

Coupled-channel analysis of charmonia

arXiv: 2312.17658v3 (v3 to be posted soon)

Satoshi Nakamura (Shandong Univ.)

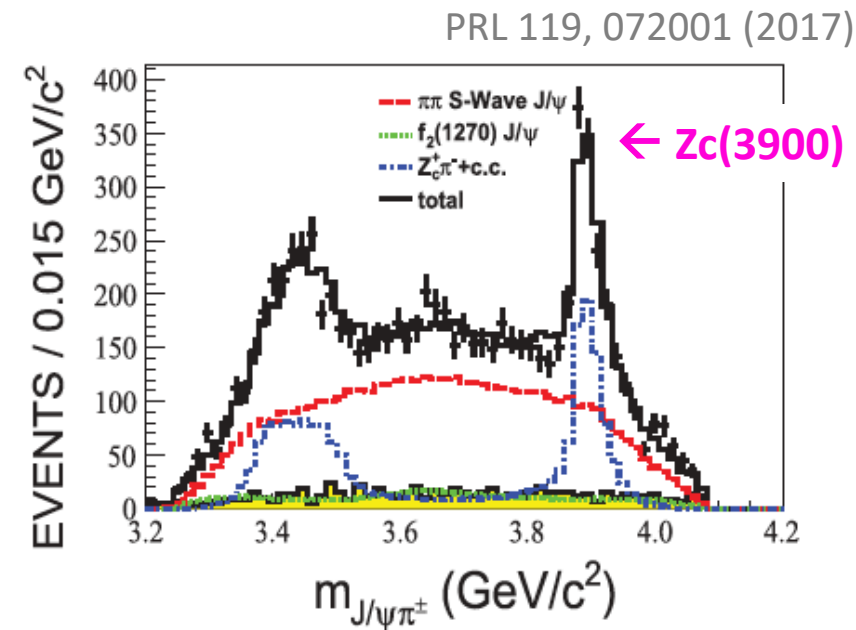
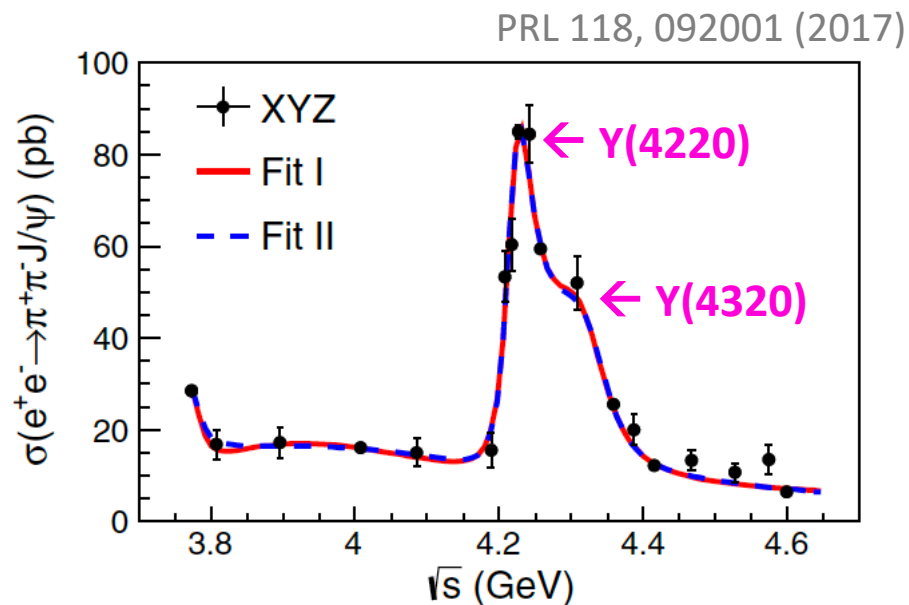
Collaborators: X.-H. Li, H.-P. Peng, X.-R. Zhou (USTC), Z.-T. Sun (SDU)

Introduction

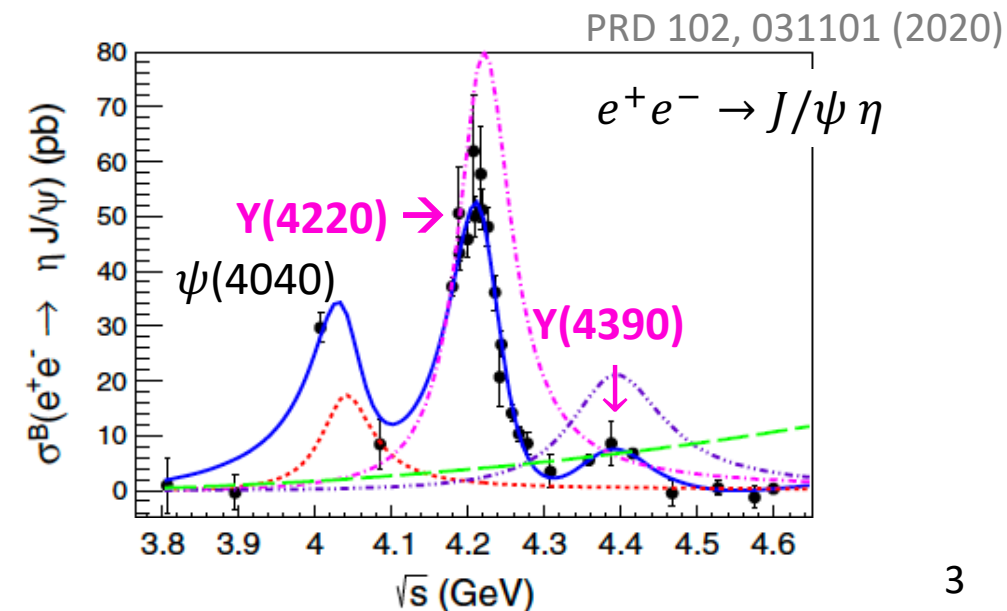
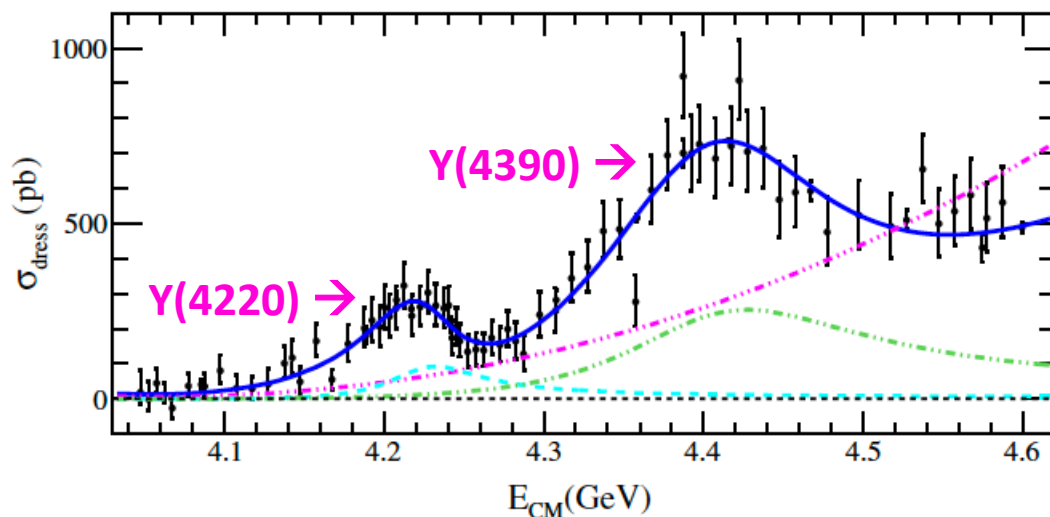
BESIII data for XYZ physics

(only a few from many)

$$e^+e^- \rightarrow J/\psi \pi^+\pi^- \rightarrow$$

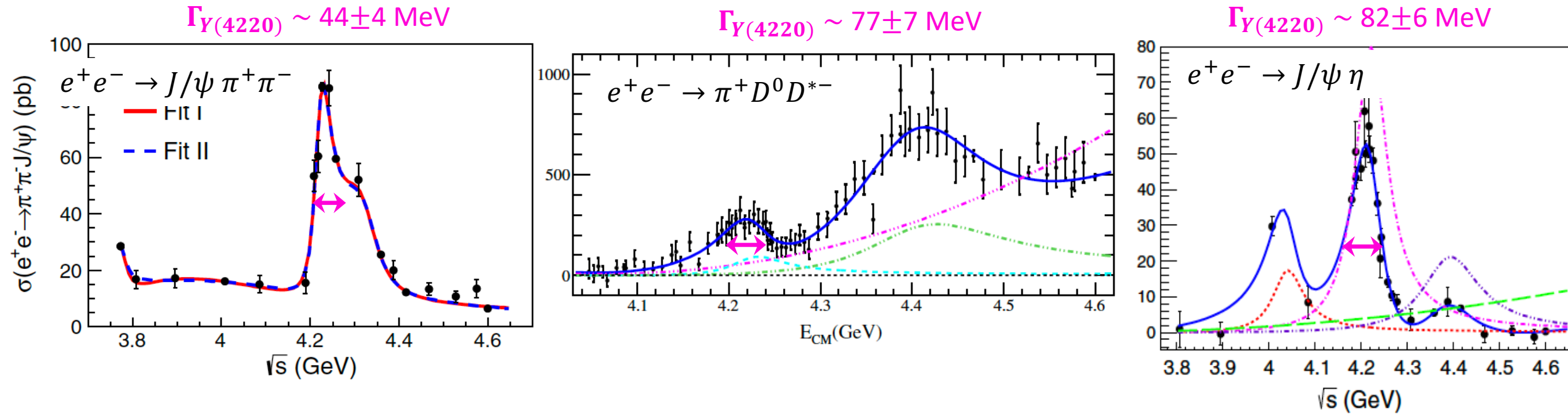


$$e^+e^- \rightarrow \pi^+ D^0 D^{*-}$$



Outstanding question in XYZ physics : Υ width problem

Why Υ states seem to have different widths for different final states ?



How to find solution to Y width problem ?

🧐 Analyze different final states with different models (usual experimental analysis; single-channel analysis)

→ no simple relation between resonance parameters from different models

→ Y width problem **created**

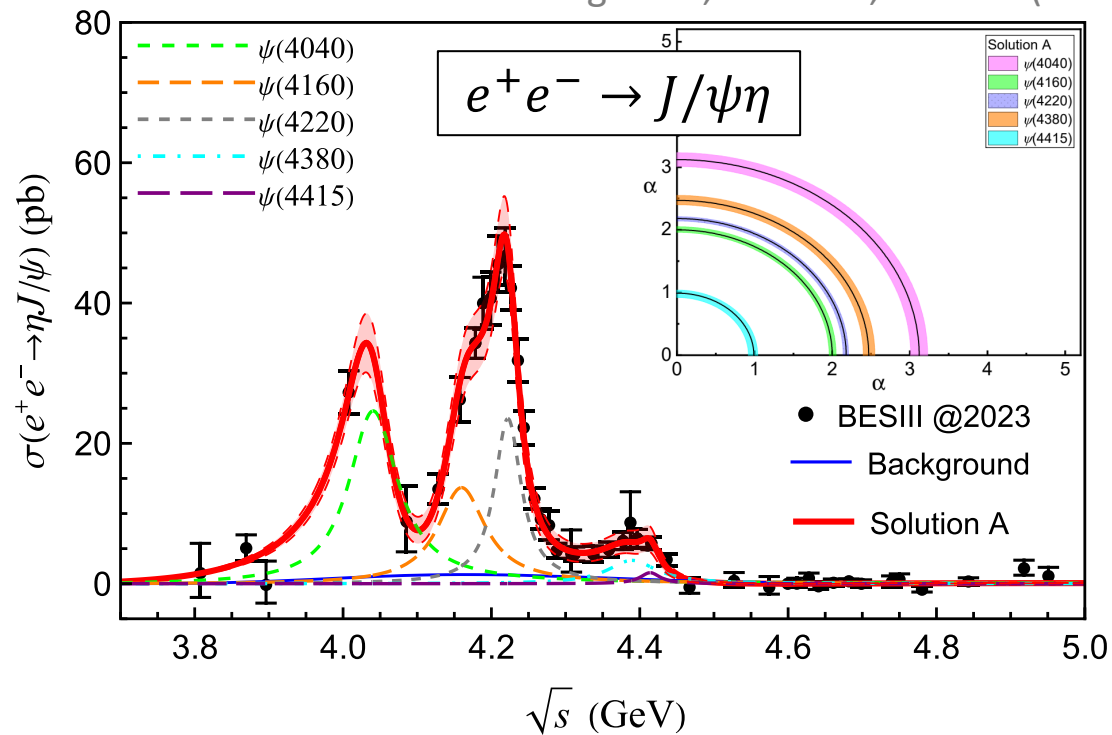
Y-width problem is artifact of single-channel analysis

How to find solution to Υ width problem ?

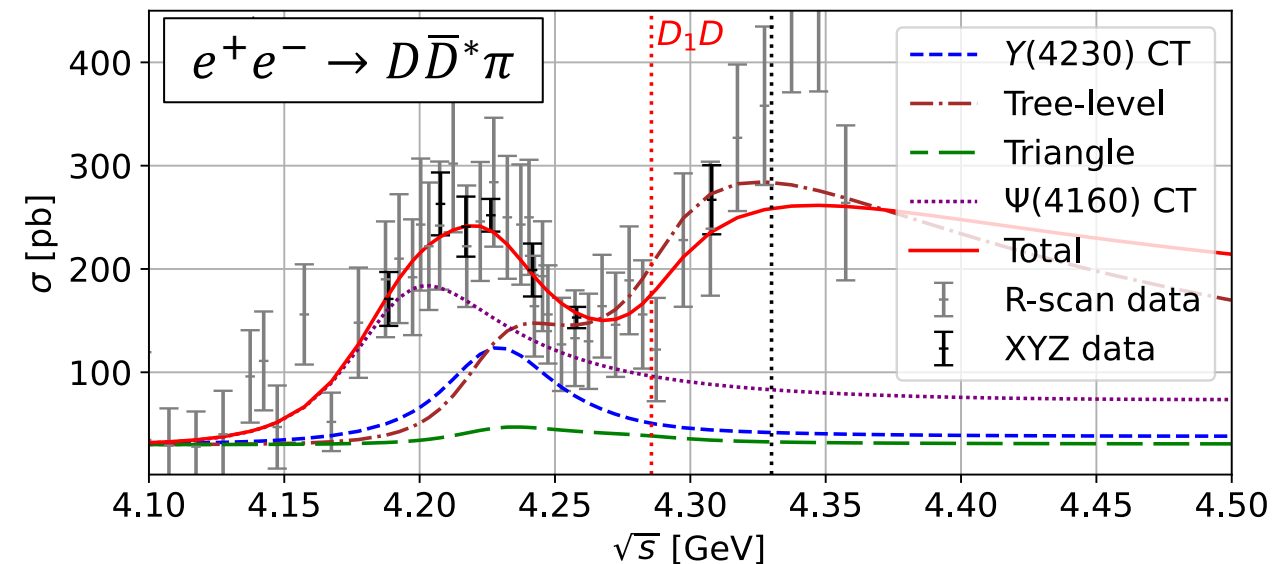
☹️ Combine a couple of charmonia to solve Υ -width problem

Narrow $\Upsilon(4220)$ from $e^+e^- \rightarrow J/\psi\pi\pi \rightarrow$ narrow $\Upsilon(4220) + \psi(4160) \rightarrow$ broad $\Upsilon(4220)$ in other processes

T.-C. Peng et al., PRD 109, 094048 (2024)



L. von Detten et al., PRD 109, 116002 (2024)



Problem: sum of Breit-Wigner amplitudes violates unitarity; more problematic for overlapping resonances

How to find solution to Υ width problem ?

😊 Analyze different final states simultaneously with a unified and (semi-)unitary model

(global coupled-channel analysis)

- * how various charmonia interfere to create different lineshapes in different final states
- * kinematical effects (threshold opening, triangle singularity) change lineshapes in some processes

→ Solution of the Υ width problem

At the same time, global analysis determines:

- (i) vector charmonium pole structure (pole locations)
- (ii) couplings of the poles with decay channels (residues)

Now is the time to conduct global analysis of $e^+e^- \rightarrow c\bar{c}$ data, and determine vector charmonium poles and residues

BESIII accumulated high-quality data for various $e^+e^- \rightarrow c\bar{c}$ processes over wide energy region covering Y

$$e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, J/\psi \eta^{(\prime)}, \chi_{c0}\omega, \Lambda_c\bar{\Lambda}_c \quad (\text{two-body final states})$$

$$e^+e^- \rightarrow \pi D^{(*)}\bar{D}^{(*)}, J/\psi\pi\pi, \psi'\pi\pi, h_c\pi\pi, J/\psi K\bar{K} \quad (\text{three-body final states})$$

$$e^+e^- \rightarrow \eta_c\rho\pi (\rho \rightarrow \pi\pi) \quad (\text{four-body final states})$$

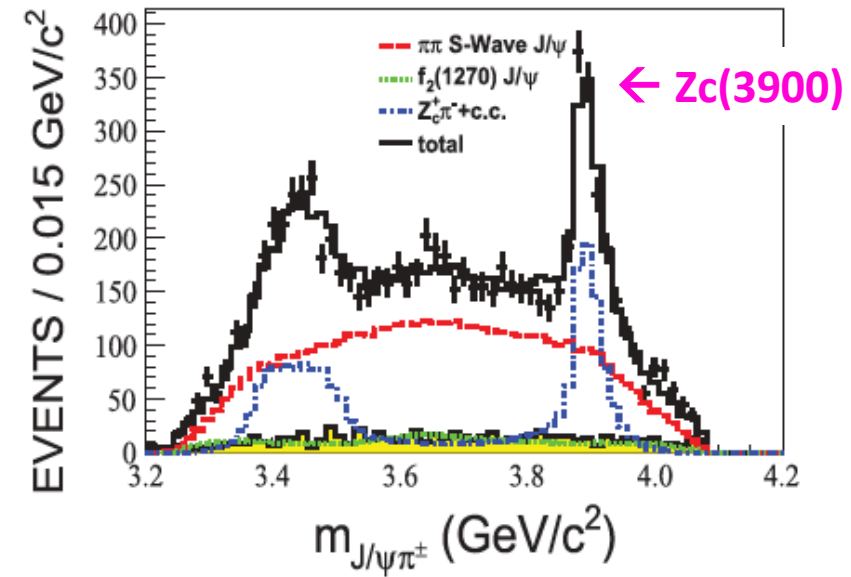
The global analysis is important not only for Y but also for well-established $\psi(4040)$, $\psi(4160)$, $\psi(4415)$ because:

- Their properties were previously determined by simple Breit-Wigner fit to inclusive ($e^+e^- \rightarrow \text{hadrons}$) R values
- Analyzing precise exclusive data \rightarrow More detailed and precise information

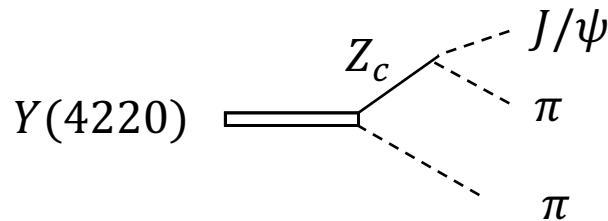
Understanding Y inevitably involves understanding Zc

$Z_c(3900)$, $Z_c(4020)$: outstanding exotic candidates including $c\bar{c}u\bar{d}$

$e^+e^- \rightarrow J/\psi \pi^+\pi^-$ at $Y(4220)$ region \rightarrow



Z_c appears as:



\rightarrow Y and Z_c properties should be highly correlated

Global $e^+e^- \rightarrow c\bar{c}$ analysis consider Z_c signals \rightarrow address Y and Z_c properties simultaneously

This work

- **Global analysis** of BESIII and Belle data in $3.75 \leq \sqrt{s} \leq 4.7$ GeV with a unified coupled-channel model

$$e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, J/\psi \eta^{(\prime)}, \chi_{c0}\omega, \Lambda_c\bar{\Lambda}_c \text{ (10 two-body final states)}$$

$$e^+e^- \rightarrow \pi D^{(*)}\bar{D}^{(*)}, J/\psi\pi\pi, \psi'\pi\pi, h_c\pi\pi, J/\psi K\bar{K} \text{ (7 three-body final states)}$$

$$e^+e^- \rightarrow \eta_c\rho\pi \text{ } (\rho \rightarrow \pi\pi) \text{ (1 four-body final states)}$$

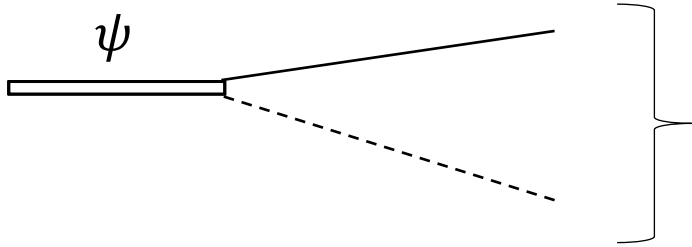
- Approximate three-body unitarity
- Fit both total cross sections and invariant mass distributions
- Extract vector charmonium (ψ , Y) and Z_c poles (mass, width)

Near-future work \rightarrow Extraction of residues (branching fractions) and solution of Y width problem

MODEL

Coupled-channels

(quasi) two-body channels included; $J^{PC} = 1^{--}$

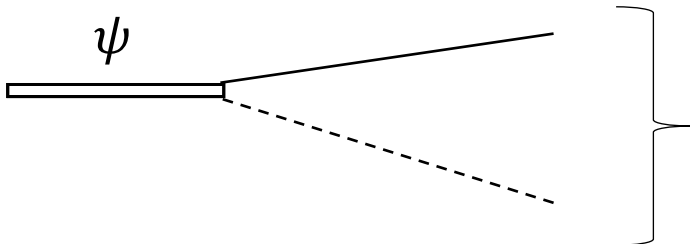


$D_1(2420)\bar{D}^{(*)}, D_1(2430)\bar{D}^{(*)}, D_2^*(2460)\bar{D}^{(*)}, D^{(*)}\bar{D}^{(*)}, D_{s1}(2536)\bar{D}_s$
 $\omega\chi_{c0}$

$D_1(2420), D_1(2430), D_2^*(2460), D^*, D_{s1}(2536), \omega \rightarrow$ Breit-Wigner (BW) propagators; mass and width from PDG

BW partially violate three-body unitarity in our three-body calculation

Otherwise the model is manifestly three-body unitary

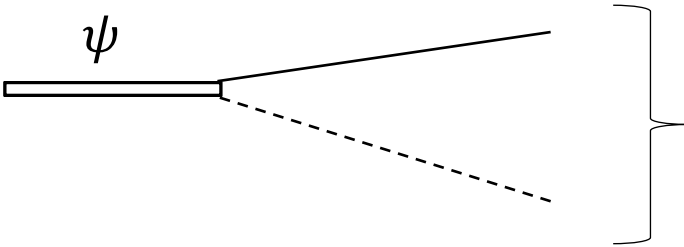


$D_s^{(*)}\bar{D}_s^{(*)}, J/\psi\eta, J/\psi\eta', \Lambda_c\bar{\Lambda}_c$

treated as stable particles

Coupled-channels

(quasi) two-body channels included; $J^{PC} = 1^{--}$



$D_0^*(2300)\bar{D}^*, f_0 J/\psi, f_2 J/\psi, f_0 \psi', f_0 h_c, Z_c \pi, Z_{cs} \bar{K}$

$D_0^*(2300), f_0, f_2, Z_c, Z_{cs}$ as (virtual) poles in two-body scattering amplitudes

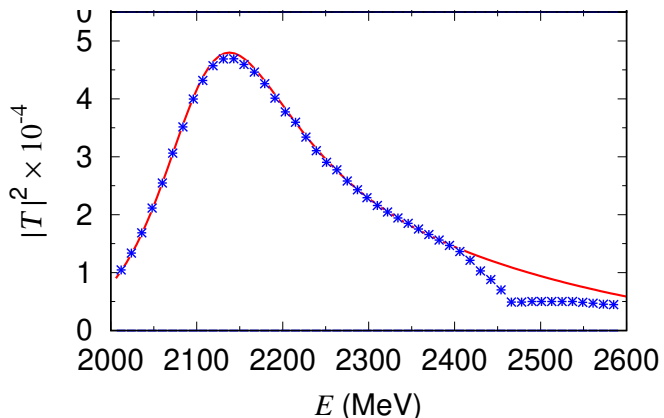
$D\pi$ s-wave amplitude fitting LQCD-based amplitude

Albaladejo et al. PLB 767 (2017)

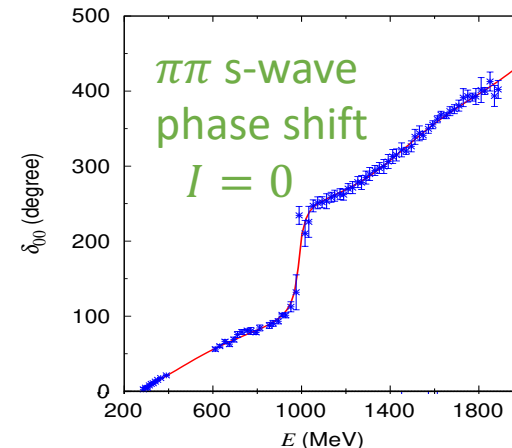
D_0^* pole :

$2104 - i 100$ MeV (ours)

$2105^{+6}_{-8} - i 102^{+10}_{-12}$ MeV
(Albaladejo et al.)



$\pi\pi$ s[d]-wave amplitude fitting empirical amplitude



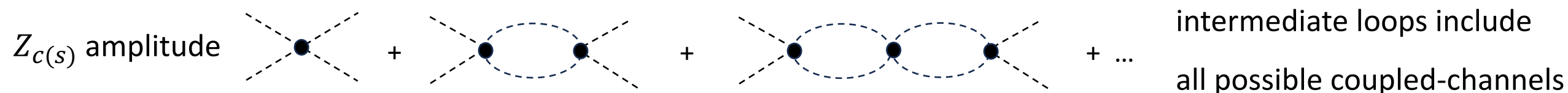
$f_0(500), f_0(980),$
 $f_0(1370), f_2(1270)$ poles
→ consistent with PDG

$Z_{c(s)}$ amplitude

$Z_c : J^{PC} = 1^{+-} \quad D^* \bar{D} - D^* \bar{D}^* - J/\psi \pi - \psi' \pi - h_c \pi - \eta_c \rho$ couple—channel scattering amplitude

$Z_{cs} : J^{PC} = 1^{+-} \quad D_s^* \bar{D} - D_s \bar{D}^* - J/\psi K$

driven by contact interactions; s-wave interactions except $h_c \pi$ p-wave interaction



$$v_{[D^* \bar{D}], [D^* \bar{D}]} = v_{D^* \bar{D}^*, D^* \bar{D}^*} = v_{[D_s^* \bar{D}], [D_s^* \bar{D}]} \quad (\text{HQSS, SU(3)})$$

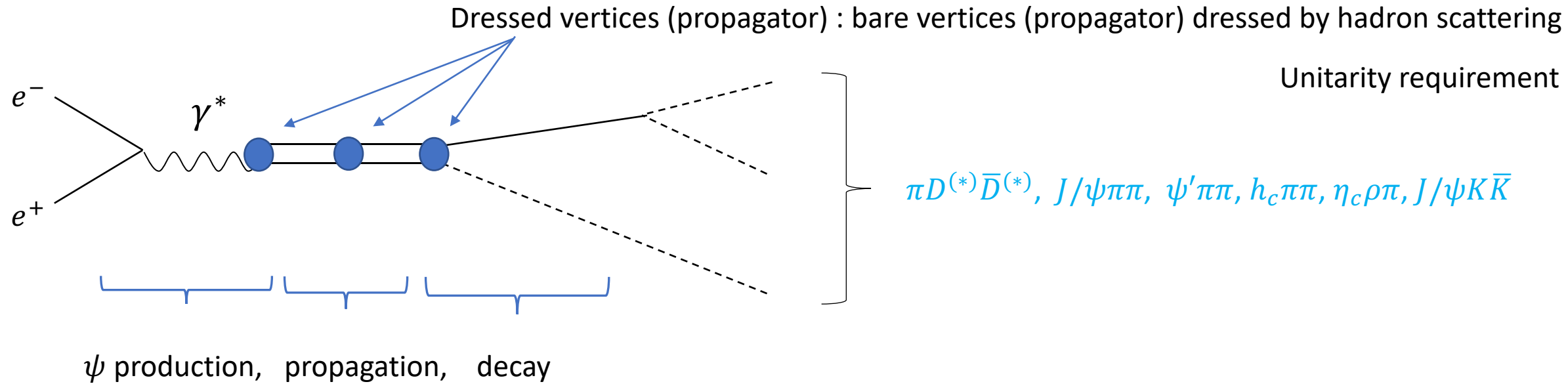
$$v_{[D^* \bar{D}], J/\psi \pi} = v_{[D_s^* \bar{D}], J/\psi K} \quad (\text{SU(3)})$$

no coupling between hidden-charm channels (e.g. $v_{J/\psi \pi, J/\psi \pi} = v_{J/\psi \pi, \psi' \pi} = 0$)

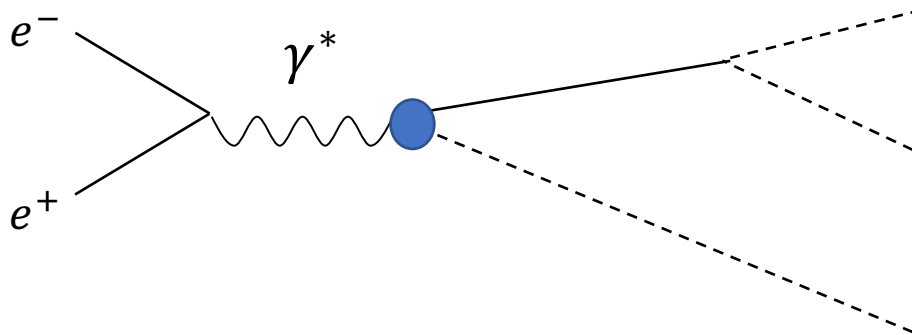
$$C = -1 \text{ basis} \quad \begin{aligned} [D^* \bar{D}] &= \frac{1}{\sqrt{2}} (D^* \bar{D} - D \bar{D}^*) \\ [D_s^* \bar{D}] &= \frac{1}{\sqrt{2}} (D_s^* \bar{D} - D_s \bar{D}^*) \end{aligned} \quad \text{SU(3)}$$

Nonzero couplings are determined by the global fit \rightarrow poles may be generated if needed by data

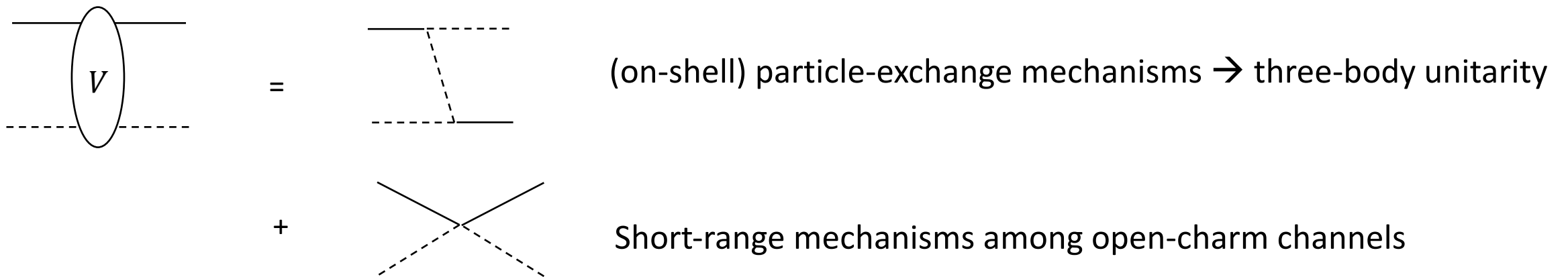
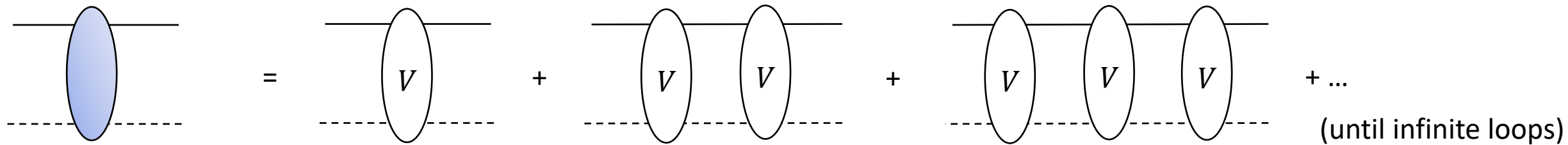
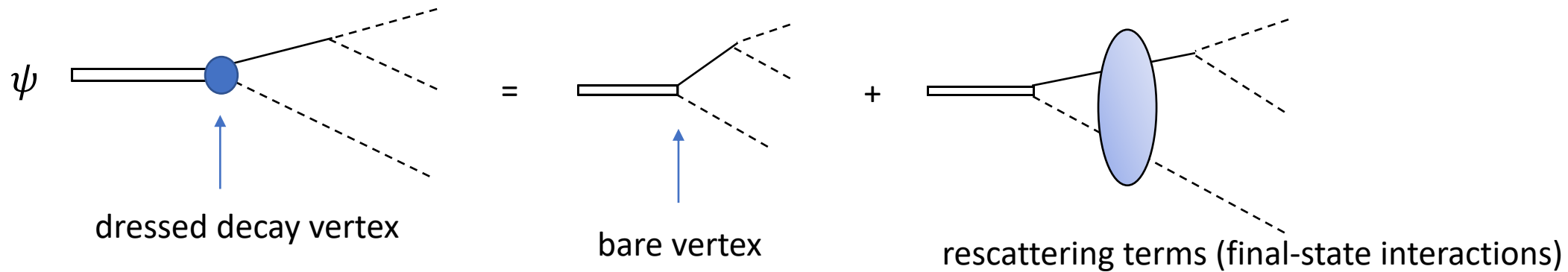
Full amplitude for $e^+e^- \rightarrow$ three-body final states



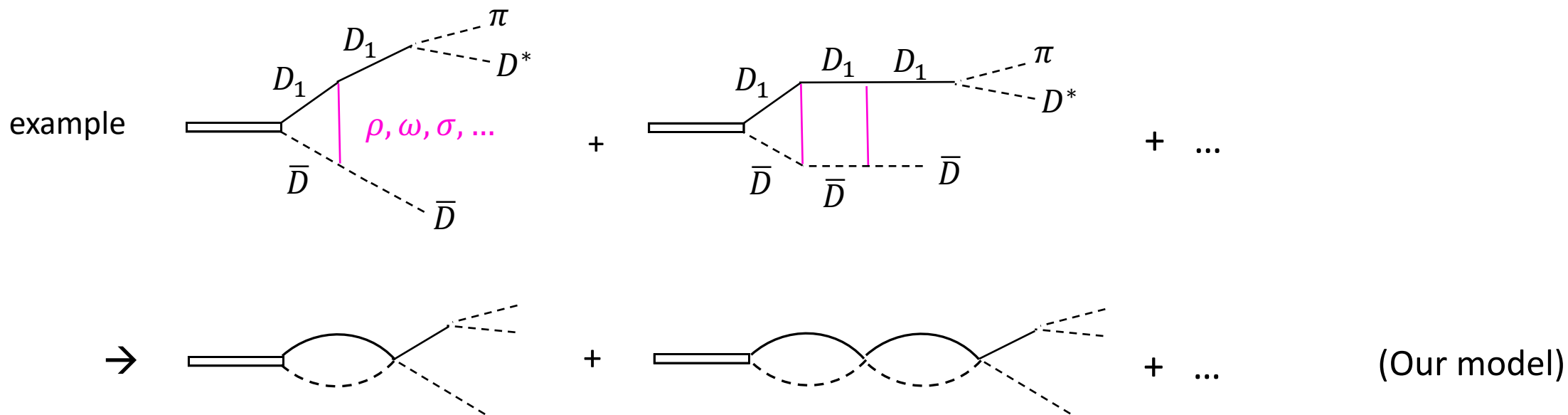
Non-resonant mechanisms are also included



Three-body decays of ψ



Short-range mechanisms among open-charm channels



Contact interactions among $D_1\bar{D}^{(*)}$, $D_2^*\bar{D}^*$, $D^{(*)}\bar{D}^{(*)}$, $D_{s1}\bar{D}_s$, $D_s^{(*)}\bar{D}_s^{(*)}$, $\Lambda_c\bar{\Lambda}_c$ channels

→ fitted to data (advantage of separable interactions)

High-precision BESIII data require these contributions (threshold cusps)

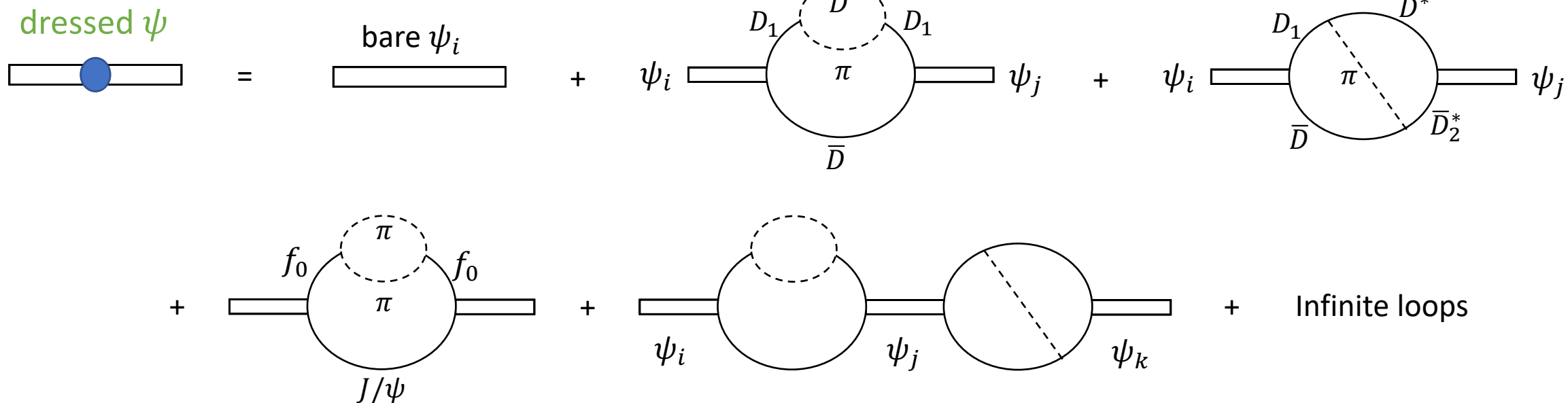
We can examine $Y(4220)$ as $D_1\bar{D}$ molecule and $Y(4360)$ as $D_1\bar{D}^*$ molecule from global analysis

→ To be done

ψ propagator

(we do not use BW)

($D^*\pi$ -loop is replaced by D_1 BW)



Charmonium poles are formed by non-perturbative couplings between bare ψ and $D_1\bar{D}$, $f_0 J/\psi$, ...
(= poles of dressed ψ propagator)

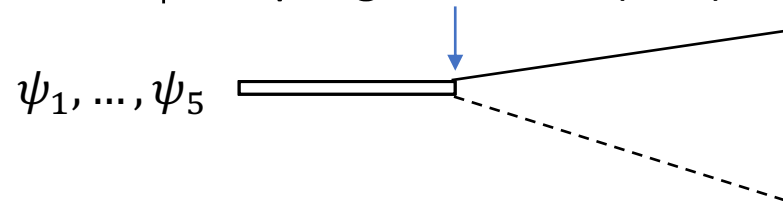
Unitary coupled-channel model : resonance pole (mass, width) and decay dynamics are explicitly related.
(unitarity requirement)

Breit-Wigner model : decay dynamics are simulated by BW mass and width parameters

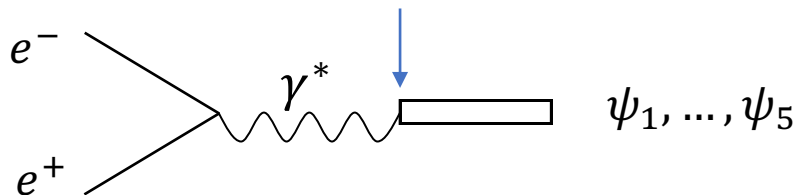
Fitting parameters in global analysis

- * bare ψ masses (5 bare states)

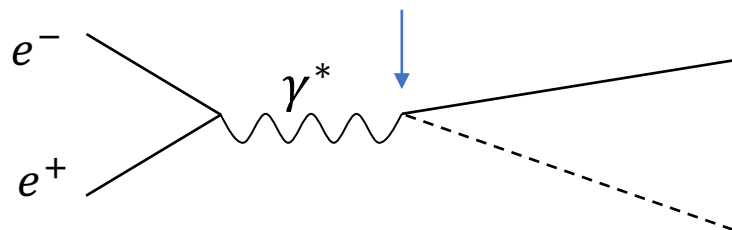
- * bare ψ coupling constants (real)



- * bare photon- ψ coupling constants (real)



- * non-resonant photon coupling constants (real)



- * $\psi(4660), \psi(4710)$ Breit-Wigner mass, width, vertices

- * coupling constants in Z_c amplitude :

$$\nu_{D^*\bar{D}, D^*\bar{D}}, \nu_{D^*\bar{D}, J/\psi\pi}, \nu_{D^*\bar{D}, \psi'\pi} \text{ etc.}$$

- * Contact-interaction strengths among open-charm channels

- * Cutoffs in non-resonant vertices for

$$\gamma^* \rightarrow D^{(*)}\bar{D}^{(*)}, D_S^{(*)}\bar{D}_S^{(*)}, \Lambda_c\bar{\Lambda}_c$$

In total, 205 fitting parameters

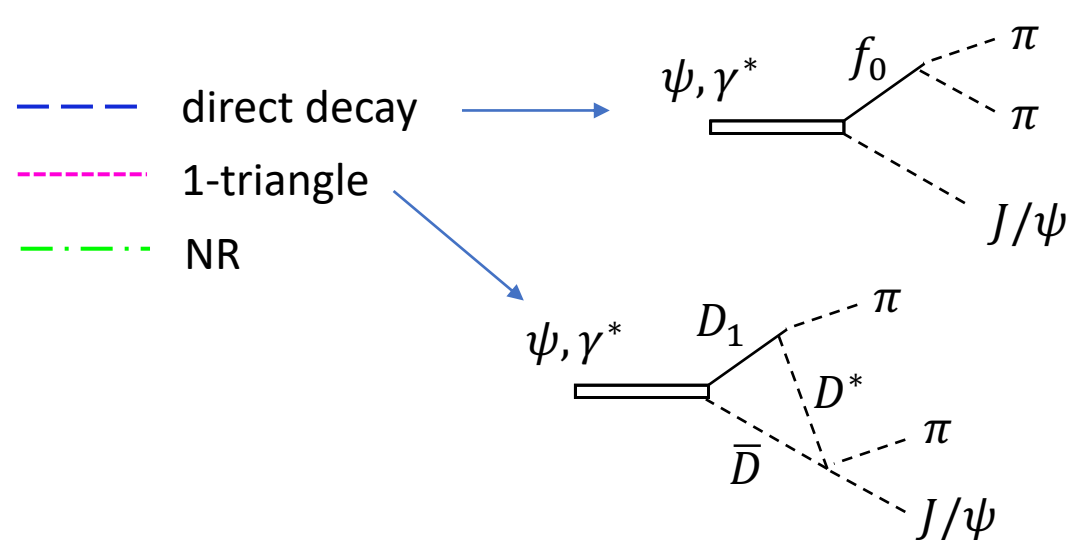
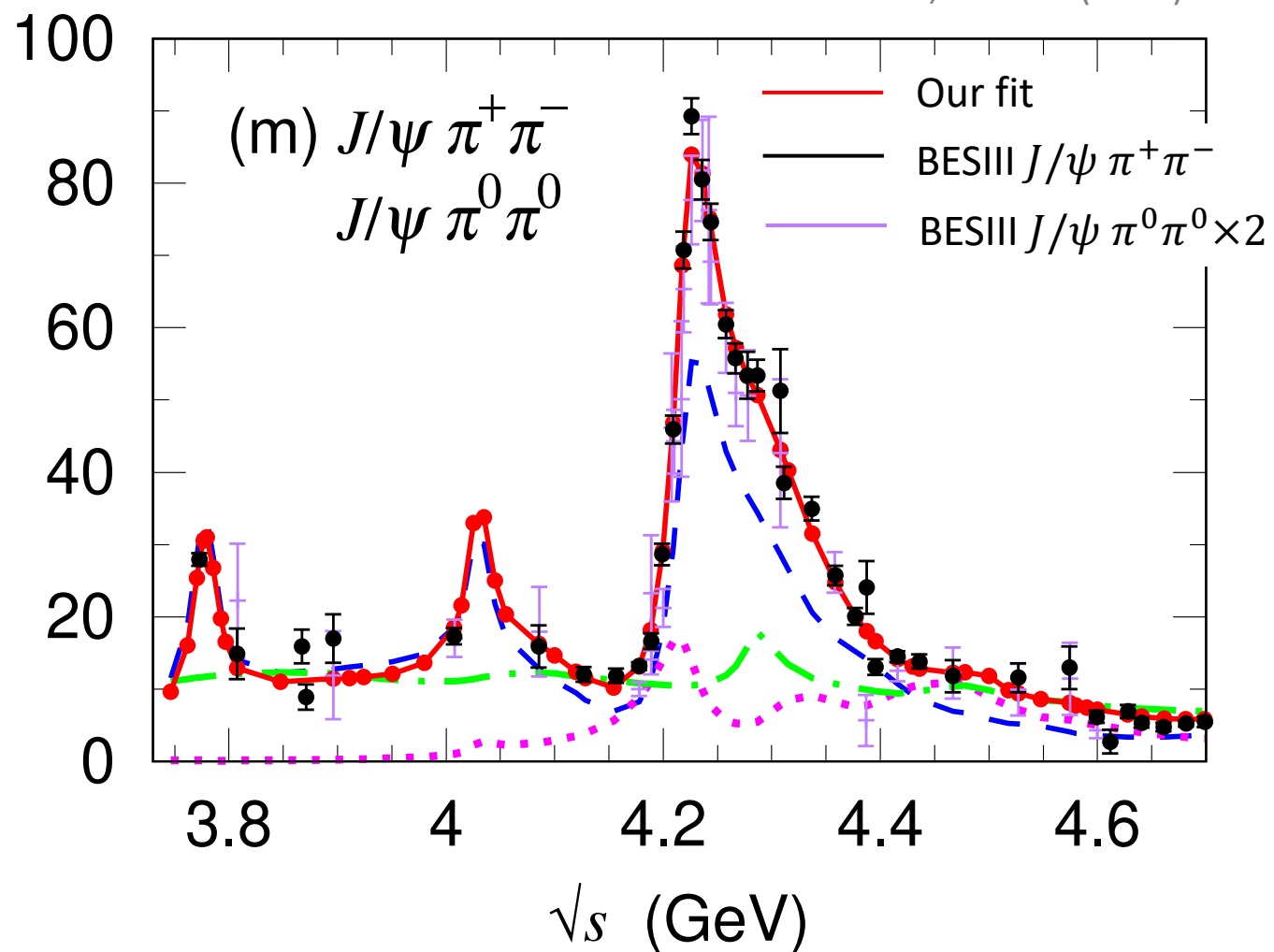
Because of including more (high precision) data,

177 (v2) \rightarrow 205 (v3) parameters

Selected fit results

$$e^+e^- \rightarrow J/\psi \pi^+\pi^-, J/\psi \pi^0\pi^0$$

Data: BESIII,
PRD 106, 072001 (2022)
PRD 102, 012009 (2020)



- Overall good agreement with data
our model is isospin symmetric
 $\sigma(J/\psi \pi^+\pi^-) = 2 \times \sigma(J/\psi \pi^0\pi^0)$
- Triangle singularity effect is seen
in NR contribution at $\sqrt{s} \sim 4.28$ GeV

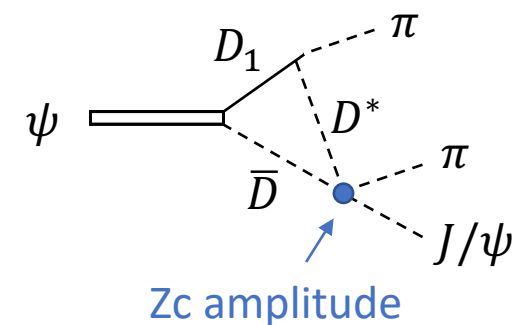
- Peaking structure at $\sqrt{s} \sim 4$ GeV is a consequence of the combined fit ($\psi(4040)$)

$$e^+e^- \rightarrow J/\psi \pi^+\pi^-$$

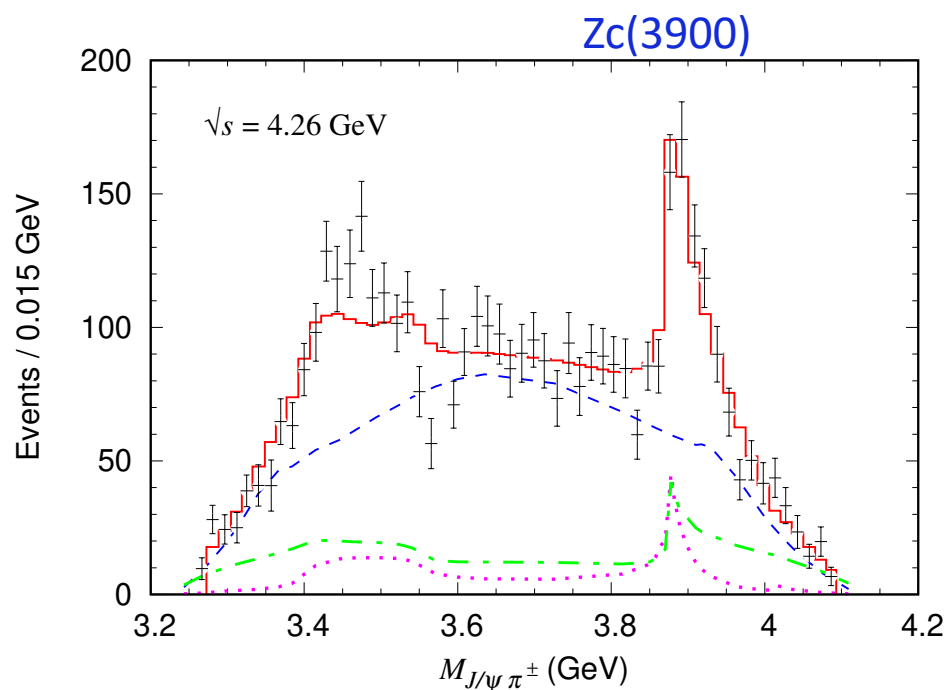
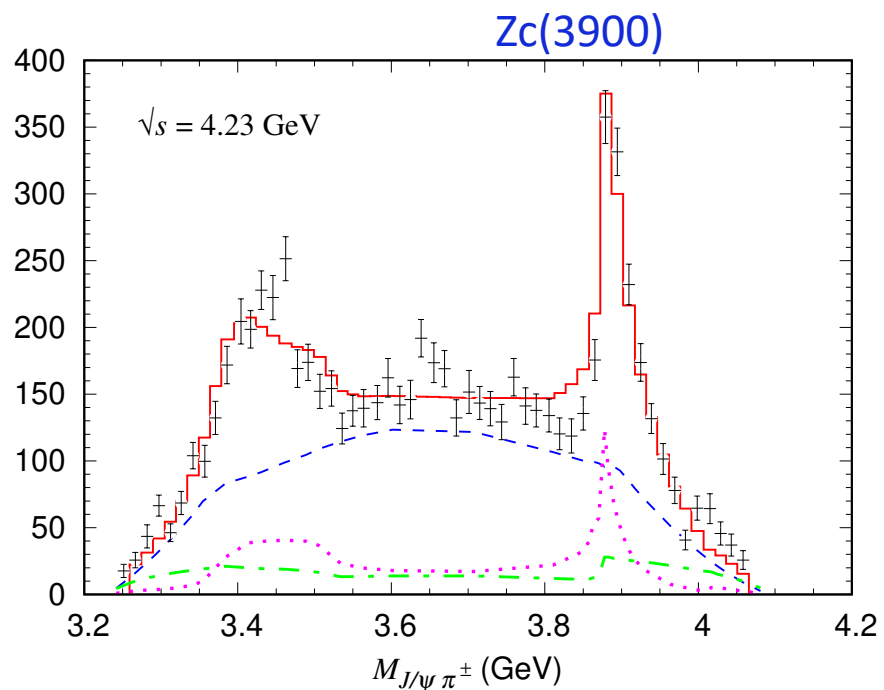
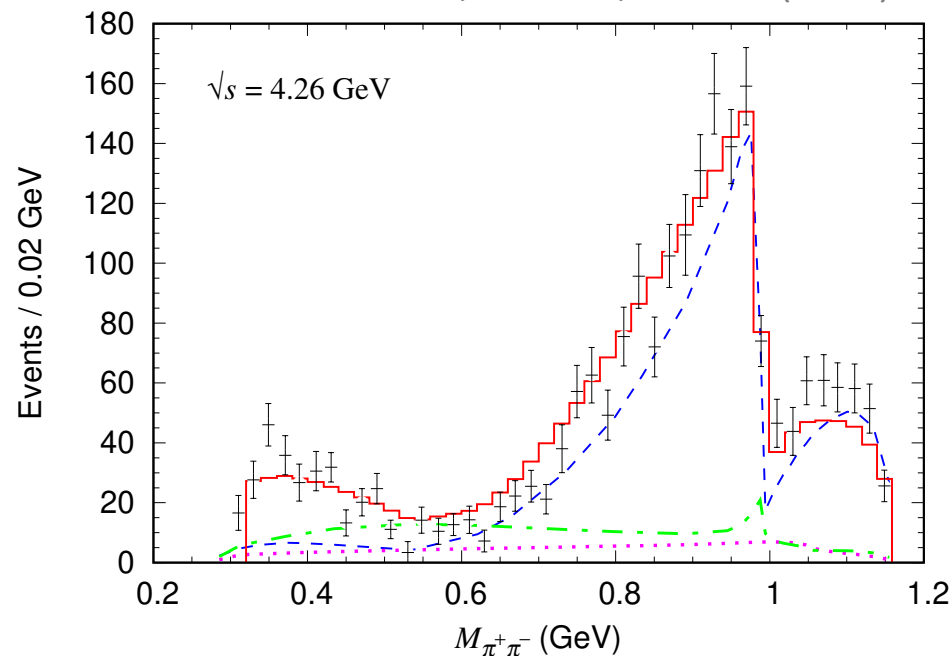
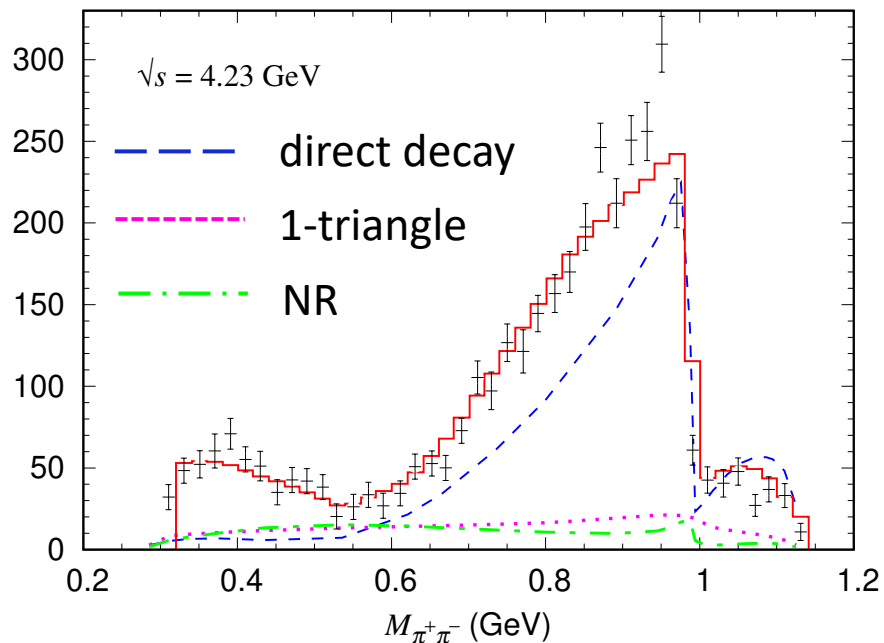
Fit to invariant masses

Zc(3900) peaks are well fitted

1-loop causes $D^*\bar{D}$ thresh. cusp
enhanced by a possible pole
(a bit off TS condition)

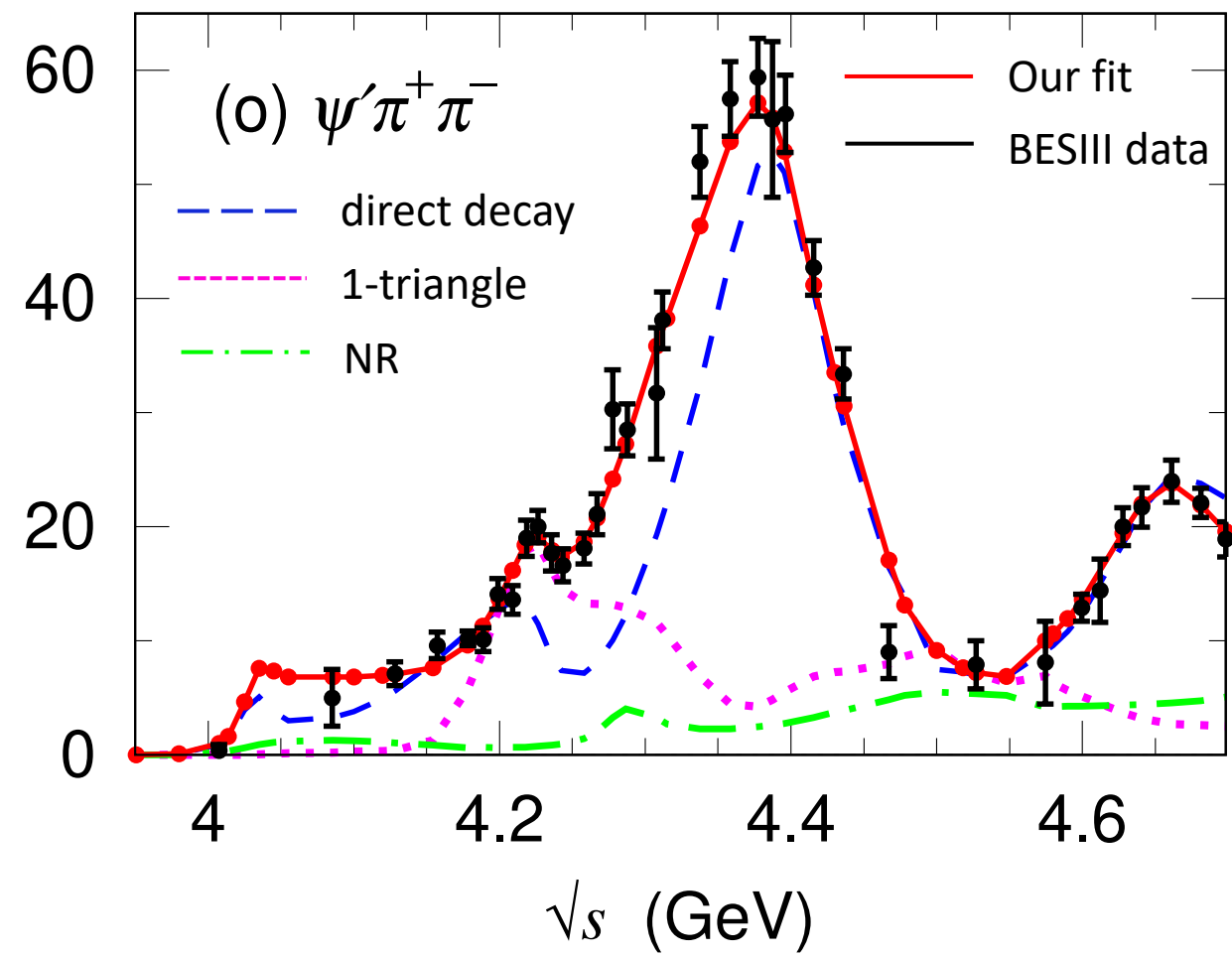


We will examine Zc(3900) pole

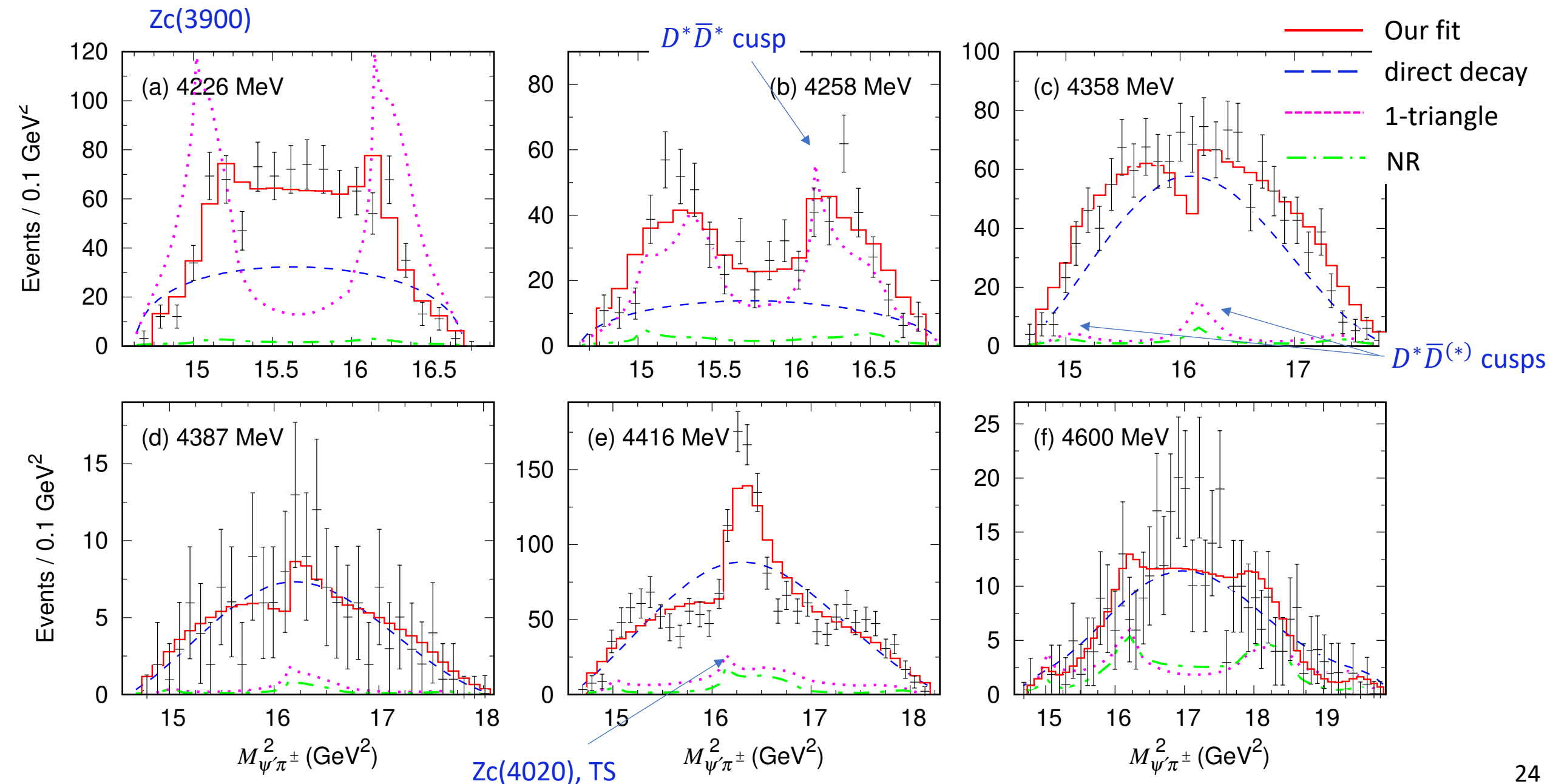


$$e^+e^- \rightarrow \psi' \pi^+ \pi^-$$

Data: BESIII, PRD 104, 052012 (2021)



- Overall good fit
- Enhancement at ~ 4.03 GeV is from $\psi(4040)$
 \leftarrow consequence of coupled-channel fit
- 1-triangle contribution is large at $\psi(4220)$ peak
- TS effect seen at ~ 4.28 GeV $\rightarrow D_1(2420)\bar{D}$ threshold in NR contribution

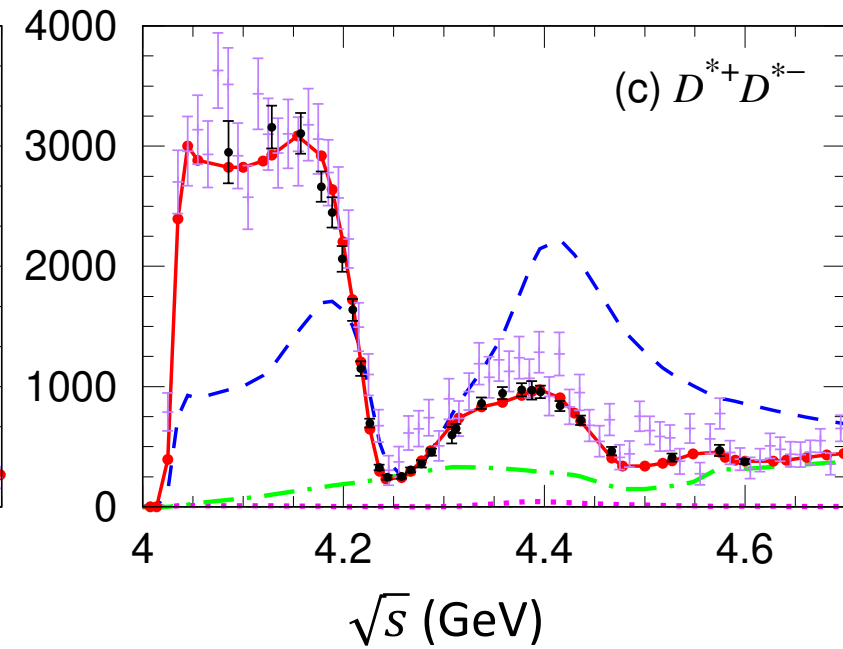
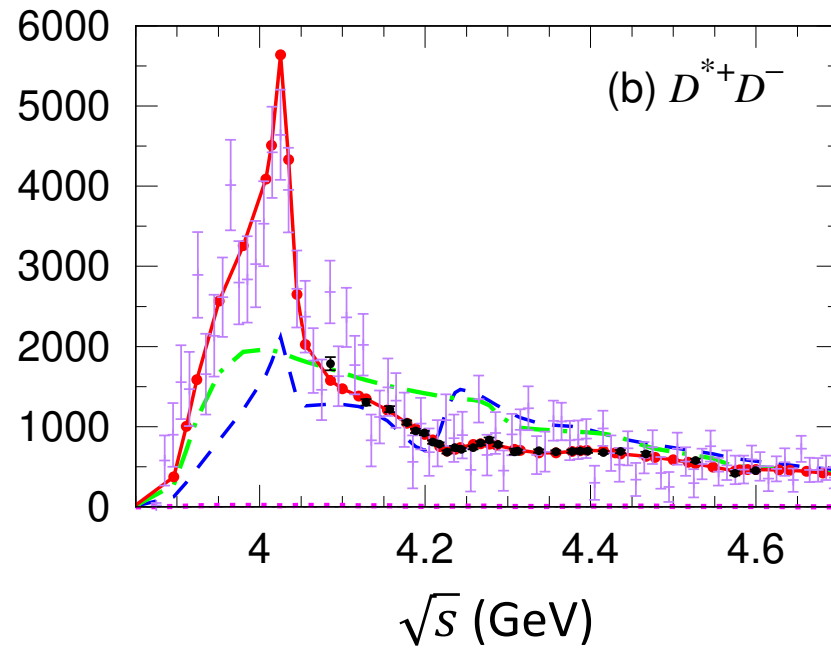
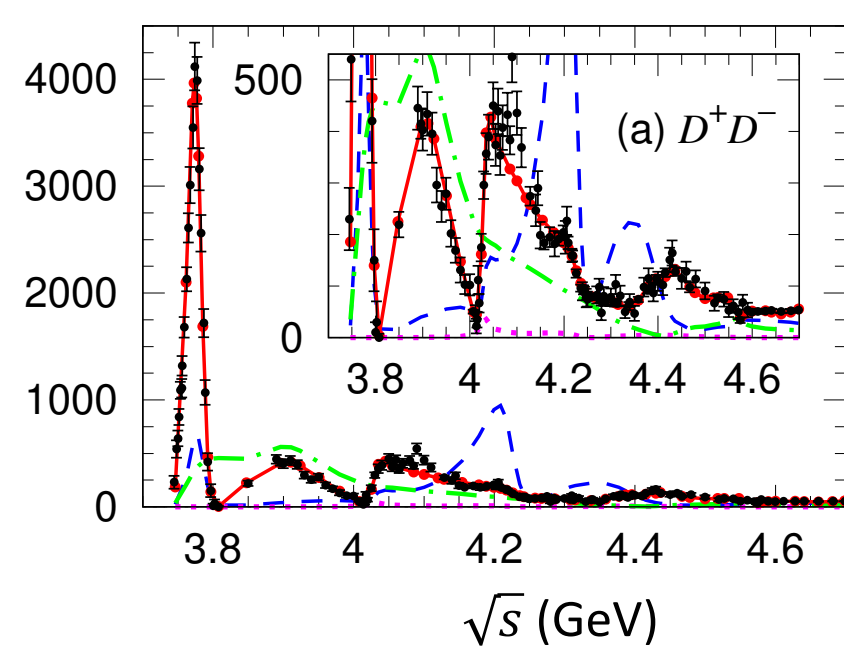


$$e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}$$

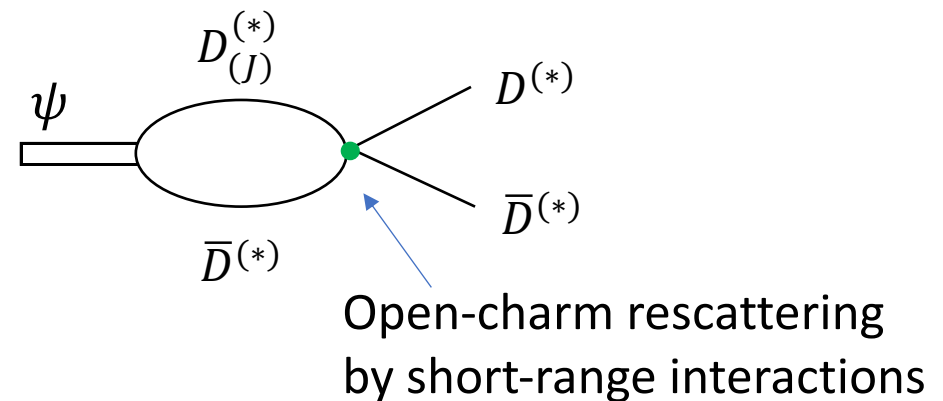
Data: BESIII, PRL 133, 081901 (2024)
JHEP 05 (2022) 155.
Belle, PRD 97, 012002 (2018)

— Our fit - - - direct decay
— BESIII data - - - 1-triangle
— Belle data - . - NR

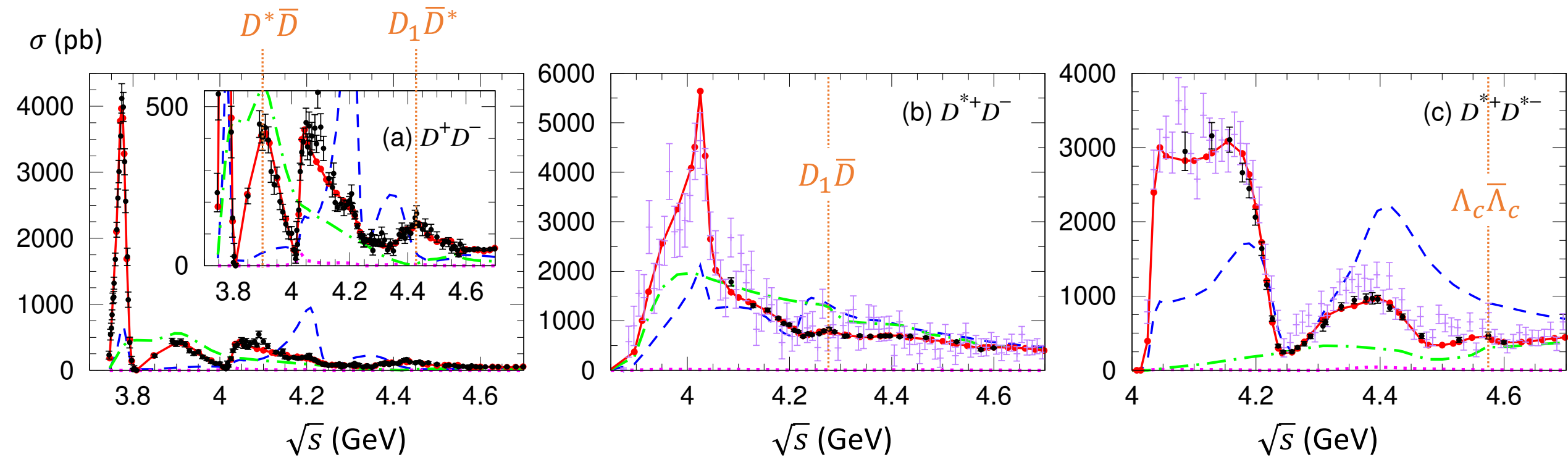
σ (pb)



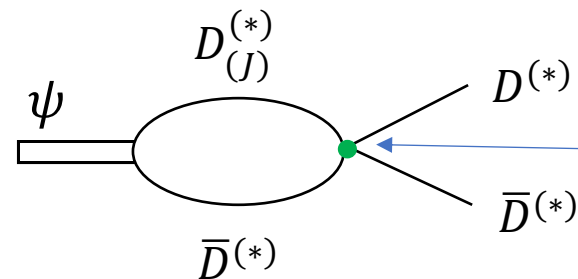
- *Precise* BESIII data are well fitted
- Contact interactions among open-charm channels important (difference between blue and red curves above)
- 1-triangle (particle exchange) is small



$$e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}$$



Fitting precise data, we need threshold cusps from



Open-charm rescattering
by short-range interactions

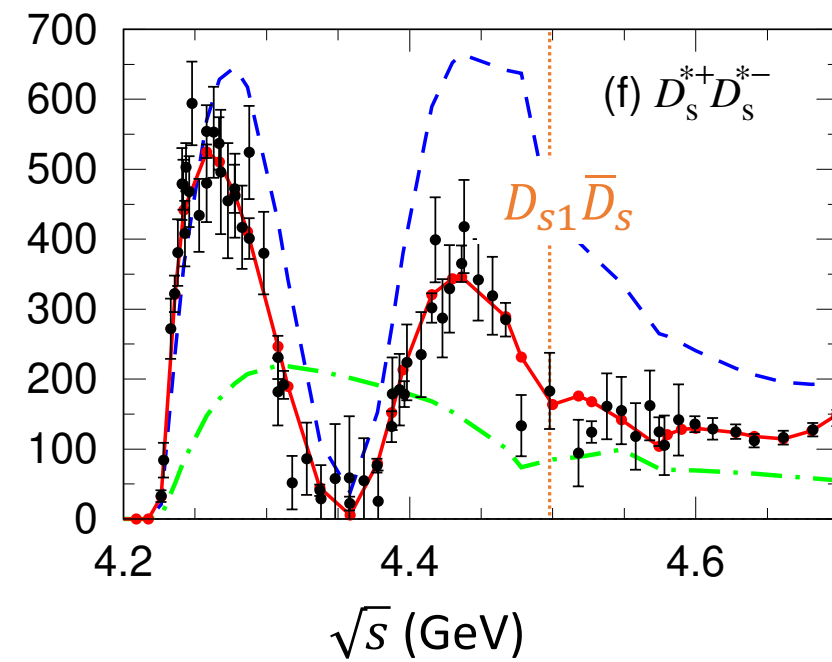
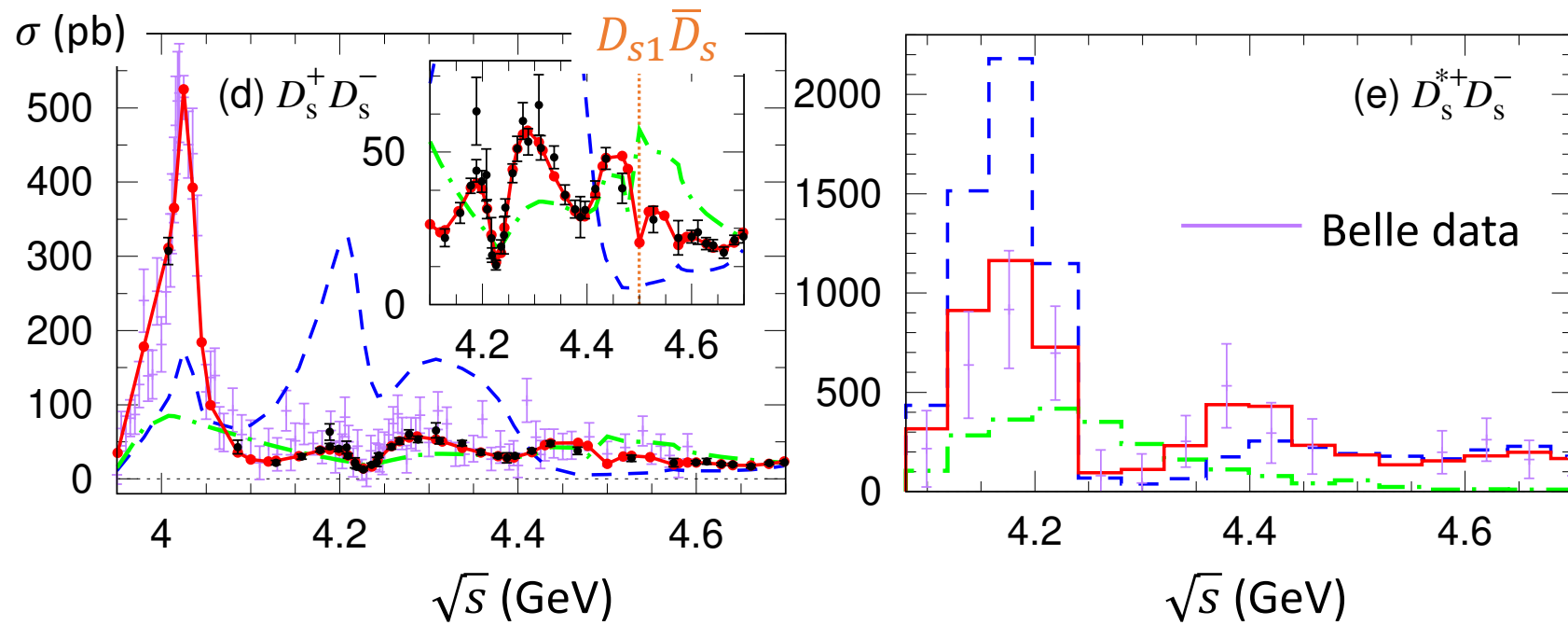
Fitting cusps \rightarrow good constraints on interactions among open-charm channels

\rightarrow good constraints on existence of $D_{(J)}^{(*)}\bar{D}^{(*)}$ molecules

$$e^+e^- \rightarrow D_s^{(*)}\bar{D}_s^{(*)}$$

— Our fit - - - direct decay
 — BESIII data - · - · - NR
 — BESIII Rscan

Data: BESIII, PRL 131, 151903 (2023)
 arXiv:2403.14998
 Belle, PRD 83, 011101 (2011)



- *Precise* BESIII data are well fitted
- Contact interactions among open-charm channels important (difference between blue and red curves above)
- $D_{s1}\bar{D}_s$ threshold cusps included to fit data

$$e^+e^- \rightarrow \Lambda_c \bar{\Lambda}_c, \pi D^* \bar{D}^*$$

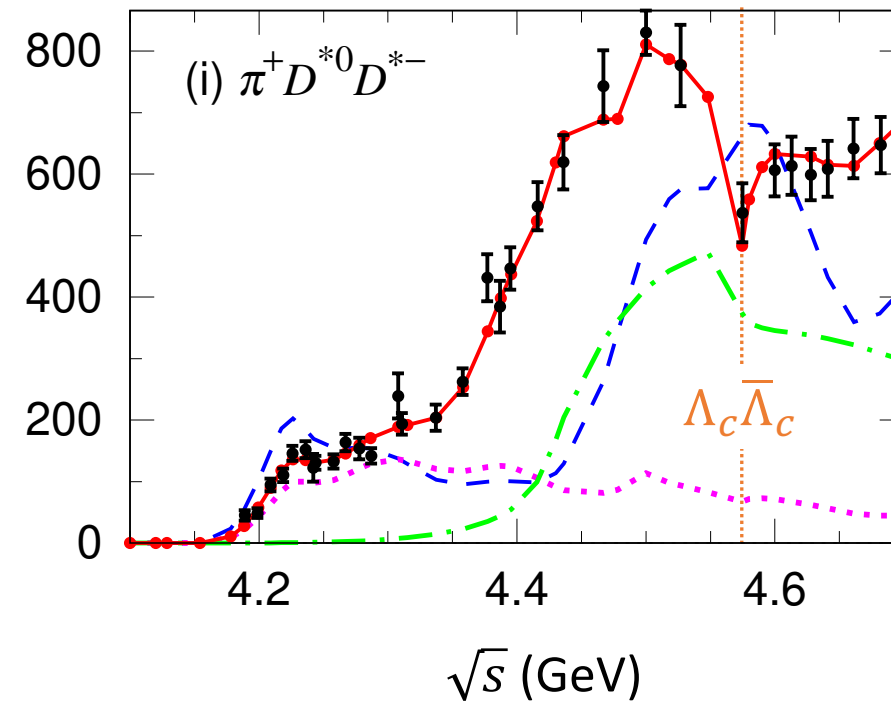
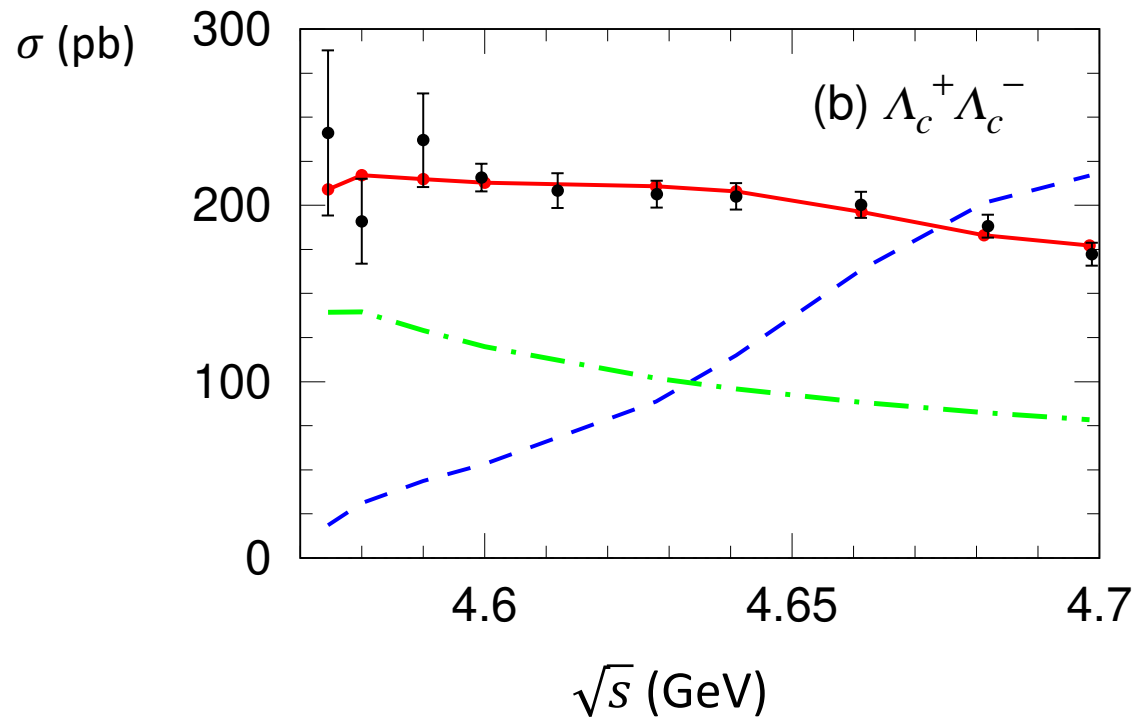
— Our fit

--- direct decay

--- 1-triangle

--- NR

Data: BESIII,
PRL 120, 132001 (2018)
PRL 131, 191901 (2023)
PRL 130, 121901 (2023)



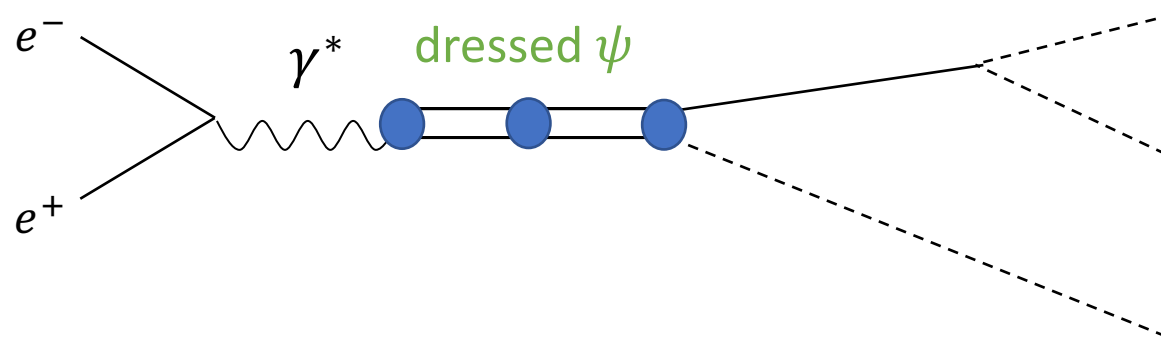
- Non-zero $e^+e^- \rightarrow \Lambda_c \bar{\Lambda}_c$ cross section at threshold \leftarrow Sommerfeld factor
- $\Lambda_c \bar{\Lambda}_c$ threshold enhancement \leftarrow attractive $\Lambda_c \bar{\Lambda}_c$ interaction (likely virtual pole near threshold)
- $\Lambda_c \bar{\Lambda}_c$ threshold cusp is important to fit $e^+e^- \rightarrow \pi D^* \bar{D}^*$ data at $\sqrt{s} \sim 4.57$ GeV

Poles and resonance properties

ψ poles from their dressed propagator

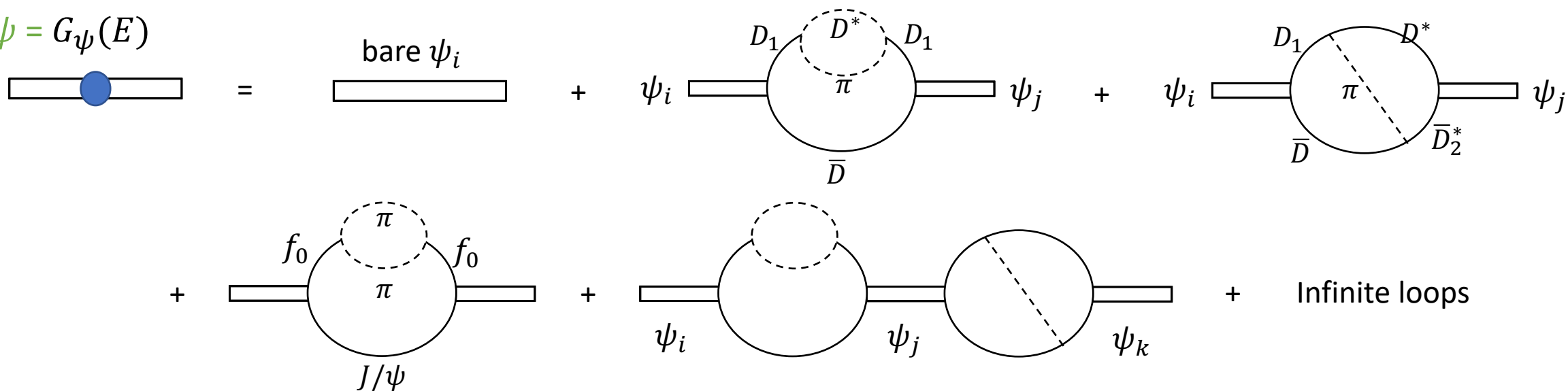
(we are not using BW)

Full amplitude



+ non-resonant

dressed $\psi = G_\psi(E)$



Search complex energy E_ψ where $G_\psi(E_\psi) = \infty$ (E_ψ : pole energy, pole position) by analytical continuation of $G_\psi(E)$

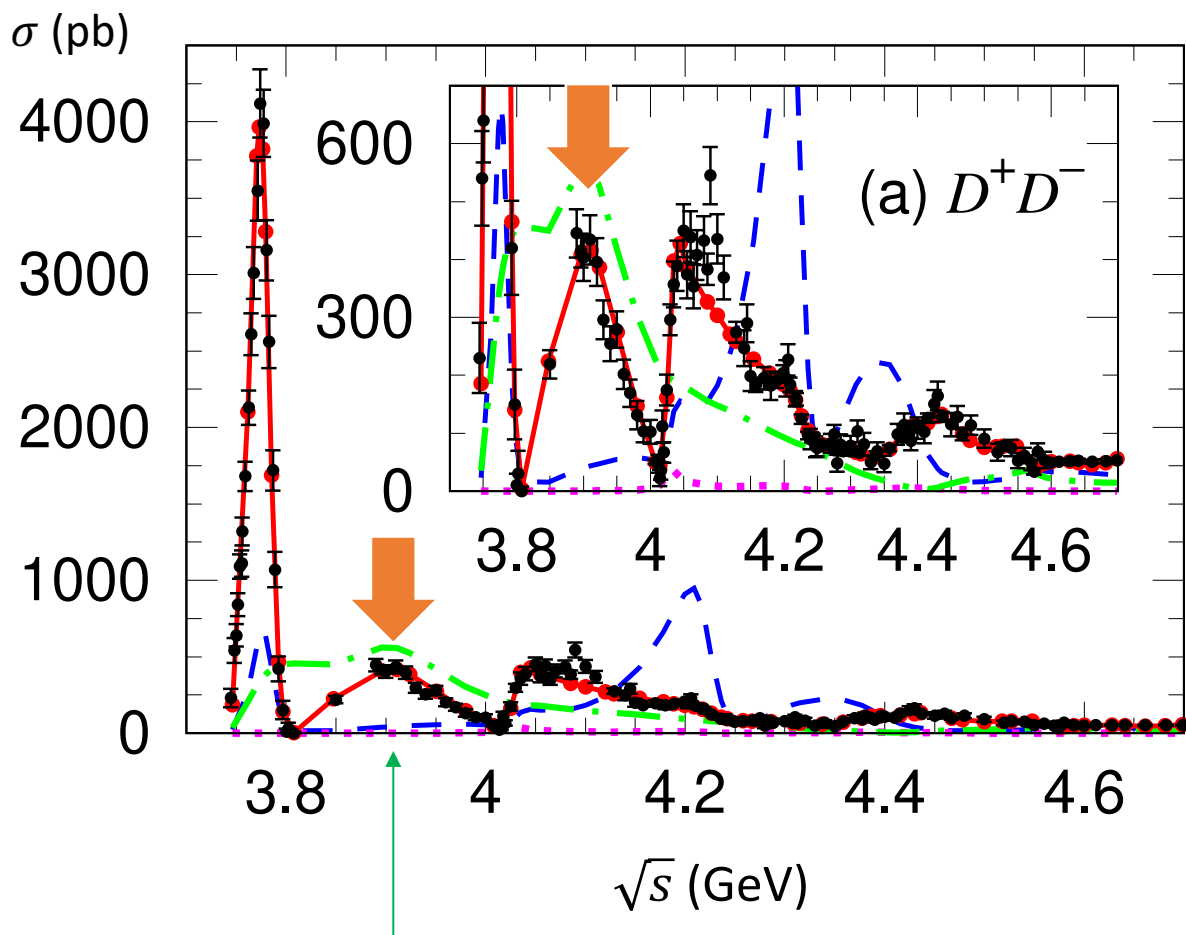
Resonance parameters (uncertainties tentative)

This work		PDG			$M = \text{Re}[E_\psi]$ $\Gamma = -2 \times \text{Im}[E_\psi]$
M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)		
3780 ± 0.5	30 ± 1.7	3778.1 ± 0.7	27.5 ± 0.9	$\psi(3770)$	7 poles from 5 bare states
4029 ± 0.3	28 ± 0.7	4039 ± 1	80 ± 10	$\psi(4040)$	
4188 ± 1.8	127 ± 2.9	4191 ± 5	70 ± 10	$\psi(4160)$	
4228 ± 0.7	44 ± 1.2	4222.5 ± 2.4	48 ± 8	$\psi(4230)$	
4306 ± 2.6	129 ± 1.9	$4298 \pm 12 \pm 26$	$127 \pm 17 \pm 10$	$Y(4320)$	← Not in PDG
4354 ± 3.1	123 ± 3.4	4374 ± 7	118 ± 12	$\psi(4360)$	
4388 ± 1.5	107 ± 3.3	4421 ± 4	62 ± 20	$\psi(4415)$	
4655 ± 1.8	135 ± 3.5	4630 ± 6	72^{+14}_{-12}	$\psi(4660)$	← BW fit

No Y-width puzzle Number of poles from our analysis is consistent with PDG + Y(4320)

Noticeable differences from PDG Mass : $\psi(4415)$
Width: $\psi(4040), \psi(4160), \psi(4415), \psi(4660)$

$\psi(4160)$ mass: 4232 MeV in our previous analysis → more precise data included → 4188 MeV now



resonance-like peak at 3.9 GeV called G(3900)

G(3900) peak in our model

Interference between $\psi(3770)$, $\psi(4040)$ and non-resonant amplitudes + $D^*\bar{D}$ threshold cusp

G(3900) state claimed in BESIII analysis of $e^+e^- \rightarrow D\bar{D}$

BESIII, PRL 133, 081901 (2024)

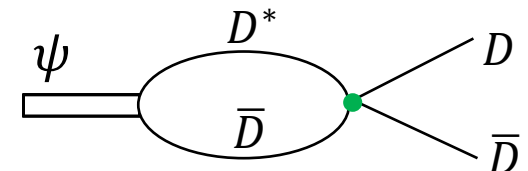
G(3900) state predicted by meson-exchange model

Z.-Y. Li et al. 2403.01727; PRL

No G(3900) pole by K-matrix analysis of $e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}$

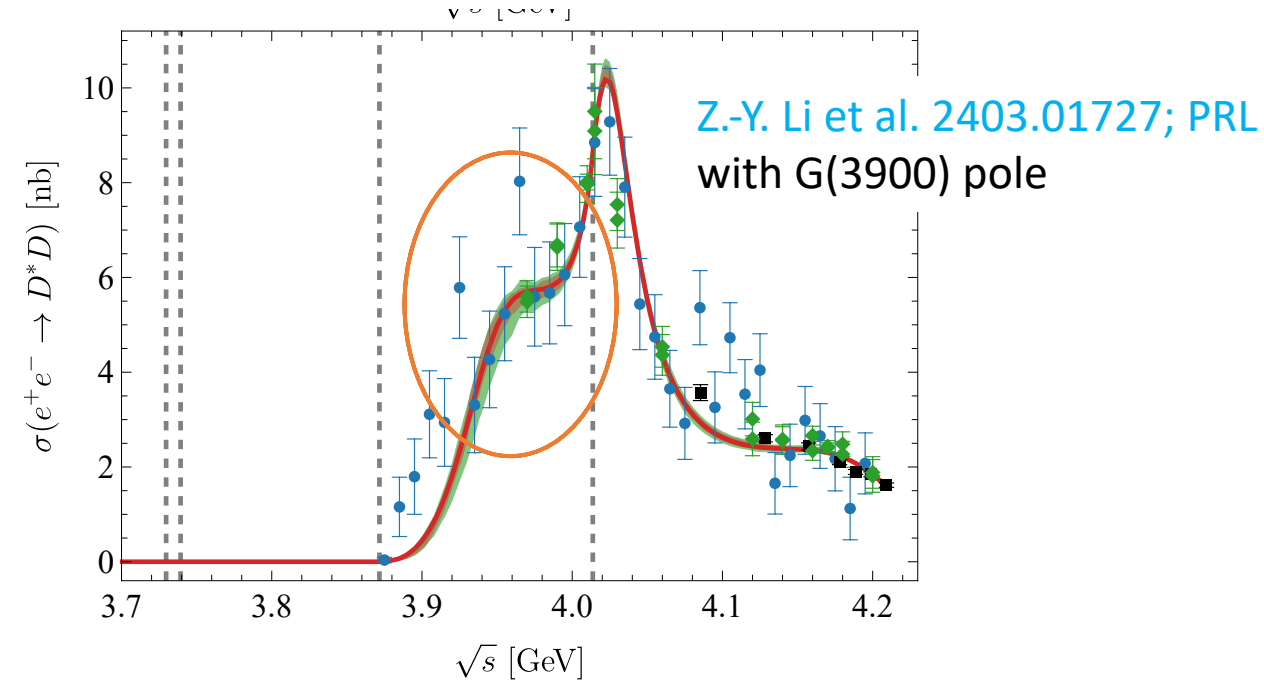
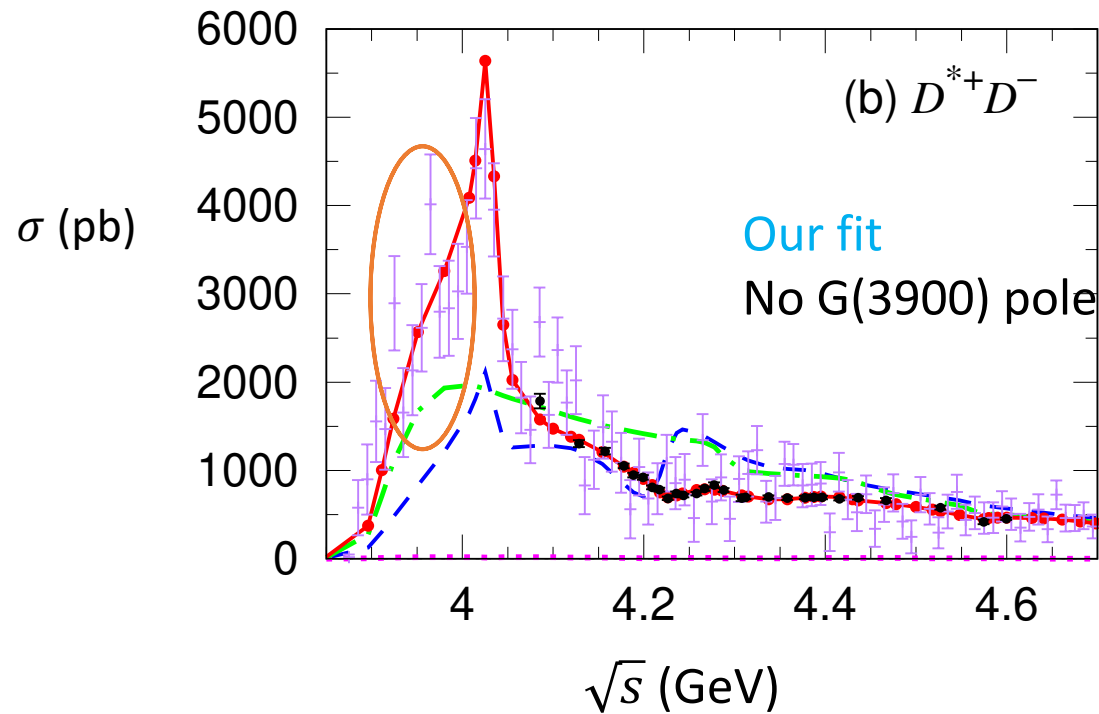
N. Husken et al., PRD 109, 11401 (2024)

No G(3900) pole from our analysis



How to pin-down existence of G(3900) ?

$e^+e^- \rightarrow D^*\bar{D}$ near threshold



Visible difference between two fits \rightarrow G(3900) effect ?

Higher precision data from BESIII could help pin-down existence of G(3900)

Is $Y(4220) D_1 \bar{D}$ molecule ?

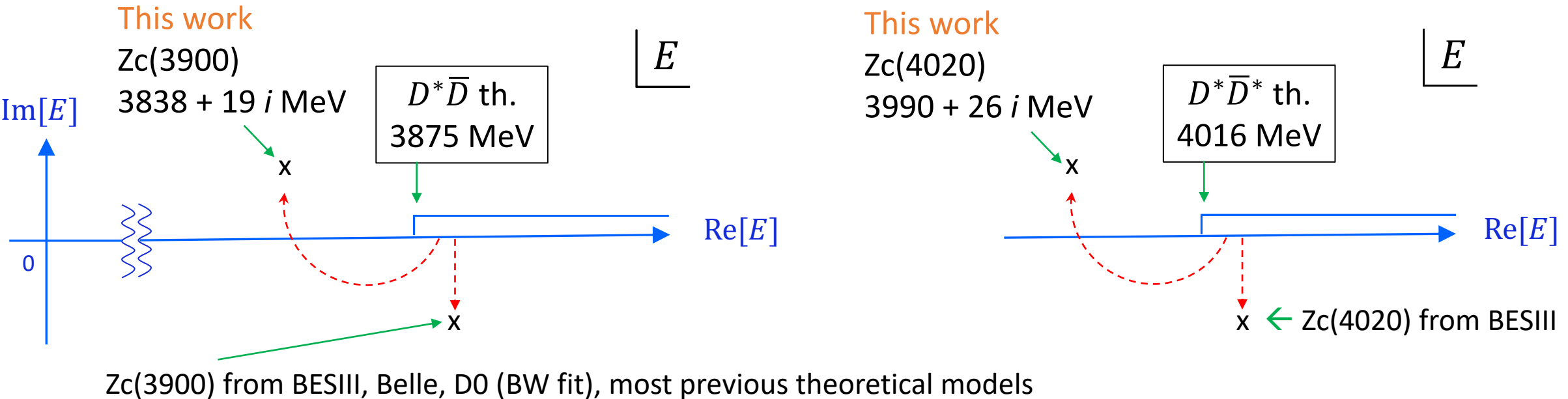
Is $Y(4360) D_1 \bar{D}^*$ molecule ?

→ To be examined soon

Zc poles

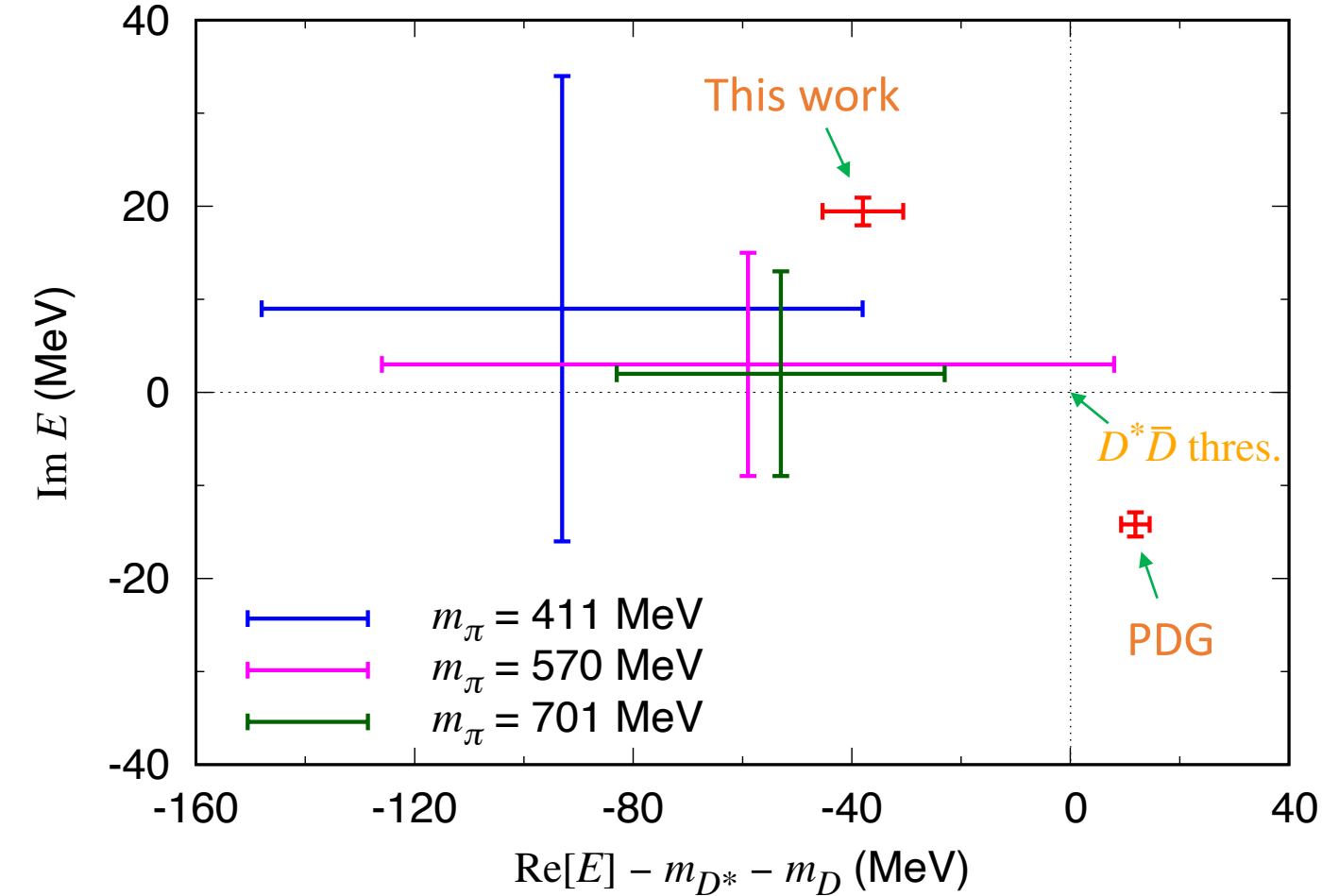
from $J^{PC} = 1^{+-} \quad D^*\bar{D} - D^*\bar{D}^* - J/\psi\pi - \psi'\pi - h_c\pi - \eta_c\rho$ couple—channel amplitude

This work	PDG [4]			RS=($s_{D^*\bar{D}}, s_{D^*\bar{D}^*}$)
E_{pole} (MeV)	RS	M (MeV)	Γ (MeV)	
$(3838 \pm 7) + (19 \pm 1)i$	<i>up</i>	3887.1 ± 2.6	28.4 ± 2.6	$Z_c(3900)$
$(3990 \pm 5) + (26 \pm 4)i$	<i>pu</i>	4024.1 ± 1.9	13 ± 5	$Z_c(4020)$



Zc from our analysis are virtual states, different from Breit-Wigner fit and most of previous theoretical analyses

$Z_c(3900)$ pole: comparison with LQCD result



$Z_c(3900)$ pole positions in $D^* \bar{D}$ unphysical sheet

LQCD ($m_\pi = 411$ MeV)

HAL QCD, J. Phys. G 45, 024002 (2018)

$m_{D^*} + m_D - (93 \pm 55 \pm 21) + (9 \pm 25 \pm 7)i$ MeV

$S(\{-k_i^*\}) = S^*({k_i})$ applied; PRD 105, 014034 (2022)

This work

$m_{D^*} + m_D - (38 \pm 7) + (19 \pm 1)i$ MeV

PDG

$m_{D^*} + m_D + (11.9 \pm 2.6) - (14.2 \pm 1.3)i$ MeV

LQCD and this work are fairly consistent (virtual poles)

Summary and perspective

Summary

- Conducted global coupled-channel analysis of most of available $e^+e^- \rightarrow c\bar{c}$ data in $\sqrt{s} = 3.75 - 4.7$ GeV
Global coupled-channel analysis is common for N^* . The $e^+e^- \rightarrow c\bar{c}$ analysis now gets closer to the standard !
- Reasonable fits are obtained overall
- Vector charmonium and Z_c poles extracted
 - 7 poles from 5 bare states; # of poles consistent with PDG + $Y(4320)$; no $G(3900)$ pole
 - Z_c poles are virtual poles at ~ 40 MeV below $D^*\bar{D}^{(*)}$ thresholds, consistent with LQCD results

Future

- Pole residues will be extracted \rightarrow address Y width problem, structure of exotic candidates Y
- Fit efficiency-corrected, background-free Dalitz plots (not 1D fit) to fully consider experimental constraints on charmonium and Z_c properties
- Include $e^+e^- \rightarrow K\bar{D}_s^{(*)}D^{(*)}$ cross sections when available \rightarrow include higher charmonium states
 \rightarrow address $Z_{cs}(3985)$ from global analysis

Backup

Previous coupled-channel analyses for Y-width puzzle

Three-body model

* M. Cleven, Q. Wang, F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Zhao, PRD 90, 074039 (2014)

Analysis of $e^+e^- \rightarrow \pi D\bar{D}^*$, $J/\psi\pi\pi$, $h_c\pi\pi$ cross section and invariant mass in $4.1 \lesssim \sqrt{s} \lesssim 4.3$ GeV [Y(4230) region]

Pioneering works, but the data were very limited

* L. Detten, C. Hanhart, V. Baru, Q. Wang, D. Winney, Q. Zhao, PRD 109, 116002 (2024)

Fitting data in Y(4230) region; more final states than the above; Y(4230) as $D_1\bar{D}$ molecule claimed

Breit-Wigner fit to cross section data

* D.-Y. Chen, X. Liu, T. Matsuki, Eur. Phys. J. C 78, 136 (2018)

Fitting $e^+e^- \rightarrow \pi D\bar{D}^*$, $J/\psi\pi\pi$, $h_c\pi\pi$ cross sections \rightarrow Y(4320) and Y(4390) unnecessary

Two-body unitary model fitted to cross section data

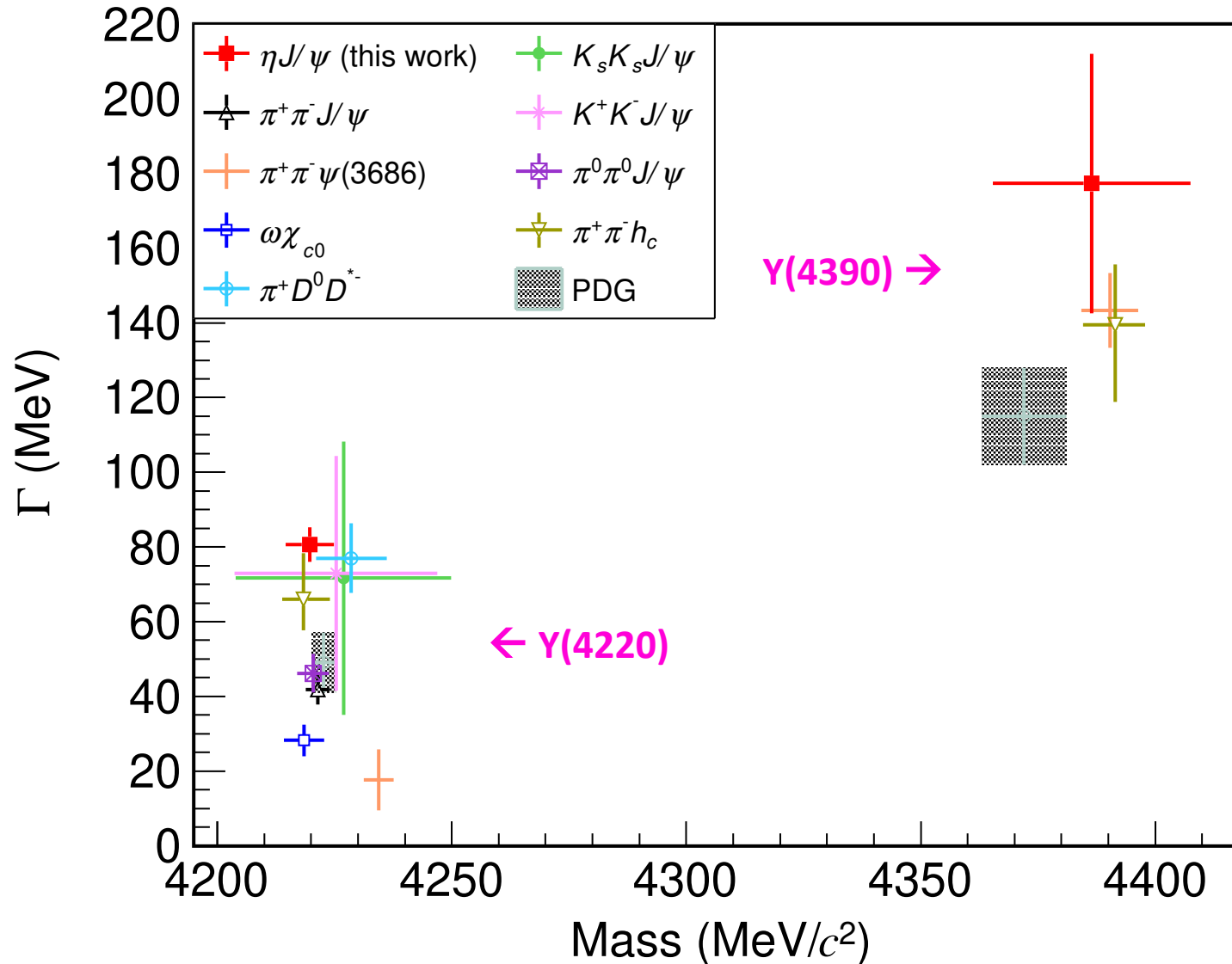
* Z.-Y. Zhou, C.-Y. Li, Z. Xiao, arXiv:2304.07052

Fitting $e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}$, $\pi D\bar{D}$ cross sections \rightarrow $\psi(4160)$ is Y(4230)

Our analysis

- more complete dataset
- more coupled-channels
- three-body unitary
- \rightarrow more reliable conclusion

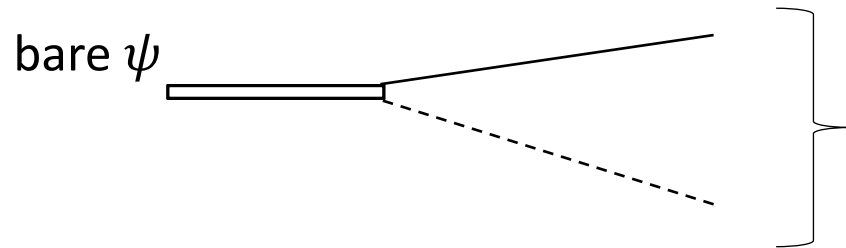
Outstanding question in XYZ physics : Υ width problem



BESIII, arXiv:2310.03361

ψ decays (bare vertices)

(quasi) two-body channels included; $J^{PC} = 1^{--}$

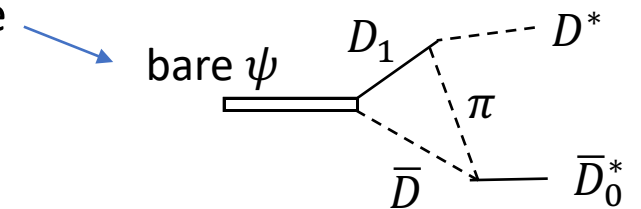


$$\underline{D_0^*(2300)\bar{D}^*}, f_0 J/\psi, f_2 J/\psi, f_0 \psi', f_0 h_c, \underline{Z_c \pi}, \underline{Z_{cs} \bar{K}}$$

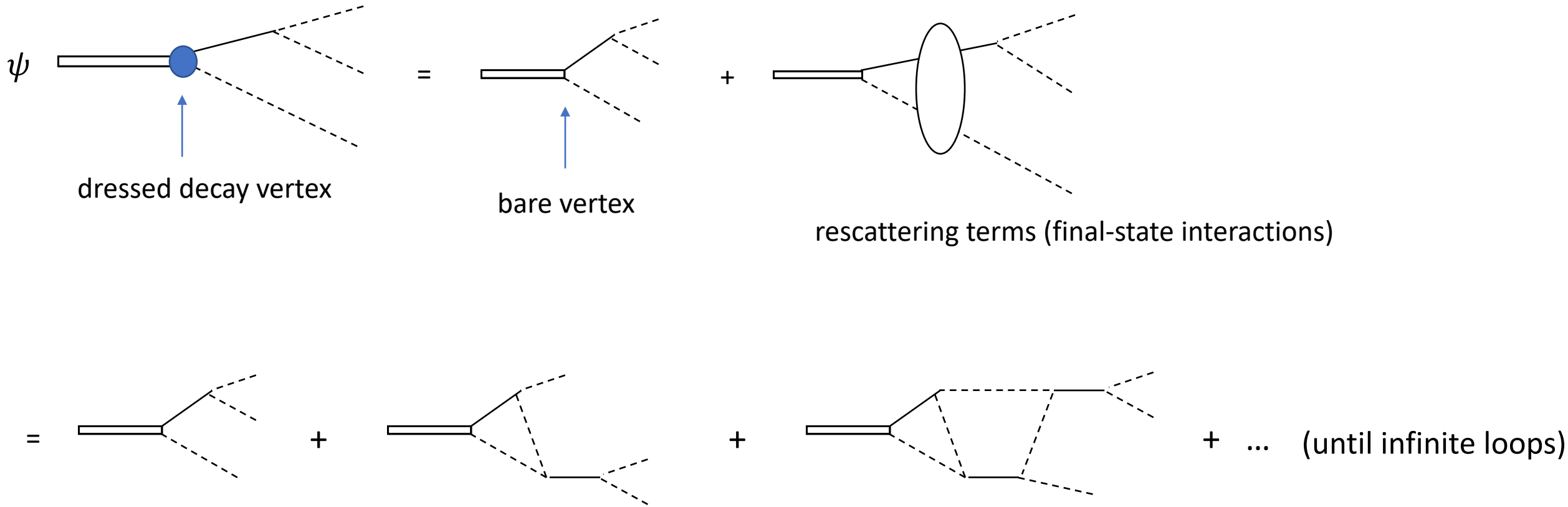
We do not include “bare $\psi \rightarrow D_0^* \bar{D}^*, Z_c \pi, Z_{cs} \bar{K}$ ”

bare ψ dominantly decays to two-body states; D_0^* and Z_c are probably not compact states

$D_0^* \bar{D}^*$ and $Z_c \pi$ channels are generated by coupled-channel effect like



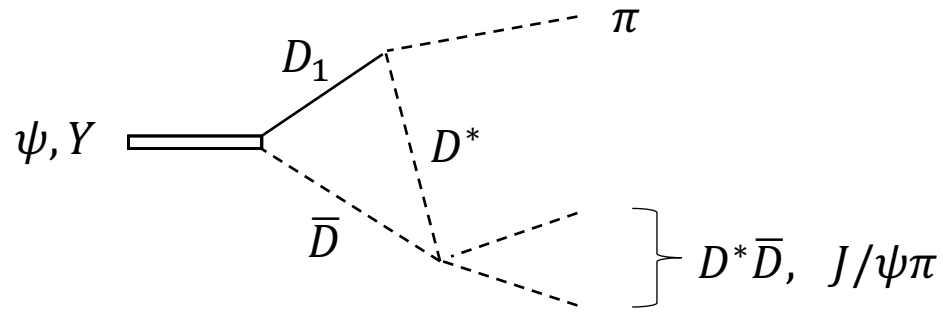
Three-body decays of ψ



Final state interactions described by solution of Faddeev equation → Coupled-channels taken into account

Rescattering mechanisms (particle exchange) required by three-body unitarity are considered

Triangle singularity (TS) from our model



Kinematical condition for TS

Energy-momentum is conserved everywhere as classical process

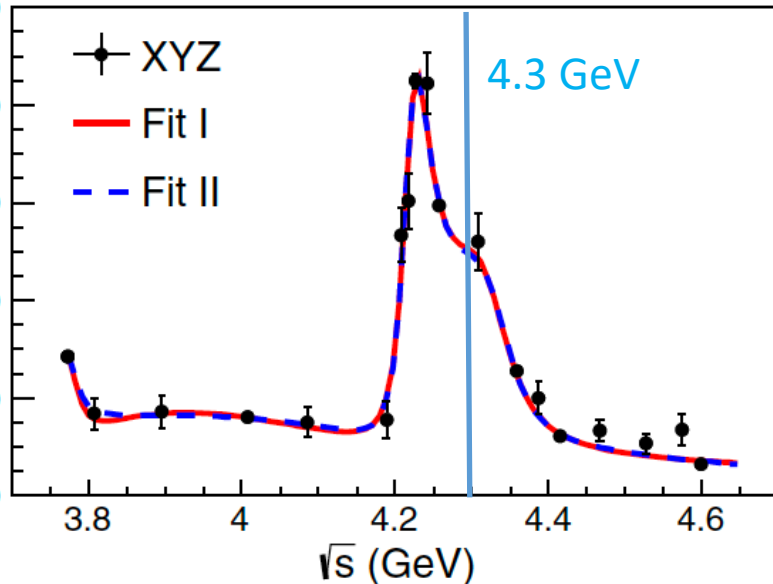
→ amplitude is significantly enhanced at

$$\sqrt{s} \sim m_{D_1} + m_{\bar{D}} \quad (\sim 4.3 \text{ GeV}) \quad \text{and} \quad M_{D^* \bar{D}} \sim m_{D^*} + m_{\bar{D}} \quad (\sim 3.88 \text{ GeV})$$

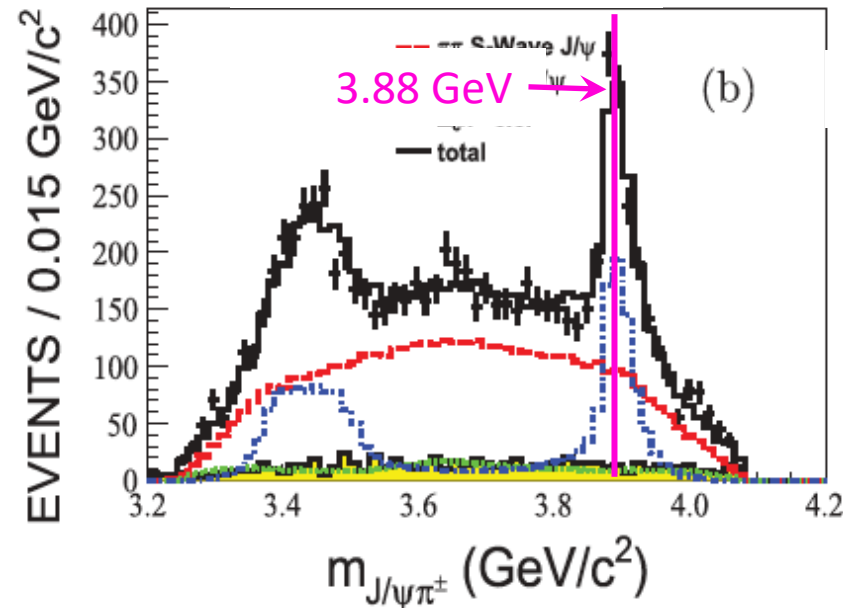
$$M_{J/\psi \pi}$$

Data show coincidence of $\Upsilon(4320)$, Z_c , and TS

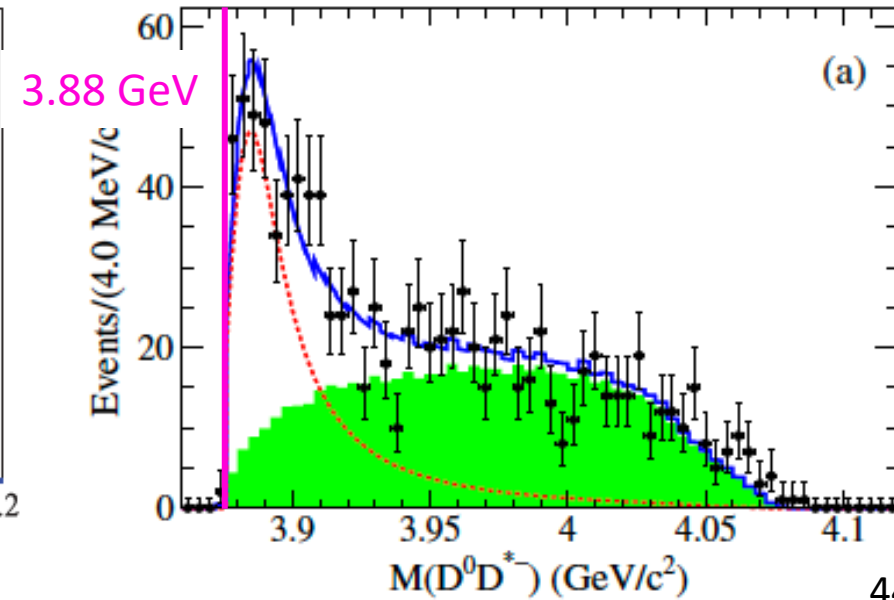
$$e^+e^- \rightarrow J/\psi \pi^+ \pi^-$$



$$e^+e^- \rightarrow J/\psi \pi^+ \pi^-$$

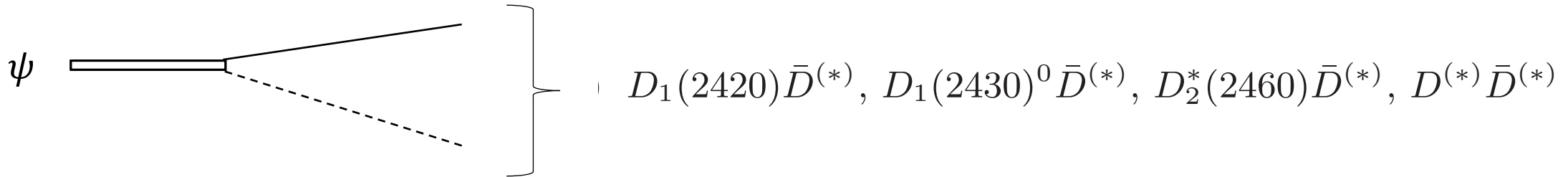


$$e^+e^- \rightarrow \pi^+ D^0 D^{*-}$$



Coupled-channels

(quasi) two-body channels included; $J^{PC} = 1^{--}$



$D_1(2420)$, $D_1(2430)$, $D_2^*(2460)$, $D^* \rightarrow$ Breit-Wigner (BW) propagators; mass and width from PDG

$D_J^{(*)} \rightarrow D^{(*)}\pi$ coupling strength is determined, assuming the following decays saturate the width

$D_1(2420) \rightarrow D^*\pi$ (mainly d-wave decay); small s-wave coupling fixed by helicity angle distribution data

$D_1(2430) \rightarrow D^*\pi$ (s-wave decay)

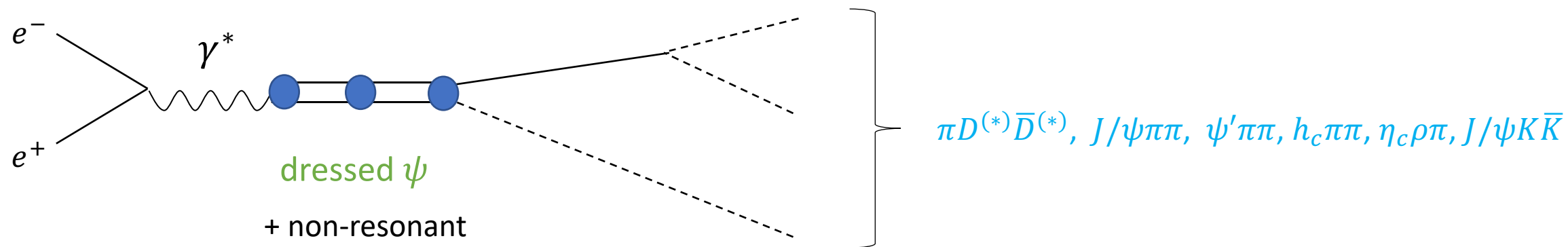
Babar, PRD 82, 111101 (2010)

$D_2^*(2460) \rightarrow D^*\pi + D\pi$; $\Gamma(D\pi)/\Gamma(D^*\pi) \sim 1.5$

$D^{*+} \rightarrow D\pi$

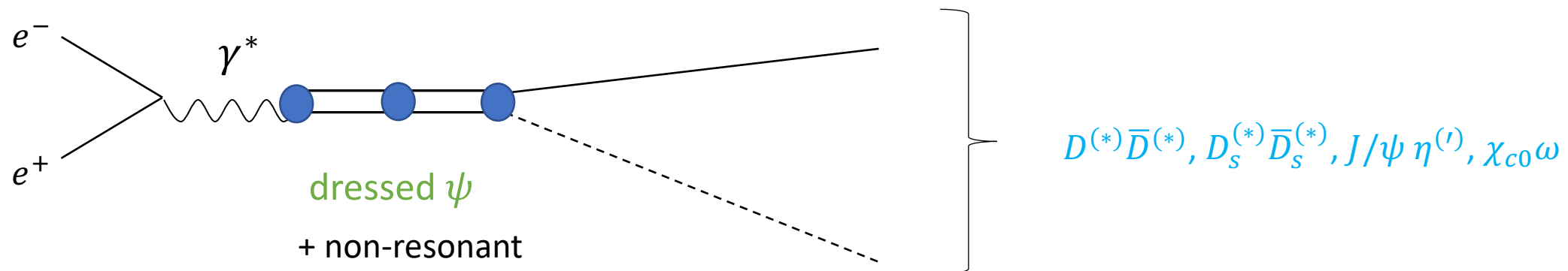
Full amplitude for three-body final states

$$e^+e^- \rightarrow \pi D^{(*)} \bar{D}^{(*)}, J/\psi \pi \pi, \psi' \pi \pi, h_c \pi \pi, \eta_c \rho \pi, J/\psi K \bar{K}$$

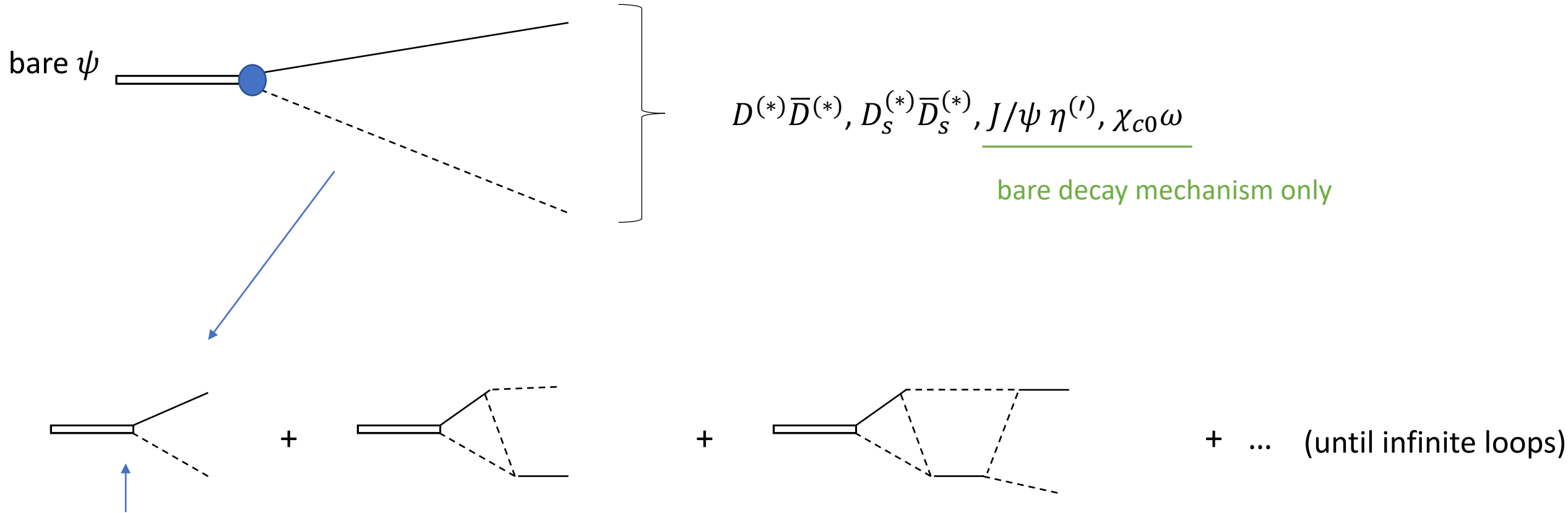


Full amplitude for two-body final states

$$e^+e^- \rightarrow D^{(*)} \bar{D}^{(*)}, D_s^{(*)} \bar{D}_s^{(*)}, J/\psi \eta^{(\prime)}, \chi_{c0} \omega$$

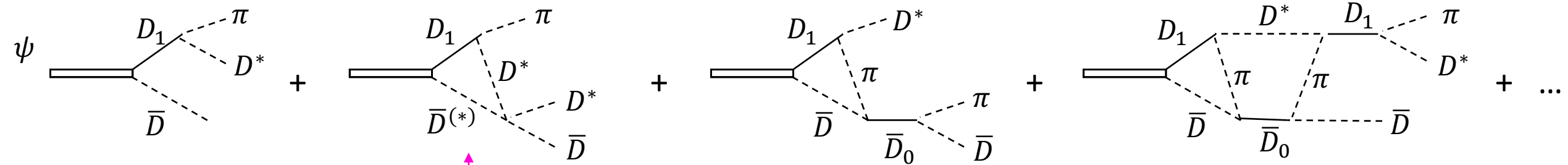


Two-body decay processes of ψ and Y



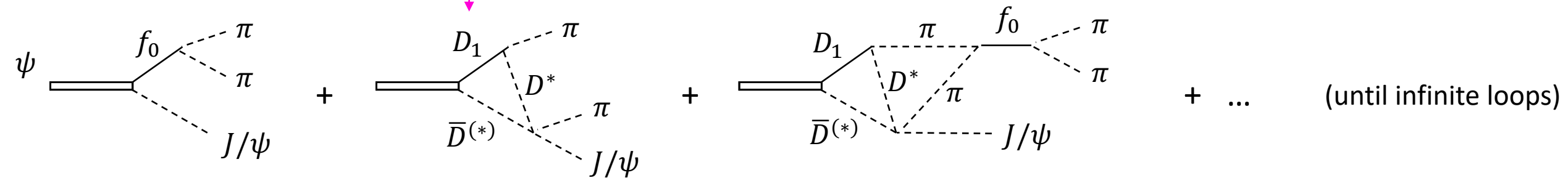
Three-body decays of ψ (some selected diagrams)

$$e^+e^- \rightarrow \pi D^* \bar{D}$$



(until infinite loops)

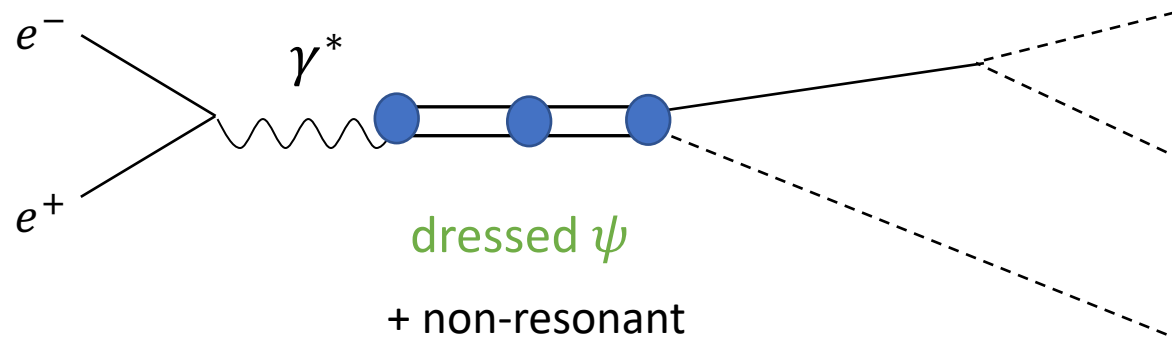
$$e^+e^- \rightarrow J/\psi \pi \pi$$



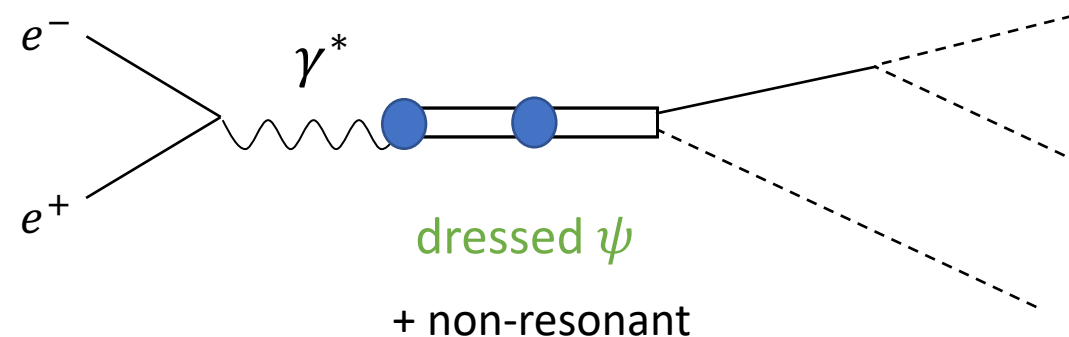
(until infinite loops)

Different processes share the same interactions \leftarrow unitarity requirement

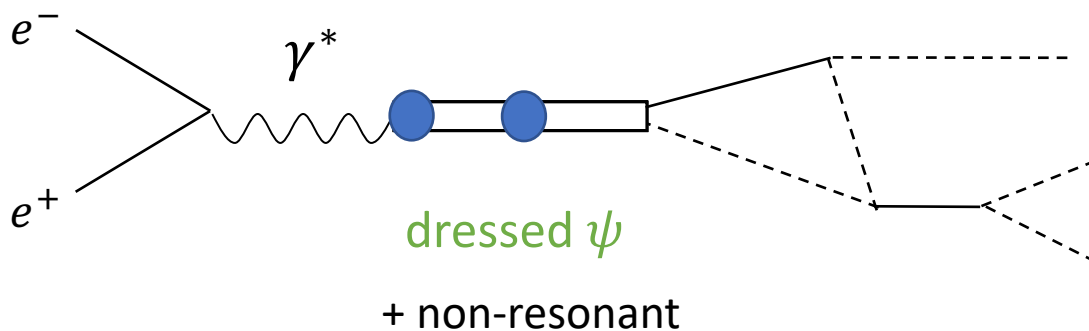
Full amplitude



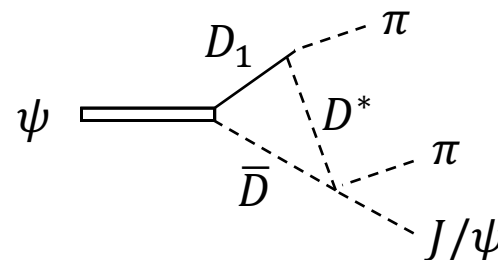
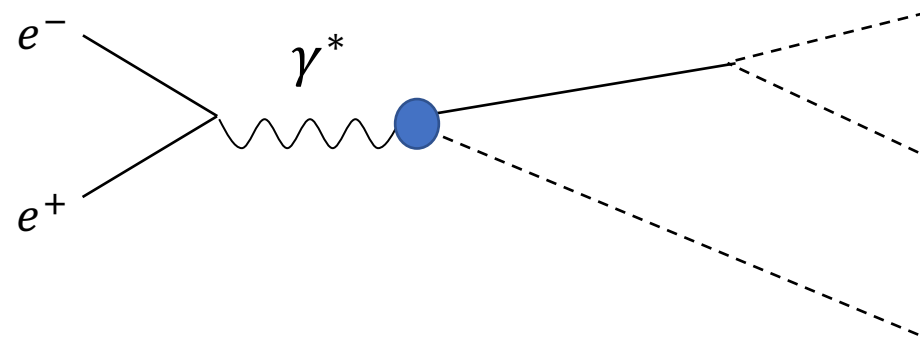
tree



1-loop



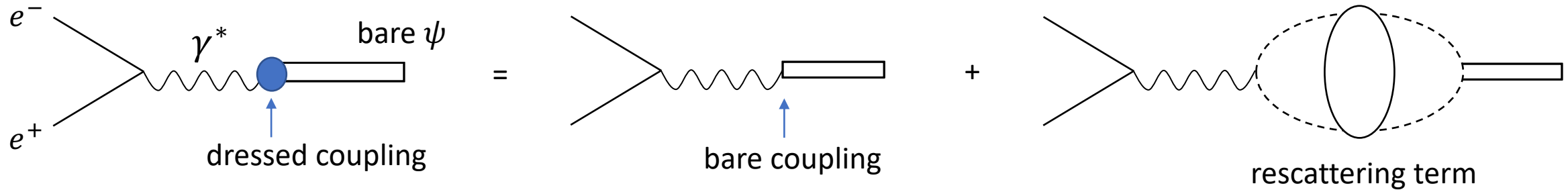
NR (non-resonant)



$D^* \bar{D}$ threshold cusp and/or
TS occurs from 1-loop

ψ production mechanisms

$e^+e^- \rightarrow c\bar{c}$ data in $3.75 \leq \sqrt{s} \leq 4.7$ GeV region \rightarrow Charmonium excitations are important mechanism



Data determine how many bare states to be included (5 bare states) and which charmonium states exist

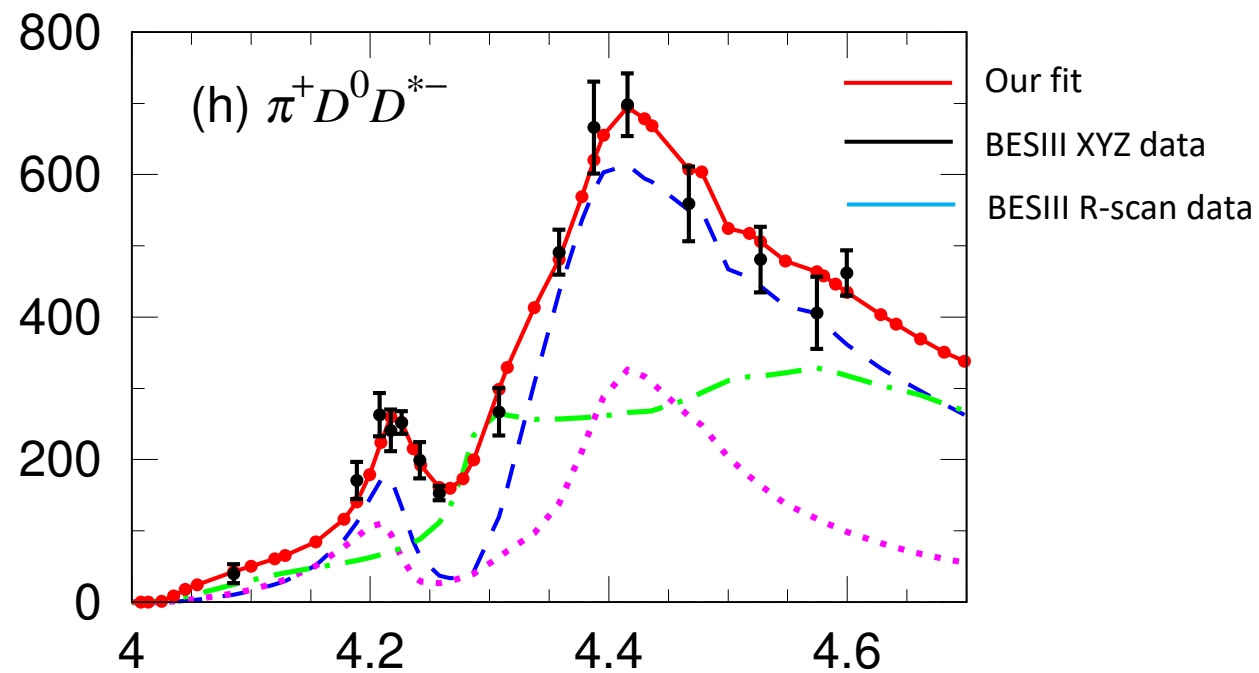
Expected states $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, $\psi(4415)$, $Y(4220)$, $Y(4360)$

Data is not sufficient for coupled-channel analysis in $\sqrt{s} > 4.6$ GeV (three-body final states including $c\bar{c}s\bar{s}$)

$Y(4660)$, $Y(4710)$ are not included in coupled-channel amplitude \rightarrow included as a Breit-Wigner amplitudes

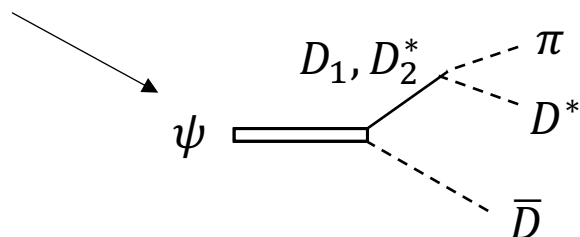
$$e^+e^- \rightarrow \pi^+ D^0 D^{*-}$$

$\sigma(\text{pb})$

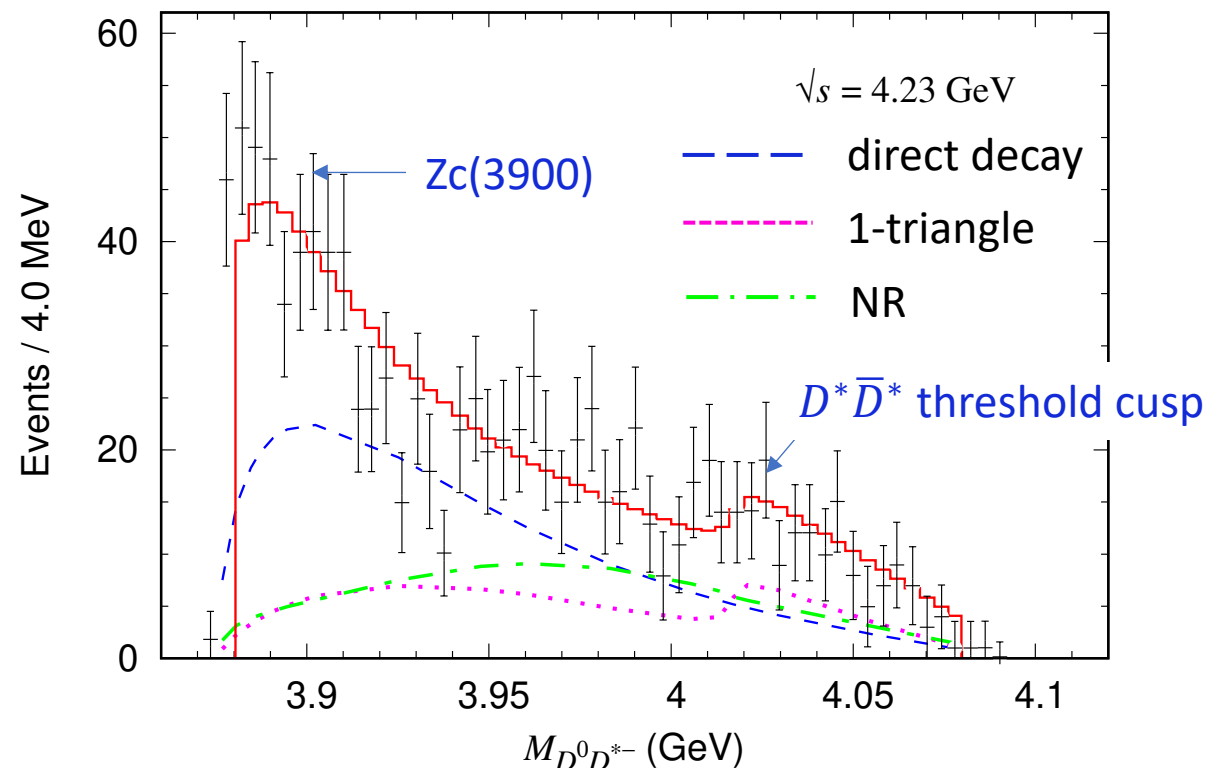


\sqrt{s} (GeV)

Tree contribution is dominant



$D^0 D^{*-}$ invariant mass distributions



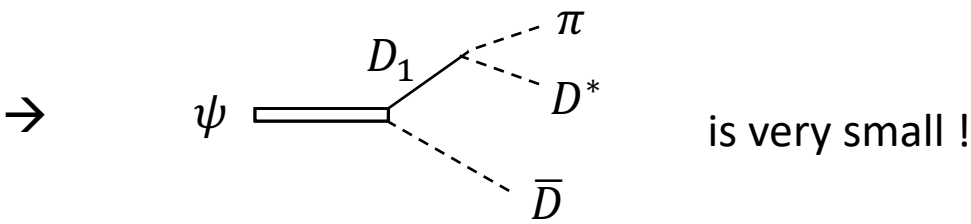
- $D^* \bar{D}^*$ threshold cusp is caused by
- $D^* \bar{D}$ threshold enhancement is mostly from tree; $\psi \rightarrow D_1 \bar{D}$

$$e^+e^- \rightarrow \pi^+ D^0 D^{*-}$$

Conflict with BESIII analysis result

Conclusion from BESIII PRD 92, 092006 (2015)

we conclude that the $D\bar{D}_1(2420)$ contribution to our observed Born cross section is smaller than its relative systematic uncertainty.



Difficult to make our model consistent with this BESIII conclusion. Why ? Insufficient information !!

Hope BESIII to conduct amplitude analysis on this process, and present detailed results and/or Dalitz plots.

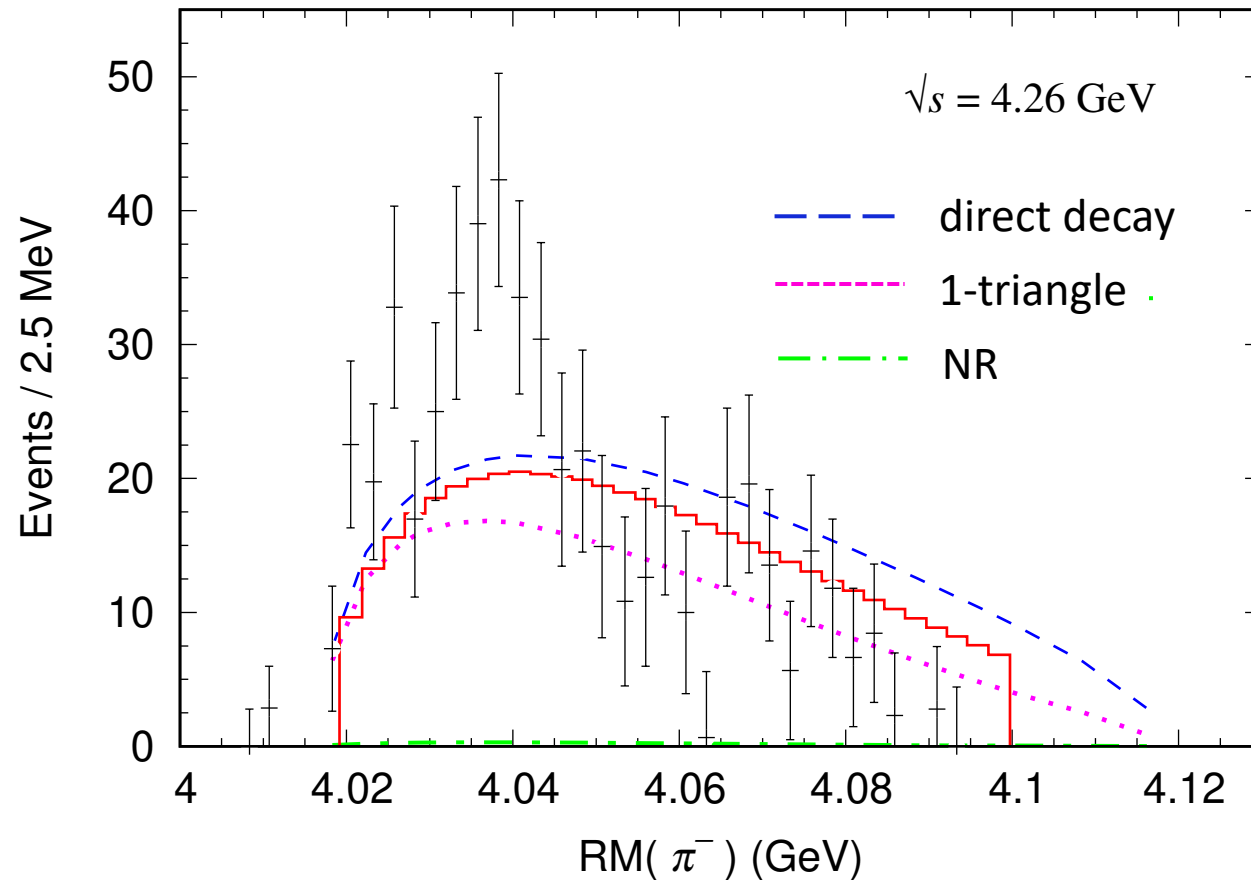
Without this information, $e^+e^- \rightarrow \pi^+ D^0 D^{*-}$ data cannot be well fitted, giving bad influence on the global fit overall

Most of previous theoretical models share the same problem

$$e^+e^- \rightarrow \pi^-(D^*\bar{D}^*)^+$$

$D^*\bar{D}^*$ invariant mass distributions (pion recoil mass)

— Our fit
— BESIII data



Fit does not seem good, however

Kinematical end of the data $\sim 4.09 \text{ GeV}$

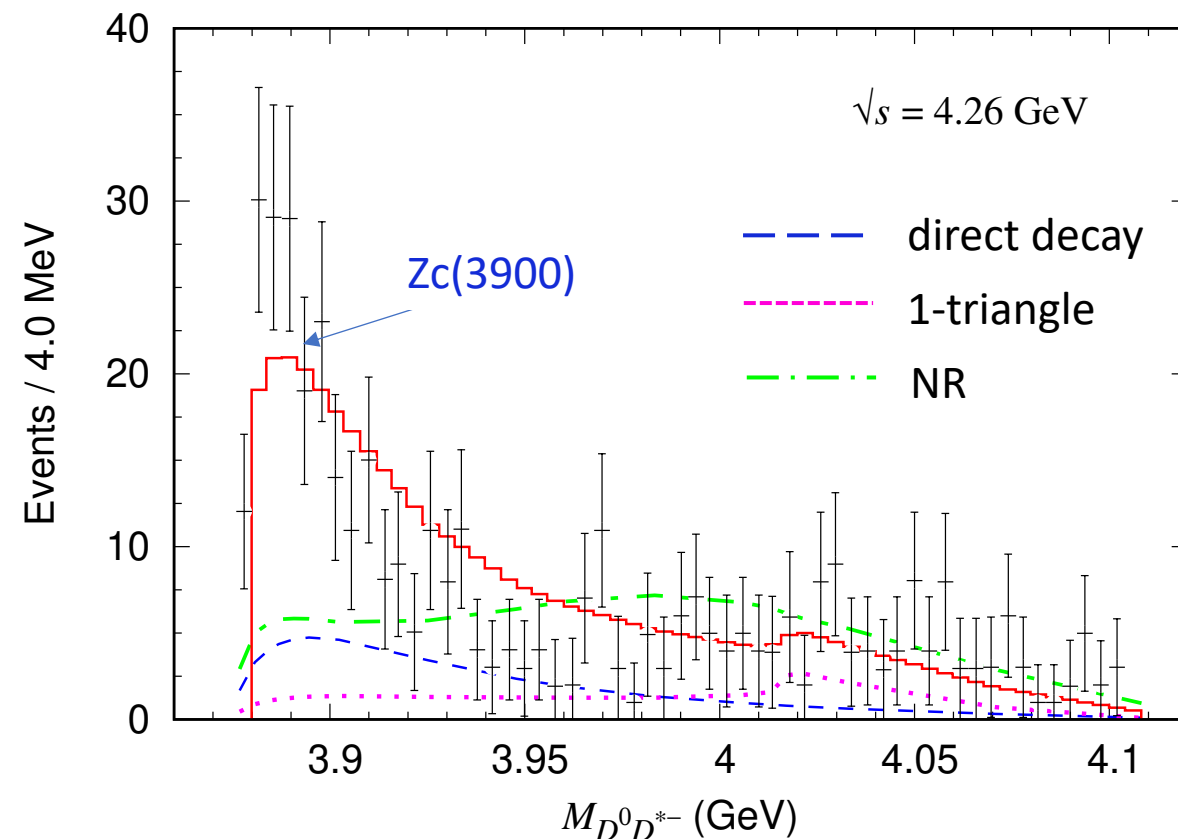
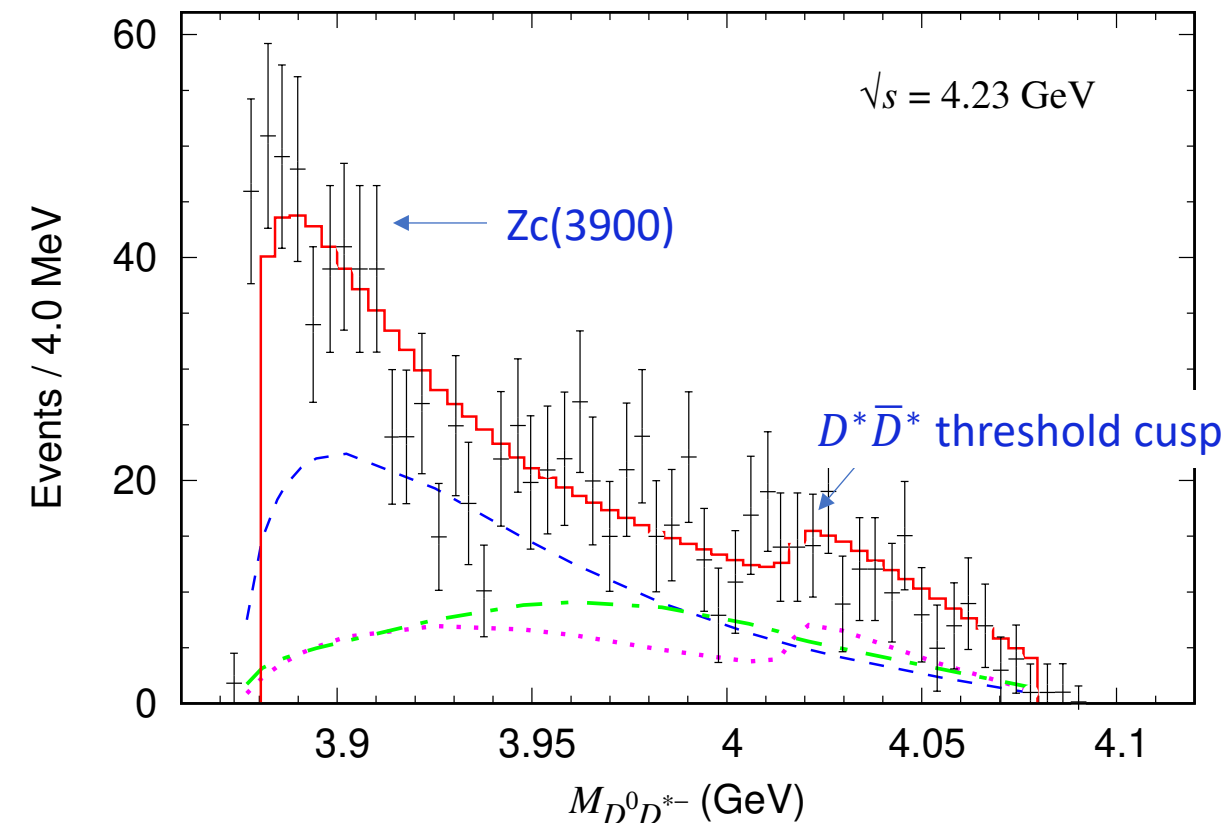
Actual kinematical end $\sim 4.12 \text{ GeV}$

→ Efficiency correction seems significant for this lineshape data

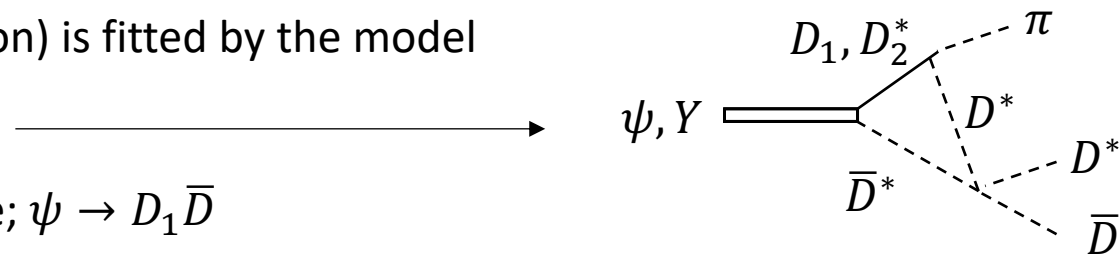
Wait for efficiency corrected data (or MC output) for future improvement of coupled-channel model

$$e^+e^- \rightarrow \pi^+ D^0 D^{*-}$$

$D^0 D^{*-}$ invariant mass distributions

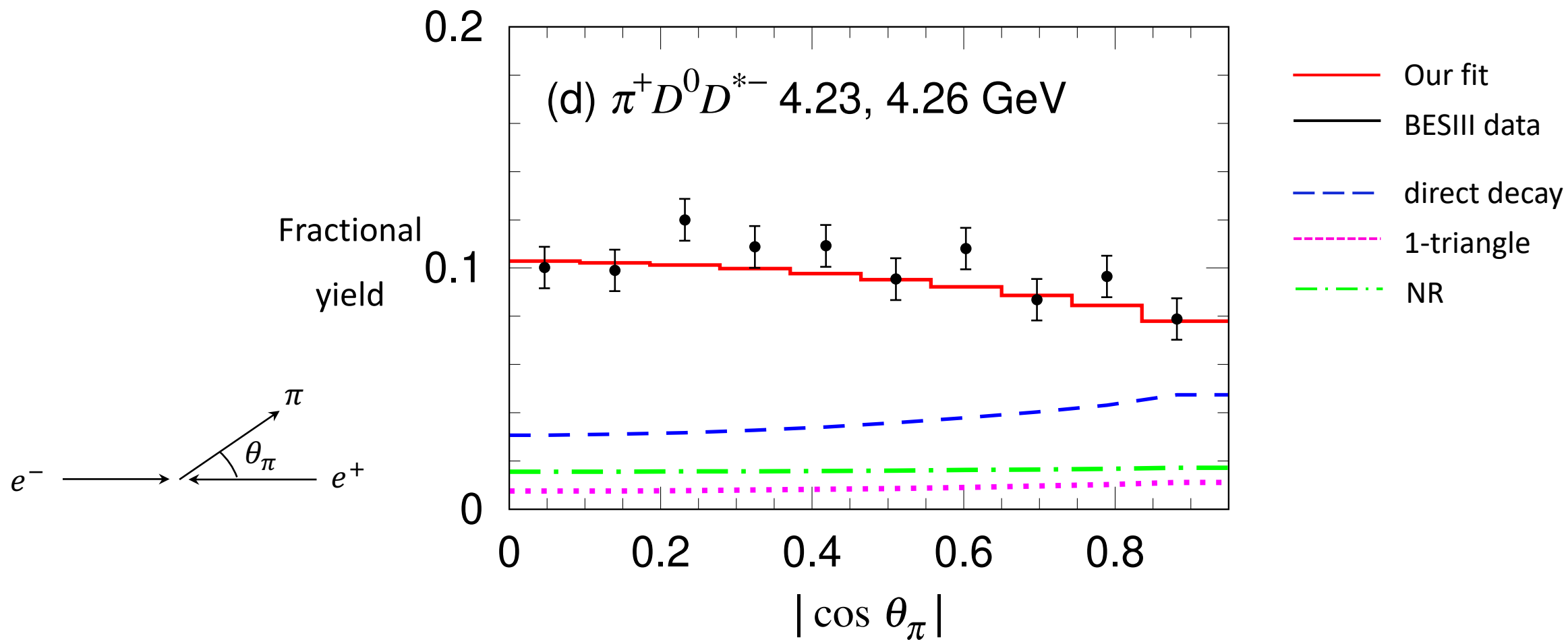


- Threshold enhancement (or $Z_c(3900)$ contribution) is fitted by the model
- $D^* \bar{D}^*$ threshold cusps are caused by
- $D^* \bar{D}$ threshold enhancement is mostly from tree; $\psi \rightarrow D_1 \bar{D}$



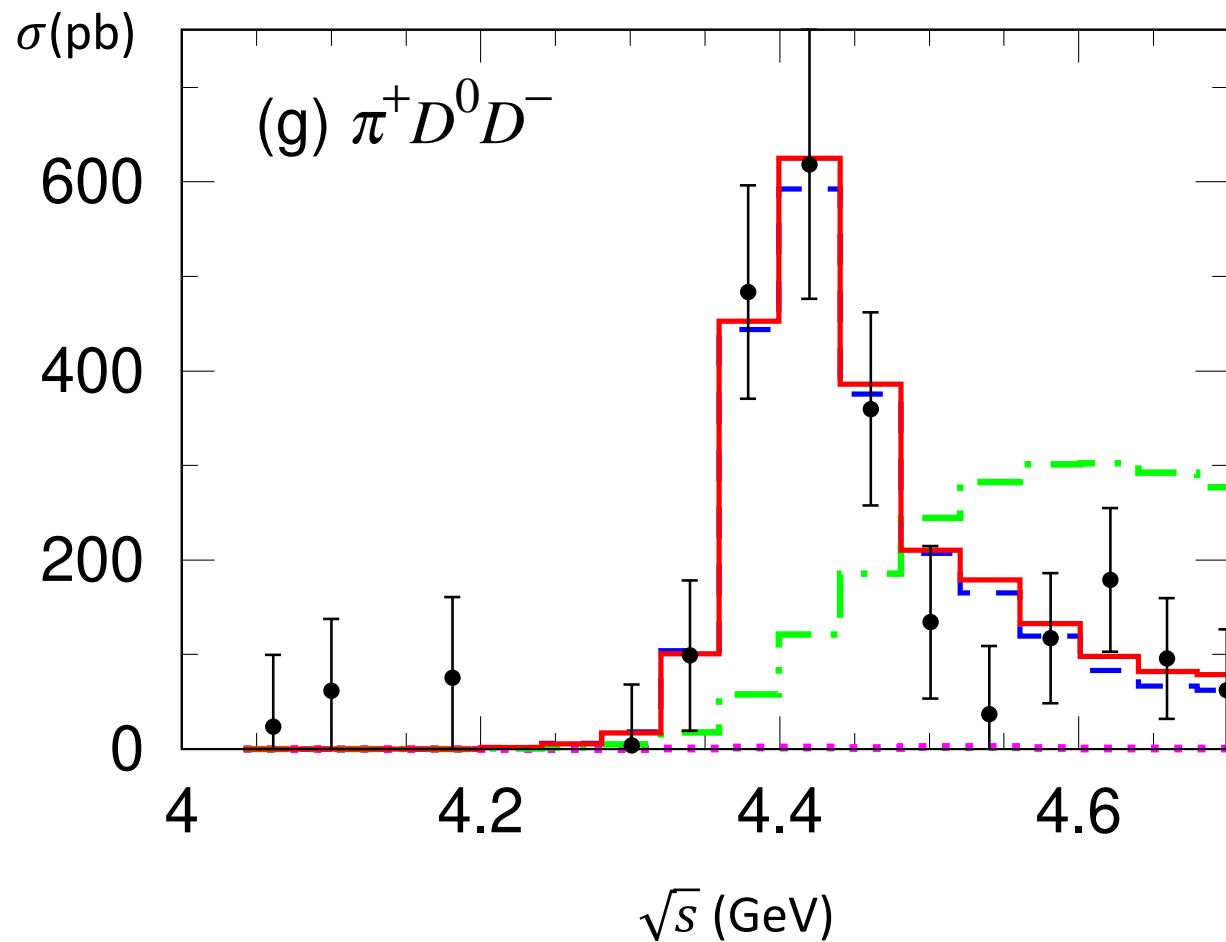
$$e^+e^- \rightarrow \pi^+ D^0 D^{*-}$$

Pion angle distributions from e^+e^- beam direction in total CM frame

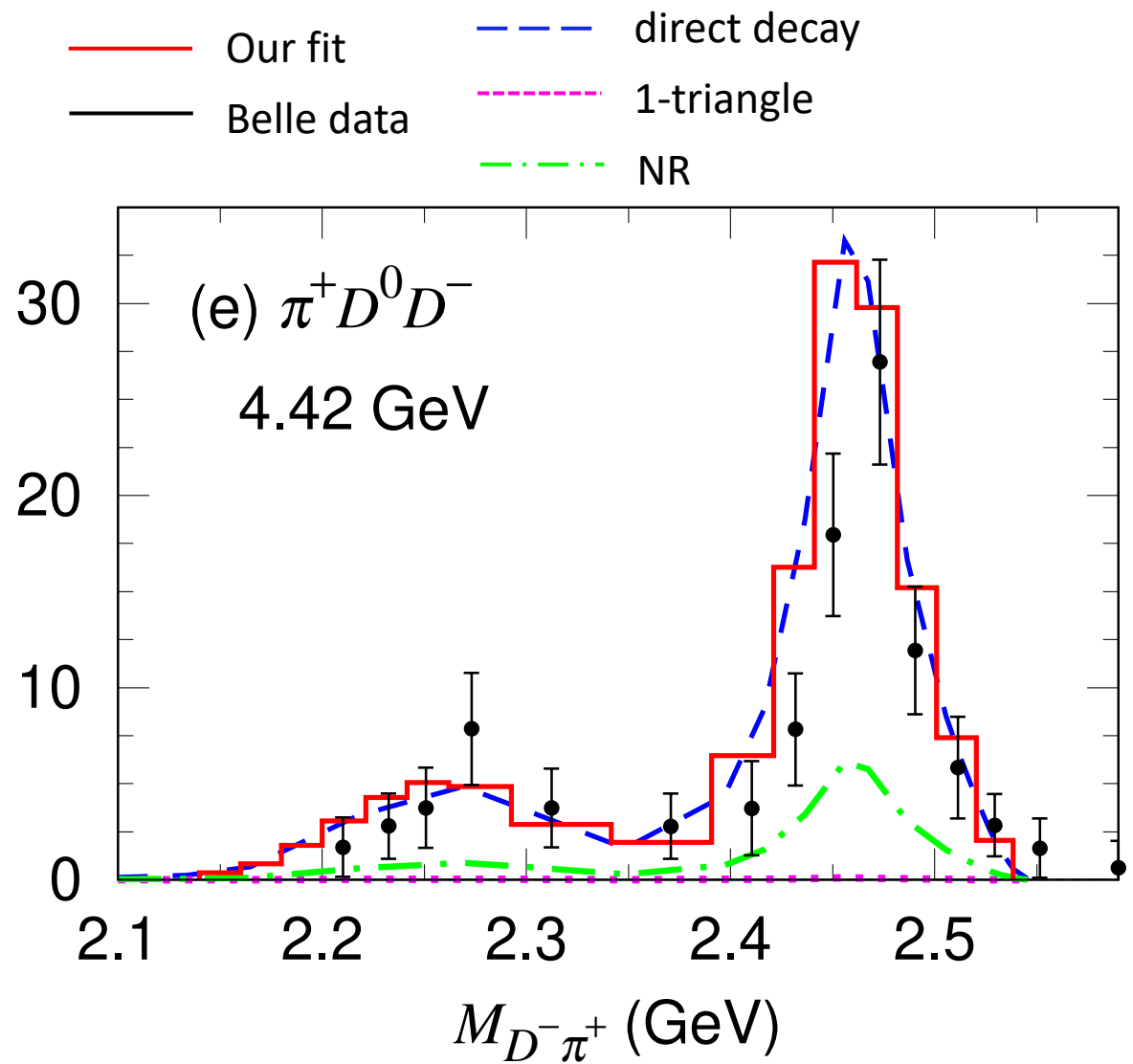


Data are average of 4.23 GeV ($N = 418$) and 4.26 GeV ($N = 239$) data

$$e^+e^- \rightarrow \pi^+ D^0 D^-$$



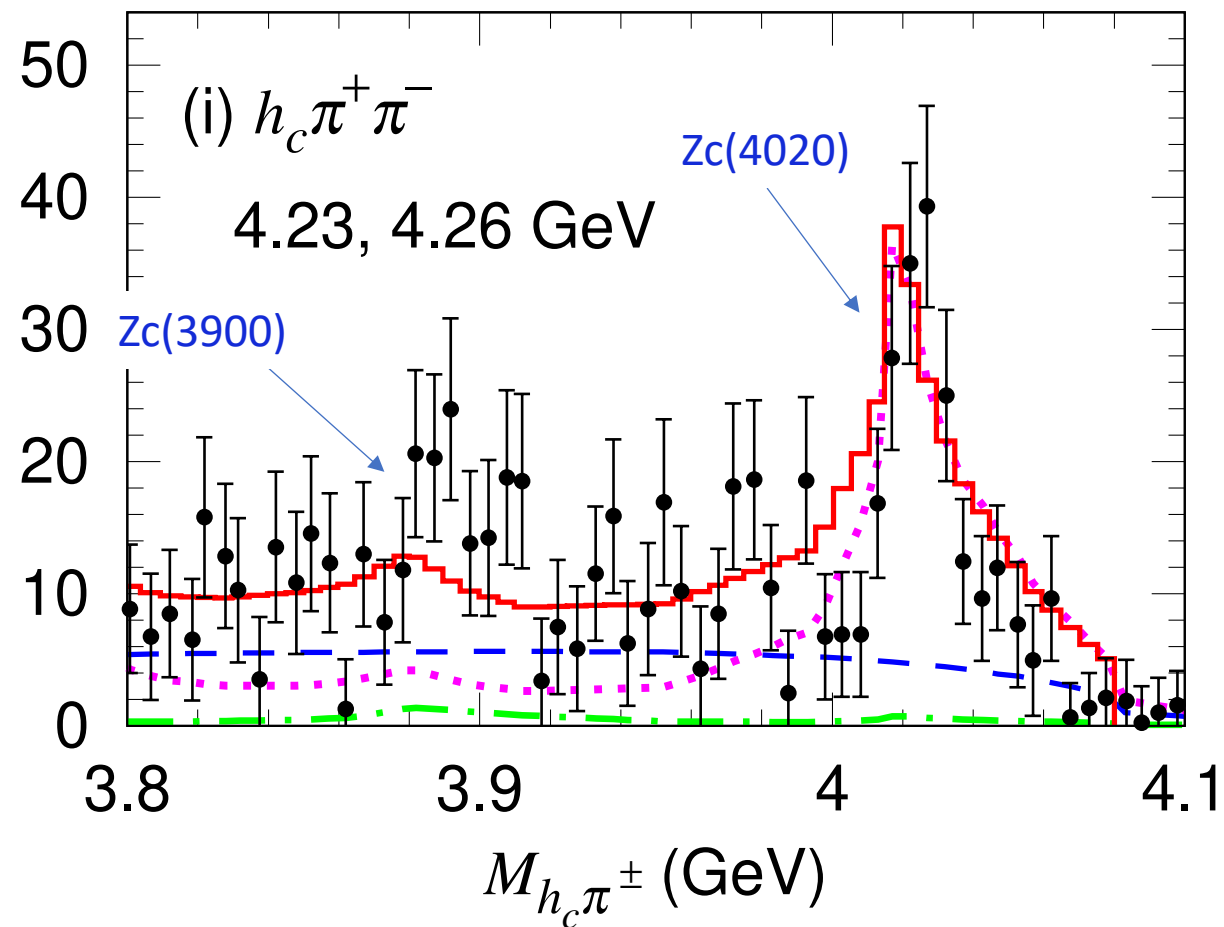
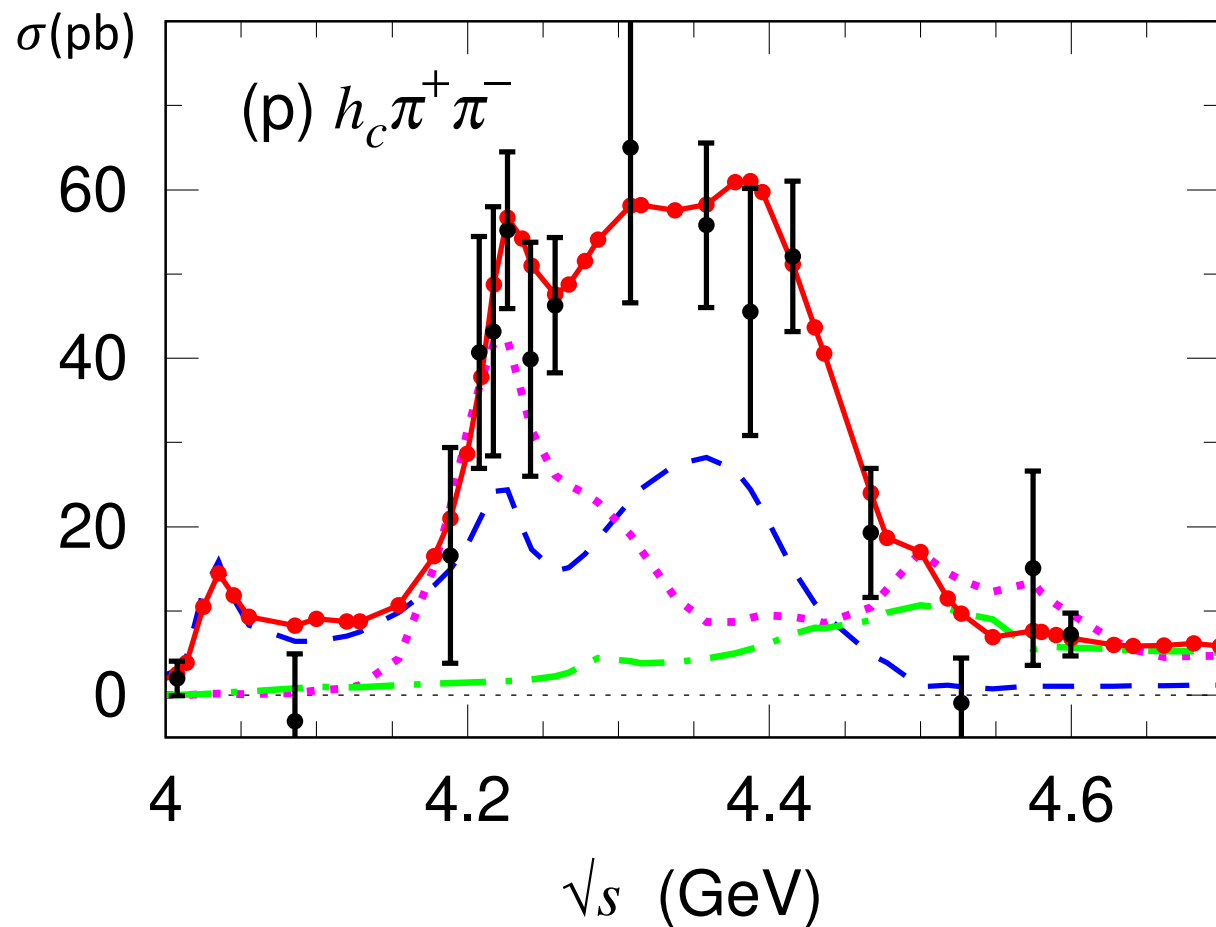
Clear $\psi(4420)$ peak is well fitted



Dominant $D_2^*(2460)$ contribution

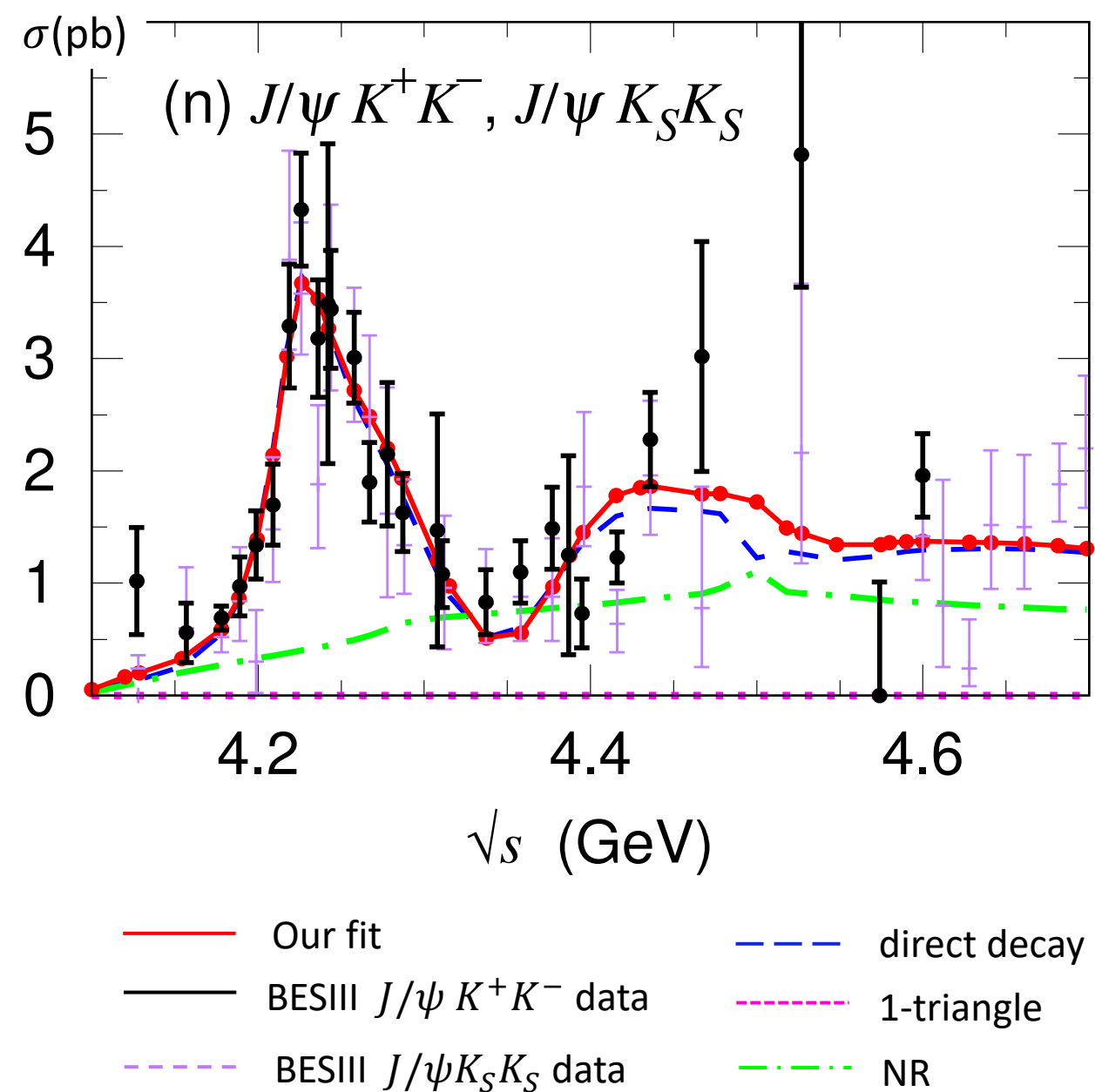
Hope to have a better quality data from BESIII ! \rightarrow important for coupled-channel analysis

$$e^+e^- \rightarrow h_c \pi^+ \pi^-$$



- Enhancement at $\sim 4.03 \text{ GeV}$ is from $\psi(4040) \leftarrow$ consequence of coupled-channel fit
- 1-triangle contribution causes threshold cusps, enhanced by Zc virtual poles

$$e^+e^- \rightarrow J/\psi K^+K^-, J/\psi K_S K_S$$

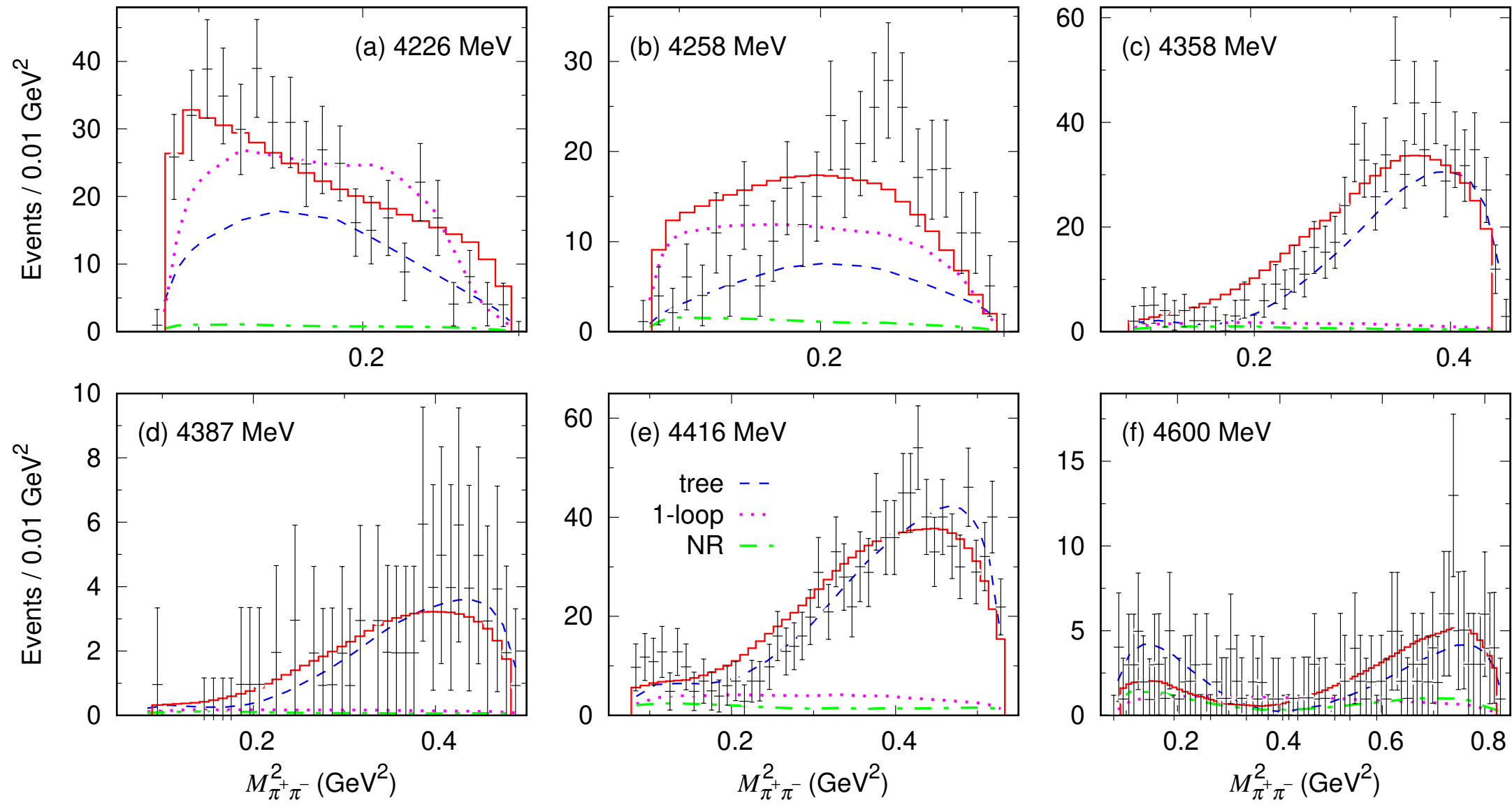


- Overall good agreement with data
(our model is isospin symmetric
 $\rightarrow \sigma(J/\psi K^+K^-) = 2 \times \sigma(J/\psi K_S K_S)$)
 - Model does not fit bump at ~ 4.5 GeV
in $J/\psi K^+K^-$ data
 - * $J/\psi K_S K_S$ data do not show the same bump
 - * data largely fluctuate and error is large
- \rightarrow our model does not have $Y(4500)$
more precise data is important to pin-down
the existence of $Y(4500)$

$$e^+e^- \rightarrow \psi' \pi^+\pi^-$$

Fit to invariant mass distributions

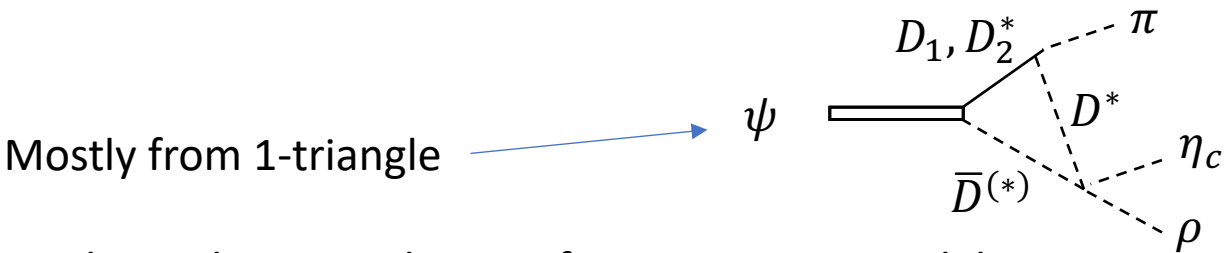
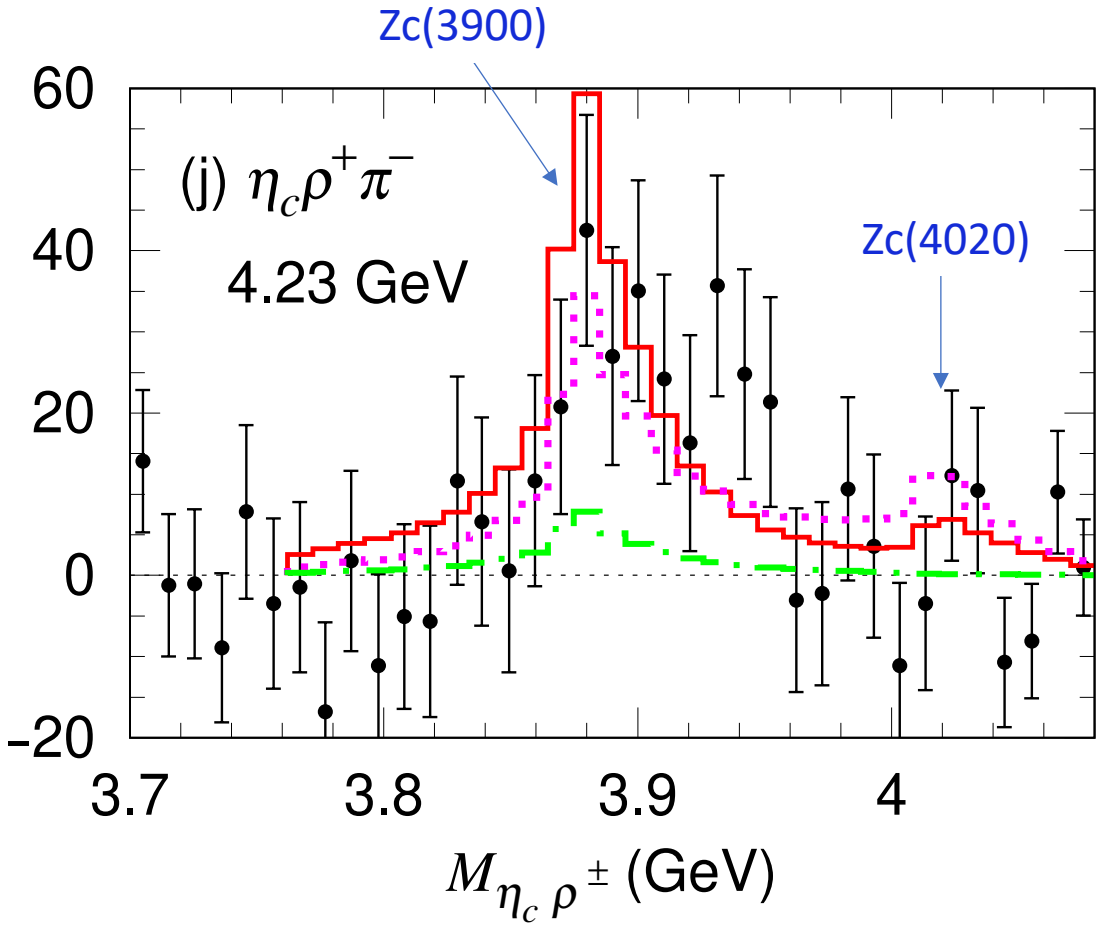
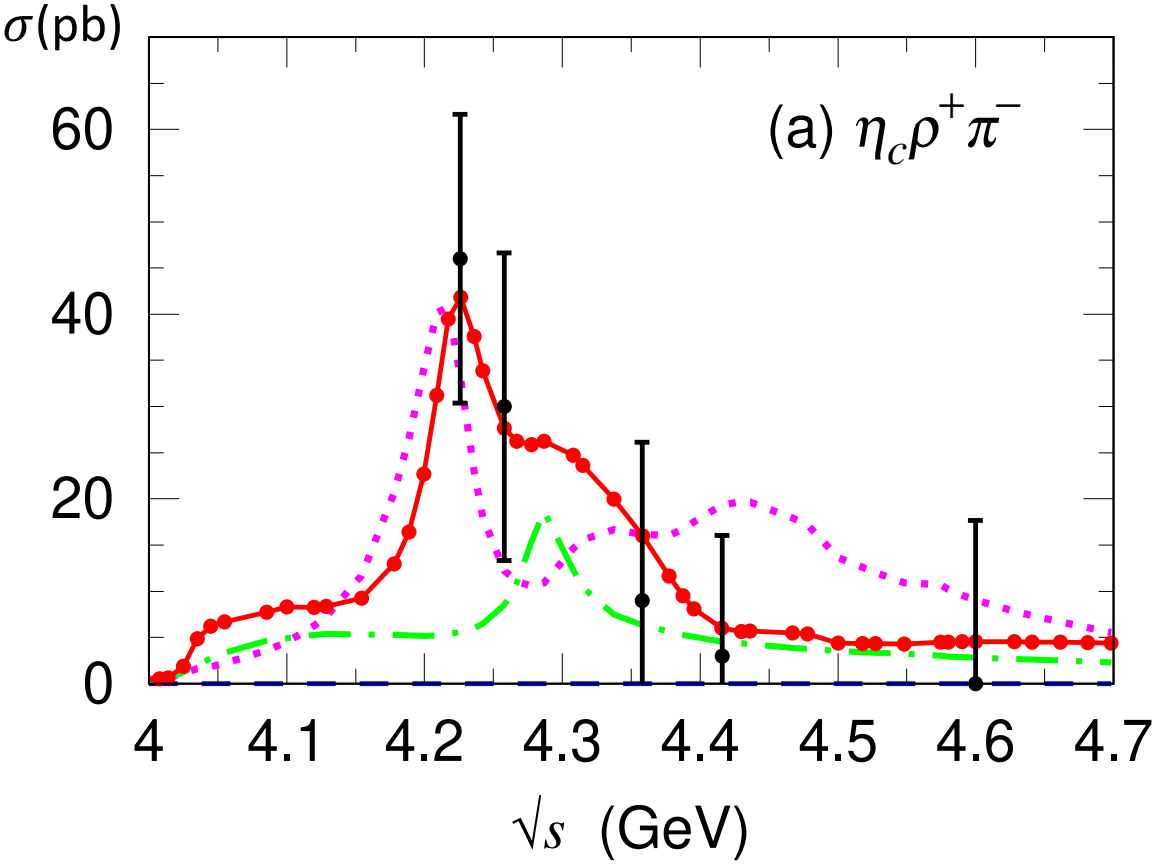
— Our fit
— BESIII data



$$e^+e^- \rightarrow \eta_c \rho^+ \pi^-$$

$\rho \rightarrow \pi\pi$ taken into account in calculation

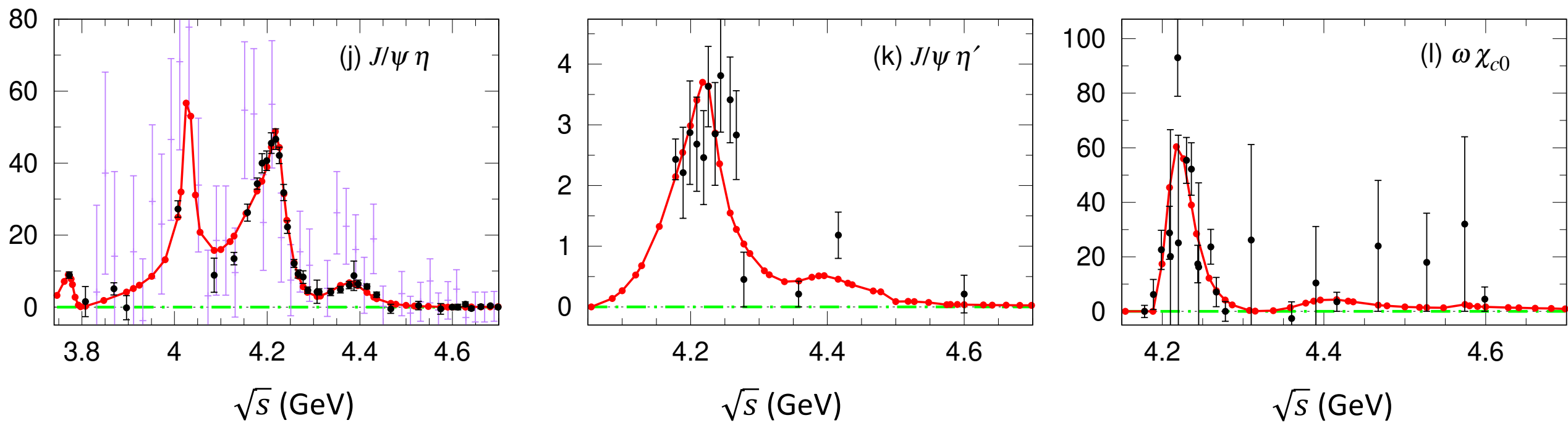
— Our fit
— BESIII data



No direct-decay mechanism for $\eta_c \rho \pi$ in our model

$Z_c(3900)$ peak is fitted

$$e^+e^- \rightarrow J/\psi \eta^{(\prime)}, \chi_{c0}\omega$$



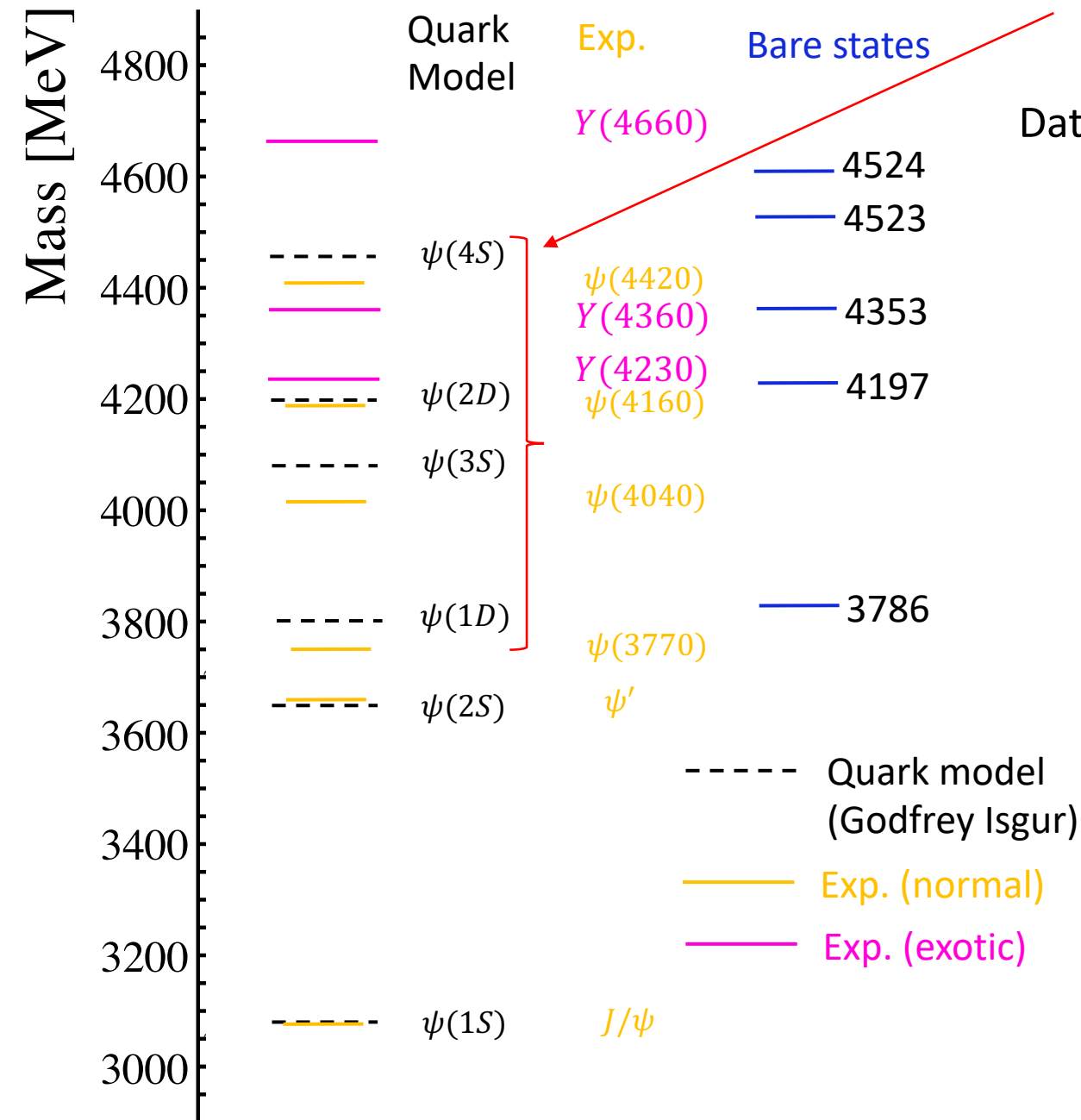
For $J/\psi \eta$, a sharp peak appears at 4.02 GeV, as a consequence of coupled-channel fit

← BESIII does not have data point, but Belle data seems to favor this result

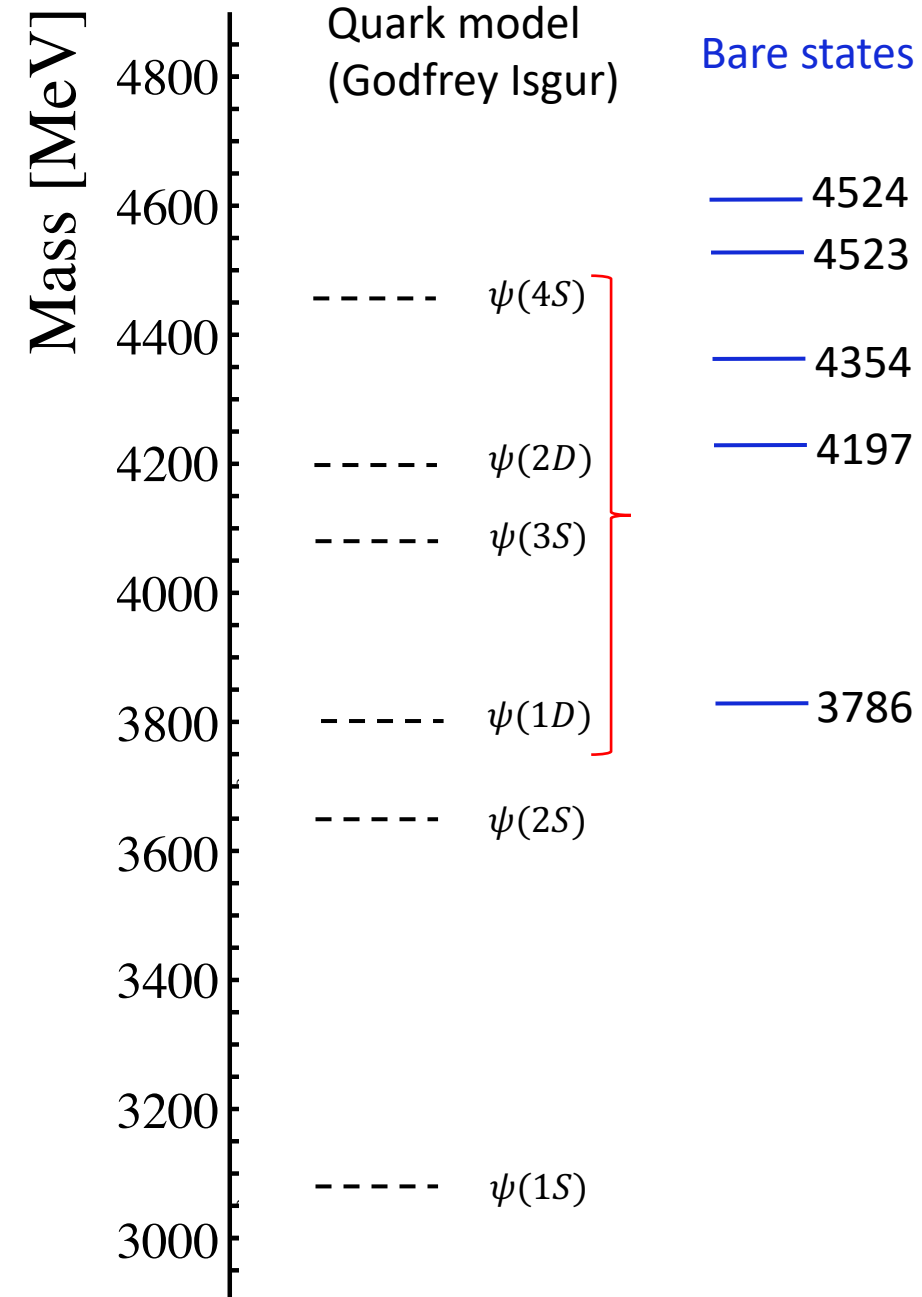
Charmonium spectrum ($J^{PC} = 1^{--}$)

Quark model predicts **four** states in the relevant energy region

Data require **five** bare states for achieving reasonable fit



Charmonium spectrum ($J^{PC} = 1^{--}$)



Quark model predicts **four** states in the relevant energy region

Data require **five** bare states for achieving reasonable fit

Conceptually, quark-model-state and our bare state is similar

→ Resonance without hadron-hadron continuum components

Very model-dependent argument/questions

One bare state is not accommodated in the quark model

→ Is it exotic bare state ?

Does it generate $Y(4230)$ and $Y(4360)$ after being dressed ?

Does it correspond to hybrid state predicted by LQCD ?

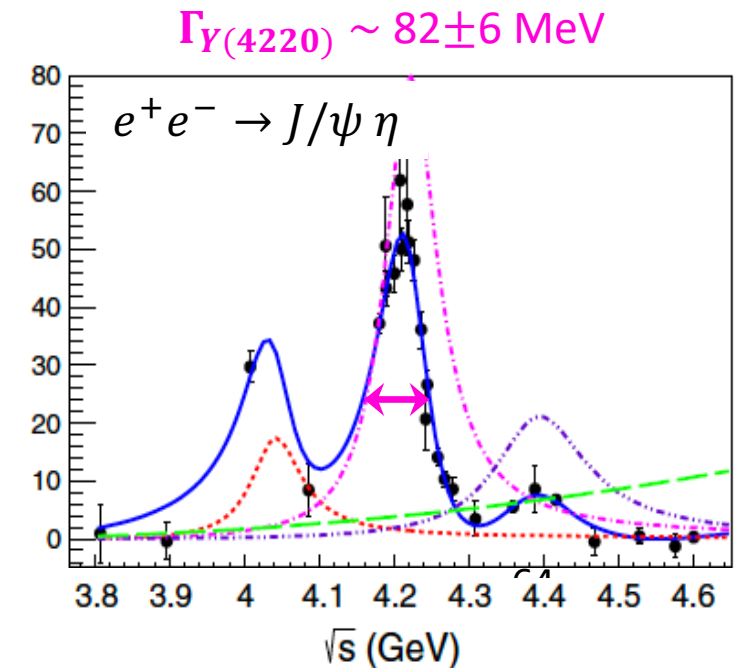
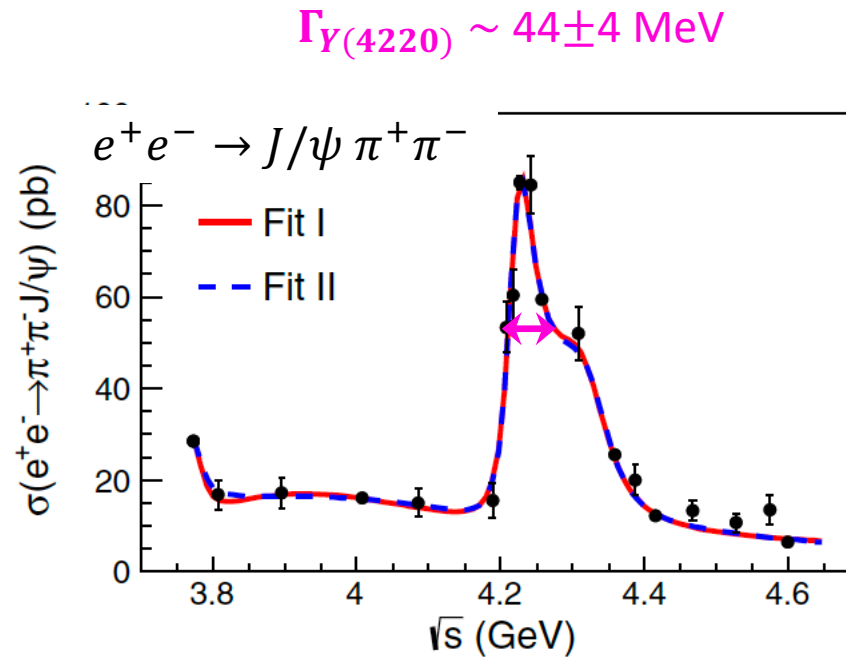
Liu et al., JHEP 07 (2012) 126

Our model alone cannot answer these interesting questions

Maybe possible by combining with structure model (quark model, etc.)

(speculation) Possible solution to Y width problem

	M (MeV)	Γ (MeV)
	3780 ± 0.5	30 ± 1.7
	4029 ± 0.3	28 ± 0.7
ψ_{wid}	4188 ± 1.8	127 ± 2.9
ψ_{nar}	4228 ± 0.7	44 ± 1.2
	4306 ± 2.6	129 ± 1.9
	4354 ± 3.1	123 ± 3.4
	4388 ± 1.5	107 ± 3.3



Two poles at $M \sim 4230$ (4380) MeV with narrow (ψ_{nar}) and wide (ψ_{wid}) widths. We can explain Y widths if:

$$\text{For } e^+e^- \rightarrow J/\psi \pi^+\pi^- \quad |g_{\psi_{\text{nar}} \rightarrow J/\psi \pi\pi}| \gg |g_{\psi_{\text{wid}} \rightarrow J/\psi \pi\pi}|$$

$$\text{For } e^+e^- \rightarrow J/\psi \eta \quad |g_{\psi_{\text{nar}} \rightarrow J/\psi \eta}| \ll |g_{\psi_{\text{wid}} \rightarrow J/\psi \eta}|$$

$g_{\psi_{\text{nar}} \rightarrow J/\psi \pi\pi}$: pole residue

Residues will be extracted in near future, and address the Y width problem

Relation between bare state and pole

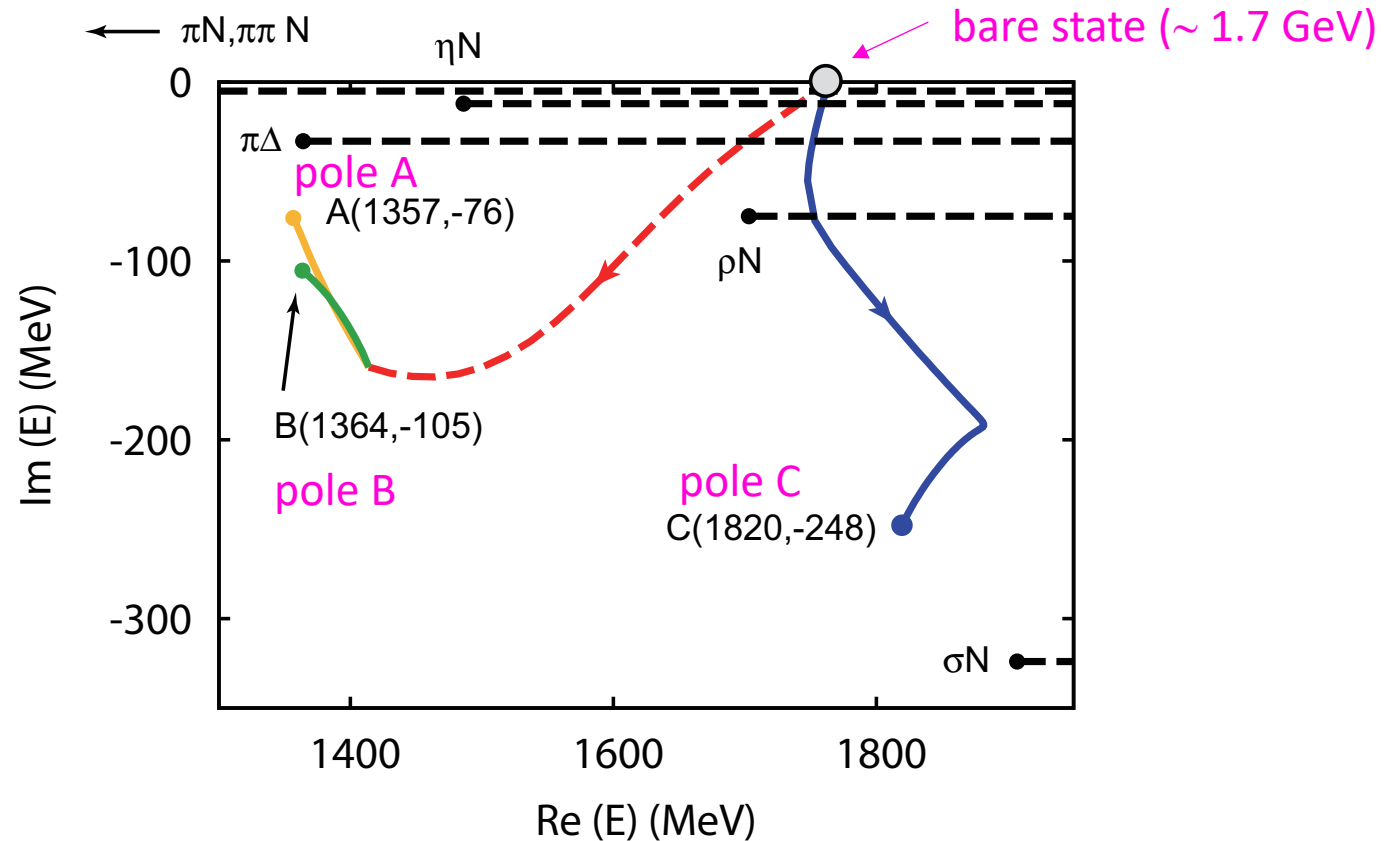
Data require **five** bare states

→ dressed by hadron continuum

→ **seven** poles

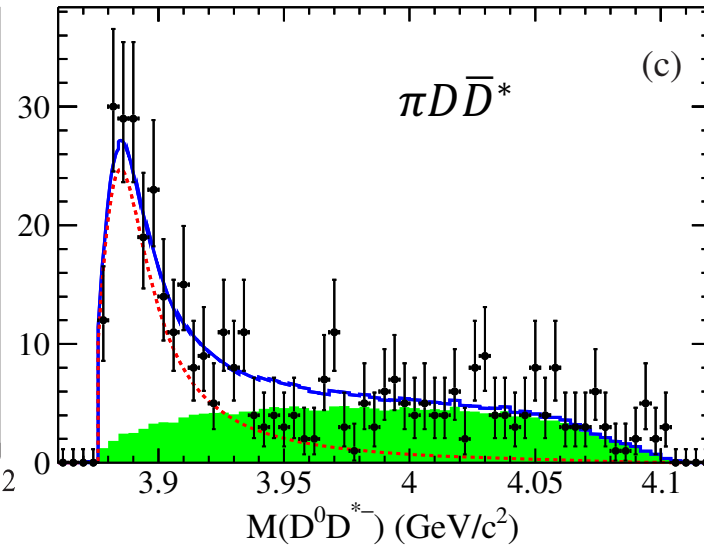
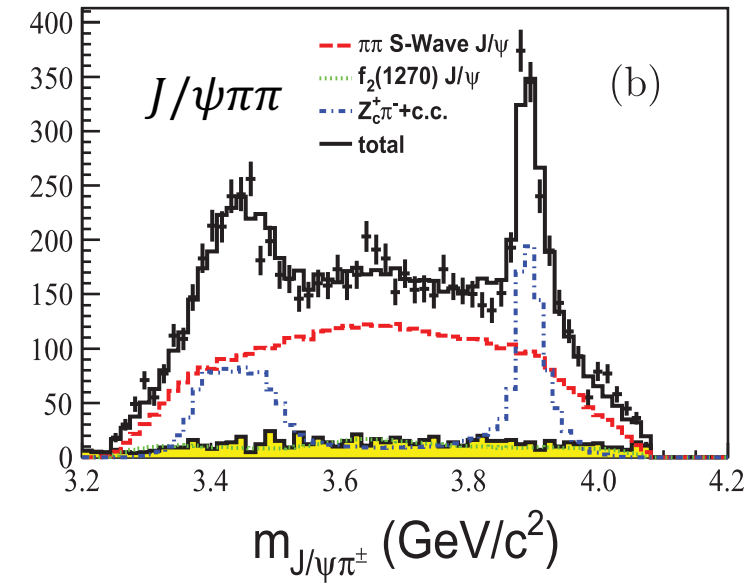
M (MeV)	Γ (MeV)
3780 ± 0.5	30 ± 1.7
4029 ± 0.3	28 ± 0.7
4188 ± 1.8	127 ± 2.9
4228 ± 0.7	44 ± 1.2
4306 ± 2.6	129 ± 1.9
4354 ± 3.1	123 ± 3.4
4388 ± 1.5	107 ± 3.3

Similar finding in nucleon resonances Suzuki et al. (EBAC) PRL 104, 042302 (2010)

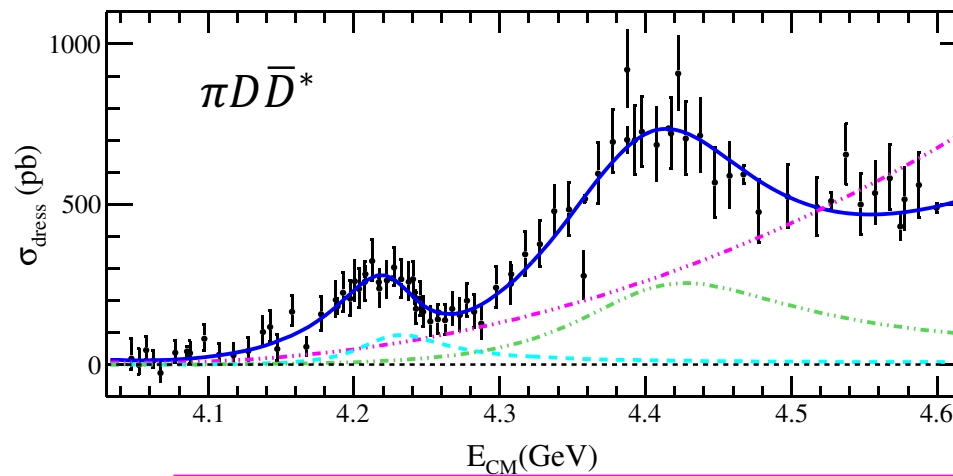
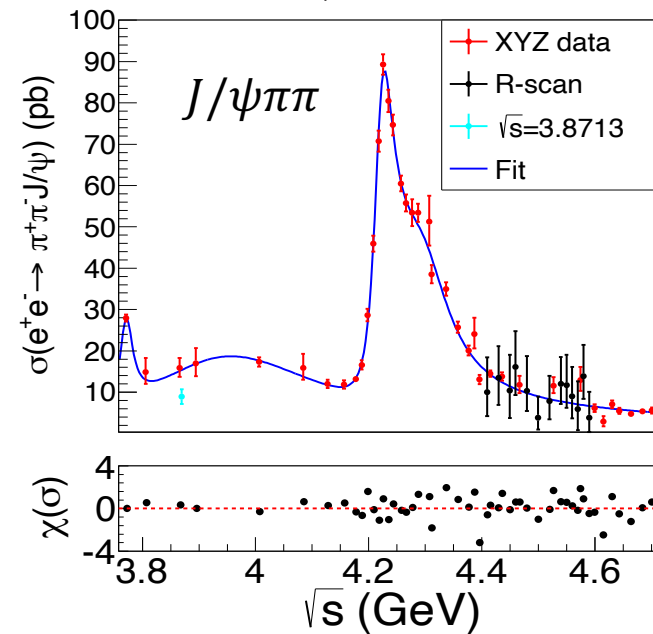


Future work : Which pair of poles come from the same bare state (mainly) ?

Common problem in previous theoretical analyses on Zc(3900)



Invariant mass distributions (left, event numbers) are fitted to determine Zc(3900) pole
 \rightarrow model's overall normalization is arbitrary but model has $\sigma(e^+e^- \rightarrow J/\psi\pi\pi)/\sigma(e^+e^- \rightarrow \pi D\bar{D}^*)$



In previous theoretical analyses,

cross sections (left) were not considered
 $\rightarrow \sigma(e^+e^- \rightarrow J/\psi\pi\pi)/\sigma(e^+e^- \rightarrow \pi D\bar{D}^*)$ from model is unchecked

Cross section data can test Zc production mechanism, Zc decay residues

$$\frac{\sigma(e^+e^- \rightarrow J/\psi\pi\pi)}{\sigma(e^+e^- \rightarrow \pi D\bar{D}^*)} \text{ should be checked to see if models are reasonable}$$

Our analysis cleared this problem

Present analysis result is consistent with lattice QCD

Previous LQCD analyses on $Z_c(3900)$ in:

Prelovsek et al. PLB 727, 172 (2013), PRD 91, 014504 (2015)

Chen et al. PRD 89, 094506 (2014)

Ikeda et al. (HAL QCD) PRL 117, 242001 (2016)

Cheung et al. (Hadron spectrum Collab.) JHEP 11, 033 (2017)

LQCD conclusion : $I = 1, J^{PC} = 1^{+-} D^* \bar{D}$ s-wave interaction is very weak,
disfavoring narrow $Z_c(3900)$ pole near $D^* \bar{D}$ threshold

Most of previous determinations of $Z_c(3900)$ pole are not consistent with LQCD

Q. Can the global analysis tell $Z_c(3900)$ is resonance or virtual state ?

The presented analysis employed energy independent interactions for Z_c amplitude

→ Only virtual or bound states are examined → virtual state works fine

Ongoing update

Z_c amplitude with resonant $Z_c(3900)$ state is implemented in the three-body coupled-channel model

→ Its performance in the global fit will be examined

$e^+e^- \rightarrow c\bar{c}$ data and coupled-channel analyses

◆—◆ BESIII
◆—◆ Belle

