$D_s \to [\pi\pi]_S e^+\nu$ and Dipion LCDAs

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Overview

I Semileptotic $D_{(s)}$decays

II Dipion Light-cone distribution amplitudes

III $D_s \to [f_0, \cdots \to] \, \pi\pi e^+ \nu$

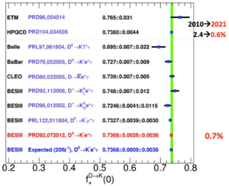
IV Conclusions and Prospects

Semileptotic $D_{(s)}$ decays

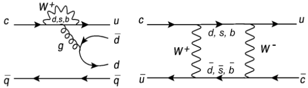
Semileptotic $D_{(s)}$ decays

play a crucial role in the precision era of particle physics

- fundamental parameters, like the CKM matrix element $|V_{cs}| = 0.975 \pm 0.006$ [PDG 2022]
  - the result measured via $D \to Kl\nu$ and $D_s \to \mu\nu_\mu$ consist with each other ($\sim 1.5\sigma$ derivation)
  - $\sim 3\sigma$ tension three years ago [PDG 2020, 2021]
  - the improvement mainly due to the high precision of $D \to K$ form factor from lattice evaluation and the f_{D_s} from the BESIII



- new physical mechanism via the FCNC
  - anomalous measured in $B \to K^* \mu^+\mu^-$, 3.6σ derivation of $d\mathcal{B}/dq^2$ in $q^2 \in [1, 6]$ GeV^2, 1.9σ derivation of $p'_5 = S_5/\sqrt{F_L(1-F_L)}$ in $[4, 8]$ GeV^2
  - a plausible effect in up-type FCNC process $c \to ull$ [Bharucha 2011.12856] SM $\mathcal{B}(D \to \pi l^+ l^-) \sim \mathcal{O}(10^{-9})$, current best-world limit $\mathcal{O}(10^{-8})$
  - first measurement of $D^0 \to \pi^+\pi^- e^+ e^-$ [LHCb-PAPER-2024-047, prelim.] $(4.53 \pm 1.38) \times 10^{-7}$ in ρ/ω and $(3.84 \pm 0.96) \times 10^{-7}$ in ϕ



Semileptotic $D_{(s)}$ decays

New physics hunter $D \to \pi\mu^+\mu^-$

- my talk in the " 超级陶粲装置研讨会" at LZU, July 8th, 2024

- Experimentail potentials

Experiment	Measurement	Sensitivity
LHCb	Angular observables	$\sim 0.2\%$ with $50\,\text{fb}^{-1}$, $\sim 0.08\%$ with $300\,\text{fb}^{-1}$
LHCb	Branching ratio	$\sim 10^{-8}$ with $50\,\text{fb}^{-1}$, $\sim 3\times10^{-9}$ with $300\,\text{fb}^{-1}$
Belle-II	Branching ratio	$\sim 10^{-8}$ (rescaling BaBar)

[talk at Towards the Ultimate Precision in Flavour Physics, Durham U.K. (2019)]

Run 4 $\sim$ 2030
Run 5 $\sim$ 2038

[BABAR Collaboration 1107.4465]

$N(D\bar{D}) \sim 10^9/\text{ab}^{-1}$ angular observables $\sim 0.2\%$

- BESIII Collaboration in the electron channel [BESIII Collaboration 1802.09752]
$\mathcal{B}(D \to \pi^+\pi^-e^+e^-) < 0.7\times10^{-5}$ with $N(c\bar{c}) = 2\times10^7$ at 3.7 GeV

3.770	1	$D^0\bar{D}^0$	3.6	3.6×10^9	
		$D^+\bar{D}^-$	2.8	2.8×10^9	
		$D^0\bar{D}^0$		7.9×10^8	Single Tag
		$D^+\bar{D}^-$		5.5×10^8	Single Tag

STCF $N(D\bar{D}) \sim 8\times10^9$ Branching ratio $\sim 10^{-8}$

- STCF is still competitive in hunting the NP via $D \to \pi\mu^+\mu^-, \pi\pi\mu^+\mu^-$

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Semileptotic $D_{(s)}$ decays

- a clean environment to study the scalar mesons
  - $f_0(1370), f_0(1500), a_0(1450), K_0^*(1430)$ form a $SU(3)$ flavor nonet
  - $f_0(500)/\sigma, f_0(980), a_0(980), K_0^*(700)/\kappa$ form another flavor nonet
    compact tetraquark and $K\bar{K}$ bound state in spectral analysis, $q\bar{q}$ is dominated in the B_s decay
  - how about the energetic $q\bar{q}$ picture $f_0(980)$ in D_s decays ?

- The solution of the above questions deduces to not only the precise pertuabtive QCD, but also **the accurate nonperturbative prediction** of the form factors
  - unstable particle are measured in the lineshape of $\pi\pi$, $K\pi$ invariant mass
  - dynamics of B_{l4} is governed by $B \to \pi\pi$ form factors, a big task of the QCD methods

- **Dipion LCDAs** are introduced in the LCSRs prediction of $B \to \pi\pi$ form factors
  [SC, Khodjamirian, Virto 1709.0173, SC 1901.06071]
  - high partial waves give few percent contributions to $B \to \pi\pi$ form factors, ρ', ρ'' and NR background contribute $\sim 20\% - 30\%$ to P-wave
  - **qualitatively explain the $|V_{ub}|$ tension (3σ)** [Belle II 2407.17403]

$$|V_{ub}|_{B\to\pi l\nu} = (3.93 \pm 0.19 \pm 0.13 \pm 0.19(\text{theo})) \times 10^{-3} \quad [\text{LQCD}]$$
$$|V_{ub}|_{B\to\pi l\nu} = (3.73 \pm 0.07 \pm 0.07 \pm 0.16(\text{theo})) \times 10^{-3} \quad [\text{LQCD} + \text{LCSRs}]$$
$$|V_{ub}|_{B\to\rho l\nu} = (3.19 \pm 0.12 \pm 0.18 \pm 0.26(\text{theo})) \times 10^{-3} \quad [\text{LCSRs}]$$

Dipion LCDAs

DiPion LCDAs

- **The study of DiPion distribution amplitude** will shine a light on **the width effect encounted in Flavor Physics** (multibody decays, $B \to [\pi\pi]\, l\nu$, $b \to sll$, $c \to ull$ $\cdots$) and **the controversial structure of scalar meson ?**

- Chiral-even LC expansion with gauge factor $[x, 0]$ [Polyakov '99, Diehl '98]

$$\langle \pi^a(k_1)\pi^b(k_2)|\bar{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_\parallel^{ab,ff'}(u, \zeta, k^2)$$

$n^2 = 0$, f, f' respects the (anti-)quark flavor, a, b indicates the electric charge

$\kappa_{+-/00} = 1$ and $\kappa_{+0} = \sqrt{2}$, $k = k_1 + k_2$ is the invariant mass of dipion state

$\tau = 1/2, \tau^3/2$ corresponds to the isoscalar and isovector 2πDAs

higher twist $\propto 1, \gamma_\mu\gamma_5$ have not been discussed yet, γ_5 vanishes due to P-parity conservation

- Three independent kinematic variables

momentum fraction u carried by anti-quark with respecting to the total momentum of DiPion state
longitudinal momentum fraction carried by one of the pions $\zeta = k_1^+/k^+$, $2q \cdot \bar{k}\, (\propto 2\zeta - 1)$, k^2

- Normalization conditions

$$\int_0^1 \Phi_\parallel^{I=1} = (2\zeta - 1)F_\pi(k^2), \int_0^1 dz\,(2u-1)\Phi_\parallel^{I=0} = -2M_2^{(\pi)}\zeta(1-\zeta)F_\pi^{\mathrm{EMT}}(k^2)$$

$F_\pi^{em}(0) = 1$, $F_\pi^{\mathrm{EMT}}(0) = 1$, $M_2^{(\pi)}$ is the moment of SPD

DiPion LCDAs

- 2πDAs is decomposed in terms of $C_n^{3/2}(2z-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=0,\mathrm{even}}^{\infty}\sum_{l=1,\mathrm{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=1,\mathrm{odd}}^{\infty}\sum_{l=0,\mathrm{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

- $B_{n\ell}(k^2,\mu)$ have similar scale dependence as the a_n of π,ρ,f_0 mesons

$$B_{n\ell}(k^2,\mu) = B_{n\ell}(k^2,\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{[\gamma_n^{(0)}-\gamma_0^{(0)}]/[2\beta_0]}$$

$$\gamma_n^{\perp(\parallel),(0)} = 8C_F\left(\sum_{k=1}^{n+1}\frac{1}{k}-\frac{3}{4}-\frac{1}{2(n+1)(n+2)}\right)$$

- Soft pion theorem relates the chirarlly even coefficients with a_n^π

$$\sum_{\ell=1}^{n+1} B_{n\ell}^{\parallel,I=1}(0) = a_n^\pi\,,\quad \sum_{\ell=0}^{n+1} B_{n\ell}^{\parallel,I=0}(0) = 0$$

- 2πDAs relate to the skewed parton distributions (SPDs) by crossing

express the moments of SPDs in terms of $B_{nl}(k^2)$ in the forward limit as

$$M_{N=\mathrm{odd}}^{\pi} = \frac{3}{2}\frac{N+1}{2N+1}B_{N-1,N}^{I=1}(0),\quad M_{N=\mathrm{even}}^{\pi} = 3\frac{N+1}{2N+1}B_{N-1,N}^{I=0}(0)$$

DiPion LCDAs

- In the vicinity of the resonance, 2πDAs reduce to the DAs of ρ/f_0

  relation between the a_n^ρ and the coefficients $B_{n\ell}$

$$a_n^\rho = B_{n1}(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1} c_m^{n1}\, m_\rho^{2m}\right], \quad c_m^{(n1)} = \frac{1}{m!}\frac{d^m}{dk^{2m}}\left[\ln B_{n1}(0) - \ln B_{01}(0)\right]$$

  f_ρ relates to the imaginary part of $B_{nl}(m_\rho^2)$ by $\quad \langle \pi(k_1)\pi(k_2)|\rho\rangle = g_{\rho\pi\pi}(k_1-k_2)^\alpha \epsilon_\alpha$

$$f_\rho^{\parallel} = \frac{\sqrt{2}\,\Gamma_\rho\, \mathrm{Im}B_{01}^{\parallel}(m_\rho^2)}{g_{\rho\pi\pi}}, \quad f_\rho^{\perp} = \frac{\sqrt{2}\,\Gamma_\rho\, m_\rho\, \mathrm{Im}B_{01}^{\perp}(m_\rho^2)}{g_{\rho\pi\pi}\, f_{2\pi}^{\perp}}$$

- The subtraction constants of $B_{n\ell}(s)$ at low s

(nl)	$B_{n\ell}^{\parallel}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2}\ln B_{n\ell}^{\parallel}(0)$	$B_{n\ell}^{\perp}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2}\ln B_{n\ell}^{\perp}(0)$
(01)	1	0	1.46 $\to$ 1.80	1	0	0.68 $\to$ 0.60
(21)	-0.113 $\to$ 0.218	-0.340	0.481	0.113 $\to$ 0.185	-0.538	-0.153
(23)	0.147 $\to$ -0.038	0	0.368	0.113 $\to$ 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

- firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]
- updated with the kinematical constraints and the new a_2^π, a_2^ρ [SC '19, '23]

DiPion LCDAs

- How to describe **the evolution from $4m_\pi^2$ to large invariant mass** $k^2 \sim \mathcal{O}(m_c^2)$? furtherly to $\mathcal{O}(m_b\lambda_{\rm QCD})$

  ◦ Watson theorem of π-π scattering amplitudes

  △ implies an intuitive way to express the imaginary part of 2πDAs

  △ leads to the Omnés solution of N-subtracted DR for the coefficients

$$B^I_{n\ell}(k^2) = B^I_{n\ell}(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1}\frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}}\ln B^I_{n\ell}(0) + \frac{k^{2N}}{\pi}\int_{4m_\pi^2}^{\infty} ds\,\frac{\delta^I_\ell(s)}{s^N(s-k^2-i0)}\right]$$

  ◦ 2πDAs in a wide range of energies is given by δ^I_ℓ and a few subtraction constants

- All discussions are at leading twist, subleading twist LCDAs are not known yet

$$D_s \to [f_0, \cdots \to]\pi\pi e^+\nu$$

$D_s \to [f_0, \cdots \to] \pi\pi e\nu$

- Semileptonic $D_{(s)}$ decays provide a clean environment to study scalar mesons
  - $D_s \to f_0 e^+ \nu$ [CLEO '09], $D_{(s)} \to a_0 e^+ \nu$ [BESIII '18, '21], $D^+ \to f_0/\sigma e^+ \nu$[BESIII '19]
  - $D_s \to f_0(\to \pi^0\pi^0, K_sK_s)e^+\nu$ [BESIII 22], $D_s \to f_0(\to \pi^+\pi^-)e^+\nu$[BESIII 23]

$$\mathcal{B}(D_s \to f_0(\to \pi^0\pi^0)e^+\nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$
$$\mathcal{B}(D_s \to f_0(\to \pi^+\pi^-)e^+\nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$
$$f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- Theoretical consideration

$$\frac{d\Gamma(D_s^+ \to f_0 l^+\nu)}{dq^2} = \frac{G_F^2|\mathbf{V_{cs}}|^2\lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2, \ D_s \to f_0 \ \mathrm{FF}$$

- Improvement with the width effect ($\pi\pi$ invariant mass spectral)

$$\frac{d\Gamma(D_s^+ \to [\pi\pi]_{\mathrm{S}}\, l^+\nu)}{dsdq^2} = \frac{1}{\pi}\frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1\beta_\pi(s)}{|m_{\mathrm{S}}^2 - s + i(g_1\beta_\pi(s)) + g_2\beta_K(s))|^2}, \ \mathrm{BESIII}$$

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_{\mathrm{S}}\, l^+\nu)}{dk^2dq^2} = \frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s}^3}\frac{\beta_{\pi\pi}(k^2)\sqrt{\lambda_{D_s}}q^2}{16\pi}\sum_{\ell=0}^{\infty} 2|F_0^{(\ell)}(q^2, k^2)|^2, \ D_s \to \pi\pi \ \mathrm{FF}$$

$D_s \to f_0$ form factor

- LCSRs calculations start with the correlation functions $j_{1,\mu} = \bar{s}\gamma_\mu\gamma_5 c$, $j_2 = \bar{c}i\gamma_5 s$

$$\Pi^{\mathrm{S}}_{\mu}(p_1, q) = i\int d^4x e^{iqx}\langle f_0(p_1)|\mathrm{T}\{j_{1,\mu}(x), j_2(0)\}|0\rangle$$

- The hadron dispersion relation of invariant amplitudes $(p_1+q)^2 \equiv p^2 > 0$

$$\Pi^{\mathrm{had}}_{\mu}(p_1,q) = \frac{\langle f_0(p_1)|j_{1,\mu}(x)|D_s(p)\rangle\langle D_s(p)|j_s(0)|0\rangle}{m^2_{D_s} - p^2} + \frac{1}{\pi}\int_{s_0^1}^{\infty} ds \frac{\rho^h_\mu(s,q^2)}{s-p^2}$$

$$= \frac{-im^2_{D_s} f_{D_s}\left[2f_+(q^2)p_{1\nu} + \left(f_+(q^2)+f_-(q^2)\right)q_\mu\right]}{(m_c+m_s)\left[m^2_{D_s} - p^2\right]} + \frac{1}{\pi}\int_{s_0^1}^{\infty} ds \frac{\rho^h_+(s,q^2)p_{1\mu} + \rho^h_-(s,q^2)q_\mu}{s-p^2}$$

- The OPE calculation in the Euclidean momenta space with negative q^2

OPE is valid for large energies of the final state mesons $\Rightarrow 0 \leqslant |q^2| \leqslant q^2_{\max} \sim m_c^2 - 2m_c\chi$, the operator product of the c-quark fields can be expanded near the LC due to the large virtuality

$$\Pi^{\mathrm{OPE}}_{\mu}(p_1,q) = \sum_t \int_0^1 du \left[T^{(t)}_\mu(u,q^2,p^2)\otimes\phi^{(t)}(u) + \int_0^u \mathcal{D}\alpha_i\, T'_\mu(u,\alpha_i,q^2,p^2)\otimes\phi_{3f_0}(\alpha_i)\right]$$

$$\equiv \frac{1}{\pi}\int_0^1 du \sum_{n=1,2}\left[\frac{\mathrm{Im}\Pi^{\mathrm{OPE}}_{+,n}(q^2,u)\,p_{1\mu} + \mathrm{Im}\Pi^{\mathrm{OPE}}_{-,n}(q^2,u)q_\mu}{u^n\left[s_2(u)-p^2\right]^n} + 3\mathrm{p}\right] \quad s_2(u) = \frac{\bar{u}m^2_{f_0} + (m_c^2 - \bar{u}q^2)}{u}$$

$D_s \to f_0$ form factor

- quark-hadron duality & Borel transformation

$$\hat{B}\left[\int_{u_0}^{1} du \frac{F(u)}{u\left[s(u)-p^2\right]}\right] = \int_{u_0}^{1} du \frac{F(u)}{u} e^{-\frac{s(u)}{M^2}}$$

$$\frac{-im_{D_s}^2 f_{D_s}\left[2f_+(q^2)p_{1\nu} + \left(f_+(q^2)+f_-(q^2)\right)q_\mu\right]}{(m_c+m_s)\left[m_{D_s}^2 - p^2\right]} = \sum_t \int_{u_0}^{1} du \left[T_\mu^{(t)}(u,q^2,p^2)\otimes\phi^{(t)}(u) + 3\text{p}\right]$$

- $D_s \to f_0$ form factors under the $\bar{s}s$ description and narrow width approximation

$$f_+(q^2) = \frac{m_c+m_s}{2m_{D_s}^2 f_{D_s}}\left\{\int_{u_0}^{1}\frac{du}{u} e^{\frac{-s_2(u)+m_{D_s}^2}{M^2}}\left[-m_c\phi(u) + um_{f_0}\phi^s(u) + \frac{m_{f_0}\phi^\sigma(u)}{3} + \frac{m_{f_0}\phi^\sigma(u)}{6}\frac{m_c^2+q^2-u^2m_{f_0}^2}{uM^2}\right] + \frac{m_{f_0}\phi^\sigma(u_0)}{6}\frac{m_c^2+q^2-u_0^2m_{f_0}^2}{m_c^2-q^2+u_0^2m_{f_0}^2} e^{\frac{-s_0^1+m_{D_s}^2}{M^2}}\right\} + 3\text{p}$$

- $M^2 \sim \mathcal{O}(um_{D_s}^2 + \bar{u}q^2 - u\bar{u}m_{f_0}^2) < s_0$
- factorisation scale $\mu_f^2 = m_{D_s}^2 - m_c^2 = 1.48^2\,\text{GeV}^2$ with $\overline{m}_c(m_c) = 1.30\,\text{GeV}$
- a compromise between the overwhelming chosen of ground state in hadron spectral that demands a small value and the convergence of OPE evaluation that prefers a large one $\Rightarrow \frac{d}{d(1/M^2)}\ln f_+(q^2) = 0$
- s_0 is usually set to close to the outset of the first excited state $s_0 \approx (m_{D_s}+\chi)^2$, which is finally determined by considering the maximal stable evolution of physical quantities on M^2

$D_s \to f_0$ form factor and $D_s^+ \to (f_0, [\pi\pi]_{\rm S})e^+\nu_e$ decay

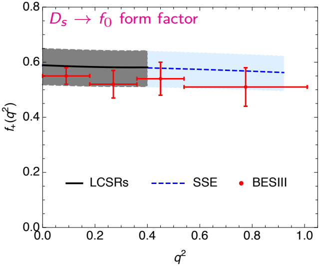
- $M^2 = 5.0 \pm 0.5\,\mathrm{GeV}^2$ and $s_0 = 6.0 \pm 0.5\,\mathrm{GeV}^2$

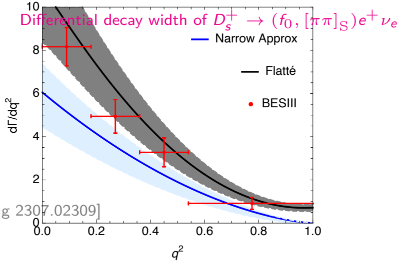
this wrok	3pSRs(07)	LFQM(09)	CLFD/DR(08)	LCSRs(10)
0.63 ± 0.04	0.96	0.87	$0.86/0.90$	0.30 ± 0.03

◦ the BESIII result in the $\pi^+\pi^-$ system $f_+(0) = 0.518 \pm 0.018 \pm 0.036$ [BESIII 23]

different input of the decay constant $\tilde{f}_{f_0} = 335$ MeV, much larger than 180 MeV in LCSRs(10)
we add the first gegenbauer expansion terms in the LCDAs, up-to-date parameters

$s\bar{s}$-$n\bar{n}$ mixing scenario of f_0 with $\theta = 20^\circ \pm 10^\circ$





[SC, Zhang 2307.02309]

◦ the uncertainty estimation is conservative, without NLO correction

◦ we need a model independent calculation, not only for the QCD understanding, but also for the future partial-wave measurement

$D_s \to [\pi\pi]_{\rm S}$ form factors

- Definition of $D_s \to [\pi\pi]_{\rm S}$ form factor

$$\langle [\pi(k_1)\pi(k_2)]_{\rm S}\,|\bar{s}\gamma_\mu(1-\gamma_5)c|D_s^+(p)\rangle = -iF_t k_\mu^t - iF_0(q^2,s,\zeta)k_\mu^0 - iF_\parallel k_\mu^\parallel$$

$$k_\mu^t = \frac{q_\mu}{\sqrt{q^2}},\, k_\mu^0 = \frac{2\sqrt{q^2}}{\sqrt{\lambda_{D_s}}}\left(k_\mu - \frac{k\cdot q}{q^2}q_\mu\right),\, k_\mu^\parallel = \frac{1}{\sqrt{k^2}}\left(\bar{k}_\mu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_{D_s}}k_\mu + \frac{4k^2(q\cdot\bar{k})}{\lambda_{D_s}}q_\mu\right)$$

- LCSRs calculations start with the correlation functions

$$\Pi_\mu^{ab}(q,k_1,k_2) = i\int d^4x e^{iq\cdot x}\langle\pi^a(k_1)\pi^b(k_2)|T\{j_{1,\mu}(x),j_2(0)\}|0\rangle$$

- Introduce a parameter angle to describe the mixing

$$[\pi\pi]_{\rm S} = |\bar{n}n\rangle\cos\theta + |\bar{s}s\rangle\sin\theta,\quad [KK]_{\rm S} = -|\bar{n}n\rangle\sin\theta + |\bar{s}s\rangle\cos\theta$$

- The chiral even two quark isoscalar 2πDAs **our knowledge of 2πDAs is still at leading twist**

$$\langle\left[\pi^a(k_1)\pi^b(k_2)\right]_{\rm S}|\bar{s}(xn)\gamma_\mu s(0)|0\rangle = 2\delta^{ab}k_\mu\sin\theta\int du e^{iux(k\cdot n)}\Phi^{l=0}_{\parallel,[\pi\pi]_{\rm S}}(u,\zeta,k^2)$$

$$\Phi^{l=0}_{\parallel,[\pi\pi]_{\rm S}} = 6u(1-u)\sum_{n=1,\rm odd}^{\infty}\sum_{l=0,\rm odd}^{n+1}B^{l=0}_{\parallel,nl}(k^2,\mu)C_n^{3/2}(2u-1)C_l^{1/2}(2\zeta-1)$$

- Do the similar LCSRs to $D_s \to f_0$ and consider the partial-wave expansion

$$F_0(q^2,k^2,\zeta) = \sum_{\ell=0}^{\infty}\sqrt{2\ell+1}\,F_{0,t}^{(\ell)}(q^2,k^2)P_\ell^{(0)}(\cos\theta_\pi)$$

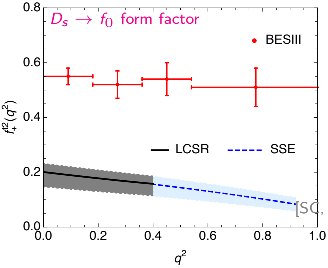
$D_s \to [\pi\pi]_{\rm S}$ form factor and $D_s \to [\pi\pi]_{\rm S}\, e^+\nu$ decay

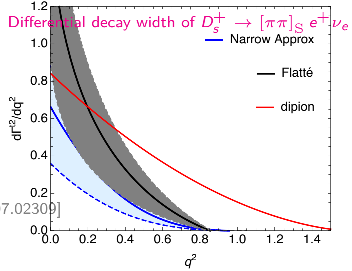
- The LCSRs ℓ'-wave $D_s \to [\pi\pi]_{\rm S}$ form factors ($\ell' = \text{even}$ & $\ell' \leqslant n+1$)

$$F_0^{(\ell')}(q^2, k^2) = \frac{m_c(m_c+m_s)\sin\theta}{m_{D_s}^2 f_{D_s}\sqrt{\lambda_{D_s}}\sqrt{q^2}} \sum_{n=1,\text{odd}}^{\infty} \frac{\beta_\pi(k^2)}{\sqrt{2\ell'+1}} J_n^0(q^2, k^2, M^2, s_0)\, B_{n\ell',\parallel}^{I=0}(k^2)$$

$$J_n^0(q^2, k^2, M^2, s_0) = 6\int_{u_0}^{1} du\, \bar{u}\, C_n^{3/2}(2u-1)\left[\lambda_{D_s} + 2uk^2\left(m_{D_s}^2 + q^2 - k^2\right)\right] e^{-\frac{s_2'(u)-m_{D_s}^2}{M^2}}$$

- Leading twist $D_s \to f_0, [\pi\pi]_{\rm S}$ form factors and $D_s \to [\pi\pi]_{\rm S}\, e^+\nu$ decay





[SC, Zhang 2307.02309]

- Twist-3 LCDAs give dominate contribution in $D_s \to f_0, [\pi\pi]_{\rm S}$ transitions

  does not indicate a breakdown of the twist expansion, the asymptotic term in the leading twist LCDAs is zero due to the charge conjugate invariance

- Further measurements would help us to understand high twist DiPion LCDAs

Conclusions and Prospects

- **The introduction of DiPion LCDAs provides an opportunity to study the width effects and the structures of nonstable mesons in H_{l4} processes**
  - a new booster on the accurate calculation in flavor physics
  - improvement study in the CKM determinations and the flavor anomalies

- DiPion LCDAs study is at leading twist so far QCD definitions and double expansion
  - determine the parameters by low energy effective theory and data constraints
  - evolution of k^2 from the threshold to large scale $\mathcal{O}(m_c^2, m_b\lambda_{QCD})$
  - universal phase shift in $\pi\pi$ scattering and heavy decay ?

- Go further to high twist LCDAs, not only to match the precise measurement
  - $B \to \pi\pi l\nu, B \to [\rho\rho \to] \to 4\pi,\ D_s \to \pi\pi l\nu, D \to K\pi\mu\nu,\ D \to \pi\pi e^+e^-$ et al.

Thank you for your patience.