Tetraquarks in quark and diquark models

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based on

MNA & T. J. Burns

- PLB **847**, 138248, 2023 (Mass Relations)
- PRD **110**, 034012, 2024 ($cc\bar{c}\bar{c}$ Tetraquarks)

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Swansea University
Prifysgol Abertawe

Contents of the Talk

Part I: Formalism

- New mass relations for tetraquarks
- Validity check

Part II: $cc\bar{c}\bar{c}$ Application

- Mass spectrum
- Interpretation of LHC States
- Decays to $D^{(*)}\bar{D}^{(*)}$ and $J/\psi J/\psi$

MNA & Burns, PRD 110, 034012, 2024

The experimental era of all-heavy tetraquark spectroscopy started by LHCb in 2020, with the first observation of an apparent $cc\bar{c}\bar{c}$ state, dubbed $X(6900)$, in the $J/\psi J/\psi$ final state [39]. Model scenarios were then considered in, for example, Refs. [2,30,40,41]. The $X(6900)$ state was subsequently confirmed by CMS which, in addition, identified two further states in $J/\psi J/\psi$ decays, reported as $X(6600)$

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experimental data on $cc\bar{c}\bar{c}$ states to the predictions of diverse theoretical approaches, aiming to identify and discriminate among various plausible model scenarios.

As well as the experiments at the LHC, the future Super τ-Charm Facility (STCF) [49], which is currently under development, will be ideal for the study of $cc\bar{c}\bar{c}$ states. The center-of-mass energy of this electron-positron collider can reach 7 GeV, which is sufficient for the production of two $c\bar{c}$ pairs, and covers the relevant mass range of the $cc\bar{c}\bar{c}$ states discovered so far, and their presumed partners. In addition to decays into charmonia pairs (such as $J/\psi J/\psi$), one also expects $cc\bar{c}\bar{c}$ states to decay into pairs of charm and anticharm mesons (such as $D^{(*)}\bar{D}^{(*)}$) via the annihilation of a $c\bar{c}$ pair into a gluon. Identifying such decays at the LHC will be difficult, due to the high background.

Physical Ansatz

Consider a $QQ\bar{q}\bar{q}$ system, $\bar{q}$ can be heavy or light anti-quark

$\hookrightarrow$ Pauli principle constrains the colour and spin of the QQ and $\bar{q}\bar{q}$ pairs
$\hookrightarrow$ For QQ pair (colour, spin) = ($\bar{\mathbf{3}}$,1) or ($\mathbf{6}$,0);
and for $\bar{q}\bar{q}$ pair ($\mathbf{3}$,1) or ($\bar{\mathbf{6}}$,0)

S-wave multiplet (subscripts are colour and superscripts are spins)

$$\begin{aligned}
|\varphi_2\rangle &= |\{(QQ)^1_{\bar{\mathbf{3}}}(\bar{q}\bar{q})^1_{\mathbf{3}}\}^2\rangle \quad 2^{+(+)} \\
|\varphi_1\rangle &= |\{(QQ)^1_{\bar{\mathbf{3}}}(\bar{q}\bar{q})^1_{\mathbf{3}}\}^1\rangle \quad 1^{+(-)} \\
|\varphi_0\rangle &= |\{(QQ)^1_{\bar{\mathbf{3}}}(\bar{q}\bar{q})^1_{\mathbf{3}}\}^0\rangle \quad 0^{+(+)} \\
|\varphi'_0\rangle &= |\{(QQ)^0_{\mathbf{6}}(\bar{q}\bar{q})^0_{\bar{\mathbf{6}}}\}^0\rangle \quad 0^{+(+)}
\end{aligned}$$

Treatment of colour basis

- Quark model: four states with $\boxed{M_0 < M_1 < M_2 < M'_0}$
- Diquark model: three S-wave states φ_2, φ_1 and φ_0 with $M_0 < M_1 < M_2$

Model Considerations

- For all-heavy tetraquarks $QQ\bar{Q}\bar{Q}$, the characteristic scale is $1/(m_Q\alpha_s)$
- Dynamics are dominated by the short-distance OGE interaction
- Potential can be treated as pair-wise, quark-level interactions

MNA *et al.* Eur.Phys.J.C 78 (2018) 8, 647

Chromomagnetic interaction model (CMI)

$$H = \overline{M} - \sum_{i<j} C_{ij}\ \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j\ \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, \tag{1}$$

$\overline{M}$ is the centre of mass (valence quarks + chromoelectric contribution)

$\boldsymbol{\lambda}_i$ colour and $\boldsymbol{\sigma}_i$ spin (Pauli) matrices

C_{ij} are (positive) parameters which depend on quark flavours

In PLB 847, 138248, 2023, we showed that the CMI model produces same results as the OGE quark potential model under some symmetry assumptions. Note

$$C_{ij} = \frac{\pi}{6}\,\frac{\alpha_s}{m^2}\,\langle \delta^3(\mathbf{r}_{ij})\rangle,$$

Model

For two flavour $QQ\bar{q}\bar{q}$ case, three couplings C_{QQ}, $C_{\bar{q}\bar{q}}$ and $C_{Q\bar{q}}$ required

For convenience, let's define a ratio

$$R = \frac{2C_{Q\bar{q}}}{C_{QQ} + C_{qq}} \qquad \text{for } \bar{q} = \bar{Q}, \qquad R = \frac{C_{Q\bar{Q}}}{C_{QQ}}$$

Express masses in terms of R

$$M_2 = \overline{M} + \frac{8}{3}\left(C_{QQ} + C_{qq}\right)(1 + R),$$
$$M_1 = \overline{M} + \frac{8}{3}\left(C_{QQ} + C_{qq}\right)(1 - R),$$

For scalars, colour mixture

$$H = \overline{M} + 2\left(C_{QQ} + C_{qq}\right)\begin{pmatrix} \frac{8}{3}(1 - 2R) & -4\sqrt{6}R \\ -4\sqrt{6}R & 4 \end{pmatrix}$$

Mixing angle

$$\theta = \tan^{-1}\left(\frac{\Delta - 1 - 4R}{6\sqrt{6}R}\right) \qquad \Delta = \sqrt{232R^2 + 8R + 1}$$

Tetraquark Mass Relations

↪New mass relation *among* tetraquark masses

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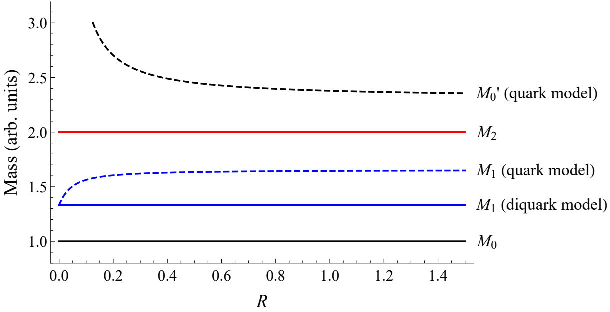
$$M_1 = M_0 + \frac{\Delta - 1}{\Delta - 1 + 8R}(M_2 - M_0) \quad (2)$$

$$M_0' = M_0 + \frac{2\Delta}{\Delta - 1 + 8R}(M_2 - M_0) \quad (3)$$

$$\Delta = \sqrt{232R^2 + 8R + 1}$$

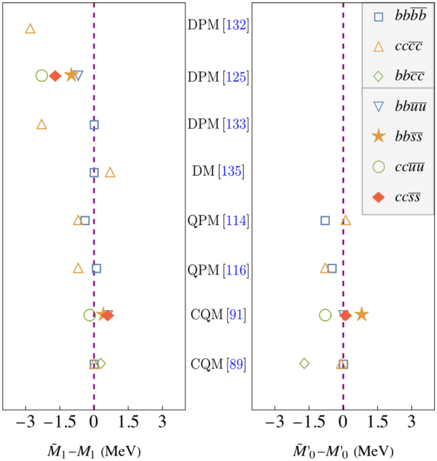
In the diquark model, $M_1 = \frac{1}{3}(2M_0 + M_2)$:

($R = 0$, Type-II diquark model Maiani & Polosa)



Validity of mass relations

M_1 model predictions, $\tilde{M}_1$ from mass relations; two flavours $QQ\bar{q}\bar{q}$ (isovectors only)

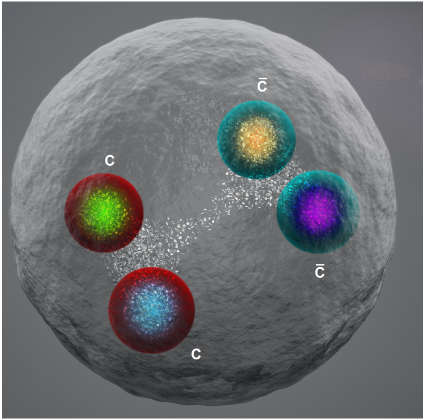


Chromomagnetic quark model CQM
Quark potential models QPM

Diquark model DM
Diquark potential model DPM

Part II

Charm-Full Tetraquarks $cc\bar{c}\bar{c}$



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$cc\bar{c}\bar{c}$ states

LHCb 2020:

The experimental era of all-heavy tetraquark spectroscopy started at LHCb with $cc\bar{c}\bar{c}$ state $X(6900)$ observed in the $J/\psi J/\psi$ final state

Sci.Bull. 65 (2020) 23, 1983-1993

CMS 2023:

The $X(6900)$ state was subsequently confirmed at CMS which, in addition, identified two further states $X(6600)$ and $X(7300)$ in $J/\psi J/\psi$ decays

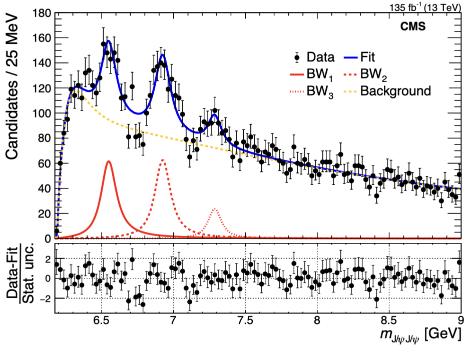
Phys.Rev.Lett. 132 (2024) 11, 111901

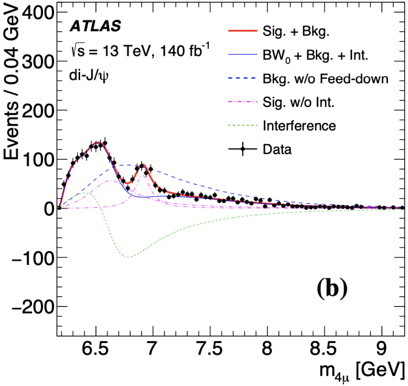
ATLAS 2023:

The $X(6900)$ was also confirmed in $J/\psi J/\psi$ and $J/\psi\psi(2S)$ at ATLAS. Hint at a lower mass peak $X(6400)$ in addition to $X(6600)$

Phys.Rev.Let. 131 (2023) 15, 151902

$cc\bar{c}\bar{c}$ states at CMS & ATLAS





Parameters: Summary

State	Parameters	LHCb 2020	CMS 2023	ATLAS 2023
$X(6900)$	M (MeV)	$6905 \pm 11 \pm 7$	$6927 \pm 9 \pm 4$	$6860 \pm 30^{+10}_{-20}$
	Γ (MeV)	$80 \pm 19 \pm 33$	$122^{+24}_{-21} \pm 18$	$110 \pm 50^{+20}_{-10}$
$X(6600)$	M (MeV)		$6552 \pm 10 \pm 12$	$6630 \pm 50^{+80}_{-10}$
	Γ (MeV)		$124^{+32}_{-26} \pm 33$	$350 \pm 110^{+110}_{-40}$
$X(6400)$	M (MeV)		"(6402 ± 15)"†	$6410 \pm 80^{+80}_{-30}$
	Γ (MeV)		?	$590 \pm 350^{+120}_{-200}$

† This entry is based on our finding.

$X(7300)$ is not included in this comparison.

Expected mass of $cc\bar{c}\bar{c}$: Naive phenomenology

From the observed masses,
$X(6xxx)$ states are most likely to have four valence charm quarks

From ccu baryon Ξ_{cc}^{++} (3621.40 ± 0.78 MeV)
the mass of $cc\bar{c}\bar{c}$ state can be estimated very roughly

- cc pair has same quantum numbers in ccu baryon and $cc\bar{c}\bar{c}$ (w/o colour mix.) $(\bar{\mathbf{3}},1)$ of (colour, spin)
- cc $(\bar{\mathbf{3}},1)$ pair mass ranges $3200 \sim 3300$ MeV e.g., PRD, 95(2017) 034011
- mass of S-wave ground state $cc\bar{c}\bar{c}$ lies in the ball park of $X(6400) \sim X(6600)$

Mass Spectrum

For $cc\bar{c}\bar{c}$ tetraquarks, two couplings C_{cc} and $C_{c\bar{c}}$ and $\overline{M}$ (2+1 parameters)

For the simplest case, $R = 1 \longrightarrow \boxed{C_{c\bar{c}} = C_{cc} \equiv C}$

From meson and baryon spectrum, $\boxed{C = 5.0 \pm 0.5 \text{ MeV}}$

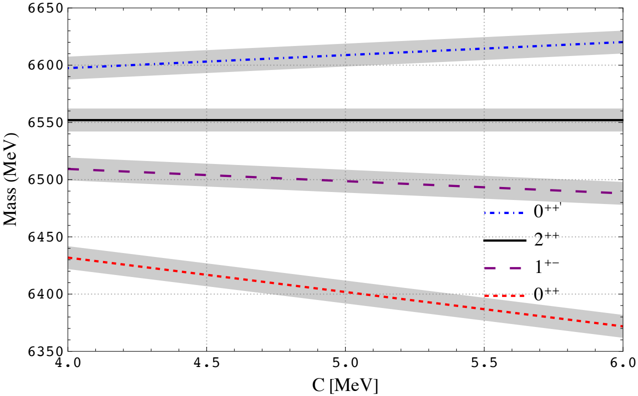
Prog.Part.Nucl.Phys. 107 (2019) 237-320

Status	Quantum Numbers	Mass (MeV)
Prediction	0^{++}	6402 ± 15
Prediction	1^{+-}	6499 ± 11
Input	2^{++}	6552 ± 10 CMS-2023
Prediction	$0^{++'}$	6609 ± 16

MNA & Burns PRD, 2024 2311.15853

Note: Also extracted C from $cc\bar{c}\bar{c}$ tetraquark spectrum, same conclusion

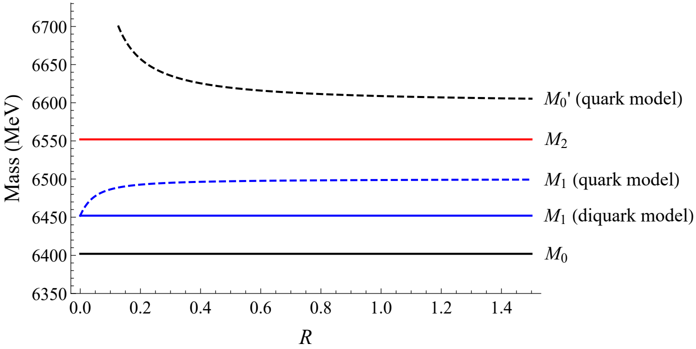
Mass Spectrum II



Mass dependence on coupling C

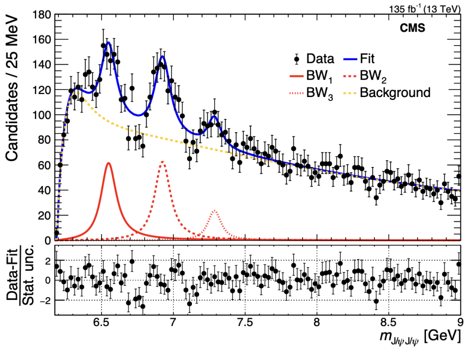
Mass Spectrum III

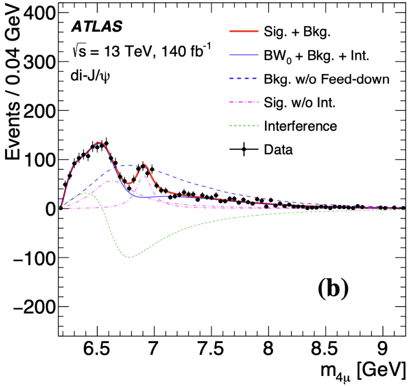
Mass dependence on ratio R



LHC states

In the $J/\psi J/\psi$, three S-wave states would be prominent: $X(6600)$ 2^{++}; $X(6400)$ 0^{++}; and $0^{++\prime}$



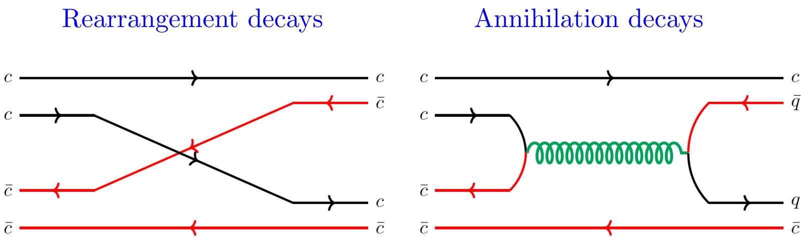


☛ $X(6400)$ needs careful treatment, CMS data show peaking behaviour around 6400 MeV, and ATLAS extracted mass for lowest peak is 6410 MeV

$0^{++\prime}$ state lies at 6609 ± 16 MeV, shoulder in CMS data? and $J/\psi\psi(2S)$ threshold

Decays of $cc\bar{c}\bar{c}$ states

Possible decays of $cc\bar{c}\bar{c}$



The allowed decays of $cc\bar{c}\bar{c}$ states to combinations of J/ψ and η_c (rearrangement) and to $D^{(*)}\bar{D}^{(*)}$ (annihilation) are constrained by C-parity.

$\hookrightarrow$ The channels accessible in S-wave are

$$2^{++} \to J/\psi J/\psi, D^*\bar{D}^* \tag{4}$$

$$1^{+-} \to J/\psi \eta_c, D\bar{D}^*, D^*\bar{D}^* \tag{5}$$

$$0^{++(\prime)} \to J/\psi J/\psi, \eta_c\eta_c, D^*\bar{D}^*, D\bar{D} \tag{6}$$

The 2^{++} state can also decay to $\eta_c\eta_c$ ($D\bar{D}$) but in D-wave, hence suppressed.

Ingredients for Decays

Spin recoupling (Fierz rearrangement)

$$\left|\{(cc)^1(\bar{c}\bar{c})^1\}^2\right\rangle = \left|\{(c\bar{c})^1(c\bar{c})^1\}^2\right\rangle, \tag{7a}$$

$$\left|\{(cc)^1(\bar{c}\bar{c})^1\}^1\right\rangle = \frac{1}{\sqrt{2}}\left|\{(c\bar{c})^0(c\bar{c})^1\}^1\right\rangle + \frac{1}{\sqrt{2}}\left|\{(c\bar{c})^1(c\bar{c})^0\}^1\right\rangle, \tag{7b}$$

$$\left|\{(cc)^1(\bar{c}\bar{c})^1\}^0\right\rangle = \frac{\sqrt{3}}{2}\left|\{(c\bar{c})^0(c\bar{c})^0\}^0\right\rangle - \frac{1}{2}\left|\{(c\bar{c})^1(c\bar{c})^1\}^0\right\rangle, \tag{7c}$$

$$\left|\{(cc)^0(\bar{c}\bar{c})^0\}^0\right\rangle = \frac{1}{2}\left|\{(c\bar{c})^0(c\bar{c})^0\}^0\right\rangle + \frac{\sqrt{3}}{2}\left|\{(c\bar{c})^1(c\bar{c})^1\}^0\right\rangle, \tag{7d}$$

Colour wavefunctions recouple as

$$\left|(cc)_{\bar{\mathbf{3}}}(\bar{c}\bar{c})_{\mathbf{3}}\right\rangle = \sqrt{\frac{1}{3}}\left|(c\bar{c})_{\mathbf{1}}(c\bar{c})_{\mathbf{1}}\right\rangle - \sqrt{\frac{2}{3}}\left|(c\bar{c})_{\mathbf{8}}(c\bar{c})_{\mathbf{8}}\right\rangle, \tag{8a}$$

$$\left|(cc)_{\mathbf{6}}(\bar{c}\bar{c})_{\bar{\mathbf{6}}}\right\rangle = \sqrt{\frac{2}{3}}\left|(c\bar{c})_{\mathbf{1}}(c\bar{c})_{\mathbf{1}}\right\rangle + \sqrt{\frac{1}{3}}\left|(c\bar{c})_{\mathbf{8}}(c\bar{c})_{\mathbf{8}}\right\rangle. \tag{8b}$$

Colour mixing

$$\begin{pmatrix} \left|0^{++}\right\rangle \\ \left|0^{++\prime}\right\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \left|\varphi_0\right\rangle \\ \left|\varphi_0'\right\rangle \end{pmatrix} \quad \text{with} \quad \theta = \tan^{-1}\left(\frac{\Delta - 1 - 4R}{6\sqrt{6}R}\right) \tag{9}$$

Rearrangement decays

Decay amplitude factorises into spin, colour, and spatial parts. For example, for $0^{++} \to \eta_c\eta_c$

$$\langle \eta_c\eta_c|\hat{H}_0|0^{++}\rangle = \phi_{\text{spin}} \times \phi_{\text{colour}} \times A(p) \tag{10}$$

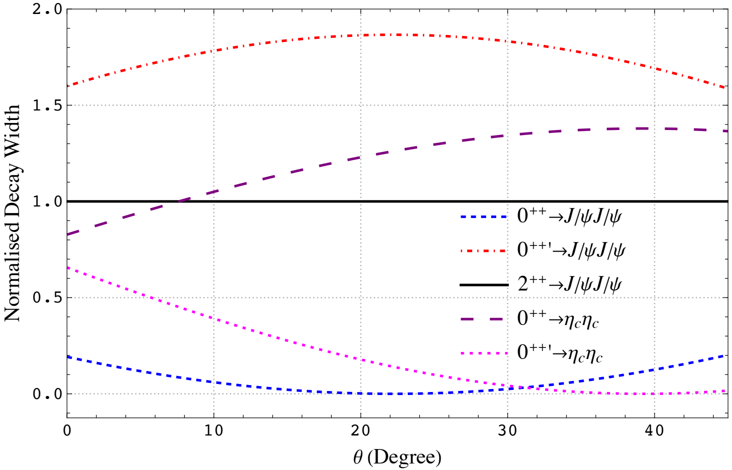
Normalised decay width

$$\frac{\Gamma(0^{++} \to \eta_c\eta_c)}{\Gamma(2^{++} \to J/\psi J/\psi)} = \frac{\omega(0^{++} \to \eta_c\eta_c)}{\omega(2^{++} \to J/\psi J/\psi)} \left(\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right)^2 \tag{11}$$

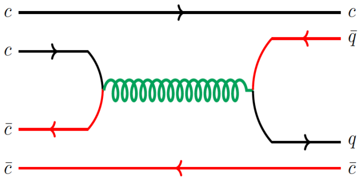
$\hookrightarrow$ For full S-wave multiplet

Final State	$\theta = 35.6^\circ$		$\theta = 0^\circ$		2^{++}	1^{+-}
	0^{++}	$0^{++\prime}$	0^{++}	$0^{++\prime}$		
$J/\psi J/\psi$	0.072	1.76	0.19	1.60	1.0	–
$\eta_c\eta_c$	1.38	0.01	0.83	0.66	~ 0	–
$J/\psi\eta_c$	–	–	–	–	–	1.08

Rearrangement decays



Annihilation decays



$\hookrightarrow$ Same strategy as rearrangement decays

$$\langle D^* \bar{D}^* | \hat{H}_2 | X_{cc\bar{c}\bar{c}} \rangle = \phi_{\text{spin}} \times \phi_{\text{colour}} \times B(p)$$

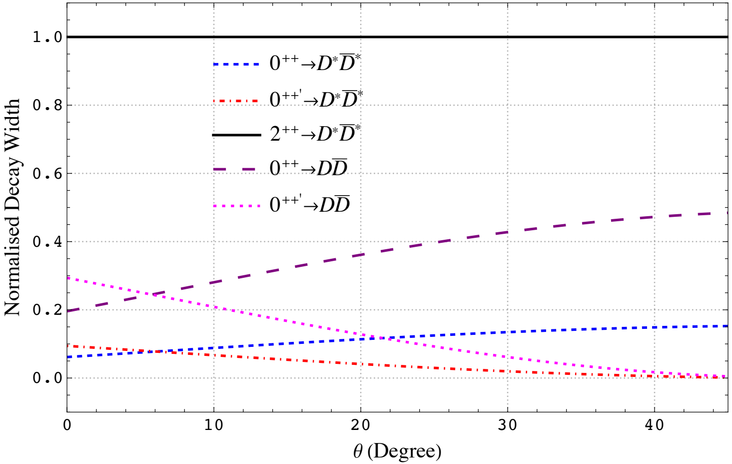
Coefficients $\{\phi_{\text{spin}}, \phi_{\text{colour}}\}$ are different than the rearrangement decays

Final State	$\theta = 35.6^\circ$		$\theta = 0^\circ$		2^{++}	1^{+-}
	0^{++}	$0^{++'}$	0^{++}	$0^{++'}$		
$D^* \bar{D}^*$	0.14	0.011	0.062	0.094	1.0	0.248
$D\bar{D}$	0.46	0.034	0.20	0.29	~ 0	–
$D\bar{D}^* + \bar{D}D^*$	–	–	–	–	–	0.252

An interesting feature, decay rate of $\{0^{++}, 0^{++'}\} \to D\bar{D} : D^*\bar{D}^* = 3 : 1$

$$\frac{\Gamma(0^{++} \to D\bar{D})}{\Gamma(0^{++} \to D^*\bar{D}^*)} \approx \frac{\Gamma(0^{++'} \to D\bar{D})}{\Gamma(0^{++'} \to D^*\bar{D}^*)} = 3.12 \qquad (12)$$

Annihilation decays



Summary

- New mass relations; existing literature confirms their validity at the MeV level
- $X(6600)$ is well described as 2^{++} S-wave $cc\bar{c}\bar{c}$ tetraquark
- The emergence of lowest scalar (0^{++}) around 6400 MeV is important to analyse further
- The decay of lowest-scalar into $\eta_c \eta_c$ is notably larger as compared to $J/\psi J/\psi$
- Annihilation decays of $cc\bar{c}\bar{c}$ states into $D^{(*)}\bar{D}^{(*)}$ would provide an independent test to the existence and their structure
- Super τ-Charm Facility STCF, with centre-of-mass energy up to 7 GeV of colliding e^+e^- can produce $cc\bar{c}\bar{c}$ states

Exciting Future for Exotic Hadron Spectroscopy!

Thanks

Swansea Uni., Singleton Park

Quark vs Diquark Models

The ratio of splittings Δ_2/Δ_1 MNA and Burns, arXiv:2311.15853

$$\Delta_1 = M_1 - M_0 \tag{13}$$

$$\Delta_2 = M_2 - M_1 \tag{14}$$

In the diquark model, $\Delta_2/\Delta_1 = 2$; in the quark model, $\Delta_2/\Delta_1 = 0.55$ with $R = 1$.