The 6th International Workshop on Future Tau Charm Facilities

FTCF, 2024, Guangzhou

TOPOLOGICAL DIAGRAMS & CHARMED BARYON DECAYS

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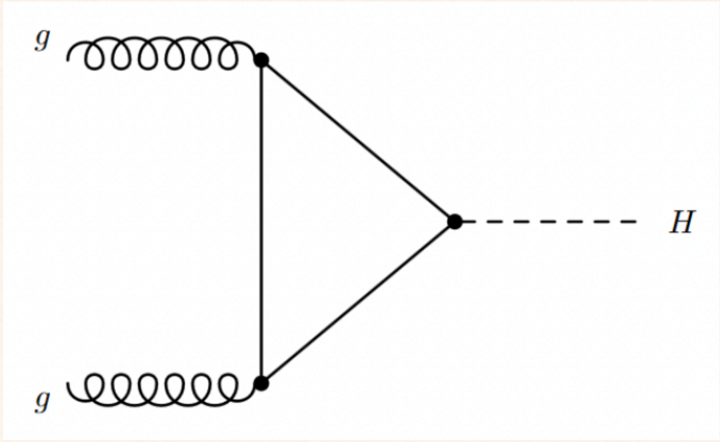
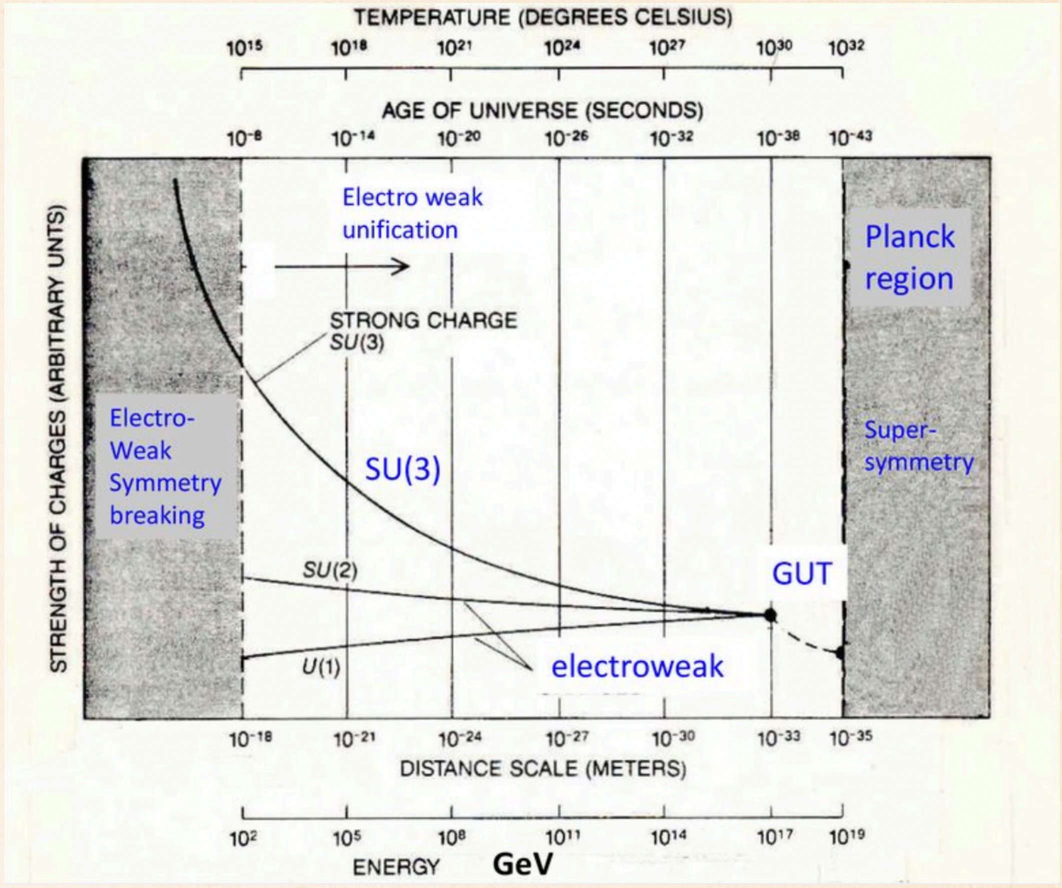
Charming Strong Interaction





?

QCD

u,d | s | c | b | H | t

CHPT

HQET

LQCD

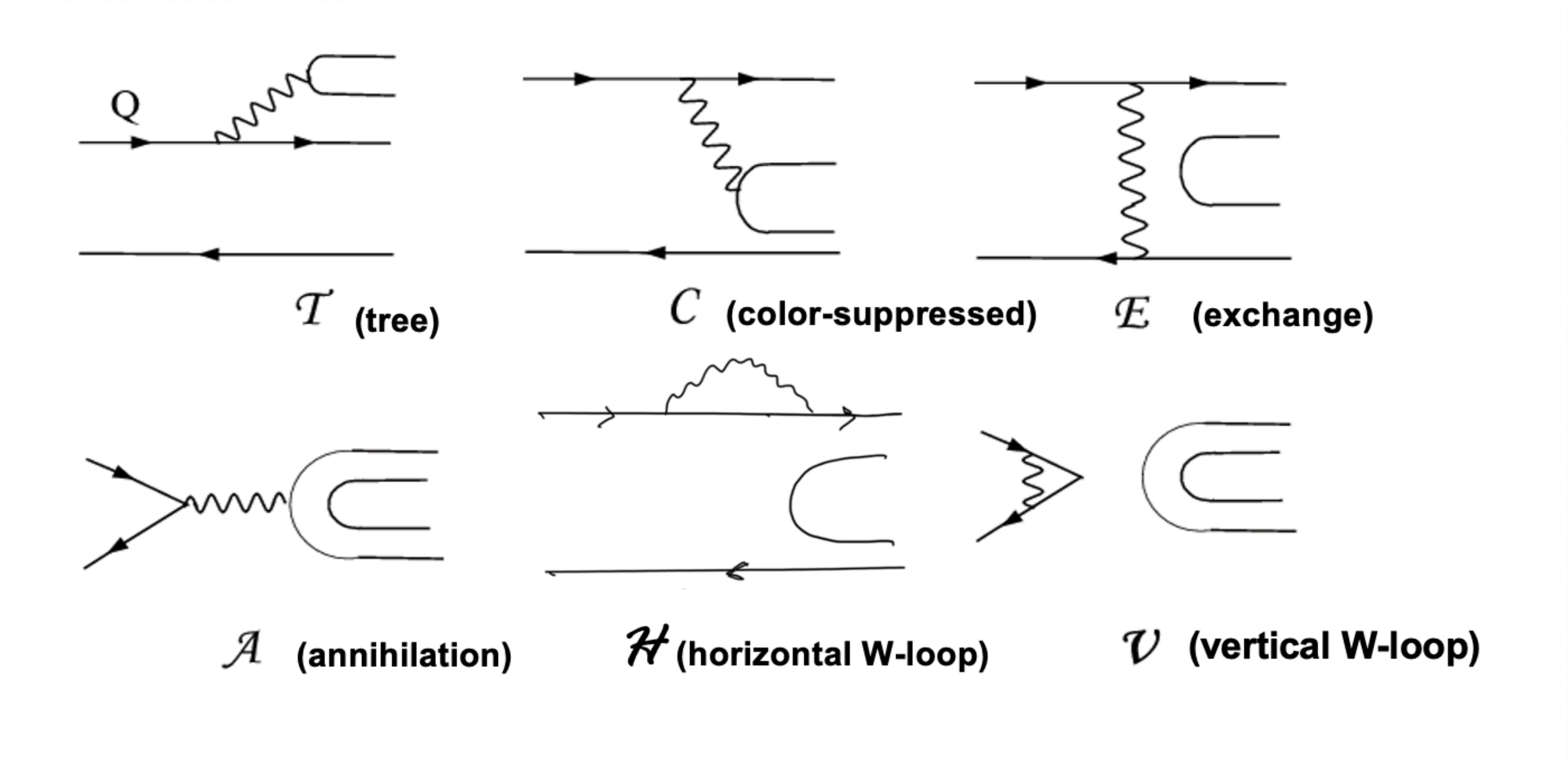
SCET

Topological Diagram Approach (TDA)

Charmed meson

two-body decays can be expressed in terms of six distinct topological diagrams [Chau (1980, 1983); Chau, Cheng(1986)]

L.-L. Chau in *Proceedings of 1980 Guangzhou Conference on Theoretical Particle Physics*



both magnitude and strong phase of each tree diagram can be determined

Its Triumph in Meson Sector

Hadronic Weak Decays of Charmed Mesons

Hai-Yang Cheng (鄭海揚)
Academia Sinica, Taipei

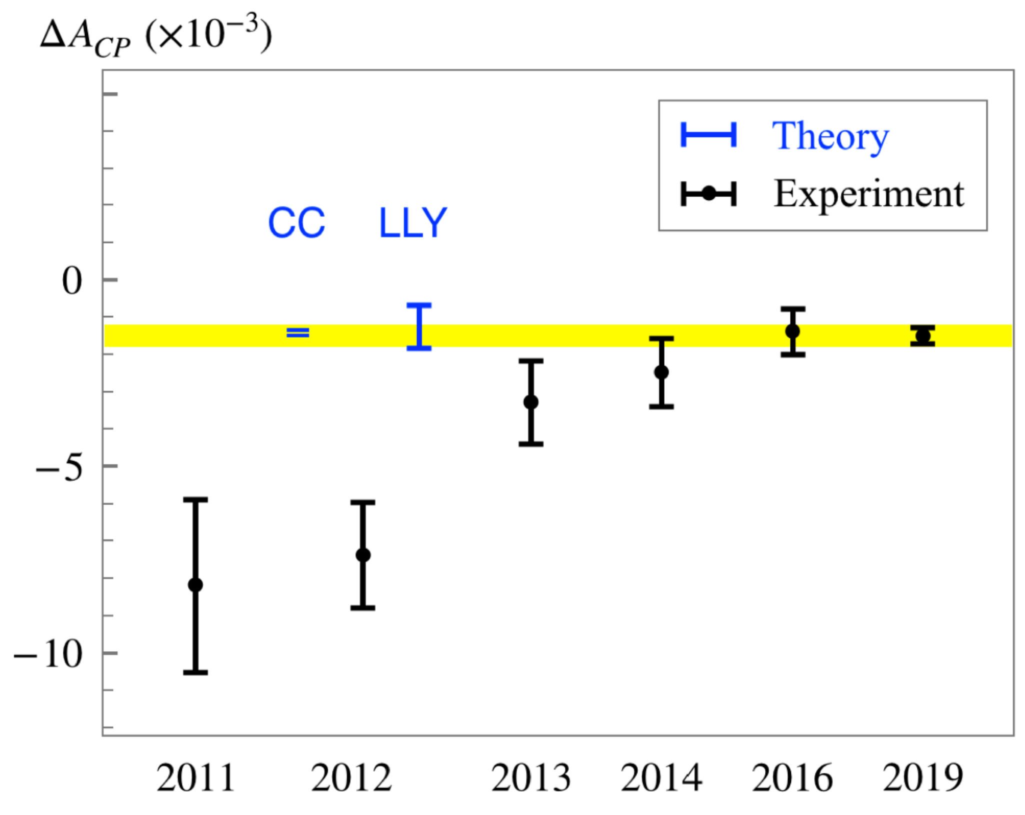
2024 BESIII Workshop on Charm Hadron Physics

May 11, 2024

Branching fractions

Direct CPV

$$\Delta A_{CP} = A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-)$$



Saur, **FSY**, Sci.Bull.2020

Th: the only predictions of O(10^{-3})

CC: topological approach + QCDF

Cheng, Chiang, 2012

LLY: factorization-assisted topology (FAT)

Li, Lu, **FSY**, 2012

Exp: LHCb, PRL122, 211803 (2019)

Topological diagrammatic approach successfully predicted the charm CPV !!!

PHYSICAL REVIEW LETTERS **122,** 211803 (2019)

Editors' Suggestion　Featured in Physics

Observation of *CP* Violation in Charm Decays

R. Aaij *et al.**
(LHCb Collaboration)

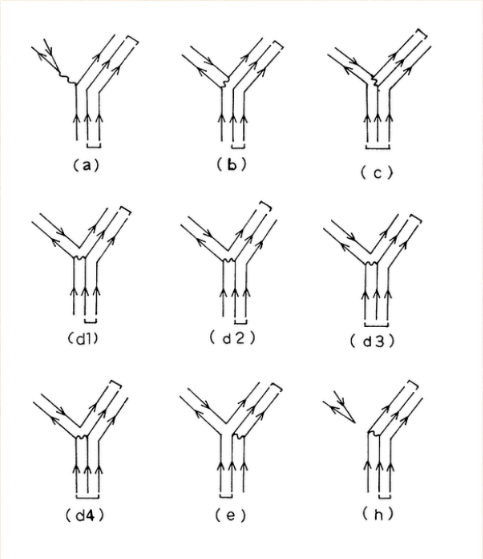
(Received 21 March 2019; revised manuscript received 2 May 2019; published 29 May 2019)

A search for charge-parity (*CP*) violation in $D^0 \to K^-K^+$ and $D^0 \to \pi^-\pi^+$ decays is reported, using pp collision data corresponding to an integrated luminosity of 5.9 fb^{-1} collected at a center-of-mass energy of 13 TeV with the LHCb detector. The flavor of the charm meson is inferred from the charge of the pion in $D^*(2010)^+ \to D^0\pi^+$ decays or from the charge of the muon in $\bar{B} \to D^0\mu^-\bar{\nu}_\mu X$ decays. The difference between the *CP* asymmetries in $D^0 \to K^-K^+$ and $D^0 \to \pi^-\pi^+$ decays is measured to be $\Delta A_{CP} = [-18.2 \pm 3.2(\mathrm{stat}) \pm 0.9(\mathrm{syst})] \times 10^{-4}$ for π-tagged and $\Delta A_{CP} = [-9 \pm 8(\mathrm{stat}) \pm 5(\mathrm{syst})] \times 10^{-4}$ for μ-tagged D^0 mesons. Combining these with previous LHCb results leads to $\boxed{\Delta A_{CP} = (-15.4 \pm 2.9) \times 10^{-4}}$, where the uncertainty includes both statistical and systematic contributions. The measured value differs from zero by more than 5 standard deviations. This is the first observation of *CP* violation in the decay of charm hadrons.

DOI: 10.1103/PhysRevLett.122.211803

Early TDA Seeking for Baryon

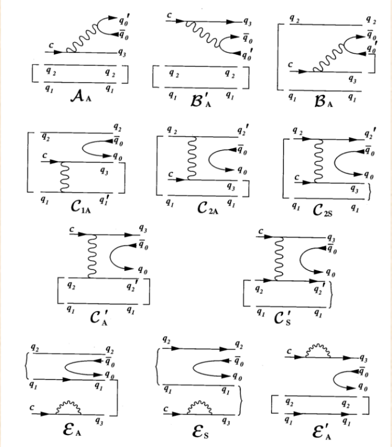
- Kohara(1991): parameterize by antisymmetric (12) and (23) for octet baryon wave function



$$\mathcal{A} = a\bar{B}^{3[ab]}B_{[ab]}M_2^1 + b\bar{B}^{1[ab]}B_{[ab]}M_2^3 + c\bar{B}^{b[13]}B_{[ab]}M_2^a + d_1\bar{B}^{a[1b]}B_{[2b]}M_a^3 + d_2\bar{B}^{b[1a]}B_{[2b]}M_a^3 + d_3\bar{B}^{a[3b]}B_{[2b]}M_a^1 + d_4\bar{B}^{b[3a]}B_{[2b]}M_a^1 + e\bar{B}^{a[13]}B_{[2b]}M_a^b + h\bar{B}^{b[13]}B_{[2b]}M_a^a ,$$

$$|\tilde{\mathcal{B}}^{m,k}(8)\rangle = \alpha|\chi^m(1/2)_{A_{12}}\rangle|\psi^k(8)_{A_{12}}\rangle + \beta|\chi^m(1/2)_{A_{23}}\rangle|\psi^k(8)_{A_{23}}\rangle$$

- Chau, Cheng, Tseng(1996): parameterize symmetric and antisymmetric (12) for octet baryon wave function



$$\boxed{|\mathcal{B}^{m,k}(8)\rangle = a|\chi^m(1/2)_{A_{12}}\rangle|\psi^k(8)_{A_{12}}\rangle + b|\chi^m(1/2)_{S_{12}}\rangle|\psi^k(8)_{S_{12}}\rangle}$$

- Kohara(1997): physics is independent of the chosen convention

Recent Baryon TDA Attempt

- He, She, Wang(2019): a general TDA description
- Zhao, Wang, Hsiao, Yu(2020);
- Hsiao, Wang, Zhao(2022): followed Kohara, parity violation terms absent
- Wang, Luo(2024): independent parametrization, penguin included

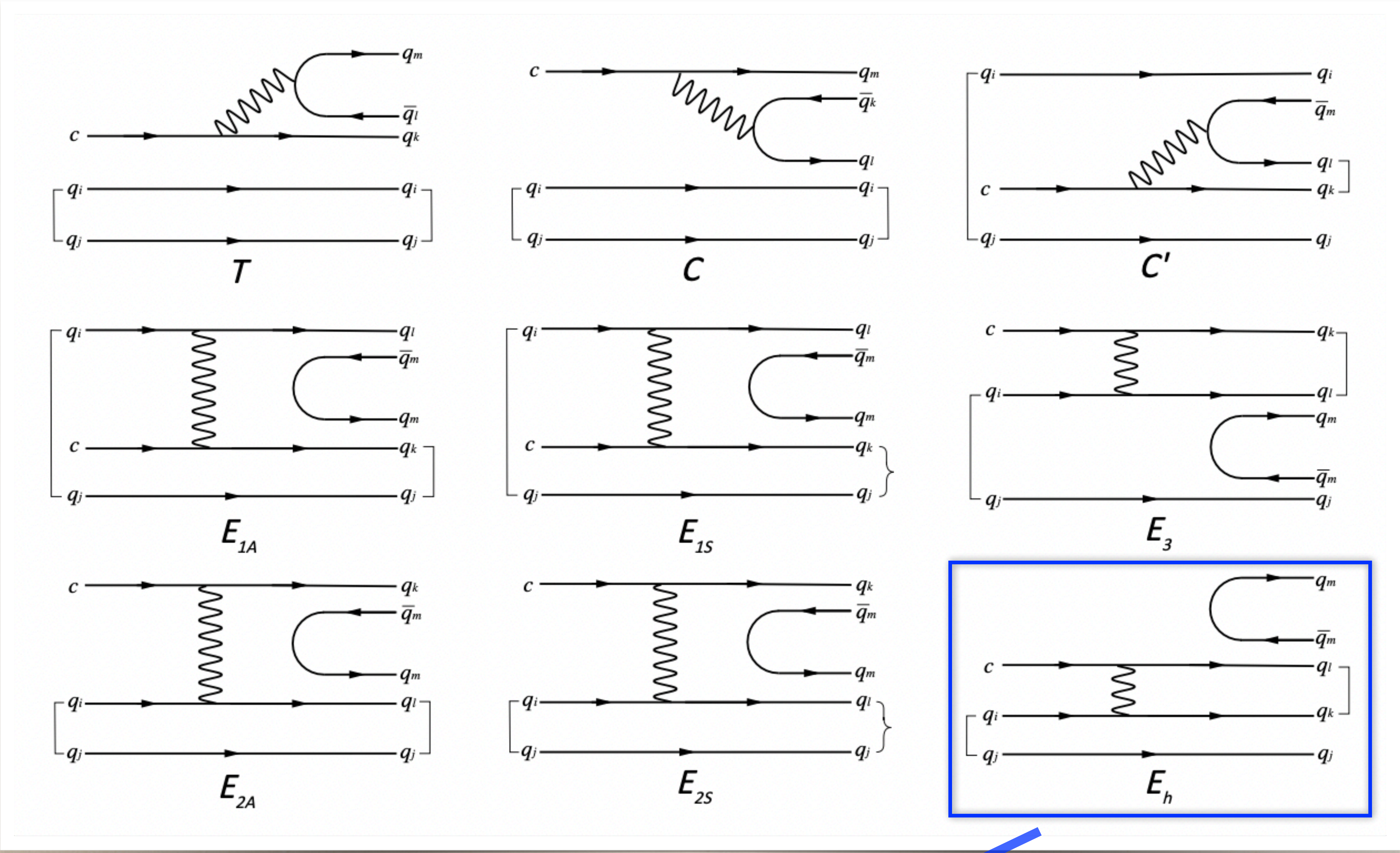
Unlike IRA, proper global fits to the measured rates and decay asymmetries in TDA were still absent

TDA: more intuitive, graphic and easier to implement model calculations

Completeness

vs. Redundancy

$$
\begin{aligned}
\mathcal{A}_{\rm TDA} = \;& T(\mathcal{B}_c)^{ij} H^{km}_l M^l_m \left[b_1\,(\mathcal{B}_8)_{ijk} + b_2\,(\mathcal{B}_8)_{ikj} + b_3\,(\mathcal{B}_8)_{jki}\right] \\
&+ C(\mathcal{B}_c)^{ij} H^{ml}_k M^k_m \left[b_4\,(\mathcal{B}_8)_{ijl} + b_5\,(\mathcal{B}_8)_{ilj} + b_6\,(\mathcal{B}_8)_{jli}\right] \\
&+ C'(\mathcal{B}_c)^{ij} H^{kl}_m M^m_i \left[b_7\,(\mathcal{B}_8)_{klj} + b_8\,(\mathcal{B}_8)_{kjl} + b_9\,(\mathcal{B}_8)_{ljk}\right] \\
&+ E_1(\mathcal{B}_c)^{ij} H^{kl}_i M^m_l \left[b_{10}\,(\mathcal{B}_8)_{jkm} + b_{11}\,(\mathcal{B}_8)_{jmk} + b_{12}\,(\mathcal{B}_8)_{kmj}\right] \\
&+ E_2(\mathcal{B}_c)^{ij} H^{kl}_i M^m_k \left[b_{13}\,(\mathcal{B}_8)_{jlm} + b_{14}\,(\mathcal{B}_8)_{jml} + b_{15}\,(\mathcal{B}_8)_{lmj}\right] \\
&+ E_3(\mathcal{B}_c)^{ij} H^{kl}_i M^m_j \left[b_{16}\,(\mathcal{B}_8)_{klm} + b_{17}\,(\mathcal{B}_8)_{kml} + b_{18}\,(\mathcal{B}_8)_{lmk}\right] \\
&+ E_h(\mathcal{B}_c)^{ij} H^{kl}_i M^m_m \left[b_{19}\,(\mathcal{B}_8)_{jkl} + b_{20}\,(\mathcal{B}_8)_{jlk} + b_{21}\,(\mathcal{B}_8)_{klj}\right],
\end{aligned}
$$



hairpin diagram
contributing to singlet η_1

$$
(\mathcal{B}_8)^i_j = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda^0 + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda^0 - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda^0 \end{pmatrix}
$$

$$
(\mathcal{B}_c)^{ij} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}
$$

$$
(\mathcal{B}_8)_{ijk} = \epsilon_{ijl}(\mathcal{B}_8^T)^l_k
$$

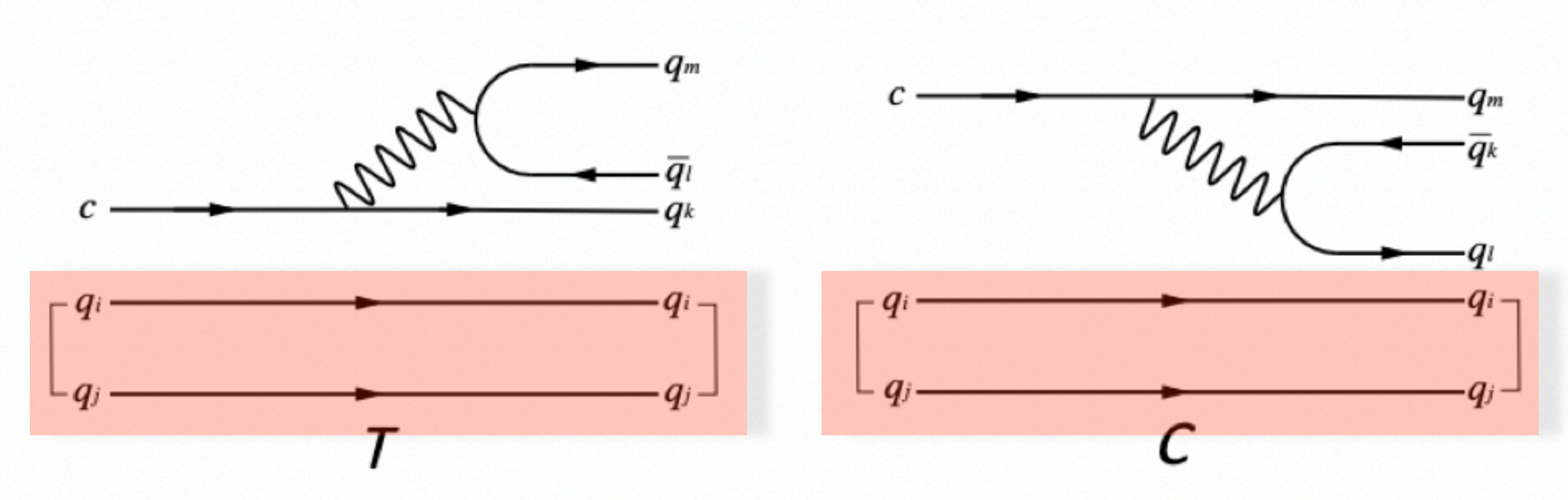
$$
M^i_j = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \overline{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \end{pmatrix}
$$

$$
H^{31}_2 = V^*_{cs}V_{ud}, \quad H^{31}_3 = V^*_{cs}V_{us}, \quad H^{21}_2 = V^*_{cd}V_{ud}, \quad H^{21}_3 = V^*_{cd}V_{us}.
$$

$$
\begin{aligned}
\mathcal{A}_{\mathrm{TDA}} = \; & T(\mathcal{B}_c)^{ij} H_l^{km} M_m^l \left[b_1\,(\mathcal{B}_8)_{ijk} + b_2\,(\mathcal{B}_8)_{ikj} + b_3\,(\mathcal{B}_8)_{jki}\right] \\
& + C(\mathcal{B}_c)^{ij} H_k^{ml} M_m^k \left[b_4\,(\mathcal{B}_8)_{ijl} + b_5\,(\mathcal{B}_8)_{ilj} + b_6\,(\mathcal{B}_8)_{jli}\right] \\
& + C'(\mathcal{B}_c)^{ij} H_m^{kl} M_i^m \left[b_7\,(\mathcal{B}_8)_{klj} + b_8\,(\mathcal{B}_8)_{kjl} + b_9\,(\mathcal{B}_8)_{ljk}\right] \\
& + E_1(\mathcal{B}_c)^{ij} H_i^{kl} M_l^m \left[b_{10}\,(\mathcal{B}_8)_{jkm} + b_{11}\,(\mathcal{B}_8)_{jmk} + b_{12}\,(\mathcal{B}_8)_{kmj}\right] \\
& + E_2(\mathcal{B}_c)^{ij} H_i^{kl} M_k^m \left[b_{13}\,(\mathcal{B}_8)_{jlm} + b_{14}\,(\mathcal{B}_8)_{jml} + b_{15}\,(\mathcal{B}_8)_{lmj}\right] \\
& + E_3(\mathcal{B}_c)^{ij} H_i^{kl} M_j^m \left[b_{16}\,(\mathcal{B}_8)_{klm} + b_{17}\,(\mathcal{B}_8)_{kml} + b_{18}\,(\mathcal{B}_8)_{lmk}\right] \\
& + E_h(\mathcal{B}_c)^{ij} H_i^{kl} M_m^m \left[b_{19}\,(\mathcal{B}_8)_{jkl} + b_{20}\,(\mathcal{B}_8)_{jlk} + b_{21}\,(\mathcal{B}_8)_{klj}\right],
\end{aligned}
$$

$$b_3 = -b_2$$

$$b_6 = -b_5$$



$$(\mathcal{B}_8)_{ijk} = \epsilon_{ijl}(\mathcal{B}_8^T)^l_k$$

$$
\begin{aligned}
\mathcal{A}_{\rm TDA} = \; & T(\mathcal{B}_c)^{ij} H_l^{km} M_m^l \left[b_1 (\mathcal{B}_8)_{ijk} + b_2 (\mathcal{B}_8)_{ikj} + b_3 (\mathcal{B}_8)_{jki}\right] \\
& + C(\mathcal{B}_c)^{ij} H_k^{ml} M_m^k \left[b_4 (\mathcal{B}_8)_{ijl} + b_5 (\mathcal{B}_8)_{ilj} + b_6 (\mathcal{B}_8)_{jli}\right] \\
& + C'(\mathcal{B}_c)^{ij} H_m^{kl} M_i^m \left[b_7 (\mathcal{B}_8)_{klj} + b_8 (\mathcal{B}_8)_{kjl} + b_9 (\mathcal{B}_8)_{ljk}\right] \\
& + E_1(\mathcal{B}_c)^{ij} H_i^{kl} M_l^m \left[b_{10} (\mathcal{B}_8)_{jkm} + b_{11} (\mathcal{B}_8)_{jmk} + b_{12} (\mathcal{B}_8)_{kmj}\right] \\
& + E_2(\mathcal{B}_c)^{ij} H_i^{kl} M_k^m \left[b_{13} (\mathcal{B}_8)_{jlm} + b_{14} (\mathcal{B}_8)_{jml} + b_{15} (\mathcal{B}_8)_{lmj}\right] \\
& + E_3(\mathcal{B}_c)^{ij} H_i^{kl} M_j^m \left[b_{16} (\mathcal{B}_8)_{klm} + b_{17} (\mathcal{B}_8)_{kml} + b_{18} (\mathcal{B}_8)_{lmk}\right] \\
& + E_h(\mathcal{B}_c)^{ij} H_i^{kl} M_m^m \left[b_{19} (\mathcal{B}_8)_{jkl} + b_{20} (\mathcal{B}_8)_{jlk} + b_{21} (\mathcal{B}_8)_{klj}\right] ,
\end{aligned}
$$

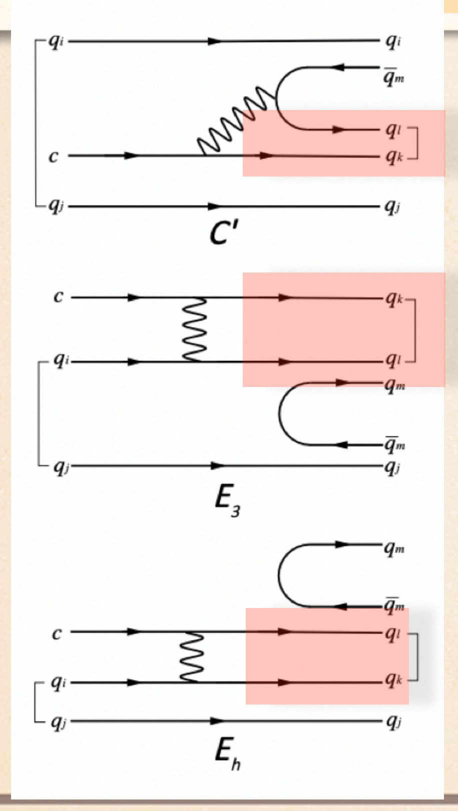
$b_3 = -b_2$

$b_6 = -b_5$

$b_9 = -b_8$

$b_{18} = -b_{17}$

$b_{20} = -b_{19}$



Korner-Pati-Woo theorem:

quark pair in a baryon produced by weak interactions
is required to be antisymmetric in flavor —> [l, k] asymmetric

$$
\begin{aligned}
\mathcal{A}_{\mathrm{TDA}} = \; & T(\mathcal{B}_c)^{ij} H^{km}_{l} M^{l}_{m} \left[b_1 (\mathcal{B}_8)_{ijk} + b_2 (\mathcal{B}_8)_{ikj} + b_3 (\mathcal{B}_8)_{jki}\right] \\
& + C(\mathcal{B}_c)^{ij} H^{ml}_{k} M^{k}_{m} \left[b_4 (\mathcal{B}_8)_{ijl} + b_5 (\mathcal{B}_8)_{ilj} + b_6 (\mathcal{B}_8)_{jli}\right] \\
& + C'(\mathcal{B}_c)^{ij} H^{kl}_{m} M^{m}_{i} \left[b_7 (\mathcal{B}_8)_{klj} + b_8 (\mathcal{B}_8)_{kjl} + b_9 (\mathcal{B}_8)_{ljk}\right] \\
& + E_1(\mathcal{B}_c)^{ij} H^{kl}_{i} M^{m}_{l} \left[b_{10} (\mathcal{B}_8)_{jkm} + b_{11} (\mathcal{B}_8)_{jmk} + b_{12} (\mathcal{B}_8)_{kmj}\right] \\
& + E_2(\mathcal{B}_c)^{ij} H^{kl}_{i} M^{m}_{k} \left[b_{13} (\mathcal{B}_8)_{jlm} + b_{14} (\mathcal{B}_8)_{jml} + b_{15} (\mathcal{B}_8)_{lmj}\right] \\
& + E_3(\mathcal{B}_c)^{ij} H^{kl}_{i} M^{m}_{j} \left[b_{16} (\mathcal{B}_8)_{klm} + b_{17} (\mathcal{B}_8)_{kml} + b_{18} (\mathcal{B}_8)_{lmk}\right] \\
& + E_h(\mathcal{B}_c)^{ij} H^{kl}_{i} M^{m}_{m} \left[b_{19} (\mathcal{B}_8)_{jkl} + b_{20} (\mathcal{B}_8)_{jlk} + b_{21} (\mathcal{B}_8)_{klj}\right] ,
\end{aligned}
$$

$b_3 = -b_2$

$b_6 = -b_5$

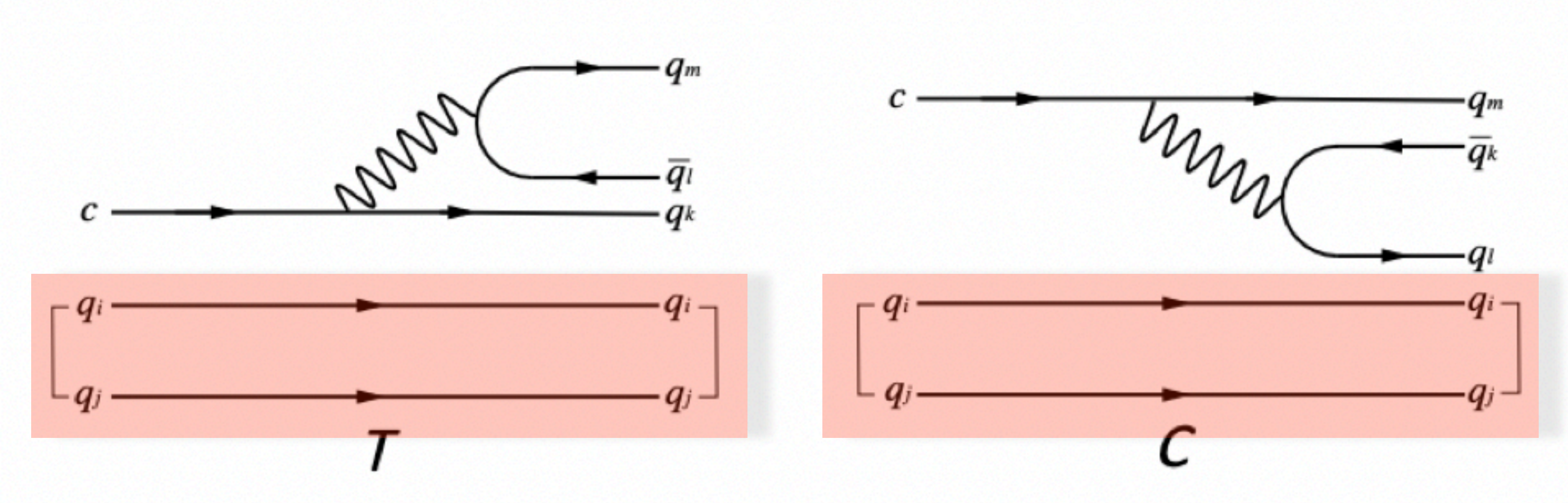
$b_9 = -b_8$

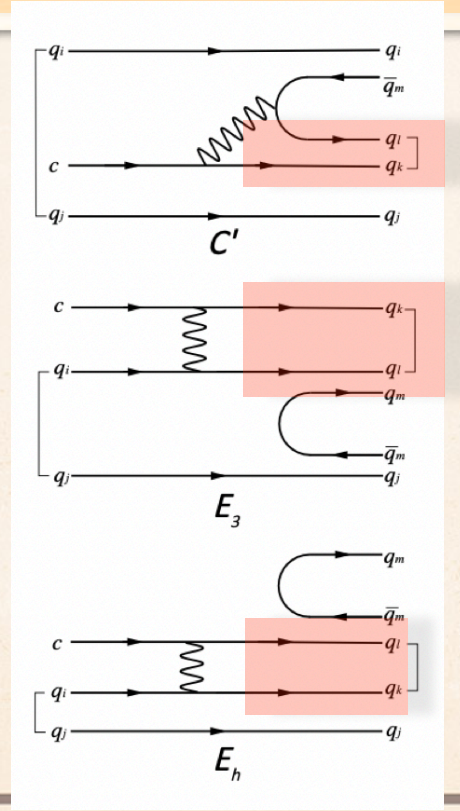
$b_{12} = b_{11}$

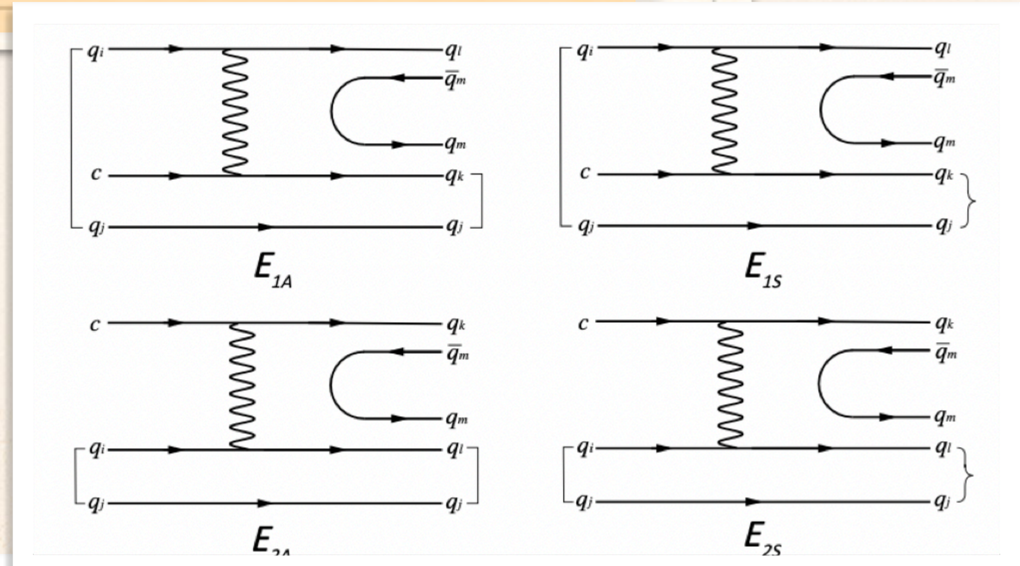
$b_{15} = b_{14}$

$b_{18} = -b_{17}$

$b_{20} = -b_{19}$



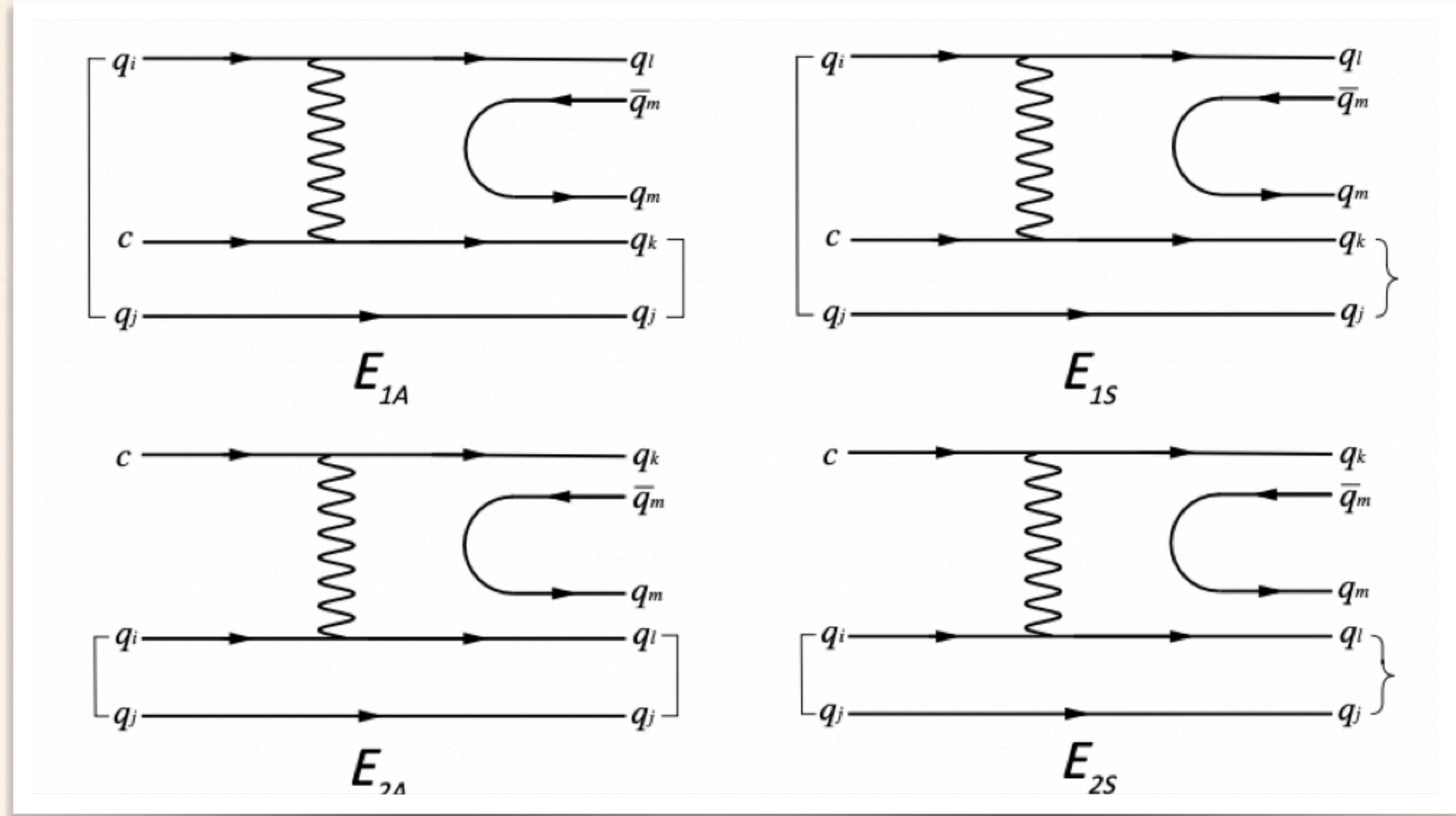




$$|\mathcal{B}^{m,k}(8)\rangle = a|\chi^m(1/2)_{A_{12}}\rangle|\psi^k(8)_{A_{12}}\rangle + b|\chi^m(1/2)_{S_{12}}\rangle|\psi^k(8)_{S_{12}}\rangle$$

A further simplification

$$
\begin{aligned}
\mathcal{A}_{\rm TDA} = \;& T(\mathcal{B}_c)^{ij} H^{km}_{l} (\mathcal{B}_8)_{ijk} M^{l}_{m} + C(\mathcal{B}_c)^{ij} H^{ml}_{k} (\mathcal{B}_8)_{ijl} M^{k}_{m} + C'(\mathcal{B}_c)^{ij} H^{kl}_{m} (\mathcal{B}_8)_{klj} M^{m}_{i} \\
& + E_{1A}(\mathcal{B}_c)^{ij} H^{kl}_{i} (\mathcal{B}_8)_{jkm} M^{m}_{l} + E_{1S}(\mathcal{B}_c)^{ij} H^{kl}_{i} M^{m}_{l} \left[(\mathcal{B}_8)_{jmk} + (\mathcal{B}_8)_{kmj}\right] \\
& \cancel{+ E_{2A}(\mathcal{B}_c)^{ij} H^{kl}_{i} (\mathcal{B}_8)_{jlm} M^{m}_{k}} + \cancel{E_{2S}(\mathcal{B}_c)^{ij} H^{kl}_{i} M^{m}_{k} \left[(\mathcal{B}_8)_{jml} + (\mathcal{B}_8)_{lmj}\right]} \\
& + E_3(\mathcal{B}_c)^{ij} H^{kl}_{i} (\mathcal{B}_8)_{klm} M^{m}_{j} + E_h(\mathcal{B}_c)^{ij} H^{kl}_{i} (\mathcal{B}_8)_{klj} M^{m}_{m},
\end{aligned}
$$



$$E_{2A} = -E_{1A}, \qquad E_{2S} = -E_{1S}.$$

KPW theorem

diagrams: 7

$$
\tilde{T} = T - E_{1S}, \quad \tilde{C} = C + E_{1S}, \quad \tilde{C}' = C' - 2E_{1S},
$$
$$
\tilde{E}_1 = E_{1A} + E_{1S} - E_3, \quad \tilde{E}_h = E_h + 2E_{1S}.
$$

Chau-Cheng-Tseng, 1996

diagrams: 5; parameters: 19

$$|\tilde{T}|_S e^{i\delta_S^{\tilde{T}}}, \quad |\tilde{C}|_S e^{i\delta_S^{\tilde{C}}}, \quad \cdots$$

$$|\tilde{T}|_P e^{i\delta_P^{\tilde{T}}}, \quad \ldots, \quad |\tilde{E}_h|_P e^{i\delta_P^{\tilde{E}_h}}$$

Amplitudes and their relations

Channel	TDA	$\widetilde{\text{TDA}}$
$\Lambda_c^+ \to \Lambda\pi^+$	$\frac{1}{\sqrt{6}}(-4T + C' + E_{1A} + 3E_{1S} - E_3)$	$\frac{1}{\sqrt{6}}(-4\tilde{T} + \tilde{C}' + \tilde{E}_1)$
$\Lambda_c^+ \to \Sigma^0\pi^+$	$\frac{1}{\sqrt{2}}(-C' - E_{1A} + E_{1S} + E_3)$	$\frac{1}{\sqrt{2}}(-\tilde{C}' - \tilde{E}_1)$
$\Lambda_c^+ \to \Sigma^+\pi^0$	$\frac{1}{\sqrt{2}}(C' + E_{1A} - E_{1S} - E_3)$	$\frac{1}{\sqrt{2}}(\tilde{C}' + \tilde{E}_1)$
$\Lambda_c^+ \to \Sigma^+\eta_8$	$\frac{1}{\sqrt{6}}(-C' + E_{1A} + 3E_{1S} - E_3)$	$\frac{1}{\sqrt{6}}(-\tilde{C}' + \tilde{E}_1)$
$\Lambda_c^+ \to \Sigma^+\eta_1$	$\frac{1}{\sqrt{3}}(-C' + E_{1A} - 3E_{1S} - E_3 - 3E_h)$	$\frac{1}{\sqrt{3}}(-\tilde{C}' + \tilde{E}_1 - 3\tilde{E}_h)$
$\Lambda_c^+ \to \Xi^0 K^+$	$E_{1A} + E_{1S} - E_3$	$\tilde{E}_1$
$\Lambda_c^+ \to p\bar{K}^0$	$2C + 2E_{1S}$	$2\tilde{C}$
$\Xi_c^0 \to \Lambda\bar{K}^0$	$\frac{1}{\sqrt{6}}(2C - C' - E_{1A} + 3E_{1S} + E_3)$	$\frac{1}{\sqrt{6}}(2\tilde{C} - \tilde{C}' - \tilde{E}_1)$
$\Xi_c^0 \to \Sigma^0\bar{K}^0$	$\frac{1}{\sqrt{2}}(2C + C' + E_{1A} + E_{1S} - E_3)$	$\frac{1}{\sqrt{2}}(2\tilde{C} + \tilde{C}' + \tilde{E}_1)$
$\Xi_c^0 \to \Sigma^+ K^-$	$-E_{1A} - E_{1S} + E_3$	$-\tilde{E}_1$
$\Xi_c^0 \to \Xi^0\pi^0$	$\frac{1}{\sqrt{2}}(-C' + 2E_{1S})$	$\frac{1}{\sqrt{2}}(-\tilde{C}')$
$\Xi_c^0 \to \Xi^0\eta_8$	$\frac{1}{\sqrt{6}}(C' + 2E_{1A} - 2E_3)$	$\frac{1}{\sqrt{6}}(\tilde{C}' + 2\tilde{E}_1)$
$\Xi_c^0 \to \Xi^0\eta_1$	$\frac{1}{\sqrt{3}}(C' + 3E_{1S} - E_{1A} + E_3 + 3E_h)$	$\frac{1}{\sqrt{3}}(\tilde{C}' - \tilde{E}_1 + 3\tilde{E}_h)$
$\Xi_c^0 \to \Xi^-\pi^+$	$2T - 2E_{1S}$	$2\tilde{T}$
$\Xi_c^+ \to \Sigma^+\bar{K}^0$	$-2C - C'$	$-2\tilde{C} - \tilde{C}'$
$\Xi_c^+ \to \Xi^0\pi^+$	$-2T + C'$	$-2\tilde{T} + \tilde{C}'$

$$A(\Lambda_c^+ \to \Lambda\pi^+) = \frac{1}{3}(-a - c - d_3 + d_4 + e) \qquad \text{Kohara ('91)}$$

$$= \frac{1}{2\sqrt{6}}(2A_A + B_A - C_{1A} + C_{2A}) - \frac{1}{2\sqrt{2}}C_{2S}. \qquad \text{CCT ('96)}$$

$$= \frac{1}{\sqrt{6}}(-T + C' + E_{1A} + 3E_{1S} - E_3) \qquad \text{This work ('24)}$$

$$= \frac{1}{\sqrt{6}}(-4\tilde{T} + \tilde{C}' + \tilde{E}_1) \qquad \text{This work ('24)}$$

1. $\tilde{T}$: $\Xi_c^0 \to \Xi^-\pi^+, \Sigma^-\pi^+, \Xi^-K^+, \Sigma^-K^+$; $\Xi_c^+ \to \Xi^0 K^+$.
2. $\tilde{C}$: $\Lambda_c^+ \to p\overline{K}^0$; $\Xi_c^0 \to \Xi^0 K^0$; $\Xi_c^+ \to \Sigma^+ K^0$.
3. $\tilde{C}'$: $\Lambda_c^+ \to \Sigma^+ K^0, \Sigma^0 K^+$; $\Xi_c^+ \to p\overline{K}^0$.
4. $\tilde{E}_1$: $\Lambda_c^+ \to \Xi^0 K^+$; $\Xi_c^0 \to \Sigma^+K^-, \Sigma^+\pi^-, pK^-, p\pi^-, n\pi^0$; $\Xi_c^+ \to p\pi^0, n\pi^+$.

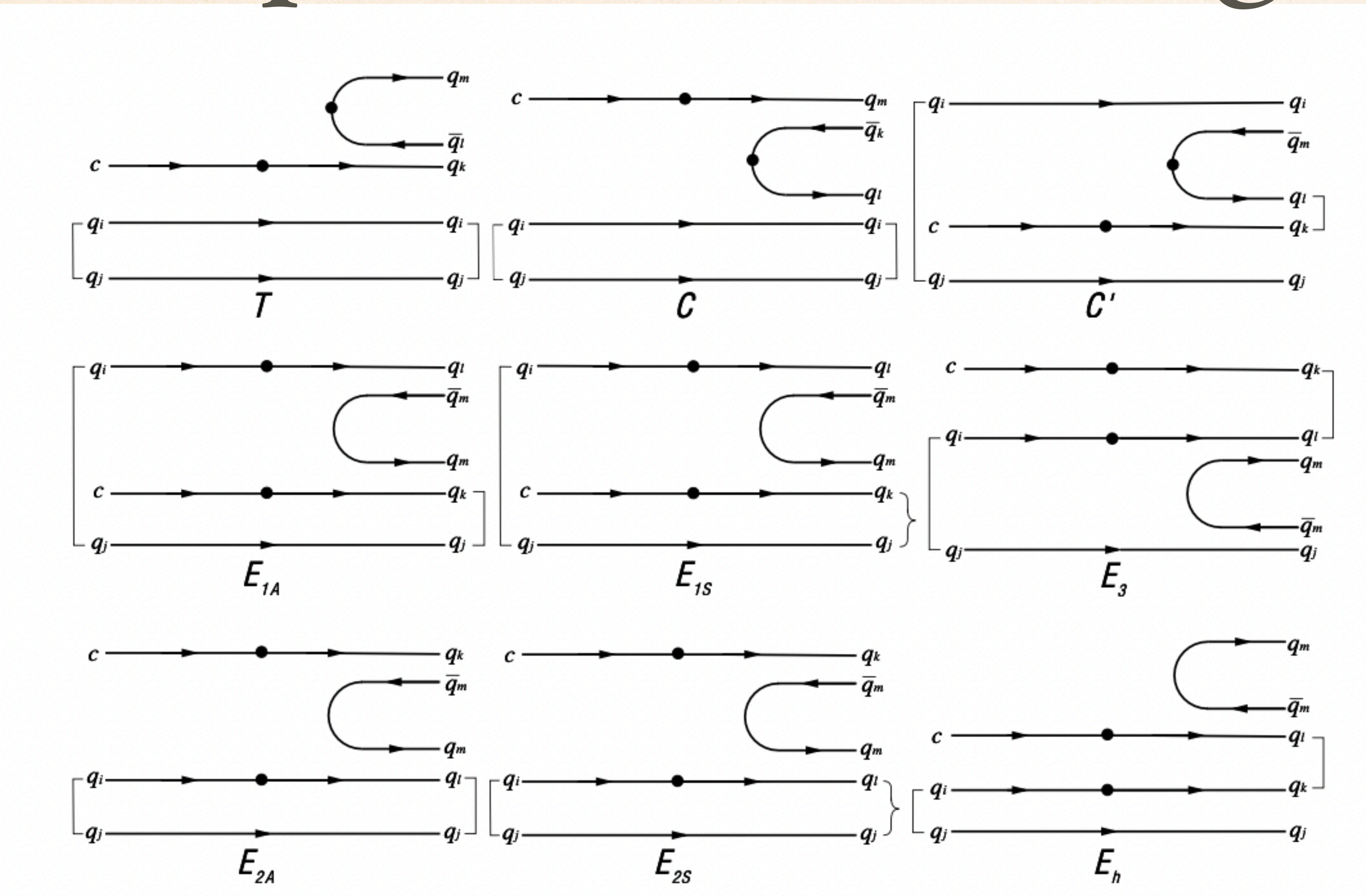
$$\frac{\tau_{\Lambda_c^+}}{\tau_{\Xi_c^0}}\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+) = 3\mathcal{B}(\Lambda_c^+ \to \Lambda\pi^+) + \mathcal{B}(\Lambda_c^+ \to \Sigma^0\pi^+) - \frac{1}{\sin^2\theta_C}\mathcal{B}(\Lambda_c^+ \to n\pi^+),$$

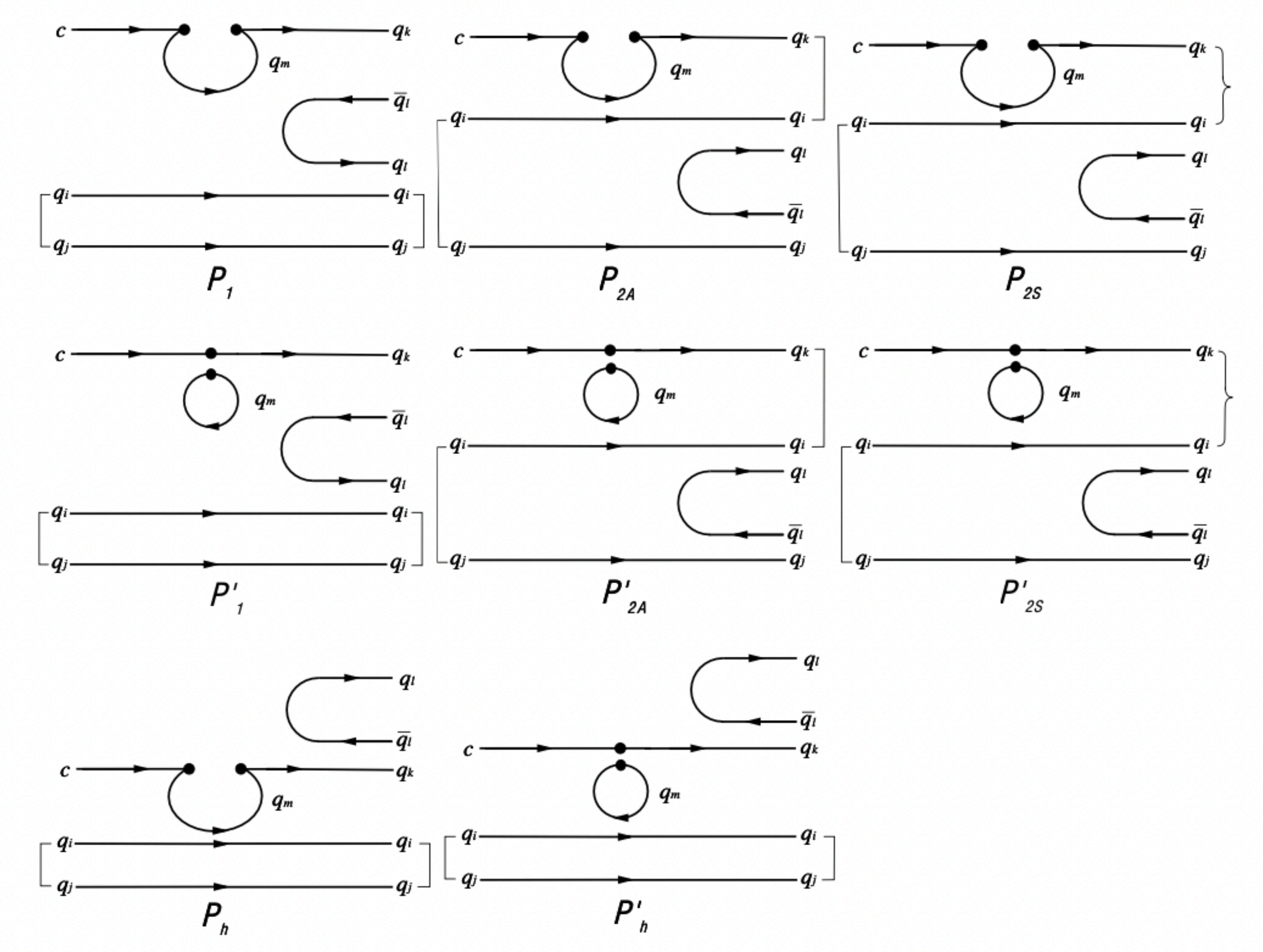
$$\frac{\tau_{\Xi_c^0}}{\tau_{\Lambda_c^+}}\mathcal{B}(\Lambda_c^+ \to p\overline{K}^0) = 3\mathcal{B}(\Xi_c^0 \to \Lambda\overline{K}^0) + \mathcal{B}(\Xi_c^0 \to \Sigma^0\overline{K}^0) - \frac{1}{\sin^2\theta_C}\mathcal{B}(\Xi_c^0 \to \Xi^0 K^0),$$

$$A(\mathcal{B}_c \to \mathcal{B}K_S^0) = -\frac{1}{\sqrt{2}}[A(\mathcal{B}_c \to \mathcal{B}\overline{K}^0) - A(\mathcal{B}_c \to \mathcal{B}K^0)],$$

$$A(\mathcal{B}_c \to \mathcal{B}K_L^0) = \frac{1}{\sqrt{2}}[A(\mathcal{B}_c \to \mathcal{B}\overline{K}^0) + A(\mathcal{B}_c \to \mathcal{B}K^0)].$$

Incorporation of Penguin





$$
\begin{aligned}
\mathcal{A}_{\mathrm{TDA}} = \;& T(\mathcal{B}_c)^{ij} H^{km}_{l} (\mathcal{B}_8)_{ijk} (M)^{l}_{m} \\
&+ C(\mathcal{B}_c)^{ij} H^{ml}_{k} (\mathcal{B}_8)_{ijl} (M)^{k}_{m} + C'(\mathcal{B}_c)^{ij} H^{kl}_{m} (\mathcal{B}_8)_{klj} (M)^{m}_{i} \\
&+ E_{1A}(\mathcal{B}_c)^{ij} H^{kl}_{i} (\mathcal{B}_8)_{jkm} (M)^{m}_{l} + E_{1S}(\mathcal{B}_c)^{ij} H^{kl}_{i} (M)^{m}_{l} \left[(\mathcal{B}_8)_{jmk} + (\mathcal{B}_8)_{kmj}\right] \\
&+ E_{2A}(\mathcal{B}_c)^{ij} H^{kl}_{i} (\mathcal{B}_8)_{jlm} (M)^{m}_{k} + E_{2S}(\mathcal{B}_c)^{ij} H^{kl}_{i} (M)^{m}_{k} \left[(\mathcal{B}_8)_{jml} + (\mathcal{B}_8)_{lmj}\right] \\
&+ E_{3}(\mathcal{B}_c)^{ij} H^{kl}_{i} (\mathcal{B}_8)_{klm} (M)^{m}_{j} + E_{h}(\mathcal{B}_c)^{ij} H^{kl}_{i} (\mathcal{B}_8)_{klj} (M)^{m}_{m} \\
&+ P_{h}(\mathcal{B}_c)^{ij} H^{mk}_{m} (\mathcal{B}_8)_{ijk} (M)^{l}_{l} + P_{1}(\mathcal{B}_c)^{ij} H^{mk}_{m} (\mathcal{B}_8)_{ijl} (M)^{l}_{k} \\
&+ P_{2A}(\mathcal{B}_c)^{ij} H^{mk}_{m} (\mathcal{B}_8)_{kil} (M)^{l}_{j} + P_{2S}(\mathcal{B}_c)^{ij} H^{mk}_{m} (M)^{l}_{j} \left[(\mathcal{B}_8)_{kli} + (\mathcal{B}_8)_{ilk}\right] \\
&+ P'_{h}(\mathcal{B}_c)^{ij} H^{km}_{m} (\mathcal{B}_8)_{ijk} (M)^{l}_{l} + P'_{1}(\mathcal{B}_c)^{ij} H^{km}_{m} (\mathcal{B}_8)_{ijl} (M)^{l}_{k} \\
&+ P'_{2A}(\mathcal{B}_c)^{ij} H^{km}_{m} (\mathcal{B}_8)_{kil} (M)^{l}_{j} + P'_{2S}(\mathcal{B}_c)^{ij} H^{km}_{m} (M)^{l}_{j} \left[(\mathcal{B}_8)_{kli} + (\mathcal{B}_8)_{ilk}\right],
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{\mathrm{TDA}} = \;& (2T - C' - 2E_{1S})(\mathcal{B}_c)_i(\mathcal{B}_8)^{i}_{j} H^{jl}_{m} M^{m}_{l} + (2C + C' + 2E_{1S})(\mathcal{B}_c)_i(\mathcal{B}_8)^{i}_{j} H^{lj}_{m} M^{m}_{l} \\
&+ C'(\mathcal{B}_c)_i(\mathcal{B}_8)^{l}_{j}(H^{ji}_{m} - H^{ij}_{m}) M^{m}_{l} + (E_{1A} - E_{1S} - E_3)(\mathcal{B}_c)_i(\mathcal{B}_8)^{j}_{m}(H^{il}_{j} - H^{li}_{j}) M^{m}_{l} \\
&+ 2E_{1S}(\mathcal{B}_c)_i(\mathcal{B}_8)^{m}_{j}(H^{jl}_{m} - H^{lj}_{m}) M^{i}_{l} - E_h(\mathcal{B}_c)_i(\mathcal{B}_8)^{l}_{j}(H^{ij}_{l} - H^{ji}_{l}) M^{m}_{m} \\
&+ (E_{1S} - E_{1A} + E_3 + 2P_1 - P_{2A} - P_{2S})(\mathcal{B}_c)_i(\mathcal{B}_8)^{i}_{m} H^{jl}_{j} M^{m}_{l} \\
&+ (-E_{1S} + E_{1A} - E_3 + 2P'_1 - P'_{2A} - P'_{2S})(\mathcal{B}_c)_i(\mathcal{B}_8)^{i}_{m} H^{lj}_{j} M^{m}_{l} \\
&+ (-E_3 + P_{2A} + P_{2S})(\mathcal{B}_c)_i(\mathcal{B}_8)^{l}_{m} H^{ji}_{j} M^{m}_{l} + (E_3 + P'_{2A} + P'_{2S})(\mathcal{B}_c)_i(\mathcal{B}_8)^{l}_{m} H^{ij}_{j} M^{m}_{l} \\
&+ (E_h + 2P_h + 2P_{2S})(\mathcal{B}_c)_i(\mathcal{B}_8)^{i}_{j} H^{lj}_{l} M^{m}_{m} + (-E_h + 2P'_h + 2P'_{2S})(\mathcal{B}_c)_i(\mathcal{B}_8)^{i}_{j} H^{jl}_{l} M^{m}_{m} \\
&- 2P_{2S}(\mathcal{B}_c)_i(\mathcal{B}_8)^{l}_{j} H^{mj}_{m} M^{i}_{l} - 2P'_{2S}(\mathcal{B}_c)_i(\mathcal{B}_8)^{l}_{j} H^{jm}_{m} M^{i}_{l},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{\mathrm{IRAa}} = \; & a_1\,(\mathcal{B}_c)_i\,(H_{\bar{6}})^{ik}_j\,(\mathcal{B}_8)^j_k\,M^l_l + a_2\,(\mathcal{B}_c)_i\,(H_{\bar{6}})^{ik}_j\,(\mathcal{B}_8)^l_k\,M^j_l + a_3\,(\mathcal{B}_c)_i\,(H_{\bar{6}})^{ik}_j\,(\mathcal{B}_8)^j_l\,M^l_k \\
& + a_4\,(\mathcal{B}_c)_i\,(H_{\bar{6}})^{jk}_l\,(\mathcal{B}_8)^i_j\,M^l_k + a_5\,(\mathcal{B}_c)_i\,(H_{\bar{6}})^{jk}_l\,(\mathcal{B}_8)^l_j\,M^i_k + a_6\,(\mathcal{B}_c)_i\,(H_{15})^{ik}_j\,(\mathcal{B}_8)^j_k\,M^l_l \\
& + a_7\,(\mathcal{B}_c)_i\,(H_{15})^{ik}_j\,(\mathcal{B}_8)^l_k\,M^j_l + a_8\,(\mathcal{B}_c)_i\,(H_{15})^{ik}_j\,(\mathcal{B}_8)^j_l\,M^l_k + a_9\,(\mathcal{B}_c)_i\,(H_{15})^{jk}_l\,(\mathcal{B}_8)^i_j\,M^l_k \\
& + a_{10}\,(\mathcal{B}_c)_i\,(H_{15})^{jk}_l\,(\mathcal{B}_8)^l_j\,M^i_k + b_1(\mathcal{B}_c)_i\,(H_3)^j\,(\mathcal{B}_8)^i_j M^l_l + b_2(\mathcal{B}_c)_i\,(H_3)^j\,(\mathcal{B}_8)^l_j M^i_l \\
& + b_3(\mathcal{B}_c)_i\,(H_3)^i\,(\mathcal{B}_8)^l_j M^j_l + b_4(\mathcal{B}_c)_i\,(H_3)^l\,(\mathcal{B}_8)^i_j M^j_l
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{H}_{\mathrm{eff}} &= \frac{G_F}{\sqrt{2}} \sum_{q_1,q_2}^{d,s} V_{cq_1} V_{uq_2} (c_1 O_1^{q_1 q_2} + c_2 O_2^{q_1 q_2}) + h.c. \\
&= \frac{G_F}{\sqrt{2}} \sum_{q_1,q_2}^{d,s} V_{cq_1} V_{uq_2} (c_+ O_+^{q_1 q_2} + c_- O_-^{q_1 q_2}) + h.c.,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{\mathrm{IRAa}} = \; & a_1\,(\mathcal{B}_c)_i\,(H_6)^{ik}_j\,(\mathcal{B}_8)^j_k\,M^l_l + a_2\,(\mathcal{B}_c)_i\,(H_6)^{ik}_j\,(\mathcal{B}_8)^l_k\,M^j_l + a_3\,(\mathcal{B}_c)_i\,(H_6)^{ik}_j\,(\mathcal{B}_8)^j_l\,M^l_k \\
& + a_4\,(\mathcal{B}_c)_i\,(H_6)^{jk}_l\,(\mathcal{B}_8)^i_j\,M^l_k + \cancel{a_5\,(\mathcal{B}_c)_i\,(H_6)^{jk}_l\,(\mathcal{B}_8)^l_j\,M^i_k} \\
& \cancel{+ a_6\,(\mathcal{B}_c)_i\,(H_{15})^{ik}_j}\,(\mathcal{B}_8)^j_k\,M^l_l + \cancel{a_7\,(\mathcal{B}_c)_i\,(H_{15})^{ik}_j\,(\mathcal{B}_8)^l_k\,M^j_l} + \cancel{a_8\,(\mathcal{B}_c)_i\,(H_{15})^{ik}_j\,(\mathcal{B}_8)^j_l\,M^l_k} \\
& + a_9\,(\mathcal{B}_c)_i\,(H_{15})^{jk}_l\,(\mathcal{B}_8)^i_j\,M^l_k + \cancel{a_{10}\,(\mathcal{B}_c)_i\,(H_{15})^{jk}_l\,(\mathcal{B}_8)^l_j}\,M^i_k.
\end{aligned}
$$

redundant dof:

$$
a'_1 = a_1 - a_5, \quad a'_2 = a_2 + a_5, \quad a'_3 = a_3 + a_5, \quad a'_4 = a_4 + a_5,
$$

$$
\begin{aligned}
\mathcal{A}^{\mathrm{tree}}_{\mathrm{IRAb}} = \; & \tilde{f}^a\,(\mathcal{B}_c)^{ik}\,(H_{\bar{6}})_{ij}\,(\mathcal{B}_8)^j_k\,M^l_l + \tilde{f}^b\,(\mathcal{B}_c)^{ik}\,(H_{\bar{6}})_{ij}\,(\mathcal{B}_8)^l_k\,M^j_l + \tilde{f}^c\,(\mathcal{B}_c)^{ik}\,(H_{\bar{6}})_{ij}\,(\mathcal{B}_8)^j_l\,M^l_k \\
& + \tilde{f}^d\,(\mathcal{B}_c)^{kl}\,(H_{\bar{6}})_{ij}\,(\mathcal{B}_8)^i_k\,M^j_l + \tilde{f}^e\,(\mathcal{B}_c)_j\,(H_{15})^{ik}_l\,(\mathcal{B}_8)^j_i\,M^l_k.
\end{aligned}
$$

Geng-He-Jin-Liu-Yang, 2024

KPW theorem:

$$
a_6 = a_7 = a_8 = a_{10} = 0
$$

$$
\tilde{T} = \frac{1}{2}(\tilde{f}^b + \tilde{f}^e), \qquad \tilde{C} = \frac{1}{2}(-\tilde{f}^b + \tilde{f}^e),
$$

$$
\tilde{C}' = \tilde{f}^b - \tilde{f}^d, \qquad \tilde{E}_1 = -\tilde{f}^c, \qquad \tilde{E}_h = \tilde{f}^a,
$$

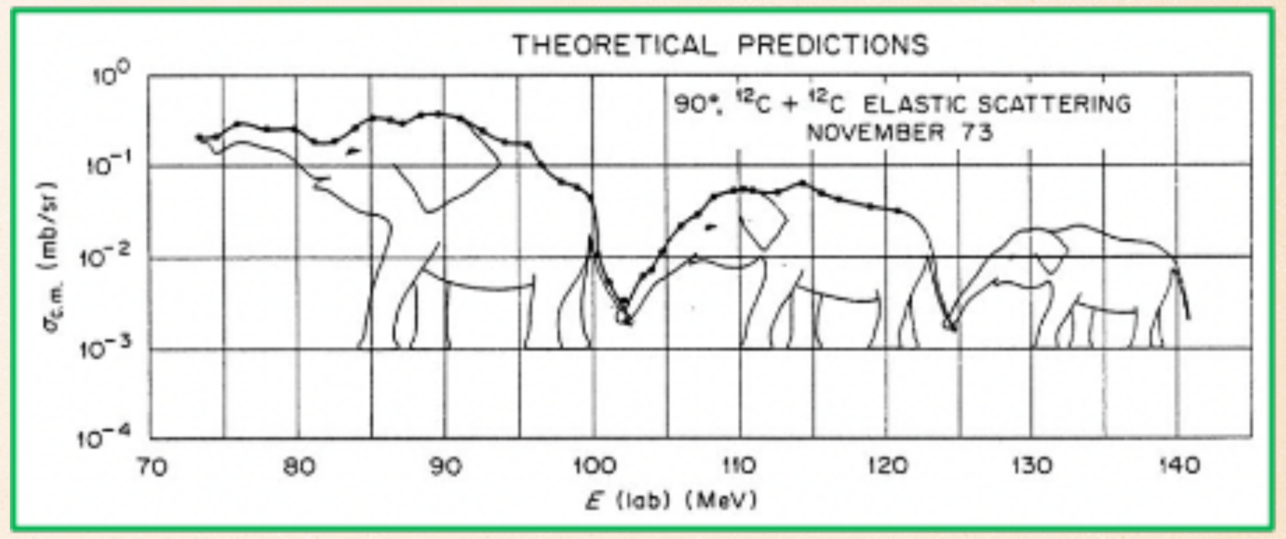
All available charmed baryon data in 2024

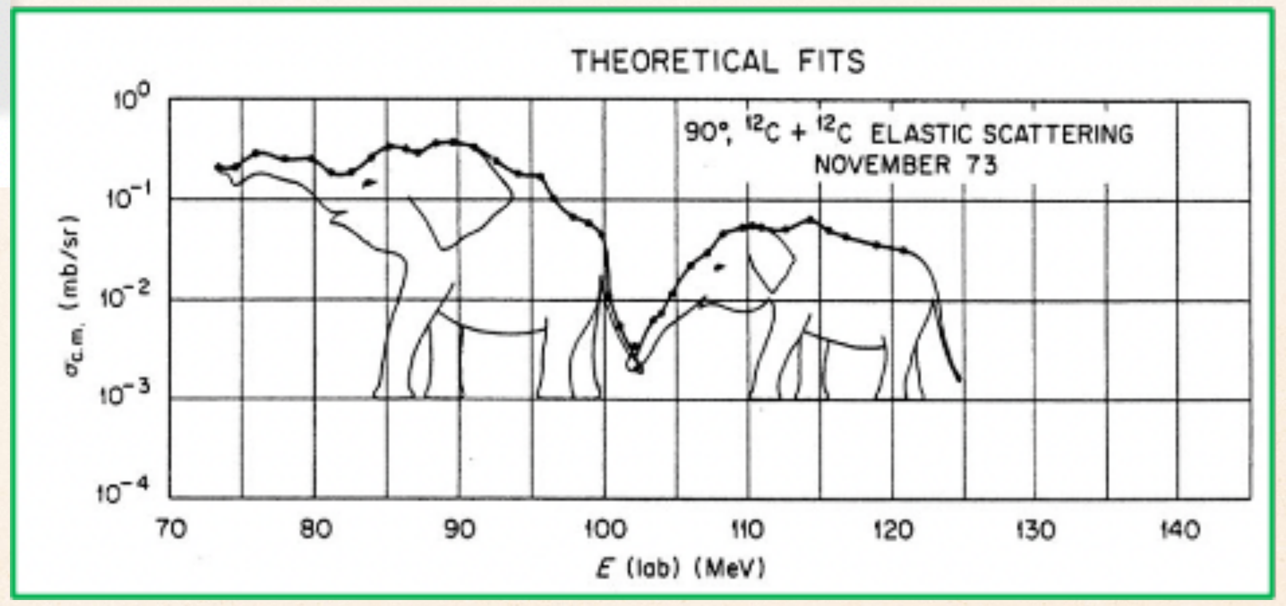
Observable	PDG	BESIII	Belle	LHCb	Average
$10^2\mathcal{B}(\Lambda_c^+\to\Lambda^0\pi^+)$	1.29 ± 0.05				1.29 ± 0.05
$10^2\mathcal{B}(\Lambda_c^+\to\Sigma^0\pi^+)$	1.27 ± 0.06				1.27 ± 0.06
$10^2\mathcal{B}(\Lambda_c^+\to\Sigma^+\pi^0)$	1.24 ± 0.09				1.24 ± 0.09
$10^2\mathcal{B}(\Lambda_c^+\to\Sigma^+\eta)$	0.32 ± 0.05				0.32 ± 0.05
$10^2\mathcal{B}(\Lambda_c^+\to\Sigma^+\eta')$	0.41 ± 0.08				0.41 ± 0.08
$10^2\mathcal{B}(\Lambda_c^+\to\Xi^0K^+)$	0.55 ± 0.07				0.55 ± 0.07
$10^4\mathcal{B}(\Lambda_c^+\to\Lambda^0K^+)$	6.42 ± 0.31				6.42 ± 0.31
$10^4\mathcal{B}(\Lambda_c^+\to\Sigma^0K^+)$	3.70 ± 0.31				3.70 ± 0.31
$10^4\mathcal{B}(\Lambda_c^+\to\Sigma^+K_S)$	4.7 ± 1.4				4.7 ± 1.4
$10^4\mathcal{B}(\Lambda_c^+\to n\pi^+)$	6.6 ± 1.3				6.6 ± 1.3
$10^4\mathcal{B}(\Lambda_c^+\to p\pi^0)$	<0.8	$1.56^{+0.75}_{-0.61}$ [40]			$1.56^{+0.75}_{-0.61}$
$10^2\mathcal{B}(\Lambda_c^+\to pK_S)$	1.59 ± 0.07				1.59 ± 0.07
$10^3\mathcal{B}(\Lambda_c^+\to p\eta)$	1.57 ± 0.12	1.63 ± 0.33 [40]			1.58 ± 0.11
$10^4\mathcal{B}(\Lambda_c^+\to p\eta')$	4.8 ± 0.9				4.8 ± 0.9
$10^2\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+)$	1.43 ± 0.27		1.80 ± 0.52 [35]		1.80 ± 0.52
$10^2\frac{\mathcal{B}(\Xi_c^0\to\Xi^-K^+)}{\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+)}$	2.75 ± 0.57				2.75 ± 0.57
$10^2\frac{\mathcal{B}(\Xi_c^0\to\Lambda K_S^0)}{\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+)}$	22.5 ± 1.3				22.5 ± 1.3
$10^2\frac{\mathcal{B}(\Xi_c^0\to\Sigma^0 K_S^0)}{\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+)}$	3.8 ± 0.7				3.8 ± 0.7
$10^2\frac{\mathcal{B}(\Xi_c^0\to\Sigma^+ K^-)}{\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+)}$	12.3 ± 1.2				12.3 ± 1.2
$10^3\mathcal{B}(\Xi_c^0\to\Xi^0\pi^0)$			6.9 ± 1.6 [20]		6.9 ± 1.6
$10^3\mathcal{B}(\Xi_c^0\to\Xi^0\eta)$			1.6 ± 0.5 [20]		1.6 ± 0.5
$10^3\mathcal{B}(\Xi_c^0\to\Xi^0\eta')$			1.2 ± 0.4 [20]		1.2 ± 0.4
$10^2\mathcal{B}(\Xi_c^+\to\Xi^0\pi^+)$	1.6 ± 0.8				1.6 ± 0.8

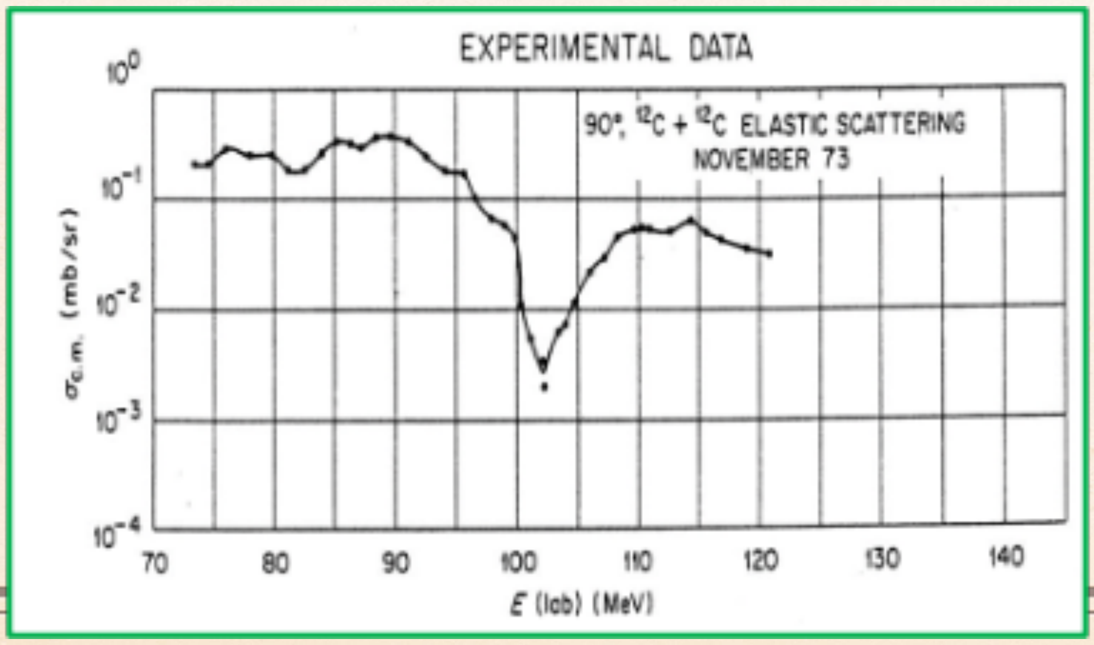
Observable	PDG	BESIII	Belle	LHCb	Average
$\alpha(\Lambda_c^+\to\Lambda^0\pi^+)$	-0.755 ± 0.006			-0.782 ± 0.010	-0.762 ± 0.006
$\alpha(\Lambda_c^+\to\Sigma^0\pi^+)$	-0.466 ± 0.018				-0.466 ± 0.018
$\alpha(\Lambda_c^+\to pK_S)$	0.18 ± 0.45			-0.744 ± 0.015	-0.743 ± 0.028
$\alpha(\Lambda_c^+\to\Sigma^+\pi^0)$	-0.484 ± 0.027				-0.484 ± 0.027
$\alpha(\Lambda_c^+\to\Sigma^+\eta)$	-0.99 ± 0.06				-0.99 ± 0.06
$\alpha(\Lambda_c^+\to\Sigma^+\eta')$	-0.46 ± 0.07				-0.46 ± 0.07
$\alpha(\Lambda_c^+\to\Lambda^0K^+)$	-0.585 ± 0.052			-0.569 ± 0.065	-0.579 ± 0.041
$\alpha(\Lambda_c^+\to\Sigma^0K^+)$	-0.54 ± 0.20				-0.54 ± 0.20
$\alpha(\Lambda_c^+\to\Xi^0K^+)$		0.01 ± 0.16 [33]			0.01 ± 0.16
$\alpha(\Xi_c^0\to\Xi^-\pi^+)$	-0.64 ± 0.05				-0.64 ± 0.05
$\alpha(\Xi_c^0\to\Xi^0\pi^0)$			-0.90 ± 0.27 [20]		-0.90 ± 0.27
$\beta(\Lambda_c^+\to\Lambda^0\pi^+)$				0.368 ± 0.021	0.368 ± 0.021
$\beta(\Lambda_c^+\to\Lambda^0K^+)$				0.35 ± 0.13	0.35 ± 0.13
$\gamma(\Lambda_c^+\to\Lambda^0\pi^+)$				0.502 ± 0.017	0.502 ± 0.017
$\gamma(\Lambda_c^+\to\Lambda^0K^+)$				-0.743 ± 0.071	-0.743 ± 0.071

LHCb 2024

Belle 2024







to interpret data

Fitted parameters

scheme I: without Belle/Belle-II 2024;
scheme II: all 38 data included

Jan. 2024

	$\lvert X_i\rvert_S$ ($10^{-2}G_F$ GeV^2)	$\lvert X_i\rvert_P$	$\delta_S^{X_i}$ (in radian)	$\delta_P^{X_i}$
$\tilde{T}$	2.37 ± 0.41	16.56 ± 0.69	–	2.76 ± 0.32
$\tilde{C}$	1.04 ± 1.08	13.82 ± 0.58	-1.97 ± 0.79	-0.37 ± 0.44
$\tilde{C}'$	2.59 ± 0.95	24.97 ± 1.67	0.29 ± 0.19	2.86 ± 0.36
$\tilde{E}_1$	4.10 ± 0.20	2.56 ± 2.21	1.18 ± 0.38	-0.96 ± 0.43
$\tilde{E}_h$	1.54 ± 1.22	19.16 ± 3.00	-1.35 ± 0.60	0.37 ± 0.41
$\tilde{f}^a$	0.81 ± 1.89	23.02 ± 4.04	–	2.12 ± 1.03
$\tilde{f}^b$	2.89 ± 1.50	30.56 ± 1.30	2.03 ± 0.61	-1.78 ± 0.98
$\tilde{f}^c$	4.20 ± 0.18	1.95 ± 2.21	-0.06 ± 1.03	-2.68 ± 1.16
$\tilde{f}^d$	0.98 ± 0.90	7.25 ± 2.07	2.72 ± 1.29	-2.55 ± 1.00
$\tilde{f}^e$	2.06 ± 0.62	4.73 ± 2.11	1.09 ± 0.99	-0.94 ± 0.99

$\chi^2_{\rm min}/\text{dof} \sim 2$

Oct. 2024

scheme I.

	$\lvert X_i\rvert_S$ ($10^{-2}G_F$ GeV^2)	$\lvert X_i\rvert_P$	$\delta_S^{X_i}$ (in radian)	$\delta_P^{X_i}$
$\tilde{T}$	4.25 ± 0.11	12.43 ± 0.30	–	2.40 ± 0.04
$\tilde{C}$	3.08 ± 0.52	11.57 ± 1.00	3.02 ± 0.12	-0.77 ± 0.20
$\tilde{C}'$	5.39 ± 0.38	18.79 ± 0.86	-0.03 ± 0.05	2.23 ± 0.11
$\tilde{E}_1$	2.90 ± 0.19	10.22 ± 0.50	-2.80 ± 0.05	1.86 ± 0.10
$\tilde{E}_h$	4.06 ± 0.53	13.82 ± 1.93	2.66 ± 0.12	-1.90 ± 0.20
$\tilde{f}^a$	4.10 ± 0.52	16.18 ± 2.34	–	1.72 ± 0.12
$\tilde{f}^b$	7.00 ± 1.36	24.52 ± 1.72	-2.78 ± 0.11	-0.30 ± 0.19
$\tilde{f}^c$	2.91 ± 0.19	10.26 ± 0.49	-2.36 ± 0.11	2.29 ± 0.14
$\tilde{f}^d$	1.59 ± 1.19	7.50 ± 3.87	-2.89 ± 0.40	0.25 ± 0.40
$\tilde{f}^e$	1.57 ± 1.39	0.71 ± 2.92	-2.44 ± 0.22	-1.64 ± 3.55

scheme II.

	$\lvert X_i\rvert_S$ ($10^{-2}G_F$ GeV^2)	$\lvert X_i\rvert_P$	$\delta_S^{X_i}$ (in radian)	$\delta_P^{X_i}$
$\tilde{T}$	4.22 ± 0.10	12.50 ± 0.28	–	2.42 ± 0.04
$\tilde{C}$	2.40 ± 0.66	12.70 ± 0.71	2.88 ± 0.59	-0.57 ± 0.15
$\tilde{C}'$	5.26 ± 0.35	19.04 ± 0.85	-0.02 ± 0.05	2.32 ± 0.11
$\tilde{E}_1$	2.86 ± 0.19	10.20 ± 0.50	-2.80 ± 0.05	1.83 ± 0.09
$\tilde{E}_h$	3.07 ± 0.47	11.80 ± 1.42	2.87 ± 0.09	-1.75 ± 0.19
$\tilde{f}^a$	3.16 ± 0.43	10.74 ± 1.73	–	1.68 ± 0.15
$\tilde{f}^b$	7.52 ± 0.30	23.27 ± 0.69	-2.98 ± 0.09	-0.56 ± 0.10
$\tilde{f}^c$	2.86 ± 0.19	10.19 ± 0.49	-2.51 ± 0.09	2.13 ± 0.12
$\tilde{f}^d$	2.34 ± 0.20	4.30 ± 0.65	3.02 ± 0.21	-0.75 ± 0.39
$\tilde{f}^e$	1.48 ± 0.32	3.82 ± 1.04	-2.05 ± 0.17	0.60 ± 0.15

	Scheme I		Scheme II	
	TDA	IRA	TDA	IRA
χ^2	34.31	33.21	59.17	57.08
$\chi^2/d.o.f.$	2.29	2.21	3.11	3.00

Part of predictions: January

Channel	$10^2\mathcal{B}$	α	$\lvert A\rvert$	$\lvert B\rvert$	$\delta_P-\delta_S$	$10^2\mathcal{B}_{\rm exp}$	$\alpha_{\rm exp}$
$\Lambda_c^+\to\Lambda^0\pi^+$	1.31 ± 0.05	-0.76 ± 0.01	2.76 ± 0.24	16.97 ± 0.38	0.21 ± 0.30	1.29 ± 0.05	-0.76 ± 0.01 [19, 26]
$\Lambda_c^+\to\Sigma^0\pi^+$	1.26 ± 0.05	-0.48 ± 0.02	4.09 ± 0.86	15.42 ± 2.32	-1.07 ± 0.04	1.27 ± 0.06	-0.47 ± 0.03 [19, 26]
$\Lambda_c^+\to\Sigma^+\pi^0$	1.27 ± 0.05	-0.48 ± 0.02	4.09 ± 0.86	15.42 ± 2.32	-1.07 ± 0.04	1.25 ± 0.09	-0.49 ± 0.03 [19, 25]
$\Lambda_c^+\to\Sigma^+\eta$	0.33 ± 0.04	-0.93 ± 0.05	2.30 ± 0.35	9.50 ± 1.16	0.34 ± 0.16	0.32 ± 0.04 [19, 25]	-0.99 ± 0.06 [25]
$\Lambda_c^+\to\Sigma^+\eta'$	0.39 ± 0.12	-0.45 ± 0.07	3.83 ± 1.44	23.00 ± 3.85	2.03 ± 0.08	0.44 ± 0.15 [19, 25]	-0.46 ± 0.07 [25]
$\Lambda_c^+\to\Xi^0K^+$	0.41 ± 0.03	-0.16 ± 0.13	3.89 ± 0.19	2.49 ± 2.13	-2.14 ± 0.63	0.55 ± 0.07	0.01 ± 0.16 [7]
$\Lambda_c^+\to\Lambda^0K^+$	0.0639 ± 0.0030	-0.56 ± 0.05	1.09 ± 0.18	3.30 ± 0.59	-0.97 ± 0.06	0.0635 ± 0.0031 [19, 26]	-0.585 ± 0.052 [26]
$\Lambda_c^+\to\Sigma^0K^+$	0.0377 ± 0.0032	-0.54 ± 0.08	0.40 ± 0.15	3.86 ± 0.26	-0.59 ± 0.43	0.0382 ± 0.0051 [19, 26]	-0.55 ± 0.20 [26]
$\Lambda_c^+\to\Sigma^+K_S$	0.038 ± 0.003	-0.54 ± 0.08	0.57 ± 0.21	5.46 ± 0.37	-0.59 ± 0.43	0.047 ± 0.014	
$\Lambda_c^+\to n\pi^+$	0.063 ± 0.009	-0.78 ± 0.13	1.00 ± 0.14	2.44 ± 0.39	0.67 ± 0.21	0.066 ± 0.013	
$\Lambda_c^+\to p\pi^0$	0.0174 ± 0.0034	-0.12 ± 0.75	0.63 ± 0.14	0.96 ± 0.68	-1.70 ± 0.87	$0.0156^{+0.0075}_{-0.0061}$ [20]	
$\Lambda_c^+\to pK_S$	1.55 ± 0.07	0.00 ± 0.30	2.08 ± 2.10	26.21 ± 1.18	-1.56 ± 0.77	1.59 ± 0.07	0.18 ± 0.45
$\Lambda_c^+\to p\eta$	0.151 ± 0.007	0.08 ± 0.37	1.04 ± 0.54	5.42 ± 0.70	-1.67 ± 1.28	0.149 ± 0.008 [19, 20, 27]	
$\Lambda_c^+\to p\eta'$	0.052 ± 0.009	-0.54 ± 0.19	0.76 ± 0.30	4.73 ± 0.73	2.28 ± 0.14	0.049 ± 0.009	
$\Xi_c^0\to\Xi^-\pi^+$	2.83 ± 0.10	-0.72 ± 0.03	4.53 ± 0.81	31.46 ± 1.34	-0.39 ± 0.32	1.80 ± 0.52 [21]	-0.64 ± 0.05
$\Xi_c^+\to\Xi^0\pi^+$	0.9 ± 0.2	-0.93 ± 0.07	2.27 ± 0.30	8.18 ± 1.17	-0.36 ± 0.23	1.6 ± 0.8	
Channel	$10^2\mathcal{R}_X$	α	$\lvert A\rvert$	$\lvert B\rvert$	$\delta_P-\delta_S$	$10^2(\mathcal{R}_X)_{\rm exp}$	$\alpha_{\rm exp}$
$\Xi_c^0\to\Xi^-K^+$	4.10 ± 0.05	-0.76 ± 0.03	1.04 ± 0.19	7.25 ± 0.31	-0.39 ± 0.32	2.75 ± 0.57	
$\Xi_c^0\to\Lambda K_S^0$	24.1 ± 1.0	-0.24 ± 0.18	3.18 ± 1.25	19.54 ± 1.76	-1.24 ± 0.29	22.9 ± 1.4 [22]	
$\Xi_c^0\to\Sigma^0K_S^0$	3.9 ± 0.7	-0.11 ± 0.67	2.74 ± 0.59	4.17 ± 2.93	-1.70 ± 0.87	3.8 ± 0.7 [22]	
$\Xi_c^0\to\Sigma^+K^-$	13.0 ± 1.1	-0.21 ± 0.17	3.89 ± 0.19	2.49 ± 2.13	-2.14 ± 0.63	12.3 ± 1.2 [22]	

if phase shift vanishes, global fit $\to \alpha\sim 1$; non-unity $\alpha\to$ non-vanishing phase shift

Part of predictions: October

scheme I.

Channel	$10^2\mathcal{B}$	α	β	γ	$\lvert A\rvert$	$\lvert B\rvert$	$\delta_P-\delta_S$	$10^2\mathcal{B}_{\rm exp}$	$\alpha_{\rm exp}$	$\beta_{\rm exp}$ $\gamma_{\rm exp}$
$\Lambda_c^+\to\Lambda^0\pi^+$	1.30 ± 0.04	-0.76 ± 0.01	0.39 ± 0.02	0.51 ± 0.01	5.57 ± 0.10	9.24 ± 0.20	2.67 ± 0.19	1.29 ± 0.05	-0.762 ± 0.006	0.368 ± 0.021
	1.30 ± 0.05	-0.76 ± 0.01	0.39 ± 0.02	0.51 ± 0.01	5.57 ± 0.10	9.23 ± 0.20	2.67 ± 0.02			0.502 ± 0.017
$\Lambda_c^+\to\Sigma^0\pi^+$	1.25 ± 0.05	-0.47 ± 0.01	0.35 ± 0.10	-0.81 ± 0.04	1.94 ± 0.22	19.20 ± 0.46	2.50 ± 0.14	1.27 ± 0.06	-0.466 ± 0.018	
	1.25 ± 0.05	-0.47 ± 0.01	0.36 ± 0.10	-0.80 ± 0.05	1.97 ± 0.23	19.14 ± 0.47	2.48 ± 0.13			
$\Lambda_c^+\to\Sigma^+\pi^0$	1.26 ± 0.05	-0.47 ± 0.01	0.35 ± 0.10	-0.81 ± 0.04	1.94 ± 0.22	19.20 ± 0.46	2.50 ± 0.14	1.24 ± 0.09	-0.484 ± 0.027	
	1.26 ± 0.05	-0.47 ± 0.01	0.36 ± 0.10	-0.81 ± 0.05	1.97 ± 0.23	19.14 ± 0.47	2.48 ± 0.13			
$\Lambda_c^+\to\Sigma^+\eta$	0.33 ± 0.04	-0.92 ± 0.04	-0.01 ± 0.15	0.40 ± 0.10	2.94 ± 0.21	6.98 ± 0.73	-3.13 ± 0.21	0.32 ± 0.05	-0.99 ± 0.06	
	0.32 ± 0.04	-0.92 ± 0.04	-0.15 ± 0.16	0.36 ± 0.12	2.87 ± 0.20	7.15 ± 0.83	-2.98 ± 0.17			
$\Lambda_c^+\to\Sigma^+\eta'$	0.39 ± 0.07	-0.44 ± 0.07	0.88 ± 0.06	0.16 ± 0.28	4.03 ± 0.78	21.52 ± 2.63	2.03 ± 0.08	0.41 ± 0.08	-0.46 ± 0.07	
	0.43 ± 0.07	-0.45 ± 0.07	0.90 ± 0.03	-0.02 ± 0.3	3.91 ± 0.80	24.80 ± 3.35	2.03 ± 0.08			
$\Lambda_c^+\to\Xi^0K^+$	0.34 ± 0.03	-0.04 ± 0.12	-0.98 ± 0.02	0.19 ± 0.09	2.76 ± 0.18	9.71 ± 0.47	-1.61 ± 0.12	0.55 ± 0.07	0.01 ± 0.16	
	0.34 ± 0.03	-0.06 ± 0.12	-0.98 ± 0.02	0.19 ± 0.09	2.77 ± 0.18	9.75 ± 0.46	-1.63 ± 0.12			
$\Lambda_c^+\to pK_S$	1.56 ± 0.06	-0.74 ± 0.03	0.56 ± 0.16	-0.37 ± 0.23	4.17 ± 0.74	15.70 ± 1.43	2.50 ± 0.14	1.59 ± 0.07	-0.743 ± 0.028	
	1.59 ± 0.06	-0.74 ± 0.03	0.44 ± 0.54	-0.51 ± 0.47	3.73 ± 1.76	16.60 ± 2.62	2.61 ± 0.49			
$\Xi_c^0\to\Xi^-\pi^+$	2.97 ± 0.09	-0.73 ± 0.03	0.67 ± 0.03	0.13 ± 0.04	8.07 ± 0.21	23.63 ± 0.57	2.40 ± 0.04	1.80 ± 0.52	-0.64 ± 0.05	
	2.96 ± 0.09	-0.72 ± 0.03	0.68 ± 0.03	0.13 ± 0.04	8.08 ± 0.21	23.47 ± 0.58	2.38 ± 0.04			
$\Xi_c^0\to\Xi^0\pi^0$	0.72 ± 0.04	-0.64 ± 0.07	0.77 ± 0.07	-0.06 ± 0.10	3.62 ± 0.26	12.63 ± 0.58	2.26 ± 0.10	$0.69\pm0.16^*$	$-0.90\pm0.27^*$	
	0.71 ± 0.04	-0.61 ± 0.08	0.79 ± 0.06	-0.04 ± 0.10	3.65 ± 0.25	12.49 ± 0.58	2.22 ± 0.10			
$\Xi_c^0\to\Xi^0\eta$	0.26 ± 0.04	0.23 ± 0.15	-0.09 ± 0.15	-0.97 ± 0.04	0.43 ± 0.27	12.58 ± 1.07	-0.36 ± 0.56	$0.16\pm0.05^*$		
	0.23 ± 0.04	0.23 ± 0.15	-0.15 ± 0.15	-0.96 ± 0.05	0.45 ± 0.26	11.75 ± 0.97	-0.57 ± 0.35			
$\Xi_c^0\to\Xi^0\eta'$	0.43 ± 0.06	-0.70 ± 0.06	0.71 ± 0.06	-0.07 ± 0.27	3.91 ± 0.75	23.72 ± 2.55	2.35 ± 0.08	$0.12\pm0.04^*$		
	0.49 ± 0.07	-0.67 ± 0.06	0.70 ± 0.09	-0.25 ± 0.28	3.73 ± 0.77	27.17 ± 3.32	2.33 ± 0.09			
$\Xi_c^+\to\Xi^0\pi^+$	1.0 ± 0.1	-0.88 ± 0.08	0.31 ± 0.10	0.36 ± 0.13	2.95 ± 0.23	6.67 ± 0.86	2.80 ± 0.45	1.6 ± 0.8		
	1.0 ± 0.1	-0.90 ± 0.07	0.29 ± 0.10	0.32 ± 0.13	2.93 ± 0.22	6.94 ± 0.85	2.83 ± 0.12			

Channel	$10^2\mathcal{R}_X$	α	β	γ	$\lvert A\rvert$	$\lvert B\rvert$	$\delta_P-\delta_S$	$10^2(\mathcal{R}_X)_{\rm exp}$	$\alpha_{\rm exp}$	$\beta_{\rm exp}$
$\Xi_c^0\to\Lambda K_S^0$	23.0 ± 0.8	-0.62 ± 0.03	0.53 ± 0.11	-0.58 ± 0.11	2.47 ± 0.34	14.07 ± 0.50	2.43 ± 0.10	22.5 ± 1.3		
	23.1 ± 0.8	-0.61 ± 0.03	0.47 ± 0.32	-0.64 ± 0.22	2.31 ± 0.72	14.28 ± 0.71	2.49 ± 0.35			
$\Xi_c^0\to\Sigma^0K_S^0$	3.8 ± 0.6	-0.40 ± 0.61	-0.92 ± 0.05	0.35 ± 0.58	1.80 ± 0.46	3.84 ± 1.65	-2.02 ± 0.62	3.8 ± 0.7		
	3.6 ± 0.7	-0.63 ± 0.17	-0.86 ± 0.26	-0.06 ± 1.59	1.47 ± 1.33	4.79 ± 3.35	-2.26 ± 0.28			
$\Xi_c^0\to\Sigma^+K^-$	13.9 ± 1.0	-0.04 ± 0.12	-0.99 ± 0.01	-0.14 ± 0.09	2.76 ± 0.18	9.71 ± 0.47	-1.61 ± 0.12	12.3 ± 1.2		
	14.0 ± 0.9	-0.06 ± 0.12	-0.99 ± 0.02	-0.14 ± 0.09	2.77 ± 0.18	9.75 ± 0.46	-1.63 ± 0.12			
$\Xi_c^0\to\Xi^-K^+$	4.39 ± 0.02	-0.72 ± 0.03	0.66 ± 0.03	0.21 ± 0.04	1.86 ± 0.05	5.44 ± 0.13	2.40 ± 0.04	2.75 ± 0.57		
	4.39 ± 0.02	-0.71 ± 0.03	0.67 ± 0.03	0.22 ± 0.04	1.86 ± 0.05	5.41 ± 0.13	2.38 ± 0.04			

Part of predictions: October

scheme II.

Channel	$10^2\mathcal{B}$	α	β	γ	$\lvert A\rvert$	$\lvert B\rvert$	$\delta_P-\delta_S$	$10^2\mathcal{B}_{\rm exp}$	$\alpha_{\rm exp}$	$\beta_{\rm exp}$ / $\gamma_{\rm exp}$
$\Lambda_c^+\to\Lambda^0\pi^+$	1.30 ± 0.05	-0.76 ± 0.01	0.39 ± 0.02	0.51 ± 0.01	5.57 ± 0.10	9.23 ± 0.20	2.67 ± 0.02	1.29 ± 0.05	-0.762 ± 0.006	0.368 ± 0.021
	1.30 ± 0.04	-0.76 ± 0.01	0.39 ± 0.02	0.51 ± 0.01	5.58 ± 0.10	9.26 ± 0.20	2.67 ± 0.02			0.502 ± 0.017
$\Lambda_c^+\to\Sigma^0\pi^+$	1.24 ± 0.05	-0.47 ± 0.01	0.32 ± 0.10	-0.82 ± 0.04	1.86 ± 0.20	19.13 ± 0.44	2.54 ± 0.14	1.27 ± 0.06	-0.466 ± 0.018	
	1.24 ± 0.05	-0.47 ± 0.01	0.33 ± 0.10	-0.82 ± 0.04	1.88 ± 0.20	19.12 ± 0.44	2.53 ± 0.14			
$\Lambda_c^+\to\Sigma^+\pi^0$	1.25 ± 0.05	-0.47 ± 0.01	0.32 ± 0.10	-0.82 ± 0.04	1.86 ± 0.20	19.13 ± 0.44	2.54 ± 0.14	1.24 ± 0.09	-0.484 ± 0.027	
	1.25 ± 0.05	-0.47 ± 0.01	0.33 ± 0.10	-0.82 ± 0.04	1.88 ± 0.20	19.12 ± 0.44	2.53 ± 0.44			
$\Lambda_c^+\to\Sigma^+\eta$	0.33 ± 0.04	-0.90 ± 0.04	-0.14 ± 0.12	0.42 ± 0.10	2.96 ± 0.19	6.83 ± 0.69	-2.99 ± 0.13	0.32 ± 0.05	-0.99 ± 0.06	
	0.33 ± 0.04	-0.89 ± 0.04	-0.08 ± 0.12	0.44 ± 0.09	2.97 ± 0.19	6.68 ± 0.69	-3.06 ± 0.13			
$\Lambda_c^+\to\Sigma^+\eta'$	0.18 ± 0.03	-0.44 ± 0.07	0.84 ± 0.14	-0.32 ± 0.34	2.12 ± 0.67	18.43 ± 1.92	2.05 ± 0.10	0.41 ± 0.08	-0.46 ± 0.07	
	0.17 ± 0.04	-0.44 ± 0.07	0.88 ± 0.08	-0.16 ± 0.38	2.28 ± 0.67	16.85 ± 2.41	2.03 ± 0.08			
$\Lambda_c^+\to\Xi^0K^+$	0.33 ± 0.03	-0.08 ± 0.11	-0.98 ± 0.02	0.18 ± 0.09	2.72 ± 0.18	9.69 ± 0.47	-1.65 ± 0.12	0.55 ± 0.07	0.01 ± 0.16	
	0.33 ± 0.03	-0.07 ± 0.11	-0.98 ± 0.02	0.18 ± 0.09	2.72 ± 0.18	9.69 ± 0.46	-1.64 ± 0.11			
$\Lambda_c^+\to pK_S$	1.59 ± 0.06	-0.74 ± 0.03	0.21 ± 0.59	-0.63 ± 0.20	3.22 ± 0.91	17.31 ± 1.02	2.86 ± 0.74	1.59 ± 0.07	-0.743 ± 0.028	
	1.56 ± 0.06	-0.74 ± 0.03	0.63 ± 0.05	-0.24 ± 0.10	4.60 ± 0.31	14.92 ± 0.67	2.44 ± 0.05			
$\Xi_c^0\to\Xi^-\pi^+$	2.97 ± 0.08	-0.74 ± 0.03	0.66 ± 0.03	0.11 ± 0.04	8.02 ± 0.20	23.76 ± 0.53	2.42 ± 0.04	1.80 ± 0.52	-0.64 ± 0.05	
	2.99 ± 0.08	-0.75 ± 0.03	0.66 ± 0.03	0.12 ± 0.04	8.06 ± 0.20	23.80 ± 0.52	2.42 ± 0.04			
$\Xi_c^0\to\Xi^0\pi^0$	0.71 ± 0.04	-0.69 ± 0.07	0.72 ± 0.07	-0.09 ± 0.10	3.53 ± 0.24	12.80 ± 0.57	2.34 ± 0.10	$0.69\pm0.16^*$	$-0.90\pm0.27^*$	
	0.72 ± 0.04	-0.69 ± 0.06	0.71 ± 0.06	-0.09 ± 0.10	3.57 ± 0.24	12.81 ± 0.56	2.34 ± 0.09			
$\Xi_c^0\to\Xi^0\eta$	0.25 ± 0.03	-0.03 ± 0.14	-0.19 ± 0.13	-0.98 ± 0.03	0.33 ± 0.22	12.38 ± 0.76	-1.73 ± 0.72	$0.16\pm0.05^*$		
	0.26 ± 0.03	-0.01 ± 0.13	-0.17 ± 0.13	-0.99 ± 0.02	0.29 ± 0.24	12.62 ± 0.62	-1.63 ± 0.74			
$\Xi_c^0\to\Xi^0\eta'$	0.23 ± 0.03	-0.71 ± 0.07	0.46 ± 0.22	-0.53 ± 0.23	2.03 ± 0.54	20.86 ± 1.91	2.56 ± 0.20	$0.12\pm0.04^*$		
	0.22 ± 0.03	-0.74 ± 0.07	0.53 ± 0.19	-0.42 ± 0.29	2.18 ± 0.59	19.25 ± 2.40	0.25 ± 0.02			
$\Xi_c^+\to\Xi^0\pi^+$	1.0 ± 0.1	-0.80 ± 0.09	0.38 ± 0.11	0.46 ± 0.11	3.02 ± 0.22	6.05 ± 0.71	2.70 ± 0.15	1.6 ± 0.8		
	1.0 ± 0.1	-0.79 ± 0.08	0.39 ± 0.10	0.46 ± 0.10	3.01 ± 0.21	6.03 ± 0.63	2.68 ± 0.13			

Channel	$10^2\mathcal{R}_X$	α	β	γ	$\lvert A\rvert$	$\lvert B\rvert$	$\delta_P-\delta_S$	$10^2(\mathcal{R}_X)_{\rm exp}$	$\alpha_{\rm exp}$	$\beta_{\rm exp}$
$\Xi_c^0\to\Lambda K_S^0$	22.8 ± 0.9	-0.63 ± 0.07	0.32 ± 0.34	-0.71 ± 0.10	2.06 ± 0.37	14.46 ± 0.33	2.67 ± 0.48	22.5 ± 1.3		
	23.0 ± 0.8	-0.67 ± 0.02	0.53 ± 0.07	-0.53 ± 0.07	2.64 ± 0.19	13.84 ± 0.38	2.47 ± 0.07			
$\Xi_c^0\to\Sigma^0K_S^0$	3.6 ± 0.6	-0.45 ± 1.02	-0.82 ± 0.11	-0.46 ± 0.81	1.11 ± 0.85	5.61 ± 1.56	-2.10 ± 1.06	3.8 ± 0.7		
	3.8 ± 0.6	0.58 ± 0.28	-0.44 ± 0.30	0.72 ± 0.10	2.05 ± 0.17	2.53 ± 0.55	-0.57 ± 0.59			
$\Xi_c^0\to\Sigma^+K^-$	13.7 ± 0.9	-0.08 ± 0.11	-0.99 ± 0.02	-0.15 ± 0.09	2.72 ± 0.18	9.69 ± 0.47	-1.65 ± 0.12	12.3 ± 1.2		
	13.6 ± 0.9	-0.07 ± 0.11	-0.99 ± 0.02	-0.15 ± 0.09	2.72 ± 0.18	9.69 ± 0.46	-1.64 ± 0.11			
$\Xi_c^0\to\Xi^-K^+$	4.38 ± 0.01	-0.73 ± 0.03	0.65 ± 0.03	0.20 ± 0.04	1.85 ± 0.05	5.47 ± 0.12	2.42 ± 0.04	2.75 ± 0.57		
	4.38 ± 0.01	-0.74 ± 0.03	0.65 ± 0.03	0.20 ± 0.04	1.86 ± 0.05	5.48 ± 0.12	2.42 ± 0.04			

A brief history of $\Lambda_c^+ \to \Xi^0 K^+$

1. Theory in 1990s, small BF & zero α, due to smallness of S-wave amplitude

2. BF was measured, not that small, $\mathscr{B}(\Lambda_c^+ \to \Xi^0 K^+) = (0.55 \pm 0.07)\,\%$

3. Recently both in SU(3) fit or dynamic model calculation, $\alpha \sim 1$

4. The puzzle with α was resolved by BESIII

PRL, 132, 031801 (2024)

$$\alpha_{\Xi^0 K^+} = 0.01 \pm 0.16(\mathrm{stat}) \pm 0.03(\mathrm{syst})$$

$$\delta_p - \delta_s = -1.55 \pm 0.25(\mathrm{stat}) \pm 0.05(\mathrm{syst})\ \mathrm{rad}$$

$$1.59 \pm 0.25(\mathrm{stat}) \pm 0.05(\mathrm{syst})\ \mathrm{rad.}$$

5. The smallness α can be accmmodated recently in both IRA and TDA

IRA: Geng et al. 2310.05491

TDA: Zhong et al. 2401.15926, 2404.01359

I. $\Lambda_c^+ \to \Xi^0 K^+$: remaining problems from BESIII

(1) two sets for amplitudes

$$\text{I.} \begin{cases} |A| = 1.6^{+1.9}_{-1.6} \pm 0.4\,, \\ |B| = 18.3 \pm 2.8 \pm 0.7\,, \end{cases} \qquad \text{II.} \begin{cases} |A| = 4.3^{+0.7}_{-0.2} \pm 0.4\,, \\ |B| = 6.7^{+8.3}_{-6.7} \pm 1.6\,, \end{cases}$$

$$\gamma = \frac{|A|^2 - \kappa^2|B|^2}{|A|^2 + \kappa^2|B|^2},$$

γ: relative size of partial waves

(2) ambiguity in sign of phase-shift

$$\boxed{(\delta_S^{X_i}, \delta_P^{X_i}) \to (-\delta_S^{X_i}, -\delta_P^{X_i})}$$

$$\delta_P - \delta_S = -1.55 \pm 0.25 \pm 0.05 \quad \text{or} \quad 1.59 \pm 0.25 \pm 0.05 \text{ rad.}$$

$$\beta = \frac{2\kappa|A^* B| \sin(\delta_P - \delta_S)}{|A|^2 + \kappa^2|B|^2}$$

β: relative sign of phase-shift

$$\Gamma = \frac{p_c}{8\pi} \frac{(m_i + m_f)^2 - m_P^2}{m_i^2} \left(|A|^2 + \kappa^2|B|^2\right)$$

$$\alpha = \frac{2\kappa|A^* B| \cos(\delta_P - \delta_S)}{|A|^2 + \kappa^2|B|^2},$$

We only had BF and α in January, so could not solve the problems.

I. $\Lambda_c^+ \to \Xi^0 K^+$: BESIII problem has been solved by LHCb

(1) two sets for amplitudes

$$\text{I.}\ \begin{cases} |A| = 1.6^{+1.9}_{-1.6} \pm 0.4\,, \\ |B| = 18.3 \pm 2.8 \pm 0.7\,, \end{cases} \qquad \text{II.}\ \begin{cases} |A| = 4.3^{+0.7}_{-0.2} \pm 0.4\,, \\ |B| = 6.7^{+8.3}_{-6.7} \pm 1.6\,, \end{cases}$$

(2) ambiguity in sign of phase-shift

$$\boxed{(\delta_S^{X_i}, \delta_P^{X_i}) \to (-\delta_S^{X_i}, -\delta_P^{X_i})}$$

$$\delta_P - \delta_S = -1.55 \pm 0.25 \pm 0.05 \text{ or } 1.59 \pm 0.25 \pm 0.05 \text{ rad.}$$

Decay	α	β	γ	Δ (radian)
$\Lambda_c^+ \to \Lambda\pi^+$	$-0.782 \pm 0.009 \pm 0.004$	$0.368 \pm 0.019 \pm 0.008$	$0.502 \pm 0.016 \pm 0.006$	$0.633 \pm 0.036 \pm 0.013$
$\Lambda_c^+ \to \Lambda K^+$	$-0.569 \pm 0.059 \pm 0.028$	$0.35 \pm 0.12 \pm 0.04$	$-0.743 \pm 0.067 \pm 0.024$	$2.70 \pm 0.17 \pm 0.04$
$\Lambda_c^+ \to pK_S^0$	$-0.744 \pm 0.012 \pm 0.009$	–	–	–

LHCb, 2409.02759

Fit 10/2024

$$|A| = 2.76 \pm 0.18,\ |B| = 9.71 \pm 0.47,$$

$$\alpha_{\Xi^0 K^+} = -0.04 \pm 0.12,$$

$$\beta_{\Xi^0 K^+} = -0.98 \pm 0.02$$

$$\delta_P - \delta_S = -1.61 \pm 0.12 \text{ rad}$$

Select the solution

3. $\Xi_c^0 \to \Xi^- \pi^+$

Observable	PDG [42]	Belle	Average
$10^2 \mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+)$	1.43 ± 0.32	$1.80 \pm 0.52^*$ [46]	1.80 ± 0.52 [46]

- $\Gamma(\Xi_c^0 \to p K^- \overline{K}^*(892)^0, \overline{K}^{*0} \to K^- \pi^+)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+)$
- $\Gamma(\Xi_c^0 \to p K^- K^- \pi^+$ (no $\overline{K}^{*0}$)$)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+)$
- $\Gamma(\Xi_c^0 \to \Lambda K_S^0)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+)$
- $\Gamma(\Xi_c^0 \to \Lambda K^- \pi^+)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+)$
- $\Gamma(\Xi_c^0 \to \Lambda \overline{K}^*(892)^0)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+)$
- $\Gamma(\Xi_c^0 \to \Sigma^0 K_S^0)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+)$
- $\Gamma(\Xi_c^0 \to \Sigma^+ K^-)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+)$
- $\Gamma(\Xi_c^0 \to \Sigma^0 \overline{K}^*(892)^0)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+)$
- $\Gamma(\Xi_c^0 \to \Sigma^+ K^*(892)^-)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+)$
- $\Gamma(\Xi_c^0 \to \Xi^- \pi^+)/\Gamma_{\text{total}}$
- $\Gamma(\Xi_c^0 \to \Xi^- \pi^+)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+ \pi^+ \pi^-)$
- $\Gamma(\Xi_c^0 \to \Omega^- K^+)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+)$
- $\Gamma(\Xi_c^0 \to \Xi^0 \phi, \phi \to K^+ K^-)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+)$
- $\Gamma(\Xi_c^0 \to \Xi^0 K^+ K^-$ nonresonant$)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+)$
- $\Gamma(\Xi_c^0 \to \Xi^- e^+ \nu_e)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+)$
- $\Gamma(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+)$

Our fit

Channel	$10^2 \mathcal{B}$	α	$\lvert A \rvert$	$\lvert B \rvert$	$\delta_P - \delta_S$
$\Xi_c^0 \to \Xi^- \pi^+$	2.83 ± 0.10	-0.72 ± 0.03	4.51 ± 0.79	31.47 ± 1.31	2.76 ± 0.32

2.97 ± 0.08	-0.74 ± 0.03	0.66 ± 0.03	0.11 ± 0.04	8.02 ± 0.20	23.76 ± 0.53	2.42 ± 0.04
2.99 ± 0.08	-0.75 ± 0.03	0.66 ± 0.03	0.12 ± 0.04	8.06 ± 0.20	23.80 ± 0.52	2.42 ± 0.04

Other support:

$$\frac{\tau_{\Lambda_c^+}}{\tau_{\Xi_c^0}} \mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) = 3\mathcal{B}(\Lambda_c^+ \to \Lambda \pi^+) + \mathcal{B}(\Lambda_c^+ \to \Sigma^0 \pi^+) - \frac{1}{\sin^2 \theta_C} \mathcal{B}(\Lambda_c^+ \to n \pi^+)$$

$\tau(\Lambda_c^+) = (202.9 \pm 1.1)$ fs,

$\tau(\Xi_c^0) = (150.5 \pm 1.9)$ fs.

	$\mathcal{B}(\Lambda_c^+ \to \Lambda \pi^+)$	$\mathcal{B}(\Lambda_c^+ \to \Sigma^0 \pi^+)$	$\mathcal{B}(\Lambda_c^+ \to n \pi^+)$
fitted results	1.31 ± 0.05	1.26 ± 0.05	0.063 ± 0.009
measured values	1.29 ± 0.05	1.27 ± 0.06	0.066 ± 0.013

$$\boxed{\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) = (2.85 \pm 0.30)\%}$$

3'. $\Xi_c^0 \to \Xi^- K^+$

Channel	$10^2\mathcal{R}_X$	α	$\lvert A\rvert$	$\lvert B\rvert$	$\delta_P - \delta_S$	$10^2(\mathcal{R}_X)_{\mathrm{exp}}$
$\Xi_c^0 \to \Xi^- K^+$	4.10 ± 0.05	-0.76 ± 0.03	1.04 ± 0.18	7.25 ± 0.30	2.76 ± 0.32	2.75 ± 0.57

Channel	TDA	$\widetilde{\mathrm{TDA}}$
$\Xi_c^0 \to \Xi^- \pi^+$	$2T - 2E_{1S}$	$2\tilde{T}$
$\Xi_c^0 \to \Xi^- K^+$	$2T - 2E_{1S}$	$2\tilde{T}$

$$\mathcal{R}_{\Xi^- K^+} = \sin^2\theta_C$$

0.045

measurement of $\Xi_c^0 \to \Xi^-\pi^+$ needs to be improved!

Reference/normalization mode

$\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)$

- ARGUS + CLEO : $(5.0 \pm 1.3)\%$ PDG 2014
- Belle: $(6.84 \pm 0.24^{+0.21}_{-0.27})\%$ obsolete
  Belle, PRL 113 (2014), 042002
- BESIII: $(5.84 \pm 0.27 \pm 0.23)\%$
  BESIII, PRL 116 (2016) , 052001

PDG 2016: $(6.35 \pm 0.33)\%$

PDG 2023 update $(6.26 \pm 0.29)\%$

$\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+)$

1. PDG 2023 updated
   $(1.43 \pm 0.32)\,\%$
2. Belle 2019
   $(1.80 \pm 0.50 \pm 0.14)\%$

	Mode (*)	Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level	P(MeV/c)
Γ_{13}	$\Xi_c^0 \to \Xi^-\pi^+$	0.0143 ± 0.0032	S=1.1	875

Category: No absolute branching fractions have been measured. The following are branching *ratios* relative to $\Xi^-\pi^+$.

The following data is related to the above value: ▸ expand all datablocks

$\Gamma(\Xi_c^0 \to pK^-\overline{K}^*(892)^0, \overline{K}^{*0} \to K^-\pi^+)/\Gamma(\Xi_c^0 \to \Xi^-\pi^+)$	Γ_2/Γ_{13}	+
$\Gamma(\Xi_c^0 \to pK^-K^-\pi^+$ (no $\overline{K}^{*0}$)$)/\Gamma(\Xi_c^0 \to \Xi^-\pi^+)$	Γ_3/Γ_{13}	+
$\Gamma(\Xi_c^0 \to \Lambda K_S^0)/\Gamma(\Xi_c^0 \to \Xi^-\pi^+)$	Γ_4/Γ_{13}	+
$\Gamma(\Xi_c^0 \to \Lambda K^-\pi^+)/\Gamma(\Xi_c^0 \to \Xi^-\pi^+)$	Γ_5/Γ_{13}	+
$\Gamma(\Xi_c^0 \to \Lambda\overline{K}^*(892)^0)/\Gamma(\Xi_c^0 \to \Xi^-\pi^+)$	Γ_6/Γ_{13}	+
$\Gamma(\Xi_c^0 \to \Sigma^0 K_S^0)/\Gamma(\Xi_c^0 \to \Xi^-\pi^+)$	Γ_9/Γ_{13}	+
$\Gamma(\Xi_c^0 \to \Sigma^+ K^-)/\Gamma(\Xi_c^0 \to \Xi^-\pi^+)$	Γ_{10}/Γ_{13}	+
$\Gamma(\Xi_c^0 \to \Sigma^0\overline{K}^*(892)^0)/\Gamma(\Xi_c^0 \to \Xi^-\pi^+)$	Γ_{11}/Γ_{13}	+
$\Gamma(\Xi_c^0 \to \Sigma^+ K^*(892)^-)/\Gamma(\Xi_c^0 \to \Xi^-\pi^+)$	Γ_{12}/Γ_{13}	+
$\Gamma(\Xi_c^0 \to \Xi^-\pi^+)/\Gamma_{\mathrm{total}}$	Γ_{13}/Γ	+

SUMMARY

- TDA approach to charmed baryon is established.
- The proper fits can be carried out.
- The unification of TDA & IRA is demonstrated.
- All the kinematic measurements: BF, alpha, beta, gamma play their roles as expected.
- Time to explore the underlying dynamics in charm system.