Geometrizing EFT Matching

Xu-Xiang Li 黎旭翔 (University of Utah)

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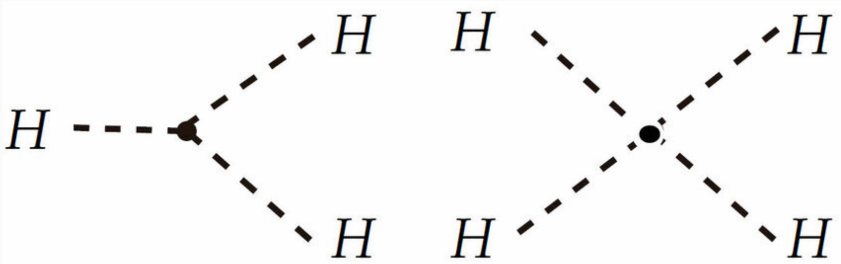
University of Science and Technology of China

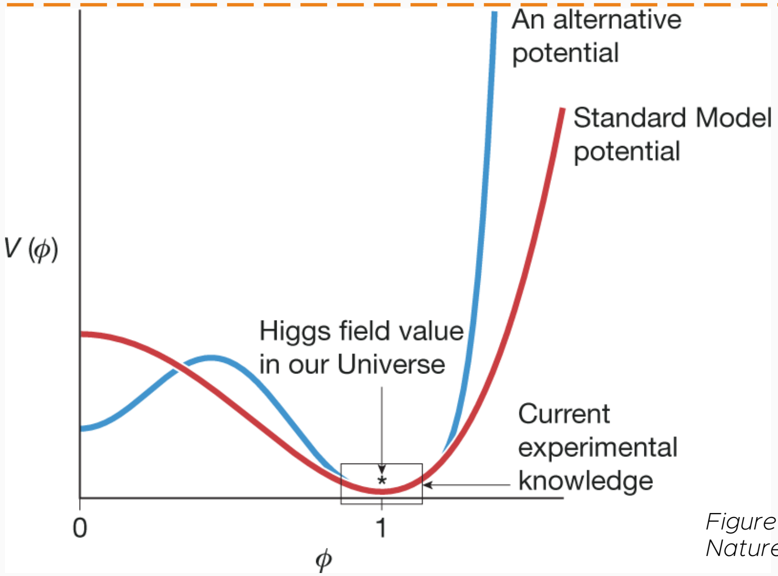
Higgs Potential 2024

Based on arXiv:**2411.04173** with Xiaochuan Lu and Zhengkang "Kevin" Zhang

BSM physics behind Higgs

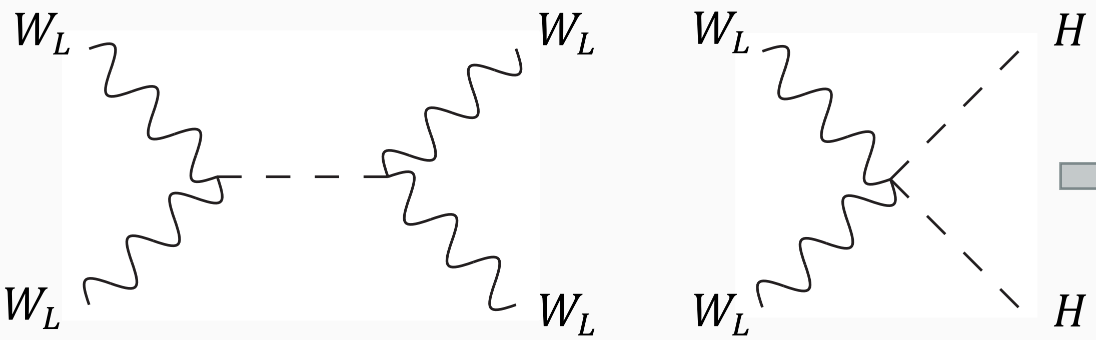


Higgs self interaction

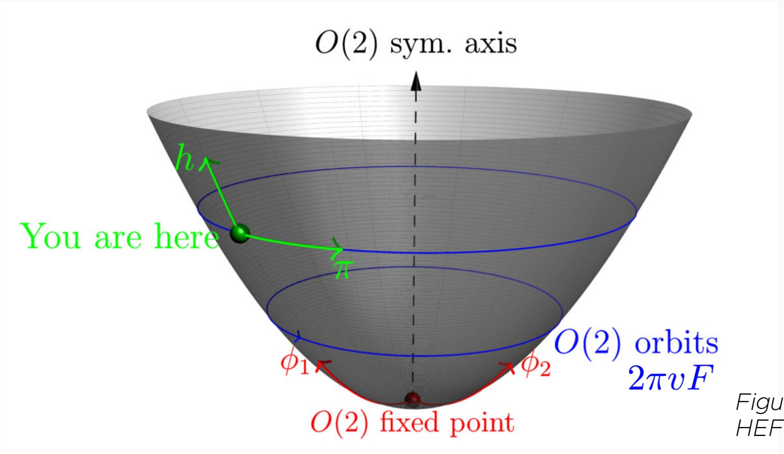


Geometry of Higgs potential

Figure from Salam, Wang and Zanderighi, Nature 607, 41–47 (2022).



Higgs-gauge boson interaction



Geometry of Higgs field space

Figure from Nathaniel Craig, HEFT 2021

Field Space Geometry in Higgs Sector

- **SM Higgs doublet under different parametrization**

Cartesian Coodinates

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} -\phi^2 - i\phi^1 \\ \phi^4 + i\phi^3 \end{pmatrix}$$

$$\Delta\mathcal{L} = \frac{1}{2}\delta_{ij}(\partial_\mu \phi^i)(\partial^\mu \phi^i) - V(|\phi|^2)$$

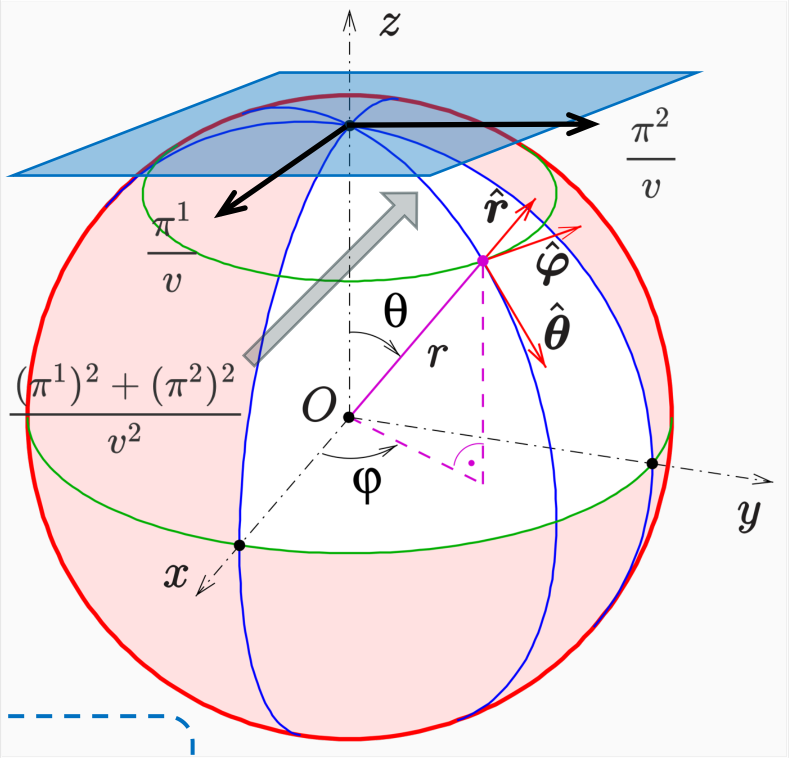
$$\Delta\mathcal{L} = (\partial_\mu H)^\dagger (\partial^\mu H) - V(H^\dagger H)$$

$$H = U(x)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$U(x) = \exp\left(-\frac{i\sigma^a \pi^a}{v}\right)$$

$$\Delta\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}\left(1+\frac{h}{v}\right)^2 g_{ab}(\pi)(\partial_\mu \pi^a)(\partial^\mu \pi^b) - V(h)$$

Spherical Coodinates



Field Space Geometry in General Scalar Theory

- **A general scalar theory with two derivatives**

$$\mathcal{L} = \frac{1}{2} g_{ij}(\varphi)(\partial_\mu \varphi^i)(\partial^\mu \varphi^j) - V(\varphi)$$

- **Under non-derivative field redefinition**

$$\phi^i \to \phi'^i = \phi'^i(\phi)$$

$$(\partial_\mu \phi^i) \to (\partial_\mu \phi'^i) = \frac{\partial \phi'^i}{\partial \phi^j}(\partial_\mu \phi^j)$$

$$g_{ij} \to g'_{ij} = \frac{\partial \phi^k}{\partial \phi'^i}\frac{\partial \phi^l}{\partial \phi'^j} g_{kl}$$

Field Space Geometry in General Scalar Theory

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⟺

Coodinate transformation

Vector

(0, 2)-tensor ⇒ **metric**

Field Space Geometry

[Alonso, Jenkins, Manohar, arXiv: 1511.00724, 1605.03602]
[Craig, Lee, arXiv: 2307.15742]
[Alminawi, Brivio, Davighi, arXiv: 2308.00017]

- **Amplitudes ~ Geometric quantities: curvatures and covariant derivatives**

$$\mathcal{A}_{ijkl} = R_{ijkl} s_{ik} + R_{ikjl} s_{ij} \quad \textbf{(if potential ignored)}$$

[Helset, Jenkins, Manohar, arXiv: 2210.08000]

Some Applications of EFT Geometry

- **Invariant form of amplitudes: field redefinition ⇔ coordinate transformation**

vielbein

covariant

$$\mathcal{A}^{\alpha_1\alpha_2\alpha_3} = \left(\prod_{a=1}^{3} e^{\alpha_a i_a}\right) \overline{V}_{;(i_1 i_2 i_3)}$$

$$\mathcal{A}^{\alpha_1\alpha_2\alpha_3\alpha_4} = \left(\prod_{a=1}^{4} e^{\alpha_a i_a}\right) \left[\overline{V}_{;(i_1 i_2 i_3 i_4)} + \frac{1}{3}\left(s_{12}\overline{R}_{i_1(i_3 i_4)i_2}\right)_{6\text{ terms}} + \left(\overline{V}_{;(i_1 i_2 j)} \frac{1}{s_{12}\overline{g}_{jk} - \overline{V}_{;jk}} \overline{V}_{;(i_3 i_4 k)}\right)_{3\text{ terms}} \right]$$

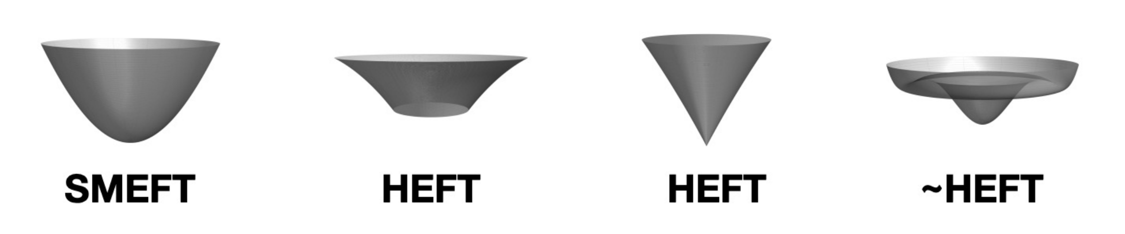
[Cohen, Craig, Lu, Sutherland, arXiv: 2108.03240]

- **Geometric soft theorems (tree level)**

$$\lim_{q\to 0} \mathcal{A}_{n+1}^{i_1\cdots i_n i} = \nabla^i \mathcal{A}_n^{i_1\cdots i_n} + \sum_{a=1}^{n} \frac{\nabla^i V^{i_a}{}_{j_a}}{(p_a+q)^2 - m_{j_a}^2}\left(1 + q^\mu \frac{\partial}{\partial p_a^\mu}\right)\mathcal{A}_n^{i_1\cdots j_a \cdots i_n}$$

[Cheung, Helset, Parra-Martinez, arXiv: 2111.03045]

- **Differentiate SMEFT vs HEFT/SMEFT**



[Cohen, Craig, Lu, Sutherland, arXiv: 2008.08597]
Nathaniel Craig, HEFT 2021

Effective Action w/ Geometry

- **Background field method**

$$\varphi = \varphi_{\rm b} + \eta$$

$$S[\varphi] = S[\varphi_{\rm b}] + \eta^i \frac{\delta S}{\delta \varphi^i}[\varphi_{\rm b}] + \frac{1}{2}\eta^i \eta^j \frac{\delta^2 S}{\delta \varphi^j \delta \varphi^i}[\varphi_{\rm b}] + \cdots$$

$$\Gamma^{(0)}[\varphi_{\rm b}] = S[\varphi_{\rm b}]$$

$$\Gamma^{(1)}[\varphi_b] = \frac{i}{2}\mathrm{Tr}\log\left(-g^{ik}\frac{\delta^2 S}{\delta \varphi^j \delta \varphi^k}[\varphi_{\rm b}]\right)$$

Effective Action w/ Geometry

- **Background field method** $\varphi = \varphi_{\rm b} + \eta$

$$S[\varphi] = S[\varphi_{\rm b}] + \eta^i \frac{\delta S}{\delta \varphi^i}[\varphi_{\rm b}] + \frac{1}{2}\eta^i \eta^j \frac{\delta^2 S}{\delta\varphi^j \delta\varphi^i}[\varphi_{\rm b}] + \cdots$$

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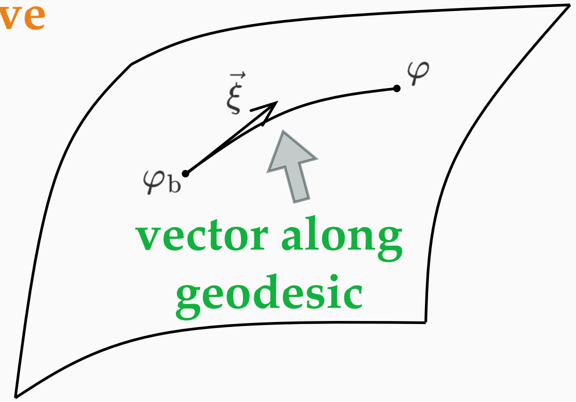
- **Geometrizing:** ordinary (functional) derivative → covariant (functional) derivative

$$\frac{\delta S}{\delta\varphi^i} \to \frac{\nabla S}{\nabla \varphi^i} = \frac{\delta S}{\delta \varphi^i} = -g_{ij}(\varphi)[\mathscr{D}_\mu(\partial^\mu\varphi)]^j - \nabla_i V(\varphi)$$

$$\frac{\delta^2 S}{\delta\varphi^i\delta\varphi^j} \to \frac{\nabla^2 S}{\nabla\varphi^i\nabla\varphi^j} = \frac{\delta^2 S}{\delta\varphi^i\delta\varphi^j} - \Gamma^k_{ij}(\varphi)\frac{\delta S}{\delta\varphi^k}\delta^{(4)}(x^i - x^j)$$

Covariant spacetime derivative

Connection



[Gaillard, Nucl. Phys. B 268, 669-692 (1986)]
[Alonso, Jenkins, Manohar, arXiv: 1605.03602]
[Alonso, West, arXiv:2207.02050]

Effective Action w/ Geometry

- **Geometrized 1PI effective action**

$$\Gamma^{(0)}_{\text{geo}}[\varphi_b] = S[\varphi_\text{b}]$$

$$\Gamma^{(1)}_{\text{geo}}[\varphi_b] = \frac{i}{2}\text{Tr}\log\left(-g^{ik}\frac{\nabla^2 S}{\nabla\varphi^j\nabla\varphi^k}[\varphi_\text{b}]\right)$$

⇓

$$-\left[g_{ij}(\phi)\mathscr{D}^2 + R_{ikjl}(\partial_\mu\phi^k)(\partial^\mu\phi^l) + \nabla_i\nabla_j V(\phi)\right]_x \delta^4(x-y)$$

- $\frac{1}{\epsilon_{\text{UV}}}$ divergent term ⇨ **RGE of EFT**
  - Also some trials to include gauge bosons and fermions

[Alonso, Jenkins, Manohar, arXiv: 1511.00724] [Alonso, Kanshin, Saa, arXiv: 1710.06848]
[Assi, Helset, Manohar, Pagès, Shen, arXiv:2307.03187] [Jenkins, Manohar, Naterop, Pagès, arXiv:2308.06315]

- **Hard region (divergent + finite)** ⇨ Matching of UV onto EFT!

*[**XXL**, Lu, Zhang, arXiv: 2411.04173]*

Geometrizing EFT Matching

- Mass hierarchy $\varphi^i = \Phi^A, \phi^a$ with $M \equiv M_\Phi \gg m_\phi$
- Integrate out some specific direction Φ^A on field space

  ⇨ EFT contains only ϕ^a, only geometry of the submanifold is considered

- Tree level

$$\mathcal{L}_{\text{EFT}}[\phi] = \mathcal{L}[\Phi_{\text{c}}[\phi], \phi]$$

Heavy EOM: scalar on EFT submanifold

$$\Phi_{\text{c}}^A = -\frac{1}{M^2}\,\delta^{AB}\left[W_{,B} + g_{Bj}\left(\mathscr{D}^\mu(\partial_\mu\varphi)\right)^j\right]\Big|_{\Phi=\Phi_{\text{c}}}$$

$$W(\Phi,\phi) = V(\Phi,\phi) - \frac{1}{2}M^2\delta_{AB}\Phi^A\Phi^B$$

Geometrizing EFT Matching

- **Mass hierarchy** $\varphi^i = \Phi^A, \phi^a$ **with** $M \equiv M_\Phi \gg m_\phi$
- **Integrate out some specific direction** Φ^A **on field space**

  ⇨ EFT contains only ϕ^a, only geometry of the submanifold is considered

- One-loop level

  ⇨ use covariant derivative expansion (CDE) technique

$$\mathcal{L}_{\mathrm{EFT}}^{[1]}[\phi] = i\frac{1}{2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4} \mathrm{tr}\log\left[(\mathscr{D}_\mu + ip_\mu)^2 + R^i{}_{kjl}(\partial_\mu\varphi^k)(\partial^\mu\varphi^l) + \nabla^i\nabla_j V\right]\Big|_{\Phi=\Phi_c[\phi]}$$

  - Divide into covariant quantities of submanifold

$$\mathscr{D}_\mu = \begin{pmatrix} \mathscr{D}_\mu^L & -i\mathcal{K}_\mu^{LH} \\ -i\mathcal{K}_\mu^{HL} & \mathscr{D}_\mu^H \end{pmatrix}$$

$$R_{abcd} = \hat{R}_{abcd} - \mathcal{K}_{ac}\mathcal{K}_{bd} + \mathcal{K}_{ad}\mathcal{K}_{bc}$$

intrinsic curvature

extrinsic curvature

Geometrizing EFT Matching

- **Mass hierarchy** $\varphi^i = \Phi^A, \phi^a$ **with** $M \equiv M_\Phi \gg m_\phi$
- **Integrate out some specific direction** Φ^A **on field space**

  ⇨ **EFT contains only** ϕ^a**, only geometry of the submanifold is considered**

- **One-loop level**

  ⇨ **use covariant derivative expansion (CDE) technique**

[Henning, Lu, Murayama, arXiv: 1412.1837]
[Cohen, Lu, Zhang, arXiv: 2011.02484]
[Fuentes-Martin, König, Pagès, Thomsen, Wilsch, arXiv:2212.04510]

$$\mathcal{L}^{[1]}_{\mathrm{EFT}}[\phi] = -\frac{i}{2}\int \mathrm{d}M^2 \int \frac{\mathrm{d}^d q}{(2\pi)^d}\,\mathrm{tr}\left[\sum_{m=0}^{\infty} \frac{1}{q^2-\mathbf{M}^2}\left\{\left[\{q^\mu, \widetilde{\mathcal{G}}'_{\mu\nu}\partial_q^\nu\} - (\widetilde{\mathcal{G}}'_{\mu\nu}\partial_q^\nu)^2\right]\frac{1}{q^2-\mathbf{M}^2}\right\}^m\right]\Bigg|_{\Phi=\Phi_c[\phi]}$$

$$-\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\int \frac{\mathrm{d}^d q}{(2\pi)^d}\,\mathrm{tr}\left[\sum_{m=0}^{\infty} \frac{1}{q^2-\mathbf{M}^2}\left\{\left[\{q^\mu, \widetilde{\mathcal{G}}'_{\mu\nu}\partial_q^\nu\} - (\widetilde{\mathcal{G}}'_{\mu\nu}\partial_q^\nu)^2\right]\frac{1}{q^2-\mathbf{M}^2}\right\}^m \left(\widetilde{\mathcal{U}} - \widetilde{\mathcal{K}}_\mu\widetilde{\mathcal{K}}^\mu + \left\{q_\mu - \widetilde{\mathcal{G}}'_{\mu\nu}\partial_q^\nu, \widetilde{\mathcal{K}}^\mu\right\}\right)\right]^n\Bigg|_{\Phi=\Phi_c[\phi]}$$

~ intrinsic curvature

~ potential + curvature

~ extrinsic curvature

Example: O(N) Sigma Model

- **Tree level:**

$$\mathcal{L}[h,\pi] = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}F^2(h)\,\hat{g}_{ab}(\pi)(\partial_\mu \pi^a)(\partial^\mu \pi^b) - V(h)$$

$$h_c = -\frac{1}{M^2}\left[(\partial^2 h) - F F_{,h}\underbrace{\hat{g}_{ab}(\partial_\mu \pi^a)(\partial^\mu \pi^b)}_{\hat{g}} + \frac{1}{2}\bar{V}_{,hhh}\,h^2 + \cdots\right]\Bigg|_{h=h_c}$$

$$\Rightarrow \mathcal{L}^{[0]}_{\mathrm{EFT}}[\pi] = \frac{1}{2}\hat{g} + \frac{\bar{F}^2_{,h}}{2M^2}\hat{g}^2 + \frac{\bar{F}^2_{,h}}{2M^4}(\partial_\mu \hat{g})^2 + \frac{\bar{F}^2_{,h}}{2M^6}\left[M^2(\bar{F}^2_{,h} + \bar{F}_{,hh}) - \frac{1}{3}\bar{F}_{,h}\bar{V}_{,hhh}\right]\hat{g}^3 + \mathcal{O}(\partial^8)$$

- **One-loop level: all in ∂'s expansion!**

$$\mathcal{U}^i{}_j \sim \begin{pmatrix} \partial^2 & \partial^4 \\ \partial^4 & \partial^2 \end{pmatrix}, \quad (\mathcal{K}_\mu)^i{}_j \sim \begin{pmatrix} & \partial^1 \\ \partial^1 & \end{pmatrix}, \quad (\mathcal{G}'_{\mu\nu})^i{}_j \sim \begin{pmatrix} 0 & \\ & \partial^2 \end{pmatrix}$$

  - **Linear sigma model (in a spherical coordinate)**

$$\mathcal{L}_{\mathrm{EFT}} = \frac{1}{2}\left[1 + \frac{\lambda}{16\pi^2}(7-6L)\right]\hat{g} - \frac{\lambda}{16\pi^2}\frac{1}{18M^2}(5-6L)\hat{g}_{ab}(\hat{\mathscr{D}}_\mu \partial_\nu \pi)^a(\hat{\mathscr{D}}^\mu \partial^\nu \pi)^b$$
$$+\frac{\lambda}{M^4}\left[1 + \frac{\lambda}{16\pi^2}(8-12L)\right]\hat{g}^2 - \frac{\lambda}{16\pi^2}\frac{\lambda}{3M^4}(13-6L)\,\hat{g}_{\mu\nu}\hat{g}^{\mu\nu} + \mathcal{O}(\partial^6)$$

a reorganization also of the result

Geometric UOLEA

- **UOLEA: Universal One-Loop Effective Action (for general UV theory)**

For …

$$\mathcal{L} = -\frac{1}{2}\Phi(D^2 + M^2 + U[\phi])\Phi$$

[Henning, Lu, Murayama, arXiv: 1412.1837]

$$\begin{aligned}
\Delta\mathcal{L}_{\text{eff,1-loop}} = \frac{c_s}{(4\pi)^2}\,\mathrm{tr}\Bigg\{ \\
&+ m^4\left[-\frac{1}{2}\left(\log\frac{m^2}{\mu^2} - \frac{3}{2}\right)\right] \\
&+ m^2\left[-\left(\log\frac{m^2}{\mu^2} - 1\right)U\right] \\
&+ m^0\left[-\frac{1}{12}\left(\log\frac{m^2}{\mu^2} - 1\right)G'^2_{\mu\nu} - \frac{1}{2}\log\frac{m^2}{\mu^2}U^2\right] \\
&+ \frac{1}{m^2}\left[-\frac{1}{60}(P_\mu G'_{\mu\nu})^2 - \frac{1}{90}G'_{\mu\nu}G'_{\nu\sigma}G'_{\sigma\mu} - \frac{1}{12}(P_\mu U)^2 - \frac{1}{6}U^3 - \frac{1}{12}UG'_{\mu\nu}G'_{\mu\nu}\right] \\
&+ \frac{1}{m^4}\bigg[\frac{1}{24}U^4 + \frac{1}{12}U(P_\mu U)^2 + \frac{1}{120}(P^2U)^2 + \frac{1}{24}\left(U^2G'_{\mu\nu}G'_{\mu\nu}\right) \\
&\qquad - \frac{1}{120}[(P_\mu U),(P_\nu U)]G'_{\mu\nu} - \frac{1}{120}[U[U,G'_{\mu\nu}]]G'_{\mu\nu}\bigg] \\
&+ \frac{1}{m^6}\left[-\frac{1}{60}U^5 - \frac{1}{20}U^2(P_\mu U)^2 - \frac{1}{30}(UP_\mu U)^2\right] \\
&+ \frac{1}{m^8}\left[\frac{1}{120}U^6\right]\Bigg\}.
\end{aligned} \tag{2.54}$$

Geometric UOLEA

- **UOLEA: Universal One-Loop Effective Action (for general UV theory)**

For …

$$\mathcal{L} = \frac{1}{2} g_{ij}(\varphi)(\partial_\mu \varphi^i)(\partial^\mu \varphi^j) - V(\varphi)$$

⇨

*[**XXL**, Lu, Zhang, arXiv: 2411.04173]*

$\mathcal{O}(\mathcal{U}^2\mathcal{P}'^2)$ terms	
$-\frac{1}{24M^2}$	$[\mathcal{P}'_\mu, \mathcal{U}]_{HH}[\mathcal{P}'^\mu, \mathcal{U}]_{HH}$
$-\frac{1}{4M^2}$	$[\mathcal{P}'_\mu, \mathcal{U}]_{HL}[\mathcal{P}'_\mu, \mathcal{U}]_{LH}$
$\mathcal{O}(\mathcal{U}^3\mathcal{P}'^2)$ terms	
$\frac{1}{4M^4}$	$\mathcal{U}_{HH}[\mathcal{P}'_\mu, \mathcal{U}]_{HL}[\mathcal{P}'^\mu, \mathcal{U}]_{LH}$
$\frac{1}{6M^4}$	$[\mathcal{P}'_\mu, \mathcal{U}]_{HH}[\mathcal{P}'^\mu, \mathcal{U}]_{HL}U_{LH} + \mathcal{U}_{HL}[\mathcal{P}'^\mu, \mathcal{U}]_{LH}[\mathcal{P}'_\mu, \mathcal{U}]_{HH}$
$-\frac{1}{4M^4}(5 - 2L)$	$[\mathcal{P}'_\mu, \mathcal{U}]_{HL}\mathcal{U}_{LL}[\mathcal{P}'^\mu, \mathcal{U}]_{LH}$
$-\frac{1}{4M^4}$	$\mathcal{U}_{HL}[\mathcal{P}'_\mu, \mathcal{U}]_{LL}[\mathcal{P}'^\mu, \mathcal{U}]_{LH} + [\mathcal{P}'_\mu, \mathcal{U}]_{HL}[\mathcal{P}'^\mu, \mathcal{U}]_{LL}\mathcal{U}_{LH}$
$\mathcal{O}(\mathcal{U}^4\mathcal{P}'^2)$ terms	
$\frac{1}{2M^6}$	$\mathcal{U}_{HL}\mathcal{U}_{LH}[\mathcal{P}'_\mu, \mathcal{U}]_{HL}[\mathcal{P}'^\mu, \mathcal{U}]_{LH}$
$\frac{1}{12M^6}(17 - 6L)$	$[\mathcal{P}'_\mu, \mathcal{U}]_{HL}\mathcal{U}_{LH}\mathcal{U}_{HL}[\mathcal{P}'^\mu, \mathcal{U}]_{LH}$
$\frac{5}{24M^6}$	$\mathcal{U}_{HL}[\mathcal{P}'_\mu, \mathcal{U}]_{LH}\mathcal{U}_{HL}[\mathcal{P}'^\mu, \mathcal{U}]_{LH} + [\mathcal{P}'_\mu, \mathcal{U}]_{HL}\mathcal{U}_{LH}[\mathcal{P}'^\mu, \mathcal{U}]_{HL}\mathcal{U}_{LH}$
$\mathcal{O}(\mathcal{K}^2)$ terms	
$\frac{1}{4}M^2$	$(\mathcal{K}_\mu\mathcal{K}^\mu)_{HH}$

+ more tables

Apply on SM+S under different field basis
⇨ Less terms contribute

Summary

- **Geometrizing EFT Matching: keep geometric covariance during EFT matching**
  - Geometrized 1PI effective action derived by expansion along geodesics
  - During EFT matching (hard region expansion), geometry on EFT submanifold survives
  - Effective Lagrangian built by geometric quantities, e.g. extrinsic/intrinsic curvature
  - O(N) sigma model: effective Lagrangian in a derivative expansion
  - Geometric UOLEA

Thank you!