# Feasibility study of radiative decay $\tau \to \pi\gamma\nu_\tau$ at STCF

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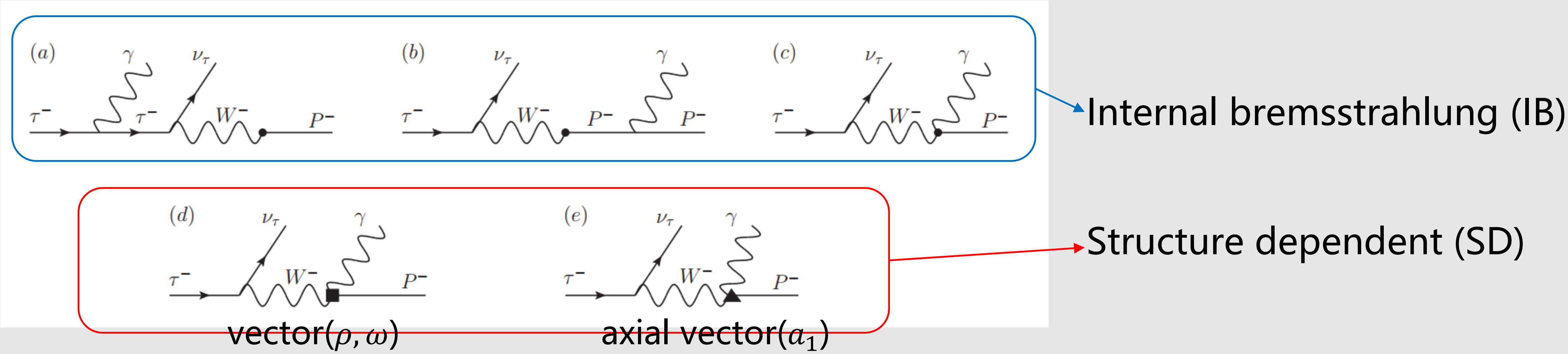
# Motivation




- None of the radiative tau decays have been measured before.
- Radiative correction becomes crucial for precision tau physics ,such as the lepton universality test in hadronic tau decays, calculation of muon g-2 using tau decays.
- Search for tau radiative decay $\tau \to \pi\gamma v_\tau$ .
- Challenge : signal has much lower branching ratio than backgrounds ($Br_{(\tau^- \to \pi^- \gamma v_\tau)}$/ $Br_{(\tau^- \to \pi^- v_\tau)} \sim 10^{-2}$ .
- ML technique is being explored to distinguish signal from backgrounds.

# Signal modelling

- Model DIY
- Theoretical calculation
- Validation

# Signal generator



Boss Version : 7.0.3

BesEvtGen-00-03-90

## 4.11 DIY

**Author:**Rong-Gang Ping
**Usage:**
BrFr daughter 1 daughter 2... DIY;

**Explanation:**

The model Doing It Yourself (DIY) is created for user to generate event by supplying decay amplitude. The DIY model is useful for the user to generate event using partial wave analysis results, or implementing coherent effects in decays, or parameterizing the physical quantities in amplitude. The DIY generates events according the decay mode specified by user with pure phase space and passing the four-vector momentum to the amplitude and calculate the decay amplitude. The maximal value of the amplitude is singled out in 20,000 events. Then it uses the reject-accepted sampling method to generate event.

# Theoretic cross section


**One-meson radiative tau decays**

Zhi-Hui Guo and Pablo Roig

Phys. Rev. D **82**, 113016 – Published 28 December 2010


$$
\begin{aligned}
\frac{\mathrm{d}^2\Gamma_{IB}}{\mathrm{d}x\,\mathrm{d}y} &= \frac{\alpha}{2\pi} f_{IB}\left(x,y,r_P^2\right) \frac{\Gamma_{\tau^-\to\nu_\tau P^-}}{\left(1-r_P^2\right)^2}\,,\\
\frac{\mathrm{d}^2\Gamma_{SD}}{\mathrm{d}x\,\mathrm{d}y} &= \frac{\alpha}{8\pi}\frac{M_\tau^4}{F_P^2}\left[|F_V(t)|^2 f_{VV}\left(x,y,r_P^2\right) + 4\Re e(F_V(t)F_A^\star(t)) f_{VA}\left(x,y,r_P^2\right) + \right.\\
&\qquad \left. 4|F_A(t)|^2 f_{AA}(x,y,r_P^2)\right] \frac{\Gamma_{\tau^-\to\nu_\tau P^-}}{(1-r_P^2)^2}\,,\\
\frac{\mathrm{d}^2\Gamma_{INT}}{\mathrm{d}x\,\mathrm{d}y} &= \frac{\alpha}{2\pi}\frac{M_\tau^2}{F_P}\left[f_{IB-V}\left(x,y,r_P^2\right)\Re e(F_V(t)) + 2 f_{IB-A}\left(x,y,r_P^2\right)\Re e(F_A(t))\right] \frac{\Gamma_{\tau^-\to\nu_\tau P^-}}{\left(1-r_P^2\right)^2}
\end{aligned}
$$

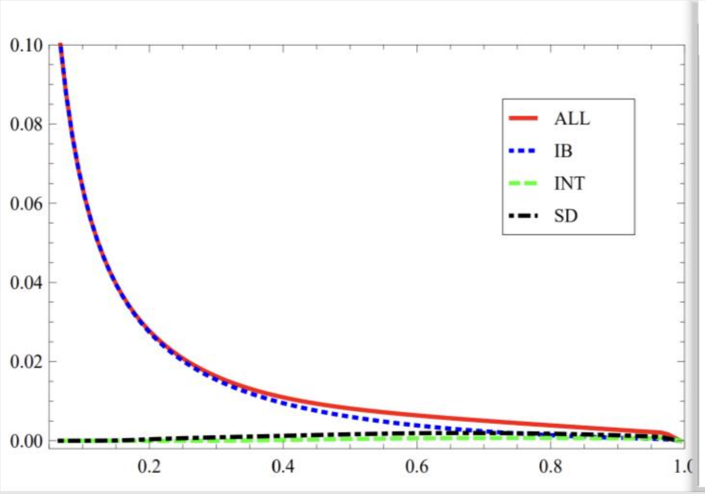
$$
\begin{aligned}
f_{IB}\left(x,y,r_P^2\right) &= \frac{\left[r_P^4(x+2) - 2r_P^2(x+y) + (x+y-1)\left(2-3x+x^2+xy\right)\right]\left(r_P^2-y+1\right)}{\left(r_P^2-x-y+1\right)^2 x^2}\\
f_{VV}\left(x,y,r_P^2\right) &= -\left[r_P^4(x+y) + 2r_P^2(1-y)(x+y) + (x+y-1)\left(-x+x^2-y+y^2\right)\right],\\
f_{AA}\left(x,y,r_P^2\right) &= f_{VV}\left(x,y,r_P^2\right),\\
f_{VA}\left(x,y,r_P^2\right) &= -\left[r_P^2(x+y) + (1-x-y)(y-x)\right]\left(r_P^2-x-y+1\right),\\
f_{IB-V}\left(x,y,r_P^2\right) &= -\frac{\left(r_P^2-x-y+1\right)\left(r_P^2-y+1\right)}{x},\\
f_{IB-A}\left(x,y,r_P^2\right) &= -\frac{\left[r_P^4 - 2r_P^2(x+y) + (1-x+y)(x+y-1)\right]\left(r_P^2-y+1\right)}{\left(r_P^2-x-y+1\right)x}.
\end{aligned}
$$

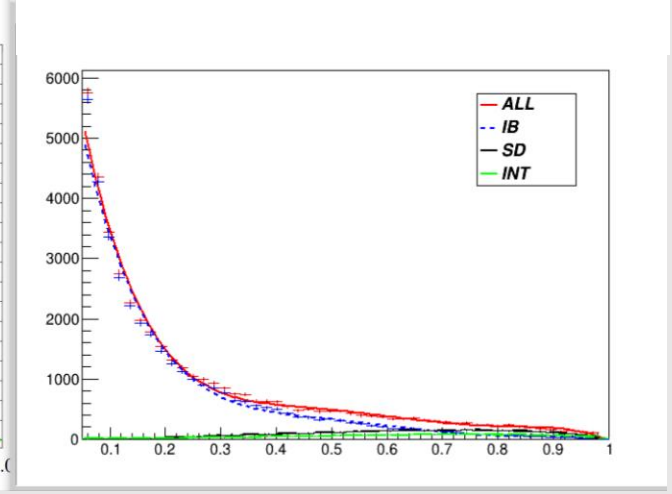
$$
\begin{aligned}
F_V^\pi(t) &= -\frac{N_C}{24\pi^2 F_\pi} + \frac{2\sqrt{2}F_V}{3F_\pi M_V}\left[(c_2-c_1-c_5)t + (c_5-c_1-c_2-8c_3)m_\pi^2\right] \times\\
&\quad \left[\frac{\cos^2\theta}{M_\phi^2}\left(1-\sqrt{2}\mathrm{tg}\theta\right) + \frac{\sin^2\theta}{M_\omega^2}\left(1+\sqrt{2}\mathrm{cotg}\theta\right)\right]\\
&\quad + \frac{2\sqrt{2}F_V}{3F_\pi M_V} D_\rho(t)\left[(c_1-c_2-c_5+2c_6)t + (c_5-c_1-c_2-8c_3)m_\pi^2\right]\\
&\quad + \frac{4F_V^2}{3F_\pi} D_\rho(t)\left[d_3 t + (d_1+8d_2-d_3)m_\pi^2\right] \times\\
&\quad \left[\frac{\cos^2\theta}{M_\phi^2}\left(1-\sqrt{2}\mathrm{tg}\theta\right) + \frac{\sin^2\theta}{M_\omega^2}\left(1+\sqrt{2}\mathrm{cotg}\theta\right)\right].
\end{aligned}
$$

$$
F_A^\pi(t) = \frac{F_V^2}{2F_\pi M_\rho^2}\left(1-\frac{2G_V}{F_V}\right) - \frac{F_A^2}{2F_\pi}D_{a_1}(t) + \frac{\sqrt{2}F_A F_V}{F_\pi M_\rho^2} D_{a_1}(t)\left(-\lambda'' t + \lambda_0 m_\pi^2\right)
$$

# Model validation

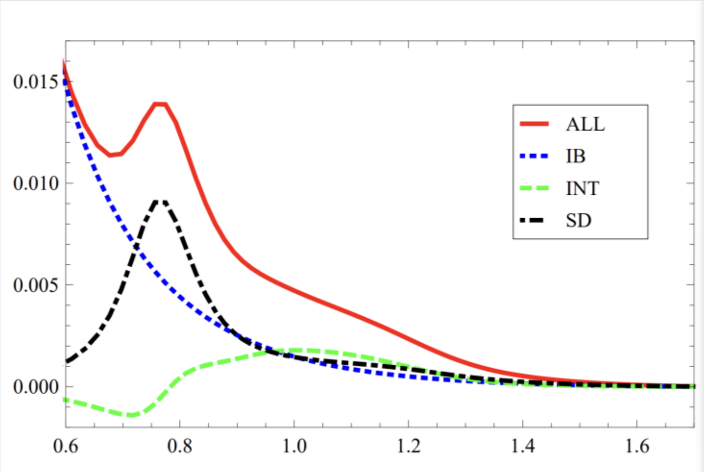
theory predict | simulation result

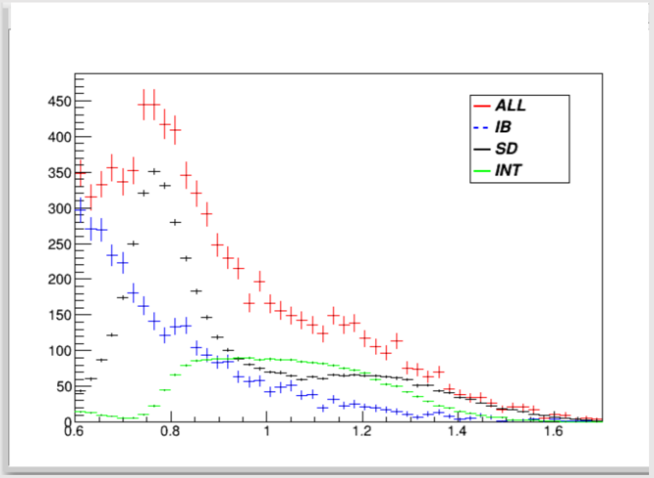



$$x := \frac{2p_\tau \cdot k}{M_\tau^2}$$

In the tau rest frame x is the photon energy Eγ




$$t = P^2 = (p_2 + k)^2$$

The variable t is the invariant mass square of the pion-photon system

# Study of event selection using BESIII detector

- Background processes
- Event selection
- Upper limit

# Inclusive MC sample at BESIII

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Boss Version : 7.0.3

$\sqrt{s}$ =4.23GeV

Luminosity($pb^{-1}$) = 1100.91

| | $\sigma(nb)$ | $N_{exp}(10^6)$ |
|---|---|---|
| di-tau | 9.86 | 10.85 |
| di-muon | 5.18 | 5.7 |
| hadronic | 20.32 | 22.37 |
| gamma gamma | 19.73 | 21.72 |
| Bhabha | 415.41 | 457.33 |

## Inclusive sample :

**di-tau** samples : tau pair generated with KKMC decay of tau with EvtGen

**di-muon** samples : generated with BABAYAGA

**Hadronic** samples : generated with LUNDARLW

# Primary selection

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Tag side : $\tau^+ \to \mu^+ \bar{\nu}_\tau \nu_\mu$

Prob side : $\tau^- \to \pi^- \gamma \nu_\tau$

**Note** : The event selection is studied with BESIII detector for now

**Criteria for selection**:

Good Mdc track = 2
(Interaction Point and Exit Angle)

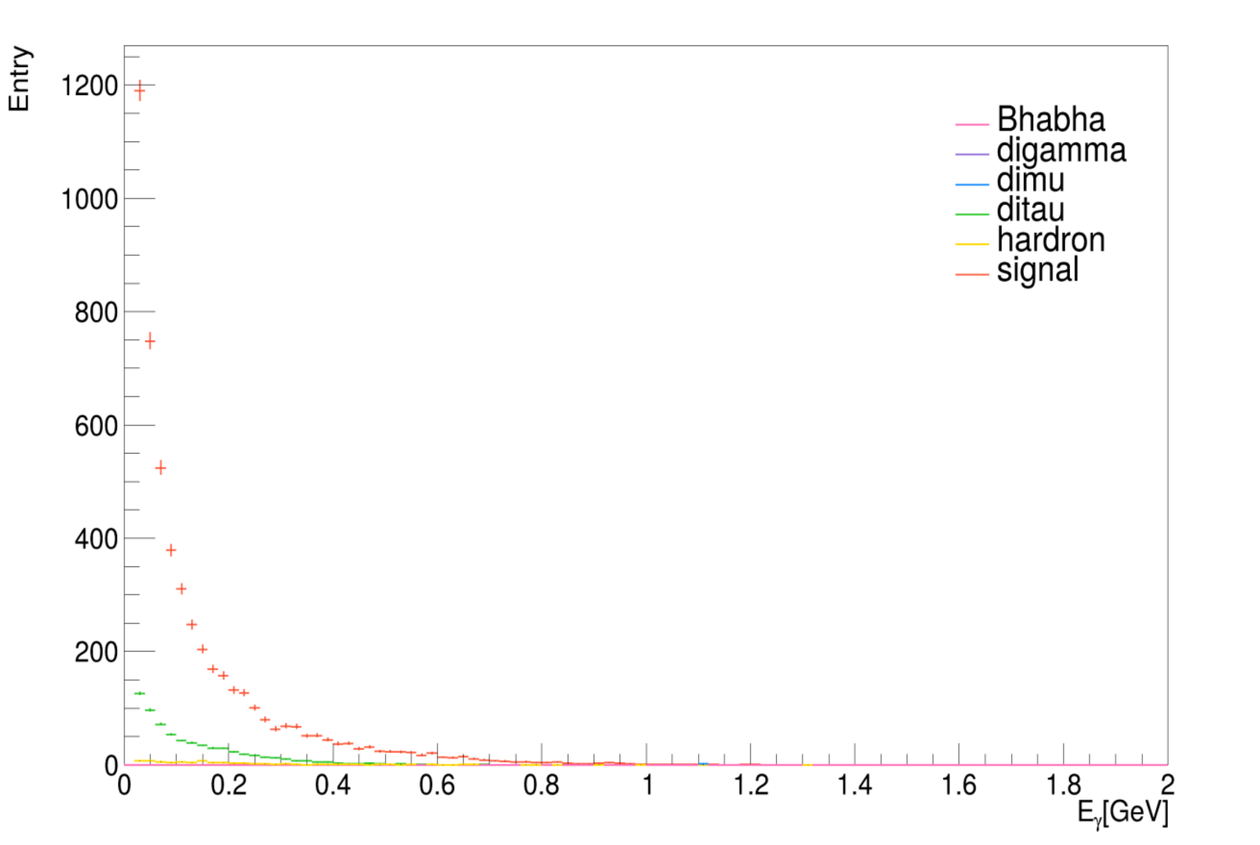
Total charge = 0

PID two track is pion and muon

Good Emc track = 2
(With Valid Shower)

# Final event selection

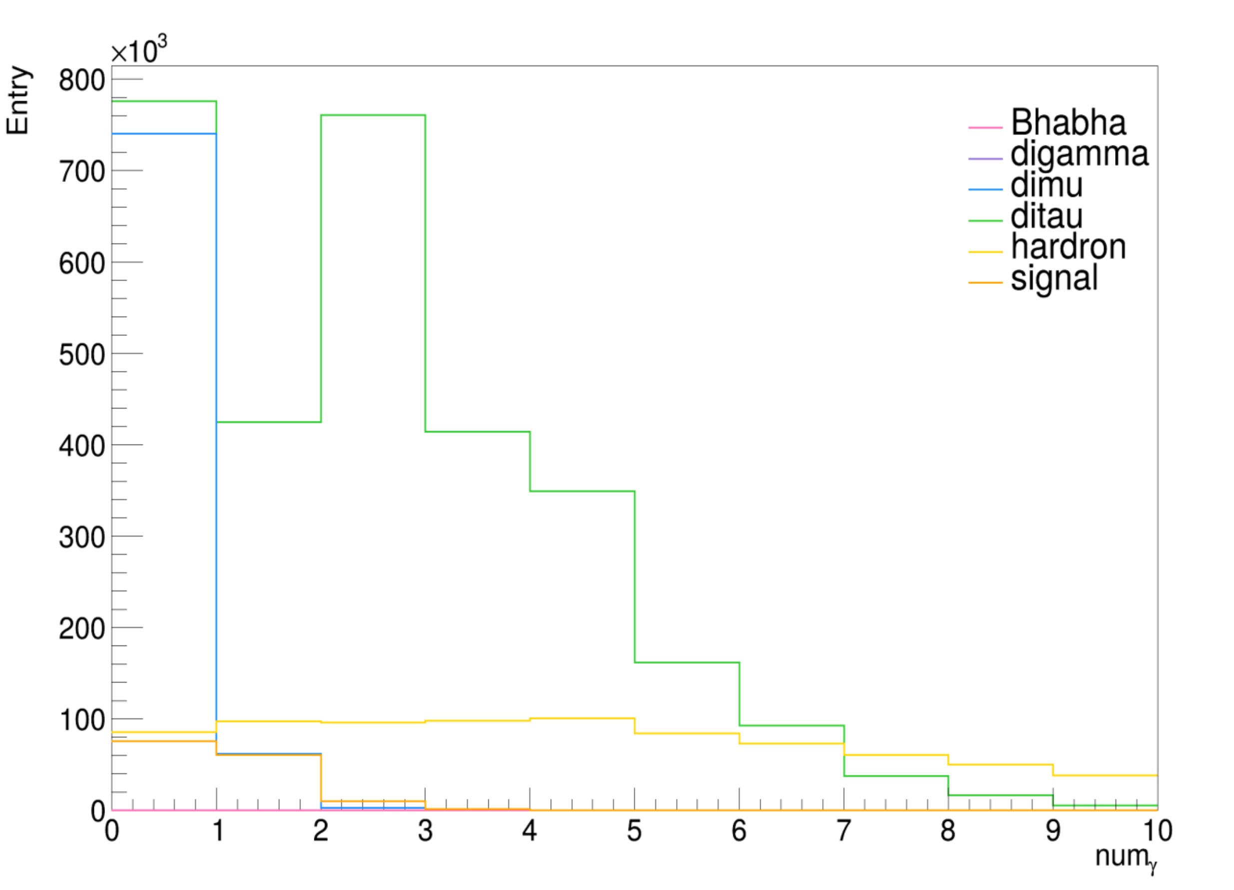
signal is scaled by 100 for better visibility




The energy distribution of photons
$E_\gamma > 0.4$ GeV

| Criteria for selection |
| --- |
| $E/p < 0.2$ |
| $Angle(\gamma,\pi) < 40$ |
| $-140 < \theta_{acop} < 140$ |
| $M^2{}_{miss} > 2\ \mathrm{GeV}^2$ |
| $N_\gamma = 1$ |
| $E_\gamma > 0.4$ GeV |
| *Not considered conjugate process* |

# Photon number constraint



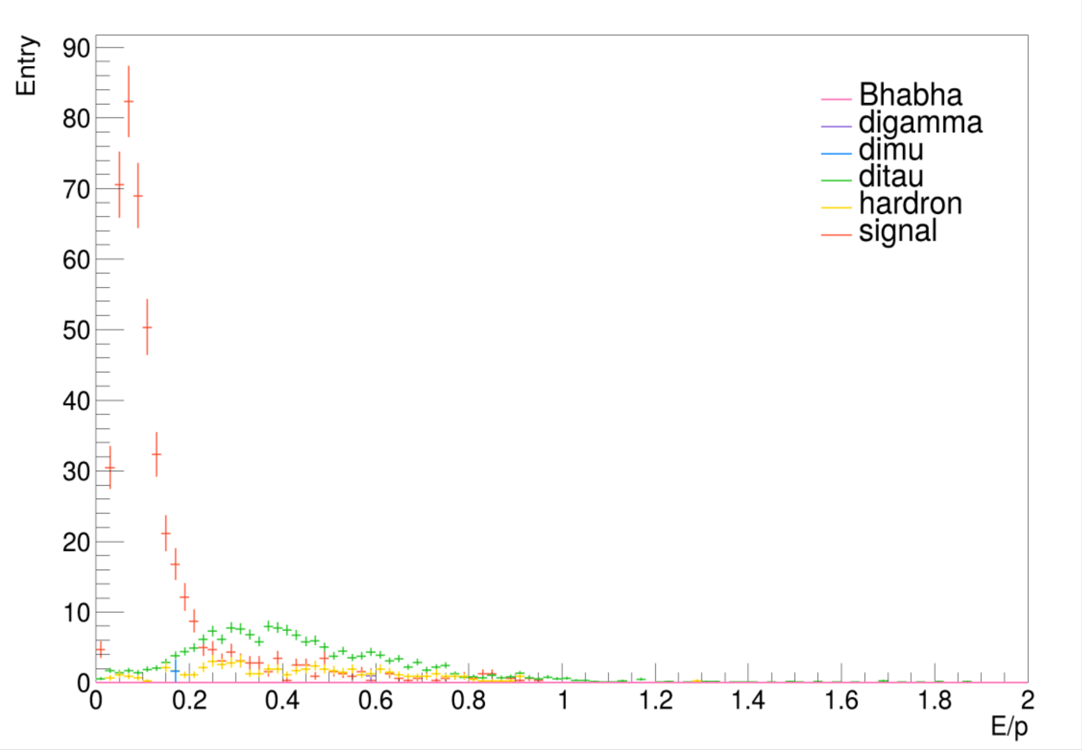

Photon number distribution

Constraint the number of photon = 1

Most of the di-muon process, di-tau process and hardronic process are removed

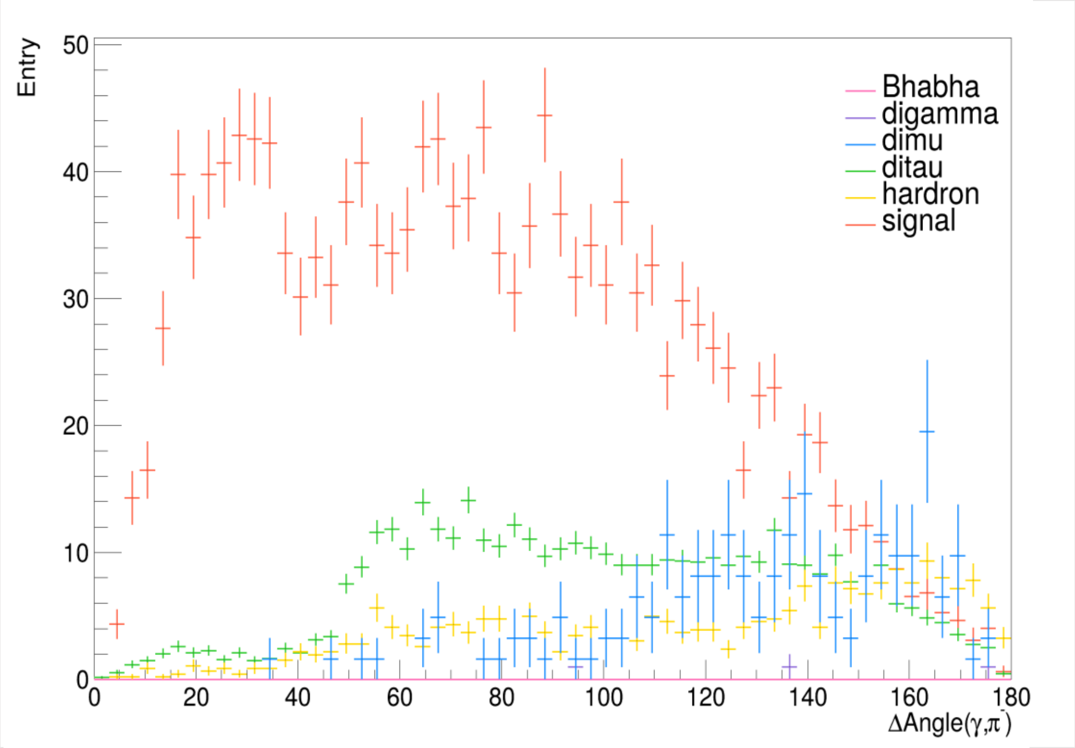
About half of the signal has been removed, and a large number of di-tau processes are still retained.

# Parameter about $\pi^-$




E/P of prob $\pi^-$ ( E is the deposited energy in EMC and p is the momentum of the track)

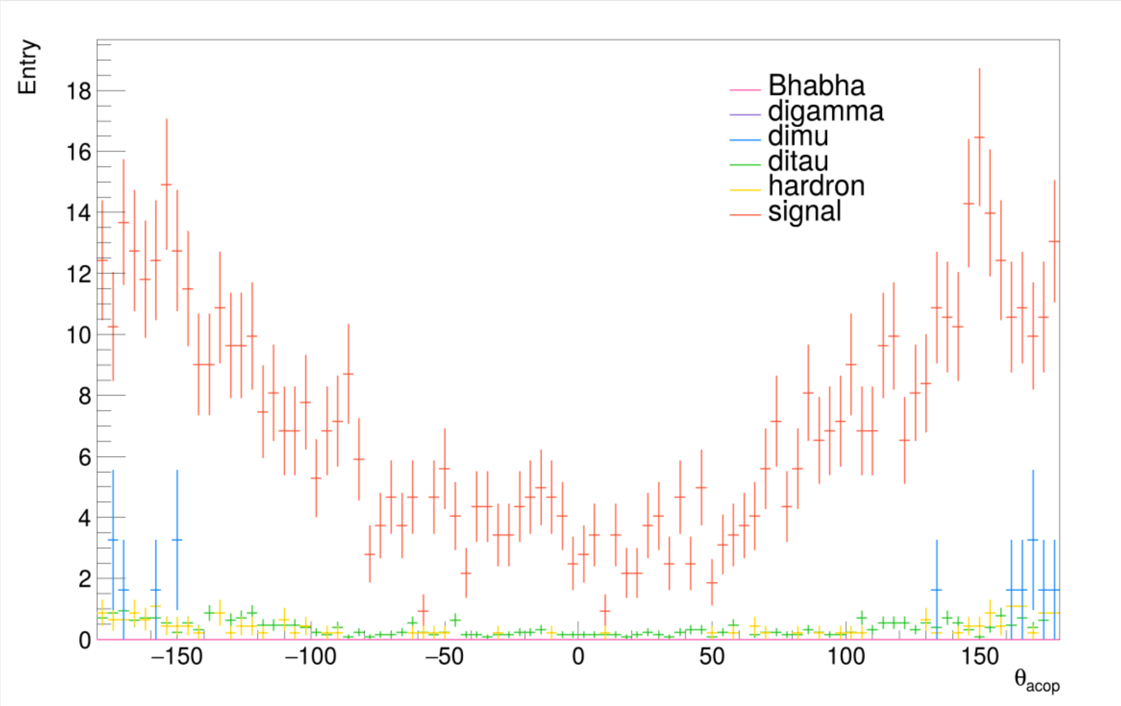
$E/p$ < 0.2



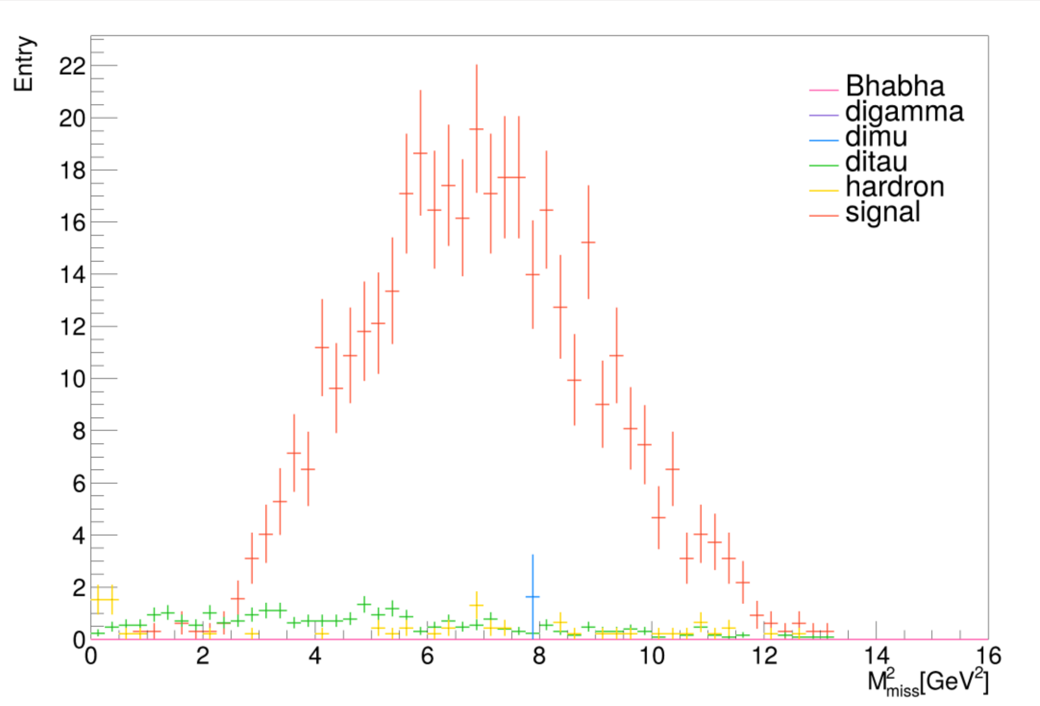

Angle between $\gamma$ and prob $\pi^-$

Angle$(\gamma,\pi)$ < 40

# Parameter about system




Acoplanarity angle (the angle between the planes formed by the momenta of two particles involved in the interaction.) -140 < $\theta_{acop}$ < 140




Missing mass squared

$$M^2_{miss} = (\sum E_{initial} - \sum E_{final})^2 - (\sum \vec{p}_{initial} - \sum \vec{p}_{final})^2$$

$$M^2_{miss} > 2\ \mathrm{GeV}^2$$

# Expected upper limit

Theoretical articles give relational formulas :

( PHYSICAL REVIEW D 103, 056017(2021) )

$$\frac{\Gamma_{(\tau^- \to \pi^- \gamma \nu_\tau)}}{\Gamma_{(\tau^- \to \pi^- \nu_\tau)}} = (1.61 \pm 0.06) \times 10^{-2}$$

The branching ratio of non-radiative process can be quoted from PDG:

$$Br_{(\tau^- \to \pi^- \nu_\tau)} = (10.82 \pm 0.05)\%$$

So upper limit on the signal can be calculated by :

$$\mu_{sig}^{up} = Br_{(measured)} \,/\, Br_{(\text{theoretical})}$$

$$= Br_{sig}^{up} \,/\, (Br_{(\tau^- \to \pi^- \nu_\tau)} \times 1.6 \times 10^{-2})$$

$\mu_{sig}^{up}$ is the signal strength

# Physics potential at the STCF



An STCF operating at CMEs ranging from 2 to 7GeV would be of great importance to the entire field of elementary particle physics.

It would address a very broad range of physics topics, including QCD tests, hadron spectroscopy, precise tests of the electroweak sector of the SM, and searches for new physics beyond the SM.

The proposed luminosity of the STCF is above $\mathbf{0.5 \times 10^{35} cm^{-2} s^{-1}}$, at this level, it is expected to deliver more than $\mathbf{1 ab^{-1}}$ of data samples each year.

| CME (GeV) | Lumi ($ab^{-1}$) | Samples | $\sigma$(nb) | No. of Events | Remarks |
|---|---|---|---|---|---|
| 4.230 | 1 | $J/\psi\pi^+\pi^-$ | 0.085 | $8.5 \times 10^7$ | |
| | | $\tau^+\tau^-$ | 3.6 | $3.6 \times 10^9$ | |
| | | $\gamma X(3872)$ | | | |

Compared with BESIII, the luminosity of STCF is increased by nearly three orders of magnitude, and although the cross-section is one-third of BESIII at the central mass of 4.23 GeV, the number taken is about 300 times that of BESIII

# Expected upper limit

$\mu_{sig}^{up}$ and luminosity have such a relationship :

$$\mu_{sig}^{up} \propto \frac{1}{\sqrt{\mathcal{L}}}$$

| | Lumi(4.23Gev) | $\mu_{sig}^{up}$@95% $C.L$ | $Br_{sig}^{up}$@95% $C.L$ | Expected signal significance |
|---|---|---|---|---|
| BESIII | 1100.81 $pb^{-1}$ | 3.357 | $5.82 \times 10^{-2}$ | 0.6$\sigma$ |
| STCF(Assuming the same event selection as BESIII) | 1 $ab^{-1}$(expected) | 0.108 | $1.94 \times 10^{-3}$ | 19.7$\sigma$ |

A Bayesian-based maximum likelihood estimator, extended from the profile likelihood approach, is used to determine the upper limit of the signal strength. Nucl. Instrum. Meth. A 551, 493–503(2005), arXiv:physics/0403059

# Event selection optimization using CNN

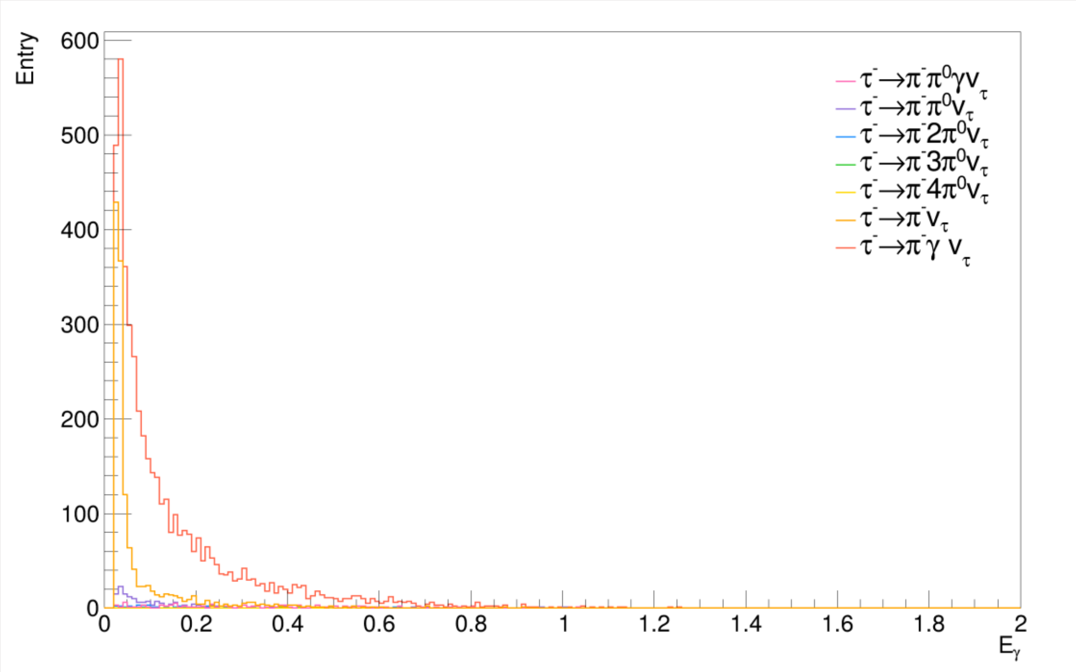
- Sample generation
- Traditional selection
- CNN method

# Tau decay mode

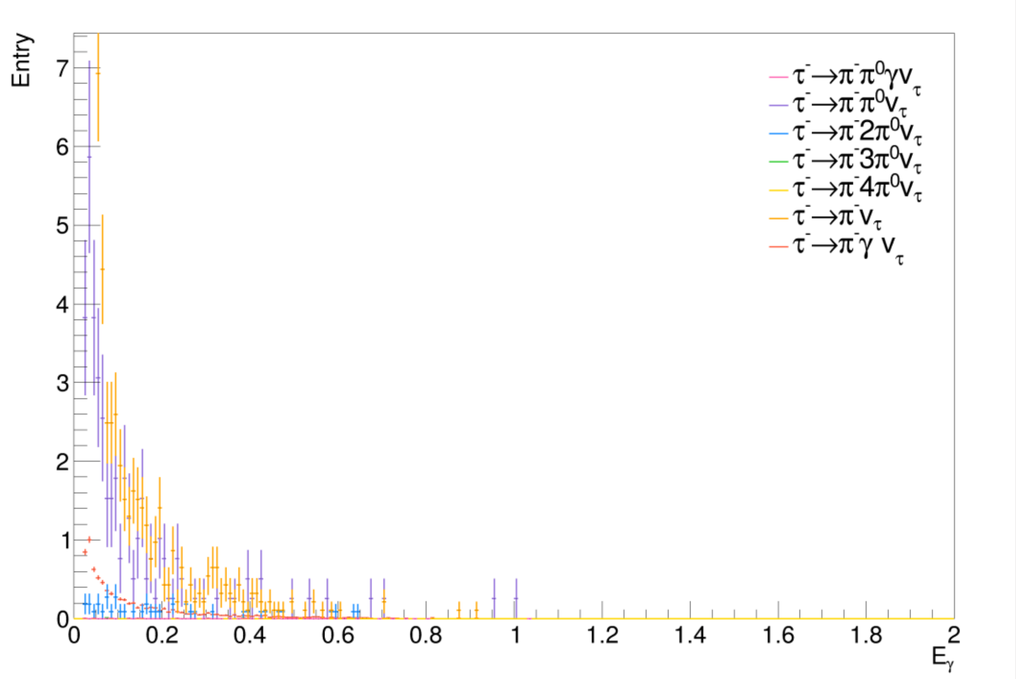
| Modes with one charged particles | Branch ratio | | |
|---|---|---|---|
| $\mu\bar{\nu}_\mu\nu_\tau$ | $(17.39 \pm 0.04)\%$ | $\pi^-2\pi^0\nu_\tau$ | $(17.39 \pm 0.04)\%$ |
| $\mu\bar{\nu}_\mu\nu_\tau\gamma$ | $(3.67 \pm 0.08) \times 10^{-3}$ | $K^-2\pi^0\nu_\tau$ | $(6.5 \pm 2.2) \times 10^{-4}$ |
| $e\bar{\nu}_e\nu_\tau$ | $(17.82 \pm 0.04)\%$ | $\pi^-3\pi^0\nu_\tau$ | $(1.04 \pm 0.07)\%$ |
| $e\bar{\nu}_e\nu_\tau\gamma$ | $(1.83 \pm 0.05)\%$ | $h^-4\pi^0\nu_\tau$ | $(1.6 \pm 0.4) \times 10^{-3}$ |
| $\pi^-\nu_\tau$ | $(10.82 \pm 0.05)\%$ | $a_1(1260)\nu_\tau \to \pi^-\gamma\nu_\tau$ | $(4.0 \pm 1.5) \times 10^{-4}$ |
| $K^-\nu_\tau$ | $(6.96 \pm 0.10) \times 10^{-3}$ | | |
| $\pi^-\pi^0\nu_\tau$ | $(25.49 \pm 0.09)\%$ | | |
| $K^-\pi^0\nu_\tau$ | $(4.33 \pm 0.15) \times 10^{-3}$ | | |

# Event selection using cuts to variables

The same number of main process events were generated with KKMC and decay of tau with EvtGen.
The distribution of photon energy is plotted using the same selection conditions as before.




Before scale



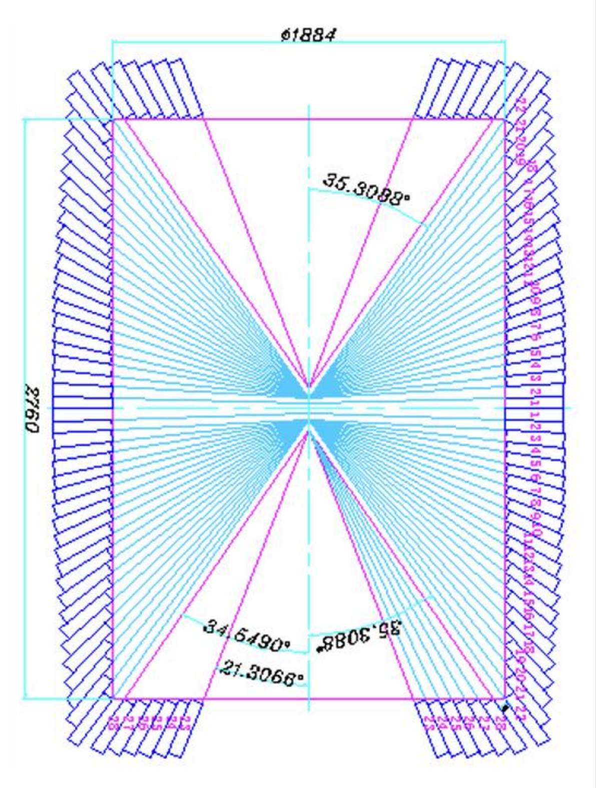

Scale by branch ratio

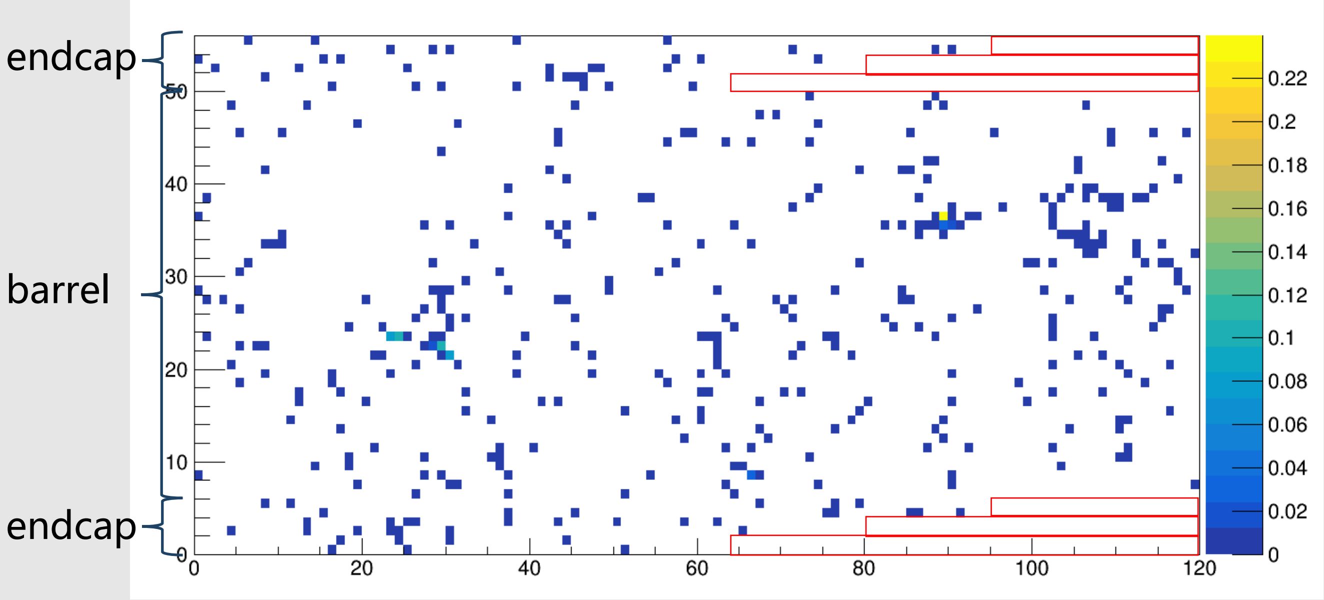
# Event selection using cuts to variables



| | $N_{gen}$ | $N_{reco}$ | $Br$ | $eff = N_{reco}/N_{gen}$ | $ratio = eff_{sig}/eff_{bkg}$ | $scale - factor$ | $N_{scale}$ |
|---|---|---|---|---|---|---|---|
| $\tau^- \to \pi^- \gamma v_\tau$ | 100000 | 4551 | 0.0017 | 0.04551 | 1 | 0.0321 | 146.04 |
| $\tau \to \pi^- \pi^0 v_\tau$ | 100000 | 158 | 0.2549 | 0.00158 | 28.8 | 4.8116 | 760.25 |
| $\tau \to \pi^- \pi^0 \gamma v_\tau$ | 100000 | 98 | 0.0014 | 0.00098 | 46 | 0.0264 | 2.58 |
| $\tau \to \pi^- 2\pi^0 v_\tau$ | 100000 | 41 | 0.0926 | 0.00041 | 111 | 1.7480 | 73.42 |
| $\tau \to \pi^- 3\pi^0 v_\tau$ | 100000 | 2 | 0.0104 | 0.00002 | 2275 | 0.1963 | 0.39 |
| $\tau \to \pi^- 4\pi^0 v_\tau$ | 100000 | 0 | 0.0016 | 0 | 0 | 0.0302 | 0 |
| $\tau^- \to \pi^- v_\tau$ | 100000 | 1247 | 0.1082 | 0.01247 | 3.65 | 2.0424 | 2549.96 |

# BESIII EMC




The EMC is divided into a barrel part and two endcaps

barrel : 120×44=5280， endcap : 2×(96,96,80,80,64,64) =960

# CNN model

The signal and one of the backgrounds are mixed

$\tau^- \to \pi^- \gamma \upsilon_\tau$

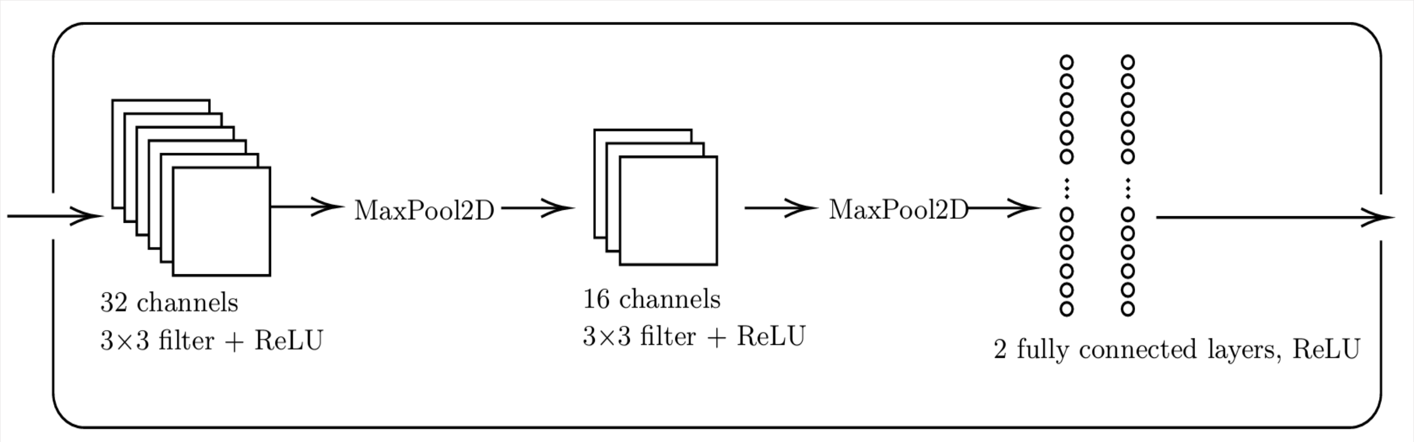
$\tau \to \pi^- \pi^0 \upsilon_\tau$

$\tau \to \pi^- \pi^0 \gamma \upsilon_\tau$

$\tau \to \pi^- 2\pi^0 \upsilon_\tau$

$\tau \to \pi^- 3\pi^0 \upsilon_\tau$

$\tau \to \pi^- 4\pi^0 \upsilon_\tau$




$P(\pi^- \gamma \nu_\tau)$

$P(background)$

Currently, binary classification between signal and each backgrounds is used. Will identify between signal and inclusive ditau MC events.

# Comparison with traditional methods

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| | $ratio_{trad}$ | $N_{scale}^{trad}$ | $ratio_{CNN}$ | $N_{scale}^{CNN}$ |
|---|---|---|---|---|
| $\tau^- \to \pi^- \gamma \upsilon_\tau$ | 1 | 146.04 | 1 | 229 |
| $\tau \to \pi^- \pi^0 \upsilon_\tau$ | 28.8 | 760.25 | 7.61 | 16018 |
| $\tau \to \pi^- \pi^0 \gamma \upsilon_\tau$ | 46 | 2.58 | 178.19 | 3.83 |
| $\tau \to \pi^- 2\pi^0 \upsilon_\tau$ | 111 | 73.42 | 459.92 | 92.99 |
| $\tau \to \pi^- 3\pi^0 \upsilon_\tau$ | 2275 | 0.39 | 1195.32 | 5.57 |
| $\tau \to \pi^- 4\pi^0 \upsilon_\tau$ | 0 | 0 | 1314.54 | 0.82 |
| $\tau^- \to \pi^- \upsilon_\tau$ | 3.65 | 2549.96 | NA | NA |

CNN can suppress the backgrounds with ≥ 3 photons more effectively.

Therefore, we want to use more means to improve the classification ability

# Summary

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- The event selection of radiative decay of $\tau \to \pi\gamma\nu_\tau$ is studied at BESIII
- The ditau backgrounds are difficult to remove due to very similar signature as signal.

- The expected upper limit of branch fraction of $\tau \to \pi\gamma\nu_\tau$ is derived at BESIII and STCF(currently using the event selection at BESIII). Expected measurement in STCF is $Br_{sig}^{up}@95\%\ C.L = 1.94 \times 10^{-3}$.
- Simulation and reconstruction using STCF detector will be performed

- Machine Learning technique is being explored to improve the background rejection
- Need to reproduce the inclusive MC samples with EMC hit map info

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Zhengzhou University

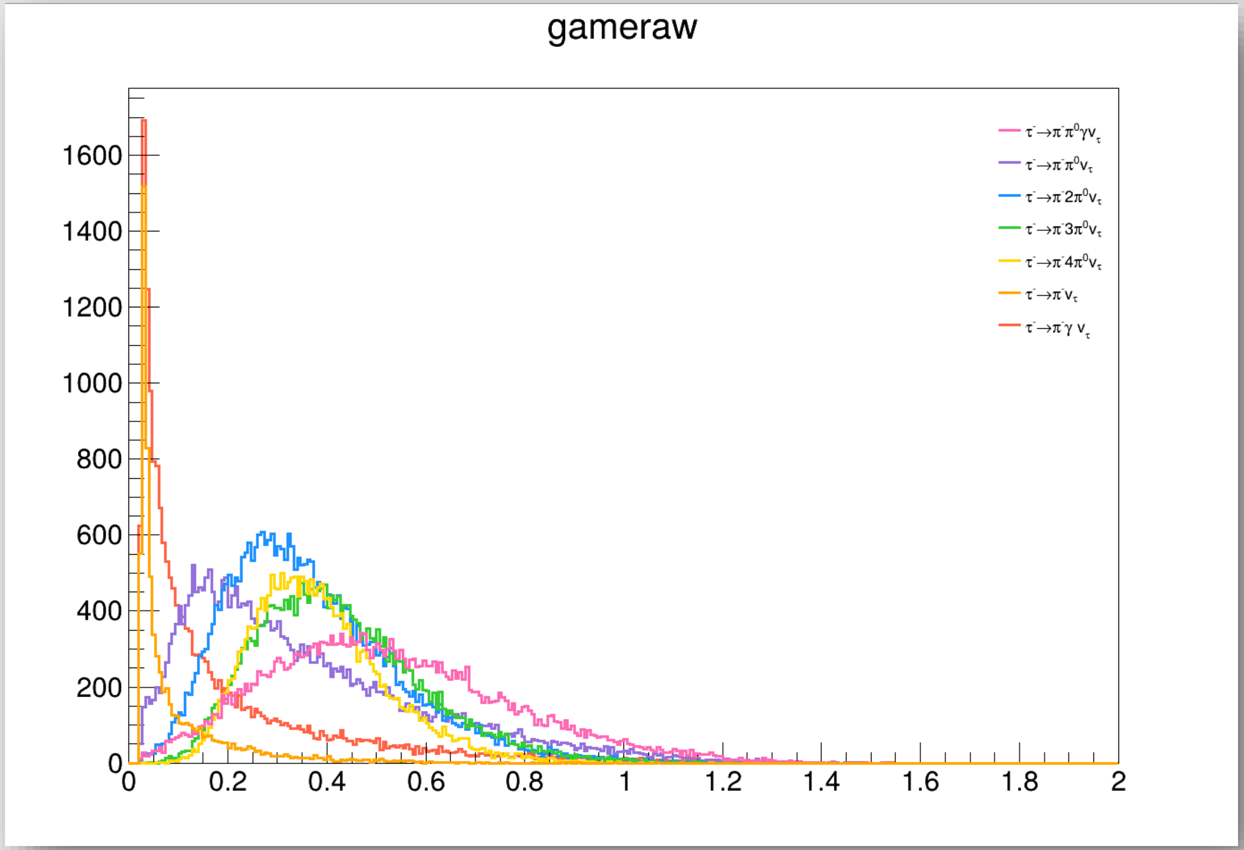
# Thanks

# Backup



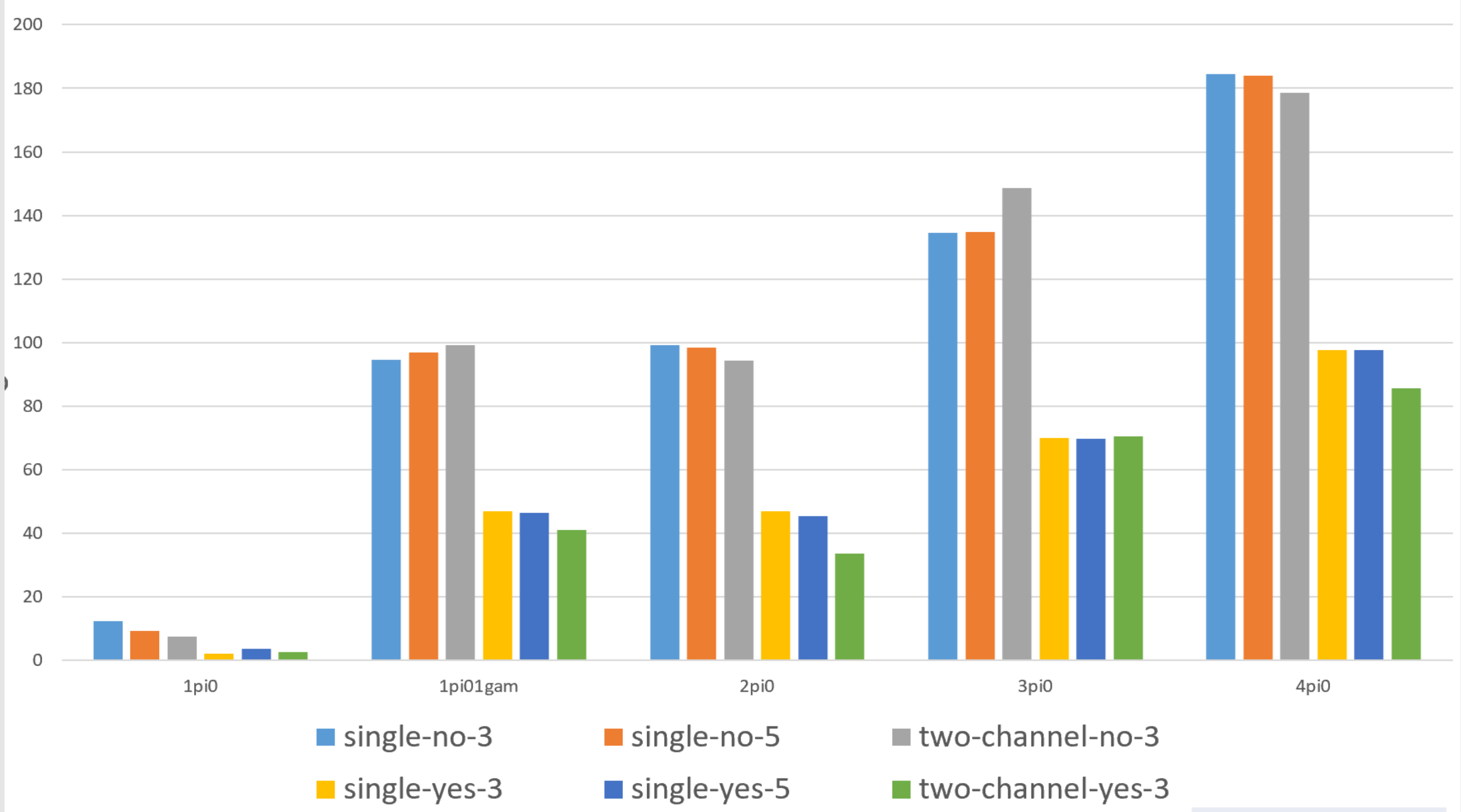
| | Ngen | Nreco |
|---|---|---|
| di-tau | 50,031,418 | 5,471,656 |
| di-muon | 3,503,867 | 1,464,702 |
| hadronic | 103,054,340 | 1,579,726 |
| gamma gamma | 21,532,505 | 394 |
| Bhabha | 3,794,461 | 770 |
| signal | 380,000 | 152,358 |

Primary efficiency




di-tau sample gamma energy

# Data Cleansing




$\tau \to \pi^- \pi^0 \upsilon$ | $\tau \to \pi^- \pi^0 \gamma \upsilon$ | $\tau \to \pi^- 2\pi^0 \upsilon$ | $\tau \to \pi^- 3\pi^0 \upsilon$ | $\tau \to \pi^- 4\pi^0 \upsilon$

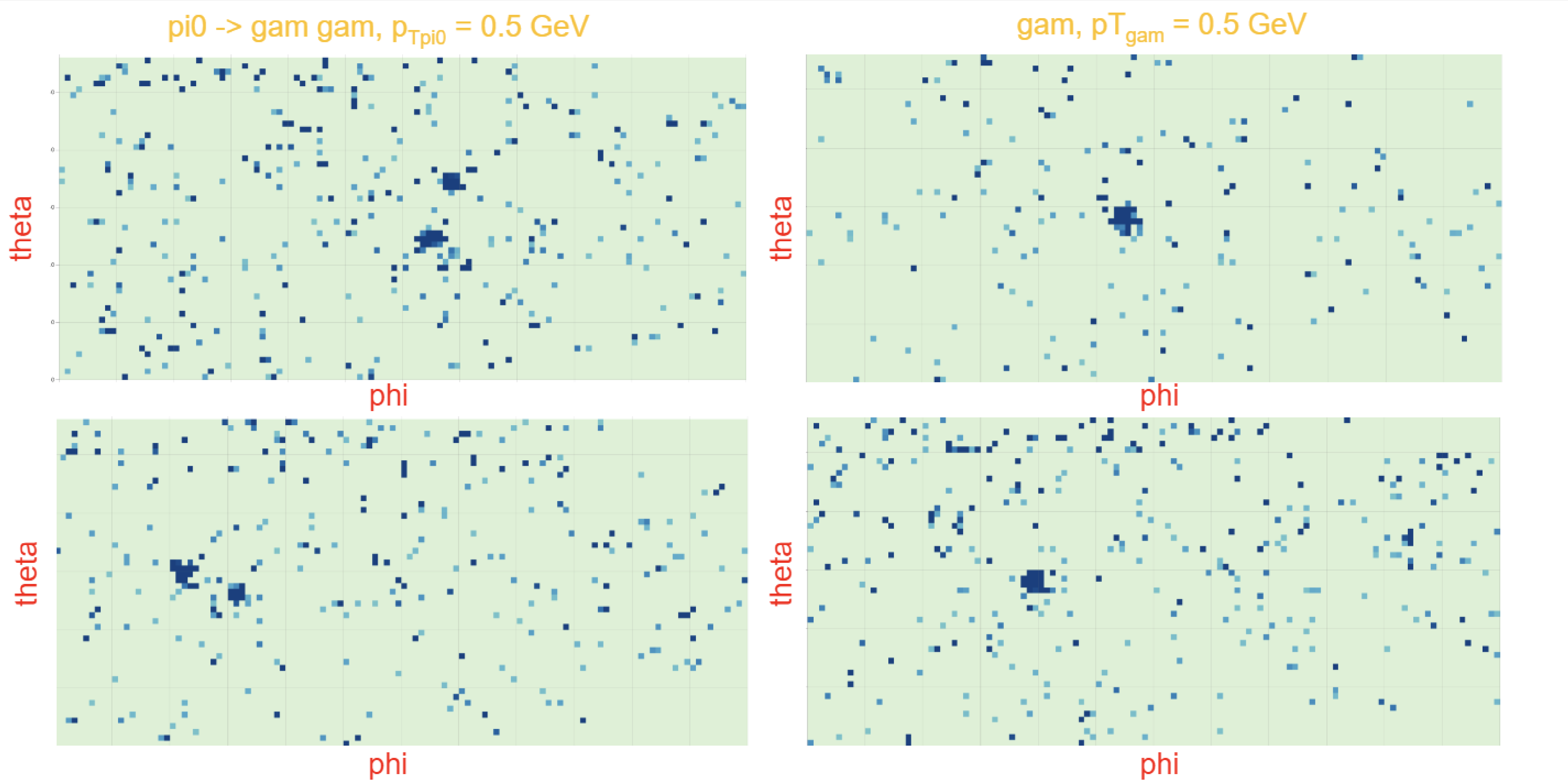
Generate samples via SingleParticleGun to remove the effect of tag side.

Record the center position of the charged track in the EMC, and remove the hit information of the surrounding 3x3 cells

Constraint the number of photon > 0

# Pi0 and gamma binary separation





# Possible backgrounds

di-tau:

| | | |
|---|---|---|
| $\tau^+ \to \mu^+ v_\mu \bar{v}_\tau \gamma$ | $\tau^- \to \pi^- v_\tau$ | signal photon is misidentified from radiative photon of tag side |
| $\tau^+ \to \pi^+ \pi^0 \bar{v}_\tau$ | $\tau^- \to \mu^- \bar{v}_\mu v_\tau$ | $\pi^0$ not successfully reconstructed and the daughter photons are regarded as signal photon (ditau process) |
| $\tau^+ \to \mu^+ v_\mu \bar{v}_\tau \gamma$ | $\tau^- \to \mu^- \bar{v}_\mu v_\tau$ | one muon is misidentified as signal pion |
| $\tau^+ \to \pi^+ \gamma \bar{v}_\tau$ | $\tau^- \to \pi^- v_\tau$ | one pion is misidentified as tag muon |

| | |
|---|---|
| di-mu with ISR: $\mu^+\mu^-\gamma$ | one muon is misidentified as tag pion, signal photon is misidentified from radiative photon (dimu process) |

| | |
|---|---|
| hadronic: $\pi^+\pi^- + (\geq 1)\pi^0$ | signal muon is misidentified from charged pion and $\pi^0$ not successfully reconstructed (hadronic process) |