Discovering tau atom and tau mass

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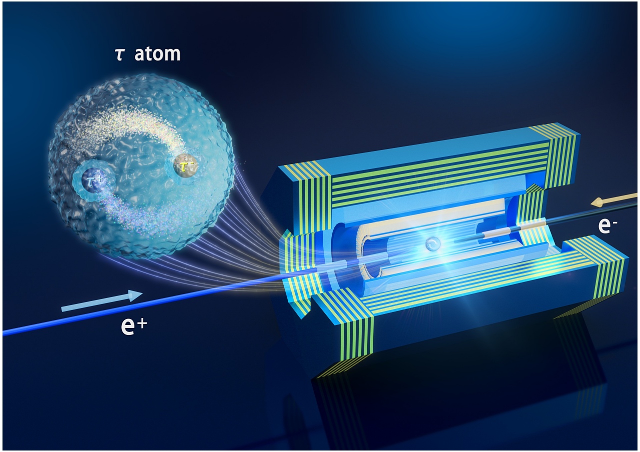
$\tau^+\tau^-$ atom



Outline

1. Introduction
   - $\tau^+\tau^-$ atom
   - τ mass
2. The frame of Calculation
3. Reduce the uncertainties
4. Summary

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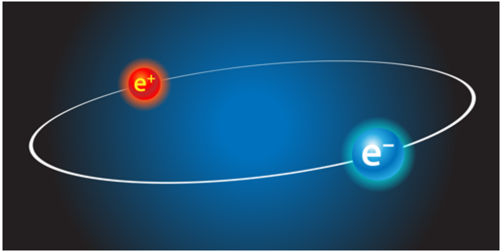
QED atom

1. QED atoms (e^+e^-, μ^+e^-, τ^+e^-, $\mu^+\mu^-$, $\tau^+\mu^-$, $\tau^+\tau^-$), composed of unstructured, point-like lepton pairs, are simpler than hydrogen formed of a proton and an electron.
2. The properties of QED atoms have been studied to test QED, fundamental symmetries, New Physics, gravity, and so on (hep-ex/0106103, 0912.0843, 1710.01833, 1802.01438, Phys.Rept. 975 (2022) 1-61).

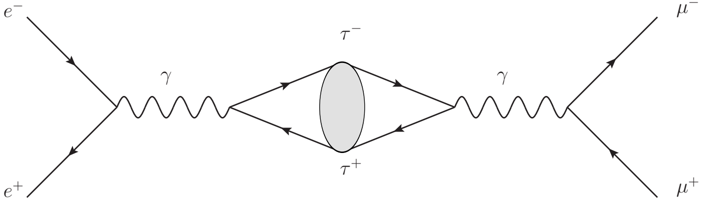
Positronium

1. Only positronium (e^+e^-) and muonium (μ^+e^-) were discovered in 1951 and 1960 respectively.
2. Positronium was discovered by Martin Deutsch in 1951.
3. Positronium can be applied to medicine and biology: Nature Reviews Physics 1 (2019)527, Rev. Mod. Phys. 95 (2023) 021002.



$\tau^+\tau^-$ atom

1. $\tau^+\tau^-$ atom is the smallest QED atom for Bohr radius is 30.4 fm (Moffat:1975uw)
2. $\tau^+\tau^-$ atom is named tauonium (Avilez:1977ai,Avilez:1978sa), ditauonium (2204.07269, 2209.11439), or true tauonium (2202.02316).
3. We name them following charmonium: $J_\tau(nS)$ for $n^{2S+1}L_J = n^3S_1$ and $J^{PC} = 1^{--}$, $\chi_{\tau J}(nP)$ for $n^{2S+1}L_J = (n+1)^3P_J$ and $J^{PC} = J^{++}$.
4. The production η_τ (2202.02316), and J_τ (2302.07365).



$e^+e^- \to J_\tau \to \mu^+\mu^-$ at STCF, 2302.07365

TABLE IV: Cross sections and expected number of events for the s-channel production of ortho-ditauonium ($\mathcal{T}_1$), and for the $\tau^+\tau^-$ and (background) $\mu^+\mu^-$ continua, in e^+e^- at $\sqrt{s} \approx m_\mathcal{T}$ at various facilities. The last column lists the expected signal statistical significance.

Colliding system, $\sqrt{s}$ ($\delta_{\sqrt{s}}$ spread), $\mathcal{L}_{\text{int}}$, experiment	σ			N			$S/\sqrt{B}$
	$\mathcal{T}_1$	$\tau^+\tau^-$	$\mu^+\mu^-$	$\mathcal{T}_1$	$\mathcal{T}_1 \to \mu^+\mu^-$	$\mu^+\mu^-$	
e^+e^- at 3.5538 GeV (1.47 MeV), 5.57 pb^{-1}, BES III	1.9 pb	117 pb	6.88 nb	10.4	2.1	38 300	0.01σ
e^+e^- at $\sqrt{s} \approx m_\mathcal{T}$ (1.24 MeV), 140 pb^{-1}, BES III	2.2 pb	103 pb	6.88 nb	310	63	$9.63 \cdot 10^5$	0.06σ
e^+e^- at $\sqrt{s} \approx m_\mathcal{T}$ (1 MeV), 1 ab^{-1}, STCF	2.6 pb	95 pb	6.88 nb	$2.6 \cdot 10^6$	$5.3 \cdot 10^5$	$6.88 \cdot 10^9$	6.4σ
e^+e^- at $\sqrt{s} \approx m_\mathcal{T}$ (100 keV), 0.1 ab^{-1}, STCF	22 pb	46 pb	6.88 nb	$2.2 \cdot 10^6$	$4.5 \cdot 10^5$	$6.88 \cdot 10^8$	17σ

1. The statistical significance, $S/\sqrt{B}$ is 6.4 σ (17 σ) with $1\ \mathrm{ab}^{-1}$ data and $\delta_W = 1(0.1)\ \mathrm{MeV}$.
2. Monochromatized beams can also provide a very precise measurement of the tau lepton mass with an uncertainty at least $\mathcal{O}$(25 keV) .

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τ

Need more precise measurements m_τ, Γ_τ, $(g-2)_\tau$ in PDG 2024

τ

$$J = \frac{1}{2}$$

Mass $m = 1776.93 \pm 0.09$ MeV

$(m_{\tau^+} - m_{\tau^-})/m_{\text{average}} < 2.8 \times 10^{-4}$, CL = 90%

Mean life $\tau = (290.3 \pm 0.5) \times 10^{-15}$ s

$c\tau = 87.03\ \mu$m

Magnetic moment anomaly = -0.057 to 0.024, CL = 95%

$\text{Re}(d_\tau) = -0.185$ to 0.061×10^{-16} $e\,$cm, CL = 95%

$\text{Im}(d_\tau) = -0.103$ to 0.0230×10^{-16} $e\,$cm, CL = 95%

m_τ and lepton universality, 1405.1076

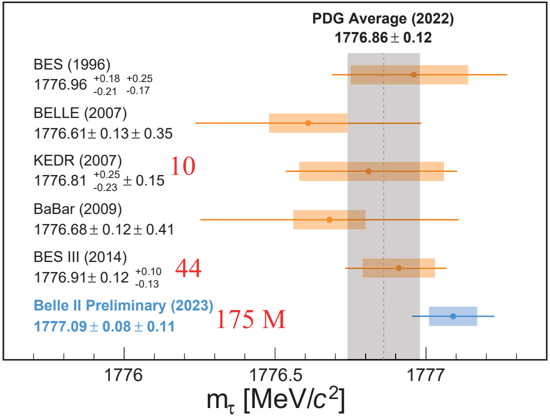
- Comparing the electronic branching fractions of τ and μ, lepton universality can be tested.

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{\tau_\mu}{\tau_\tau}\left(\frac{m_\mu}{m_\tau}\right)^5 \frac{B(\tau \to e\nu\bar{\nu})}{B(\mu \to e\nu\bar{\nu})}(1+F_W)(1+F_\gamma),$$

- BESIII measurement, 1405.1076

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = 1.0016 \pm 0.0042,$$

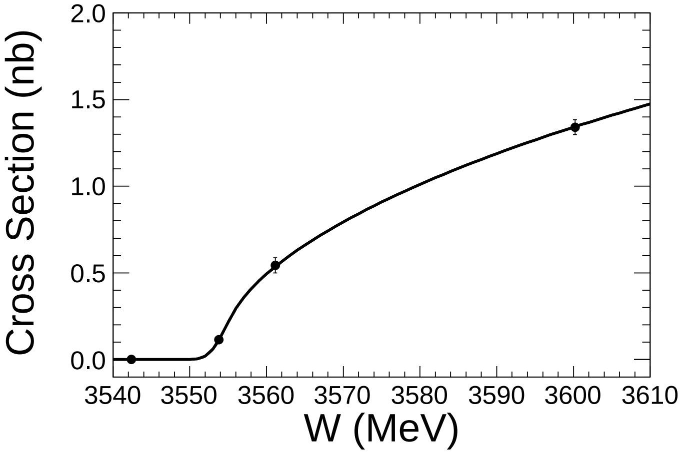
Measured m_τ, 175 M enents with 190 fb^{-1}, Belle II 2305.19116



$$\delta m_\tau = \left(\frac{\partial\sigma}{\partial m_\tau}\right)^{-1} \cdot \sqrt{\frac{\sigma}{\mathcal{L}}}$$

m_τ measurement at BESIII, 1405.1076

Scan	E_{CM} (MeV)	$\mathcal{L}$(nb^{-1})
J/ψ	3088.7	78.5 ± 1.9
	3095.3	219.3 ± 3.1
	3096.7	243.1 ± 3.3
	3097.6	206.5 ± 3.1
	3098.3	223.5 ± 3.2
	3098.8	216.9 ± 3.1
	3103.9	317.3 ± 3.8
τ	3542.4	4252.1 ± 18.9
	3553.8	5566.7 ± 22.8
	3561.1	3889.2 ± 17.9
	3600.2	9553.0 ± 33.8
ψ'	3675.9	787.0 ± 7.2
	3683.7	823.1 ± 7.4
	3685.1	832.4 ± 7.5
	3686.3	1184.3 ± 9.1
	3687.6	1660.7 ± 11.0
	3688.8	767.7 ± 7.2
	3693.5	1470.8 ± 10.3

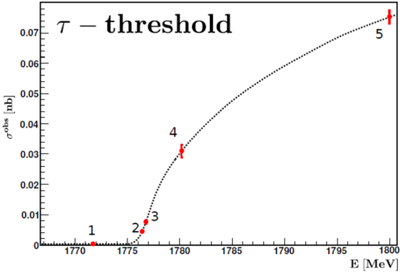


Statistical uncertainty < 45 keV, systematic uncertainty 90 keV, 1812.10056

Three energy regions:

- Low energy region
  Point 1, 14 pb^{-1}, to determine background
- Near threshold
  Point 2, 39 pb^{-1} and point 3, 26 pb^{-1}, to determine tau mass
- High energy region
  Point 4, 7 pb^{-1} for X^2 check
  Point 5, 14 pb^{-1} to determine detection efficiency

Total lum. ~100pb^{-1},
uncertainty: 0.1MeV



We obtain more than 130 pb^{-1} tau scan data!

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$e^+e^- \to \tau^+\tau^- \to X^+Y^-\bar{\nu}\nu$ around the $\tau^+\tau^-$ production threshold

1. Cross sections in BESIII, 1405.1076

$$\sigma(E_{\rm CM}, m_\tau, \delta_w^{\rm BEMS}) = \frac{1}{\sqrt{2\pi}\delta_w^{\rm BEMS}} \int_{2m_\tau}^{\infty} dE'_{\rm CM} e^{\frac{-(E_{\rm CM}-E'_{\rm CM})^2}{2(\delta_w^{\rm BEMS})^2}} \int_0^{1-\frac{4m^2}{E'^2_{\rm CM}}} dx F(x, E'_{\rm CM}) \frac{\sigma_1(E'_{\rm CM}\sqrt{1-x}, m_\tau)}{|1-\prod(E_{\rm CM})|^2}$$

2. Updated cross sections

$$\sigma_{ex}(W, m_\tau, \Gamma_\tau, \delta_w) = \int_{m(J_\tau)}^{\infty} dW' \frac{e^{-\frac{(W-W')^2}{2\delta_w^2}}}{\sqrt{2\pi}\delta_w} \int_0^{1-\frac{m(J_\tau)^2}{W'^2}} dx F(x, W') \frac{\bar{\sigma}(W'\sqrt{1-x}, m_\tau, \Gamma_\tau)}{|1-\Pi(W'\sqrt{1-x})|^2}.$$

3. Difference: shift $2m_\tau$ to $m(J_\tau)$ in the range of integration and add Γ_τ as a variable of the cross sections after including $J_\tau(nS)$ atom.

$\bar{\sigma}(W, m_\tau, \Gamma_\tau)$, orthogonal complete normalized basis, 1312.4791

1. $\bar{\sigma}(W, m_\tau, \Gamma_\tau)$

$$\bar{\sigma}(W, m_\tau, \Gamma_\tau) = \frac{4\pi\alpha^2}{3W^2}\frac{24\pi}{W^2}\mathrm{Im}\left[G_{X^+Y^-\bar{\nu}\nu}(0,0,W-2m_\tau)\right],$$

2. $G_{X^+Y^-\bar{\nu}\nu}(\vec{r}, \vec{r}\,', E)$ represents a Green function of $\tau^+\tau^-$ currents in the non-relativistic effective theory, where $\tau^+\tau^-$ decay to $X^+Y^-\bar{\nu}\nu$

$$G_{X^+Y^-\bar{\nu}\nu}(\vec{r}, \vec{r}\,', E) = \sum_n \frac{\psi_n(\vec{r})\psi_n^*(\vec{r}\,')}{E_n - E - i\epsilon} Br[n \to X^+Y^-\bar{\nu}\nu] + \int \frac{d^3\vec{k}}{2\pi^3}\frac{\psi_{\vec{k}}(\vec{r})\psi_{\vec{k}}^*(\vec{r}\,')}{E_{\vec{k}} - E - i\epsilon},$$

3. Then

$$\bar{\sigma}(W) = \bar{\sigma}^{J_\tau}(W) + \bar{\sigma}(W)_{con.}$$

Breit-Wigner formula

1. Green function approach to bound states is consistent with Breit-Wigner formula for a narrow bound states

$$\bar{\sigma}^{J_\tau}(W) = \sum_n \frac{6\pi^2}{W^2}\delta(W - m(J_\tau(nS)))\Gamma(J_\tau(nS) \to e^+e^-)Br(J_\tau(nS) \to X^+Y^-\not{E})$$

Decay mode of $J_\tau(nS)$

$$
\begin{aligned}
\Gamma_{\rm total}(J_\tau(nS)) &= \Gamma_{\rm Annihilation}(J_\tau(nS)) + \Gamma_{\rm Weak}(J_\tau(nS)) + \Gamma_{\rm E1}(J_\tau(nS)) \\
\Gamma_{\rm Annihilation}(J_\tau(nS)) &= (2+R)\Gamma(J_\tau(nS) \to e^+e^-) \\
\Gamma_{\rm Weak}(J_\tau(nS)) &= 2\Gamma(\tau \to \nu X^-)
\end{aligned}
$$

Parameters

1. Parameters

$$m_\tau = m_\tau^{\rm PDG} = 1776.86\,{\rm MeV}, \quad R = 2.342 \pm 0.0645,$$
$$\Gamma_\tau = 2.2674 \pm 0.0039\,{\rm meV}, \qquad \delta_W = 1\ {\rm MeV},$$
$$\varepsilon_{X^+Y^-\not{E}} = (8 \pm 0.2)\%, \quad \varepsilon_{\mu^+\mu^-} = 45\%,$$
$$\alpha(0) = 1/137.036, \qquad \Delta\alpha_{had}(m_{J_\tau}) = (74 \pm 7) \times 10^{-4}.$$

2. The resulting NLO expression for $\bar{\sigma}^{J_\tau}(W)$ is given by

$$\bar{\sigma}^{J_\tau}(W) = (3.12 \pm 0.02)\,\delta\left(\frac{W - 2m_\tau + 13.8\ {\rm keV}}{1\ {\rm MeV}}\right)\ {\rm pb},$$

where $13.8\ {\rm keV} = \sum_n B_n Br^{J_\tau(nS)}_{X^+Y^-\not{E}} \Gamma^{J_\tau(nS)}_{e^+e^-} / \sum_n Br^{J_\tau(nS)}_{X^+Y^-\not{E}} \Gamma^{J_\tau(nS)}_{e^+e^-}$.

Decay width of $J_\tau(nS)$

TABLE II: The decay data of $J_\tau(nS)$ in meV.

n	$\Gamma^{J_\tau(nS)}_{e^+e^-}$	$2\Gamma_\tau$	$\Gamma^{J_\tau(nS)}_{E1}$	$\Gamma^{J_\tau(nS)}_{\text{total}}$	$\Gamma^{J_\tau(nS)}_{e^+e^-} Br^{J_\tau(nS)}_{X^+Y^-\not{E}}$
1	6.484	4.535	0.0000	32.695	0.899
2	0.808	4.535	0.0000	8.044	0.455
3	0.239	4.535	0.0072	5.573	0.195
$\sum_{n=1}^{\infty}$					1.795 ± 0.012

Cross sections from $J_\tau(nS)$

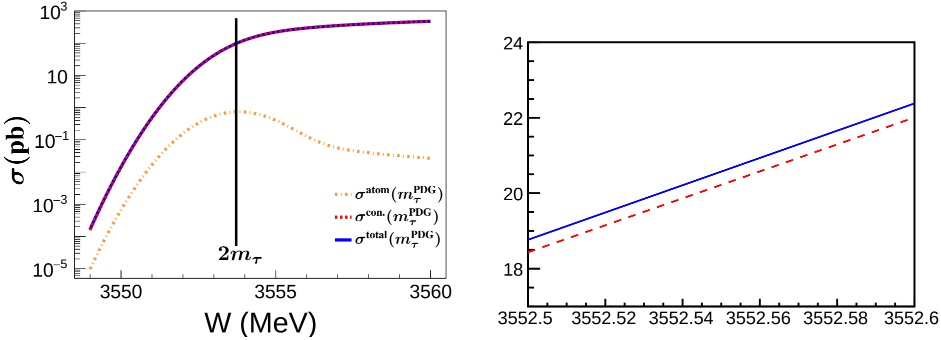
1. Then we get the cross section $\bar{\sigma}(W, m_\tau, \Gamma_\tau)$

$$\bar{\sigma}(W) = (3.12 \pm 0.02)\delta\left(\frac{W - 2m_\tau + 13.8\mathrm{keV}}{\mathrm{MeV}}\right)\ \mathrm{pb} + \theta(W - 2m_\tau)\bar{\sigma}_{con.}(W)$$

2. Continue $\bar{\sigma}_{con.}(2m_\tau)$

$$\bar{\sigma}_{Continue}(2m_\tau) = 236\ \mathrm{pb}$$

Cross sections from $J_\tau(nS)$



Reduce the uncertainties

1. The measured cross secitons

$$\sigma^{X^+Y^-\not{E}}(W) = \frac{N^{X^+Y^-\not{E}}(W)}{\mathcal{L}\varepsilon}$$

2. Uncertaintiy of ISR($\sim 0.5\%$ @ BESIII), the vacuum polarization factor (0.14%), and the integrated luminosity ($\sim 0.5\%$@ BESIII) are all larger than 0.1%.
3. Systematical uncertainty of cross section measurement at STCF maybe $> 0.2\%$.
4. The significance of 5σ require $S/\sqrt{(\Delta_{stat.}(B+S))^2 + (\Delta_{syst.}(B+S))^2} > 5$.
5. If Ignore statistical uncertainty, systematic significance of 5σ require $S/B > 1\%$ at STCF.

Uncertainty of J/ψ decay: 10 B events, 0.5% uncertainty

$J/\psi(1S)$

$I^G(J^{PC}) = 0^-(1^{--})$

Mass $m = 3096.900 \pm 0.006$ MeV

Full width $\Gamma = 92.6 \pm 1.7$ keV (S = 1.1)

$J/\psi(1S)$ DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
hadrons		(87.7 ± 0.5) %		–
virtual $\gamma \to$ hadrons		(13.50 ± 0.30) %		–
ggg		(64.1 ± 1.0) %		–
γgg		(8.8 ± 1.1) %		–
e^+e^-		(5.971 ± 0.032) %		1548
$e^+e^-\gamma$	[$hhaa$]	$(8.8 \pm 1.4) \times 10^{-3}$		1548
$\mu^+\mu^-$		(5.961 ± 0.033) %		1545

Reduce the uncertainties

1. We introduce $R_{X^+Y^-\not{E}}$, ratio of the cross sections, as

$$R_{X^+Y^-\not{E}}(W,\delta_W,m_\tau)=\frac{\sigma(W,m_\tau,\Gamma_\tau,\delta_W)}{\sigma^{\mu^+\mu^-}(W,\delta_W)}.$$

Here, $\sigma^{\mu^+\mu^-}(W,\delta_W)$ is calculated with $\bar{\sigma}^{\mu^+\mu^-}(W)=\frac{4\pi\alpha^2(1+3\alpha/4\pi)}{3W^2}$.

2. The measurement is

$$\mathcal{R}_{X^+Y^-\not{E}}(W,\delta_W,m_\tau)=\frac{N_{X^+Y^-\not{E}}}{N_{\mu^+\mu^-}}.$$

Fit approach

1. The least squares method

$$\chi^2 = \sum_i \left(\frac{\mathcal{R}_i^{\rm data} - \hat{\mathcal{R}}_i(m_\tau)}{\Delta \mathcal{R}_i^{\rm data}} \right)^2 .$$

2. $\hat{\mathcal{R}}_i(m_\tau)$ is the theoretical fit function with J_τ. The expected m_τ can be determined from the minimum value of χ^2.
3. To quantify the significance of the J_τ, another fit is performed by excluding the $\bar{\sigma}^{J_\tau}$ in $\hat{\mathcal{R}}_i$. This leads to a new minimum value $\chi^2_{\rm without\ J_\tau}$ at a new τ mass.
4. The significance of the J_τ atom can be calculated from $\Delta\chi^2_{J_\tau} = \chi^2_{\rm without\ J_\tau} - \chi^2$.

Determine energy points

1. The least squares method

$$\chi^2 = \sum_{i=1}^{3} \chi_i^2 = \sum_{i=1}^{3} \left(\frac{\mathcal{R}_i^{\mathrm{data}} - \hat{\mathcal{R}}_i(m_\tau)}{\Delta \mathcal{R}_i^{\mathrm{data}}} \right)^2 ,$$

2. Where $\mathcal{R}_i^{\mathrm{data}} = \frac{N_{X^+Y^-\not{E},\, i}^{\mathrm{data}}}{N_{\mu^+\mu^-,\, i}^{\mathrm{data}}}$ and $\Delta\mathcal{R}_i^{\mathrm{data}}$ is its statistical uncertainty (the systematic uncertainty is discussed below).
3. The values of $\frac{\chi_i^2}{\mathcal{L}_i}$ are relatively large at $W = 3552.56$ and 3555.83 MeV.
4. An additional energy point of 3549.00 MeV is needed to obtain the whole lineshape of the $e^+e^- \to X^+Y^-\not{E}$ cross section.

Numbers of the events

TABLE III: Numbers of $e^+e^- \to X^+Y^-\not{E}$ and $\mu^+\mu^-$ events and their statistical uncertainties in the pseudoexperiments with $m_\tau = m_\tau^{\mathrm{PDG}}$.

i	$\mathcal{L}_i/\mathrm{fb}^{-1}$	W_i/MeV	$N^{\mathrm{data}}_{X^+Y^-\not{E},\,i}$	$N^{\mathrm{data}}_{\mu^+\mu^-,\,i}$
1	5	3549.00	$0.1^{+1.2}_{-0.1}$	$(1.1764 \pm 0.0003) \times 10^7$
2	500	3552.56	$(8.772 \pm 0.009) \times 10^5$	$(1.17394 \pm 0.00003) \times 10^9$
3	1000	3555.83	$(2.4052 \pm 0.0005) \times 10^7$	$(2.34331 \pm 0.00005) \times 10^9$

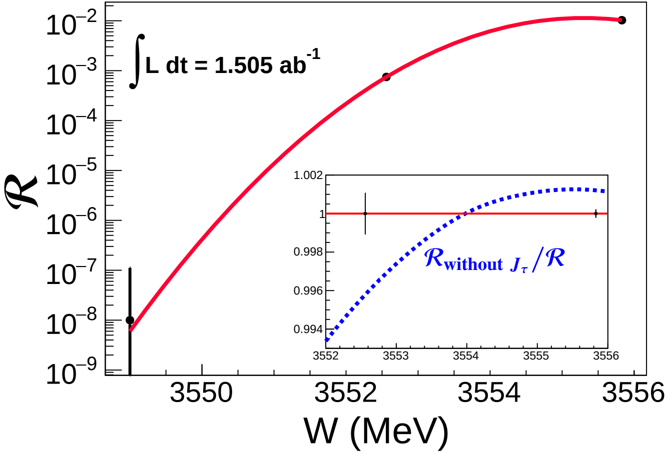
Determine χ^2 and m_τ

1. A least-square fit is applied

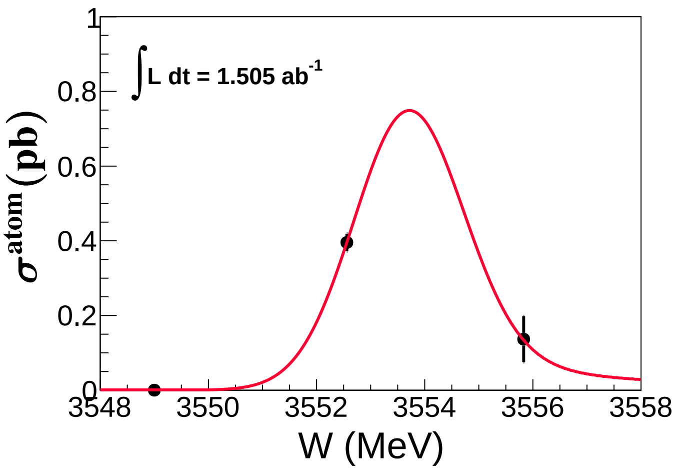
$$\chi^2 = \sum_{i=1}^{3} \left(\frac{\mathcal{R}_i^{\mathrm{data}} - \hat{\mathcal{R}}_i(m_\tau)}{\Delta \mathcal{R}_i^{\mathrm{data}}} \right)^2 ,$$

2. Where $\mathcal{R}_i^{\mathrm{data}} = \frac{N_{X^+Y^-\not{E},\,i}^{\mathrm{data}}}{N_{\mu^+\mu^-,\,i}^{\mathrm{data}}}$ and $\Delta\mathcal{R}_i^{\mathrm{data}}$ is its statistical uncertainty.
3. And $\hat{\mathcal{R}}_i(m_\tau)$ is the expected ratio at the τ mass m_τ to be determined from the fit.

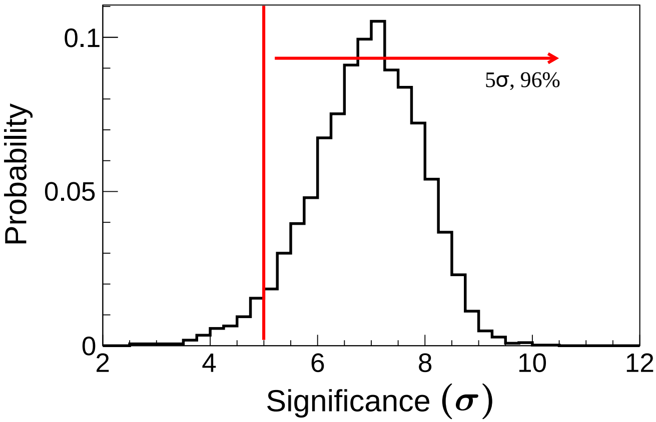
Ratio of the events



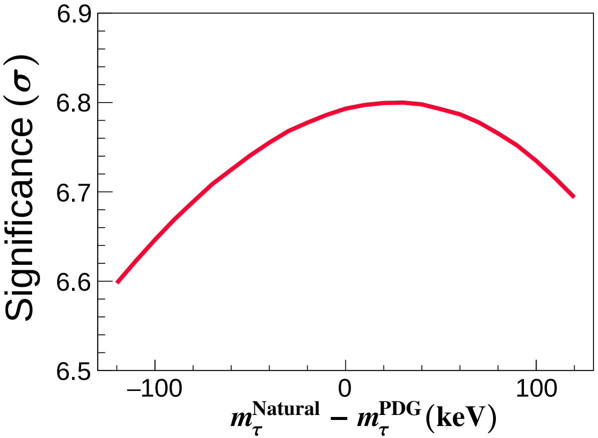
The cross section of J_τ



The statistical significance distribution in 10^5 sets pseudoexperiments



The significance of $J_\tau(nS)$ as a function of $m_\tau^{\mathrm{Natural}} - m_\tau^{\mathrm{PDG}}$.



The significance of $J_\tau(nS)$ in 10^5 sets pseudoexperiments

1. The average value of χ^2/ndf is 0.7/2 with $J_\tau(nS)$, and 51/2 without $J_\tau(nS)$.
2. Considering the systematic uncertainties, the average signal significance of J_τ is 6.7σ, which is 6.8σ without systematic uncertainties.
3. These data samples correspond to 350 (175) days' runtime at the STCF(SCTF).
4. If the δ_W is reduced to $0.1\mathrm{MeV}$, the required integrated luminosity is only 66 fb^{-1}.

m_τ

1. With these data samples, a high precision τ mass is obtained

$$m_\tau = (1\,776\,860.00 \pm 0.25\,(\mathrm{stat.}) \pm 0.99\,(\mathrm{syst.}))\ \mathrm{keV}.$$

2. The fit with the $J_\tau(nS)$ contribution removed gives a shift of -4 keV relative to the nominal fit with both the bound state and continuum contributions.

The systematic uncertainties σ_{m_τ}

1. The uncertainty of the energy scale W is estimated according to the VEPP-4M, which had a characteristic uncertainty of 1.5 keV in the beam energy in the $\psi(2S)$ mass scan (hep-ex/0306050). The uncertainty of W_2 (W_3) is estimated to be $1.5\sqrt{2} = 2.12$ keV, leading to 0.72 (0.35) keV in σ_{m_τ}.
2. σ_{m_τ} from energy spread and energy scale are 16 keV and $^{+22}_{-86}$ keV from BESIII (1405.1076), and 25 keV and 40 keV from KEDR (JETP Lett. 85 (2007) 347-352). Take the maximum ratio of $16/22 \sim 0.73$, leading to $0.73 \times \sqrt{0.72^2 + 0.35^2}$ =0.59 keV in σ_{m_τ}.
3. $\varepsilon_{X^+Y^-\not{E}} = (8.0 \pm 0.2)\%$ lead to 0.04 keV in σ_{m_τ}.
4. By exchanging the NLO correction with the NNLO correction in the calculation of the $e^+e^- \to X^+Y^-\not{E}$ cross sections, which is included in 0.07 keV in σ_{m_τ} due to the theoretical accuracy.

The systematic uncertainties σ_{m_τ}

TABLE IV: The systematic uncertainties of the m_τ (σ_{m_τ}) in keV.

Sources	σ_{m_τ}/keV
Energy scale of W_2	0.72
Energy scale of W_3	0.35
Energy spread δ_W	0.59
Efficiency	0.04
Theory	0.07
Systematic uncertainties	0.99

Summary

1. We show that the $\tau^+\tau^-$ atom can be observed with a significance larger than 5σ with a $1.5\ \mathrm{ab}^{-1}$ data sample at STCF or SCTF, by measuring the cross section ratio of the processes $e^+e^- \to X^+Y^-\not{E}$ and $e^+e^- \to \mu^+\mu^-$.
2. With the same data sample, the τ lepton mass can be measured with a precision of 1 keV, a factor of 100 improvement over the existing world best measurements.
3. We propose to measure the relative rate $\mathcal{R} = \frac{N_{X^+Y^-\not{E}}}{N_{\mu^+\mu^-}}$ rather than the absolute cross section so that the uncertainties are controlled at a low level since those in VP, ISR, and luminosity determinations are canceled.

Thank you for your listening!