Search for the Electric Dipole Moment of the Tau Lepton at the Super Tau-Charm Facility

Xulei Sun[1] Yongcheng Wu[2] Xiaorong Zhou[1]
[1]University of Science and Technology of China
[2]Nanjing Normal University

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Electric Dipole Moment (EDM)

EDM: Current strongest CP violation test.

Non-zero EDM ⇒ Violation of Parity (P) and Time (T) reversal symmetry

Under the condition of CPT symmetry conservation, T violation ⇒ CP violation ⇒ **matter-antimatter asymmetry**[1]

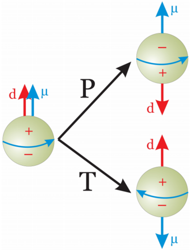

Figure 1: Non-zero EDM ⇒ P, T violation $\overset{\text{CPT}}{\Rightarrow}$ CP violation

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[1] K. Kleinknecht, Symmetries, in: Uncovering CP Violation, Springer Tracts in Modern Physics, vol. 195, Springer, Berlin, Heidelberg, 2003.

Electric Dipole Moment (EDM)

Table 1: Theoretical and experimental values of the intrinsic EDM of τ

Source	EDM	Description
Standard Model	$10^{-37} e\,\mathrm{cm}$	Insufficient to explain the observed asymmetry
New Physics Models	$10^{-19} e\,\mathrm{cm}$	Promising for precise experimental measurement to discover New Physics
Current Most Precise Experiment[2]	Upper limit $10^{-17} e\,\mathrm{cm}$	Obtained by the Belle experiment through the study of $e^+e^- \to \tau^+\tau^-$ at the KEKB collider

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[2] Belle Collaboration, An improved search for the electric dipole moment of the τ lepton, J. High Energy Phys. 2022 (2022) 1-17.

Super Tau-Charm Facility (STCF)

STCF can produce a large amount of data in the **tau-charm region** (3.5×10^9, 1.7×10^9 τ pairs at $\sqrt{s} = 4.2\,\mathrm{GeV}$, $7.0\,\mathrm{GeV}$ per year).[3]

Compared to the high-energy region of the LEP experiment and the $10\,\mathrm{GeV}$ of the Belle experiment, in the **tau-charm region**:

- Fewer radiative τ pair events;
- Accurate hadron production simulations;
- High τ production rate;
- Precise charged particle identification and photon reconstruction
  $\Rightarrow$ high τ reconstruction efficiency.

The first measurement of τ EDM in this region!

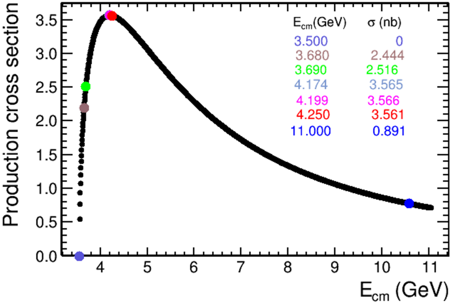

Figure 2: τ pair production cross section at different CMEs

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[3] M. Achasov, X.C. Ai, L.P. An, et al., STCF conceptual design report (Volume 1): Physics & detector, Front. Phys. 19 (2024) 14701.

EDM Measurement

CP-violating effective Lagrangian for τ-pair production in the vertex:

$$\mathcal{L}_{\mathrm{CP}} = -i d_\tau^{\mathrm{NP}} \bar{\tau} \sigma_{\mu\nu} \gamma_5 \tau F^{\mu\nu} \left(d_\tau^{\mathrm{NP}} = d_\tau - d_\tau^{\mathrm{SM}} \right) \tag{1}$$

Squared spin density matrix for $e^+(\boldsymbol{p}) e^-(-\boldsymbol{p}) \rightarrow \tau^+(\boldsymbol{k}, \boldsymbol{S}_+) \tau^-(-\boldsymbol{k}, \boldsymbol{S}_-)$:

$$\mathcal{M}^2_{\mathrm{prod}} = \mathcal{M}^2_{\mathrm{SM}} + \mathrm{Re}\left(d_\tau\right) \mathcal{M}^2_{\mathrm{Re}} + \mathrm{Im}\left(d_\tau\right) \mathcal{M}^2_{\mathrm{Im}} + \left|d_\tau\right|^2 \mathcal{M}^2_{d^2} \tag{2}$$

$$\begin{aligned} \mathcal{M}^2_{\mathrm{SM}} = \frac{e^4}{k_0^2} \Big[& k_0^2 + m_\tau^2 + \left|\boldsymbol{k}^2\right| (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{p}})^2 - \boldsymbol{S}_+ \cdot \boldsymbol{S}_- |\boldsymbol{k}|^2 \left(1 - (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{p}})^2\right) \\ & + 2\left(\hat{\boldsymbol{k}} \cdot \boldsymbol{S}_+\right)\left(\hat{\boldsymbol{k}} \cdot \boldsymbol{S}_-\right)\left(|\boldsymbol{k}|^2 + \left(k_0 - m_\tau\right)^2 (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{p}})^2\right) + 2k_0^2 \left(\hat{\boldsymbol{p}} \cdot \boldsymbol{S}_+\right)\left(\hat{\boldsymbol{p}} \cdot \boldsymbol{S}_-\right) \\ & - 2k_0 \left(k_0 - m_\tau\right)(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{p}})\left(\left(\hat{\boldsymbol{k}} \cdot \boldsymbol{S}_+\right)\left(\hat{\boldsymbol{p}} \cdot \boldsymbol{S}_-\right) + \left(\hat{\boldsymbol{k}} \cdot \boldsymbol{S}_-\right)\left(\hat{\boldsymbol{p}} \cdot \boldsymbol{S}_+\right)\right) \Big] \end{aligned} \tag{3}$$

$$\mathcal{M}^2_{\mathrm{Re}} = 4\frac{e^3}{k_0}|\boldsymbol{k}| \left[-\left(m_\tau + \left(k_0 - m_\tau\right)(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{p}})^2\right)\left(\boldsymbol{S}_+ \times \boldsymbol{S}_-\right) \cdot \hat{\boldsymbol{k}} + k_0 (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{p}})\left(\boldsymbol{S}_+ \times \boldsymbol{S}_-\right) \cdot \hat{\boldsymbol{p}} \right]$$

$\boldsymbol{p}$: e^+ momentum; $\hat{\boldsymbol{k}}$, $\hat{\boldsymbol{p}}$: unit vectors.[4]

⇒ It is important to **reconstruct τ momentum $\boldsymbol{k}$ and spin $\boldsymbol{S}_+$**.

[4] Belle Collaboration, Search for the electric dipole moment of the τ lepton, Phys. Lett. B 551 (2003) 16-26.

EDM Measurement

For $\tau \to \rho\nu_\tau \to \pi\pi^0\nu_\tau$, **spin $\boldsymbol{S}_\pm$ can be determined by momenta**[5]:

$$\boldsymbol{S}_\pm = \mp \frac{1}{(k_\pm H^\pm) - m_\tau^2 (p_{\pi^\pm} - p_{\pi^0})^2} \left(\mp H_0^\pm \boldsymbol{k} + m_\tau \boldsymbol{H}^\pm + \frac{\boldsymbol{k}\left(\boldsymbol{k} \cdot \boldsymbol{H}^\pm\right)}{k_0 + m_\tau} \right) \tag{4}$$

$$\left(H^\pm\right)^\nu = 2\left(p_{\pi^\pm} - p_{\pi^0}\right)^\nu \left(p_{\pi^\pm} - p_{\pi^0}\right)^\mu \left(k_\pm\right)_\mu + \left(p_{\pi^\pm} + p_{\pi^0}\right)^\nu \left(p_{\pi^\pm} - p_{\pi^0}\right)^2$$

k_0: τ energy; $k_\pm = (k_0, \pm\boldsymbol{k})$; $H^\pm = \left(H_0^\pm, \boldsymbol{H}^\pm\right)$: four-vector; $k_\pm H^\pm$: four-vector product; $p_{\pi^\pm}$, p_{π^0}: four-momenta.

$\Rightarrow$ It is important to **reconstruct $\boldsymbol{k}$**!

[5] A. Posthaus, P. Overmann, A method to determine the electroweak mixing angle from Z decays to tau leptons using optimal observables, J. High Energy Phys. 1998 (1998) 001.

EDM Measurement

Optimal observables (matrix element method):

$$\mathcal{O}_{\mathrm{Re}} = \frac{\mathcal{M}^2_{\mathrm{Re}}}{\mathcal{M}^2_{\mathrm{SM}}}, \quad \mathcal{O}_{\mathrm{Im}} = \frac{\mathcal{M}^2_{\mathrm{Im}}}{\mathcal{M}^2_{\mathrm{SM}}} \tag{5}$$

It can be proven[6]:

$$\langle \mathcal{O}_{\mathrm{Re}} \rangle = a_{\mathrm{Re}} \cdot \mathrm{Re}\,(d_\tau) + b_{\mathrm{Re}}, \quad \langle \mathcal{O}_{\mathrm{Im}} \rangle = a_{\mathrm{Im}} \cdot \mathrm{Im}\,(d_\tau) + b_{\mathrm{Im}} \tag{6}$$

Electron-positron collision ⇒ Final state particle parameters ⇒ τ momentum & spin ⇒ Optimal observables ⇒ EDM

[6] Belle Collaboration, An improved search for the electric dipole moment of the τ lepton, J. High Energy Phys. 2022 (2022) 1-17.

EDM Measurement

Optimal channel: $\rho\rho$ mode ($e^+e^- \to \tau^+\tau^-, \tau^\pm \to \pi^\pm\pi^0\nu_\tau$)

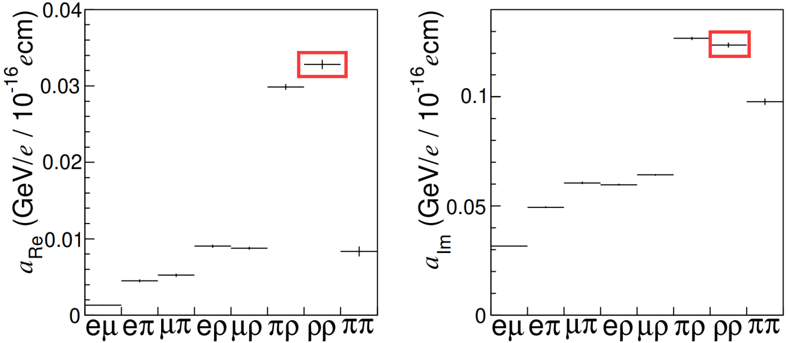

Figure 3: Sensitivity of τ EDM in different decay channels[7]

[7] Belle Collaboration, An improved search for the electric dipole moment of the τ lepton, J. High Energy Phys. 2022 (2022) 1-17.

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Event Selection

Signal:

$$e^+e^- \to \tau^+\tau^- \; (\tau^\pm \to \pi^\pm\pi^0\nu_\tau, \pi^0 \to 2\gamma)$$

branch fraction: $(25\%)^2 = 6.2\%$

Main background decays:

$$\tau^\pm \xrightarrow{17.8\%} \nu_\tau e^\pm \nu_e, \quad \tau^\pm \xrightarrow{17.4\%} \nu_\tau \mu^\pm \nu_\mu,$$

$$\tau^\pm \xrightarrow{10.8\%} \pi^\pm \nu_\tau, \quad \tau^\pm \xrightarrow{9.3\%} \pi^\pm\pi^0\pi^0\nu_\tau$$ [8]

[8] R.L. Workman, et al. (Particle Data Group), Review of Particle Physics, PTEP 2022 (2022) 083C01.

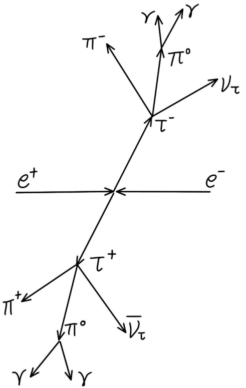

Figure 4: The production and signal decay of τ

Event Selection

Table 2: Event selection results for $5,567,300$ simulated events

Step		Percentage of previous step		Signal purity
		Inclusive events	Signal events	
0	Total events	-	-	**6.2%**
1	Number of charged tracks = 2, total charge = 0	58.3%	76.5%	8.1%
2	Number of photons = 4	7.2%	23.7%	26.7%
3	Number of π^+ = 1, Number of π^- = 1	81.8%	92.1%	30.0%
4	Passed the particle pairing	25.2%	52.3%	62.5%
5	Passed event-level machine learning selection	57.9%	73.4%	79.3%
6	Passed the τ momentum reconstruction	97.0%	97.7%	**80.0%**

Overall efficiency: 0.49%. Signal efficiency: 6.3%. Signal purity: 80.0%.

Main background: multi-π^0 processes, such as $\tau^\pm \to \pi^\pm\pi^0\pi^0\nu_\tau$ (about 14%).

Event Selection

STCF $\sqrt{s} = 4.2\,\mathrm{GeV}$: 3.5×10^9 τ pairs (**per year**)

$\Rightarrow$ signal events ($\rho\rho$ mode): 2.2×10^8

$\Rightarrow$ signal yield after selection: $\mathbf{1.4 \times 10^7}$ (efficiency: 6.3%, purity: 80.0%)

Comparison: Belle experiment $\sqrt{s} = 10\,\mathrm{GeV}$ (**up to 2022**)

$\Rightarrow$ signal events ($\rho\rho$ mode): 5.2×10^7

$\Rightarrow$ signal yield after selection: $\mathbf{3.3 \times 10^6}$ (efficiency: 6.3%, purity: 82.4%)[9]

The current feasibility study shows that the selection efficiency is comparable to that of the B factory.

After 10 years of operation, the STCF will collect 1.4×10^8 tau pairs after reconstruction, which is **2 orders of magnitude higher** than Belle.

The event selection at STCF can be further optimized with the vertex detector.

[9] Belle Collaboration, An improved search for the electric dipole moment of the τ lepton, J. High Energy Phys. 2022 (2022) 1-17.

Particle Pairing

How are final-state photons **paired** with the corresponding π^+ or π^-? (i.e. Which τ does each photon come from?)

Loop all possible pairing schemes for **kinematic fitting**.

Kinematic **constraints**:

- Energy-momentum conservation;
- Mass constraints of the intermediate states τ and π^0.

Revise the known quantities according to constraints, solve for the unknown quantities—the neutrino (ν) energy-momentum, and obtain χ^2—the deviation of fitted and original values over the error.

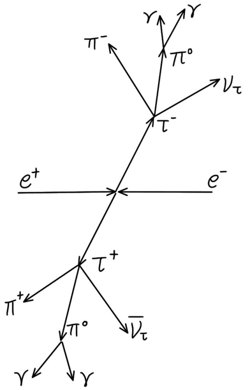

Figure 5: The production and signal decay of τ

Particle Pairing

- Correct pairing: χ^2 and $p_\nu = p_{\nu_{(1)}} + p_{\nu_{(2)}}$ are relatively small;
- Incorrect pairing: probably constraint equations are unsolvable or solutions are non-physical $\Rightarrow$ larger χ^2 or p_ν.

Filter: $\chi^2 < 10$ & $p_\nu < \frac{E^{\text{Total}}}{2}$. **Pairing correct rate: $\mathbf{82.5\%}$** (for signal events).

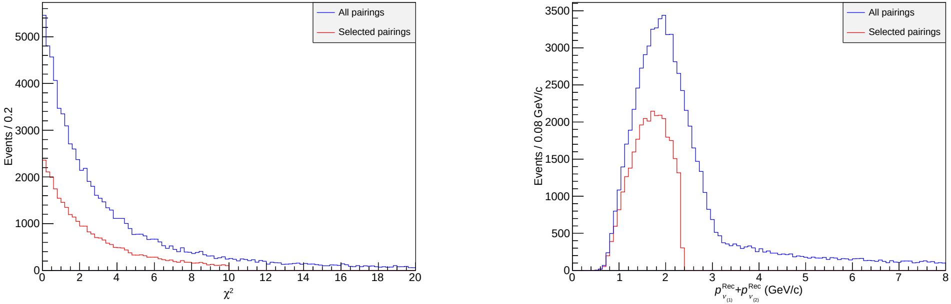

Figure 6: Distribution of χ^2 and $p_\nu = p_{\nu_{(1)}} + p_{\nu_{(2)}}$ for selected pairings and all pairings.

Event-level Machine Learning Selection

Input variables: momenta of π^+, π^-, and four photons (in order).

BDTG Cut $= 0.2$, retaining about 73% of signal events and 32% of background events.

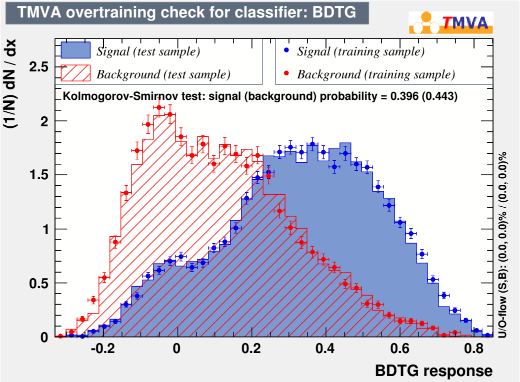

Figure 7: BDTG response

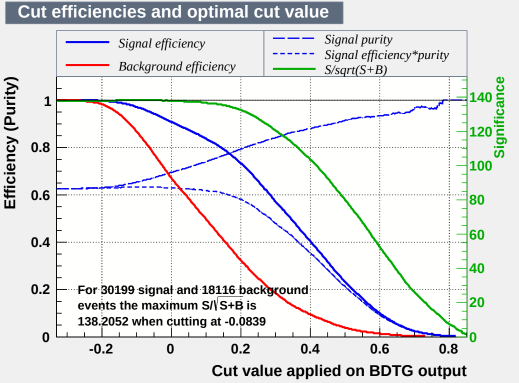

Figure 8: BDTG cut efficiency

τ Momentum Reconstruction

In the center-of-mass frame:

$$\boldsymbol{p}^{\text{Total}} = \mathbf{0} \tag{7}$$

$$E_{\tau^+} = E_{\tau^-} = E_\tau = \frac{E^{\text{Total}}}{2} \tag{8}$$

The angle $\theta_\pm$ between the vectors $\boldsymbol{p}_{\tau^\pm}$ and $\boldsymbol{p}_{h^\pm} = \boldsymbol{p}_{\pi^\pm} + \boldsymbol{p}_{\pi^0}$[10]:

$$\cos\theta_\pm = \frac{\gamma x_\pm - \left(1 + r_\pm^2\right)/2\gamma}{\beta\sqrt{\gamma^2 x_\pm^2 - r_\pm^2}} \tag{9}$$

$$x_\pm = \frac{E_{h^\pm}}{E_\tau} = \frac{E_{\pi^\pm} + E_{\pi^0}}{E_\tau}, \quad r_\pm = \frac{m_\pi}{m_\tau}, \quad \gamma = \frac{E_\tau}{m_\tau} \tag{10}$$

[10] J.H. Kühn, Tau kinematics from impact parameters, Phys. Lett. B 313 (1993) 458.

τ Momentum Reconstruction

Relative deviation of **transverse momentum** $p_T = \sqrt{p_x^2 + p_y^2}$: FWHM 0.10

Opening angle: Peak 7°, RMS 10°

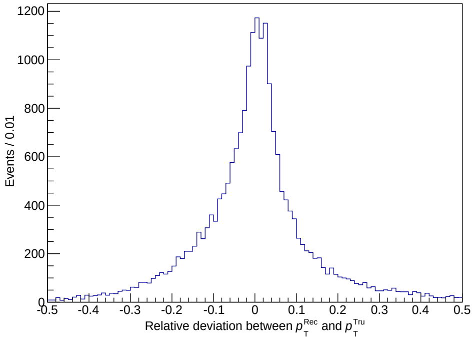

Figure 9: Relative deviation between the reconstructed and truth transverse momentum

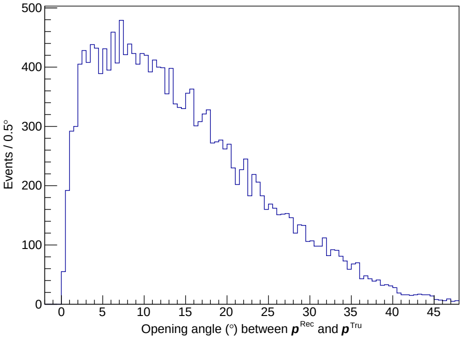

Figure 10: Opening angle between the reconstructed and truth momentum

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Optimal Observable

Use MadGraph with custom UFO files to set various EDMs and simulate the production and decay of τ leptons for different EDM d_τ with fast simulation.

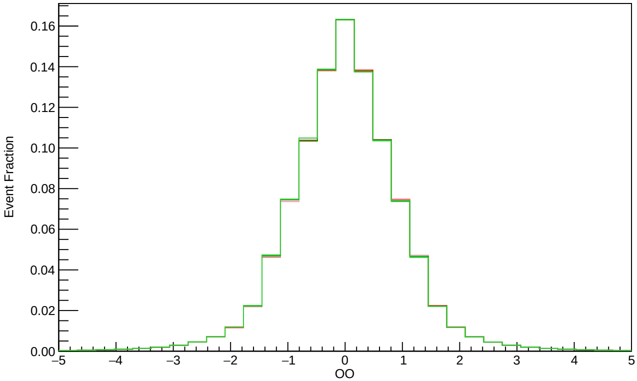
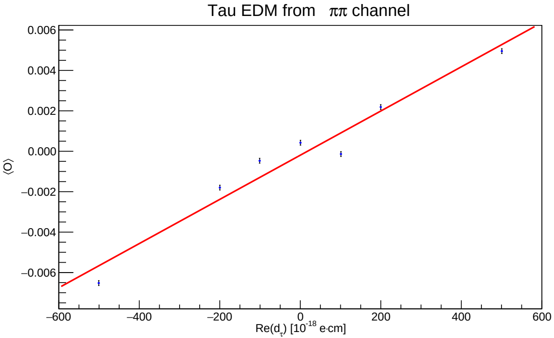


Figure 11: Distribution of $\mathcal{O}_{\rm Re}$ for different d_τ (Red: $d_\tau < 0$; Green: $d_\tau > 0$)

Optimal Observable



uncertainty of $\langle \mathcal{O}_{\mathrm{Re}} \rangle$: 3.818×10^{-5}
slope: $1.093 \times 10^{13}/(e\,\mathrm{cm})$

Figure 12: Relationship between $\langle \mathcal{O}_{\mathrm{Re}} \rangle$ and $\mathrm{Re}(d_\tau)$

Estimated sensitivity (after 10 years of operation at STCF):

- From $\pi\pi$ channel: $\boldsymbol{|d_\tau| < 3.49 \times 10^{-18}\, e\,\mathrm{cm}}$.
- From $\rho\rho$ channel: is still running due to large statistics.

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Summary and Outlook

Summary:

- Signal events were selected through **machine learning**.
  ⇒ Signal efficiency: 6.3%; Signal purity increased from 6.2% to 80.0%
  ⇒ STCF's statistics will be **2 orders of magnitude higher** than Belle's.
- **Kinematic fitting** was introduced to pair particles.
  ⇒ Correct pairing rate: $\mathbf{82.5\%}$ (for signal events)
- Symbolic computation was employed to **derive τ momentum**.
- **Optimal observable** and its relationship with the EDM were obtained.
  ⇒ Estimated sensitivity from $\pi\pi$ channel: $\boldsymbol{|d_\tau| < 3.49 \times 10^{-18}\, e\,\mathrm{cm}}$

Plan:

- Finish the running in the **$\rho\rho$ channel**.
- Use the **vertex detector** at STCF to improve the particle pairing and τ momentum reconstruction.

Thank you!