η and η' physics

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What is special about the $\eta - \eta'$ system

QCD symmetry breaking: $SU(3)_L \times SU(3)_R \times U(1)_B \; [\times U(1)_A] \to SU(3)_V \times U(1)_B$

- $\eta(\eta')$ emerge as an octet (would-be singlet in large-N_c) Goldstone boson (GB) η_8, η_0
- In real world, $\frac{m_s - m_{u,d}}{\Lambda_{QCD}} \neq 0 \Rightarrow \eta_8 - \eta_0$ mix into $\eta - \eta'$
- Properties of chiral symmetry breaking, physics and **$U(1)_A$ physics**

GBs play a central role in describing QCD dynamics at low energies

- Peculiarities of the η, η' and new physics:
  - $\Gamma(\eta \to 3\pi)$ isospin-breaking $(m_d - m_u) \sim \Gamma(\eta \to 2\gamma)$; $\Gamma_\eta = 1.31(5)$ keV is small
  - $\Gamma(\eta' \to \eta 2\pi)$ phase-space, $\Gamma(\eta^{(\prime)} \to 4\pi \sim \rho\rho)$ suppressed, $\Gamma_{\eta'} = 188(6)$ keV is moderate
  - $I^G J^{PC} = 0^+ 0^{-+}$ and C, P, T eigenstate with negligible SM $\not{CP}$ contribution

Sensitivity to weakly coupled New Physics and CP tests

Channel	Expt. branching ratio	Discussion
$\eta \to 2\gamma$	39.41(20)%	chiral anomaly, η–η' mixing
$\eta \to 3\pi^0$	32.68(23)%	$m_u - m_d$
$\eta \to \pi^0\gamma\gamma$	$2.56(22) \times 10^{-4}$	χPT at $O(p^6)$, leptophobic B boson, light Higgs scalars
$\eta \to \pi^0\pi^0\gamma\gamma$	$< 1.2 \times 10^{-3}$	χPT, axion-like particles (ALPs)
$\eta \to 4\gamma$	$< 2.8 \times 10^{-4}$	$< 10^{-11}$ [55]
$\eta \to \pi^+\pi^-\pi^0$	22.92(28)%	$m_u - m_d$, C/CP violation, light Higgs scalars
$\eta \to \pi^+\pi^-\gamma$	4.22(8)%	chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation
$\eta \to \pi^+\pi^-\gamma\gamma$	$< 2.1 \times 10^{-3}$	χPT, ALPs
$\eta \to e^+e^-\gamma$	$6.9(4) \times 10^{-3}$	theory input for $(g-2)_\mu$, dark photon, protophobic X boson
$\eta \to \mu^+\mu^-\gamma$	$3.1(4) \times 10^{-4}$	theory input for $(g-2)_\mu$, dark photon
$\eta \to e^+e^-$	$< 7 \times 10^{-7}$	theory input for $(g-2)_\mu$, BSM weak decays
$\eta \to \mu^+\mu^-$	$5.8(8) \times 10^{-6}$	theory input for $(g-2)_\mu$, BSM weak decays, P/CP violation
$\eta \to \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs
$\eta \to \pi^+\pi^-e^+e^-$	$2.68(11) \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \to \pi^+\pi^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \to e^+e^-e^+e^-$	$2.40(22) \times 10^{-5}$	theory input for $(g-2)_\mu$
$\eta \to e^+e^-\mu^+\mu^-$	$< 1.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \to \mu^+\mu^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \to \pi^+\pi^-\pi^0\gamma$	$< 5 \times 10^{-4}$	direct emission only
$\eta \to \pi^\pm e^\mp \nu_e$	$< 1.7 \times 10^{-4}$	second-class current
$\eta \to \pi^+\pi^-$	$< 4.4 \times 10^{-6}$ [56]	P/CP violation
$\eta \to 2\pi^0$	$< 3.5 \times 10^{-4}$	P/CP violation
$\eta \to 4\pi^0$	$< 6.9 \times 10^{-7}$	P/CP violation

$\eta - \eta'$ topics in this talk

- Many interesting physics: SM+BSM
- Left table from *Phys.Rept. 945 (2022) 1-105*, "Precision tests of fundamental physics with η and η' mesons" by Gan, Kubis, Passemar & Tullin,
- I will focus on (i) SM/QCD physics for g-2 and (ii) BSM physics: CP-violation
- Note that relevant progress in g-2 contect from $e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-\eta^{(\prime)}$ accessible at STCF.
- Lattice also making progress (2106.05398) and some puzzles with experiment (2305.04570)!
- See also S. Gozalez-Solis talk at 2024 Int'l workshop on future Tau Charm facilities for other channels

Past present and future of η/η' factories

- Fixed target experiments (i.e. $pd \to \eta^3 He$ or $\gamma p \to \eta p$)

  WASA $\sim 5 \times 10^8 \eta$ (past) (EPJ Web Conf. 199) MAMI $\sim 10^8 \eta, 10^6 \eta'$ (past) (2007.00664)

- e^+e^- colliders through $e^+e^- \to R(R = \phi, J/\psi) \to \eta^{(\prime)}\gamma(+\eta^{(\prime)}\phi)$

	$N(J/\psi, \phi)$	$\times(\mathrm{BR}_{\eta\gamma+\eta\phi})$	$\times(\mathrm{BR}_{\eta'\gamma+\eta'\phi})$	Ref
STCF/year	3.4×10^{12}	$(3.8 + 1.6) \times 10^9$	$(1.8 + 0.2) \times 10^{10}$	2303.15790
BESIII	10^{10}	$(1.1 + 0.5) \times 10^7$	$(5.2 + 0.7) \times 10^7$	1912.05983
KLOEII	2.4×10^{10}	$(3.1 + 0) \times 10^8$	$(1.5 + 0) \times 10^6$	1904.12034

- BESIII is a driving force in η, η' (precision) physics $\Rightarrow$ STCF $\times 300$ stat.
- SCTF potential of 10^{10} η, η' mesons (largest η' factory unless full REDTOP)
- Also future fixed-target experiments

JEF (approved) $\sim 10^8$ $\eta, \eta'/200$ days (PR12-14-004) REDTOP (proposal) $\sim 10^{13(11)}\eta^{(\prime)}$ (2203.07651)

Also $e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-\eta^{(\prime)}$ (KLOE/BESIII/BaBar/Belle/STCF)

Section 1

η and η' transition form factors

The $\eta^{(\prime)}$ transition form factors: introduction

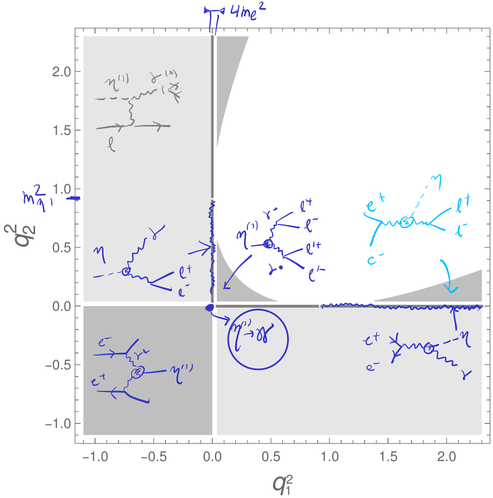
- $F_{P\gamma^*\gamma^*}(q_1^2, q_2^2)$ describe $\gamma^*(q_1)\gamma^*(q_2) \to \eta$

$$i\int d^4x e^{iq_1x} \langle 0|T\{j_\mu(x), j_\nu\}|\eta\rangle = \epsilon_{\mu\nu q_1 q_2} F_{P\gamma^*\gamma^*}(q_1^2, q_2^2)$$

- Relevant to $\eta - \eta'$ mixing via $F_{P\gamma^*\gamma^*}(0,0)$
- Essential input for computing HLbL to muon g-2 (low spacelike region $\sim Q^2 < 2\ \mathrm{GeV}^2$)

- Exclusive processes in pQCD (Brodsky-Lepage '80 10.1103/PhysRevD.22.2157); access to ϕ_P

$$F_{P\gamma^*\gamma^*}(-Q_1^2, -Q_2^2) \to 2F_P \,\mathrm{tr}(\mathcal{Q}^2\lambda^P) \int dx \frac{\phi_P(x)}{xQ_1^2 + \bar{x}Q_2^2}$$



The $\eta - \eta'$ mixing

- The η, η' are an octet-singlet admixture due to $SU(3)_F$-breaking $(m_s - \hat{m} \neq 0)$
- Defining mixing in terms of $\eta_{8,0}$ requires Lagrangian with $\eta_{8,0}$ fields (i.e. large-N_c χPT)
- In practice, $\eta - \eta'$ mixing refers to their decay constants (physical defined without $\mathcal{L}$)

$$\langle 0| A^a_\mu |P(q)\rangle = iq_\mu F^a_P, \qquad A^a_\mu = \bar{q}\gamma_\mu\gamma^5\frac{\lambda^a}{2}q$$

- Naive mixing suggests

$$\begin{pmatrix} F^8_\eta & F^0_\eta \\ F^8_{\eta'} & F^0_{\eta'} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} F_8 & 0 \\ 0 & F_0 \end{pmatrix} \text{ (3 pars) and } \begin{pmatrix} F_8 & 0 \\ 0 & F_0 \end{pmatrix} \sim \langle 0| A^a_\mu |\eta_{8,0}\rangle = F^a_{\eta_{8,0}}$$

- In real world, each F^a_P is independent

$$\begin{pmatrix} F^8_\eta & F^0_\eta \\ F^8_{\eta'} & F^0_{\eta'} \end{pmatrix} = \begin{pmatrix} F_8\cos\theta_8 & -F_0\sin\theta_0 \\ F^8_8\sin\theta_8 & F_0\cos\theta_0 \end{pmatrix}$$

How to extract them? $\Rightarrow$ Phenomenology (next)

The $\eta - \eta'$ mixing

- $F_{P\gamma\gamma}(0,0)$ intimately related to the ABJ anomaly in the **chiral limit**

$$\partial^\rho \langle V_\mu V_\nu A^a_\rho \rangle \sim \frac{N_c \operatorname{tr}(\mathcal{Q}^2\lambda^a)}{4\pi^2}\epsilon_{\mu\nu q_1 q_2} \Rightarrow F^a_\eta F_{\eta\gamma^*\gamma^*}(0,0) + F^a_{\eta'} F_{\eta'\gamma^*\gamma^*}(0,0) = \frac{N_c \operatorname{tr}(\mathcal{Q}^2\lambda^a)}{4\pi^2}$$

- Which implies the solution

$$\begin{pmatrix} F_{\eta\gamma\gamma} \\ F_{\eta'\gamma\gamma} \end{pmatrix} = \begin{pmatrix} F^8_\eta & F^0_\eta \\ F^8_{\eta'} & F^8_{\eta'} \end{pmatrix}^{-1} \frac{N_c}{4\pi^2} \begin{pmatrix} \operatorname{tr}(\mathcal{Q}^2\lambda^8) \\ \operatorname{tr}(\mathcal{Q}^2\lambda^0) \end{pmatrix} \Rightarrow F_{\eta\gamma\gamma}(0,0) = \frac{1}{4\pi^2}\frac{c_8 F^0_{\eta'} - c_0 F^8_{\eta'}}{F^0_{\eta'}F^8_\eta - F^8_{\eta'}F^0_\eta}$$

- In real world corrections [large-N_c χPT]: $c_8 = \frac{1+\frac{K_2}{3}(7M_\pi^2 - 4M_K^2)}{\sqrt{3}}, \quad c_0 = \sqrt{\frac{8}{3}}[1 + \Lambda_3 + \frac{K_2}{3}(2M_\pi^2 + M_K^2)]$
- On the opposite regime $\lim_{Q^2\to\infty} Q^2 F_{P\gamma\gamma} \to 2N_c[F^8_P \operatorname{tr}(\mathcal{Q}^2\lambda^8) + F^0_P \operatorname{tr}(\mathcal{Q}^2\lambda^0)]$
- Low+High regimes of $F_{\eta^{(\prime)}\gamma^*\gamma^*}$ access to $\eta - \eta'$ mixing! EMGS (1512.07520) (MeV units)

$F_8 = 117(2), F_0 = 105(5), \theta_8 = -21(2)^\circ, \theta_0 = -7(2)^\circ$

$F_8 = 115(3), F_0 = 100(4), \theta_8 = -26(3)^\circ, \theta_0 = -8(2)^\circ$ **Lattice** (2106.05398)

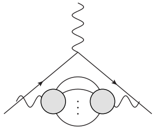
The HLbL contribution to the muon g-2

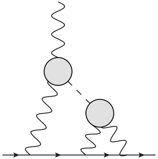
- $(g-2)_\mu$ probe of new physics[a]
$a_\mu^{\rm th} = 116591810(43)\times 10^{-11}$ $a_\mu^{\rm exp} = 16592055(24)\times 10^{-11}$

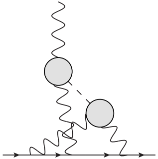
- It's a 5σ tension. Errors dominated by HVP ($\sim 7000 \times 10^{-11}$) ; then, HLbL ($\sim 100 \times 10^{-11}$) ; theory must improve error for future ± 16 (10^{-11} units) exp. uncertainty

- The leading HLbL contribution due to pseudoscalar poles and their TFFs (low spacelike region)

- Next, the 2 approaches in the WP to outline how and necessities







$$a_\mu^{HLbL;P} = \sum_i \int dQ_1 dQ_2 dc_\theta \, \frac{T_i(Q_1^2, Q_2^2, c_\theta)}{Q_{12}^2 + m_P^2}$$
$$\times F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2) F_{P\gamma^*\gamma^*}(Q_{12}^2, 0)$$

[a] Aoyama et al, Phys.Rept.887 (2020), Muon g-2 Coll. Phys.Rev.Lett.131(2023)16

The HLbL contribution to the muon g-2

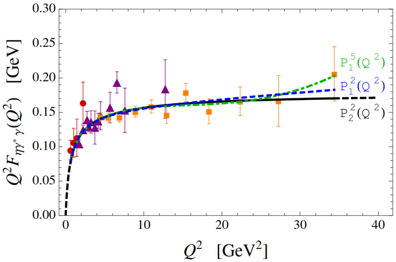
- One approach in WP: Canterbury/Padé approximants[a]

$$P_M^N(x) = \frac{Q_N(x)}{R_M(x)},\ P_1^0 = \frac{a_0}{1+b_1 x},\ P_2^1 = \frac{a_0+a_1 x}{1+b_1 x+b_2 x^2}$$

coefficients to match Taylor series

$$C_1^0(x,y) = \frac{a_{0,0}}{1+b_{1,0}(x+y)+b_{11}xy}$$ pretty similar

- Use sequences $P_{N+1}^N(x)$; improves with $N\uparrow$ (as Taylor exp.)
- Taylor coeffs for $F_{P\gamma^*\gamma}(-Q^2,0)$ from data fitting



[a] P. Masjuan, PSP, Phys.Rev.D95(2017)5

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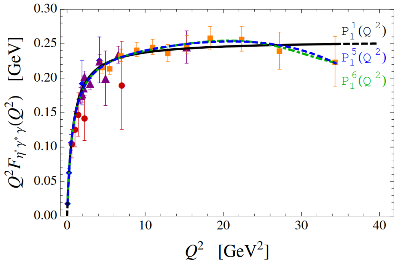

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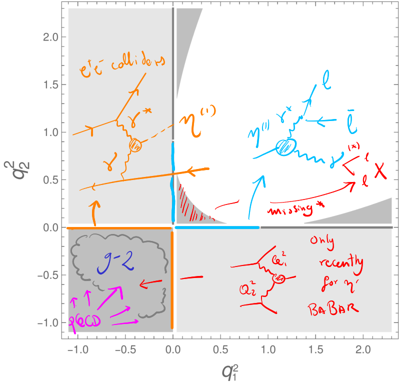
coefficients to match Taylor series

$$C^0_1(x,y) = \frac{a_{0,0}}{1+b_{1,0}(x+y)+b_{11}xy}$$ pretty similar

- Use sequences $P^N_{N+1}(x)$; improves with $N\uparrow$ (as Taylor exp.)
- Taylor coeffs for $F_{P\gamma^*\gamma}(-Q^2,0)$ from data fitting
- Excellent prediction at low q^2 timelike (Dalitz decays)



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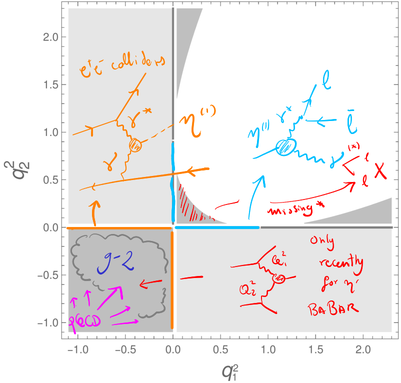
- Taylor coeffs for $F_{P\gamma^*\gamma}(-Q^2,0)$ from data fitting
- Excellent prediction at low q^2 timelike (Dalitz decays)
- $F_{P\gamma^*\gamma}(-Q_1^2,-Q_2^2)$ no data: pQCD asymptotics

$$F_{P\gamma^*\gamma}(-Q^2,-Q^2) = 2\,\mathrm{tr}(\mathcal{Q}^2\lambda^a)F_P^a Q^{-2} + \ldots$$



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The HLbL contribution to the muon g-2

- Other approach in WP are dispersion relations

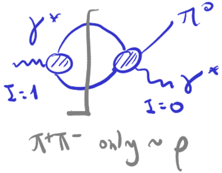
$$F_{P\gamma^*\gamma}(q^2,0) = \frac{1}{\pi}\int \frac{\mathrm{Im}\, F_{P\gamma^*\gamma}(s,0)}{s-q^2} \quad \text{(similar for } q_2^2 \neq 0\text{)}$$

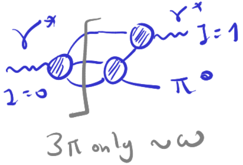
- $\mathrm{Im}\, F_{P\gamma^*\gamma}(s,0)$ unknown but for lowest unitarity cuts $F_{P\gamma^*\gamma^*}(q_1^2,q_2^2)$ for low q_i^2 possible (1808.04823)

- Recently, also η's (2411.08098) for lowest unitarity cuts $F_{P\gamma^*\gamma^*}(q_1^2,q_2^2)$ for low q_i^2 possible for $I=1$

- Capture $\rho, \omega\phi$-dominated processes. For singly virtual, $F_{P\gamma^*\gamma}(-Q^2,0)$, use data and effective pole.

- Doubly-virtual not $\rho(\omega,\phi)$-dominated. Again (different) use of pQCD.





Heavier states $\sim$ VMD

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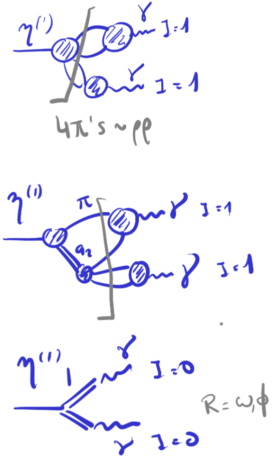

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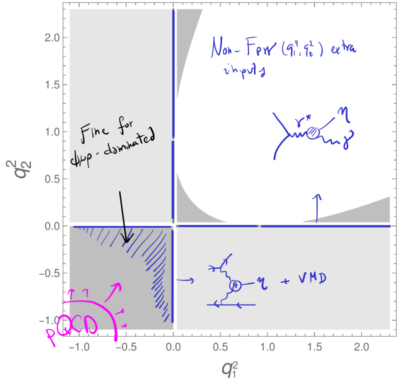
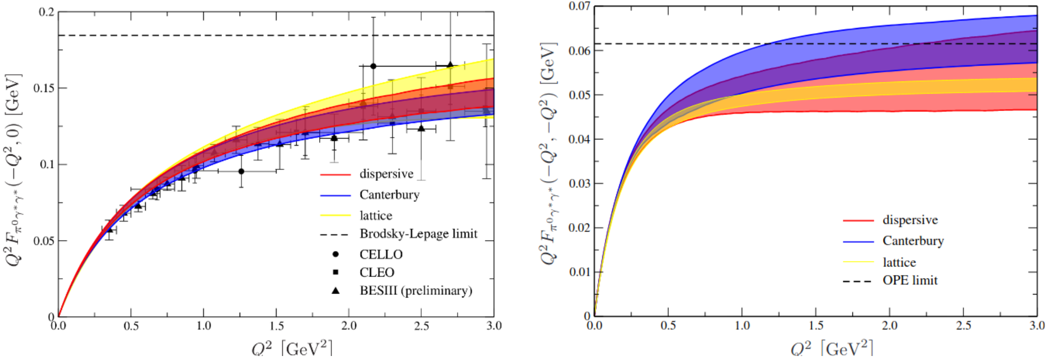


The HLbL contribution to the muon g-2

- How do they compare?



π^0 Fig. from (2006.04822); Canterbury lacks full syst. in the plot

The HLbL contribution to the muon g-2

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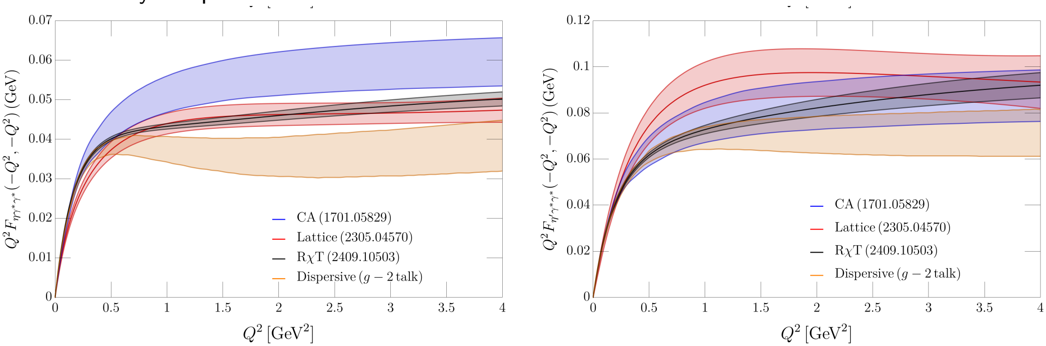


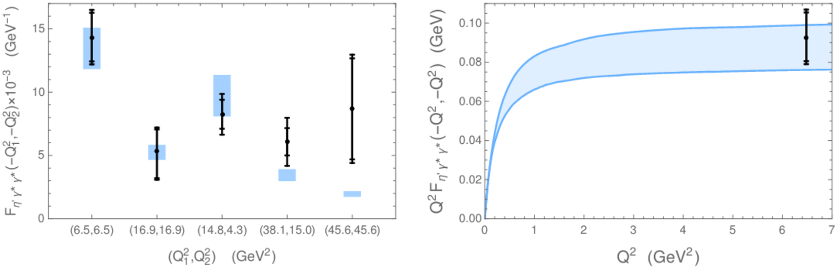
Figs. from S. Gonzalez-Solis (2409.10503); RχT fit to data+lattice (DR: 7th g-2 conference, S. Holz)

CAs without full systematic. Recall lattice smaller $\Gamma(\eta \to 2\gamma)$

Clearly exp. data necessary to improve our understanding!

The HLbL contribution to the muon g-2

- How do they compare?



Note the exceptional BaBar η' data (1808.08038) too large Q^2 to offer insight; fig from (2006.04822)

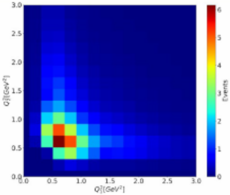
Could STCF help in this respect?

The HLbL contribution to the muon g-2

- How do they compare?

$\gamma^*\gamma^* \rightarrow \eta, \eta'$

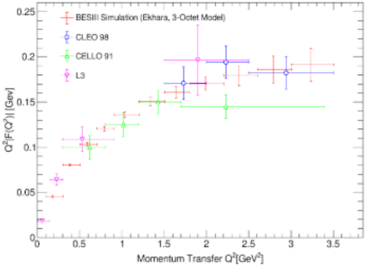
- First studies by Maurice Anderson
  - Using combination of previously available data



Simulation of expected yields for 13 fb^{-1} of data at $\sqrt{s} \geq 3.773$ GeV

- To be done on full data set

Simulations using full 20 fb^{-1}



- Preliminary BESIII results (B. Liu, Y. Ji at QNP24) (also EPJWebConf.303(2024)01001)

Data in relevant region; STCF could perform better!!

The HLbL contribution to the muon g-2

Final numbers for π^0, η, η' poles (10^{-11} units)

CAs

$a_\mu^{\pi^0} = 63.6(1.3)(0.6)(2.3), \quad a_\mu^{\eta} = 16.3(1.0)(0.5)(0.9), \quad a_\mu^{\eta'} = 14.5(0.7)(0.4)(1.5)$

DRs

$a_\mu^{\pi^0} = 62.6(3.0), \quad a_\mu^{\eta} = 14.7(9), \quad a_\mu^{\eta'} = 13.5(7)$

Lattice BMW (2305.04570)

$a_\mu^{\pi^0} = 57.8(1.8)(0.9), \quad a_\mu^{\eta(*)} = 11.6(1.6)(0.5)(1.1), \quad a_\mu^{\eta'} = 15.5(3.9)(1.1)(1.3)$

$^{(*)}$note $F_{\eta\gamma\gamma}^{\mathrm{BMW,ETM}} = 0.22(3)\ \mathrm{GeV}^{-1}$ vs. $F_{\eta\gamma\gamma}^{\mathrm{PDG}} = 0.274(5)\ \mathrm{GeV}^{-1}$

Lattice ETM (2308.12458,2212.06704)

$a_\mu^{\pi^0} = 56.7(3.2), \quad a_\mu^{\eta(*)} = 13.8(5.2)(1.5)$

Lattice Mainz (1903.09471)

$a_\mu^{\pi^0} = 59.7(3.6) \Rightarrow a_\mu^{\pi^0\,\mathrm{latt+exp}2\gamma} = 62.3(2.3)$

- We need DV measurements and $\Gamma(\eta \to 2\gamma)$ (tensions with lattice):

STCF can help in this respect

Section 2

CP-vioaltion in η, η' decays

Motivation

- η, η' mesons are $I^G J^{PC} = 0^+0^{-+}$ C, P eigenstates $\Rightarrow$ natural candidates for C, P tests.
In addition, **almost SM background-free but** price to pay

C-even, P-odd case **highly constrained** by electric dipole moments (EDMs)[1]

- Timely to assess how promising such cases *really* are to set priorities in experimental programmes (**e.g. necessary statistics to be competitive**).

This talk: C-even, P-odd $\eta^{(\prime)}$ (semi)leptonic decays

How to link such decays to nEDM? Make use of the **SMEFT** (at LO D=6, C-even P-odd)

Links $\eta \to \{\mu^+\mu^-,\ \mu^+\mu^-\gamma,\ \mu^+\mu^-\bar{\ell}\ell,\ \pi^0\mu^+\mu^-,\ \pi^+\pi^-\mu^+\mu^-\}$ to nEDM

Most relevant effects in —essentially— **only 3 Wilson Coefficients**!

[1]See 2212.07794, 2111.02417, 2307.02533, 1903.11617 for C-odd P-even.

The SMEFT

- Warsaw basis (JHEP 10 (2010) 085); focus on QCD+QED kind, not EW

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \widetilde{\varphi})$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{l}_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{\varphi}\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{\varphi}\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{q}_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{u}_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{d}_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$

Table 2: Dimension-six operators other than the four-fermion ones.

Hadronic $\not{CP}$ quark EDMs $\not{CP}$ QED $\not{CP}$

The SMEFT

- Warsaw basis (JHEP 10 (2010) 085); focus on QCD+QED kind, not EW

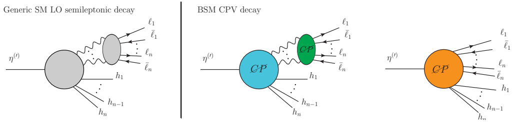
$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu \tau^I q_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu \tau^I l_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

Hadronic $\not{CP}$ lepton-quark $\not{CP}$

The SMEFT

- The SMEFT approach to $\not{CP}$ in η decays



Part 1: $\not{CP}$ in η decays

- Compute the SM: EM matrix elements $\langle h_1 \ldots h_n | \, T\{j^{\mu_1}_{EM} \ldots j^{\mu_n}_{EM}\} \, |\eta\rangle$
- Compute contributions from SMEFT $\not{CP}$ operators (matrix elements!)

$$\not{CP} \text{ interference } |M_{SM} + M^{\not{CP}}_{BSM}|^2 = |M_{SM}|^2 + 2\,\mathrm{Re}\, M_{SM} M^{\not{CP}}_{BSM} + \ldots$$

Identify $\not{CP}$ observables and **estimate exp. sensitivity to WCs (statistics!)**

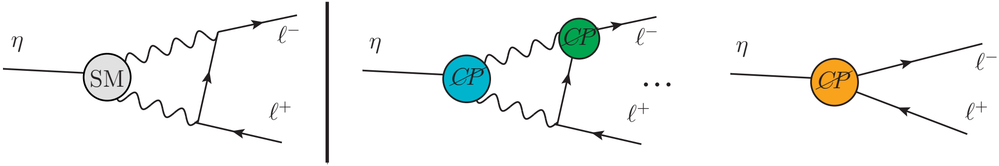
Part 2: Bounds from other processes

- Compute EDM contribution from SMEFT $\not{CP}$ operators $\Rightarrow$ **bounds!**

Wilson Coefficients: $\not{CP}$ in η vs. EDMs

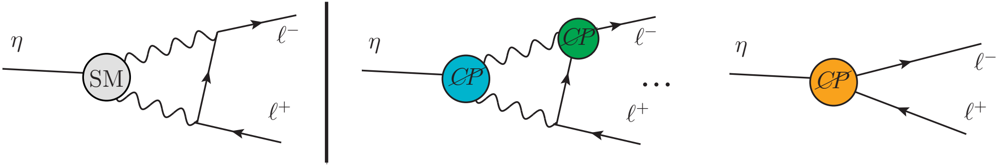
$\eta \to \mu^+\mu^-$ decays: basics

- Consider SM and $\not{CP}$ SMEFT contributions (details later)

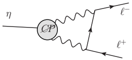


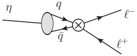
$\eta \to \mu^+\mu^-$ decays: basics

- Consider SM and $\not{CP}$ SMEFT contributions (details later)



- Checked that $\not{CP}$ in QED highly suppressed by μEDM; hadronic ones:





$$F_{\eta\gamma\gamma}(\eta F\tilde{F}) + \epsilon_{\not{CP}} F_{\eta\gamma\gamma}(\eta FF)$$

$$\mathcal{O}^{(1)}_{\ell equ} \Rightarrow -\,\mathrm{Im}\, c^{(1)2211}_{\ell equ} \frac{G_F}{\sqrt{2}} (\bar{\mu}\mu)(\bar{u} i\gamma^5 u)$$
$$\mathcal{O}_{\ell edq} \Rightarrow -\,\mathrm{Im}\, c^{22jj}_{\ell edq} \frac{G_F}{\sqrt{2}} (\bar{\mu}\mu)(\bar{d}_j i\gamma^5 d_j)$$

$\Rightarrow$ Looping yields $g_{SM} \bar{u} i\gamma^5 v + g_{\not{CP}} \bar{u} v$

$\Rightarrow \frac{G_F}{\sqrt{2}} c_i \langle 0 | \bar{q} i\gamma^5 v | \eta \rangle \bar{u} v \sim g_{\not{CP}} \bar{u} v$

$\eta \to \mu^+\mu^-$ decays: basics

- After hadronization essentially

$$\mathcal{L}_{eff} = g_{SM}\eta\bar{\ell}i\gamma^5\ell + g_{\not{CP}}\eta\bar{\ell}\ell$$

$$|\mathcal{M}|^2 = |\mathcal{M}_{SM}|^2 + |\mathcal{M}_{\not{CP}}|^2 + 2\,\mathrm{Re}\,\mathcal{M}_{SM}\mathcal{M}^*_{\not{CP}}$$

$\eta \to \mu^+\mu^-$ decays: basics

- After hadronization essentially

$$\mathcal{L}_{eff} = g_{SM}\eta\bar{\ell}i\gamma^5\ell + g_{\not{CP}}\eta\bar{\ell}\ell$$

$$|\mathcal{M}|^2 = |\mathcal{M}_{SM}|^2 + |\mathcal{M}_{\not{CP}}|^2 + 2\,\mathrm{Re}\,\mathcal{M}_{SM}\mathcal{M}^*_{\not{CP}}$$

- The $\not{CP}$ violation in the interference term: vanishes if summing over spins $(\boldsymbol{n}, \bar{\boldsymbol{n}})$!

$$2\,\mathrm{Re}\,\mathcal{M}_{SM}\mathcal{M}^*_{\not{CP}} \Rightarrow \frac{m_\eta^2}{2}\left[\mathrm{Re}(g_{SM}g^*_{\not{CP}})(\bar{\boldsymbol{n}} \times \boldsymbol{n}) \cdot \boldsymbol{\beta}_\ell + \mathrm{Im}(g_{SM}g^*_{\not{CP}})\boldsymbol{\beta}_\ell \cdot (\boldsymbol{n} - \bar{\boldsymbol{n}})\right]$$

Solution	$\Rightarrow$	Account for spins: asymmetries (so far only REDTOP)
How?	$\Rightarrow$	$\mu^\pm$ decay ($e^\pm$ preferentially along(against) $\mu^\pm$ spin)

$$A_L = \frac{\beta_\mu}{3}\frac{\mathrm{Im}\,A\tilde{g}_{\not{CP}}}{|A|^2}, \qquad \tilde{g}_{\not{CP}} = -\frac{g_{\not{CP}}}{2m_\mu\alpha^2 F_{\eta\gamma\gamma}}, \quad A \sim \mathrm{SM}$$

$\eta \to \mu^+\mu^-$ decays: basics

- After hadronization essentially

$$\mathcal{L}_{eff} = g_{SM}\eta\bar{\ell}i\gamma^5\ell + g_{\not{CP}}\eta\bar{\ell}\ell$$

$$|\mathcal{M}|^2 = |\mathcal{M}_{SM}|^2 + |\mathcal{M}_{\not{CP}}|^2 + 2\,\mathrm{Re}\,\mathcal{M}_{SM}\mathcal{M}^*_{\not{CP}}$$

$$A_L = 0.11\epsilon_{\not{CP}} - \mathrm{Im}[2.7(c^{(1)2211}_{\ell equ} + c^{(1)2211}_{\ell edq}) - 4.1c^{(1)2222}_{\ell edq}] \times 10^{-2}$$

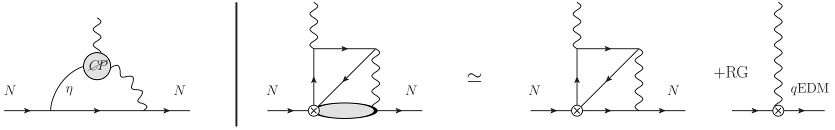
Sensitivities: assume SM gaussian noise. At REDTOP ($2 \times 10^{12}\eta$)

$$\Delta\epsilon_{\not{CP}} = 10^{-3}, \Delta\,\mathrm{Im}\,c^{(1)2211}_{\ell equ} = \Delta\,\mathrm{Im}\,c^{(1)2211}_{\ell edq} = 0.007, \Delta\,\mathrm{Im}\,c^{(1)2222}_{\ell edq} = 0.005$$

Is it below nEDM/other bounds?

$\eta \to \mu^+\mu^-$ decays: nEDM bounds

- QCD vs Quark-Lepton means 1-loop vs 2-loops



- We find, in absolute values, (note $\not{CP}$ in QCD potentially more stringent!)

$$\epsilon_{\not{CP}} < 2 \times 10^{-7}, \qquad c^{(1)2211}_{\ell equ} < 0.001, \qquad c^{(1)2211}_{\ell edq} < 0.002, \qquad c^{(1)2222}_{\ell edq} < 0.02$$

- Recall previous section

$$\Delta\epsilon_{\not{CP}} = 10^{-3}, \Delta c^{(1)2211}_{\ell equ} = \Delta c^{(1)2211}_{\ell edq} = 0.007, \Delta c^{(1)2222}_{\ell edq} = 0.005$$

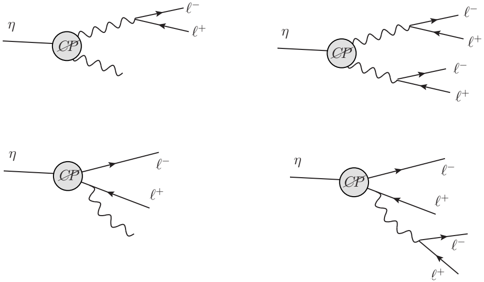
The strange does overcome nEDM! (and constraints from D_s decays)

Note for $\ell = e$, yet stronger bounds from atomic physics

Takeout message: quark-lepton "direct" and most promising

Dalitz and Double-Dalitz decays

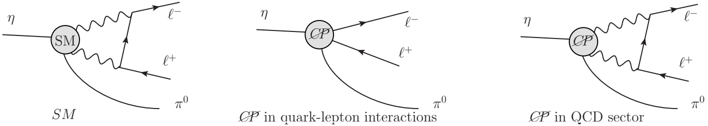
- Less promising since involve α suppressions (Dalitz: polarization; double-Dalitz: triple-product)



- One finds for SD/DD: $\Delta\epsilon_{CP} = 10^{-2}/10^{-3}$, $\Delta\,\mathrm{Im}\, c_{\mathcal{O}}^{22st} = 1/40$

$\eta \to \pi^0 \mu^+ \mu^-$ decays

- Proceed following previous section (JHEP 05 (2022) 147)

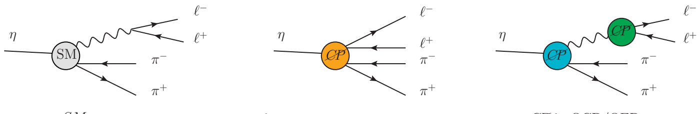


- Focus on SMEFT Quark-Lepton operators (same operators appear)
- Again, with 3 particles in final state, μ polarimetry required
- Hadronize corresponding $\langle\pi^0|\,\bar{q}q\,|\eta\rangle$ matrix elements ($\bar{s}s$ isospin suppressed)

Process	Asymmetry	Im $c_{\ell edq}^{2222}$	Im $c_{\ell equ}^{(1)2211}$	Im $c_{\ell edq}^{2211}$
$\eta \to \pi^0\mu^+\mu^-$	A_L	0.7	0.07	0.07
$\eta' \to \pi^0\mu^+\mu^-$	A_L	11	2.4	2.5
$\eta' \to \eta\mu^+\mu^-$	A_L	5	68	79
$\eta \to \mu^+\mu^-$	A_T	0.005	0.007	0.007

$\eta^{(\prime)} \to \pi^+\pi^-\mu^+\mu^-$ decays

- Proceed as previously (tensor quark-lepton operators EDM at 1 loop)



- $\not{CP}$ in QCD irrelevant from nEDM [Gan,Kubis,Passemar,Tulin '22]
- NO polarimetry: A_ϕ ($\pi^+\pi^-$-$\mu^+\mu^-$ plane angle)

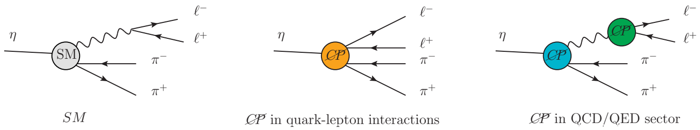
$$\mathrm{Re}\,\mathcal{M}^*_{\mathrm{SM}}\mathcal{M}_{\mathrm{BSM}} = \frac{4\sqrt{2}e^2 m_\mu G_F}{s_\ell}\epsilon_{p_1p_2p_3p_4}\,\mathrm{Re}\left[\mathcal{F}^*_{\eta^{(\prime)}}\langle\pi^+\pi^-|\frac{1}{2}\,\mathrm{Im}\left(c^{(1)2211}_{\ell equ}+c^{2211}_{\ell edq}\right)P^q + \mathrm{Im}\,c^{2222}_{\ell edq}P^s|\eta^{(\prime)}\rangle\right]$$

We find $A_\phi \propto \sin\phi$ **vs.** $A_\phi \propto \sin 2\phi$ from $\not{CP}$ in QCD
Experiments should include this!

[1] M. Zillinger, B. Kubis, PSP, 2210.14925

$\eta^{(\prime)} \to \pi^+\pi^-\mu^+\mu^-$ decays

- Proceed as previously (tensor quark-lepton operators EDM at 1 loop)



- $\not{CP}$ in QCD irrelevant from nEDM [Gan,Kubis,Passemar,Tulin '22]
- NO polarimetry: A_ϕ ($\pi^+\pi^-$-$\mu^+\mu^-$ plane angle)
- Relevant outcome for expt. (REDTOP: $N_\eta = 5 \times 10^{12}$, $N_{\eta'} = 4 \times 10^8$)

$$A_\phi^\eta = 47(14) \times 10^{-5} \left(\mathrm{Im}\, c_{\ell equ}^{(1)2211} + \mathrm{Im}\, c_{\ell edq}^{2211} \right) - 0.4(2.2) \times 10^{-5}\, \mathrm{Im}\, c_{\ell edq}^{2222},$$
$$A_\phi^{\eta'} = 2.9(5) \times 10^{-5} \left(\mathrm{Im}\, c_{\ell equ}^{(1)2211} + \mathrm{Im}\, c_{\ell edq}^{2211} \right) - 1.4(5) \times 10^{-5}\, \mathrm{Im}\, c_{\ell edq}^{2222},$$

$$(\eta/\eta')\Delta\, \mathrm{Im}\, c_{\ell equ}^{(1)2211} = \Delta\, \mathrm{Im}\, c_{\ell edq}^{2211} = 12/36 \qquad \Delta\, \mathrm{Im}\, c_{\ell edq}^{2222} = 1584/77$$

- Unfortuantely, well above nEDM bounds

[1] M. Zillinger, B. Kubis, PSP, 2210.14925

Outlook and summary

- STCF is a huge "η, η' factory"
- η, η' physics unique in different aspects: SM (QCD) and BSM
- Unique acces to the $U(1)_A$ QCD sector, $m_d - m_u$, chiral symm. breaking, $\eta - \eta'$ mixing
- One key point are transition form factors
  - Key ingredient for $(g-2)_\mu \Rightarrow$ doubly-virtual and $\Gamma(\eta^{(\prime)} \to 2\gamma)$ (tensions with lattice!)
  - Also relevant for $\eta - \eta'$ mixing (also relevant for $(g-2)_\mu$)
  - Accessed via $\eta \to \ell^+\ell^-\gamma$, $\eta \to \ell^+\ell^-\ell^+\ell^-$ and $e^+e^- \to e^+e^-\eta^{(\prime)}$ (colliders only!)
- Another interesting point is CP-violation (focused on C-even, P-odd)
  - If heavy physics, only through μ polarimetry in $\eta \to \mu^+\mu^-$
  - Novel triple product asymmetries in $\eta \to \mu^+\mu^-\pi^+\pi^-$ (yet discarded from SMEFT)