$X(3872)$ properties and the existence of W_{c1}

Teng Ji

In collaboration with X.-K. Dong, F.-K. Guo, C. Hanhart and U.-G. Meißner.

HISKP, Bonn University

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The Thriving Hadron Spectrum: Challenges! Chances!!!

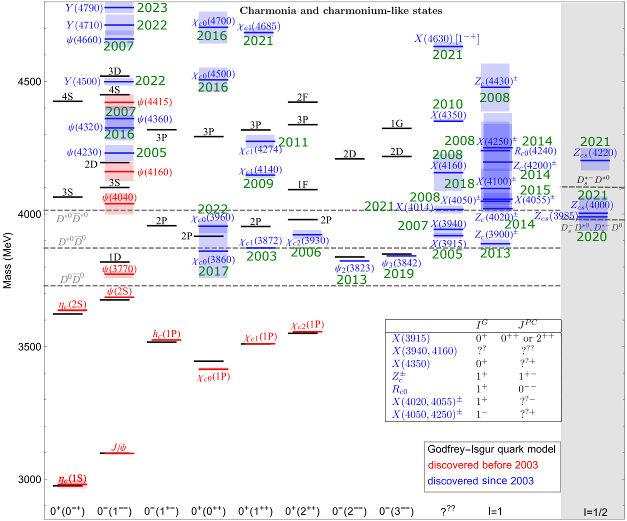


Figure 1: Spectrum of charmonium(-like) states up to 2023. Plot courtesy of Feng-Kun Guo.

What are they?

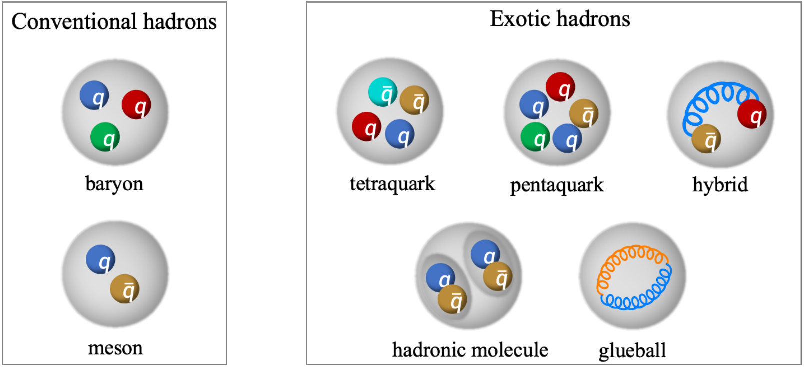


Figure 2: Illustration of hadron structures. Plot courtesy of Feng-Kun Guo.

Low-energy QCD, non-perturbative.

Recent reviews: A. Esposito, et al. arXiv:1611.07920; F.-K. Guo, et al. arXiv:1705.00141; S. L. Olsen, et al. arXiv:1708.04012; Y.-R. Liu, et al. arXiv:1903.11976; N. Brambilla, et al. arXiv:1907.07583; Hua-Xing Chen, et al. arXiv:2204.02649.

(Many others not listed here. Sorry!)

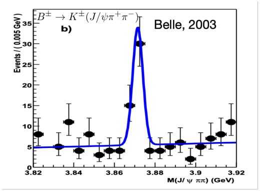
Contents

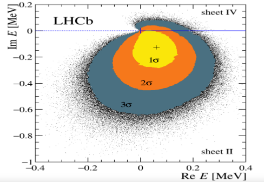
Profile of the Superstar $X(3872)$

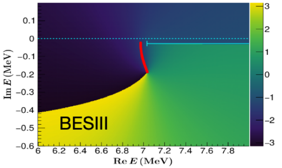
- "Born" in 2003 at Belle. Followed by many other experiments.
  ~~One of~~ the most famous member in exotic hadron community. Most studied.
- Observed channels: $J/\psi\rho/\omega(2\pi/3\pi)$, $J/\psi\gamma$, $\psi(2S)\gamma$, $\chi_{cJ}(1P)\pi^0$, $D^0\bar{D}^{*0}(\to D^0\bar{D}^0\pi^0/\gamma)$.
- Mass and width very different from $c\bar{c}$.
  - Mass, 3871.64 ± 0.06 MeV(PDG), exactly at $D^0\bar{D}^{*0}$ threshold.
  - Tiny decay width, 1.19 ± 0.21 MeV(PDG).
  - Breit-Wigner, well-known IMPROPER!
  - Flatté, LHCb, arXiv:2005.13419, BESIII, arXiv:2309.01502,

$$E_X \sim \begin{cases} (25 - 140i)\ \text{keV (LHCb)} \\ (0 - 190i)\ \text{keV (BESIII)} \end{cases} .$$

    Improved but NOT enough!
  - Call for more reliable parameterization.
    Unitarity and analyticity; Coupled channels; $D\bar{D}\pi$ 3-body effects; Production information.







(a) sheet I: $E_I = 7.04 - 0.19i$ MeV

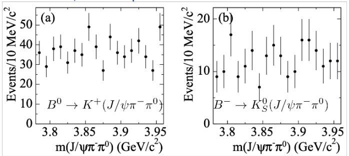
Profile of the Superstar $X(3872)$

- $J^{PC} = 1^{++}$ LHCb, arXiv:1302.6269. Not $\chi_{c1}(2P)$.
- Isospin-0, no charged partner observed.
- Large isospin breaking in the decay, Br($J/\psi 3\pi$)/Br($J/\psi 2\pi$)

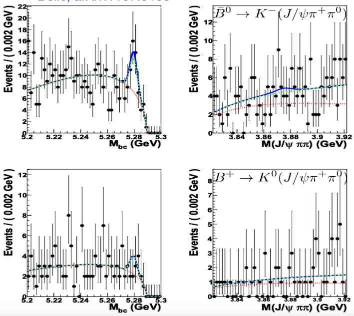
$$\begin{cases} 1.0 \pm 0.4 \pm 0.3 & \text{Belle}, \\ 0.7 \pm 0.3\,(1.7 \pm 1.3) & \text{BaBar } B^+\,(B^0), \\ 1.6^{+0.4}_{-0.3} \pm 0.2 & \text{BESIII}, \end{cases}$$

- PhSp difference. $R_X \equiv g_{X\psi\rho}/g_{X\psi\omega}$ M. Suzuki, arXiv:hep-ph/0508258. How large?
- $I = 1\ W_{c1}$, mild cusp as virtual states. χEFT Z. H. Zhang, et al. arXiv:2404.11215, LQCD M. Sadl, et al. arXiv:2406.09842. Data lacks sufficient statistics.
- Her nature: more likely a $D^0\bar{D}^{*0}$-D^+D^{*-} molecule but still in debate. Compositeness. Productions. Decays.

BaBar, arXiv: hep-ex/0412051



Belle, arXiv:1107.0163



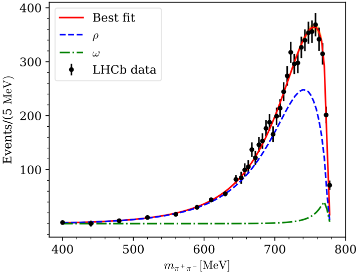
Isospin breaking in the $X(3872)$ decays

- Br($J/\psi 3\pi$)/Br($J/\psi 2\pi$), mediated by ρ and ω, BW,
  $R_X = 0.29 \pm 0.02$ M. Suzuki, arXiv:hep-ph/0508258
  $R_X = 0.30 \pm 0.07$ E. Braaten, et al. arXiv:hep-ph/0507163.
- Distribution of 3π and 2π, ρ-ω mixing,
  $R_X = 0.26^{+0.08}_{-0.05}$ C. Hanhart, et al. arXiv:1111.6241.
- Much more precious 2π distribution, coupled 2π-3π, ρ and ω BW,
  $R_X = 0.29 \pm 0.04$ LHCb, arXiv:2204.12597.
- ρ and ω BW + dipole FF, for running or constant ρ width,
  $R_X = (0.25 \text{ or } 0.30) \pm 0.01$ H.-N. Wang, et al. arXiv:2206.14456.
- Best available ρ spectral function,
  Omnès instead of BW, (Diff. by ~0.03)

$$\mathcal{M}_{2\pi} = q_\pi \, P(s)\,\Omega(s) \left[1 - \frac{\epsilon_{\rho\omega}}{R_X} G_\omega(s)\right],$$

$$\varepsilon_{\rho\omega} = 3.4 \times 10^{-3}\ \mathrm{GeV}^2.$$

  $R_X = 0.27 \pm 0.03$ J. M. Dias, et al. arXiv:2409.13425.
- Possible W_{c1}^0 contribution?



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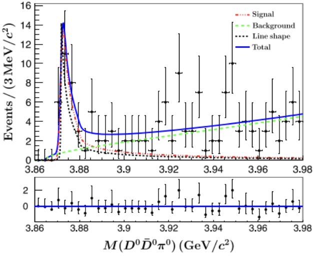
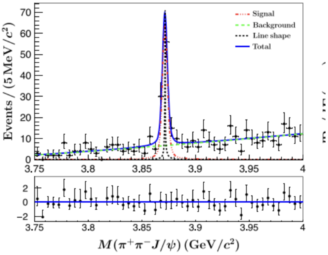
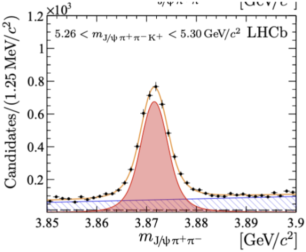
Dynamics of the $X(3872)$

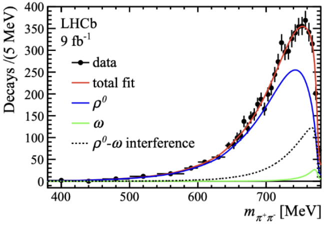


(a)

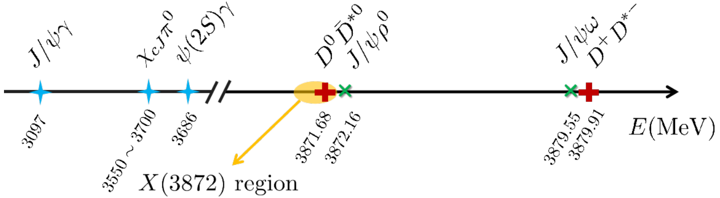


(b)



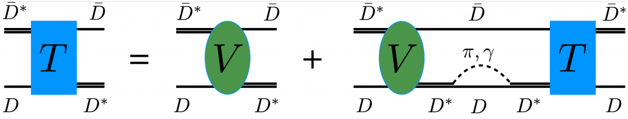


Dynamics of the $X(3872)$



- Hadron-hadron interactions: Pole of scattering amplitudes.
- Elastic channels, strongly coupled to $X(3872)$. Tiny PhSp, large BR. $D^0\bar{D}^{*0}$, D^+D^{*-}. $(D\bar{D}\pi/\gamma)$.
- Inelastic channels, weakly coupled to $X(3872)$. EM or OZI suppressed.
  - $J/\psi\rho$ and $J/\psi\omega$. Close to $X(3872)$. $J/\psi 2\pi$ and $J/\psi 3\pi$;
  - $J/\psi\gamma$, $\psi(2S)\gamma$, $\chi_{cJ}(1P)\pi^0$: far beyond the $X(3872)$ energy region.

Lippmann-Schwinger equation



- $D^0\bar{D}^{*0}$-D^+D^{*-} scattering amplitude,

$$T_{\alpha\beta}(E;p',p) = V_{\alpha\beta}(E;p',p) + \int_0^\Lambda \frac{l^2 dl}{2\pi^2} V_{\alpha\mu}(E;p',l)G_{\mu\nu}(E;l)T_{\nu\beta}(E;l,p).$$

- Two-body loop propagators, D^* self-energy considered,

$$G_{\alpha\beta}(E;l) = \frac{\delta_{\alpha\beta}}{E - \Delta_{\alpha 1} - \frac{l^2}{2\mu_\alpha} + \frac{i}{2}[\Gamma_\alpha(E;l) + \Gamma_\alpha^{\text{rad}}]}.$$

$\Gamma_\alpha(E;l)$, $\Gamma_\alpha^{\text{rad}}$, the strong and radiative widths of channel-α.

- Hard cutoff $\Lambda = 1$ GeV. Small cutoff-dependence of physical quantities, numerically checked for $\Lambda = 0.6 \sim 1.4$ GeV.
- Three contributions to the potential,

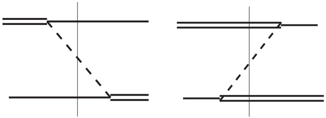
$$V(E;p',p) = V^{\text{ct}} + V^{\pi}(E;p',p) + V^{\text{inel}}(E).$$

Potentials

- Contact interaction, HQSS, 2 LECs, C. Hidalgo-Duque, et al., arXiv:1210.5431.

$$V^{\rm ct} = \frac{1}{2}\begin{pmatrix} C_{0X}+C_{1X} & C_{0X}-C_{1X} \\ C_{0X}-C_{1X} & C_{0X}+C_{1X} \end{pmatrix}.$$

- OPE, onshell, TOPT
  Chiral Lag. for $D^*D\pi$ coupling, $\Gamma_{D^*\to D\pi}$.



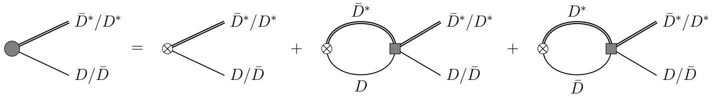
- Effective potential from inelastic loops, C. Hanhart, et al., arXiv:1507.00382

$$V^{\rm inel}_{\alpha\beta}(E) = v_{\alpha j}(s)G_{jk}(E)v_{k\beta}(s) \to -i\, v_{\alpha j}(s)\rho_{jk}(E,s)v_{k\beta}(s).$$

$\sum_i$ 

  - Real part absorbed by $V^{\rm ct}$.
  - Elastic-inelastic transitions, $v_{\alpha j}(s) = (1+as)u_{\alpha k}\begin{pmatrix} 1 & \epsilon_{\rho\omega}G_\rho(s) \\ \epsilon_{\rho\omega}G_\omega(s) & 1 \end{pmatrix}_{kj}$
  - ρ-ω mixing considered C. Hanhart, et al. arXiv:1111.6241.
  - Dispersive treatment of ρ, Omnès instead of BW J. M. Dias, et al. arXiv:2409.13425.
  - Other channels, PhSp flat, constant contributions. Neglected.

Production amplitudes

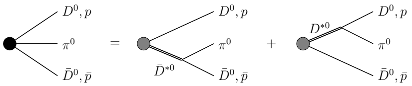


- Production of elastic channels

$$U_\alpha(E,p) = P_\alpha + \int_0^\Lambda \frac{l^2 \mathrm{d}l}{2\pi^2} P_\mu G_{\mu\nu}(E,l) T_{\nu\alpha}(E;l,p).$$

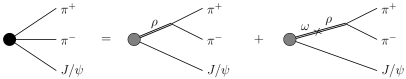
- Production of $D^0\bar{D}^0\pi^0$

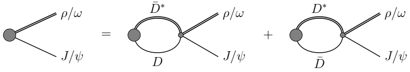
$$T_{D^0\bar{D}^0\pi^0}(E,p,\bar{p}) = U_\alpha(E,p)F_\alpha(p) + U_\alpha(E,\bar{p})F_\alpha(\bar{p}),$$



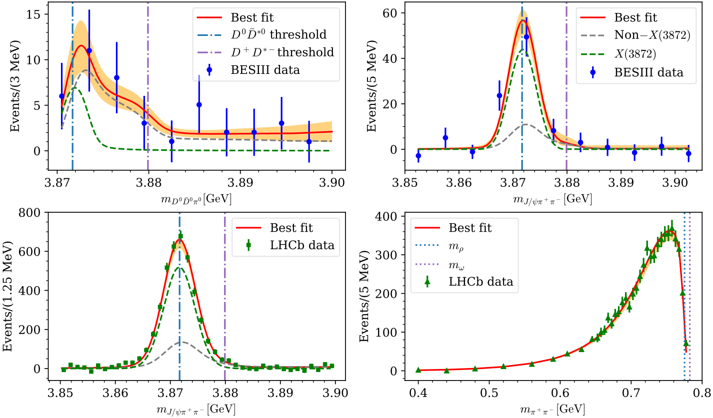
- Production of $J/\psi\pi\pi$

$$T_{J/\psi\pi\pi}(E,s) = \int_0^\Lambda \frac{l^2 \mathrm{d}l}{2\pi^2} U_\alpha(E,l) G_{\alpha\beta}(E,l) H_\beta(s),$$



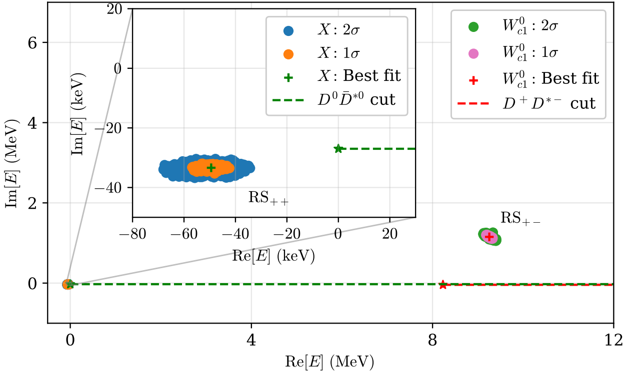


Fitted lineshapes



Experimental resolutions and efficiencies considered.
BESIII data on $e^+e^- \to \gamma(D^0\bar{D}^{*0}\pi^0/J/\psi\pi^+\pi^-)$ BESIII, arXiv:2309.01502,
LHCb data on $B^+ \to K(^+J/\psi\pi^+\pi^-)$ LHCb, arXiv:2005.13422, arXiv:2204.12597.
$\chi^2/\text{dof} = 71.5/82 = 0.83$.

Pole positions of $X(3872)$ and W_{c1}^0



- Bound state of $D^0\bar{D}^{*0}$ with binding energy $E_X = (-49.4^{+7.1}_{-7.7} - (33.4^{+1.4}_{-1.}))$ keV
- Another pole W_{c1}^0 on unphysical RS, virtual state, $E_W = (1.0 \pm 0.1 + i\,(1.1 \pm 0.1))$ MeV relative to D^+D^{*-} threshold.
- For comparison $E_X \sim \begin{cases} (25 - i\,140) \text{ keV (LHCb)} \\ (0 - i\,190) \text{ keV (BESIII)} \end{cases}$ with uncertainty $\mathcal{O}(100 \text{ keV})$.

Decay widths of $X(3872)$

- From pole position,

$$\Gamma_{X\to D^0\bar{D}^{*0}} + \Gamma_{X\to J/\psi V} \approx 67\ \mathrm{keV}.$$

- Tiny binding energy, $\Gamma_{X\to D^0\bar{D}^{*0}}$ constraint by D^{*0} width,

$$\Gamma_{X\to D^0\bar{D}^{*0}} = \Gamma_{X\to D^0\bar{D}^0\pi^0} + \Gamma_{X\to D^0\bar{D}^0\gamma} \sim \Gamma_{D^{*0}} \approx 55\ \mathrm{keV}.$$

- The rest for $J/\psi V$,

$$\Gamma_{X\to J/\psi V} = \Gamma_{X\to J/\psi 2\pi} + \Gamma_{X\to J/\psi 3\pi} \approx 12\ \mathrm{keV}$$

- $\frac{\mathrm{Br}(X(3872)\to J/\psi\gamma+\psi(2S)\gamma+\chi_{c1}\pi^0)}{\mathrm{Br}(X(3872)\to J/\psi\pi^+\pi^-)} \sim 0.79 + 0.79\times 1.67 + 0.88 = 3.0$, we estimate

$$\Gamma_X = 70 \sim 110\ \mathrm{keV}.$$

- More precious data are needed for other channels.

Isospin breaking of the $X(3872)$.

- Coupling of $X(3872)$ to each channel, $g_{X,\alpha} g_{X,\beta} = \lim_{E\to E_X}(E - E_X)T_{\alpha\beta}$,

$$g_{X,D^0\bar{D}^{*0}} = 0.25, \quad g_{X,D^+D^{*-}} = 0.15.$$

- Coupling of W_{c1}^0 to each channel, $g_{W,\alpha} g_{W,\beta} = \lim_{E\to E_W}(E - E_W)T_{\alpha\beta}$,

$$g_{W,D^0\bar{D}^{*0}} = 0.2 - 0.3i, \quad g_{W,D^+D^{*-}} = -0.3 - 0.5i$$

- By couplings,

$$\begin{aligned} X &\propto (g_{X,D^0\bar{D}^{*0}} + g_{X,D^+D^{*-}})|I=0\rangle + (g_{X,D^0\bar{D}^{*0}} - g_{X,D^+D^{*-}})|I=1\rangle \\ &= 0.97|I=0\rangle + 0.24|I=1\rangle \\ W &= (-0.11 - 0.82i)|I=0\rangle + (0.53 + 0.18i)|I=1\rangle \end{aligned}$$

- For $X(3872)$, mass eigenstate $\neq$ isospin eigenstate, non-negligible isospin-1 component.
- By calculating the ratio of $X \to J/\psi\rho$ and $X \to J/\psi\omega$ amplitudes,

$$R_X = 0.29 \pm 0.03.$$

Compositeness of $X(3872)$

- ERE at complex $D^0\bar{D}^{*0}$ threshold, $1/f = 1/a_0 - ik + r_0/2\,k^2 + \cdots$.

$$a_0 = (-19.4 \pm 1.5 + i\,(1.6 \pm 0.3))\ \text{fm},$$
$$r_0 = (-0.6 \pm 0.1 + i\,(-1.1 \pm 0.1))\ \text{fm}.$$

For comparison,
$a_0 \simeq -33$ fm, $-2.0 \lesssim r_0 \lesssim -5.3$ fm, LHCb, A. Esposito, et al. arXiv:2108.11413
$a_0 = -16.5^{+7.0+5.6}_{-27.6-27.7}$ fm, $r_0 = -4.1^{+0.9+2.8}_{-3.3-4.4}$ fm, BESIII, arXiv:2309.01502

- Compositeness I. Matuschek, et al. arXiv:2007.05329

$$\bar{X}_A = \left(1 + 2\left|\frac{\text{Re}\,r_0 + \Delta r_{0,\text{IB}}}{\text{Re}\,a_0}\right|\right)^{-\frac{1}{2}} = 0.94 \pm 0.03$$

- Isospin breaking correction, V. Baru, et al. arXiv:2110.07484

$$\Delta r_{0,\text{IB}} = \frac{g_2^2}{g_1^2}\sqrt{\frac{\mu_2}{2\mu_1^2\Delta_{21}}} = 0.52\ \text{fm}.$$

Contents

Summary

- Properties of the $X(3872)$ are studied precisely, in $J/\psi\pi^+\pi^-$ and $D^0\bar{D}^0\pi^0$ channels, LHCb 2022 and BESIII 2023.
- More reliable parameterization: unitarity and analyticity; coupled channels ($D\bar{D}^*$, $J/\psi V$); $D\bar{D}\pi$ 3-body effects (D^* dynamical widths, OPE).
  - Bound state of $D^0\bar{D}^{*0}$, $E_X = -49 \pm 7$ keV.
  - $\Gamma_X = 70 \sim 110$ keV. Partial decay widths. Data on other channels.
  - $R_X = 0.29 \pm 0.03$, much larger than $c\bar{c}$, ~ 0.05.
  - Isospin partners $W_{c1}^{0,\pm}$, virtual states.
    Mild threshold cusps in $J/\psi\pi^\pm\pi^0$ channels.

Thanks for your attention!

Backup Slides

Parameterization of near-threshold lineshapes

Two channels, LO NREFT X.-K. Dong, et al. arXiv:2011.14517

- At leading order

$$T(E) = 8\pi\Sigma_2 \begin{pmatrix} -\frac{1}{a_{11}} + ik_1 & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & -\frac{1}{a_{22}} - \sqrt{-2\mu_2 E - i\epsilon} \end{pmatrix}^{-1}$$

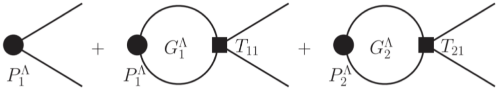
$$= -\frac{8\pi\Sigma_2}{\det} \begin{pmatrix} \frac{1}{a_{22}} + \sqrt{-2\mu_2 E - i\epsilon} & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \frac{1}{a_{11}} - ik_1 \end{pmatrix},$$

with $\det = (1/a_{11} - i k_1)\left(1/a_{22} + \sqrt{-2\mu_2 E - i\epsilon}\right) - 1/a_{12}^2$.

- Production amplitude of channel-1, the observing channel,

$$P_1^\Lambda[1 + G_1^\Lambda T_{11}(E)] + P_2^\Lambda G_2^\Lambda(E) T_{21}(E) = P_1 T_{11}(E) + P_2 T_{21}(E).$$

Energy dependence only in T_{11} and T_{21}.



- For symmetry-related two-channel system, see Z.-H. Zhang, et al. arXiv:2407.10620.

Near-(2nd)threshold lineshapes

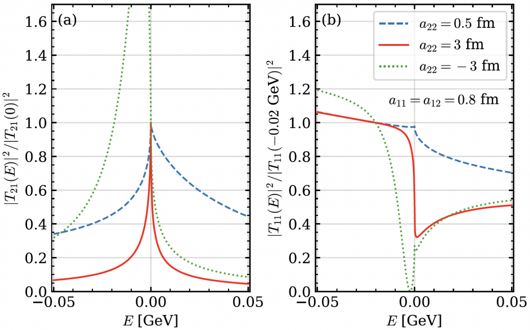
Highly depends on production ratio. X.-K. Dong, et al. arXiv:2011.14517

(a) T_{21} driven, $P_2 \gg P_1$

- Virtual state: peak at threshold;
- Bound state: peak below threshold;
- Attraction: near-threshold peak.

(b) T_{11} driven, $P_1 \gg P_2$

- Universal dip for strong S-wave attraction in channel-2;
- $T_{11} \propto \frac{1}{a_{22}} + \sqrt{-2\mu_2 E - i\epsilon}$.



Internal structure of the $X(3872)$

- Far beyond quark model predictions for $\chi_{c1}(2P)$.

Compact tetraquark $[c\bar{c}q\bar{q}]$

- Fine tuning.
- Three partners. No evidence.
- Large isospin breaking.
- Strong coupling to $D^0\bar{D}^{*0}$.

$D^0\bar{D}^{*0}$ molecule

- Fine tuning, but not that much.
- Prompt production. Short range.
- $\frac{\mathrm{Br}(X(3872)\to\psi'\gamma)}{\mathrm{Br}(X(3872)\to J/\psi\gamma)}$. Short range.
- Large negative r_0. Small here.

- Compositeness $X(\bar{X}_A) = 1 - Z$, S. Weinberg, Phys.Rev.137,B672(1965).

$$a_0 = -2\left(\frac{1-Z}{2-Z}\right)\frac{1}{\gamma} + \mathcal{O}\left(\frac{1}{\beta}\right), r_0 = -\left(\frac{Z}{1-Z}\right)\frac{1}{\gamma} + \mathcal{O}\left(\frac{1}{\beta}\right),$$

  - $Z = 0, 1$ for pure molecule or compact,

$$a \to -\frac{1}{\gamma} \& r \to \frac{N_r}{\beta}\ (Z \to 1), \quad a \to -\frac{N_a}{\beta} \& r \to -\infty\ (Z \to 0).$$

  - Large negative r_0 $\Rightarrow$ compact component in $X(3872)$?
    LHCb, BESIII and Belle data. LHCb, arXiv:2204.12597; BESIII, arXiv:2309.01502; A. Esposito, et al. arXiv:2108.11413; H. Xu, et al.arXiv:2401.00411. Flatté, large uncertainty.
  - To extract X precisely using more reliable parameterization.

TOPT of OPE

$$V^{\pi}(E;p',p) = \frac{g^2}{6F_\pi^2}\begin{pmatrix} \frac{1}{2}V^{SS}_{D^0\pi^0\bar{D}^0} & V^{SS}_{D^0\pi^+D^-} \\ V^{SS}_{D^+\pi^-\bar{D}^0} & \frac{1}{2}V^{SS}_{D^+\pi^0D^-} \end{pmatrix},$$

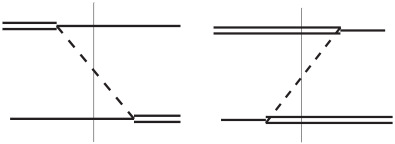
with

$$V^{SS}_{\alpha\beta\zeta} = \frac{1}{2}\int_{-1}^{1} dz \frac{q^2}{D^{\rm TOPT}_{\alpha\beta\zeta}(E;p',p,z)},$$

$$\frac{1}{D^{\rm TOPT}_{\alpha\beta\zeta}} = \frac{1}{2E_\beta(q)}\left(\frac{1}{D^R_{\alpha\beta\zeta}} + \frac{1}{D^A_{\alpha^*\beta\zeta^*}}\right),$$

$$D^R_{\alpha\beta\zeta} = \sqrt{s} - E_\alpha(p) - E_\beta(q) - E_\zeta(p') + i\varepsilon,$$

$$D^A_{\alpha^*\beta\zeta^*} = \sqrt{s} - E_{\alpha^*}(p') - E_\beta(q) - E_{\zeta^*}(p) + i\varepsilon.$$



Energy-dependent widths of D^*

$$\Gamma_1(E;l) = \Gamma_{[D^{*0}\to D^0\gamma]} + \frac{g^2 M_{D^+}}{12\pi F_\pi^2 M_{D^{*0}}}\left[\Sigma_{D^+\pi^- D^0}(E;l,\mu_\pm) - \Sigma_{D^+\pi^- D^0}(0;0,\mu_\pm)\right] + \frac{g^2 M_{D^0}}{24\pi F_\pi^2 M_{D^{*0}}}\Sigma_{D^0\pi^0 D^0}(E;l,\mu_0)\,,$$

$$\Gamma_2(E;l) = \Gamma_{[D^{*+}\to D^+\gamma]} + \frac{g^2 M_{D^+}}{24\pi F_\pi^2 M_{D^{*+}}}\Sigma_{D^+\pi^0 D^-}(E;l,\mu_\pm) + \frac{g^2 M_{D^0}}{12\pi F_\pi^2 M_{D^{*+}}}\Sigma_{D^0\pi^+ D^-}(E;l,\mu_\pm)\,.$$

with

$$\Gamma_{[D^{*0}\to D^0\gamma]} = 19.5\ \text{keV}, \quad \Gamma_{[D^{*+}\to D^+\gamma]} = 1.3\ \text{keV},$$

$$\Sigma_{\alpha\beta\zeta}(E;l,\mu) = \left[2\mu_{\alpha\beta}\left(\sqrt{s} - M_\alpha - M_\beta - M_\zeta - \frac{l^2}{2\mu}\right)\right]^{3/2},$$

with $\mu_{\alpha\beta} = M_\alpha M_\beta/(M_\alpha + M_\beta)$.

Event distributions

$$\frac{\mathrm{d\,Br}[D^0\bar{D}^0\pi^0]}{\mathrm{d}E} = \frac{1}{32\pi^3}\int_0^{p_{\max}}\frac{p\mathrm{d}p}{\omega_{D^0}(p)}\int_{\bar{p}_{\min}}^{\bar{p}_{\max}}\frac{\bar{p}\mathrm{d}\bar{p}}{\omega_{\bar{D}^0}(\bar{p})}\times\left(\left|T_{D^0\bar{D}^0\pi^0}(E,p,\bar{p})\right|^2 + f_{\mathrm{bg}}\right),$$

$$\frac{\mathrm{d\,Br}[J/\psi\pi\pi]}{\mathrm{d}E} = \int_{2m_{\pi^+}}^{E-m_{J/\psi}}\frac{\mathrm{d}m_{2\pi}^2}{2\pi}\int\mathrm{d}\Phi_{2\pi}\frac{k_{J/\psi}(E,m_{2\pi})}{4\pi E}\left|T_{J/\psi\pi\pi}(E,m_{2\pi}^2,p_+)\right|^2,$$

$$\frac{\mathrm{d\,Br}[J/\psi\pi\pi]}{\mathrm{d}m_{\pi\pi}} = \int\mathrm{d}E\int\mathrm{d}\Phi_{2\pi}\frac{k_{J/\psi}(E,m_{2\pi})m_{2\pi}}{4\pi^2 E}\left|T_{J/\psi\pi\pi}(E,m_{2\pi}^2,p_+)\right|^2,$$

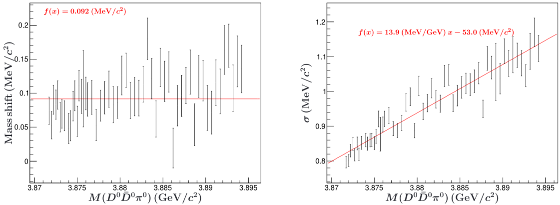
Resolutions and efficiencies considered.
BESIII:

$$\epsilon_{D^0\bar{D}^0\pi^0} = 1.31\times 10^{-3},\ \epsilon_{J/\psi\pi^+\pi^-} = 3.78\times 10^{-2}.$$

LHCb:

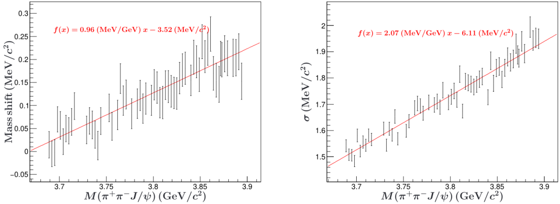
$$\epsilon(m_{\pi\pi}) = 0.966 + 1.345(m_{\pi\pi}-0.7) + 1.607(m_{\pi\pi}-0.7)^2,$$
$$\sigma(m_{\pi\pi}) = 2.39(1-\exp(-m_{\pi\pi}/0.2203)) - 5.4\exp(-m_{\pi\pi}/0.2203)\ \ [\mathrm{MeV}].$$

BESIII resolutions and mass shifts



(a) (b)

FIG. 1. The (a) mass shift and (b) mass resolution of $D^0\bar{D}^0\pi^0$. The red lines are the fit results.



(a) (b)

Fitted parameter values

Table 1: The parameter values and the correlation matrix from the best fit.

Parameters	Values	Covariance matrix									
P_1^{B}	2.8 ± 0.8	1.00									
$P_2^{\mathrm{B}}/P_1^{\mathrm{B}}$	0.5 ± 0.5	-0.84	1.00								
P_1^{L}	5.3 ± 1.0	0.76	-0.36	1.00							
$P_2^{\mathrm{L}}/P_1^{\mathrm{L}}$	0.49 ± 0.04	-0.15	0.10	-0.24	1.00						
C_{0X}	-2.95 ± 0.05	0.31	-0.19	0.38	-0.50	1.00					
C_{1X}	-10.8 ± 0.3	-0.07	0.07	-0.06	0.38	-0.91	1.00				
f_{bg}	3 ± 6	-0.36	0.15	-0.41	0.08	-0.18	0.05	1.00			
a	0.87 ± 0.35	0.12	-0.09	0.12	-0.36	0.76	-0.79	-0.07	1.00		
u	-0.81 ± 0.23	0.60	-0.30	0.76	-0.28	0.74	-0.61	-0.33	0.67	1.00	
σ	2.72 ± 0.06	0.03	0.00	0.06	0.09	-0.23	0.18	-0.02	-0.18	-0.01	1.00