Coupled channel analysis of vector charmonia

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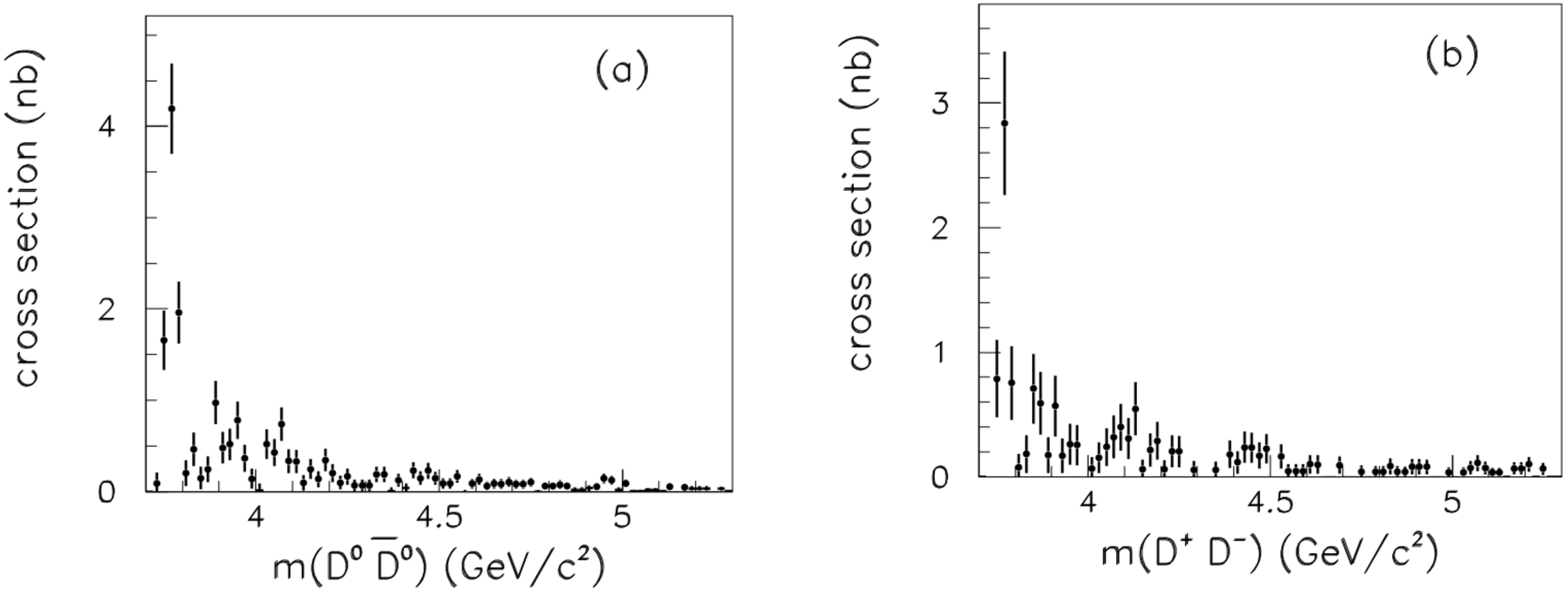
The experimental status of vector charmonia

- $e^+e^- \to \text{hadrons} \;\Rightarrow\; N_c = 3$
- The cross sections of a pair of open charm mesons take a large fraction
- Study the normal vector charmonia and exotic vector charmonium-like states



- Integrated luminosity 384 fb^{-1}
- $G(3900)$ $\quad M(G(3900)) = 3943 \pm 17_{\text{stat}} \pm 12_{\text{syst}}$ MeV

BaBar@2007, arXiv:0710.1371

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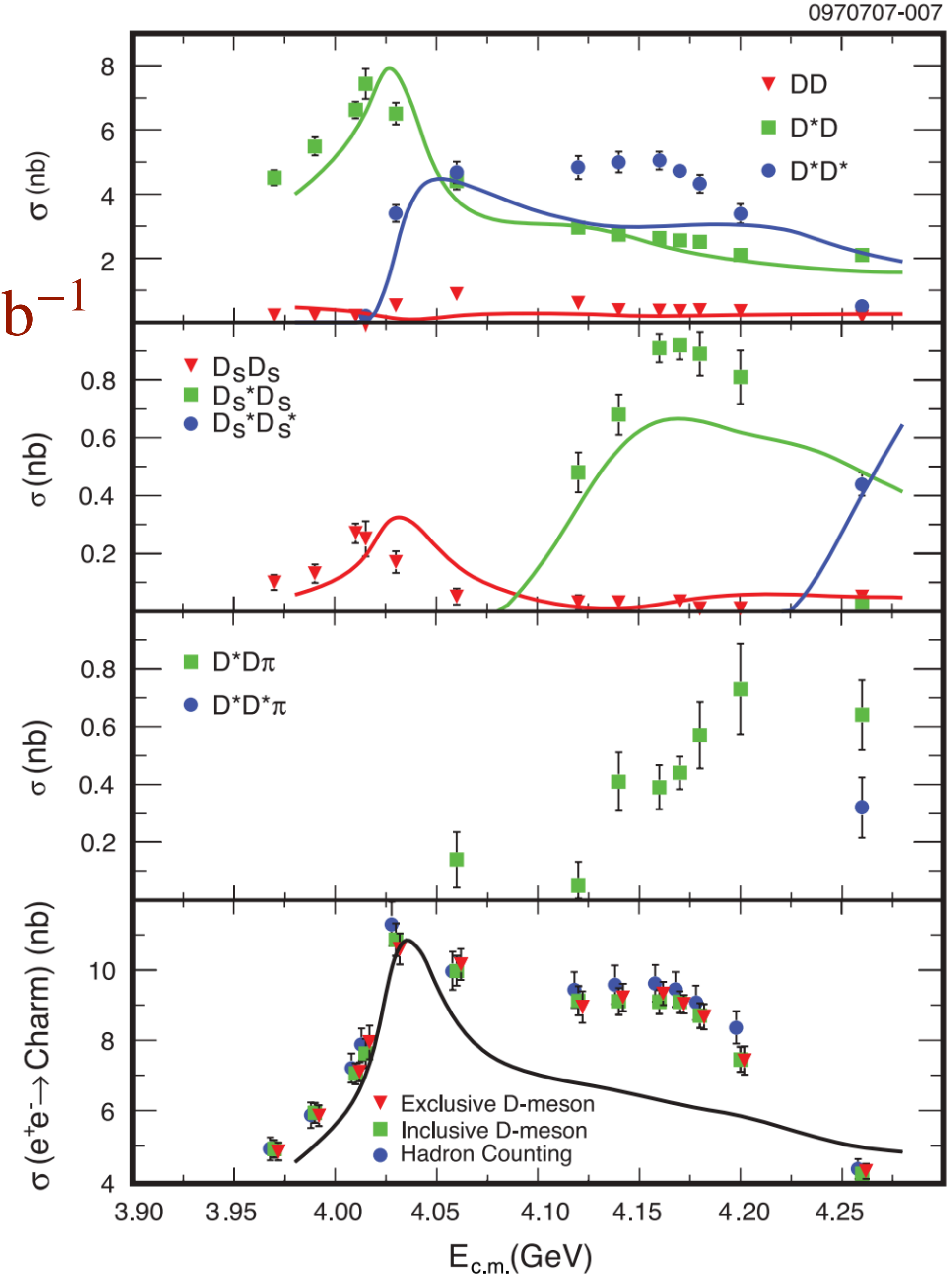
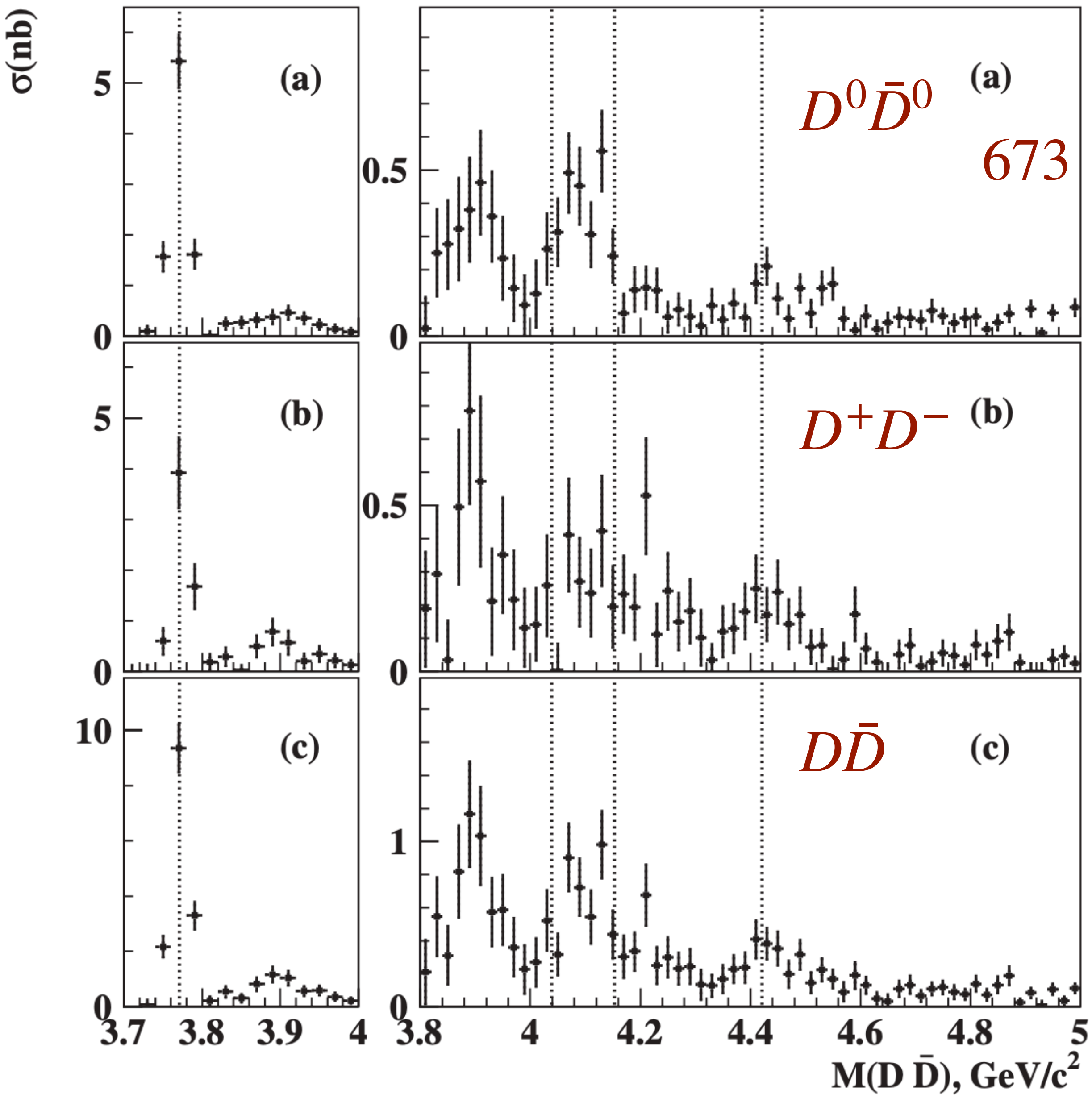


60 pb^{-1}

CLEO@2009, PRD80(2009)072001



$D^0\bar{D}^0$

673 fb^{-1}

D^+D^-

$D\bar{D}$

Belle@2008, PRD77(2008)011103(R)

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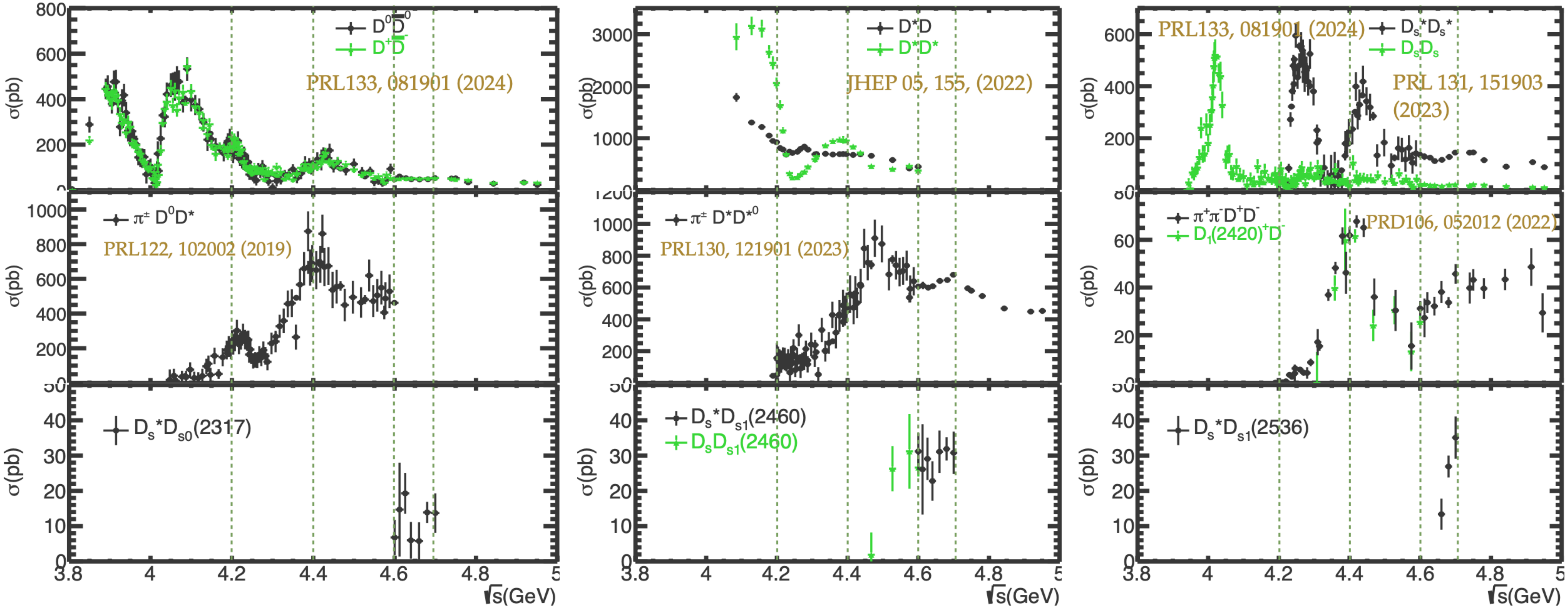


- High precision measurement from BESIII

Slides from Yuping Guo on the 1st Sino-Thai Workshop on Hadron Physics

The theoretical status of vector charmonia

- The non-$D\bar{D}$ decay width of $\psi(3770)$

Zhang, Li, Zhao, PRL102(2009)172001, Liu, Zhang, Li, PLB675(2009)441 ,

Hanhart, Kürten, Reboud, et al., EPJC84(2024)483, Shamov, Todyshev, PLB769(2017)187

- The time-like electromagnetic form factor of $D^* \to D$

Zhang, Zhao, PRD81(2010)074016

- The resonance parameters of given vector charmonium states

Zhang, Zhao, PRD81(2010)034011, Chen, Zhao, PLB718(2013)1369, Cao, Lenske, arXiv:1410.1375,

Coito, Giacosa, NPA981(2019)38, Uglov, Kalashnikova, Nefediev, et al., JETP Lett.105(2017)1,

Shamov, Todyshev, PLB769(2017)187, Hüsken, Lebed, Mitchell, et al., PRD109(2024)114010,

Hanhart, Kürten, Reboud, et al., EPJC84(2024)483

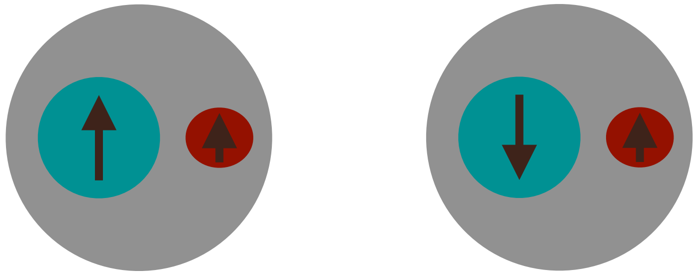
- New exotic vector charmonium-like state

Phys.Rev.C, 98(2018)044002, Du, Meiβner, QW, PRD94(2016)096006

The Heavy Quark Spin Symmetry

The Heavy Quark Spin Symmetry

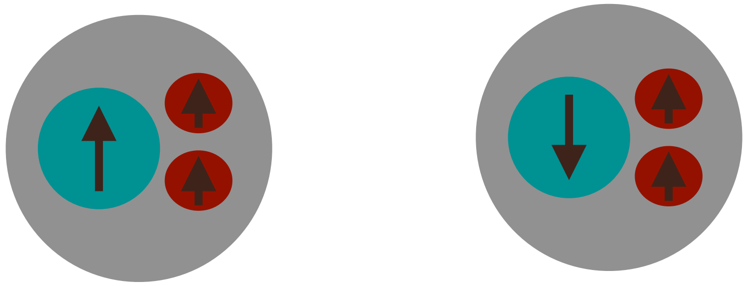
$m_Q \to \infty$ the strong interaction independent of the spin of heavy quark



$J = s_l + \frac{1}{2}$ $J = s_l - \frac{1}{2}$

$m_{D^*} - m_D = 142$ MeV

$s_l = \frac{1}{2}^-$ doublet



$J = s_l + \frac{1}{2}$ $J = s_l - \frac{1}{2}$

$m_{\Sigma_c^*} - m_{\Sigma_c} = 64$ MeV

$s_l = 1^+$ doublet

Spin rearrangement $\bar{D}^{(*)}$ $D^{(*)}$

$$\left([\bar{Q}q_{J_{l_1}}]_{j_1} l [Q\bar{q}_{J_{l_2}}]_{j_2}\right)_J \sim \sum_{HL} \mathscr{C}^{j_1 j_2 J}_{j_{l_1} j_{l_2} HL} \left((\bar{Q}Q)_H (q\bar{q}l)_L\right)_J$$

Hadron basis Heavy-Light basis

The Heavy-Light decomposition

- Spin rearrangement

Du, Meiβner, QW, PRD94(2016)096006

$$\left| l\left(\left[s_{l_1}s_{Q_1}\right]_{j_1}\left[s_{l_2}s_{Q_2}\right]_{j_2}\right)_s\right\rangle_J = \sum_{s_l,s_Q,s_q}(-1)^{l+s_q+s_Q+J}\hat{s}_q\hat{s}_Q\hat{j}_1\hat{j}_2\hat{s}\hat{s}_l\begin{Bmatrix} s_{l_1} & s_{Q_1} & j_1 \\ s_{l_2} & s_{Q_2} & j_2 \\ s_q & s_Q & s \end{Bmatrix}$$

$$\times\begin{Bmatrix} l & s_q & s_l \\ s_Q & J & s \end{Bmatrix}\left|\left(l\left[s_{l_1}s_{l_2}\right]_{s_q}\right)_{s_l}\left[s_{Q_1}s_{Q_2}\right]_{s_Q}\right\rangle_J$$

- s_{Q_i}, s_{l_i} the spins of heavy (anti)quark and light (anti)quark of the *i*th hadron
- l the relative orbital angular momentum between the two hadrons
- j_1, j_2 the spin of the two hadrons
- s_Q, s_q the spins of heavy quark pair and light quark pair
- s_l light degree of freedom
- J the angular momentum of the system

The decomposition of the P-wave $D^{(*)}\bar{D}^{(*)}$ pair

- The decomposition of $1^{-\mp}$ $D^{(*)}\bar{D}^{(*)}$ pair

Du, Meiβner, QW, PRD94(2016)096006

$J^{PC} = 1^{--}$ $\quad D^{(*)}\bar{D}^{(*)}:$

$$|D\bar{D}\rangle_{1^{--}} = \frac{1}{2}|0 \otimes 1\rangle + \frac{1}{2\sqrt{3}}|1 \otimes 0\rangle - \frac{1}{2}|1 \otimes 1\rangle + \frac{1}{2}\sqrt{\frac{5}{3}}|1 \otimes 2\rangle,$$

$$|\, D\bar{D}^* + \text{ c.c. } \rangle_{1^{--}} = -\frac{1}{\sqrt{3}}|1 \otimes 0\rangle + \frac{1}{2}|1 \otimes 1\rangle + \frac{1}{2}\sqrt{\frac{5}{3}}|1 \otimes 2\rangle,$$

$$\left|D^*\bar{D}^*\right\rangle_{1^{--}}^{s=0} = \frac{1}{2}\sqrt{3}\,|0 \otimes 1\rangle - \frac{1}{6}|1 \otimes 0\rangle + \frac{1}{2\sqrt{3}}|1 \otimes 1\rangle - \frac{\sqrt{5}}{6}|1 \otimes 2\rangle,$$

$$\left|D^*\bar{D}^*\right\rangle_{1^{--}}^{s=2} = \frac{\sqrt{5}}{3}|1 \otimes 0\rangle + \frac{1}{2}\sqrt{\frac{5}{3}}|1 \otimes 1\rangle + \frac{1}{6}|1 \otimes 2\rangle\,.$$

$J^{PC} = 1^{-+}$ $\quad D^{(*)}\bar{D}^{(*)}:$

$$|\, D\bar{D}^* + \text{ c.c. } \rangle_{1^{-+}} = -\frac{1}{\sqrt{2}}|0 \otimes 1\rangle + \frac{1}{\sqrt{2}}|1 \otimes 1\rangle$$

$$\left|D^*\bar{D}^*\right\rangle_{1^{-+}}^{s=1} = \frac{1}{\sqrt{2}}|0 \otimes 1\rangle + \frac{1}{\sqrt{2}}|1 \otimes 1\rangle$$

The decomposition of the P-wave $D^{(*)}\bar{D}^{(*)}$ pair

- Coefficient collected in a matrix

Du, Meißner, QW, PRD94(2016)096006

$$g^{1^{--}} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2\sqrt{3}} & -\frac{1}{2} & \frac{1}{2}\sqrt{\frac{5}{3}} \\ 0 & -\frac{1}{\sqrt{3}} & \frac{1}{2} & \frac{1}{2}\sqrt{\frac{5}{3}} \\ \frac{1}{2}\sqrt{3} & -\frac{1}{6} & \frac{1}{2\sqrt{3}} & -\frac{\sqrt{5}}{6} \\ 0 & \frac{\sqrt{5}}{3} & \frac{1}{2}\sqrt{\frac{5}{3}} & \frac{1}{6} \end{pmatrix}$$

- Low-Energy constants

$$C_1 \equiv V_{01} = \langle 0 \otimes 1 | \hat{H} | 0 \otimes 1 \rangle,$$
$$C_2 \equiv V_{10} = \langle 1 \otimes 0 | \hat{H} | 1 \otimes 0 \rangle,$$
$$C_3 \equiv V_{11} = \langle 1 \otimes 1 | \hat{H} | 1 \otimes 1 \rangle,$$
$$C_4 \equiv V_{12} = \langle 1 \otimes 2 | \hat{H} | 1 \otimes 2 \rangle,$$

- Include $\psi(2S), \psi(1D), \psi(3S), \psi(2D)$ bare charmonia

- The potential is

$$V = \begin{pmatrix} v_{ij} & v_{i\beta} \\ v_{\alpha j} & 0 \end{pmatrix}$$

$$v_{ij} = g_{ik}^{1^{--}} g_{jk}^{1^{--}} C_k, \quad v_{i\beta} = g_{il}^{1^{--}} \mu_{l\beta}, \quad v_{\alpha j} = g_{jk}^{1^{--}} \mu_{k\alpha}$$

- $i, j = 1{,}2{,}3{,}4$ for

  $D\bar{D}_{1^{--}}, (D\bar{D}^* + c\,.\,c.)_{1^{--}}, D^*\bar{D}^{*s=0}_{1^{--}}, D^*\bar{D}^{*s=2}_{1^{--}},$

- $\alpha, \beta = 1{,}2{,}3{,}4$ for

  $\psi(2S), \psi(1D), \psi(3S), \psi(2D)$

The reduction of LSE

- The 8 × 8 T-matrix is reduced to 4 × 4 T-matrix   Du, Meiβner, QW, PRD94(2016)096006

$$T_{ij} = \hat{V}_{ij} - \hat{V}_{ik} G_k T_{kj} \qquad \hat{V}_{ij} \equiv V_{ij} - V_{i\alpha} S_\alpha V_{\alpha j}$$

- The physical production amplitude satisfies

$$\mathcal{U}_i = \hat{\mathcal{F}}_i - \hat{V}_{ij} G_j \mathcal{U}_j \qquad \hat{\mathcal{F}} \equiv \mathcal{F}_i - V_{i\alpha} S_\alpha f_\alpha$$

S_α the propagator of the αth bare state

G_i the propagator of the ith two-hadron channel

- The $e^+e^- \to D^{(*)}\bar{D}^{(*)}$ amplitude $\quad \mathcal{M} = \bar{v}\left(p_+\right)\left(-ie\gamma_\mu\right)u\left(p_-\right)\frac{iP_\gamma^{\mu\nu}(p)}{s}\mathcal{T}_\nu$

- The $e^+e^- \to D^{(*)}\bar{D}^{(*)}$ differential cross section

$$\frac{d\sigma_i}{d\cos\theta} = \frac{\left|p_{D^{(*)}}\right|}{64\pi s^{3/2}}\left|\mathcal{M}_i\right|^2$$

$$\left|\mathcal{M}_1\right|^2 = \mathcal{U}_1^2 \frac{32\pi\alpha}{s}\left|p_D\right|^2\left(1 - \cos^2\theta\right)$$

$$\left|\mathcal{M}_2\right|^2 = \mathcal{U}_2^2 \frac{32\pi\alpha}{s}\left|p_D\right|^2\left(1 + \cos^2\theta\right)$$

$$\left|\mathcal{M}_4\right|^2 = \mathcal{U}_4^2 \frac{112\pi\alpha}{5s}\left|p_{D^*}\right|^2\left(1 - \frac{1}{7}\cos^2\theta\right)$$

$$\left|\mathcal{M}_3\right|^2 = \mathcal{U}_3^2 \frac{32\pi\alpha}{s}\left|p_{D^*}\right|^2\left(1 - \cos^2\theta\right)$$

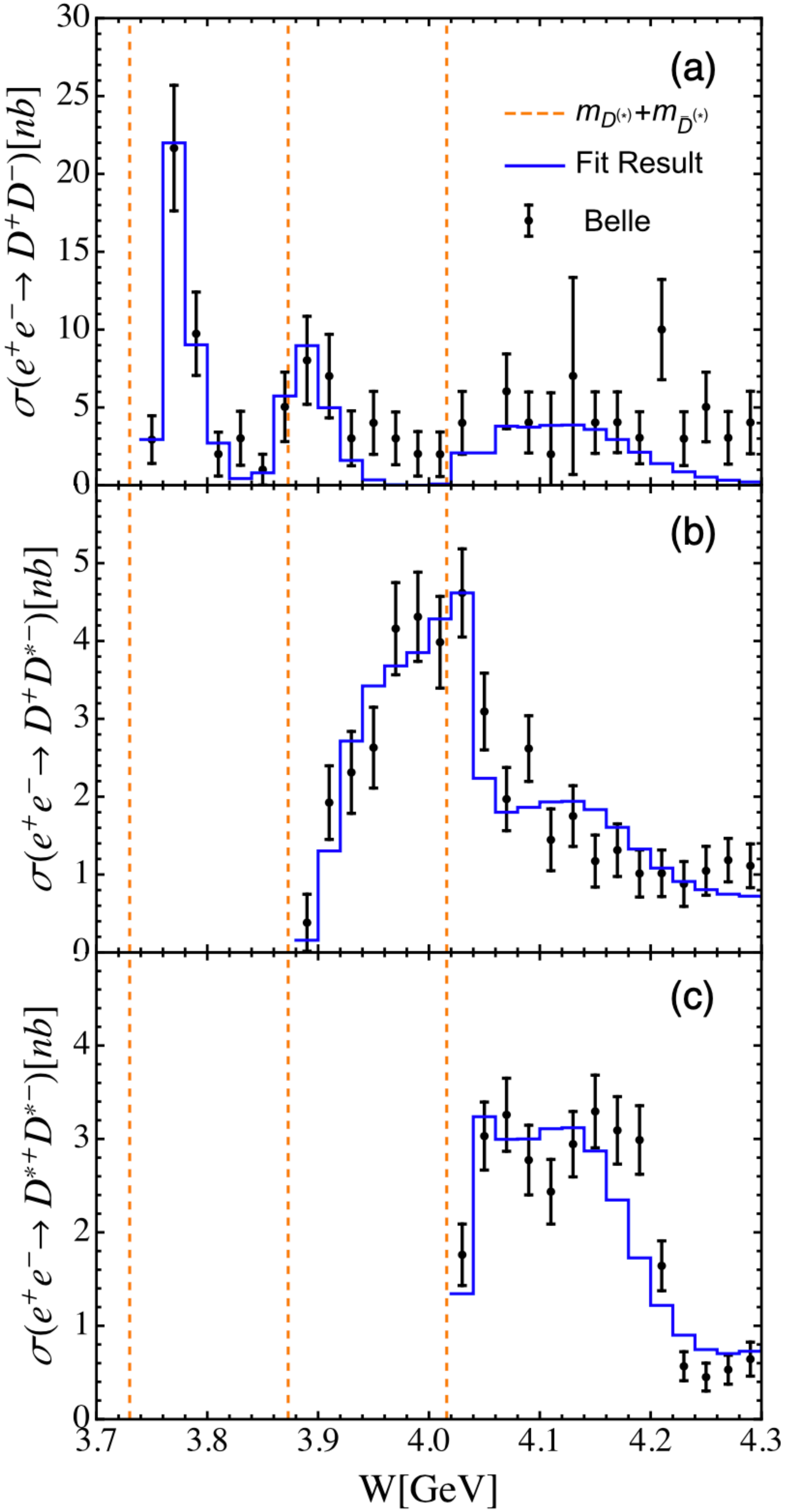
The results

Du, Meiβner, QW, PRD94(2016)096006

- The fitted parameters

$C_1(\mathrm{GeV}^{-2})$	$C_2(\mathrm{GeV}^{-2})$	$C_3(\mathrm{GeV}^{-2})$	$C_4(\mathrm{GeV}^{-2})$
79.70 ± 1.15	5.79 ± 0.22	43.90 ± 0.50	49.28 ± 1.37
$g_{2S}(\mathrm{GeV}^0)$	$g_{3S}(\mathrm{GeV}^0)$	$g_{1D}(\mathrm{GeV}^0)$	$g_{2D}(\mathrm{GeV}^0)$
0.90 ± 0.05	15.69 ± 0.04	3.65 ± 0.11	8.66 ± 0.15
$g^0_{2S}(\mathrm{GeV}^2)$	$g^0_{3S}(\mathrm{GeV}^2)$	$g^0_{1D}(\mathrm{GeV}^2)$	$g^0_{2D}(\mathrm{GeV}^2)$
0.22 ± 0.15	-0.17 ± 0.01	-0.05 ± 0.03	-0.15 ± 0.01
$f^0_S(\mathrm{GeV}^0)$	$f^0_D(\mathrm{GeV}^0)$	$a(3.9\ \mathrm{GeV})$	$\chi^2/\mathrm{d.o.f.}$
-1.55 ± 0.09	0.53 ± 0.08	0.56 ± 0.01	1.47

- $g_{2S}, g_{3S}, g_{1D}, g_{2D}$ the couplings between $D^{(*)}\bar{D}^{(*)}$ and $\psi(2S), \psi(3S), \psi(1D), \psi(2D)$
- $g^0_{2S}, g^0_{3S}, g^0_{1D}, g^0_{2D}$ the couplings between γ^* and $\psi(2S), \psi(3S), \psi(1D), \psi(2D)$
- $g^0_S \equiv \langle 1 \otimes 0 | \hat{H} | \gamma^* \rangle$



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Du, Meiβner, QW, PRD94(2016)096006

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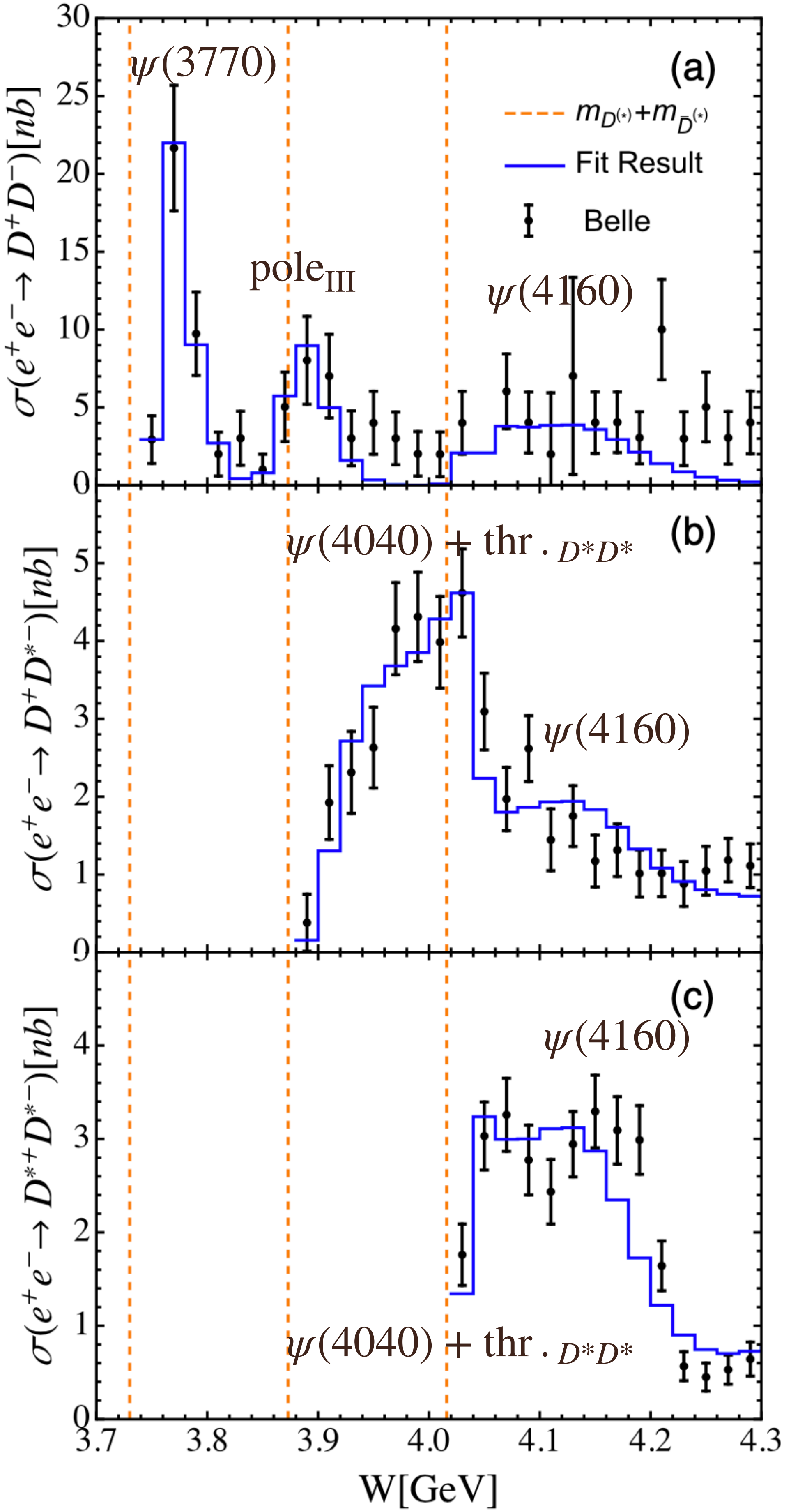
- The pole positions

Sheet	Poles (GeV)	$\lvert g_{D\bar{D}}\rvert$	$\lvert g_{D\bar{D}^*}\rvert$	$\lvert g_{D^*\bar{D}^*_{s=0}}\rvert$	$\lvert g_{D^*\bar{D}^*_{s=2}}\rvert$
II	$3.764 \pm i0.006$	13.53	9.48	5.88	16.78
III	$3.879 \pm i0.035$	4.40	10.96	7.63	18.15
IV	$4.034 \pm i0.014$	2.90	2.23	12.52	12.85



The results

Du, Meiβner, QW, PRD94(2016)096006

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$\psi(3770)$

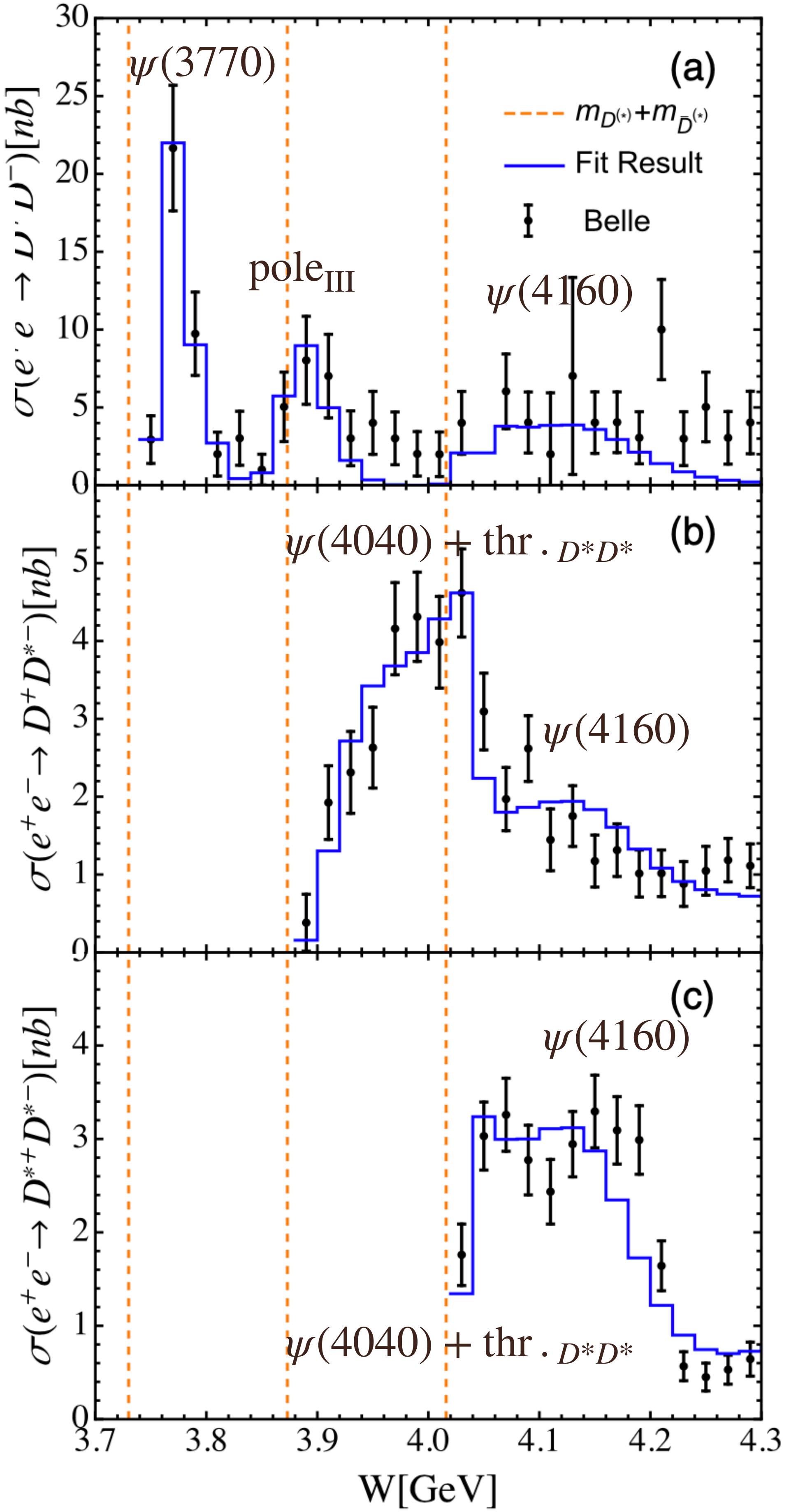
$\psi(4040)$

- The signals of charmonia are different
- The couplings ratios in HQSS

$$\text{S - wave charmonium}: \quad \frac{1}{2\sqrt{3}} : \frac{1}{\sqrt{3}} : \frac{1}{6} : \frac{\sqrt{5}}{3}$$

$$\text{D - wave charmonium}: \quad \frac{1}{2}\sqrt{\frac{5}{3}} : \frac{1}{2}\sqrt{\frac{5}{3}} : \frac{\sqrt{5}}{6} : \frac{1}{6}$$

- None of them are pure S/D-wave charmonium
- $|1 \otimes 0\rangle, |1 \otimes 2\rangle$ to J/ψ+S-wave and D-wave π



The results

- The angular distribution in $D^*\bar{D}^*$ channel

spin=0 $$\left|\mathcal{M}_3\right|^2 = \mathcal{U}_3^2 \frac{32\pi\alpha}{s} \left|p_{D^*}\right|^2 \left(1-\cos^2\theta\right)$$

spin=2 $$\left|\mathcal{M}_4\right|^2 = \mathcal{U}_4^2 \frac{112\pi\alpha}{5s} \left|p_{D^*}\right|^2 \left(1-\frac{1}{7}\cos^2\theta\right)$$

- The asymmetry for the fraction of the cross sections with $\theta < 60^\circ$ and $\theta > 120^\circ$

$$\mathscr{A}(E) \equiv \frac{\int_{-1.0}^{-0.5} \frac{d\sigma(E)}{d\cos\theta} d\cos\theta + \int_{0.5}^{1.0} \frac{d\sigma(E)}{d\cos\theta} d\cos\theta}{\sigma(E)}$$

$$\mathscr{A}_{s=0}(4.04\ \text{GeV}) = 0.31$$

$$\mathscr{A}_{s=2}(4.04\ \text{GeV}) = 0.48$$

- Our prediction at 4.04 GeV is $$\frac{d\sigma_{s=0}/d\cos\theta}{d\sigma_{s=2}/d\cos\theta} = \frac{0.41\left(1-\cos^2\theta\right)}{0.23\left(1-\frac{1}{7}\cos^2\theta\right)}$$

- Assume $\sigma_{s=0} = 1.59$ nb $\sigma_{s=2} = 1.23$ nb $\Rightarrow$ $\mathscr{A}(4.04\ \text{GeV}) = 0.39$

Du, Meiβner, QW, PRD94(2016)096006

The results

- The 1^{-+} state potential

$$V^{1^{-+}} = \begin{pmatrix} \frac{1}{2}C_1 + \frac{1}{2}C_3 & -\frac{1}{2}C_1 + \frac{1}{2}C_3 \\ -\frac{1}{2}C_1 + \frac{1}{2}C_3 & \frac{1}{2}C_1 + \frac{1}{2}C_3 \end{pmatrix}$$

- The Riemann sheets

Du, Meiβner, QW, PRD94(2016)096006

$$\text{I}\, \mathrm{Im}\, q_{D\bar{D}^*} > 0,\ \ \mathrm{Im}\, q_{D^*\bar{D}^*} > 0,\ \text{for } E < \mathrm{thr}._{DD^*}$$

$$\text{II}\, \mathrm{Im}\, q_{D\bar{D}^*} < 0,\ \ \mathrm{Im}\, q_{D^*\bar{D}^*} > 0,\ \text{for } \mathrm{thr}._{DD^*} < E < \mathrm{thr}._{D^*D^*}$$

$$\text{III}\, \mathrm{Im}\, q_{D\bar{D}^*} < 0,\ \ \mathrm{Im}\, q_{D^*\bar{D}^*} < 0,\ \text{for } E > \mathrm{thr}._{D^*D^*}$$

- The pole positions

Sheets	Poles (GeV)	$\lvert g_{D\bar{D}^*}\rvert$	$\lvert g_{D^*\bar{D}^*_{s=1}}\rvert$
II	$3.915 \pm i0.003$	7.91	3.48

- The expected measured channels (the interference pattern similar to the two Z_bs)

$$\mathscr{A}\left(e^+e^- \to \gamma 1^{-+} \to \gamma J/\psi 3\pi\right) \propto \frac{g_{\gamma 1} g_{J/\psi 1}}{E - E_1 + i\Gamma_1/2} + \frac{g_{\gamma 2} g_{J/\psi 2}}{E - E_2 + i\Gamma_2/2},$$

$$\mathscr{A}\left(e^+e^- \to \gamma 1^{-+} \to \gamma \eta_c 4\pi\right) \propto \frac{g_{\gamma 1} g_{\eta_c 1}}{E - E_1 + i\Gamma_1/2} + \frac{g_{\gamma 2} g_{\eta_c 2}}{E - E_2 + i\Gamma_2/2}$$

Bondar, Garmash, Milstein, Mizuk, Voloshin, PRD84(2011)054010

Summary

- A coupled channel analysis of the $e^+e^- \to D\bar{D}, D\bar{D}^* + c\,.\,c\,.\,, D^*\bar{D}^*$ processes in HQSS and SU(2) within the energy region [3.7,4.25] GeV
- Pole positions: $\psi(3770):\ \ 3.764 \pm i0.006\ \text{GeV}$,

  $\psi(4040):\ \ 4.034 \pm i0.014\ \text{GeV}$
- Pole position of $G(3900)$, $3.879 \pm i0.035\ \text{GeV}$
- In J^{-+} channel, another exotic state with pole position $3.915 \pm i0.003\ \text{GeV}$
- Propose asymmetry in $D^*\bar{D}^*$ channel to distinguish the $s=0$ and $s=2$ contributions

Thank you very much for your attention!

Outlook

- High precision data from BESIII
- SU(2) ⇒SU(3)
- Whether the pole position of $G(3900)$ is a dynamically generated state or a shift of normal charmonium?
- Predict the unmeasured line shapes, for instance the $e^+e^- \to D_s^+ D_s^{*-}$ process
- Extract the non-open charmed meson widths
- Exotic states in the $J^{PC} = 1^{-+}$ channel

Thank you very much for your attention!