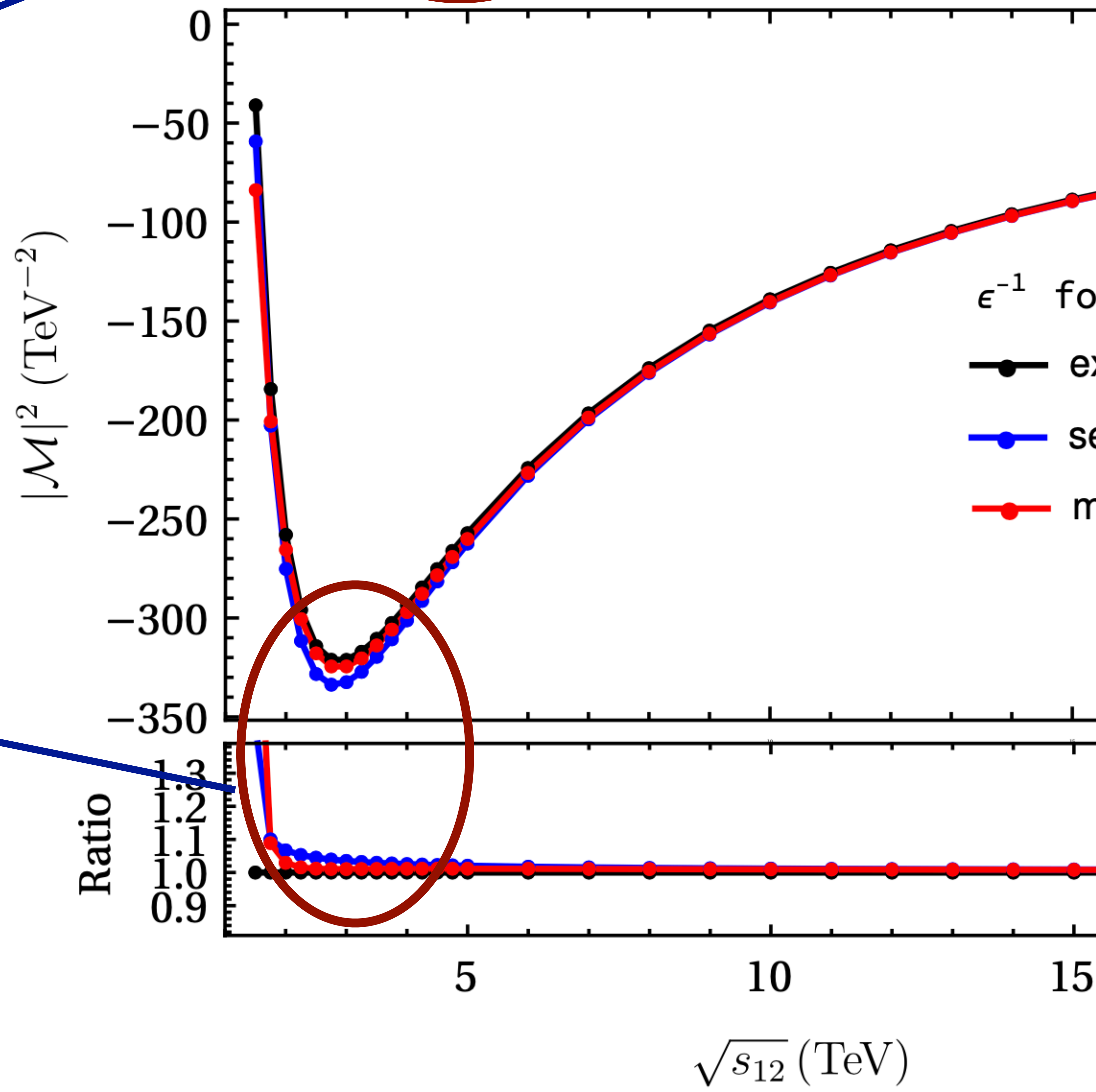
- ➤ Two-loop amplitudes at high energies are ready
- ➤ Combine with low energy approximations (threshold / soft Higgs)?
- ➤ Differential cross sections (IR subtraction)?

Towards sub-leading factorization

$$\left|\mathcal{M}^{\text{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\{p\})\right\rangle + \mathcal{O}\left(\frac{m^2}{s_{ij}}\right)$$

Power corrections to the factorization formula

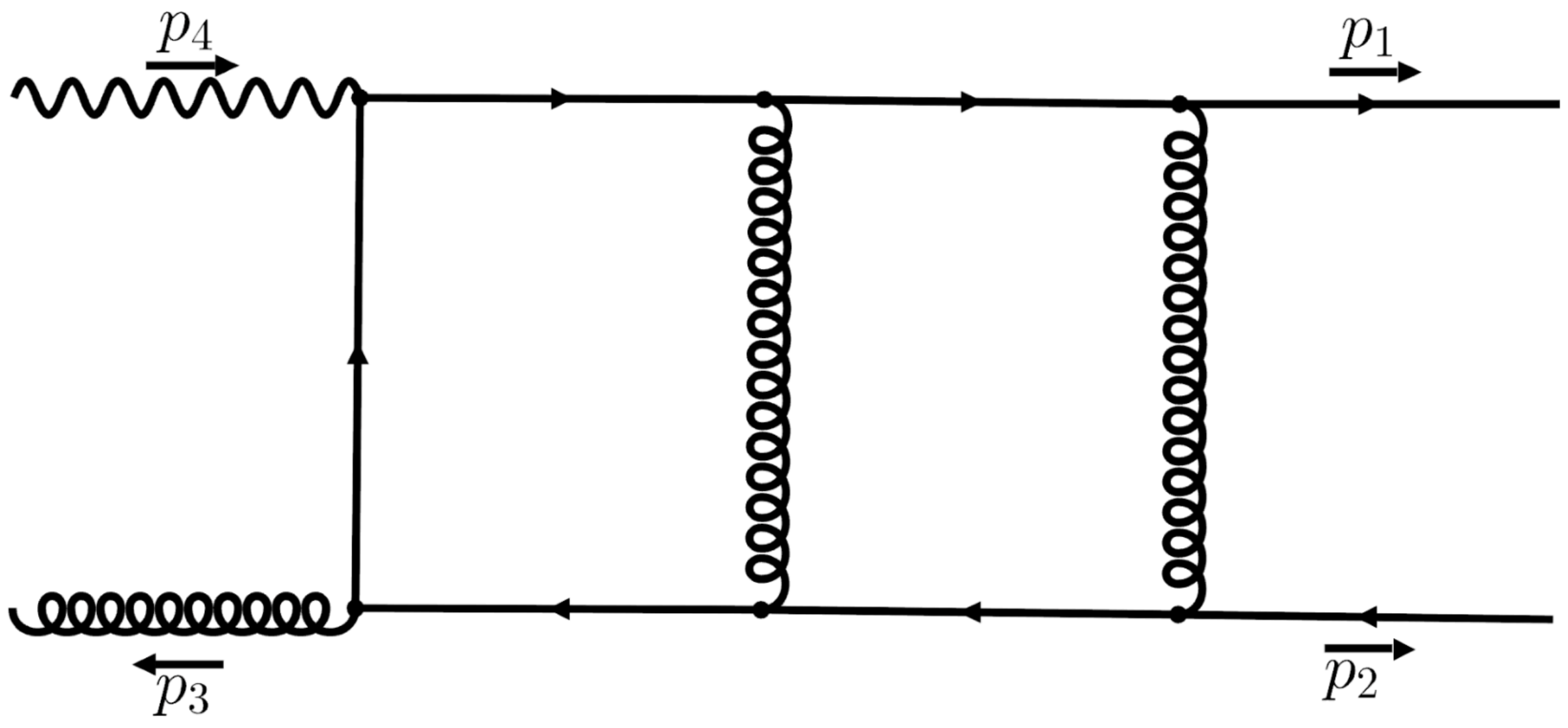
Important for intermediate energy range



Towards sub-leading factorization

$$\left|\mathcal{M}^{\text{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\{p\})\right\rangle + \mathcal{O}\left(\frac{m^2}{s_{ij}}\right)$$

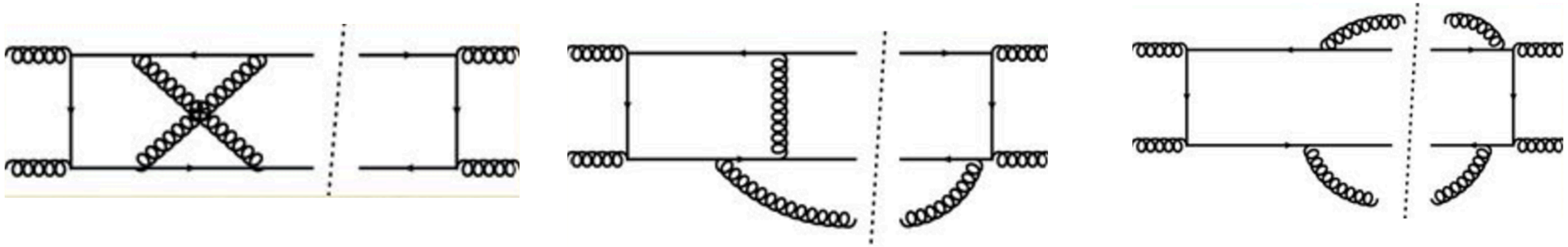
Ongoing: analyzing sub-leading corrections in $1 \to 3$ form factors using two methods



- Small-mass expansion
- Method of regions

Towards NNLO differential cross sections

For the NNLO cross section, we need to combine three contributions: double virtual, virtual+real and double real



One may employ Q_T subtraction, originally designed for colorless final states

Catani, Grazzini, hep-ph/0703012

$$\mathrm{d}\sigma_{\mathrm{NNLO}} = \mathrm{d}\sigma_{\mathrm{NNLO}}^{Q_T\to 0} + \left(\mathrm{d}\sigma_{\mathrm{NLO}}^{1-\mathrm{jet}} - \mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{CT}}\right)$$

Described by a factorization formula in the small Q_T limit

Towards NNLO differential cross sections

The Q_T factorization formula for colored final-states   Zhu, Li, Li, Shao, LLY: 1208.5774, 1307.2464

$$d\sigma \sim B_i \otimes B_j \otimes S_{ij} H_{ij} + \mathcal{O}(Q_T^2/Q^2)$$

Universal beam functions, NNLO available

Gehrmann, Lübbert, LLY: 1209.0682, 1403.6451

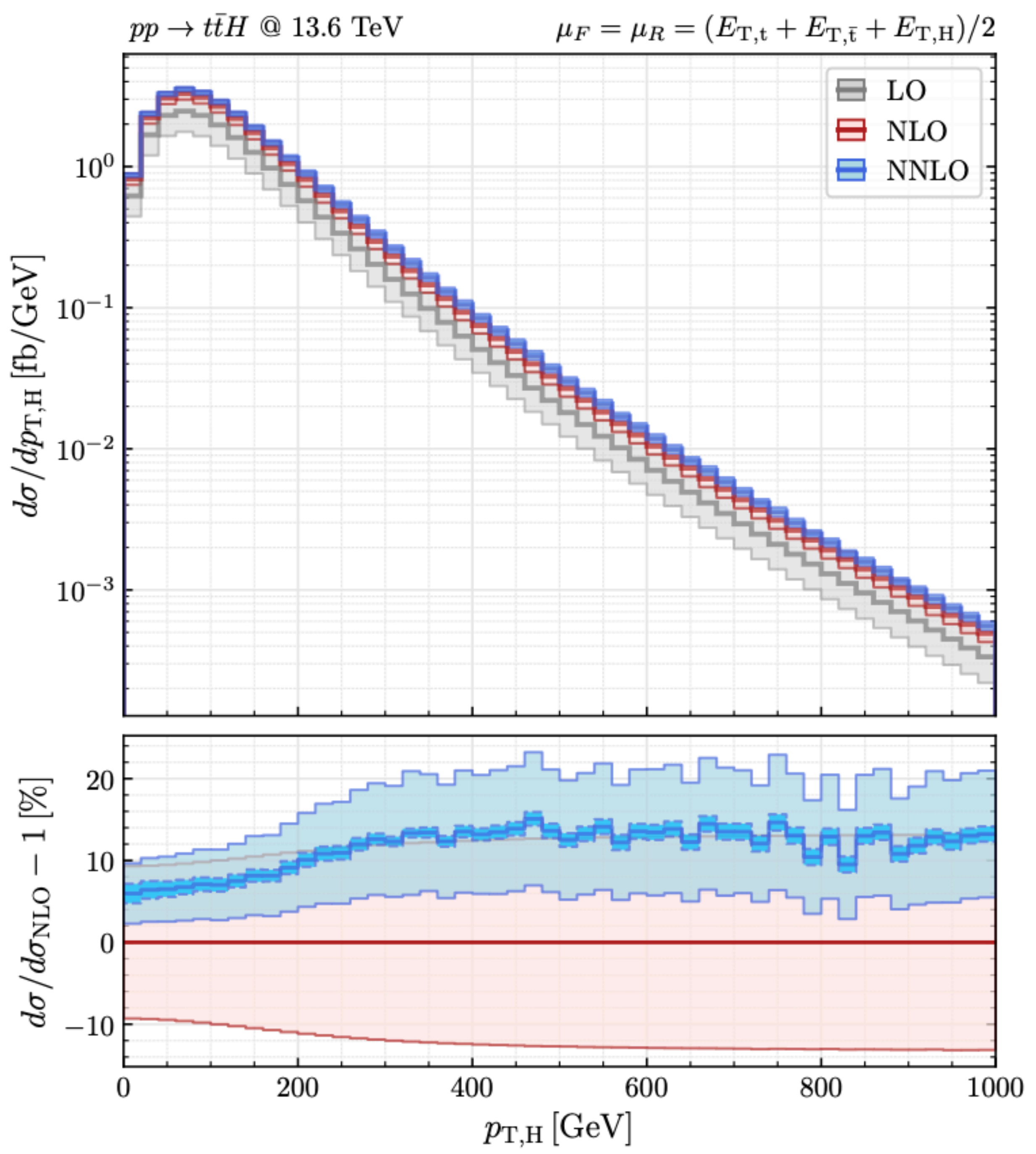
Hard functions, from our calculation

Process-dependent soft functions

Partial NNLO analytic results in Liu, Monni: 2411.13466

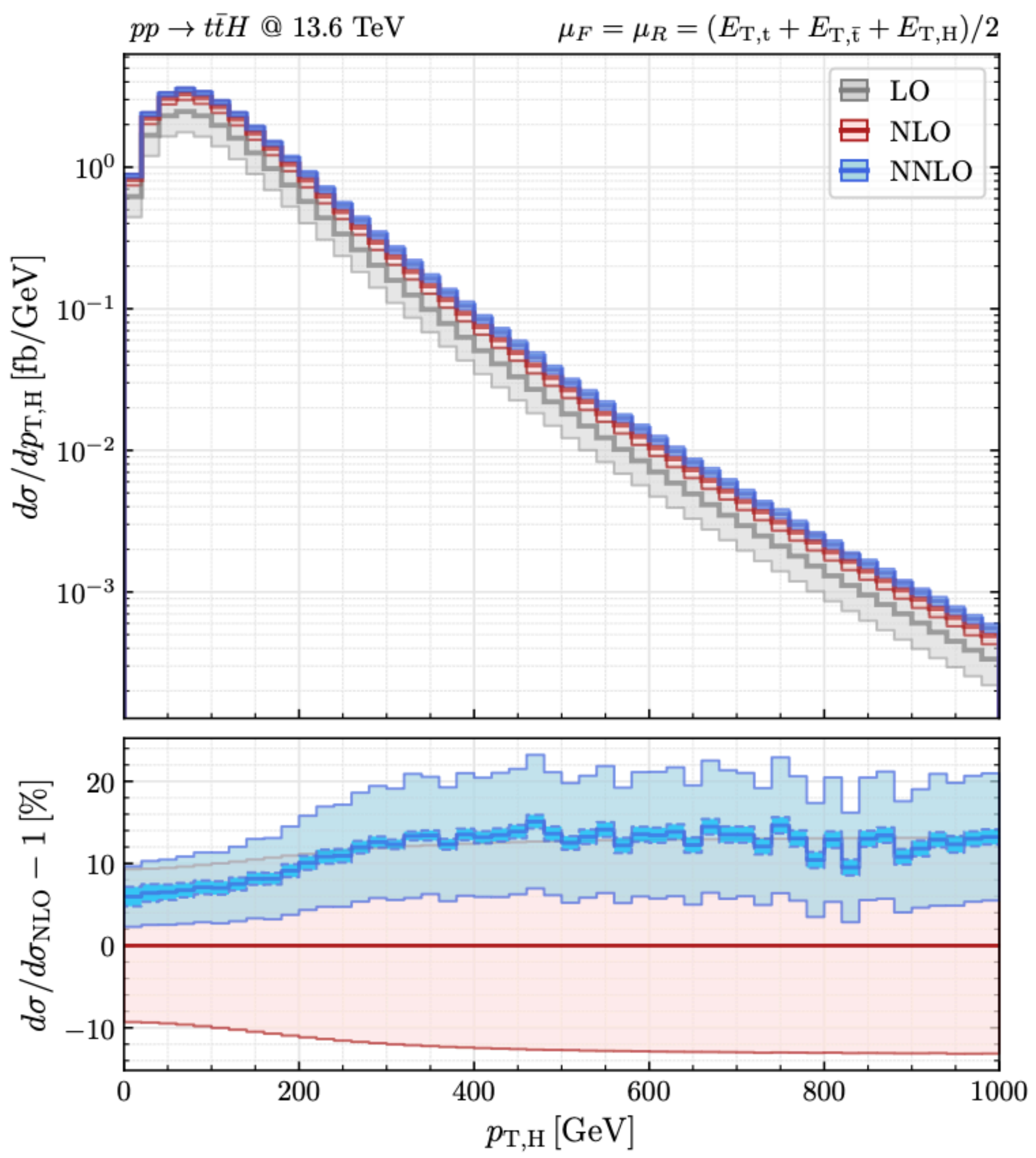
Towards NNLO differential cross sections

A parallel study in Devoto et al.: 2411.15340



Towards NNLO differential cross sections

A parallel study in Devoto et al.: 2411.15340



Leading color

Sub-leading color

- Only the leading color part, but sub-leading terms not negligible according to our study
- Remains to see the effects at the level of cross sections

exact / semi-	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
A^g	6.189970	−38.61149	185.2507	−502.3539	
	6.140920	−38.20303	180.8479	−466.0329	277.4175
B^g	−9.662043	67.19163	−504.2190	2664.419	
	−9.528888	66.04951	−485.4858	2302.867	−8009.039
C^g		−26.39205	137.5097	122.6492	
		−26.08095	129.1071	−282.4970	−363.2452
D^g			5.872808	−190.6910	
			5.665436	−179.9663	1016.432
E_l^g		−7.221632	30.41847	−83.58398	
		−7.164407	30.11041	−81.90409	122.4383
E_h^g			−7.090949	55.10048	
			−0.2277007	7.477590	−38.94969
F_l^g		11.27238	−51.40444	145.7744	
		11.11704	−50.54900	139.8451	−187.9518
F_h^g			13.78273	−106.0896	
				−10.76121	124.8742
G_l^g			13.19603	−19.72861	
			13.04047	−17.39200	−19.68276
G_h^g				18.82390	
					−16.44998
H_l^g			1.375549	−1.554947	
			1.364649	−1.537369	1.772607
H_{lh}^g				2.363650	
				0.07590022	−1.707018
H_h^g					
					0.2450480
I_l^g			−2.147121	2.581333	
			−2.117531	2.532773	−2.311413
I_{lh}^g				−4.594245	
					3.483198
I_h^g					
Total	4.316971	−34.94825	146.5483	−257.9766	
	4.288068	−34.65125	145.6813	−275.2426	−381.0666

Summary and outlook

- The tTH production is important for probing the top quark Yukawa coupling
- Towards NNLO prediction at high energies
  - Two-loop IR poles
  - High energy factorization formula for QCD amplitudes
  - Applied to tTH production: approximate two-loop amplitudes
  - Future: sub-leading corrections to the factorization formula
  - Future: combine with real emissions for NNLO cross sections
- A complete two-loop calculation? Requires new integral reduction techniques!

Summary and outlook

- The tTH production is important for probing the top quark Yukawa coupling
- Towards NNLO prediction at high energies
  - Two-loop IR poles
  - High energy factorization formula for QCD amplitudes
  - Applied to tTH production: approximate two-loop amplitudes
  - Future: sub-leading corrections to the factorization formula
  - Future: combine with real emissions for NNLO cross sections
- A complete two-loop calculation? Requires new integral reduction techniques!

Thank you!