Semi-Graph Neural Network Method for Muon Reconstruction in Neutrino Telescope

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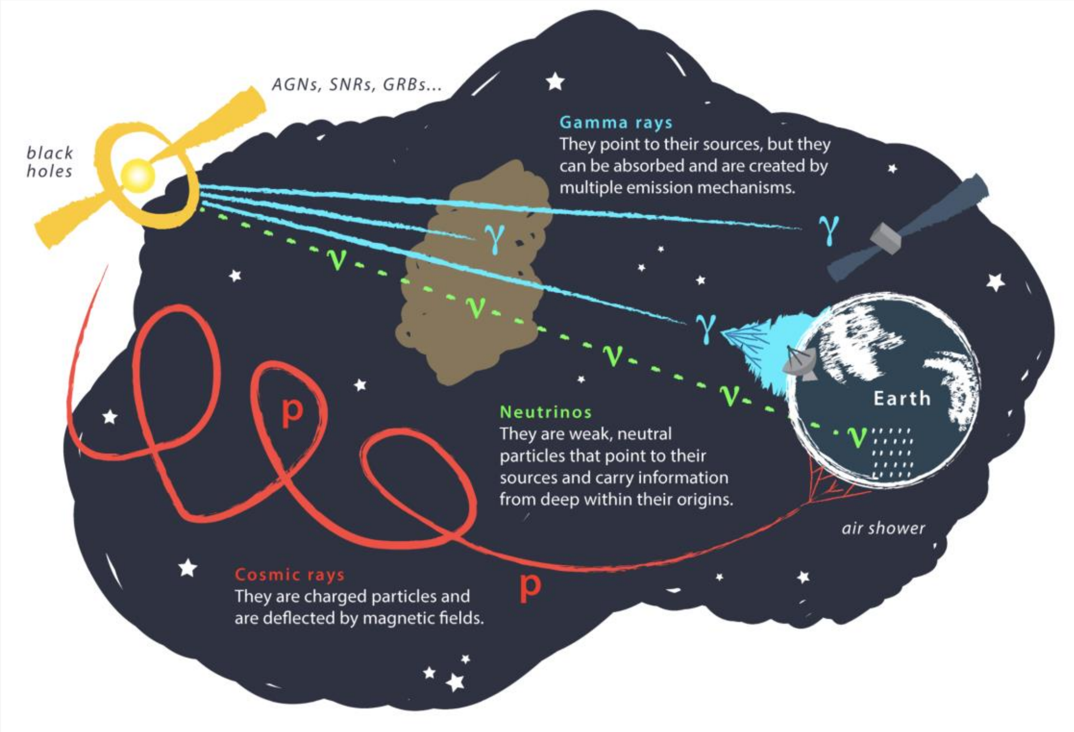
Neutrino Astronomy

➡Probe origins of cosmic ray using neutrino

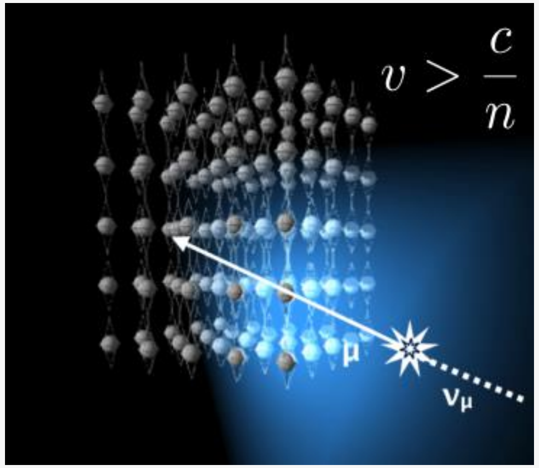


Juan Antonio Aguilar and Jamie Yang. IceCube/WIPAC

Astrophysical neutrino:

- **Small flux**
  $E_\nu \Phi_\nu < 2 \times 10^{-8} GeV cm^{-2} s^{-2} sr^{-1}$
- **Small cross section**
  $\sigma \sim 10^{-33} cm^2$ for $E_\nu \sim 10 PeV$

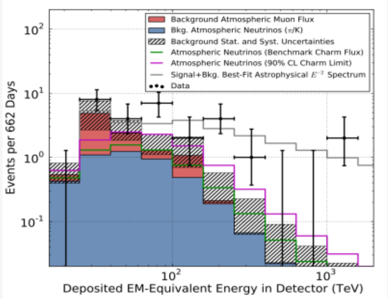
Sea water / Ice as target



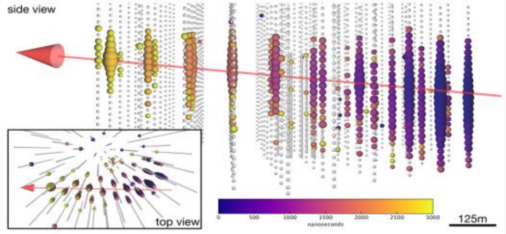
- Neutral-current interaction
  $\nu_l + N \rightarrow \nu_l + X$
- Charged-current interaction
  $\nu_l + N \rightarrow l + X$

Neutrino Astronomy

Science. 342 6161 (2013)



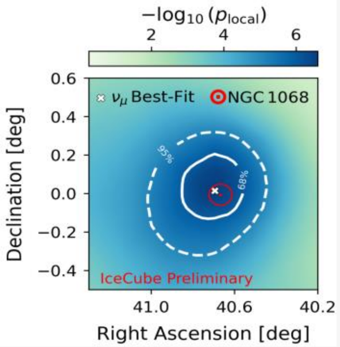
IceCube: neutrino telescope under the South Pole



- Detect extraterrestrial high-energy **diffuse neutrino flux**

Science. 361 6398 (2018)

- Evidence for neutrino emission from a **flaring blazar**

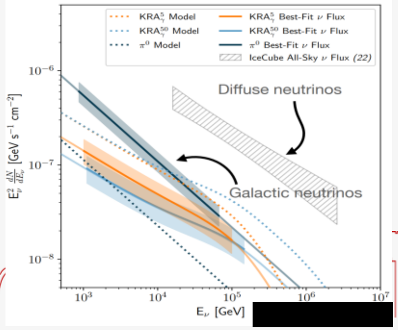


- **Glashow resonance event** at 2.3σ
- Neutrino emission from **active galaxy NGC 1068 reaches 4σ**

Science 378 538-543 (2022)

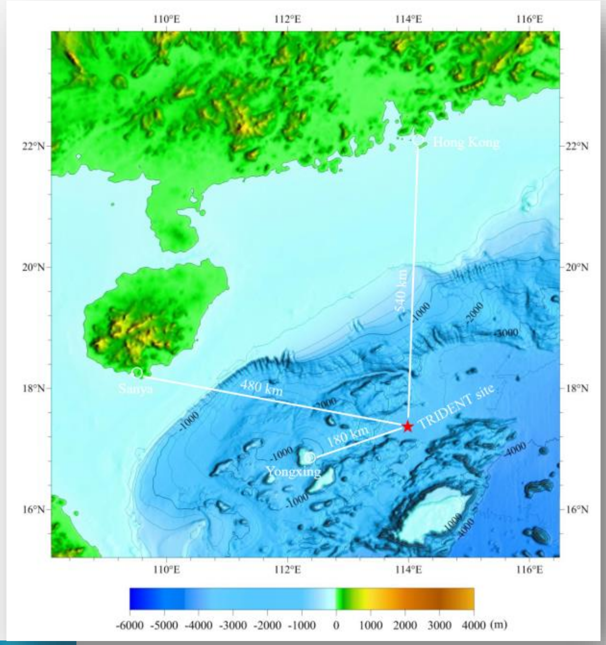
- Galactic plane **>TeV neutrinos** observed at a 4.5σ

Science 380 1338 (2023)

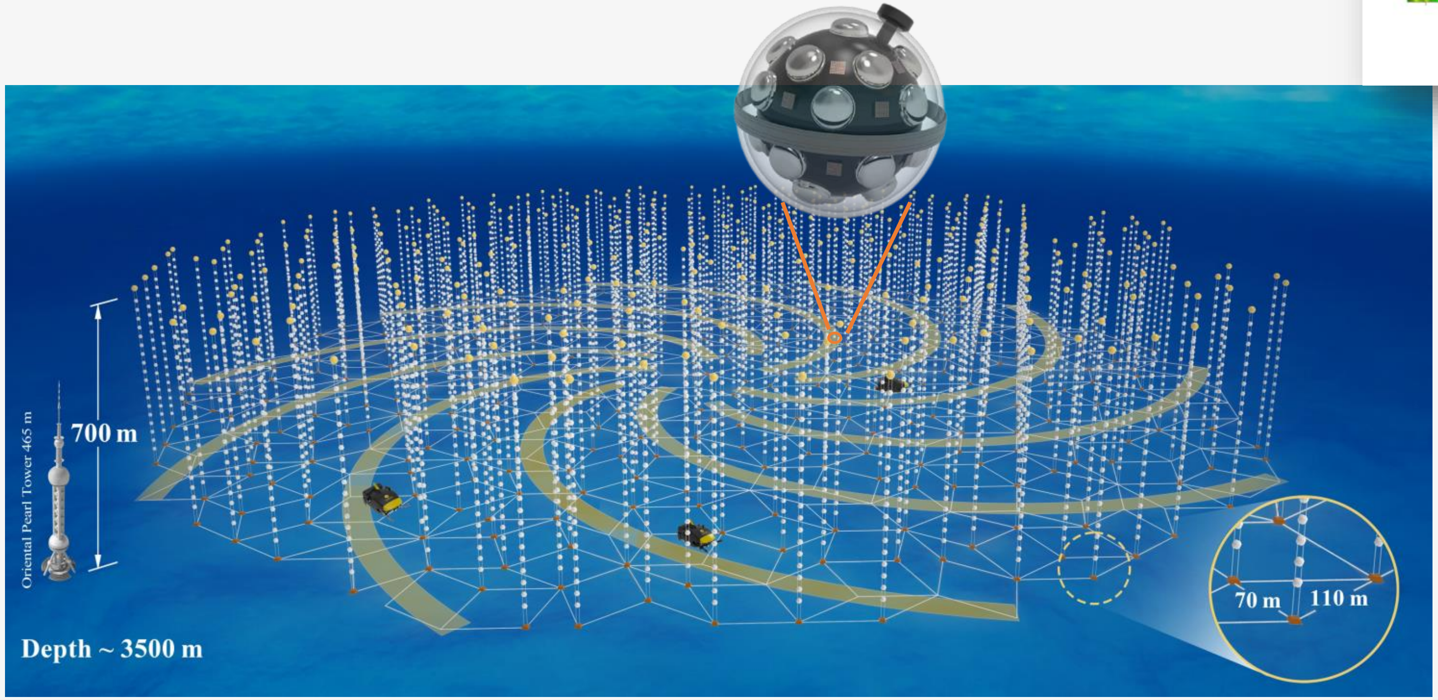


TRIDENT

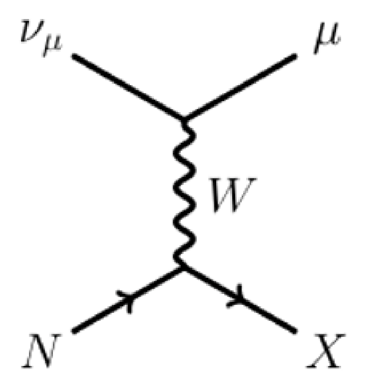
- **TRIDENT**: TRoplcal DEep-sea Neutrino Telescope *Nat Astron (2023)*
- **Water depth**: $\sim 3500m$
- **Number of strings**: ~ 1000 (20 hDOMs per string)
- **Inter-DOM distance**: 35m
- **Detection Volume**: $\sim 8\ km^3$

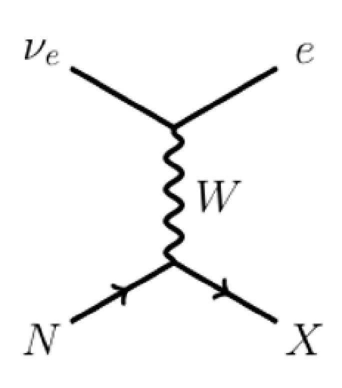


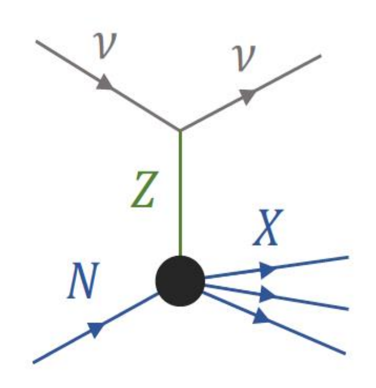
hDOM = PMTs + SiPM

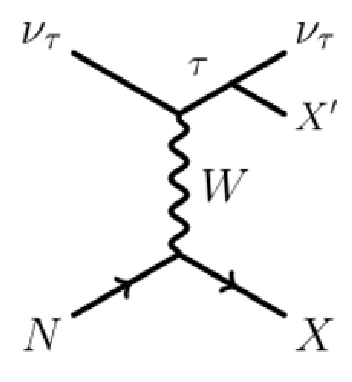


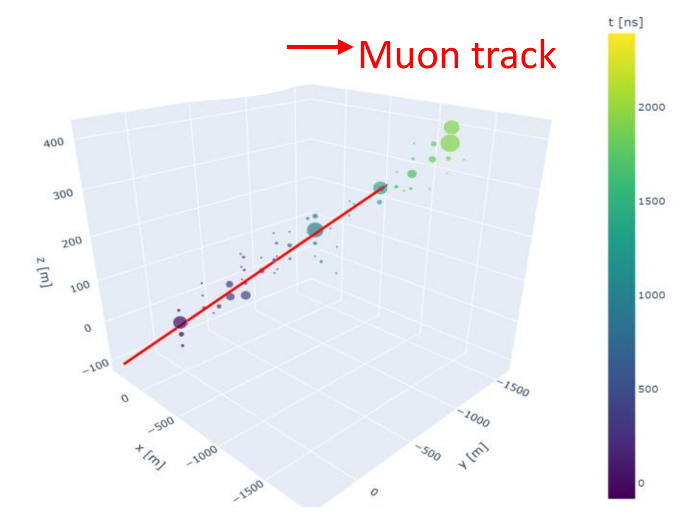
Neutrino Events

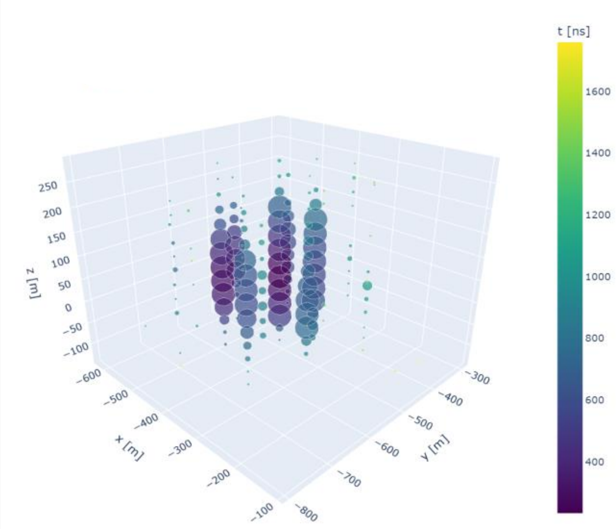


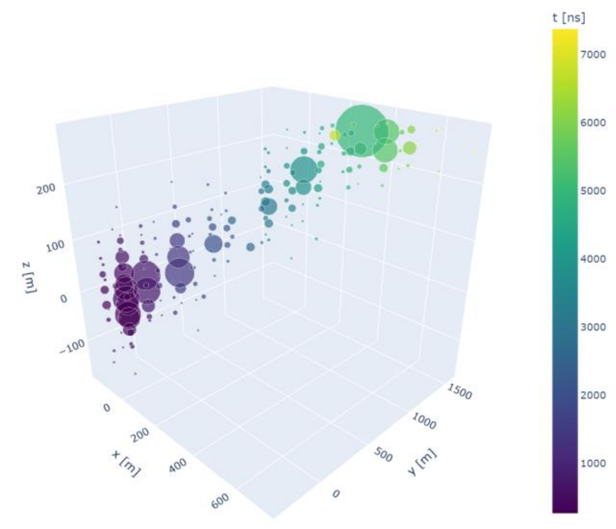












Track

$\nu_\mu + N \to \mu + X$

Cascade

$\nu_e + N \to e + X$
$or\ \nu + N \to \nu + X$

Double bang

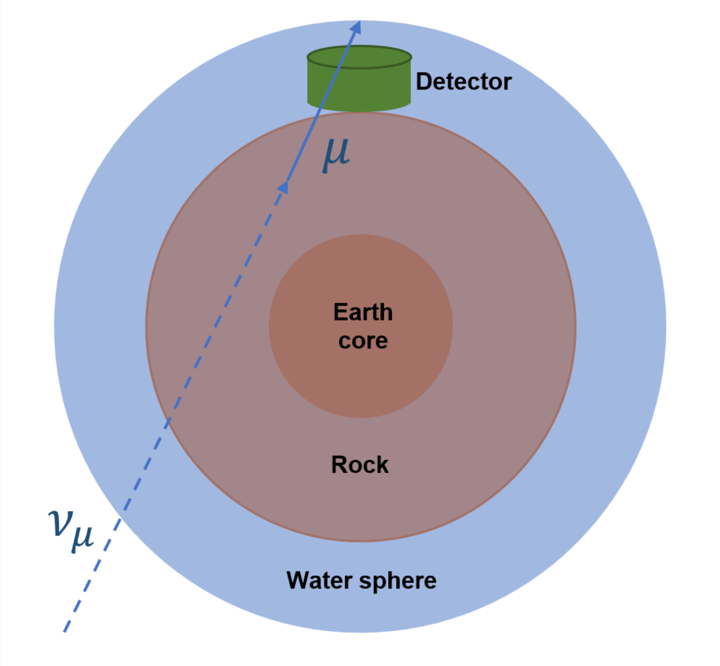
$\nu_\tau + N \to \tau + X$
$\& \ \tau \to \nu_\tau + X'$

Monte Carlo Simulation

Neutrino event generator

Based on **CORSIKA8** (arxiv:2208.14240):

- A preliminary earth model is built.
- Scattering of ν and p is simulated with **PYTHIA8**.
- Propagation of μ is simulated with **PROPOSAL**.

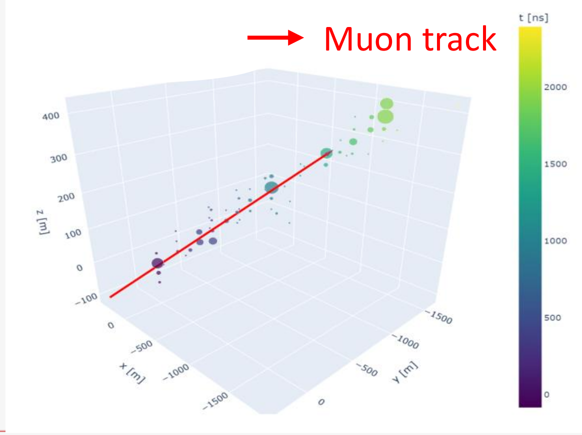


Preliminary earth model

Detector simulation

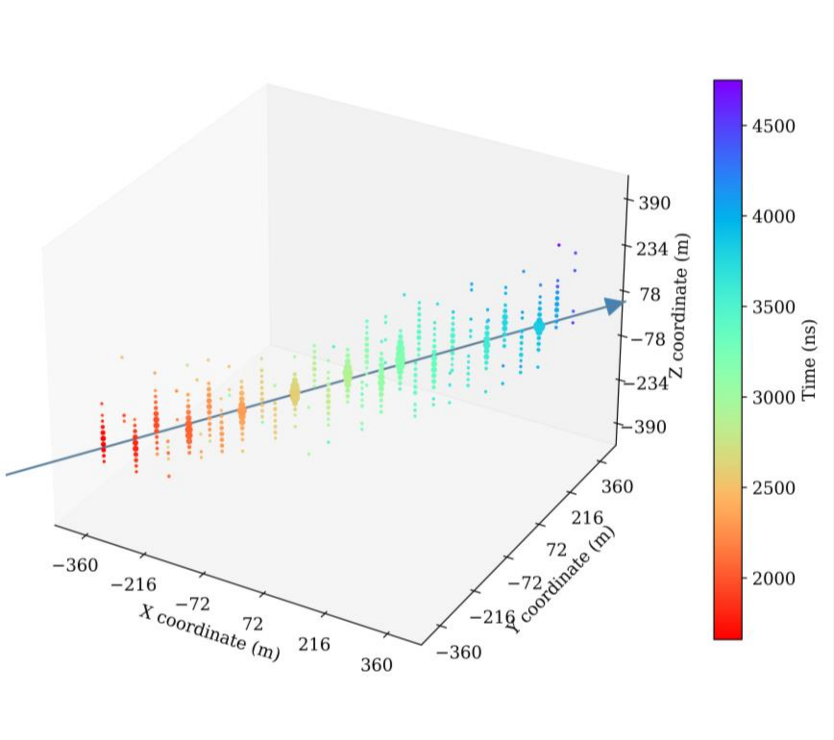
Based on **Geant4**:

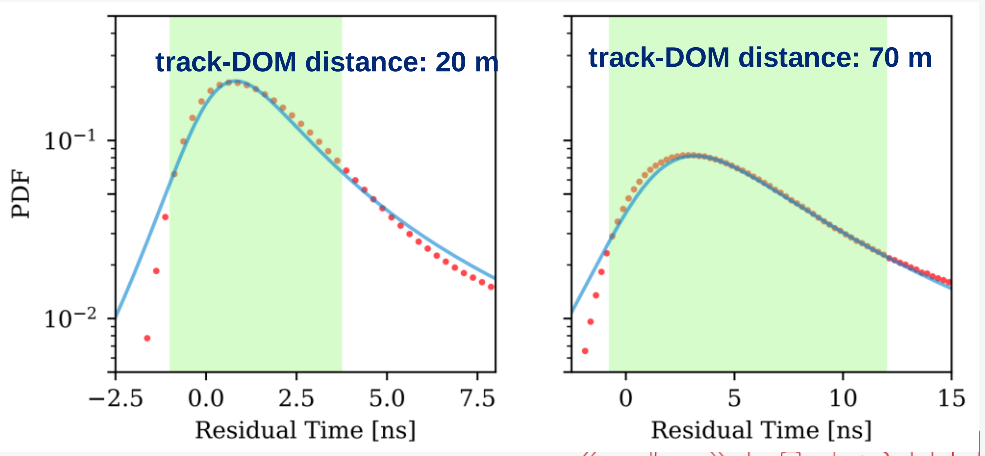
- Simulate the propagation of secondary particles.
- Accelerate Cherenkov photons simulation with OptiX.



Track-like Events

- Induced by high-energy muons.
- Obvious **geometric feature**:
  - DOMs surrounding muon track are likely to be triggered.
- Sensitive to **distance** between track and DOM.
- Sensitive to **photon arrival time**.





Residual Time:

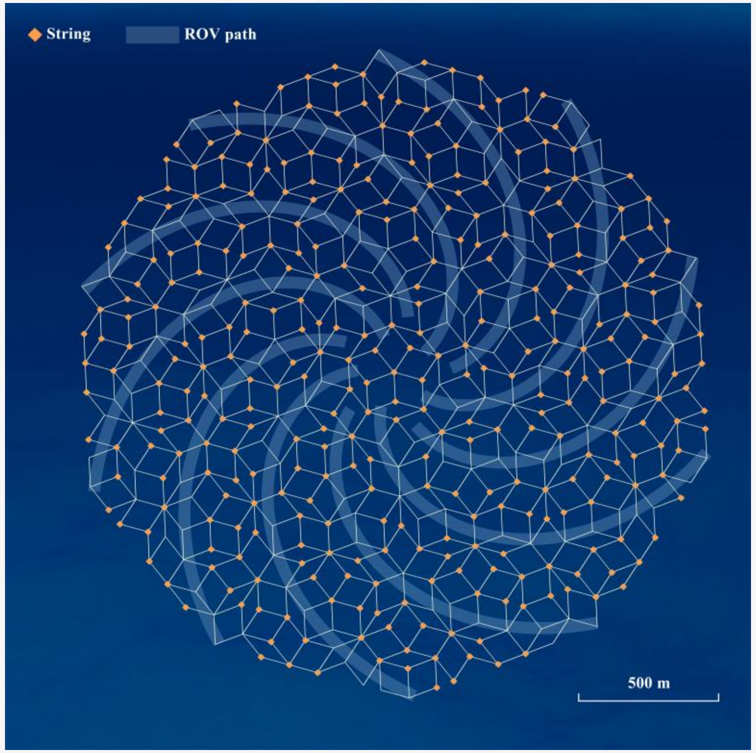
$$\boldsymbol{photon\ arrival\ time\ -\ geometric\ time}$$

Neural Networks in Neutrino Telescope

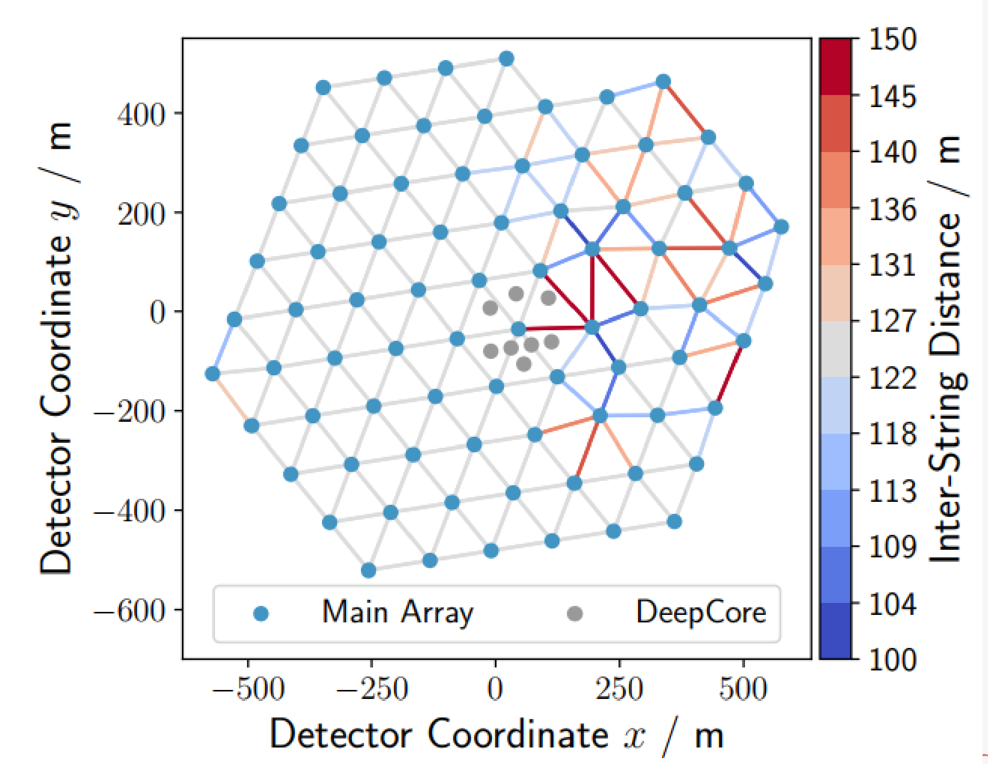
Challenges for neural networks in neutrino telescope

- Irregular detector geometry: Penrose geometry for TRIDENT

Top view of TRIDENT



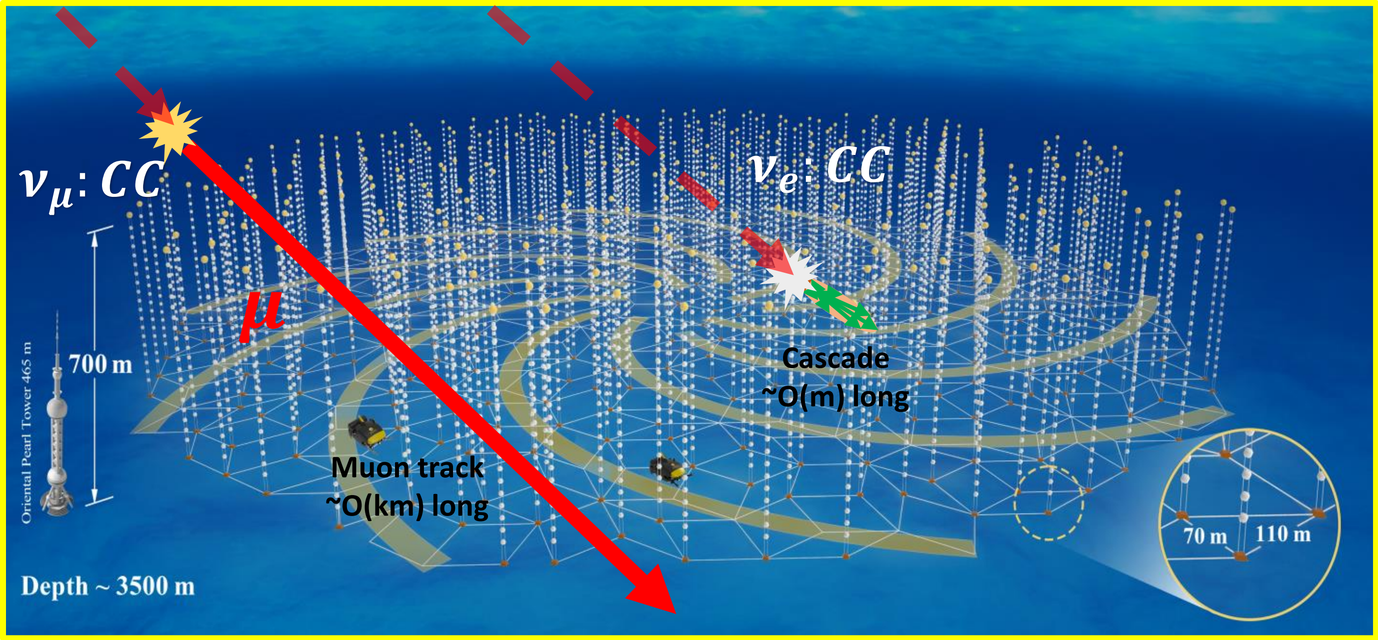
Top view of IceCube



Neural Networks in Neutrino Telescope

Challenges for neural networks in neutrino telescope

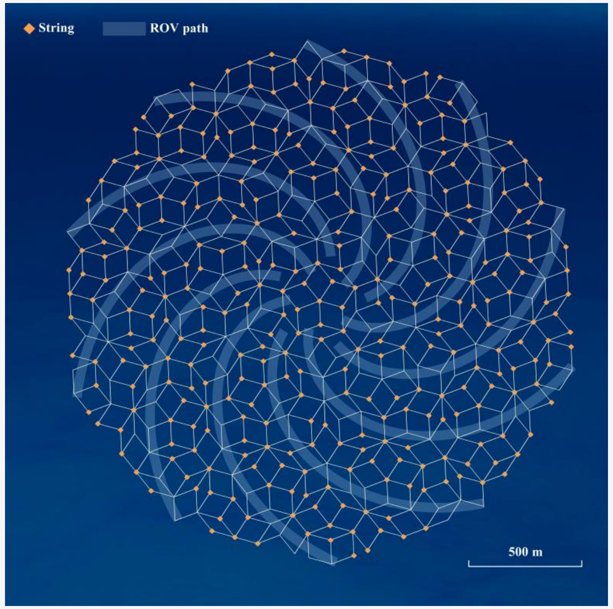
- Irregular detector geometry: Penrose geometry for TRIDENT
- High dimensionality: (x, y, z, t) for each pixel
- Sparsity: only a small fraction of hDOMs are triggered by signal



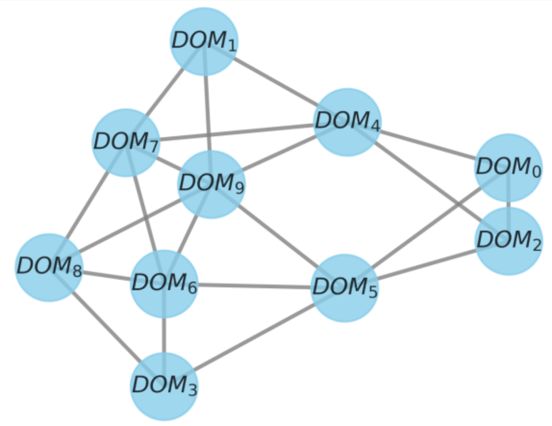
Neural Networks in Neutrino Telescope

Challenges for neural networks in neutrino telescope

- Obvious geometric features
- While CNN is inefficient for neutrino telescopes



Top view of TRIDENT



Use **point cloud** to represent neutrino events:

- **Triggered DOMs** → Nodes of point cloud
- **Location of DOMs** → Coordinate of nodes, pos_i.
- **DOM-measured time** → Features of nodes, x_i.

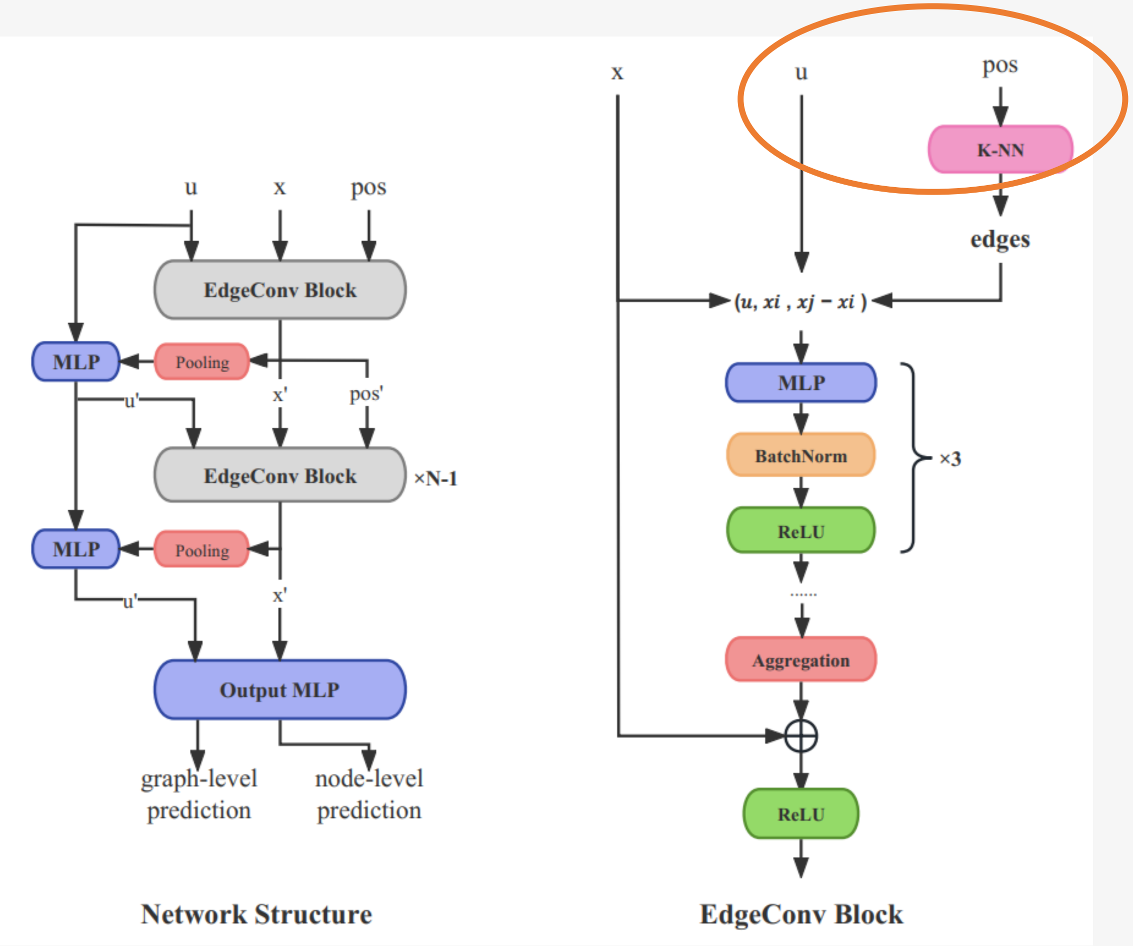
Compared GNN and SSCNN (arxiv:1706.01307) performance:

- GNN outperforms SSCNN in terms of angular resolution in track-like events.

Network Structure

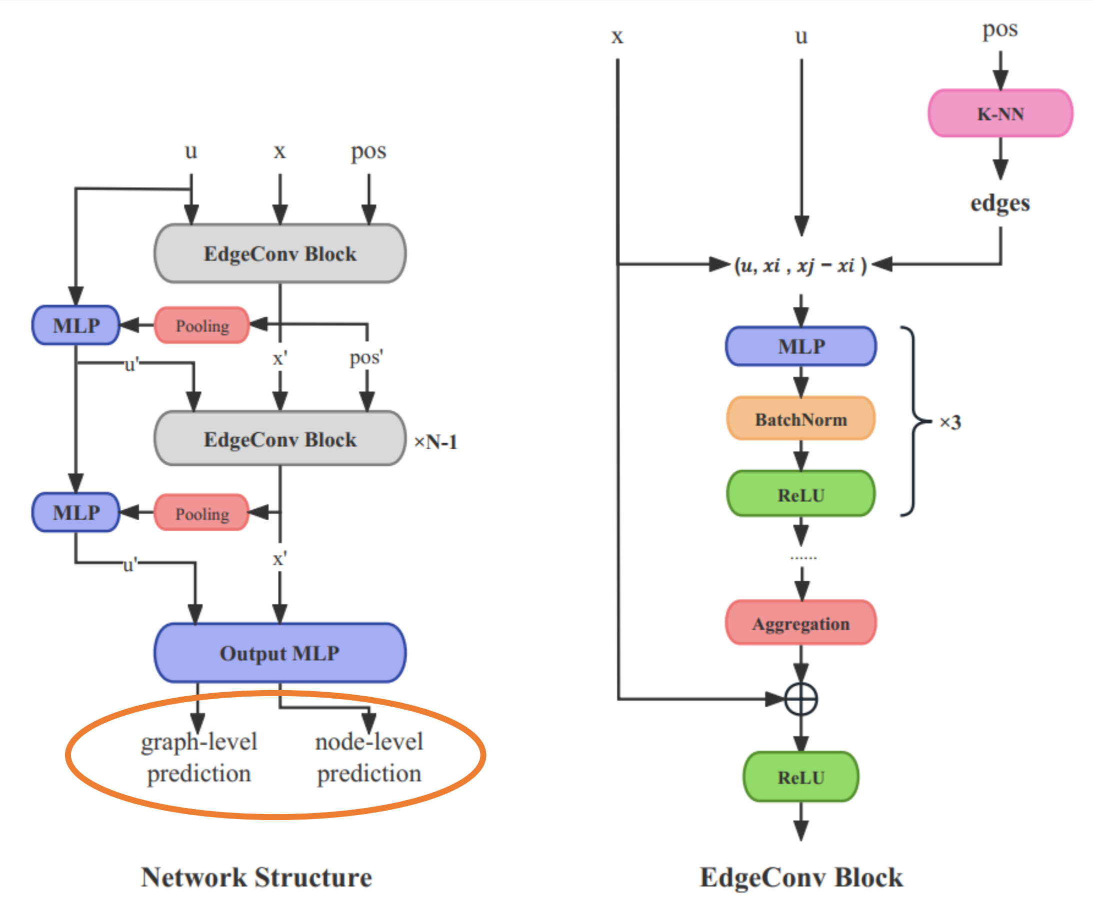
Core structure: **EdgeConv** block.

- Aware of local geometrical features: edges constructed with K-NN
- Aware of global features u



Network Structure

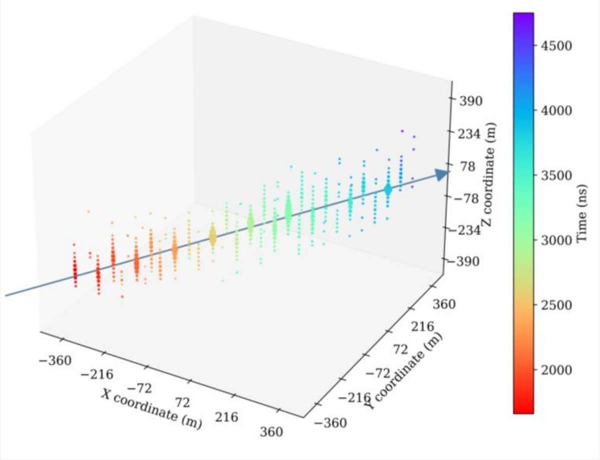
- Depending on task, make either graph-level prediction or node-level prediction.



Network In & Out

DOM$_\mathrm{i}$: $[hit_0, hit_1 \dots hit_N]$
$(hit_i = [x_i, y_i, z_i, t_i])$

Node$_\mathrm{i}$: $\{\mathrm{Median(t)}, \mathrm{x_i}, \mathrm{y_i}, \mathrm{z_i}, \mathrm{num\_hits}, \}$



Node definition

- $Photon\ hit := [x, y, z, t]$ relative to the first photon hit
- $DOM := [hit_0, hit_1, \dots, hit_N]$
- $Node := [\mathrm{Median(t)}, \mathrm{x_i}, \mathrm{y_i}, \mathrm{z_i}, \mathrm{num\_hits}]$
  - $\mathrm{Median(t)}$: less affected by dark noise.
  - $\mathrm{num\_hits}$: importance of the node

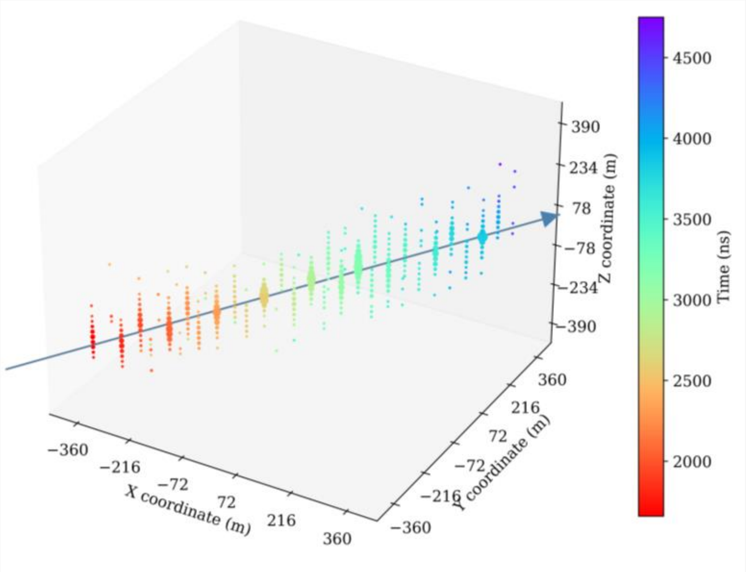
Edge definition

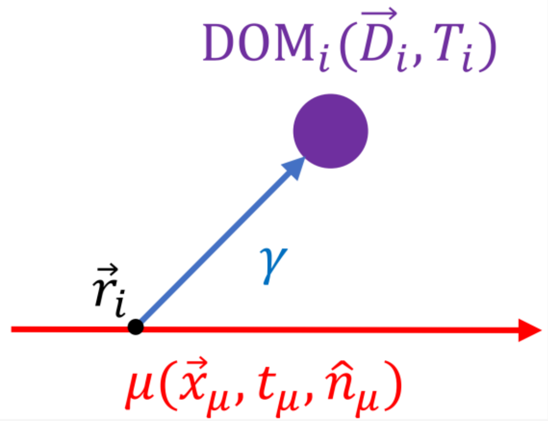
- **Dynamically** connect edge using position/node feature (K-NN)

Network In & Out

Workflow of prediction:

- **Semi-GNN method**:
  - Network predicts positions of photon emission, $\vec{\boldsymbol{r}}_{\boldsymbol{i}}$
  - Linear fit on predicted $\vec{\boldsymbol{r}}_{\boldsymbol{i}}$
  - Obtain $\hat{\boldsymbol{n}}_{\boldsymbol{\mu}}$
- More aware of geometric feature by employing linear fit.
- More robust against outliers.
- Goodness of reconstruction can be quantified.





μ emits photon and triggers $\mathrm{DOM_i}$

Method Performance

- Two semi-GNN models are trained:
  - **Large model**: 8 EdgeConv blocks, with 10M trainable parameters in total.
  - **Lite model**: 3 EdgeConv blocks, with 543k trainable parameters in total.

As a reference:

- **Full GNN / traditional GNN**:
  - Same network structure as semi-GNN.
  - Output $\hat{\boldsymbol{n}}_{\boldsymbol{\mu}}$ directly
- **Likelihood**: three steps

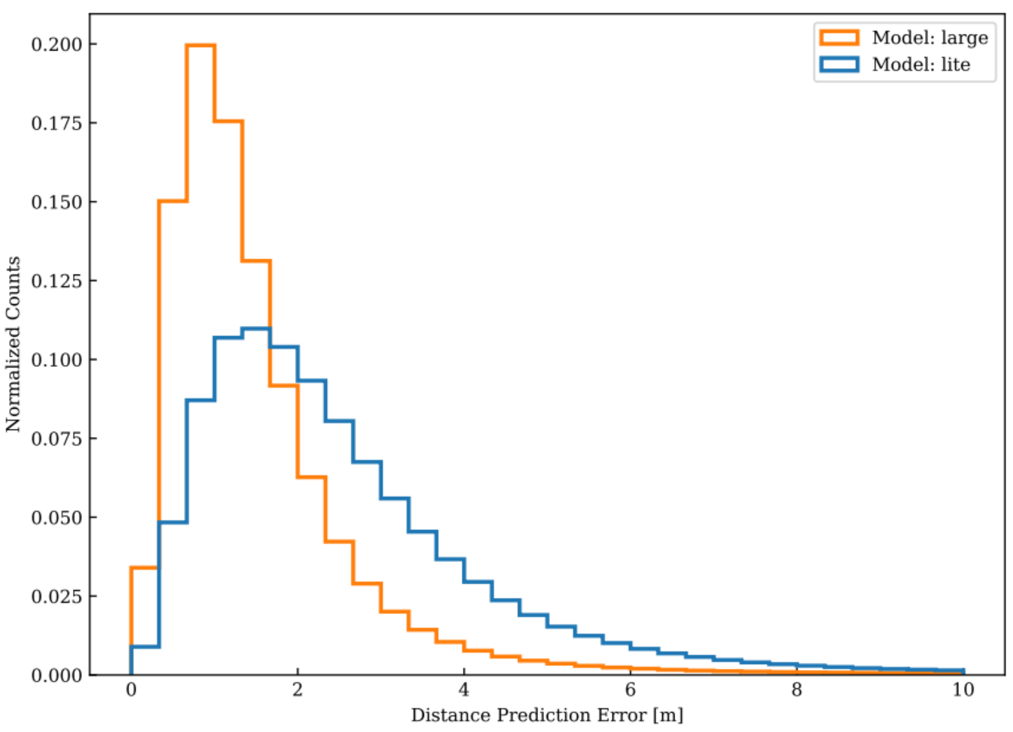
  1. Line fitting 2. Fitting on a coarse $\boldsymbol{PDF}(\textbf{residual time})$ 3. Fitting on a fine-tuned $\boldsymbol{PDF}(\textbf{residual}\ \boldsymbol{time})$

Method Performance

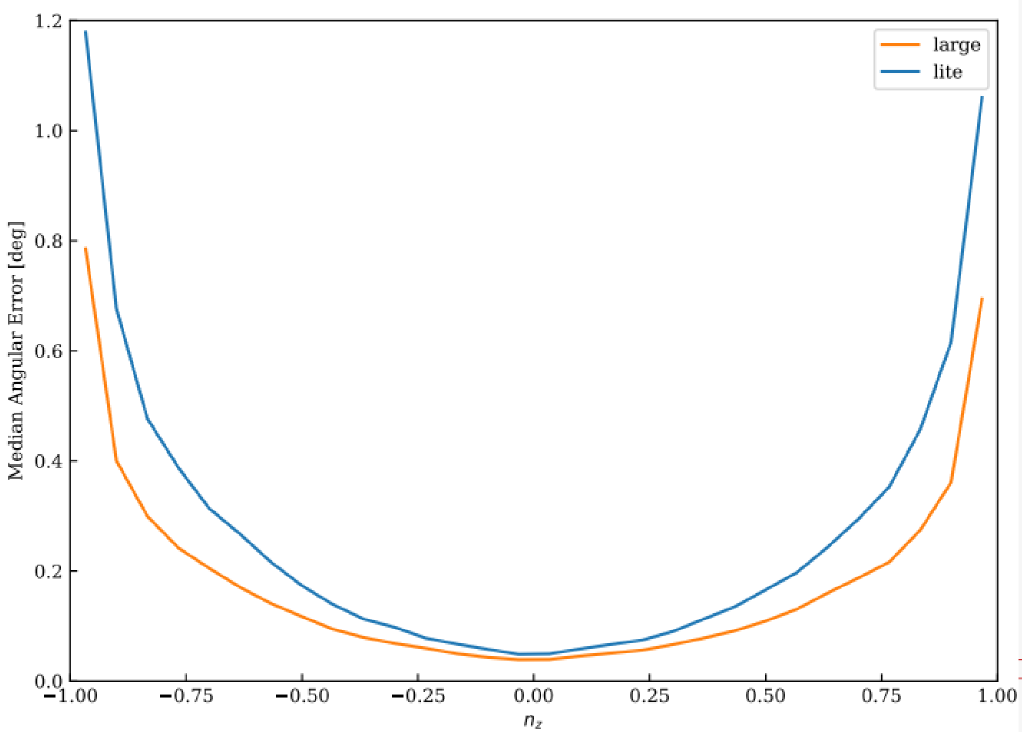
First look at performance of Semi-GNN by comparing Large & Lite model:

- Comparing the truth $\vec{r}_i$ with the predicted ones, a **distance error** of $\sim\mathbf{1}\boldsymbol{m}$ is achieved by the large model.

Distance Prediction Error

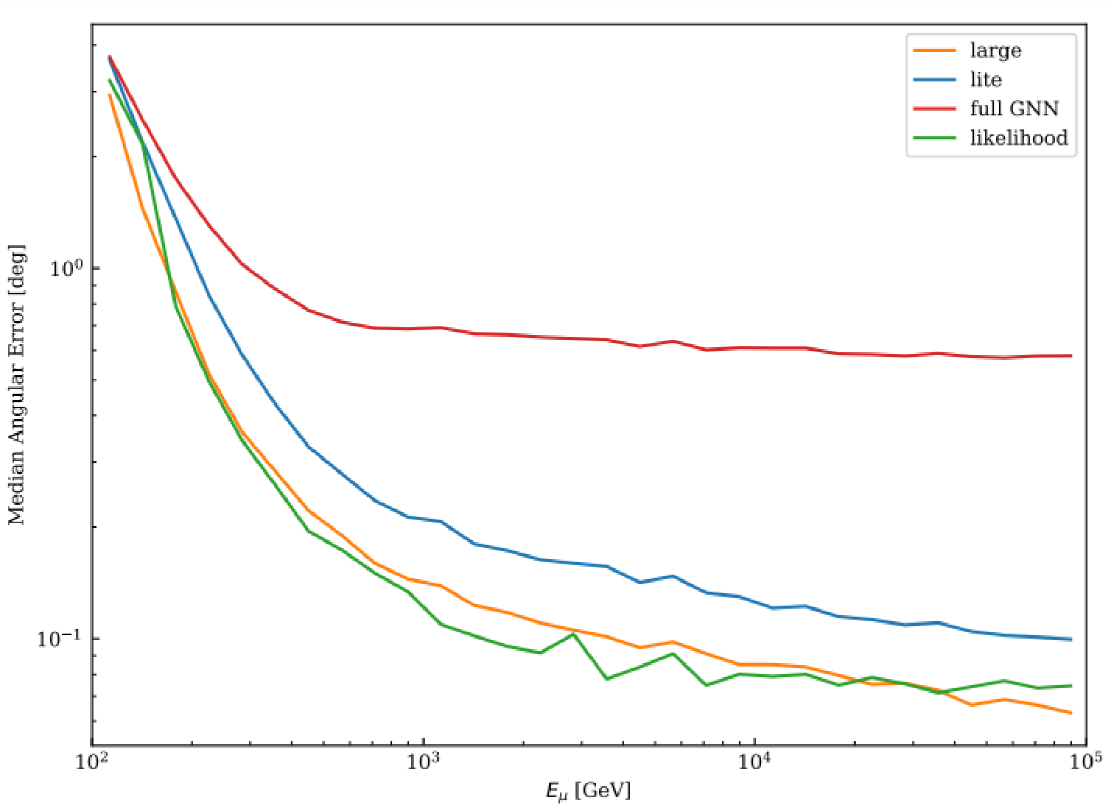


Angular Prediction error V.S. $\cos\theta_\mu$



Method Performance

Angular resolution V.S. E_μ



- **Angular resolution**: median of angles between $\hat{n}_\mu$ and $\hat{n}_{recon}$
- **Large model** outperforms **full GNN** by $\sim$6 times.
- Similar to **likelihood method**, a resolution of $\sim 0.1°$ is reached in high-energy region.

Method Performance

- Runtime of evaluation
- Using **GPU**
  - large model is ~1,000 times faster than likelihood method
  - lite model is ~10,000 times faster than likelihood method

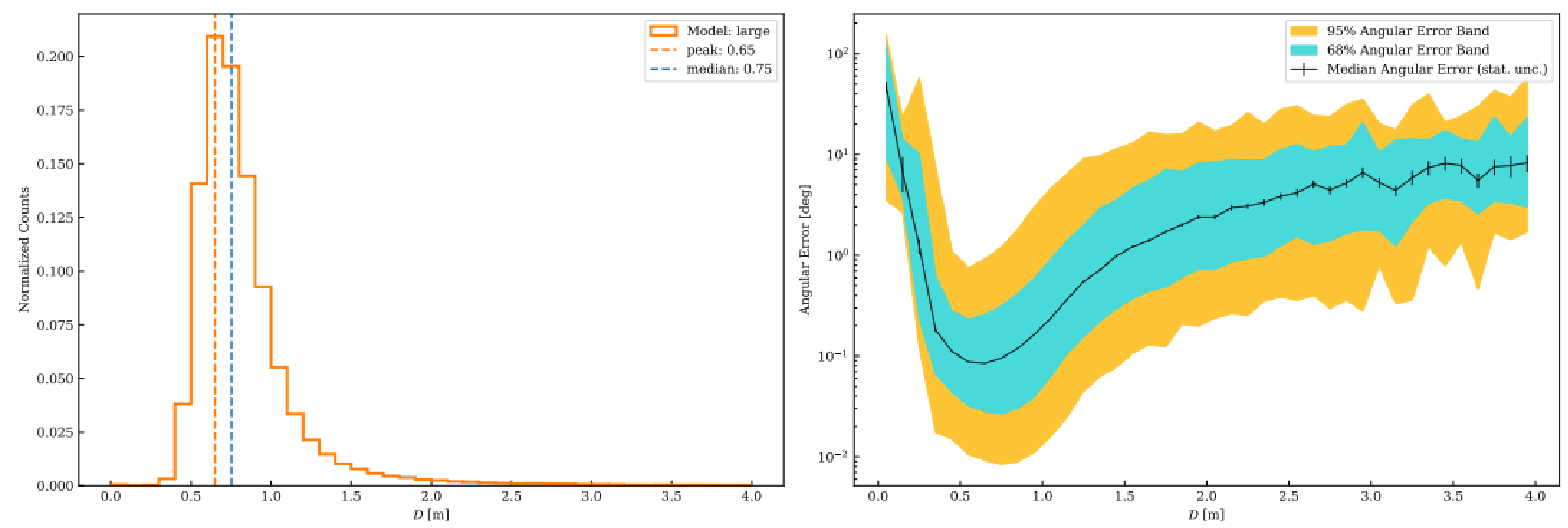
Method	Time (0.1-1 TeV) [ms]	Time (1-10 TeV) [ms]	Time (10-100 TeV) [ms]
Likelihood	1552.30	1259.86	919.14
GNN lite (GPU)	**0.19**	**0.21**	**0.29**
GNN large (GPU)	0.38	0.78	2.37
GNN lite (CPU)	5.05	12.53	30.44
GNN large (CPU)	54.71	152.48	181.80

Goodness of Reconstruction

- Outputs of the model, $\vec{r}_i$, are expected to be align along a straight line
- Define

$$\mathcal{D} \coloneqq \sum_{i \in \mathrm{DOM}} \mathrm{Distance}(\textbf{predicted } \vec{\boldsymbol{r}}_{\boldsymbol{i}}, \textbf{predicted track}) \, / \, \mathrm{num\_DOM}$$

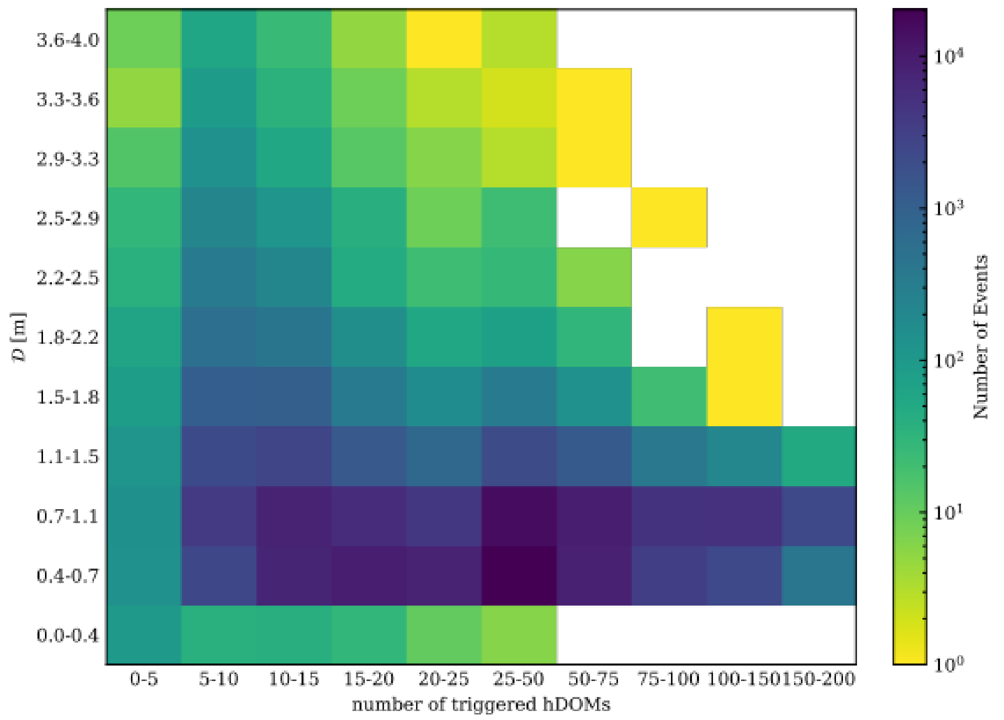
- $\mathcal{D}$ works as a metric indicating goodness of reconstruction.

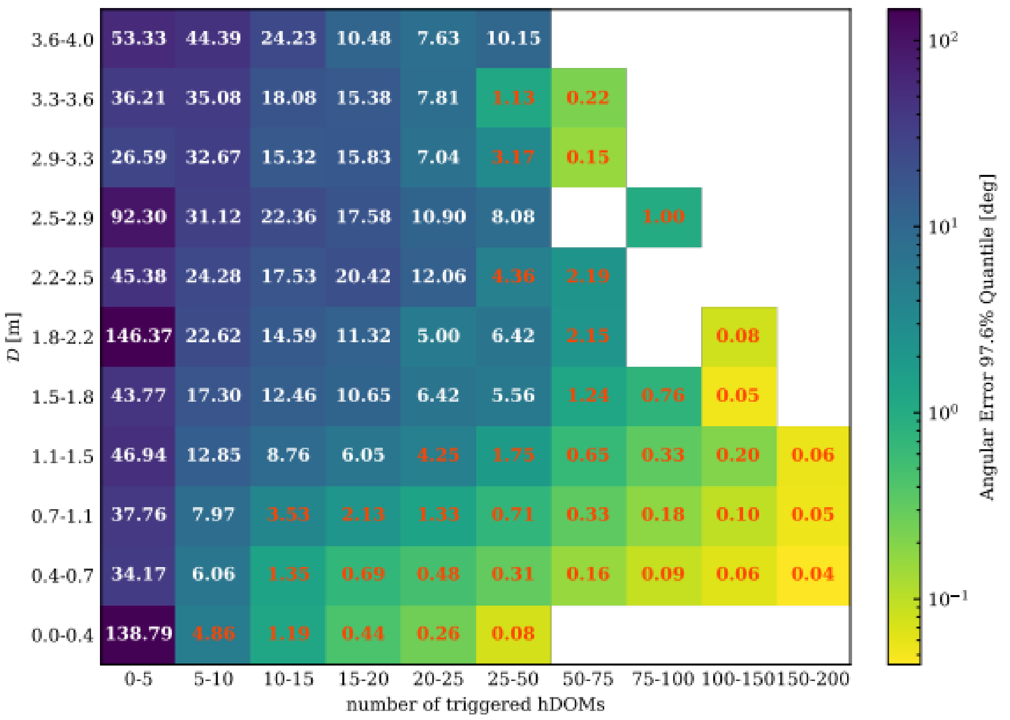


Goodness of Reconstruction

Define well-reconstructed region:

- Combine $\mathcal{D}$ with number of triggered DOMs
- Events with lower $\mathcal{D}$ and higher num DOMs tends to have small angular error





Summary

A semi-GNN method is designed to reconstruct direction of track-like events in neutrino telescopes.

- **Reconstruction accuracy** outperforms full-GNN method **by 6 times** in high-energy region, and achieves a angular resolution of **0.1°** , **similar accuracy as likelihood method**.
- **Runtime cost for reconstruction** is smaller than likelihood method by **3~4 orders of magnitude**.
- **Goodness of reconstruction** can be quantified using $\mathcal{D}$.
- **Well-reconstructed region** where angular errors 97.6% of events are less than 5° can be further defined.

Thanks for your listening

Goodness of Reconstruction

- Angular error with number of triggered hDOMs