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Overview and perspective of light baryon radii and nucleon form factors

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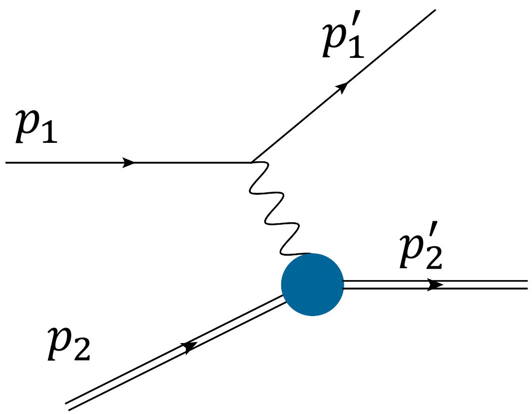
Based on
Yong-Hui Lin, FKG, U.-G. Meißner, PLB 856, 138887 (2024); PLB 858, 139023 (2024);
Xiong-Hui Cao, FKG, Qu-Zhi Li, De-Liang Yao, arXiv:2411.13398

17-21 Nov. 2024

Electric form factor

- Electron scattering off a charge distribution



charge density

$$\propto \int d^3r \left[\int d^3r' \frac{e^2 \rho(\vec{r}')}{|\vec{r} - \vec{r}'|} \right] e^{-i\vec{q}\cdot\vec{r}}$$

$$\propto \frac{e^2}{\vec{q}^2} \int d^3r' \, \rho(\vec{r}') \, e^{-i\vec{q}\cdot\vec{r}'}$$

$$= \frac{e^2}{-q^2} \underbrace{\int d^3r' \, \rho(\vec{r}') \, e^{-i\vec{q}\cdot\vec{r}'}}$$

in the Breit frame, $q^2 = -\vec{q}^2$

Form factor $F(q^2) = F(-\vec{q}^2)$ is the Fourier transform of the charge density in the Breit frame

$$\rho(\vec{r}) = \int \frac{d^3q}{(2\pi)^3} F(-\vec{q}^2) \, e^{-i\vec{q}\cdot\vec{r}}$$

- Charge radius

$$\langle r^2 \rangle = \frac{\int d^3\vec{r} r^2 \rho(\vec{r})}{\int d^3\vec{r} \rho(\vec{r})} = -6 \frac{F'(0)}{F(0)} \Rightarrow F(-\vec{q}^2) = F(0) \left(1 - \frac{\langle r^2 \rangle}{6} \vec{q}^2 + \cdots \right)$$

we have used $\int d^3\vec{r} \rho(\vec{r}) = F(0)$ and $\int d^3\vec{r} r^2 \rho(\vec{r}) = -6F'(0) \equiv -6 \frac{dF(-\vec{q}^2)}{d\vec{q}^2} |_{\vec{q}^2=0}$

Proton EM form factor

- Nucleon electromagnetic form factor

$$\langle N(p')|J^{\nu}_{\text{em}}|N(p)\rangle = \bar{u}(p')\left[\gamma^{\nu}F_1(q^2) - \frac{iF_2(q^2)}{2m_N}\sigma^{\mu\nu}q_{\mu} + i(\gamma^{\nu}q^2\gamma_5 - 2m_N q^{\nu}\gamma_5)F_A(q^2) - \frac{F_3(q^2)}{2m_N}\sigma^{\mu\nu}q_{\mu}\gamma_5\right]u(p)$$

Lorentz invariant form factors (FFs)

F_1: Dirac FF; F_2: Pauli FF; F_A: P-violating anapole FF; F_3: P, CP-violating electric dipole FF

Sachs FFs $(t = q^2)$ Ernst, Sachs, Wali, PR 119, 1105 (1960); Sachs, PR 126, 2256 (1962)

$$G_E(t) = F_1(t) + \frac{t}{4m^2}F_2(t), \qquad G_M(t) = F_1(t) + F_2(t)$$

Fourier transforms of the charge and magnetization distributions in the Breit frame

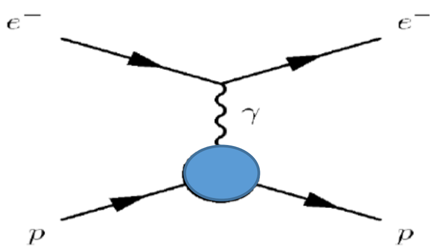
$$G_E(t) = G_E(0)\left(1 + \frac{\langle r_E^2\rangle}{6}t + \cdots\right)$$

$$G_E(0) = e_N \quad (\text{charge}), \qquad G_M(0) = \mu_N \quad (\text{magnetic moment})$$

Therefore, for a precise measurement of $\langle r_E^2\rangle$, we need as small t as possible

Proton EM form factor

- Electron scattering

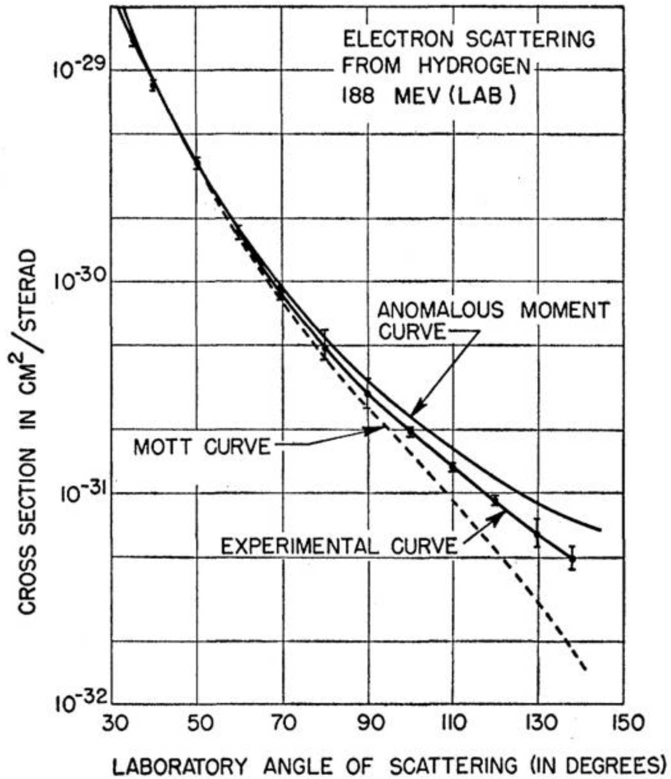


Electron Scattering from the Proton

Robert Hofstadter and Robert W. McAllister

Phys. Rev. **98**, 217 – Published 1 April 1955

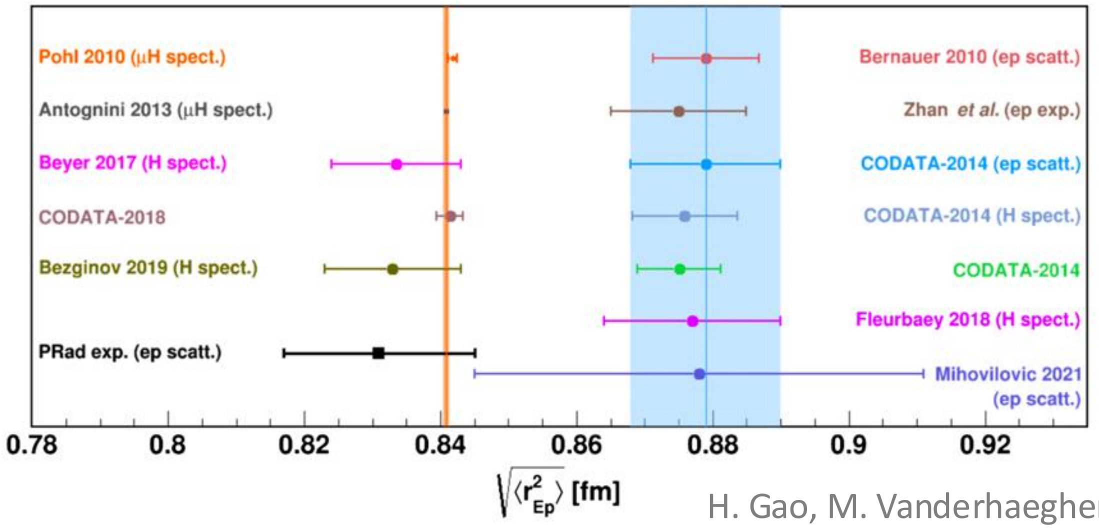
well by the following choices of size. At 188 Mev, the data are fitted accurately by an rms radius of $(7.0\pm2.4)\times10^{-14}$ cm. At 236 Mev, the data are well fitted by an rms radius of $(7.8\pm2.4)\times10^{-14}$ cm. At 100 Mev the data are relatively insensitive to the radius but the experimental results are fitted by both choices given above. The 100-Mev data serve therefore as a valuable check of the apparatus. A compromise value fitting all the experimental results is $(7.4\pm2.4)\times10^{-14}$ cm. If the proton were a spherical ball of charge, this rms radius would indicate a true radius of 9.5×10^{-14} cm, or in round numbers 1.0×10^{-13} cm. It is to be noted that if our interpretation is correct the Coulomb law of force has not been violated at distances as small as 7×10^{-14} cm.



Proton charge radius

- Spectroscopy method:

  measuring the charge radius from Lamb shift of (muonic) hydrogen atom

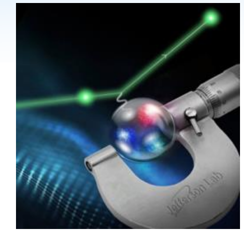


H. Gao, M. Vanderhaeghen, RMP 94, 015002 (2022)

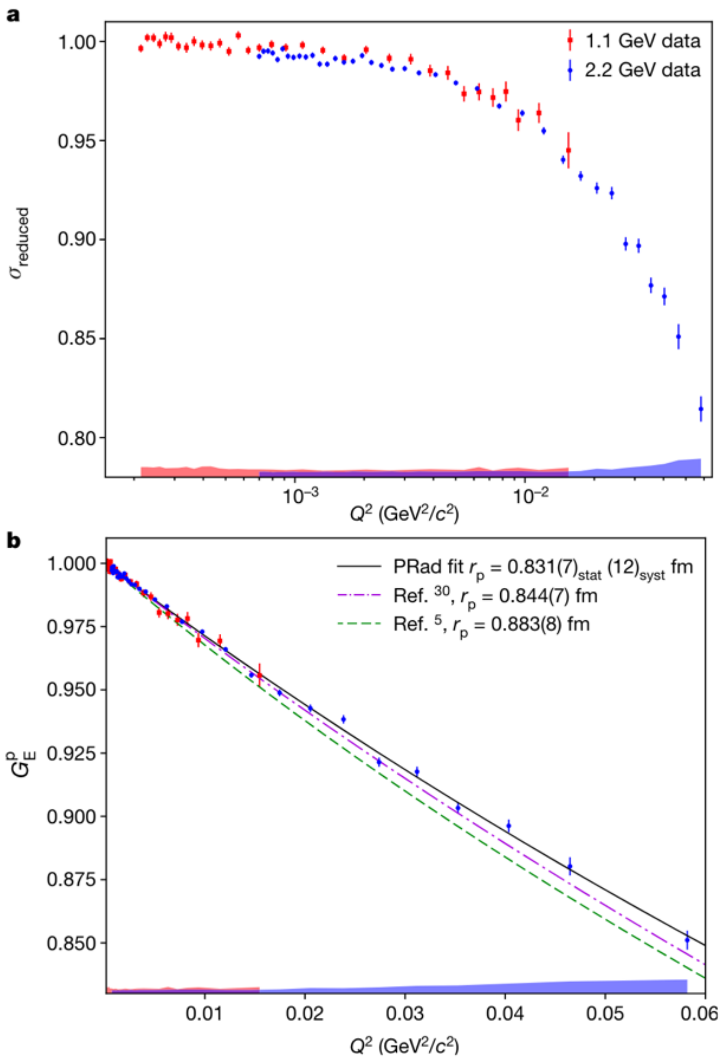
FIG. 18. The proton charge radius $\langle r^2_{Ep} \rangle^{1/2}$ as extracted from electron-scattering and spectroscopic experiments since 2010 and before 2020 together with CODATA-2014 and CODATA-2018 recommended values. Note the reinterpreted result from the Mainz ISR experiment was scheduled for publication in 2021. From Jingyi Zhou.

- The latest CODATA value: $0.84075(64)$ fm R. Mohr et al., arXiv:2409.03787

PRad measurement



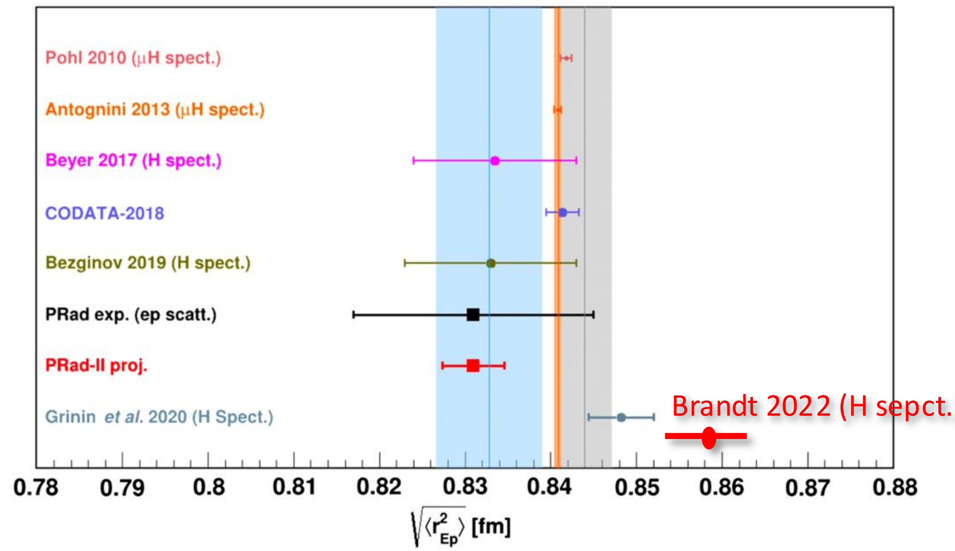
- ep scattering with $Q^2 \in [2.1 \times 10^{-4}, 0.06]$ GeV^2 in the spacelike region W. Xiong et al. [PRad], Nature 575, 147 (2019)



$$r_p = 0.831 \pm 0.007_{stat} \pm 0.012_{syst}\ \mathrm{fm}$$

- Prad-II will cover $Q^2 \in [4 \times 10^{-5}, 0.06]$ GeV^2

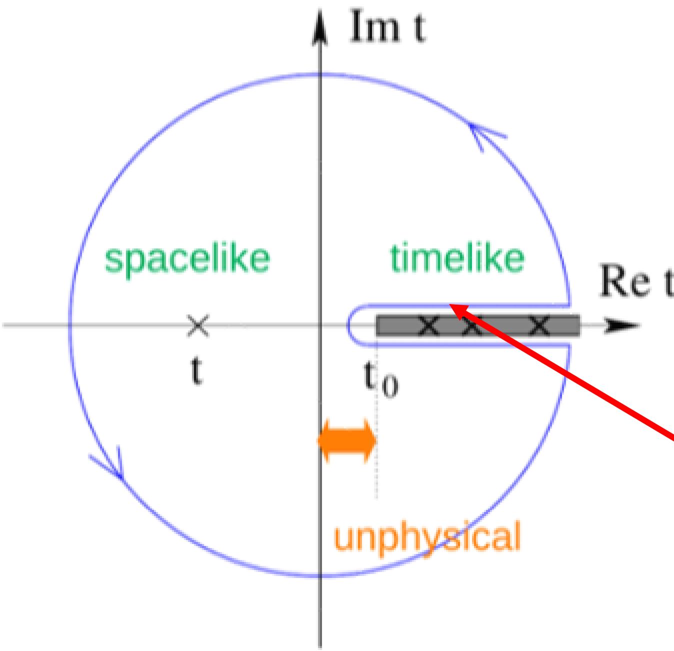
H. Gao, M. Vanderhaeghen, RMP 94, 015002 (2022);
A. Gasparian et al. [PRad-II], arXiv:2009.10510;
private communication with W.-Z. Xiong



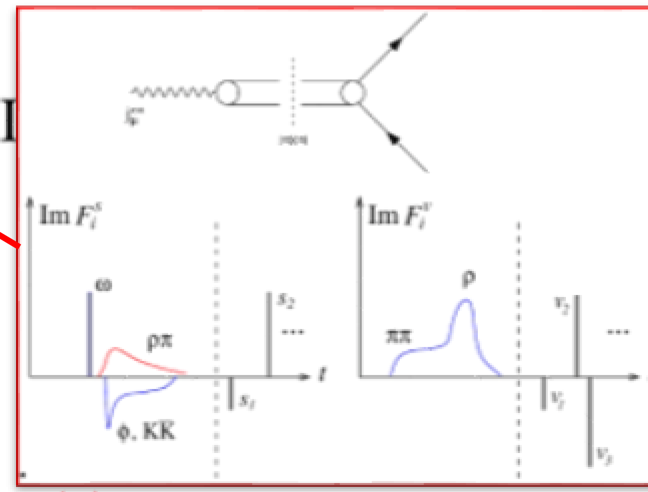
Proton radius puzzle not yet completely solved.

Dispersive approach

Y.-H. Lin, talk at HAPOF-28 (https://indico.itp.ac.cn/event/100/)



$$F(t) = \frac{1}{\pi}\int_{t_0}^{\infty} \frac{\mathrm{Im}\, F(t')}{t' - t - i\epsilon}\, dt'$$

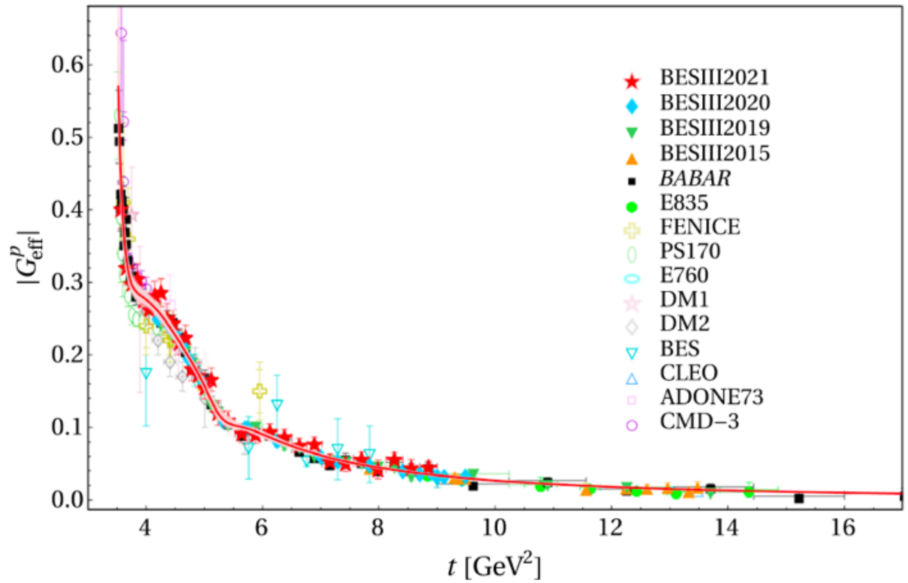


The spectral function $\mathrm{Im}F(t)$ are central quantities.

07/23/21 8 YHL

Dispersive approach

Y.-H. Lin, H.-W. Hammer, U.-G. Meißner, PRL 128, 052002 (2022)



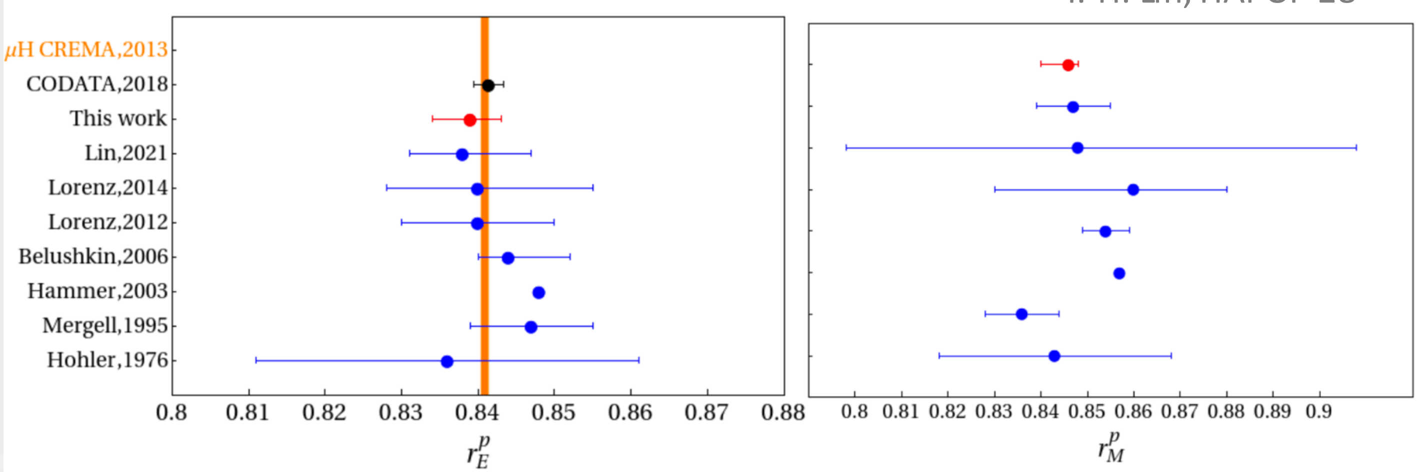
$$|G_{\rm eff}| \equiv \sqrt{\frac{|G_E|^2 + \xi |G_M|^2}{1+\xi}}$$

$$r_E^p = 0.839 \pm 0.002^{+0.002}_{-0.003} \,{\rm fm},$$

$$r_M^p = 0.846 \pm 0.001^{+0.001}_{-0.005} \,{\rm fm}$$

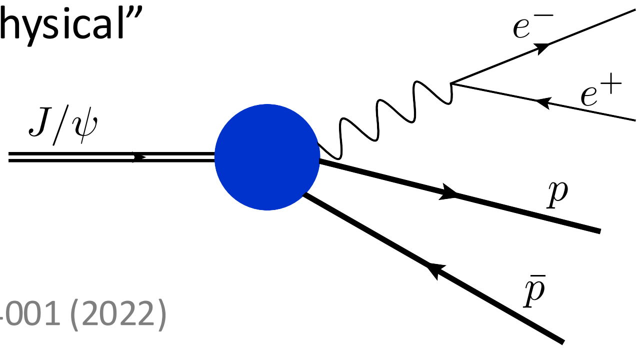
- Comparing to existing DR determination

Y.-H. Lin, HAPOF-28



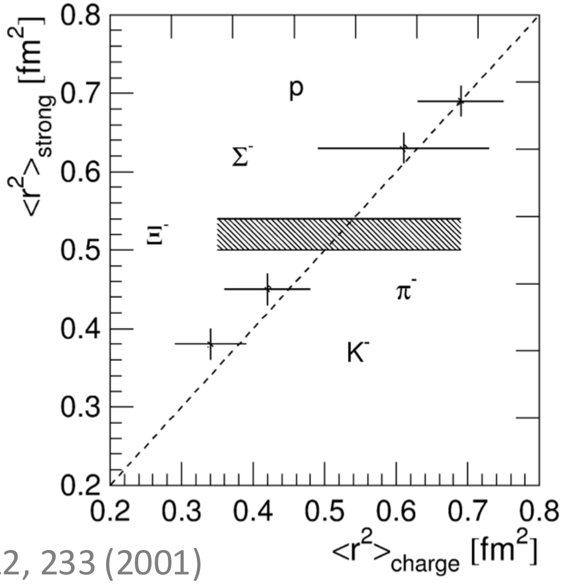
Dalitz decay

Yong-Hui Lin, FKG, U.-G. Meißner, PLB 156, 138887 (2024)



- Possibility to measure the proton FFs in the time-like "unphysical" region?
  - Dalitz decay $J/\psi \to p\bar{p}e^+e^-$ by measuring the e^+e^- distribution
  - BESIII has 10^{10} J/ψ and 2.7×10^9 ψ' BESIII, CPC 46, 074001 (2022)
  - STCF can collect 3.4×10^{12} J/ψ and 6.4×10^{11} ψ' per year STCF, Front. Phys. 19, 14701 (2024)
  - Can reach very small $q^2 \sim 4\,m_e^2 = 1.05 \times 10^{-6}\ \mathrm{GeV}^2$
    - e^+ and e^- can be efficiently detected as long as they have transverse momenta larger than a few tens of MeV (~ 50 MeV at BESIII from H.-B. Li)
    - Collinear e^+e^- $\Rightarrow$ threshold kinematics
  - More similar Dalitz decays:
    - $J/\psi \to \pi^+\pi^-e^+e^-, K^+K^-e^+e^-$
    - $\psi' \to \Xi^-\bar{\Xi}^+e^+e^-$, $\psi' \to \Sigma^\pm\bar{\Sigma}^\mp e^+e^-$
    - …

Among all hyperons, only the Σ^- charge radius has been measured: 0.78 ± 0.10 fm, with Σ^- beam scattering off atomic electrons



SELEX (E781), PLB 522, 233 (2001)

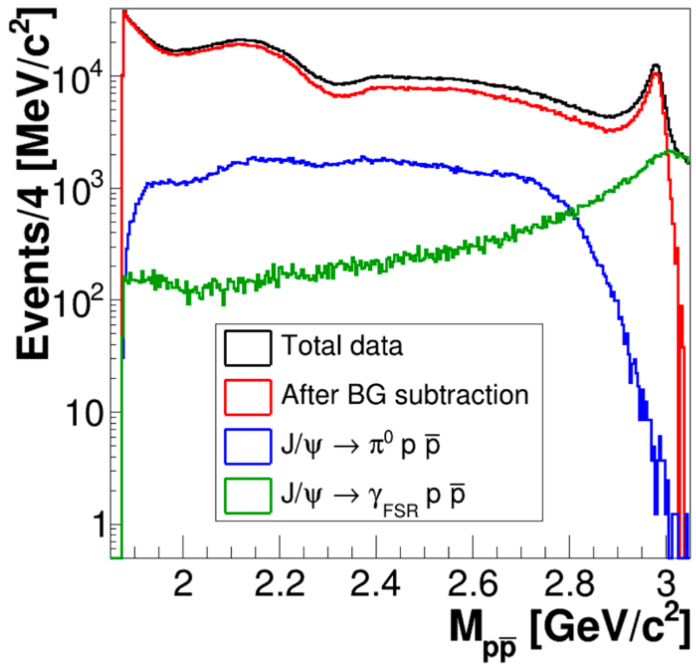
Dalitz decay

Take $J/\psi \to p\bar{p}e^+e^-$ as an example

Problems:

- Requires final-state radiation (FSR) virtual photon, only a small portion from the whole decay events
  - method subtracting the major background and/or partial-wave analysis
- For FSR photon, measures transition FFs from some intermediate state A to $p\gamma^*$, proton is only part of A
  - to identify a region dominated by the proton pole
  - For large $\mathrm{m}_{p\bar{p}}$, both $m_{p\gamma^*}$ and $m_{\bar{p}\gamma^*}$ are small, proton and antiproton pole dominance may work

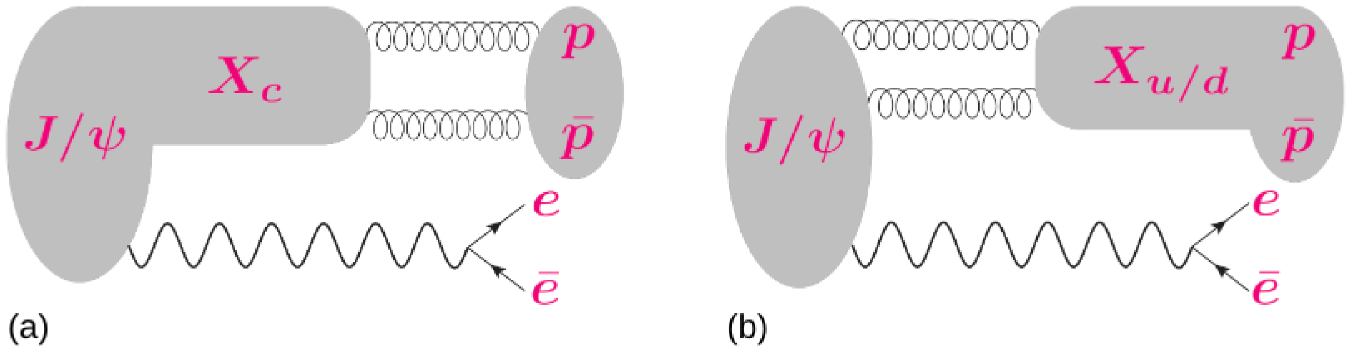
$J/\psi \to p\bar{p}\gamma$



R. Kappert, PhD thesis, Groningen U. (2022)

Decay mechanisms

- Virtual photon emitted
  - from (anti-)charm quark, type X: diagrams (a) and (b)
    - $c\bar{c} \to$ two gluons
      - type-X_c: η_c
      - type-$X_{u/d}$: light meson resonances such as $X(1835)$, …
    - isospin symmetric: $\mathcal{A}_X(p\bar{p}e^+e^-) = \mathcal{A}_X(n\bar{n}e^+e^-)$ up to $\mathcal{O}(1\%)$
      - Isospin breaking effects: from quark mass difference $\mathcal{O}\left(\frac{m_d-m_u}{\Lambda_{\rm QCD}}\right)$ or from virtual photons $\mathcal{O}(\alpha)$
      - Similarly, $\mathcal{A}_X(\Xi^-\bar{\Xi}^+e^+e^-) = \mathcal{A}_X(\Xi^0\bar{\Xi}^0e^+e^-)$, … ; + easier to detect neutral hyperons than neutrons

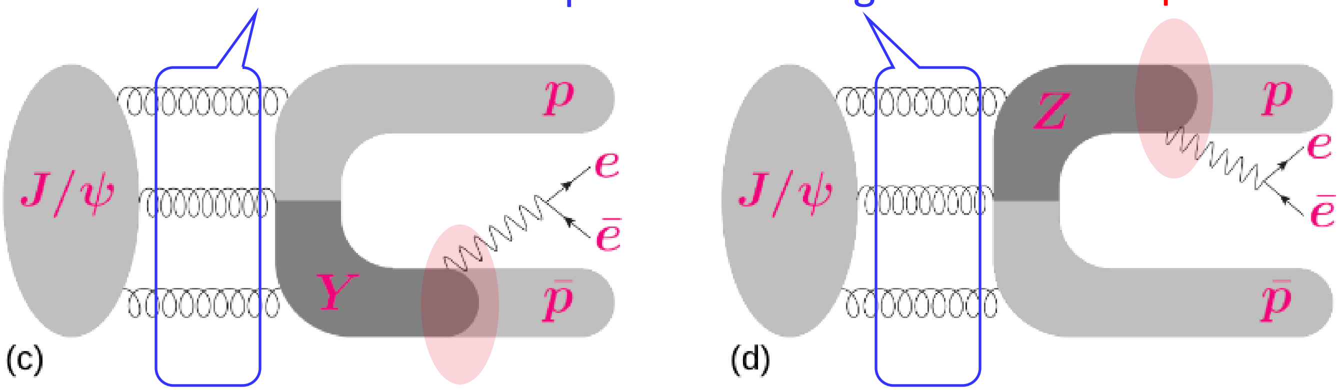


Decay mechanisms

- Virtual photon emitted
  - from anti-light and light quarks, types Y and Z: diagrams (c) and (d)
    - three gluons or a virtual photon
    - FSR $\gamma^* \to e^+e^-$
    - if proton is replaced by neutron, the FSR contribution is negligible at small q^2: zero charge, $\langle (r_E^n)^2 \rangle = -0.1155(17)\ \mathrm{fm}^2$ PDG2024
    - proton FF = antiproton FF

can also be a virtual photon exchange

contains proton FF



contains antiproton FF

Subtraction of background

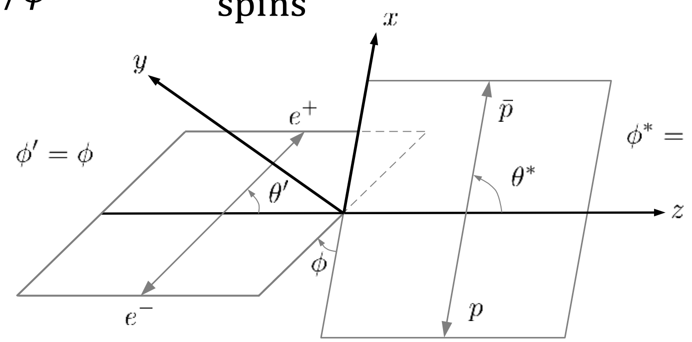
- Differential decay widths

$$\frac{d\Gamma(J/\psi \to p\bar{p}e^+e^-)}{dm_{e^+e^-}dm_{p\bar{p}}d\cos\theta_p^* d\cos\theta_e' d\phi} = \frac{|\vec{k}_{e^+e^-}||\vec{k}_p^*||\vec{k}_{e^-}'|}{(2\pi)^6 16 M_{J/\psi}^2} \frac{C(q^2)}{3} \sum_{\text{spins}} |\mathcal{M}|^2$$

$$|\mathcal{M}|^2 = |\mathcal{M}_{Y+Z}|^2 + 2\,\mathrm{Re}(\mathcal{M}_{Y+Z}\mathcal{M}_X^*) + |\mathcal{M}_X|^2$$

$$i\mathcal{M}_{(i)} = H_{(i)}^{\mu} \frac{-ig_{\mu\nu}}{q^2} \left[-ie\bar{u}_{s_{e^-}}(p_1)\gamma^\nu v_{s_{e^+}}(p_2)\right]$$

hadronic part — leptonic part



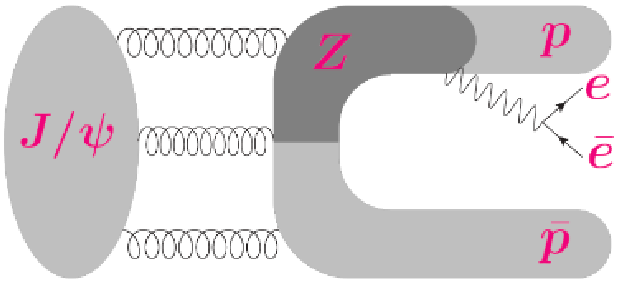
- ☐ Sommerfeld factor resums poles of e^+e^- Coulomb bound states:

$$C(q^2) = \frac{y}{1 - e^{-y}}, \qquad y = \frac{\pi\alpha m_e}{k_e'}$$

- Background subtraction
  - ☐ For $J/\psi \to n\bar{n}e^+e^-$: $\mathcal{M} \approx \mathcal{M}_X$
    - ➢ Background subtraction can in principle be achieved by subtracting out the $J/\psi \to n\bar{n}e^+e^-$ (properly normalized) event distribution
  - ☐ Signal part: $|\mathcal{M}_{\text{signal}}|^2 \equiv |\mathcal{M}_{Y+Z}|^2 + 2\,\mathrm{Re}(\mathcal{M}_{Y+Z}\mathcal{M}_X^*)$ all contains proton FF in the specific kinematic region

Selection of kinematic region

$$|p\rangle\langle p| + |N\pi\rangle\langle N\pi| + \cdots$$



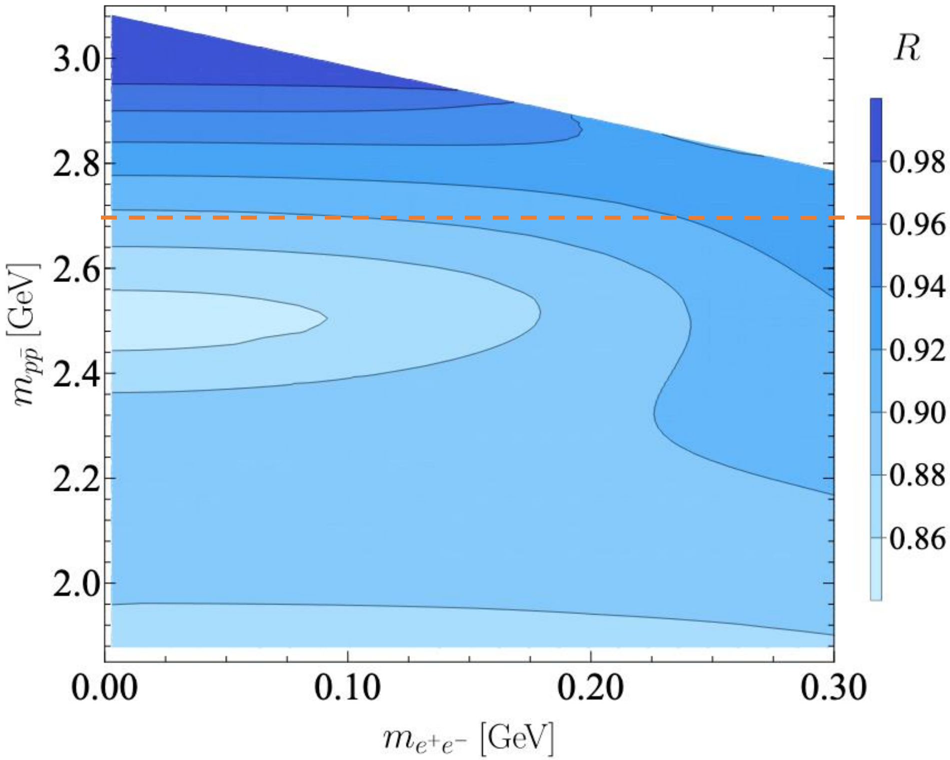
- Identify a region dominated by the proton and antiproton poles
  - Large $m_{p\bar{p}}$ ⇒ small $m_{p\gamma^*}$ and $m_{\bar{p}\gamma^*}$ ⇒ (anti-)proton dominance
  - Approximate $|N\pi\rangle\langle N\pi| + \cdots$ by the lowest $N\pi$ resonance Δ^+: $J/\psi \to \Delta^+ p + \mathrm{c.c.}$, $\Delta^+ \to p\gamma^*$, check the region where the Δ contribution can be neglected

$$\frac{dR_{N/(N+\Delta)}}{dm_{e^+e^-}dm_{p\bar{p}}} = \int d\cos\theta_p^*\, d\cos\theta_e'\, d\phi\, \frac{d\Gamma_{Y+Z}^{N}}{d\Gamma_{Y+Z}^{N+\Delta}}$$

- For $\psi' \to \Sigma^+\bar{\Sigma}^- e^+e^-$ and $\psi' \to \Xi^-\bar{\Xi}^+ e^+e^-$, consider the $J^P = 3/2^+$ $\Sigma(1385)$ and $\Xi(1530)$ to estimate contributions from higher states

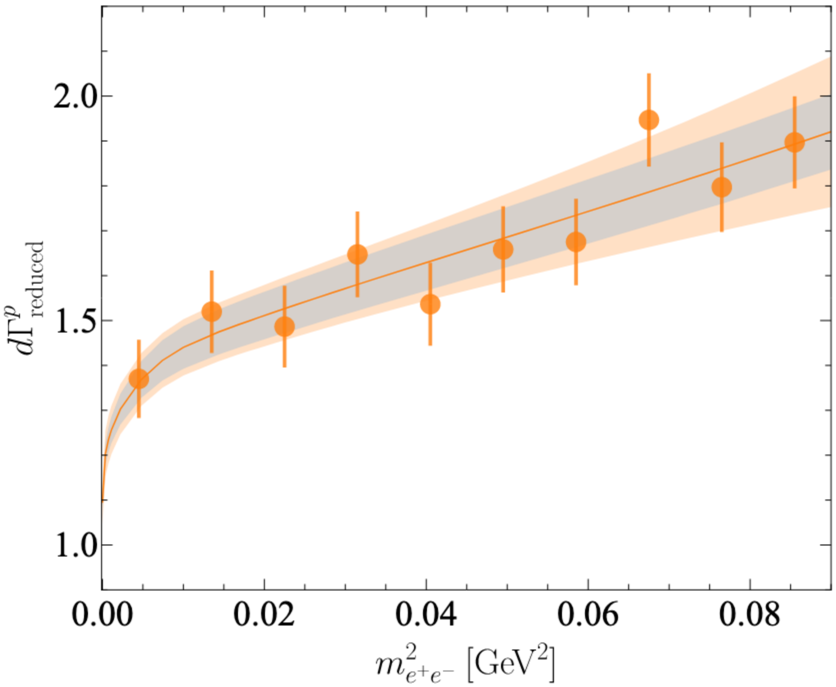
Selection of kinematic region

- Lower bound of the ratio $\frac{dR_{N/(N+\Delta)}}{dm_{e\bar{e}}dm_{p\bar{p}}}$ from types Y+Z
  - always larger than 90% for $m_{p\bar{p}} \gtrsim 2.7$ GeV



Sensitivity study for proton

- Estimate of the number of events
  - Consider only the signal part $\left|\mathcal{M}_{\text{signal}}\right|^2 \equiv |\mathcal{M}_{Y+Z}|^2 + 2\,\text{Re}(\mathcal{M}_{Y+Z}\mathcal{M}_X^*)$
  - For type-X, consider only the η_c contribution
  - $\sim 3\times 10^3$ events (BESIII) for $m_{e^+e^-} < 0.3$ GeV, $m_{p\bar{p}} > 2.7$ GeV
- Sensitivity to the proton charge radius r_E^p of the $m_{e^+e^-}$ distribution normalized to a pointlike-proton assumption



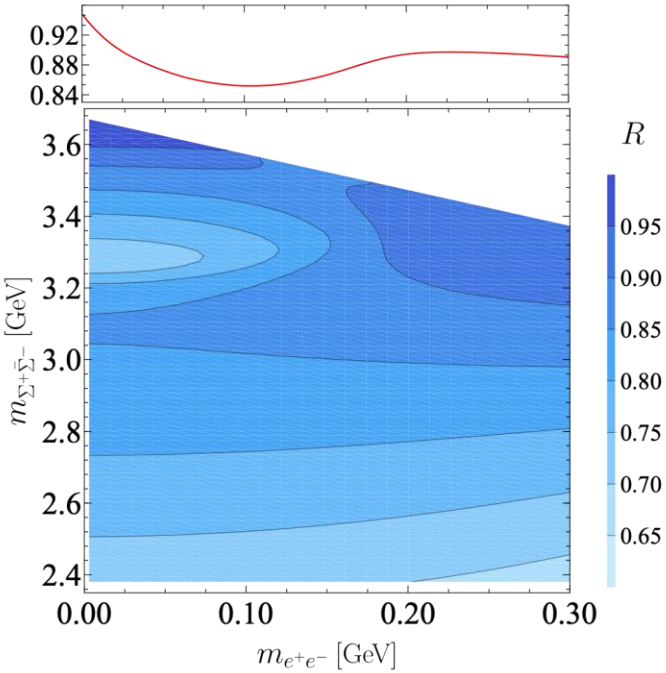
Using 0.84 fm as input to generate synthetic Monte Carlo data, obtained

- 0.71(9) fm for BESIII configuration
- 0.845(7) fm for STCF configuration

Sensitivity study for hyperons

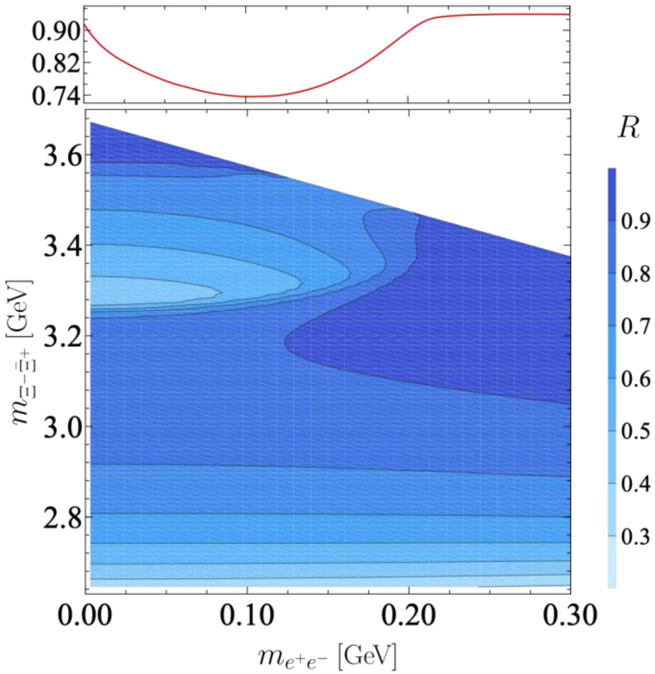
- For $\psi' \to \Sigma^+\bar{\Sigma}^- e^+e^-$

Σ-pole dominates above 2.8 GeV



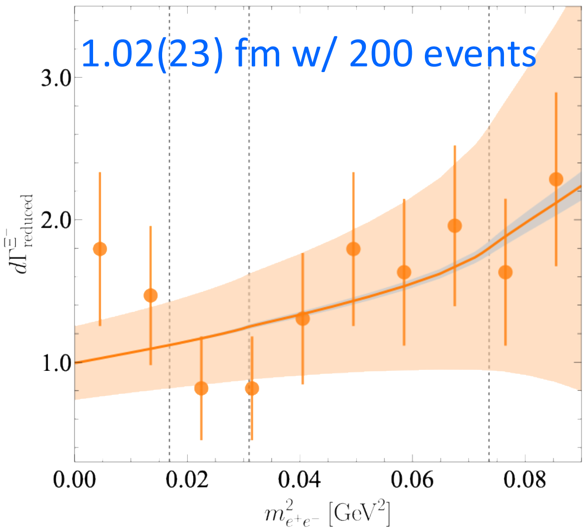
- For $\psi' \to \Xi^-\bar{\Xi}^+ e^+e^-$

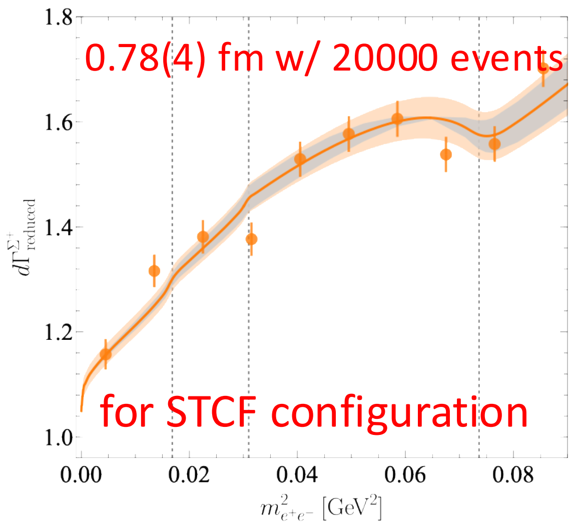
Ξ-pole dominates above 2.9 GeV



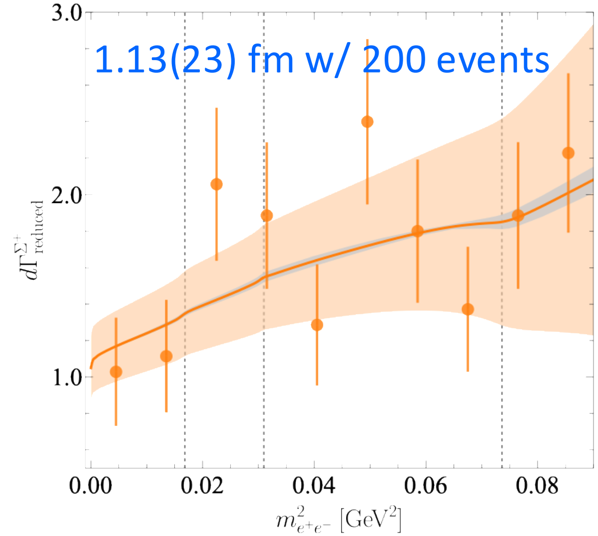
Sensitivity study for hyperons

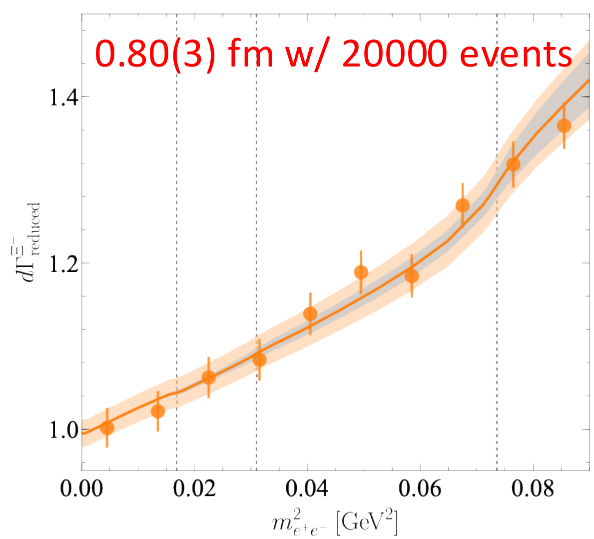
- For $\psi' \to \Sigma^+\bar{\Sigma}^- e^+e^-$ using 0.8 fm as input





- For $\psi' \to \Xi^-\bar{\Xi}^+ e^+e^-$ using 0.8 fm as input





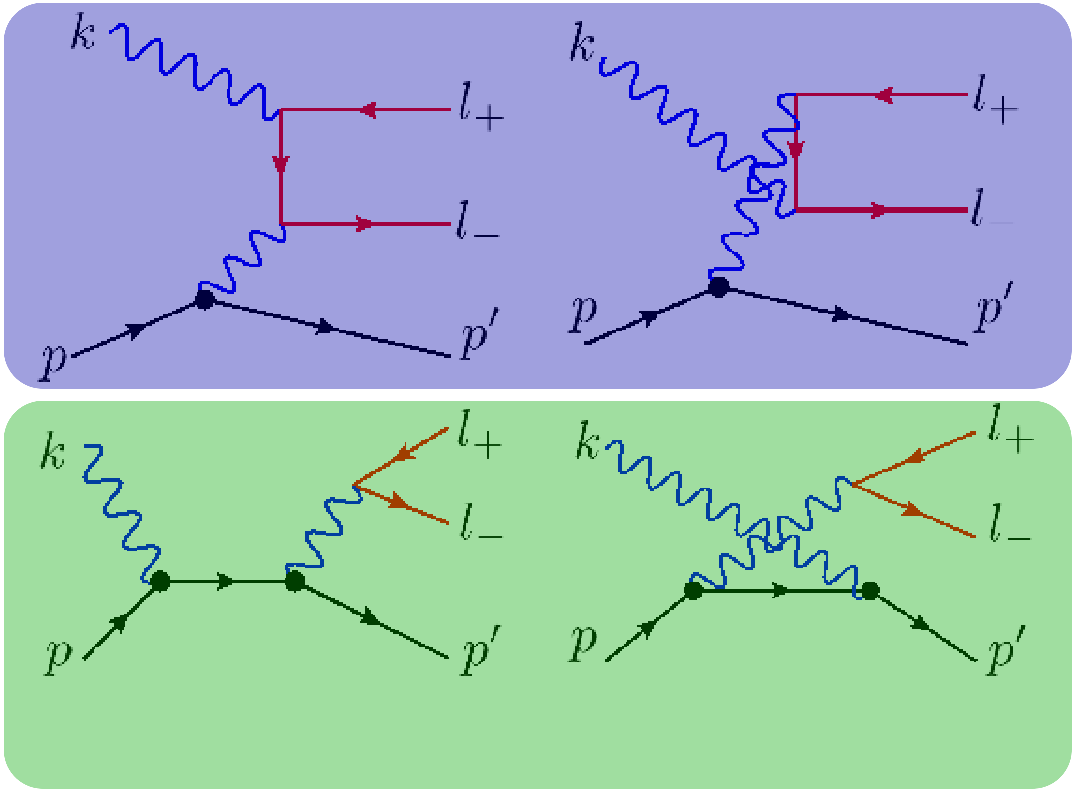
Proton radius from dimuon photoproduction

Yong-Hui Lin, FKG, U.-G. Meißner, PLB 858, 139023 (2024)

- The muon-proton scattering value of the proton charge radius has not been measured
  - Planned experiments
    - MUSE @PSI, $Q^2 \in [0.002, 0.07]\ \mathrm{GeV}^2$
    - AMBER @CERN, $Q^2 \in [0.001, 0.02]\ \mathrm{GeV}^2$
  - It may be extracted from the dimuon photoproduction $\gamma p \to p\mu^+\mu^-$

Bethe-Heitler (BH)

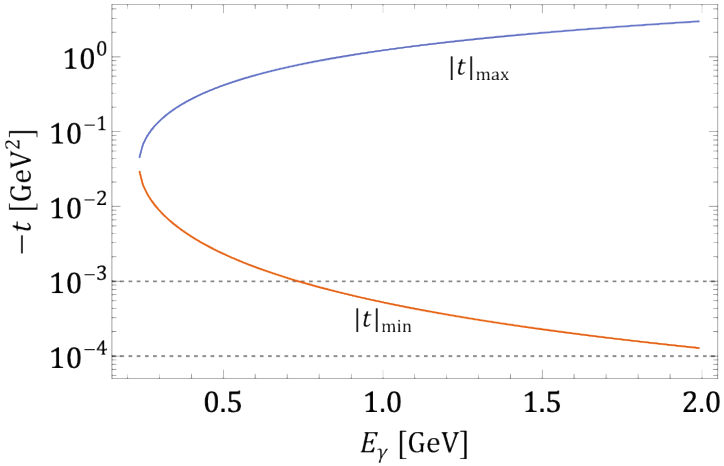
Compton scattering

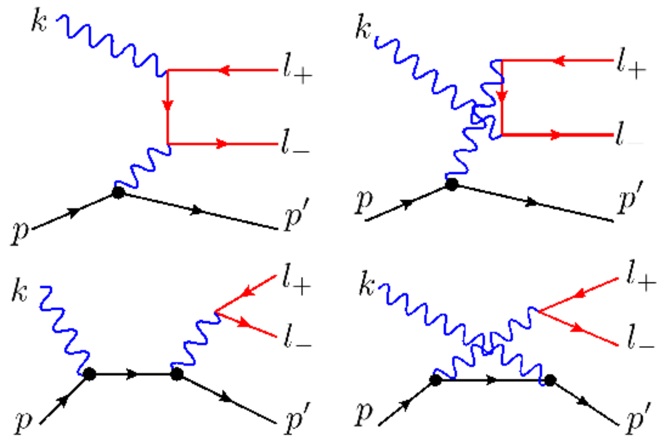


Proton radius from dimuon photoproduction

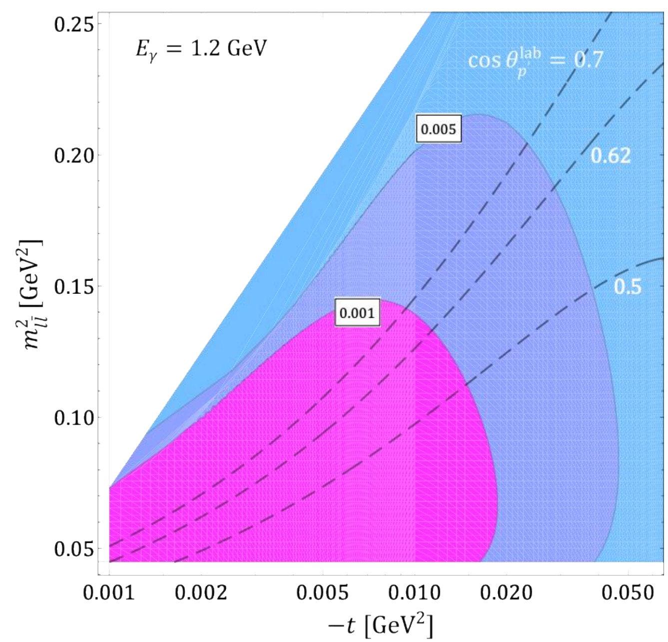
Yong-Hui Lin, FKG, U.-G. Meißner, PLB 858, 139023 (2024)

- Dimuon photoproduction $\gamma p \to p\mu^+\mu^-$
  - Kinematical region of $-t = Q^2$
    - Can reach 10^{-3} GeV^2 for $E_\gamma^{\rm lab} > 0.8$ GeV



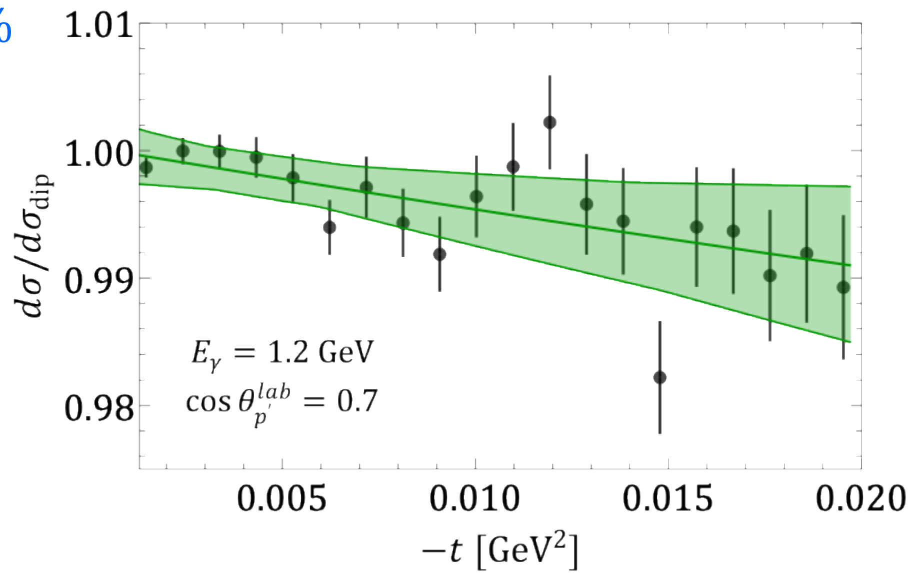


- The BH mechanism overwhelmingly dominates for small Q^2 and low $m_{l\bar{l}}^2$



Proton radius from dimuon photoproduction

- Dimuon photoproduction $\gamma p \to p\mu^+\mu^-$
  - Integrated cross section for $t \in [0.001, 0.02]\ \mathrm{GeV}^2$: $\mathcal{O}(100)$ nb
  - 5×10^6 events for a flux of 10^7 photons/s on a TPC target ($\sim$ 1 m long) in a few months
    - LEPS2 *(BL31LEP)*: $E_\gamma \in [1.3, 2.4]$ GeV, $< 2 \times 10^7$ photons/s (with 355-nm UV Solid-state laser) https://www.rcnp.osaka-u.ac.jp/Divisions/np1-b/?LEPS2_%28BL31LEP%29
    - Much higher photon intensity at JAEA-ERL
    - …
  - Sensitivity with 5×10^6 events: $\sim 1\%$
    - Input: 0.84 fm
    - Extracted: 0.848(8) fm



Nucleon gravitational FFs

- Nucleon gravitational FFs

Review: M. Polyakov, P. Schweitzer, IJMPA 22, 1830025 (2018)

$$\langle N(p')|\hat{T}^{\mu\nu}|N(p)\rangle$$
$$= \frac{1}{4m_N}\bar{u}(p')\left[A(t)P^\mu P^\nu + J(t)\left(iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho\right) + D(t)\left(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right)\right]u(p)$$

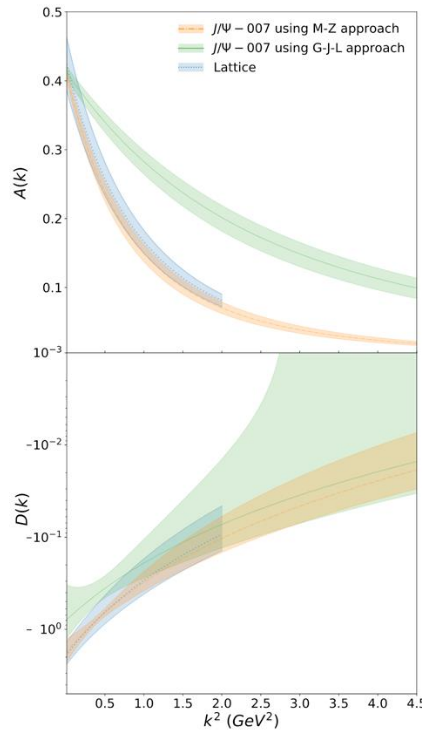
- ☐ Trace FF: $\Theta(t) = m_N\left[A(t) - \frac{t}{4m_N^2}\left(A(t) - 2J(t) + 3D(t)\right)\right]$
- ☐ Normalizations:

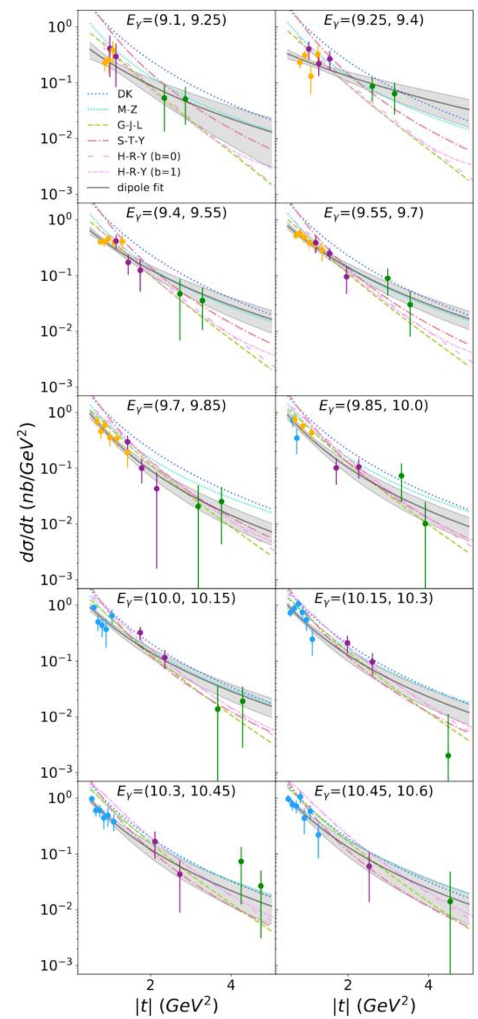
$\Theta(0) = m_N,\ A(0) = 1,\ J(0) = \frac{1}{2}$ (spin),

D-term: $D(0)$

- Extractions using near-threshold J/ψ photoproduction data based on two models:
  - ☐ Holographic QCD
  - ☐ GPD + VMD

Duran et al. [J/ψ-007], Nature 615, 813 (2023)

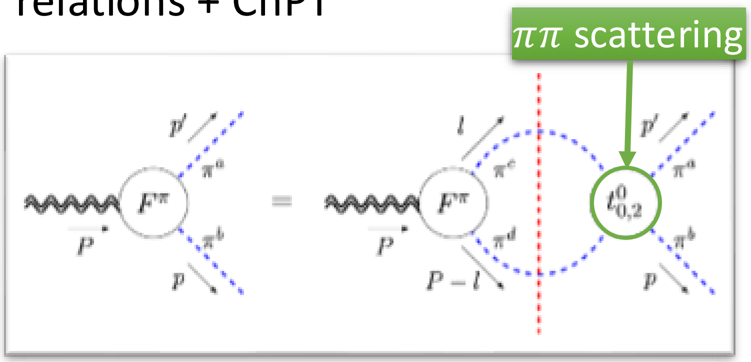


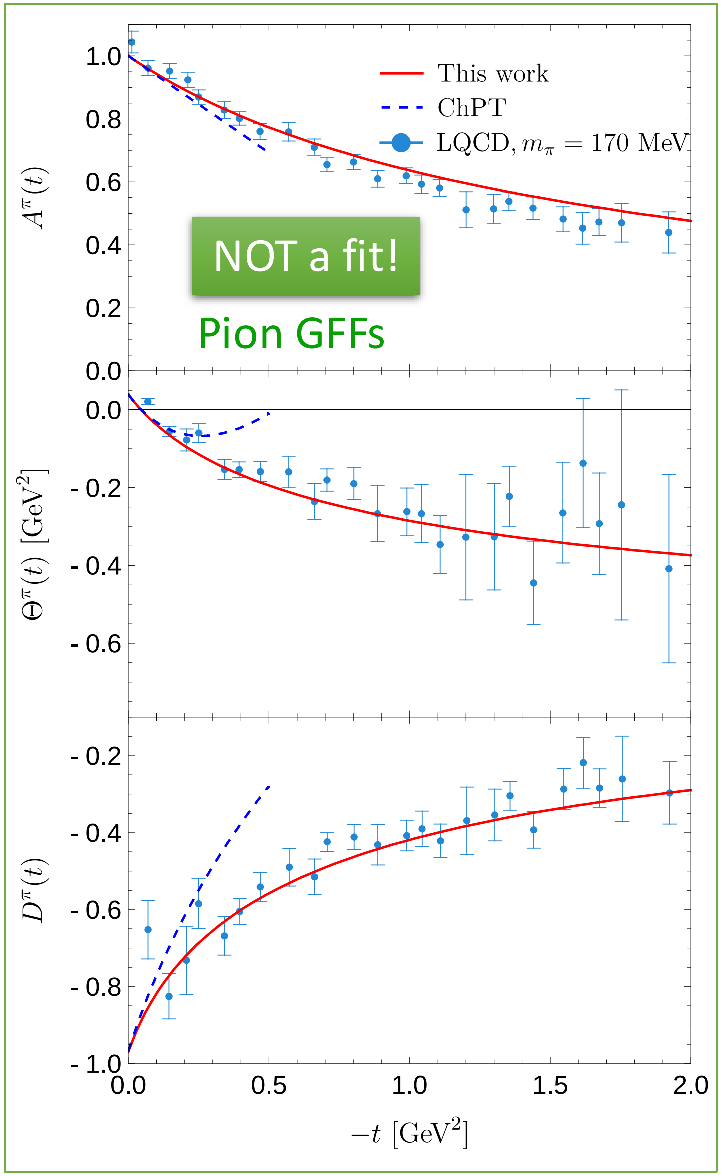


Nucleon gravitational FFs

Xiong-Hui Cao, FKG, Qu-Zhi Li, De-Liang Yao, arXiv:2411.13398

- Gravitational FFs (GFFs) be model-independently predicted using data-driven dispersion relations + ChPT





ChPT: Donoghue, Leutwyler, ZPC 52, 343 (1991)
LQCD: Hackett et al., PRD 108, 114504 (2023)

Nucleon gravitational FFs

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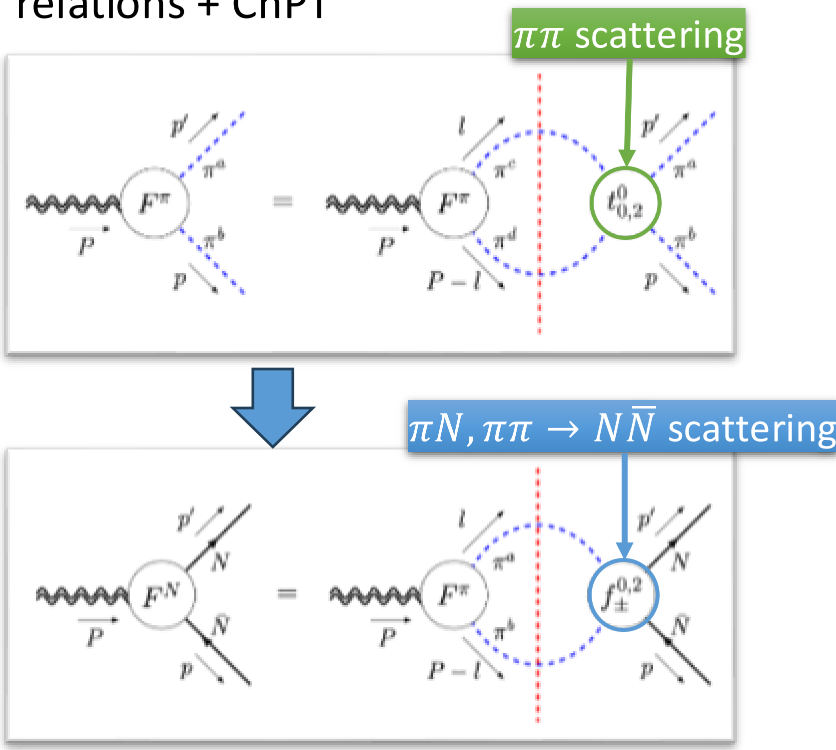
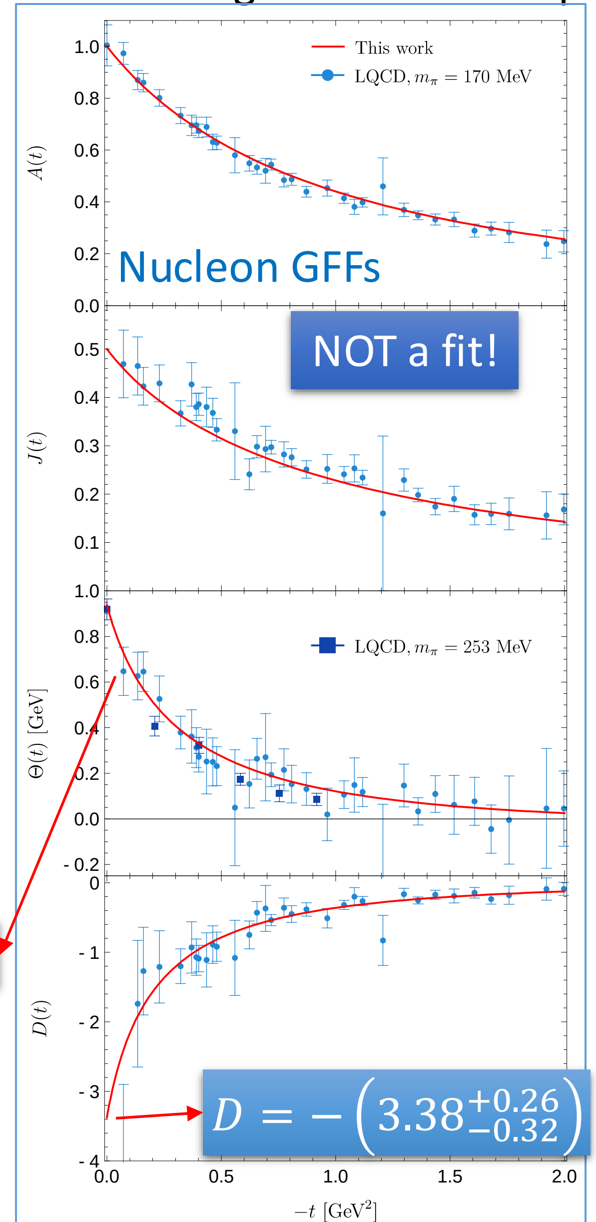


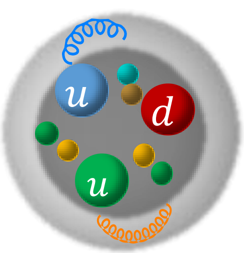
LQCD (170 MeV): Hackett et al., PRL 132, 251904 (2024);
LQCD (253 MeV): B. Wang et al., PRD 109, 094504 (2024)



$$\langle r^2_{\rm mass}\rangle = (0.97^{+0.02}_{-0.03}\ {\rm fm})^2$$

$$\vee$$

$$\langle r^2_{{\rm E},p}\rangle = (0.84075(64){\rm fm})^2$$



Summary and outlook

- Novel proposals for measuring the charge radii of stable charged baryons
  - from the time-like region using the Dalitz decays $J/\psi \to p\bar{p}e^+e^-$, $\psi' \to \Sigma^+\bar{\Sigma}^-e^+e^-$ and $\psi' \to \Xi^-\bar{\Xi}^+e^+e^-$
    - can reach $|q^2| \sim 1.05 \times 10^{-6}\ \mathrm{GeV}^2$, smaller than all ep scattering experiments
    - measurements of Ξ^-, Σ^+ radii w/o hyperon beams
  - the muon-proton scattering value of the proton charge radius from $\gamma p \to p\mu^+\mu^-$ using the Bether-Heitler mechanism w/o using muon beams
- Nucleon gravitational FFs precisely determined using data-driven dispersive approach
  - challenging to measure experimentally

Thank you for your attention!