

Tetraquarks in quark and diquark models

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based on

MNA & T. J. Burns

- PLB **847**, 138248, 2023 (Mass Relations)
- PRD **110**, 034012, 2024 ($cc\bar{c}\bar{c}$ Tetraquarks)

The 6th International Workshop on Future Tau Charm Facilities (FCTF 2024),
November 19, 2024, Guangzhou, China



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Tetraquarks $QQ\bar{q}\bar{q}$

FCTF-2024 Guangzhou

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Contents of the Talk

Part I: Formalism

- New mass relations for tetraquarks
- Validity check

The experimental era of all-heavy tetraquark spectroscopy started by LHCb in 2020, with the first observation of an apparent $cc\bar{c}\bar{c}$ state, dubbed $X(6900)$, in the $J/\psi J/\psi$ final state [39]. Model scenarios were then considered in, for example, Refs. [2,30,40,41]. The $X(6900)$ state was subsequently confirmed by CMS which, in addition, identified two further states in $J/\psi J/\psi$ decays, reported as $X(6600)$

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Part II: $cc\bar{c}\bar{c}$ Application

- Mass spectrum
- Interpretation of LHC States
- Decays to $D^{(\prime)}\bar{D}^{(\prime)}$ and $J= J=$

MNA & Burns, PRD 110, 034012, 2024

experimental data on $cc\bar{c}\bar{c}$ states to the predictions of diverse theoretical approaches, aiming to identify and discriminate among various plausible model scenarios.

As well as the experiments at the LHC, the future Super τ -Charm Facility (STCF) [49], which is currently under development, will be ideal for the study of $cc\bar{c}\bar{c}$ states. The center-of-mass energy of this electron-positron collider can reach 7 GeV, which is sufficient for the production of two $c\bar{c}$ pairs, and covers the relevant mass range of the $cc\bar{c}\bar{c}$ states discovered so far, and their presumed partners. In addition to decays into charmonia pairs (such as $J/\psi J/\psi$), one also expects $cc\bar{c}\bar{c}$ states to decay into pairs of charm and anticharm mesons (such as $D^{(\prime)}\bar{D}^{(\prime)}$) via the annihilation of a $c\bar{c}$ pair into a gluon. Identifying such decays at the LHC will be difficult, due to the high background.

Physical Ansatz

Consider a $QQ\bar{q}\bar{q}$ system, \bar{q} can be **heavy or light** anti-quark

! Pauli principle constrains the colour and spin of the QQ and $\bar{q}\bar{q}$ pairs

! For QQ pair (colour, spin) = $(\bar{\mathbf{3}}, 1)$ or $(\mathbf{6}, 0)$;

and for $\bar{q}\bar{q}$ pair $(\mathbf{3}, 1)$ or $(\bar{\mathbf{6}}, 0)$

S-wave multiplet (subscripts are colour and superscripts are spins)

$$'_2 = f(QQ)_{\mathbf{3}}^1 (\bar{q}\bar{q})_{\mathbf{3}}^1 g^2 \quad 2^{+(+)}$$

$$'_1 = f(QQ)_{\mathbf{3}}^1 (\bar{q}\bar{q})_{\mathbf{3}}^1 g^1 \quad 1^{+()}$$

$$'_0 = f(QQ)_{\mathbf{3}}^1 (\bar{q}\bar{q})_{\mathbf{3}}^1 g^0 \quad 0^{+(+)}$$

$$'_0^{\prime} = jf(QQ)_{\mathbf{6}}^0 (\bar{q}\bar{q})_{\mathbf{6}}^0 g^0 \quad 0^{+(+)}$$

Treatment of colour basis

- Quark model: four states with $|M_0 < M_1 < M_2 < M_0^{\prime}|$
- Diquark model: three S-wave states $'_2, '_1$ and $'_0$ with $M_0 < M_1 < M_2$

Model Considerations

- For all-heavy tetraquarks $QQ\bar{Q}\bar{Q}$, the characteristic scale is $1/(m_Q - m_s)$
- Dynamics are dominated by the short-distance OGE interaction
- Potential can be treated as pair-wise, quark-level interactions

MNA *et al.* *Eur.Phys.J.C* 78 (2018) 8, 647

Chromomagnetic interaction model (CMI)

$$H = \bar{M} \sum_{i < j} C_{ij} \lambda_i \lambda_j \sigma_i \sigma_j; \quad (1)$$

\bar{M} is the centre of mass (valence quarks + chromoelectric contribution)

λ_i colour and σ_i spin (Pauli) matrices

C_{ij} are (positive) parameters which depend on quark flavours

In [PLB 847, 138248, 2023](#), we showed that the CMI model produces same results as the OGE quark potential model under some symmetry assumptions. Note

$$C_{ij} = \frac{s}{6} \frac{1}{m^2} \langle \chi^3(\mathbf{r}_{ij}) \rangle;$$

Model

For two flavour $QQq\bar{q}$ case, three couplings C_{QQ} , C_{qq} and C_{Qq} required
For convenience, let's define a ratio

$$R = \frac{2C_{Qq}}{C_{QQ} + C_{qq}} \quad \text{for } \bar{q} = \bar{Q}; \quad R = \frac{C_{QQ}}{C_{QQ}}$$

Express masses in terms of R

$$M_2 = \bar{M} + \frac{8}{3} C_{QQ} + C_{qq} (1 + R),$$
$$M_1 = \bar{M} + \frac{8}{3} C_{QQ} + C_{qq} (1 - R),$$

For scalars, colour mixture

$$H = \bar{M} + 2 C_{QQ} + C_{qq} \left(\frac{8}{3} (1 - \frac{2R}{6R}) \right) \quad \frac{4}{4} \frac{\rho}{6R}$$

Mixing angle

$$\theta = \tan^{-1} \frac{\Delta}{\frac{1}{6} \frac{4R}{6R}} \quad \Delta = \frac{\rho}{232R^2 + 8R + 1}$$

Tetraquark Mass Relations

./ New mass relation *among* tetraquark masses

MNA & Burns, PLB 847, 138248, 2023

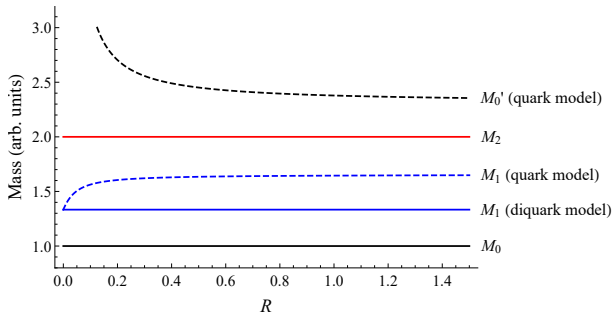
$$M_1 = M_0 + \frac{\Delta}{1 + 8R} (M_2 - M_0) \quad (2)$$

$$M_0^\theta = M_0 + \frac{2\Delta}{1 + 8R} (M_2 - M_0) \quad (3)$$

$$\Delta = \rho \sqrt{232R^2 + 8R + 1}$$

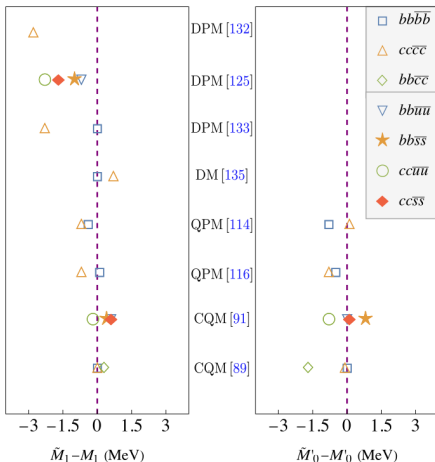
In the diquark model, $M_1 = \frac{1}{3} (2M_0 + M_2)$:

($R = 0$, Type-II diquark model Maiani & Polosa)



Validity of mass relations

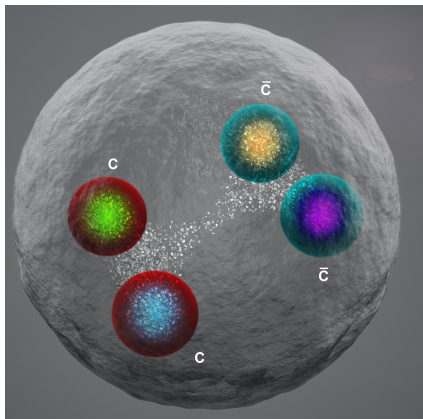
M_1 model predictions, \tilde{M}_1 from mass relations; two flavours $QQ\bar{q}\bar{q}$ (isovectors only)



Chromomagnetic quark model CQM
Quark potential models QPM

Diquark model DM
Diquark potential model DPM

Charm-Full Tetraquarks $cccc$



$cc\bar{c}\bar{c}$ states

LHCb 2020:

The experimental era of all-heavy tetraquark spectroscopy started at LHCb with $cc\bar{c}\bar{c}$ state $X(6900)$ observed in the $J^P = J^P$ final state

[Sci.Bull. 65 \(2020\) 23, 1983-1993](#)

CMS 2023:

The $X(6900)$ state was subsequently confirmed at CMS which, in addition, identified two further states $X(6600)$ and $X(7300)$ in $J^P = J^P$ decays

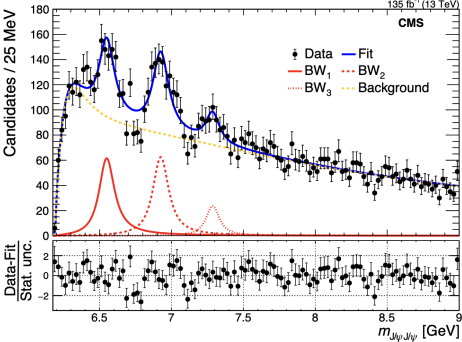
[Phys.Rev.Lett. 132 \(2024\) 11, 111901](#)

ATLAS 2023:

The $X(6900)$ was also confirmed in $J^P = J^P$ and $J^P = (2S)$ at ATLAS. Hint at a lower mass peak $X(6400)$ in addition to $X(6600)$

[Phys.Rev.Let. 131 \(2023\) 15, 151902](#)

$c\bar{c}c\bar{c}$ states at CMS & ATLAS



Parameters: Summary

State	Parameters	LHCb 2020			CMS 2023			ATLAS 2023	
$X(6900)$	M (MeV)	6905	11	7	6927	9	4	6860	30^{+10}_{-20}
	Γ (MeV)	80	19	33	122^{+24}_{-21}		18	110	50^{+20}_{-10}
$X(6600)$	M (MeV)				6552	10	12	6630	50^{+80}_{-10}
	Γ (MeV)				124^{+32}_{-26}		33	350	110^{+110}_{-40}
$X(6400)$	M (MeV)				“(6402 15)” ^y			6410	80^{+80}_{-30}
	Γ (MeV)				?			590	350^{+120}_{-200}

^y This entry is based on our finding.

$X(7300)$ is not included in this comparison.

Expected mass of $cc\bar{c}\bar{c}$: Naive phenomenology

From the observed masses,
 $X(6XXX)$ states are most likely to have four valence charm quarks

From ccu baryon Ξ_{cc}^{++} (3621.40 ± 0.78 MeV)
the mass of $cc\bar{c}\bar{c}$ state can be estimated very roughly

- cc pair has same quantum numbers in ccu baryon and $cc\bar{c}\bar{c}$ (w/o colour mix.) $(\bar{\mathbf{3}}, 1)$ of (colour, spin)
- cc $(\bar{\mathbf{3}}, 1)$ pair mass ranges 3200 – 3300 MeV e.g., [PRD, 95\(2017\) 034011](#)
- mass of S -wave ground state $cc\bar{c}\bar{c}$ lies in the ball park of $X(6400)$ – $X(6600)$

Mass Spectrum

For $c\bar{c}c\bar{c}$ tetraquarks, **two couplings** C_{cc} and $C_{\bar{c}\bar{c}}$ and \bar{M} (2+1 parameters)

For the simplest case, $R = 1$! $| \overline{C_{cc} = C_{\bar{c}\bar{c}} \quad C} |$

From meson and baryon spectrum, $| \overline{C = 5:0 \quad 0:5 \text{ MeV}} |$

Prog.Part.Nucl.Phys. 107 (2019) 237-320

Status	Quantum Numbers	Mass (MeV)
Prediction	0^{++}	6402 15
Prediction	1^+	6499 11
Input	2^{++}	6552 10 CMS-2023
Prediction	$0^{++ \ 0}$	6609 16

MNA & Burns PRD, 2024 2311.15853

Note: Also extracted C from $c\bar{c}c\bar{c}$ tetraquark spectrum, same conclusion

Mass Spectrum II

Mass dependence on coupling g

Mass Spectrum III

Mass dependence on ratio R

LHC states

In the $J = J =$, three S-wave states would be prominent: X (6600) 2^{++} ; X (6400) 0^{++} ; and $0^{++ 0}$

→ X (6400) needs careful treatment, CMS data show peaking behaviour around 6400 MeV, and ATLAS extracted mass for lowest peak is 6410 MeV
 $0^{++ 0}$ state lies at 6609 ± 16 MeV, shoulder in CMS data? and $J =$ (2S) threshold

Decays of 0^{++} states

Possible decays of $c\bar{c}$

Rearrangement decays

Annihilation decays

The allowed decays of $c\bar{c}$ states to combinations of $J = 0$ and $J = 1$ (rearrangement) and to $D^{(+)D^{(-)}}$ (annihilation) are constrained by C -parity.

! The channels accessible in S-wave are

$$2^{++} \rightarrow J = J = 0; D^+ D^+ \quad (4)$$

$$1^+ \rightarrow J = 0; D D^+; D^+ D^+ \quad (5)$$

$$0^{++(0)} \rightarrow J = J = 0; c^+ c^+; D^+ D^+; D D^+ \quad (6)$$

The 2^{++} state can also decay to $c^+ c^+$ (DD) but in D-wave, hence suppressed.

Ingredients for Decays

Spin recoupling (Fierz rearrangement)

$$f(cc)^1(cc)^1g^2 = f(cc)^1(cc)^1g^2 ; \quad (7a)$$

$$f(cc)^1(cc)^1g^1 = \rho \frac{1}{2} f(cc)^0(cc)^1g^1 + \rho \frac{1}{2} f(cc)^1(cc)^0g^1 ; \quad (7b)$$

$$f(cc)^1(cc)^1g^0 = \frac{\rho}{2} f(cc)^0(cc)^0g^0 + \frac{1}{2} f(cc)^1(cc)^1g^0 ; \quad (7c)$$

$$f(cc)^0(cc)^0g^0 = \frac{1}{2} f(cc)^0(cc)^0g^0 + \frac{\rho}{2} f(cc)^1(cc)^1g^0 ; \quad (7d)$$

Colour wavefunctions recouple as

$$j(cc)_3(cc)_3 = \sqrt{\frac{1}{3}} j(cc)_1(cc)_1 - \sqrt{\frac{2}{3}} j(cc)_8(cc)_8 ; \quad (8a)$$

$$j(cc)_6(cc)_6 = \sqrt{\frac{2}{3}} j(cc)_1(cc)_1 + \sqrt{\frac{1}{3}} j(cc)_8(cc)_8 ; \quad (8b)$$

Colour mixing

$$\begin{pmatrix} 0^{++} \\ 0^{++} \end{pmatrix} = \begin{pmatrix} \cos & \sin \\ \sin & \cos \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{with} \quad \tan \theta = \frac{1}{\rho} \frac{4R}{6\bar{R}} \quad (9)$$

Rearrangement decays

Decay amplitude factorises into spin, colour, and spatial parts. For example, for $0^{++} \rightarrow c\bar{c}$

$$\langle c\bar{c} | \hat{H}_0 | 0^{++} \rangle = \text{spin} \times \text{colour} \times A(\rho) \quad (10)$$

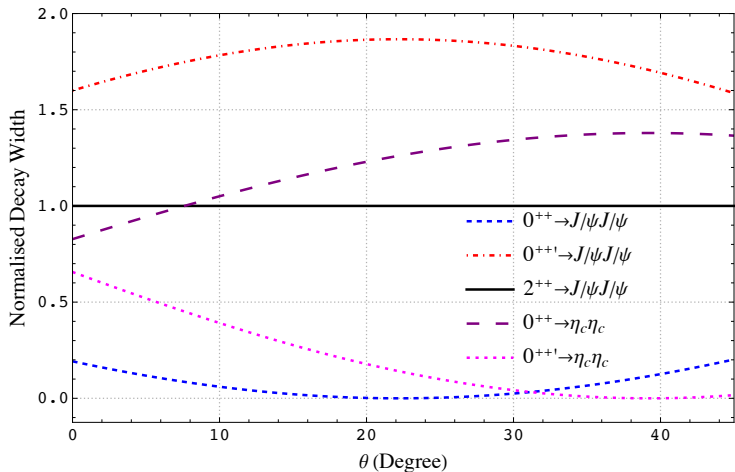
Normalised decay width

$$\frac{\Gamma(0^{++} \rightarrow \eta_c \eta_c)}{\Gamma(2^{++} \rightarrow J/\psi J/\psi)} = \frac{\omega(0^{++} \rightarrow \eta_c \eta_c)}{\omega(2^{++} \rightarrow J/\psi J/\psi)} \left[\frac{\rho_3}{2} \cos \theta + \frac{\rho_2}{2} \sin \theta \right]^2 \quad (11)$$

For full S-wave multiplet

Final State	$\theta = 35.6$		$\theta = 0$		2 ⁺⁺	1 ⁺
	0 ⁺⁺	0 ⁺⁺ ⁰	0 ⁺⁺	0 ⁺⁺ ⁰		
$J/\psi J/\psi$	0.072	1.76	0.19	1.60	1.0	
$\eta_c \eta_c$	1.38	0.01	0.83	0.66	0	
$J/\psi \eta_c$						1.08

Rearrangement decays

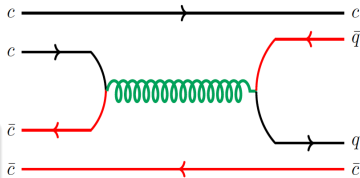


Annihilation decays

! Same strategy as rearrangement decays

$$hD \bar{D} j \hat{H}_2 j X_{cc\bar{c}\bar{c}} i = \text{spin} \quad \text{colour} \quad B(\rho)$$

Coefficients $f_{\text{spin}; \text{colour}} g$ are different than the rearrangement decays



Final State	$\theta = 35.6$		$\theta = 0$		2^{++}	1^+
	0^{++}	0^{++^0}	0^{++}	0^{++^0}		
$D \bar{D}$	0.14	0.011	0.062	0.094	1.0	0.248
$D \bar{D}$	0.46	0.034	0.20	0.29	0	
$D \bar{D} + \bar{D} D$						0.252

An interesting feature, decay rate of $f_{0^{++}; 0^{++^0}} g$! $D \bar{D} : D \bar{D} = 3 : 1$

$$\frac{\Gamma(0^{++} ! D \bar{D})}{\Gamma(0^{++} ! D \bar{D})} \quad \frac{\Gamma(0^{++^0} ! D \bar{D})}{\Gamma(0^{++^0} ! D \bar{D})} = 3.12 \quad (12)$$

Annihilation decays

Summary

New mass relations; existing literature confirms their validity at the MeV level

$X(6600)$ is **well described** as $\bar{c}^* c$ S-wave $c\bar{c}c\bar{c}$ tetraquark

The emergence of lowest scalar (0^+) around 6400 MeV is important to **analyse further**

The decay of lowest-scalar into $c\bar{c}c\bar{c}$ is notably **larger** as compared to $J=J=$

Annihilation decays of $c\bar{c}c\bar{c}$ states into $D^{(*)}D^{(*)}$ would provide an independent test to the existence and their structure

Super \bar{c} -Charm Facility STCF, with centre-of-mass energy up to **7 GeV** of colliding e^+e^- can **produce** $c\bar{c}c\bar{c}$ states

| Exciting Future for Exotic Hadron Spectroscopy! |

Thanks

1/2 Swansea Uni., Singleton Park

Quark vs Diquark Models

The ratio of splittings $\Delta_2 = \Delta_1$

MNA and Burns, arXiv:2311.15853

$$\Delta_1 = M_1 \quad M_0 \quad (13)$$

$$\Delta_2 = M_2 \quad M_1 \quad (14)$$

In the diquark model, $\Delta_2/\Delta_1 = 2$; in the quark model, $\Delta_2/\Delta_1 = 0.55$ with $R = 1$.

