Hadronic molecules with exotic $J^{PC} = 0^{--}$

Xiang-Kun Dong
In collaboration with T. Ji, F.-K. Guo, U.-G. Meißner, and B.-S. Zou

Based on PRL 129(2022)10,102002 and PLB 853(2024)138646.

HISKP, Bonn University

Nov. 20, 2024

FTCF2024-Guangzhou
Nov. 17-21, 2024

Contents

$\psi(4230)$ related hadronic molecules

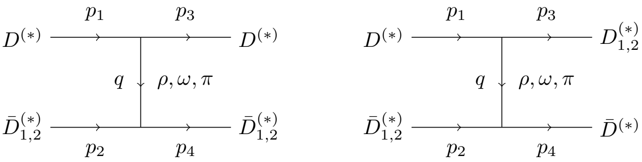
- $Y(4260)$ observed in $e^+e^- \to \gamma_{\rm ISR} J/\psi\pi^+\pi^-$ by BaBar.
- Candidate of exotic state, properties different from $c\bar{c}$.
- Strong coupling to $D\bar{D}_1$, hadronic molecules.
- HQSS implies other molecular states of $D^{(*)}\bar{D}_{1,2}$.

Table: The hadronic molecules considered in this work and their possible experimental candidates.

Molecule	Components	J^{PC}	Candidates	Mass (GeV)	E_B (MeV)
$\psi(4230)$	$\frac{1}{\sqrt{2}}(D\bar{D}_1 - \bar{D}D_1)$	1^{--}	$\psi(4230)$	$4.220 \pm 0.015^\dagger$	67 ± 15
$\psi(4360)$	$\frac{1}{\sqrt{2}}(D^*\bar{D}_1 - \bar{D}^*D_1)$	1^{--}	$\psi(4360)$	$4.368 \pm 0.013^\dagger$	62 ± 14
$\psi(4415)$	$\frac{1}{\sqrt{2}}(D^*\bar{D}_2 - \bar{D}^*D_2)$	1^{--}	$\psi(4415)$	$4.421 \pm 0.004^\dagger$	49 ± 4
$\psi_0(4360)$	$\frac{1}{\sqrt{2}}(D^*\bar{D}_1 - \bar{D}^*D_1)$	0^{--}	-	-	-

- $\psi(4230), \psi(4360)$ & $\psi(4415)$ as inputs.
- $\mathcal{C}|D\rangle = |\bar{D}\rangle,\ \mathcal{C}\,|D^*\rangle = -\left|\bar{D}^*\right\rangle,\ \mathcal{C}\,|D_1\rangle = \left|\bar{D}_1\right\rangle,\ \mathcal{C}\,|D_2^*\rangle = -\left|\bar{D}_2^*\right\rangle.$

Meson-exchange interaction



(a) (b)

- Meson-exchange potential

$$\mathcal{M}_{ij}^{P} = \alpha_{ij}^{P} \frac{1}{\boldsymbol{q}^2 + m_P^2} + \beta_{ij}^{P} \frac{\boldsymbol{q}^2}{\boldsymbol{q}^2 + m_P^2} = A_{ij}^{P} \frac{1}{\boldsymbol{q}^2 + m_P^2} + B_{ij}^{P},$$

$$\mathcal{M}_{ij}^{V} = \alpha_{ij}^{V} \frac{1}{\boldsymbol{q}^2 + m_V^2} + \beta_{ij}^{V} \frac{\boldsymbol{q}^2}{\boldsymbol{q}^2 + m_V^2} = A_{ij}^{V} \frac{1}{\boldsymbol{q}^2 + m_V^2} + B_{ij}^{V}.$$

- HQSS $\Rightarrow$ 4 independent contact terms for isoscalar $D^{(*)}\bar{D}_{1,2}^{(*)}$ system

$$F_{Ij_\ell}^{d} \equiv \left\langle \frac{1}{2}, \frac{3}{2}, j_\ell \left| \hat{\mathcal{H}}_I \right| \frac{1}{2}, \frac{3}{2}, j_\ell \right\rangle, \quad F_{Ij_\ell}^{c} \equiv \left\langle \frac{1}{2}, \frac{3}{2}, j_\ell \left| \hat{\mathcal{H}}_I \right| \frac{3}{2}, \frac{1}{2}, j_\ell \right\rangle$$

where $j_\ell = 1, 2$ is the spin of light quarks.

Contact terms

- P, V-exchange $\Rightarrow \delta$ potential in position, short-distance interaction.
- Resonance saturation: The interaction is saturated by meson exchange.
- t-channel $\Rightarrow$ two kinds of δ potential $\Rightarrow$ two parameters c, d.
- "u"-channel $\Rightarrow$ another two contact terms. Not included.
- Introduce c, d for renormalization, the potential read

$$V_{ij} = -\frac{1}{4\sqrt{M_1 M_2 M_3 M_4}} \left(A_{ij}^P \frac{1}{\boldsymbol{q}^2 + m_P^2} + A_{ij}^V \frac{1}{\boldsymbol{q}^2 + m_V^2} + d B_{ij}^P + c B_{ij}^V \right)$$

- Gaussian form factor

$$V_{ij}\left(\boldsymbol{k}', \boldsymbol{k}\right) \to V_{ij}\left(\boldsymbol{k}', \boldsymbol{k}\right) e^{-\boldsymbol{q}^2/\Lambda^2} \tag{1}$$

with $\boldsymbol{q}^2 = \boldsymbol{k}^2 + \boldsymbol{k}'^2 - 2kk' \cos\theta$.

- Poles from LSE

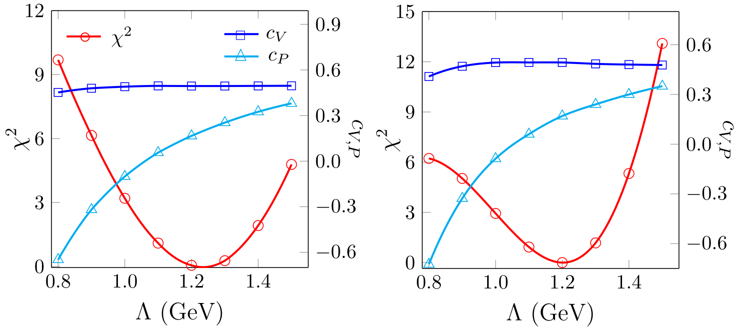
$$T_{ij}\left(E; \boldsymbol{k}', \boldsymbol{k}\right) = V_{ij}\left(\boldsymbol{k}', \boldsymbol{k}\right) + \sum_n \int \frac{\mathrm{d}^3 l}{(2\pi)^3} \frac{V_{in}\left(\boldsymbol{k}', \boldsymbol{l}\right) T_{nj}(E; \boldsymbol{l}, \boldsymbol{k})}{E - l^2/\left(2\mu_n\right) - \Delta_{n1} + i\epsilon}$$

t-channel results

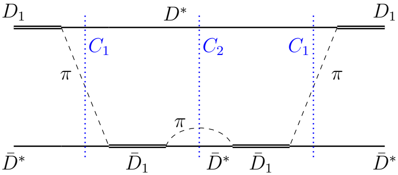
- Adjusting c, d to reproduce the binding energy of $\psi(4230), \psi(4360)$ & $\psi(4415)$,

$$\chi^2 = \sum_i \left(\frac{E_{B,ii} - E^{\text{cen}}_{\text{exp},ii}}{E^{\text{err}}_{\text{exp},ii}} \right)^2 .$$

- Single channel. Predicted ψ_0 binding energy, 72.4 ± 17.4 MeV.
- Little coupled-channel effects on predicted ψ_0.



$D\bar{D}^*\pi$ 3-body effects



- Cut C_1, OPE. TOPT. Left-hand cut

$$\frac{1}{q^2 - m_\pi^2 + i\epsilon} \to \frac{1}{2E(m_\pi, \boldsymbol{q})}\left(\frac{1}{d_1} + \frac{1}{d_2}\right)$$

$$d_i = \sqrt{s} - E(m_\pi, \boldsymbol{q}) - E(m_i, \boldsymbol{k}) - E(m_i, \boldsymbol{k}')$$

- Cut C_2, D_1 self-energy. Right-hand cut.

$$\Gamma_{D_1}(E, \boldsymbol{l}) = \frac{g_S^2}{4}(m_{D_1}^2 - m_{D^*}^2)^2 \frac{p_{\rm cm}}{8\pi m_{D^*\pi}^2},$$

Assumed in S-wave. $g_S = g_{S0} = 2.0\ \text{GeV}^{-1}$ and $g_S = g_{S1} = \sqrt{31/12}\, g_{S0}$ for uncertainty.

3-body effects on pole positions of $\psi_{(0)}$

Table: Pole positions relative to the $D^*\bar{D}_1$ threshold in units of MeV with $c_V = 0.50$, $c_P = 0.18$ from the single t-channel fitting. "C_2" means the D_1 self-energy considered while the u-channel pion exchange not and "$C_1\&C_2$" means both contributions included.

System	1^{--}		0^{--}	
t-channel	-63.5 ± 13.8		-72.4 ± 17.4	
g_S	g_{S0}	g_{S1}	g_{S0}	g_{S1}
C_2	$-61.5 - 3.5i$	$-61.5 - 9.2i$	$-70.0 - 3.5i$	$-70.0 - 8.9i$
$C_1\&C_2$	$-65.8 - 6.6i$	$-73.1 - 14.2i$	$-65.8 - 0.30i$	$-59.4 - 1.1i$

- Binding energies change $\mathcal{O}(10\text{ MeV})$ with 3-body effects.
- Called $\psi_0(4360)$ with mass 4366 ± 18 MeV.
- For 1^{--}, $D\bar{D}^*\pi$ partial width $\sim \Gamma_{D_1}$.
- For 0^{--}, $D\bar{D}^*\pi$ partial width is tiny. C and P conservation.
- Limited decay channels for 0^{--}, total decay width much smaller than 10 MeV.

Experimental search

- $\psi_0(4360)$ production channel in e^+e^- annihilation at $\sqrt{s} \sim 5$ GeV is P-wave $\eta\psi_0(4360)$. High chances in STCF with $e^+e^- \to \eta D\bar{D}^*$.
- Hard to distinguish from $\eta\psi(4360)$ with only invariant mass distribution of, e.g., $D\bar{D}^*$, $J/\psi\eta$. Angular distribution is necessary.
- $e^+e^- \to \gamma^* \to \eta(p_1) + \psi_0(p_2)$, $\mathcal{M}_0 \propto \epsilon(\gamma^*) \cdot q$
- $e^+e^- \to \gamma^* \to \eta(p_1) + \psi(p_2)$, $\mathcal{M}_1 \propto \epsilon_{\alpha\beta\gamma\delta}\epsilon^\alpha(\gamma^*)\epsilon^{*\beta}(\psi)P^\gamma q^\delta$.
- Sum over initial and final polarizations we have

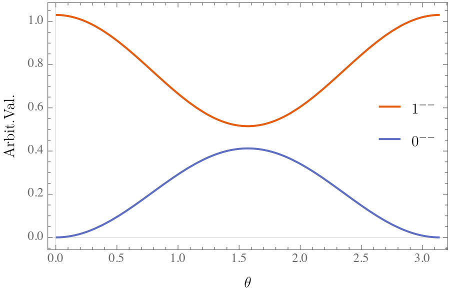
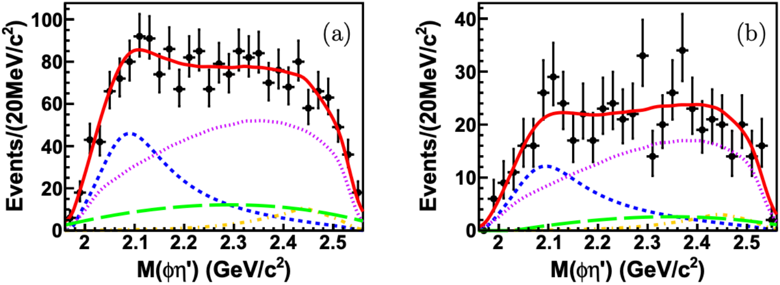


Figure: Angular distribution of $e^+e^- \to \eta\psi_{(0)}$. θ is the angle between the outgoing η and initial e^+e^- beam.

Contents

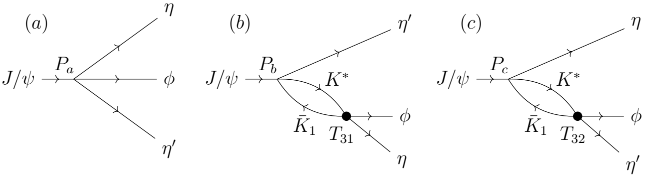
Experimental signals in hidden-strangeness system?

- Similar pattern is expected in hidden-strangeness sector.
- $D^* \bar{D}_1 \Rightarrow K^* \bar{K}_1$ and $J/\psi\eta^{(\prime)} \Rightarrow \phi\eta^{(\prime)}$.
- $J/\psi \to \phi\eta\eta'$



- $\phi\eta$ distribution not published.
- Let's fit it with $K^* \bar{K}_1(1270)$ channel.

Fitting framework



- Production amplitude of $J/\psi \to \phi\eta'\eta$

$$T_{J/\psi\to\phi\eta'\eta} = P_a q_\eta \tilde{q}_{\eta'} + P_b G_{33} T_{31} q_{\eta'} + P_c G_{33} T_{32} q_\eta,$$

with $J/\psi\eta$-$J/\psi\eta'$-$K^*\bar{K}_1$ scattering amplitudes,

$$\begin{aligned} T_{33} &= V_{33} + V_{33} G_{33} T_{33} + \mathcal{O}\left(V_{31}^2, V_{32}^2\right), \\ T_{31} &= T_{33} V_{33}^{-1} V_{31} + \mathcal{O}\left(V_{31}^3, V_{32}^3\right), \\ T_{32} &= T_{33} V_{33}^{-1} V_{32} + \mathcal{O}\left(V_{31}^3, V_{32}^3\right). \end{aligned}$$

- The differential decay width of J/ψ,

$$\frac{\mathrm{d}\Gamma_{J/\psi\to\phi\eta'\eta}}{\mathrm{d}M_{\phi\eta'}} = \int \mathrm{d}M_{\phi\eta}^2 \frac{2M_{\phi\eta'}}{256\pi^3 m_{J/\psi}^3} \left|T_{J/\psi\to\phi\eta'\eta}\right|^2 + \alpha f_{\mathrm{bg}}(M_{\phi\eta'}).$$

Fit results

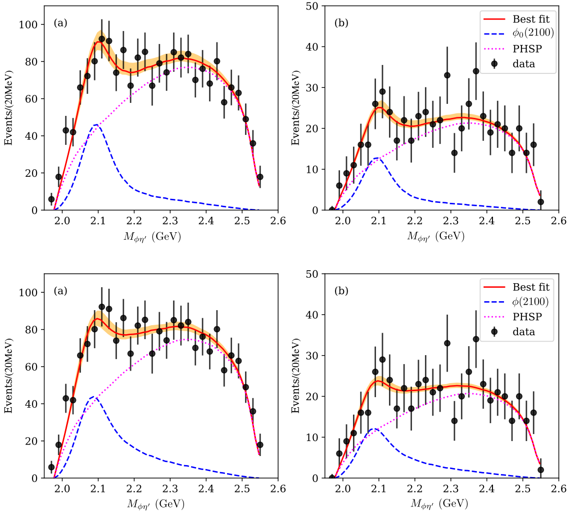


Figure: $K^*\bar{K}_1$ rescattering only in $J/\psi\eta'$ channel, $P_b = 0$.

Fit results

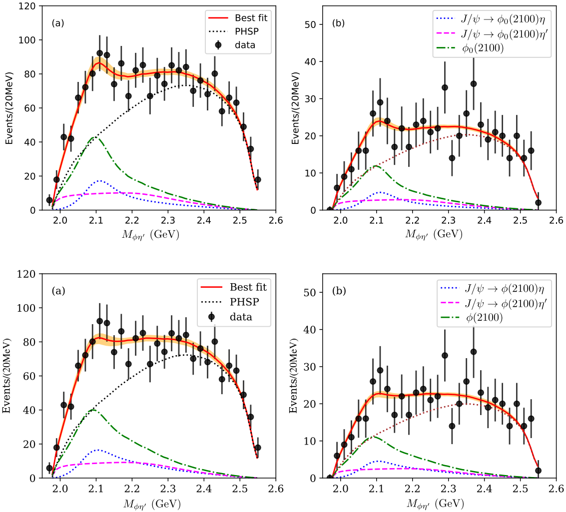
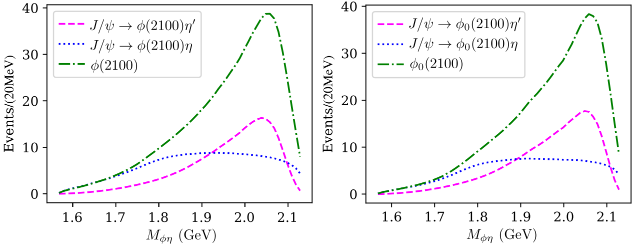


Figure: $K^*\bar{K}_1$ rescattering in both $J/\psi\eta'$ and $J/\psi\eta$ channels, P_b free.

Lineshape in $\phi\eta$ channel



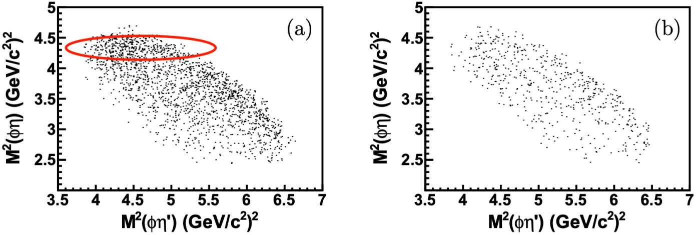


FIG. 3. Dalitz plots for modes I (a) and II (b).

Angular distribution

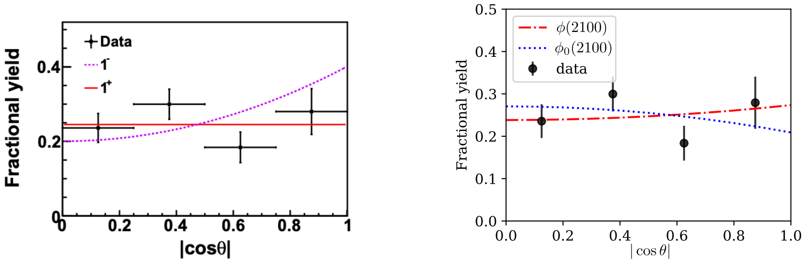


Figure: The η polar angular distribution. Left: BESIII analysis; Right: Our analysis.

- $J^P = 0^-$ is excluded in BESIII analysis.
- PHSP process contribution dominates, S-wave, flat,

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta} \propto \frac{1}{4}\begin{cases} \left(\tilde{\alpha}_1 + \frac{3}{4}(1-\tilde{\alpha}_1)(1+\cos^2\theta)\right) & \text{for } 1^{--} \\ \left(\tilde{\alpha}_0 + \frac{3}{4}(1-\tilde{\alpha}_0)(1-\cos^2\theta)\right) & \text{for } 0^{--} \end{cases}$$

  $\tilde{\alpha}_1 = 0.815$ and $\tilde{\alpha}_0 = 0.835$ are the fraction of the PHSP process.
- 0^{--} is also plausible.
- Using the complete set of J/ψ events in BESIII.

Contents

Summary

- No $J^{PC} = 0^{--}$ states observed up to now. Exotic quantum numbers.
- $\psi(4230), \psi(4360)$ & $\psi(4415)$ as 1^{--} $D^{(*)}D_{1,2}$ molecules $\Rightarrow$ Robust prediction of narrow 0^{--} $\psi_0(4360)$.
  - Meson-exchange;
  - 3-body effects.
- Searched for in $e^+e^- \to \eta\psi_0 \to \eta(J/\psi\eta^{(\prime)}/D\bar{D}^*)$. High chances in STCF.
- Hints of the 0^{--} hadronic molecule in hidden-strangeness sector, to be confirmed by using the full set of J/ψ events in BESIII.

Thanks for your attention!