Precise measurement of beam energy in colliders

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Contents

Methods of beam energy measurement

- Using the magnetic field along the orbit. The field along the orbit can be calculated based on currents or indirectly determined using NMR.
  Accuracy: $\Delta E/E \gtrsim 10^{-3}$.

$$E = \frac{e}{2\pi}\oint B_\perp dl$$

- Spectrometer: based on the deflection of particles in a specially calibrated magnet. This requires measuring the magnetic field within the magnet and the beam's orbit (BPM).
  Accuracy: $\Delta E/E \gtrsim 10^{-4}$

$$E = \frac{e}{\Delta\theta}\int B_\perp dl$$

- Using the edge of the inverse Compton scattering spectrum: This requires a detector made of ultra-pure germanium and an infrared laser (10μm).
  Accuracy: $\Delta E/E \gtrsim 10^{-5}$

$$E = \frac{\omega_{max}}{2}\left(1 + \sqrt{1 + \frac{m^2}{\omega_{max}\omega_0}}\right),$$

- Resonant spin depolarization method.
  Accuracy: $\Delta E/E \gtrsim 10^{-6}$

$$E = mc^2\left(\frac{g-2}{2}\right)^{-1} \times \left(\frac{\Omega_s}{\omega_0} - 1\right)$$

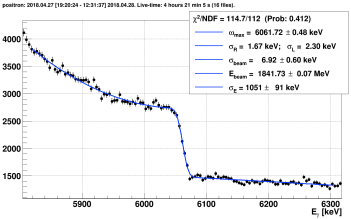
Compton backscattering for beam energy measurement

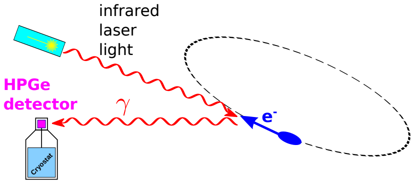
Maximum photon energy

$$\omega_{max} = \frac{4E^2\omega_0}{m^2 + 4E\omega_0} \approx 4\gamma^2\omega_0$$

The beam energy

$$E = \frac{\omega_{max}}{2}\left(1 + \sqrt{1 + \frac{m^2}{\omega_0\omega_{max}}}\right)$$





- Taiwan Light Source (1996)
- BESSY-I (1997), BESSY-II (2002)
- VEPP-4M (2006)
- BEPC-II (2011)
- VEPP-2000 (2013)

Need HpGe detector calibration by γ-sources

Resonant Depolarization (RD) Method

The most precise method of beam energy measurement

- $\Delta E/E \sim 10^{-6}$
- Suggested and firstly applied in BINP (Novosibirsk) at 1971
  Baier, Sov. Phys. Usp. 14 695–714 (1972)
- Used in experiments of precise mass measurement in the wide energy range
  Skrinskii, Shatunov, Sov. Phys. Usp. 32 548–554 (1989)
- Energy calibration for some synchrotron light sources: BESSY-I, BESSY-II, ALS ,SLS, KARA, *SOLEIL*

Used in experiments of precise mass measurement

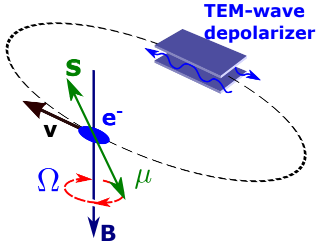
Particle	Experiment		Date
Φ, $K^{\pm}$	VEPP-2M	OLYA	1975-1979
J/ψ, $\psi(2S)$	VEPP-4	OLYA	1980
$\Upsilon(1S)$,$\Upsilon(2S)$,$\Upsilon(3S)$	VEPP-4	MD-1	1982-1986
$\Upsilon(1S)$	CESR	CUSB	1984
$\Upsilon(2S)$	DORIS II	ARGUS, Crystal Ball	1983
K^0, ω	VEPP-2M	CMD	1987
Z	LEP	ALEPH, DELPHI, L3, OPAL	1993
J/ψ, $\psi(2S)$, τ, D^0, $D^{\pm}$ $\psi(3770)$	VEPP-4M	KEDR	2003-2015

The idea of the resonant depolarization method

Frenkel, Thomas (1926), Bargmann, Michel, Telegdi (1959)

$$\frac{ds^i}{d\tau} = 2\mu F^{ij} s_j - 2\mu' u^i F^{jk} u_j s_k$$

$$\frac{d\vec{s}}{dt} = \underbrace{2\mu \frac{\vec{s}\times\vec{B}'}{\gamma}}_{\text{dynamic}} + \underbrace{(\gamma-1)\frac{\vec{s}\times[\vec{v}\times\dot{\vec{v}}]}{v^2}}_{\text{kinematic (Thomas precession)}}$$



$$\Omega = \omega_0\left(1 + \frac{E}{m_e}\frac{\mu'}{\mu_0}\right) = \omega_0 n \pm \omega_d, \quad n \in \mathbb{Z}$$

$$\delta(\mu'/\mu_0) \approx 1.03\times10^{-10} \quad \delta m_e \approx 2.94\times10^{-10}$$

$$E = (440.648\,462\,134 \pm 0.000\,000\,137)\,[\text{MeV}] \times \left(n - 1 \pm \frac{\omega_d}{\omega_0}\right)$$

Stages of RD energy measurement

1. Preparation of polarized beam via Sokolov-Ternov effect of radiative polarization
2. Beam polarization observation.
3. Scanning the depolarizer frequency within a specified range, defined or guided by an approximate knowledge of the beam energy.
4. Determination the moment of depolarization and extracting the precession frequency and beam energy.

Radiative polarization

Sokolov-Ternov effect (1963)

Sokolov, Ternov, Dokl.Akad.Nauk SSSR 153 (1963) no.5, 1052-1054

Intensity of SR with spin flip

$$W^{\uparrow\downarrow} \approx W_0 \frac{4}{3} \left(\frac{\omega_c}{E}\right)^2$$

$$\tau_p = P_0 \frac{\lambda_C}{\alpha c} \frac{1}{\gamma^2} \left(\frac{H_0}{H}\right)^3 \qquad P_0 = \frac{8\sqrt{3}}{15} \approx 92.4\%$$

First observation

- VEPP-2 (Novosibirsk) in 1970

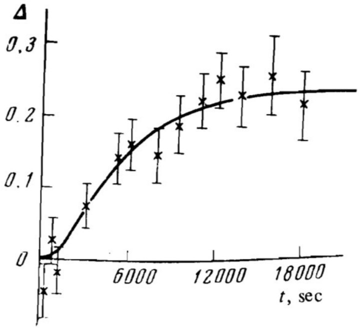
  Baier, Sov. Phys. Usp. 14 695–714 (1972)

- ACO storage ring (Orsay) in 1972

  Duff, Marin, Masnou, Sommer, Preprint, Orsay 4-73(1973)

Radiative polarization at VEPP-2M observed with Touschek polarimeter, $\tau = 70$ min (1974)

Serednyakov, Skrinskii, Tumaikin, Shatunov, JETP, V44, No. 6, p.1063 (1976)



$$P(t) = P\frac{\tau}{\tau_p}\left(1 - e^{-t/\tau}\right); \qquad \tau = \frac{\tau_d \tau_p}{\tau_p + \tau_d}$$

Depolarizing resonances

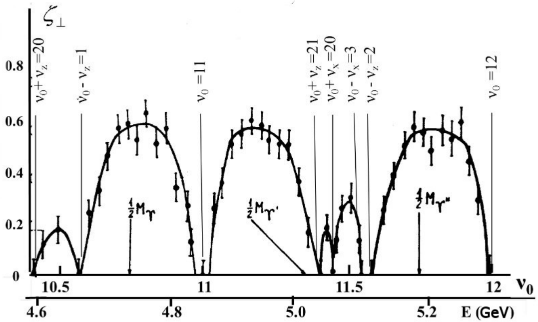
$$\nu = \frac{\Omega}{\omega_0} - 1 = k \cdot \nu_x + l \cdot \nu_y + m \cdot \nu_s + n \quad k, l, m, n \in \mathbb{Z}$$

- Stochastic depolarization

$$\tau_d \sim \left(\nu_0^2 \sum \frac{|w_k|^2}{(\nu_0 - \nu_k)^4} \right)^{-1}$$

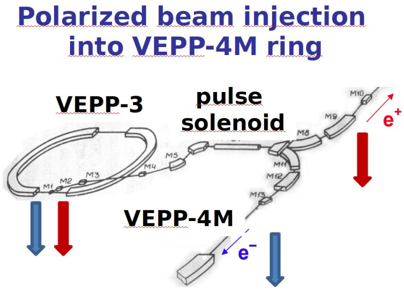
- Difficult to accelerate polarized beam due to resonance cross
- Spin precession shift

$$\delta\nu \sim \frac{1}{2} \sum \frac{|w_k|^2}{\nu_0 - \nu_k}$$



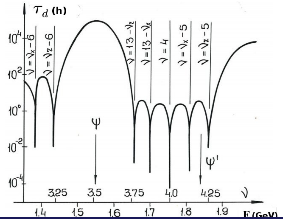
Equilibrium polarization degree measurement at VEPP-4 with laser polarimeter.

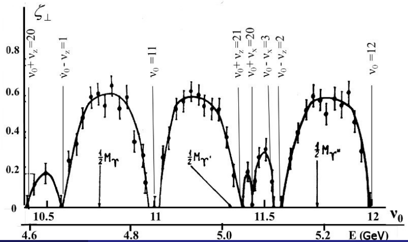
Obtaining polarization at VEPP-4M



Polarization time

Ring	VEPP-3	VEPP-4M
τ_p [h]	$\frac{12}{E[GeV]^5}$	$\frac{1540}{E[GeV]^5}$
τ_p @ 1.55 GeV	1.34 h	172 h
τ_p @ 1.85 GeV	0.56 h	71 h
τ_p @ 4.1 GeV		80 min
τ_p @ 4.73 GeV		39 min





- Good beam polarization for J/ψ, $\psi(2S)$, $\Upsilon(1S)$, $\Upsilon(3S)$
- Problem with τ lepton energy region (close to integer $\nu = 4$ resonance)

Polarization measurement

- Fixed target
  - Mott scattering (spin orbit coupling, $100 kev < E < 5$ MeV): JLab
  - Moller scattrinc (atomic electron, $\lesssim 1$ GeV): JLab, BINP,...
- Touschek (intrabeam scattering) polarimeter (BINP, BESSY-I/II, ALS, SLS...). Best for lower energies $E < 2$ GeV
- Compton backscattering (better for high energies $E > 5GeV$)
  - laser: Cornell (CESR), DESY (DORIS), BINP (VEPP-4), SLAC (SLD) ...
  - synchrotron light from clashing (positron) beam: BINP (VEPP-4)
- Synchrotron spin-light: BINP (VEPP-4)

Synchrotron Spin-light polarimeter

Classical synchrotron light

$$W_0 = \frac{2}{3}\frac{e^2 c}{R^2}\gamma^4$$

Magnet dipole synchrotron light

$$W_{md} = \frac{2}{3}\frac{\mu_0^2}{c^3}\omega_0^4\zeta^2 \propto \hbar^2$$

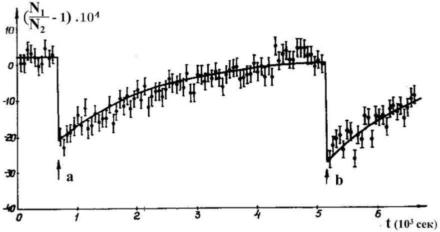
Interference between them

$$W_{mixed} = 2\sqrt{W_0 W_{md}} \propto \hbar$$

For $\omega/\omega_c > 10$, $B = 1\,T$, $E = 10 \div 100$ GeV

$$\delta = \frac{W_{mixed}}{W_0} \sim \zeta\omega/E \approx 10^{-4} \div 10^{-3}$$

- Suggested by Korchuganov, Kulipanov, Mezentsev (1977)
- Implemented at BINP (1982) (Belomestnykh, Bondar et al)



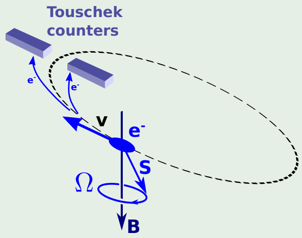
Touschek polarimeter

- Proposal to use beam lifetime to detect polarization in 1968 (flat beam calculation)
  Baier, Khoze, Atomnaya Énergiya, V25, No.5, pp. 440–442 (1968)
- Tumaikin's proposal to use scint. counters (1970)
- Calculation for 2D beam
  Serednyakov, Skrinskii, Tumaikin, Shatunov, JETP, V44, No. 6, p.1063 (1976)
- With some relativistic corrections (1978)
  Baier, Katkov, Strakhovenko, Dokl.Akad.Nauk SSSR, 1978, V241,No4, P.797–800
- with Coulomb effects (2011)
  Strakhovenko, Phys. Rev. ST Accel. Beams 14, 012803

Itra-beam scattering ($e^-e^- \to e^-e^-$) scattering

$$d\sigma = d\sigma_0\left(1 - (\vec{s}_1\vec{s}_2)\frac{\sin^2\theta}{1+3\cos^2\theta}\right)$$

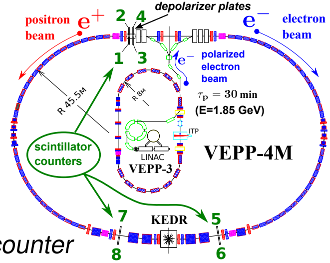
$$\frac{dN}{dt} \approx A\frac{N^2}{V\gamma^2(\Delta p/p)^2}(1-P^2\eta)$$

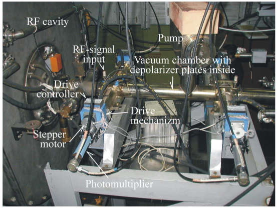


Touschek polarimeter at VEPP-4M

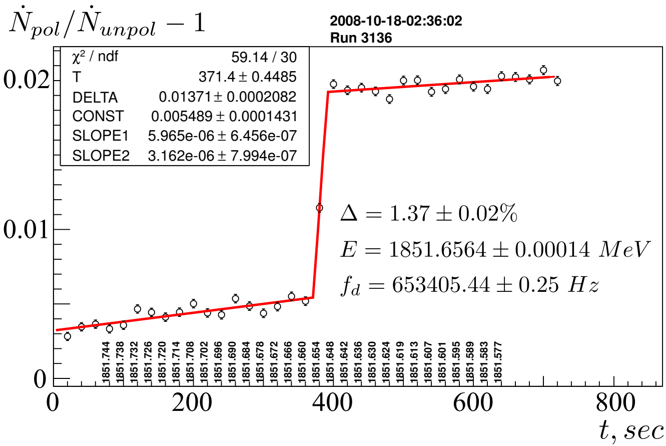
8 movable scintillator counters located inside vacuum chamber at different places of VEPP-4M

Energy range	$1.5 \div 2.0$ GeV
Beam current	> 0.1 mA
Number of bunches (electron or positron)	4
Count rate	1 MHz
	$50\,\mathrm{kHz/mA^2}/counter$
Compensation technique	$\Delta = \dot{N}_{pol}/\dot{N}_{unpol} - 1$
Depolarization effect	$\Delta = 1 \div 3\,\%$
Polarization degree	$\approx 80\%$
Stat accuracy	1 keV (10^{-6})
Number of calibration at same bunches	3
Calibration duration	2 hours
Number of energy calibrations since 2001	≈ 4000

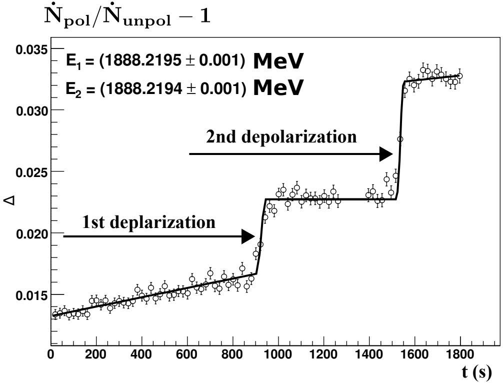




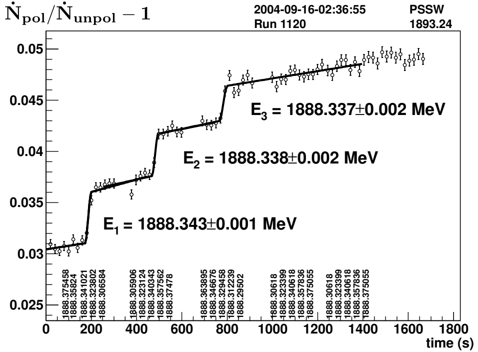
Energy calibration example



Several calibrations with same polarized bunch



Double jump

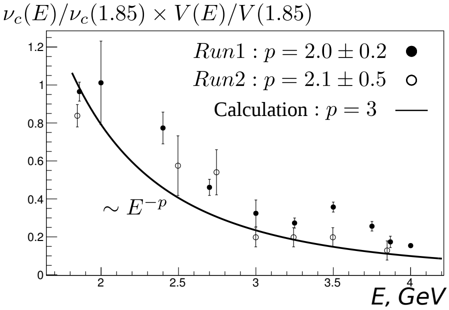


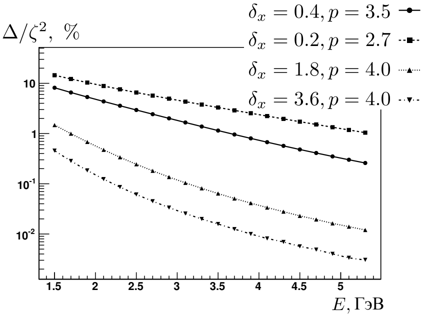
Triple jump

Double up-down scan increase reliability of energy calibration.
Suppress cases of calibration at side 50 Hz spin resonances

$$\dot{N} \propto \frac{I_{beam}^2}{E^{2\div3} V_{beam}} \propto \frac{1}{E^{5\div6}}$$

$$\Delta \approx \frac{0.5\%}{\delta q_x \delta q_y} \zeta^2 \propto \frac{1}{E^4}$$





Small count rate and polarization effect for $E = 5$ GeV

$\dot{N} \approx 10$kHz for $I = 10$ mA $\qquad \Delta \approx 0.3\%$

Need alternative method of polarization measurement

Compton backscattering polarimeter

- Suggested in BINP in 1969:

  Baier, Khoze, Sov.J.Nucl.Phys. V9, p238 (1969)

- First implemented at SPEAR (1979)

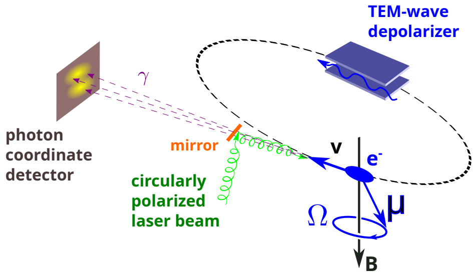
  Gustavson et al, NIM, V165, No2, p177 (1979)

- VEPP-4 (1982)

  Vorob'ev et al, Proc. All-union conference on charged particle accelerators. (1983)

- Tikhonov (1982): SR from clashing beam as source of circular polarized light
- at LEP for Z boson mass measurement (1993)

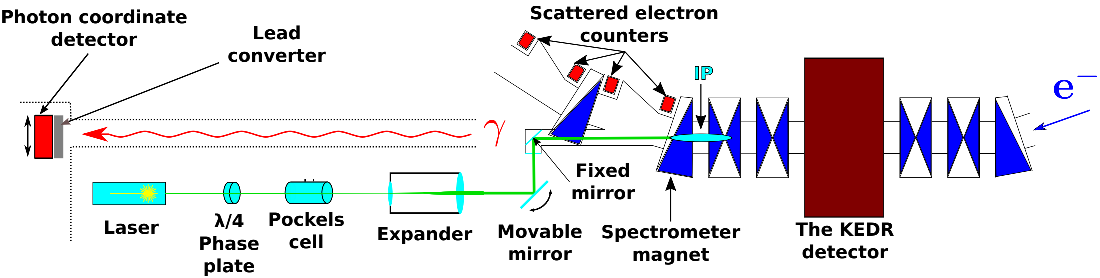
Up-down scattering asymmetry for left-right photon backscattering on vertically polarized electron beam

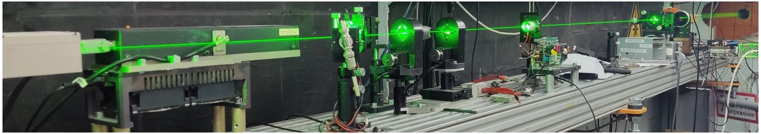


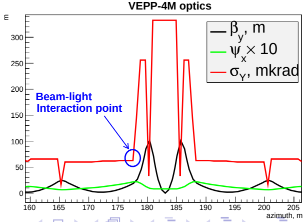
$$A = \frac{N_{up} - N_{down}}{N_{up} + N_{down}} \approx -\frac{3}{4}\frac{E\omega_0}{m_e^2}VP$$

ω_0 is the initial photon energy, V is the Stokes parameter of circular polarization (± 1)

Laser polarimeter at VEPP-4M

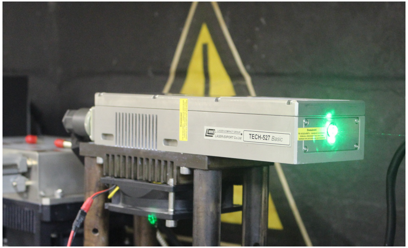


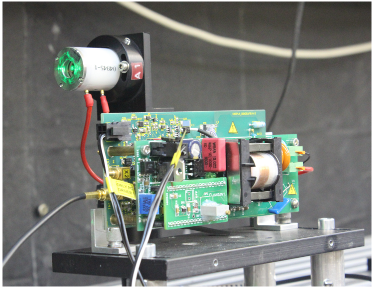




Laser and polarization

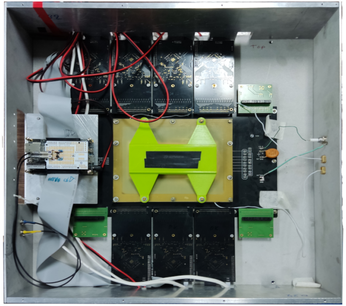
- Nd:YLF with frequency doubling
- Wavelength: 527 nm
- Pump frequency up to 4 kHz
- Average power: 2 W
- Pulse duration: 5 ns (1.5 m)
- Pulse instability: 2 ns



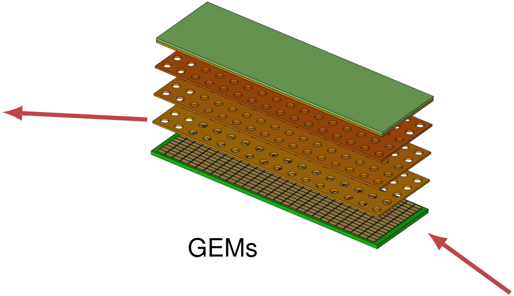


- Circular polarization is prepared using a $\lambda/4$ phase plate.
- Switching between left and right circular polarization is achieved using a KD*P Pockels cell. Half-wave voltage is 3 kV.
- Switching modes for each laser pulse:
  - Switching from n-left to n-right, where n=1-16.
  - Pseudo-random switching based on a linear feedback shift register (LFSR).

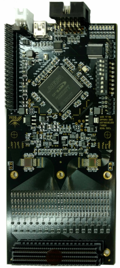
Coordinate Photon Detector



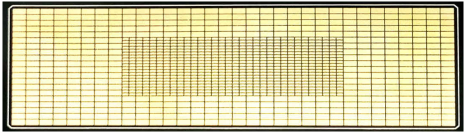
Detector Configuration



GEMs



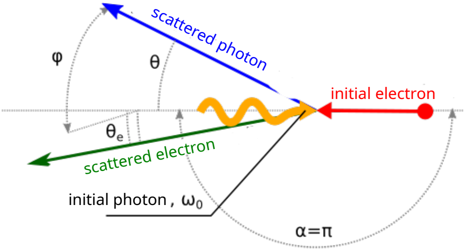
Readout board



Readout electrodes

- Gaseous (Ar/CO_2) electron multiplier: 3 layers
- Sensitive area 40 $\times$ 128 mm
- 640 channels in the center + 512 channels on the periphery
- Max trigger rate 4 kHz

Compton Backscattering Cross Section



P — transverse polarization of electrons
Q — Stokes parameter of linear polarization of photons
V — Stokes parameter of circular polarization of photons
β — inclination of the plane of linear polarization
$\kappa = 4\gamma\omega/m_e$ — photon "hardness"

$$\frac{d\sigma(P,Q,V,\varphi,\beta)}{d\Omega_{\text{lab}}} = 2\gamma^2 r_e^2 \left[\frac{1}{1+\gamma^2\theta^2+\kappa}\right]^2 \left\{2 + \frac{\kappa^2}{(1+\gamma^2\theta^2)(1+\gamma^2\theta^2+\kappa)} - \frac{4\gamma^2\theta^2}{(1+4\gamma^2\theta^2)^2}\left(1 - Q\cos(2[\varphi-\beta])\right) + \frac{2\kappa PV\gamma\theta\sin\varphi}{(1+\gamma^2\theta^2)(1+\gamma^2\theta^2+\kappa)}\right\}.$$

Linear polarization γ

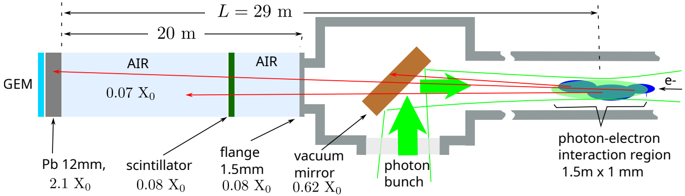
Circular polarization γ

Data Processing: Deconvolution Method

Detector respond

$$D^{L,R}(x,y) = \frac{dN^{L,R}(x,y)}{dxdy} = \int B(x,y,\theta'_x,\theta'_y)C^{L,R}(\theta'_x,\theta'_y)d\theta'_x d\theta'_y \approx \int B(x-x',y-y')C^{L,R}\left(\frac{x'}{L},\frac{y'}{L}\right)\frac{dx'dy'}{L^2}$$

$$D^{L,R}(x,y) \approx B(x,y)\otimes\tilde{C}^{L,R}(x,y),$$



Compton cross section

$$C(\theta_x,\theta_y)^{LR} = \frac{d\sigma^{L,R}}{d\theta_x d\theta_y}(\theta_x,\theta_y),$$

$$\hat{D} = \mathcal{F}\left[\frac{D^L}{N_L}+\frac{D^R}{N_R}\right], \quad \hat{C} = \mathcal{F}[\tilde{C}^L+\tilde{C}^R],$$

$$B^*(x,y) = \mathcal{F}^{-1}\left(\frac{\hat{D}}{\hat{C}+\delta}\cdot R\right),$$

$$R = \frac{|\hat{C}|^2}{|\hat{C}|^2 + k\sum|\hat{C}|^2},$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ are the forward and inverse 2D Fourier transform. $\delta \approx 10^{-12}$ is a regularization parameter to suppress zeros in the denominator, and R is the Wiener regularization function.

Two-dimensional Fitting

- Preliminary data filtering: filling with average values for turned-off channels.
- Discrete Fourier Transform in an extended area: (32×20) → (96×60) to suppress edge effects of Furier transformation.
- Minimization of χ^2:

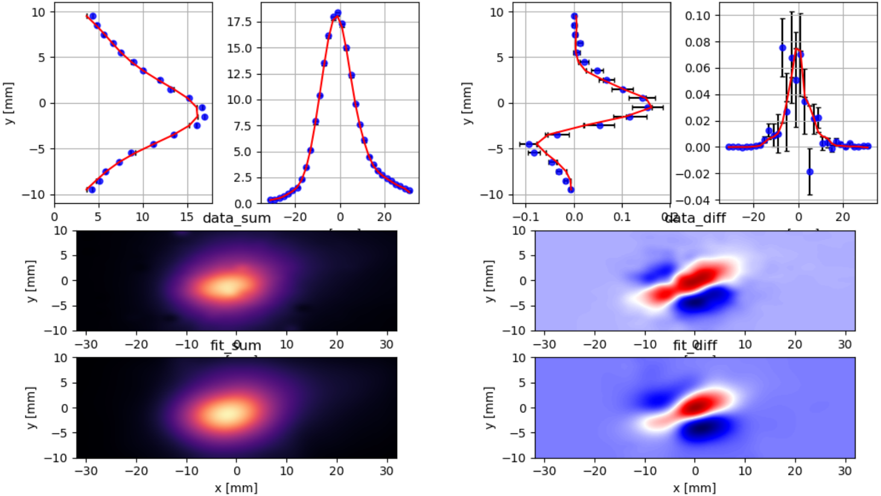
$$\chi^2 = \sum_{x,y} \frac{(\Delta D(x,y) \cdot [B^* \otimes C] - D(x,y) \cdot [B^* \otimes \Delta C])^2}{(B^* \otimes C - B^* \otimes \Delta C)^2 \cdot D^L(x,y)/N_L^2 + (B^* \otimes C + B^* \otimes \Delta C)^2 \cdot D^R(x,y)/N_R^2},$$

  where $\Delta D(x,y) = D^L/N_L - D^R/N_R$, $\Delta C(x,y) = C^L - C^R$.
- Five free parameters: P, Q, β, $\delta N = (N_L - N_R)/(N_L + N_R)$. Regularization parameter $k_{reg} = 10^{-4}$ is manually chosen and fixed.

2D fit example

Fit results (fit method 3)

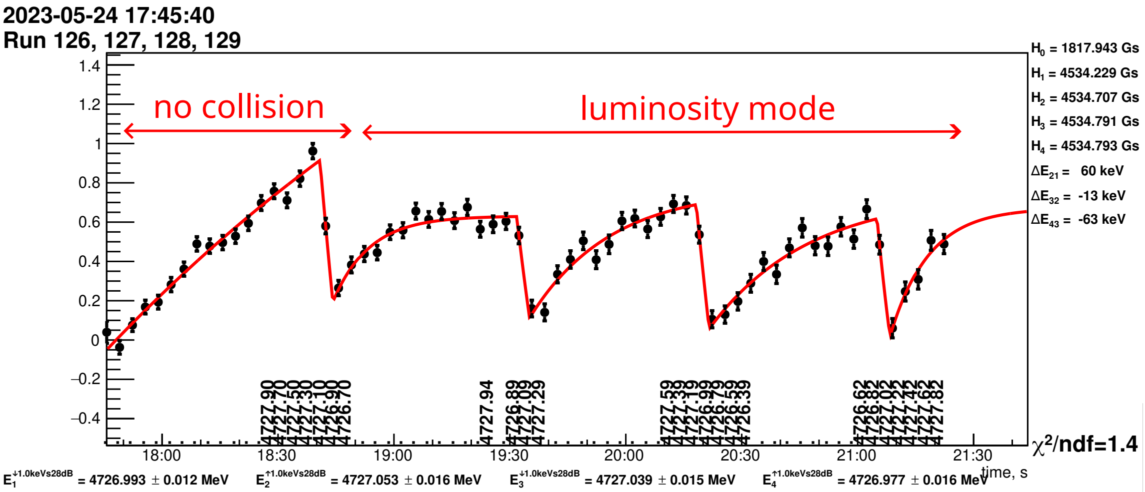
begin: 2023-05-24 18:34:00
end: 2023-05-24 18:34:50
$\chi^2/ndf = 717/636 = 1.13$
$prob(\chi^2) = 0.0143$
$L = 29.90$ m
$P = 0.808 \pm 0.077$
$Q = -0.494 \pm 0.014$
$\beta = 40.57 \pm 0.73^\circ$
$DN = 0.002 \pm 0.001$
$k_{reg} = 1.0e-4$



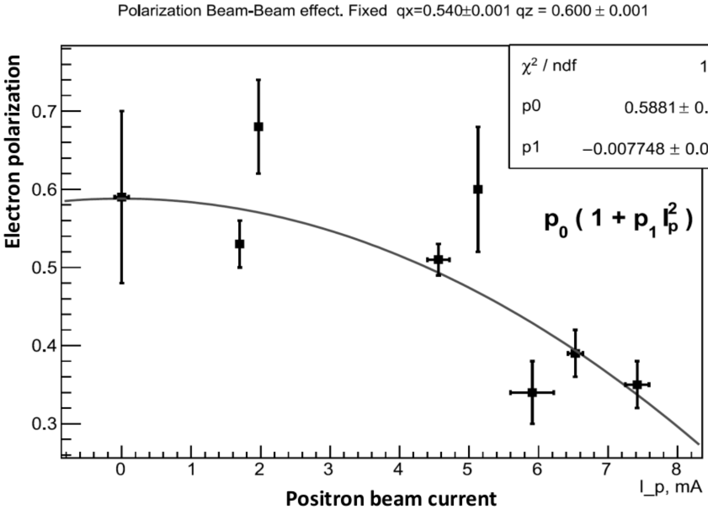
Energy Calibration Procedure

1. Beam preparation (average duration 45 minutes)
    - Reset of previous beams, cycle in VEPP-4
    - Accumulation of electrons in VEPP-3, acceleration, and transfer to VEPP-4
    - Accumulation of positrons in VEPP-3, acceleration, and transfer to VEPP-4
    - Acceleration of beams from 1.9 GeV to 4.7 GeV.
2. Relaxation of fields and radiation polarization (approximately 45 minutes). Beams are separated.
3. Luminosity and data collection (2 hours) by the KEDR detector with simultaneous energy calibrations. A total of 3 calibrations per run with alternating scanning directions.

Energy measurements by Laser polarimeter during $\Upsilon(1S)$ scan

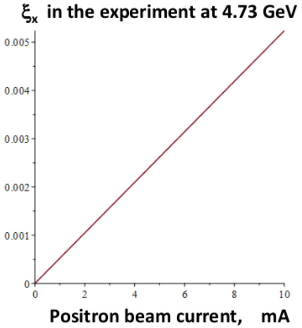


Measured beam-beam depolarization effect (BBD) at $\Upsilon(1S)$ energy region



Critical beam-beam parameters (luminosity) at VEPP-4M:

$\xi_x / \xi_y \approx 0.02/0.04$



The depolarizing effect of the counter beam was manifested at currents several times lower than the critical one

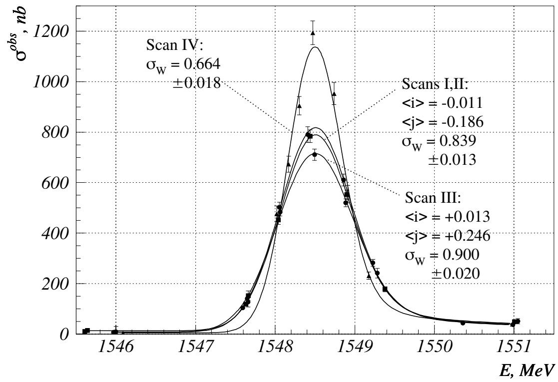
Energy calibration accuracy

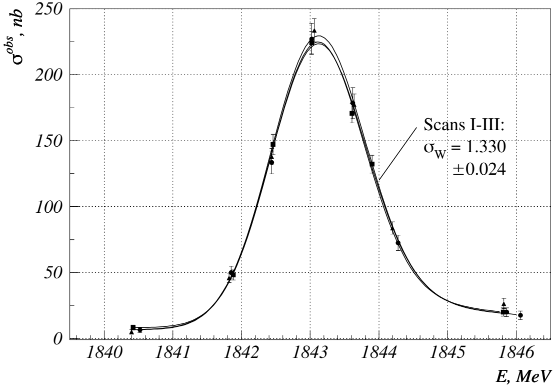
1. Measurement of the spin precession frequency by resonance depolarization
   - 50 Hz side spin resonances due to pulsation in magnets
   - Spin line width
   - Energy drift (temperatures, tides, etc...)
2. Calculation of average beam energy
3. Calculation of beam energy at the interaction point
4. Calculation of luminosity weighted average c.m. energy

More about corrections and errors to center of mass energy

Bogomyagkov, et al., RUPAC-2006-MOAP02.
Nikitin, RUPAC-2006-MOAP01.
Bogomyagkov, et al., Conf. Proc. C 070625 (2007) 63.

J/ψ ($0.7, pb^{-1}$), $\psi(2S)$ ($1.0\,pb^{-1}$) mass measurement with KEDR detector





$$M_{J/\psi} = 3096.900 \pm 0.002 \pm 0.006\ MeV$$
$$M_{\psi(2S)} = 3686.099 \pm 0.004 \pm 0.009\ MeV$$

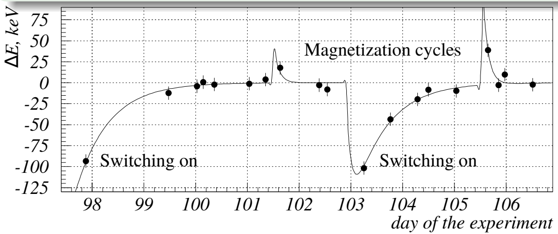
KEDR Collaboration / Phys.Lett.B 573 (2003) 63–79
Anashin et al. / Phys.Lett.B 749 (2015) 50–56

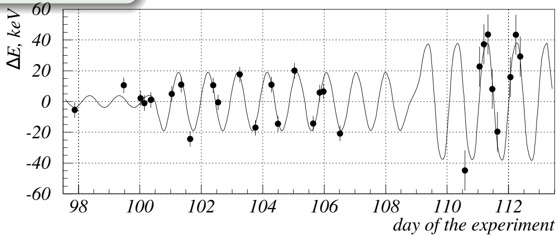
Energy interpolation between calibrations

Energy prediction function

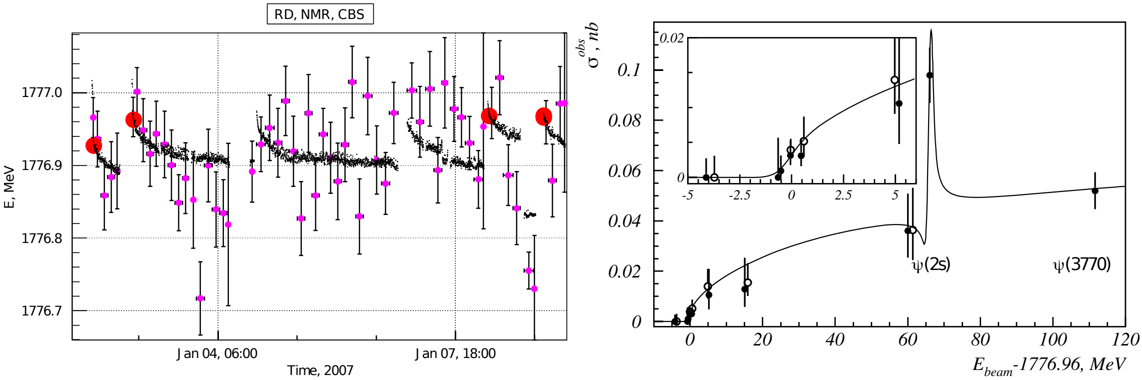
$$E = \alpha_H \cdot H_{\text{NMR}} \cdot (1 + \alpha_T \cdot (T_{\text{ring}} - T_{\text{NMR}})) \times f(T_{\text{ring}}, T_{\text{air}}, T_{\text{water}}) +$$
$$+ A(t) \cdot \cos\left(\frac{2\pi t}{\tau_{\text{day}}} - \varphi(t)\right) +$$
$$\delta E_{\text{on}} \cdot \exp\left(-\frac{t_{\text{on}}}{\tau_{\text{on}}}\right) + \delta E_{\text{cycle}} \cdot \exp\left(-\frac{t_{\text{cycle}}}{\tau_{\text{cycle}}}\right) + E_0(\Delta i, t),$$

Energy prediction $6 \div 8$ *keV* with 218 energy calibrations





Energy calibration in tau mass experiment

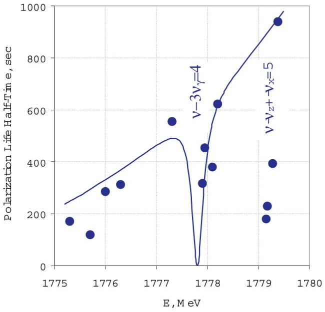


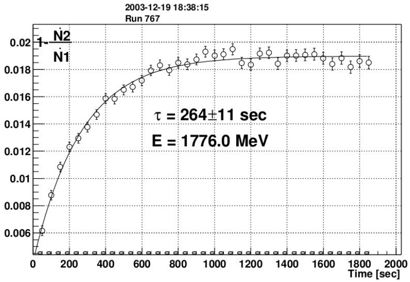
$$M_\tau = 1776.69^{+0.17}_{-0.19} \pm 0.15$$

A.G.Shamov / Nuclear Physics B (Proc. Suppl.) 189 (2009) 21–23

Small polarization lifetime at tau threshold

- Tau threshold (1.78 GeV) close to $\nu = 4$ integer spin resonance (E=1.76 GeV). No polarization in VEPP-3.
- Special effort to increase polarization lifetime at tau threshold were done.





- Polarization at 1.85 GeV and decelerate to tau threshold
- Energy calibration after 30 min magnetic field relaxation

Depolarization model

- Froissart-Stora exact solution for single crossing of isolated resonance with harmonic amplitude w and spin detune speed $\dot{\epsilon}$ (ω_0 is the revolution frequency).

$$\Delta\zeta = 2\zeta\left(\exp\left\{-\frac{\pi|w|^2\omega_0}{2|\dot{\epsilon}|}\right\} - 1\right) \approx -\frac{\pi|w|^2\omega_0}{|\dot{\epsilon}|}\zeta, \quad |\Delta\zeta/\zeta| \ll 1$$

- For spin $\zeta(\nu, \dot{\nu}, \nu_s(t))$ and depolarizer line $w(\nu, \nu_d(t))$ distributions, spin detune $\dot{\epsilon} = \dot{\nu}_d - \dot{\nu}$

$$\dot{\zeta} = -\pi\omega_0 \int_{-\infty}^{\infty} \frac{|w(\nu, \nu_d)|^2}{|\dot{\epsilon}|}\zeta(\epsilon, \dot{\epsilon}, \epsilon_s)|\dot{\epsilon}|d\nu d\dot{\nu} = -\pi\omega_0 \int_{-\infty}^{\infty} |w(\nu, \nu_d(t))|^2\zeta\left(\nu, \nu_s(t)\right) d\nu$$

- Monotonic scan case: $\dot{\epsilon} = \dot{\nu}_d - \dot{\nu}_s = const$

$$\zeta(t) = \zeta_0 \exp\left(-\frac{\pi\omega_0|w|^2}{|\dot{\nu}_d - \dot{\nu}_s|}\Theta(t)\Big|_{t_0}^{t}\right)$$

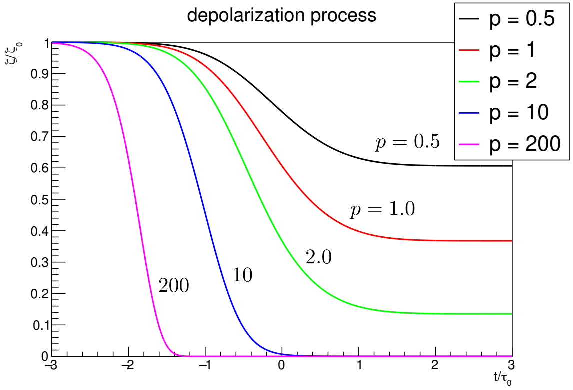
- For Gaussian spin&depolarizer distributions

$$\Theta(t) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{\dot{\epsilon}(t - t_d)}{\sqrt{2}\sigma}\right)\right]$$

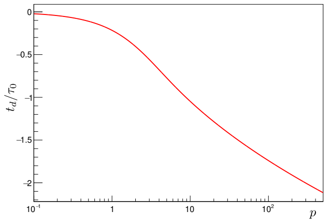
$\Theta(t)$ is a step-like dimensionless function

- Depolarization when "strength":

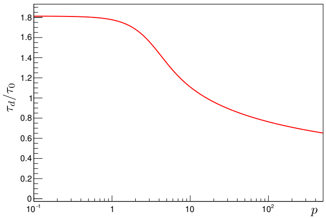
$$p = \frac{\pi\omega_0|w|^2}{|\dot{\nu}_d - \dot{\nu}_s|} = \frac{|w|^2 2\pi^2 f_0\,[\text{Hz}]}{\dot{E}\,[\text{keV/s}]} \times 440648\,[\text{keV}] \gtrsim 1$$



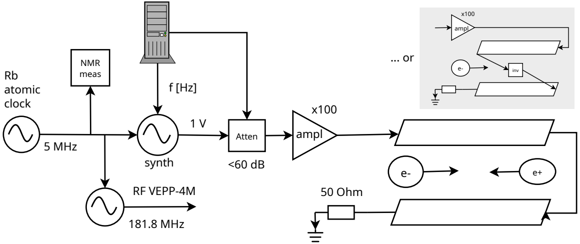
- Depolarization moment shift for different p



- Depolarization time for different p



TEM-wave depolarizer



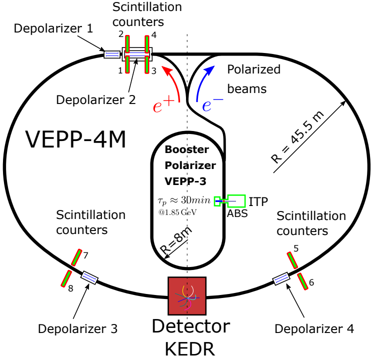
$$|w| = \nu|F^\nu|\frac{U\,l_d}{2\pi d_d B_\rho} = \nu|F^\nu|\frac{\Delta\varphi_\perp}{2\pi}$$

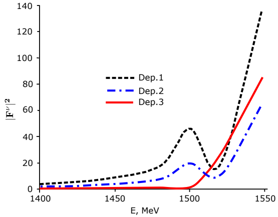
$$\frac{\Delta E}{E} = -\frac{1}{\alpha}\frac{\Delta\omega_0}{\omega_0}$$

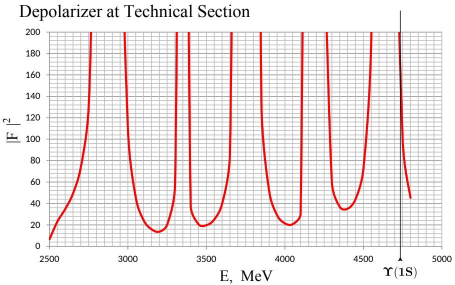
α is the momentum compaction factor

- Common Rb standard of 10^{-10} at VEPP-4M provides an energy stability of 10^{-8} ($\alpha = 0.017$).
- To compensate for the FCC-ee energy drift of about 1 keV/s due to tidal effects by tuning the RF frequency, a frequency standard of at least 10^{-13} ($\alpha \sim 10^{-5}$) would likely be required.

Depolarizer location and spin response function F^{ν}

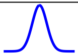
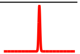
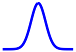
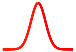
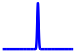
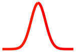






Due to the large values of ν and the spin response factor (DKS, 1979), it is beneficial at high energies. The factor F^{ν} was measured for the first time in the VEPP-4 experiment to study resonant spin diffusion in the field of a counter TEM wave in the early 80s.

Scan modes applied on VEPP-4M

Type	w	dE/dt keV/c	dynamic depol width, keV	τ_d s	ΔE	Relative line width depolarizer	spin
rough	$\sim 10^{-6}$	10	2.3	~ 1	10 keV		
normal	5×10^{-7}	0.3	0.4	~ 1	2 keV		
fine	4×10^{-8}	0.005	0.05	~ 100	2 eV		

- **rough**: quick energy measurements with wide scan range, low accuracy.
- **normal**: most precise calibrations in narrow $q\bar{q}$ resonance peaks. Systematic error is about spin width
- **fine**: precise comparison of spin frequencies of electron and positron. But unknown systematic error depending on spin line shape

Spin line width

Spin line half-width due to radiative diffusion of spin precession phase

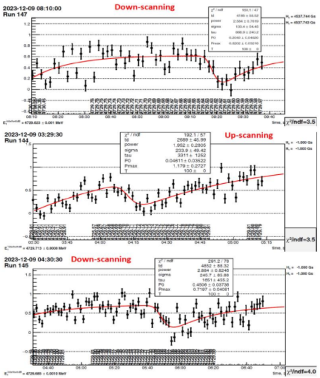
$$\varepsilon_{\text{diff}} = \nu \frac{\alpha}{2} \sigma_\gamma^2$$

Spin line half-width due to sextupoles

$$\varepsilon_{\text{nl}} = \nu \left\langle B'' \left(\sigma_{x\beta}^2 + \sigma_{x\gamma}^2 \right) \right\rangle$$

	E GeV	f_0 kHz	σ_ν spin tune spread due to energy spread [turn^{-1}]	ν_γ synchrotron tune [turn^{-1}]	σ_ν / ν_γ modulation index	$\lambda_\gamma / 2\pi$ radiation decrement [rad^{-1}]	ε_{nl} due to non-linearity [turn-1]	$\varepsilon_{\text{diff}}$ due to radiative diffusion [turn^{-1}]	$\frac{\sqrt{\varepsilon_{\text{nl}}^2 + \varepsilon_{\text{diff}}^2}}{\nu}$	Spin line half- width [keV]
VEPP-4M	1.85 4.73	820	0.0013 0.0072	~0.01 0.015	~0.13 ≈0.5	1.8e-6 3.0e-5	~2e-6 ~2.5e-5	2.3e-7 1.5e-5	≈5e-7 ≈2.7e-6	≈1 ≈13
LEP	45.6	11	0.061	0.083	0.73	4.7e-4	-	3.4e-4	~3e-6	~140
FCC-ee	45.6 80	3	0.039 0.120	0.025 0.051/0.080	1.56 2.37/1.50	1.25e-4 6.8e-4	~7.3e-5 -	2e-4 1.6e-3/1.0e-3	≈2.3e-6 8.8e-6/5.6e-6	≈108 705/450

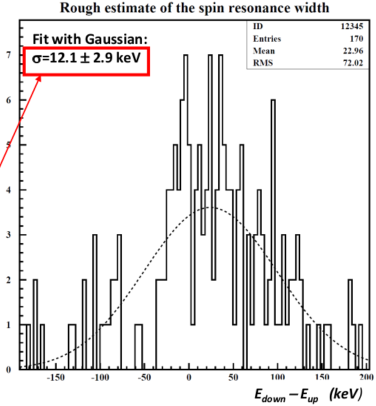
Special Fine up-down scanning at 0.05 keV/s



Fit of long-drawn jumps

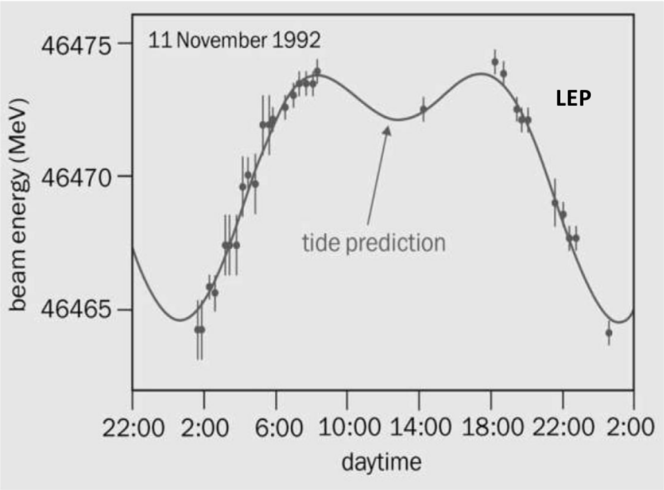
Run	χ^2/ndf	p	σ, sec	σ, keV
144	3.5	2.0±0.3	234±49	11.7±2.6
145	4.0	2.9±0.8	245±86	12.3±4.3
147	3.5	2.6±0.8	133±54	6.7±2.7
	average	2.2±0.3	196±33	9.8±1.7

Routine RD up-down scans during Y-running at 1 keV/s



Both experimental results are close to the theoretical estimate of the spin half-linewidth σ ≈13 keV

Effect of Earth tides on beam energy drift in supercolliders



High tides The evolution of the beam energy at LEP due to Earth tides, showing the measurements from resonant depolarisation (red points), and the predictions of a model. At FCC-ee the Earth tides, if uncorrected, will induce energy changes that are an order of magnitude larger. Source: J Wenninger

From CERN Courier article by
A. Blondel, J. Keintzel and Guy Wilkinson.
The power of polarisation for FCC-ee physics.
16 Nov. 2022

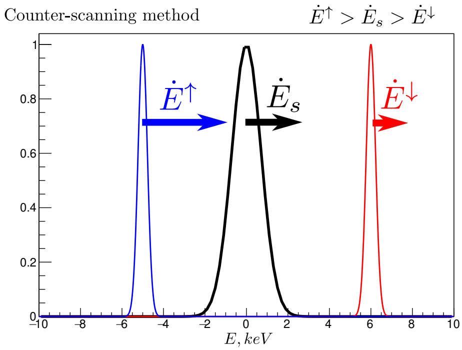
"The gravitational pull of the moon distorts the tunnel in "Earth tides ..."

Energy drift at LEP during Z-running:
" ... around 10 MeV over a few hours..." .
Expected drift at FCC-ee: "20 times larger"
(about 7 keV/s)

"...At FCC-ee these distortions will be combatted by adjustment of the radio frequency (RF) cavities, as is now routinely done in the LHC. ..."

Counter-scanning method

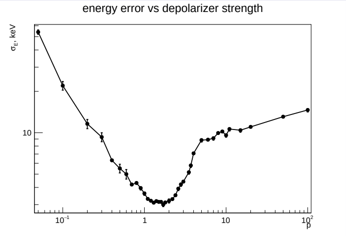
- Model dependent depolarization moment shift requires depolarization independent depolarization of two bunches with counter scanning.
- In absence of energy drift determination and averaging of the moment of half polarization changes would give true energy value.
- But in case of energy drift one need to apply some model and use joint fit of counter scanning.
- This allow one to determine energy and drift speed at some time point.



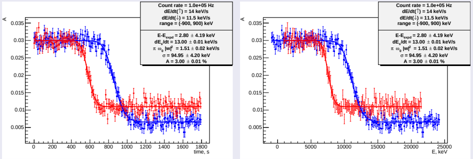
Counter-scanning simulation result for FCCee: $\sigma_E \approx 3$ keV, $\dot{E}_s = 13 \pm 0.01$ keV

$$\sigma_E \sim \frac{|\dot{E}_d - \dot{E}_s|}{\sqrt{\dot{N}}} \frac{(T^2 + \tau_d^2)^{1/4}}{\zeta_0(1 - e^{-p})}$$

Optimal depolarizer strength $p = 1.5 \div 2$



$p = 1.5$, $\dot{E}^{\uparrow\downarrow} = 14, 11.5$ keV/s, $\dot{E}_s = 13$ keV/s



Summary

- Resonant depolarization method is the most precise method of beam energy calibration ($\simeq 10^{-6}$)
- Requires polarized beam
- Need special time to measure spin precession frequency
- Need beam energy interpolation between calibrations. NMR,temperatures, moon phase...
- Requires calculation of the c.m. energy from measured spin precession frequency.
- Counter-scanning method of simultaneously measurement of the beam energy and energy drift speed.
- Possible BBD effects could alter longitudinal polarization of future colliders

THANK YOU