Precise measurement of total cross section of e^+e^- →hadrons

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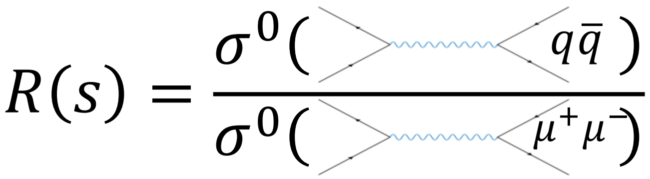
FTCF2024
Guangzhou

November 18, 2024

R(s)



$$R(s) = \frac{\sigma^0(\;q\bar{q}\;)}{\sigma^0(\;\mu^+\mu^-)}$$

In the zeroth order of QCD and zero quark masses:

$$R^{(0)}(s) = 3\sum_f q_f^2$$

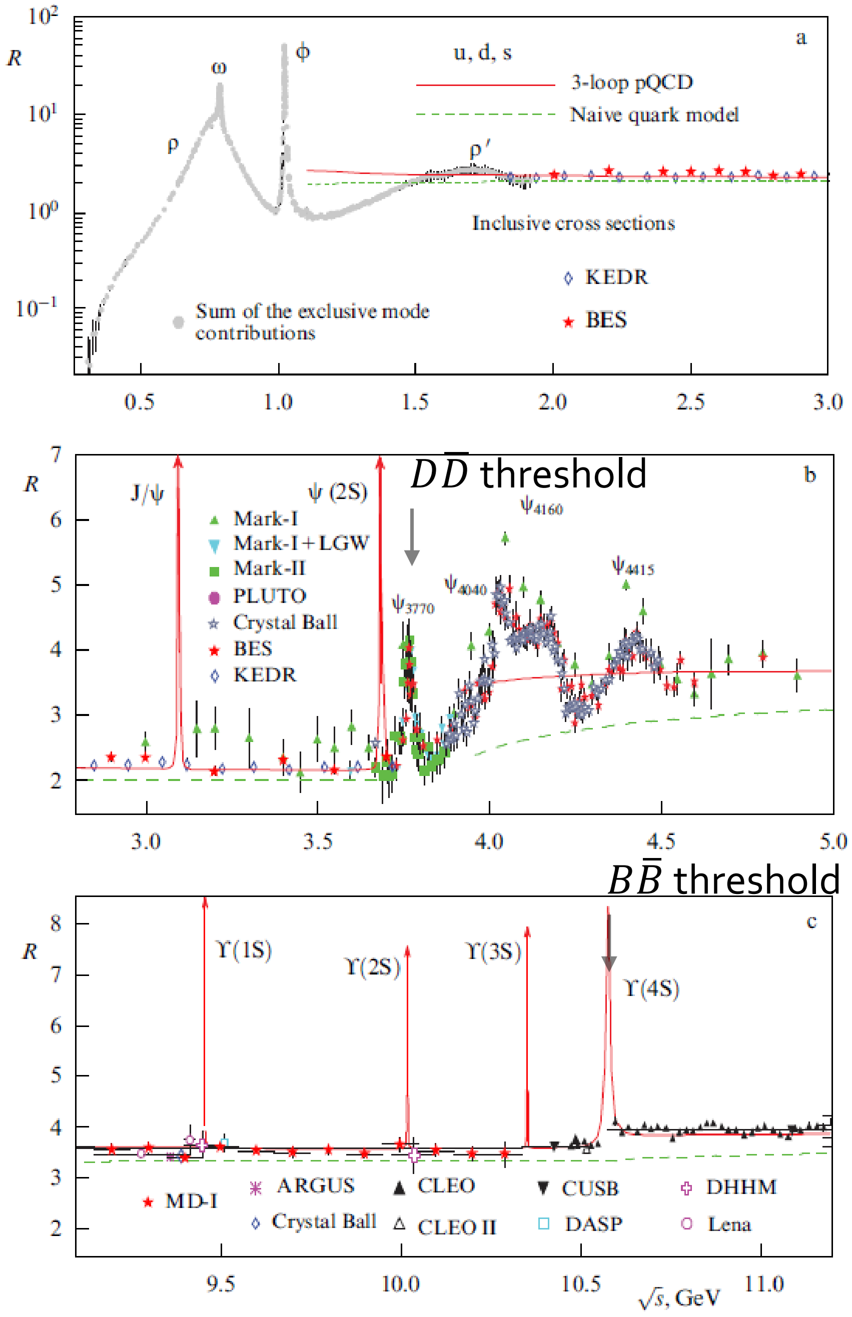
$$R(u,d,s) = \frac{6}{3}$$

$$R(u,d,s,c) = \frac{10}{3}$$

$$R(u,d,s,c,b) = \frac{11}{3}$$

Full pQCD calculation includes NNLO contribution, quark masses, running α_s,…

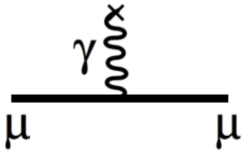
Important for $a_\mu, \alpha(M_Z^2), \alpha_s(s)$, …



Muon (g-2): the basics

Gyromagnetic ratio g connects magnetic moment μ and spin s

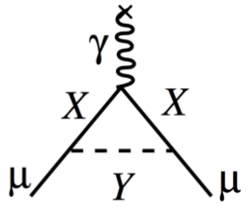
$$\vec{\mu}_S = g\frac{e}{2m}\vec{S}$$



For point-like particle $\boldsymbol{g} = \mathbf{2}$

Anomalous magnetic moment $\boldsymbol{a}$ arises in higher-orders

$$a = (g-2)/2$$



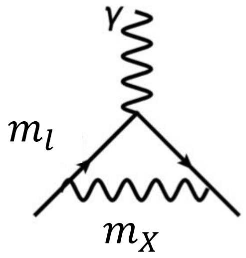
$a_e \approx a_\mu \approx \frac{\alpha}{2\pi} \approx 10^{-3}$ (QED dominated)

Idea of experiment: by comparing measured value of $\boldsymbol{a}$ with the theory prediction we probe extra contributions beyond theory expectations

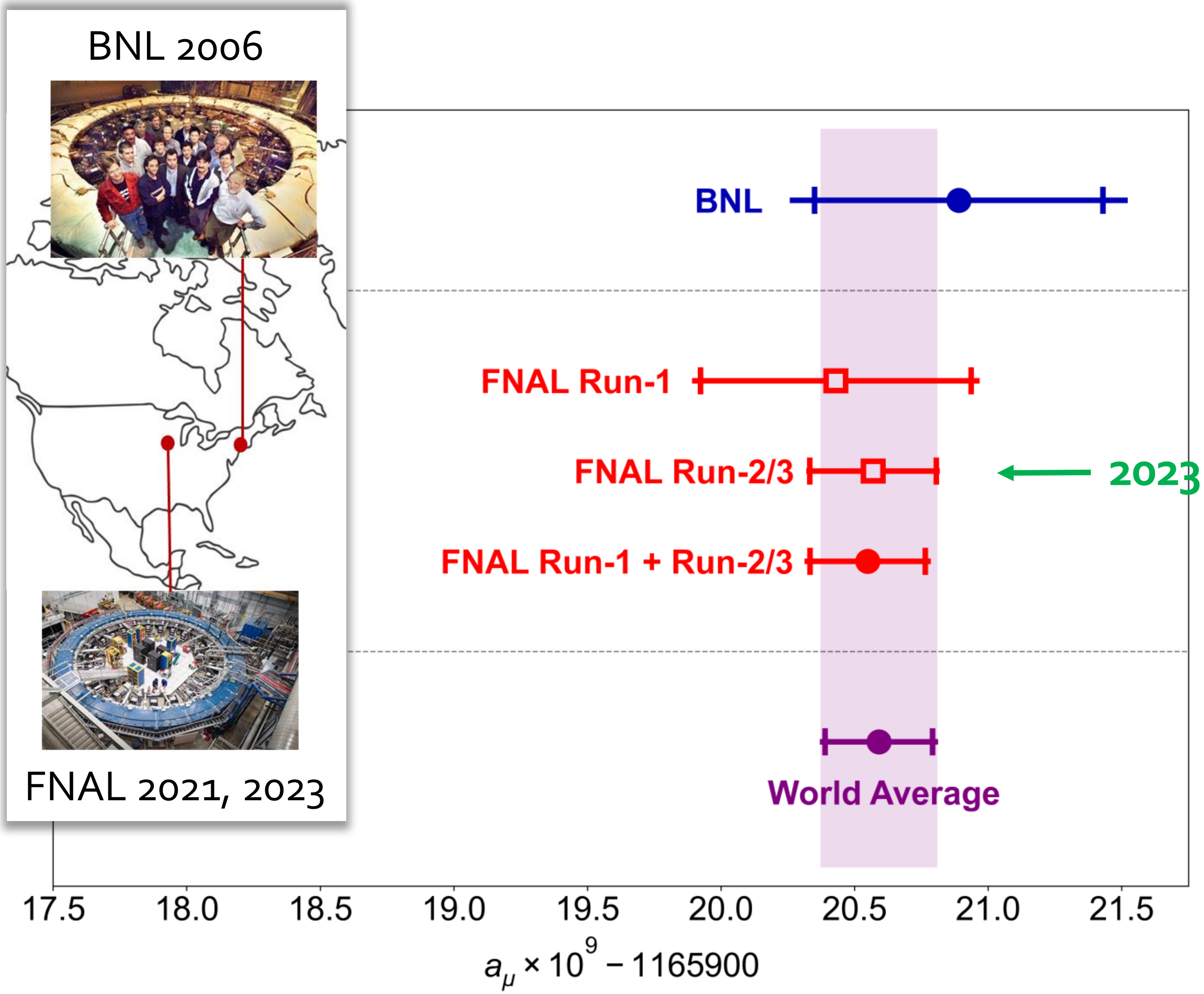
$$a_\mu(strong)/a_\mu(QED) \approx 6 \times 10^{-5} \qquad a_\mu(weak)/a_\mu(QED) \approx 10^{-6}$$

Why muon? For massive fields there is natural scaling, which enhances contribution to a_μ by $\left(m_\mu/m_e\right)^2 \sim 43000$ compared to a_e

$$\Delta a \sim \left(\frac{m_l}{m_X}\right)^2$$



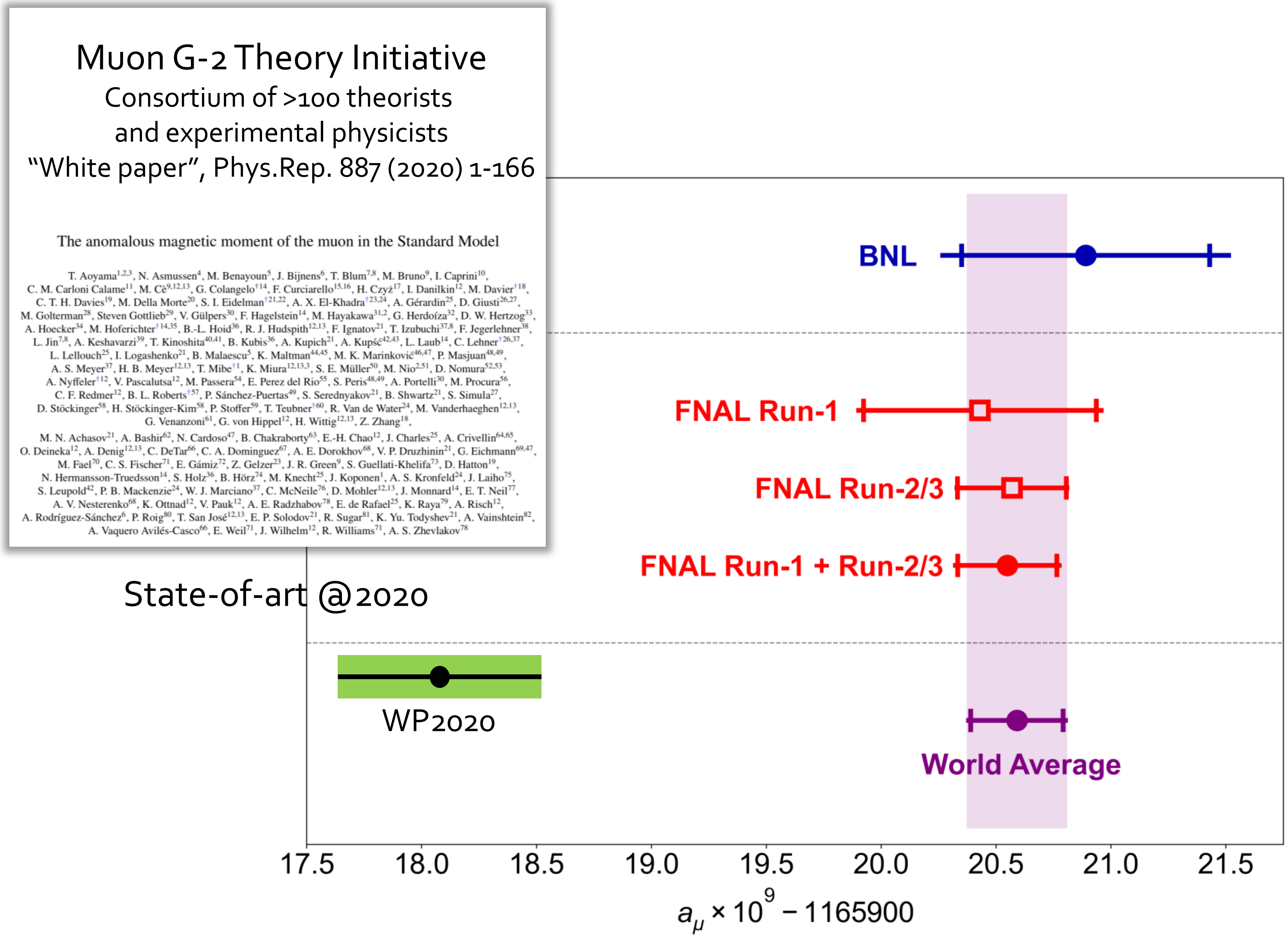
Muon G-2 2023 result



2023 result

$$a_\mu(\mathrm{Exp}) = 0.00\,116\,592\,059(22) \quad [190\ \mathrm{ppb}]$$

Experiment vs SM prediction



Muon G-2 Theory Initiative
Consortium of >100 theorists
and experimental physicists
“White paper”, Phys.Rep. 887 (2020) 1-166

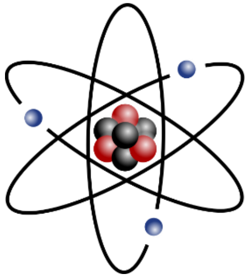
The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama[1,2,3], N. Asmussen[4], M. Benayoun[5], J. Bijnens[6], T. Blum[7,8], M. Bruno[9], I. Caprini[10], C. M. Carloni Calame[11], M. Cè[9,12,13], G. Colangelo[†14], F. Curciarello[15,16], H. Czyż[17], I. Danilkin[12], M. Davier[†18], C. T. H. Davies[19], M. Della Morte[20], S. I. Eidelman[†21,22], A. X. El-Khadra[†23,24], A. Gérardin[25], D. Giusti[26,27], M. Golterman[28], Steven Gottlieb[29], V. Gülpers[30], F. Hagelstein[14], M. Hayakawa[31,2], G. Herdoíza[32], D. W. Hertzog[33], A. Hoecker[34], M. Hoferichter[†14,35], B.-L. Hoid[36], R. J. Hudspith[12,13], F. Ignatov[21], T. Izubuchi[37,8], F. Jegerlehner[38], L. Jin[7,8], A. Keshavarzi[39], T. Kinoshita[40,41], B. Kubis[36], A. Kupich[21], A. Kupść[42,43], L. Laub[14], C. Lehner[†26,37], L. Lellouch[25], I. Logashenko[21], B. Malaescu[5], K. Maltman[44,45], M. K. Marinković[46,47], P. Masjuan[48,49], A. S. Meyer[37], H. B. Meyer[12,13], T. Mibe[†1], K. Miura[12,13,3], S. E. Müller[50], M. Nio[2,51], D. Nomura[52,53], A. Nyffeler[†12], V. Pascalutsa[12], M. Passera[54], E. Perez del Rio[55], S. Peris[48,49], A. Portelli[30], M. Procura[56], C. F. Redmer[12], B. L. Roberts[†57], P. Sánchez-Puertas[49], S. Serednyakov[21], B. Shwartz[21], S. Simula[27], D. Stöckinger[58], H. Stöckinger-Kim[58], P. Stoffer[59], T. Teubner[†60], R. Van de Water[24], M. Vanderhaeghen[12,13], G. Venanzoni[61], G. von Hippel[12], H. Wittig[12,13], Z. Zhang[18],

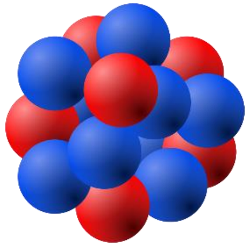
M. N. Achasov[21], A. Bashir[62], N. Cardoso[47], B. Chakraborty[63], E.-H. Chao[12], J. Charles[25], A. Crivellin[64,65], O. Deineka[12], A. Denig[12,13], C. DeTar[66], C. A. Dominguez[67], A. E. Dorokhov[68], V. P. Druzhinin[21], G. Eichmann[69,47], M. Fael[70], C. S. Fischer[71], E. Gámiz[72], Z. Gelzer[23], J. R. Green[9], S. Guellati-Khelifa[73], D. Hatton[19], N. Hermansson-Truedsson[14], S. Holz[36], B. Hörz[74], M. Knecht[25], J. Koponen[1], A. S. Kronfeld[24], J. Laiho[75], S. Leupold[42], P. B. Mackenzie[24], W. J. Marciano[37], C. McNeile[76], D. Mohler[12,13], J. Monnard[14], E. T. Neil[77], A. V. Nesterenko[68], K. Ottnad[12], V. Pauk[12], A. E. Radzhabov[78], E. de Rafael[25], K. Raya[79], A. Risch[12], A. Rodríguez-Sánchez[6], P. Roig[80], T. San José[12,13], E. P. Solodov[21], R. Sugar[81], K. Yu. Todyshev[21], A. Vainshtein[82], A. Vaquero Avilés-Casco[66], E. Weil[71], J. Wilhelm[12], R. Williams[71], A. S. Zhevlakov[78]

State-of-art @2020

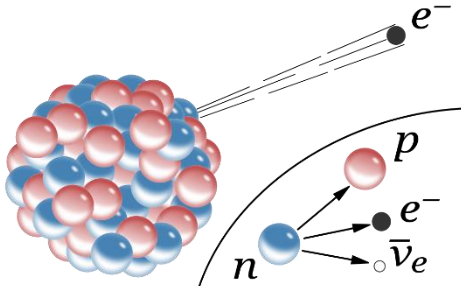
SM prediction for a_μ



Electromagnetic interactions



Strong interactions



Weak interactions

0.001 165 847 19 (0.1)

0.000 000 069 37 (43)

0.000 000 001 54 (1)

$$a_\mu = 0.001\,165\,918\,10\ (43)$$

The uncertainty is dominated by contribution of strong interactions

Hadronic contribution

Leading order (LO)

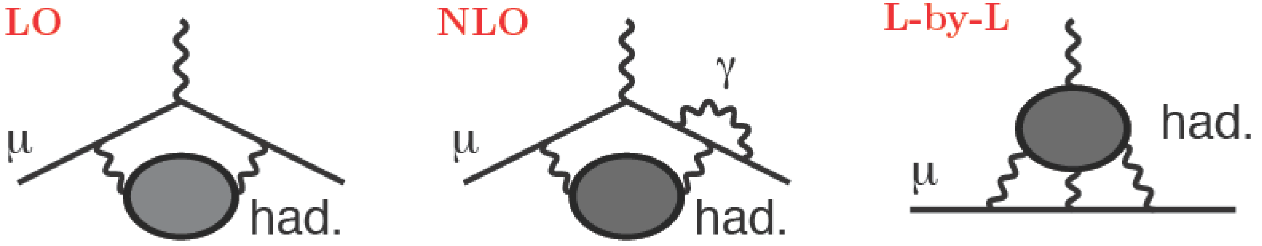
Next-to-leading order (NLO)

Light-by-light (LBL)

WP(2020)

$$(6\,931 \pm 40)\times 10^{-11} \qquad (-98.3 \pm 0.7)\times 10^{-11} \qquad (92 \pm 19)\times 10^{-11}$$

$$a_\mu^{\text{had}} = a_\mu^{\text{had,VP LO}} + a_\mu^{\text{had,VP NLO}} + a_\mu^{\text{had,Light-by-Light}}$$



350 ppb (0.6%)

10 ppb

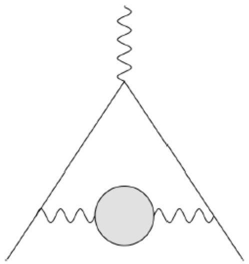
160 ppb

Underestimated!

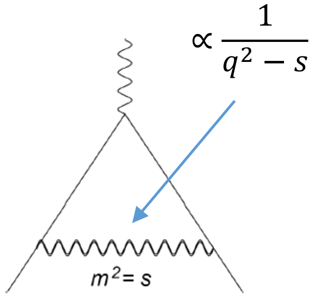
Compare to experimental accuracy of 190 ppb

HVP: what do we need to measure

Dispersion relation:



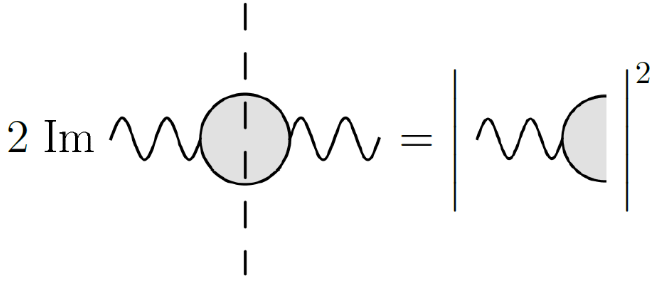
$$= \int_0^\infty \frac{ds}{s}\frac{1}{\pi}\mathrm{Im}\Pi'(s) \times$$



$a_\mu^{had}(LO)$

$\frac{\alpha}{\pi}K_\mu(s)$

Optical theorem:



$$\mathrm{Im}\,\Pi'(s) = \frac{s}{4\pi\alpha}\sigma^0(e^+e^- \to \gamma \to hadrons + \cdots)$$

Lets put everything together:

This is what we need to measure

$$a_\mu^{had}(LO) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}R(s)K_\mu(s)$$

$$R(s) = \frac{\sigma^0(e^+e^- \to \gamma \to hadrons)}{4\pi\alpha^2/3s}$$

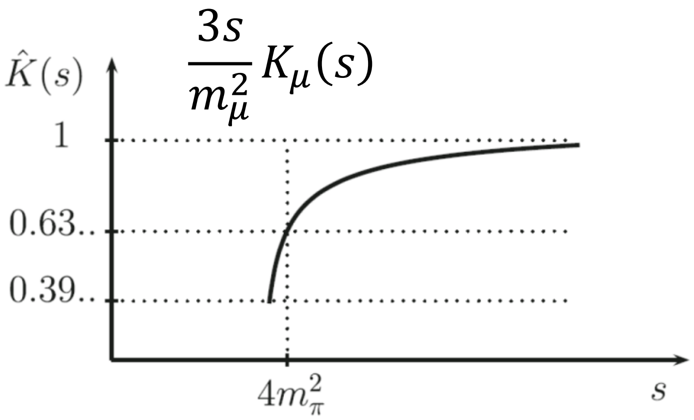
$\sigma^0(e^+e^- \to \mu^+\mu^-)$

$s = (\text{c.m. energy})^2$

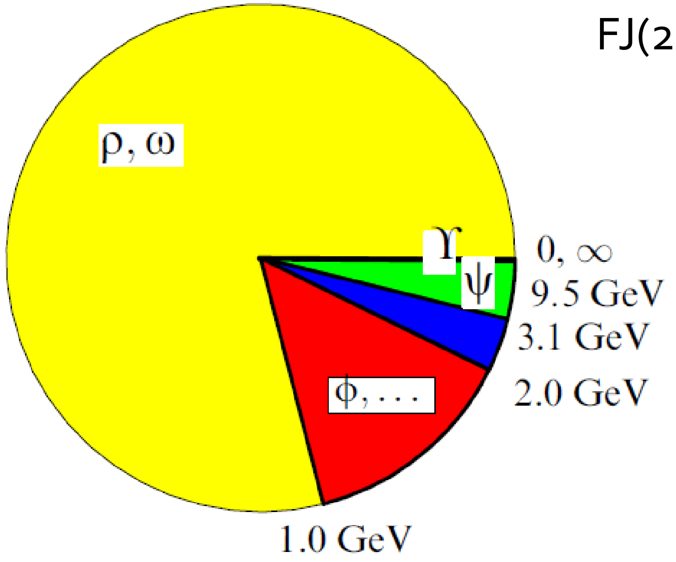
Contribution of various energies

In a_μ^{had} integral, the main contribution comes from low energies

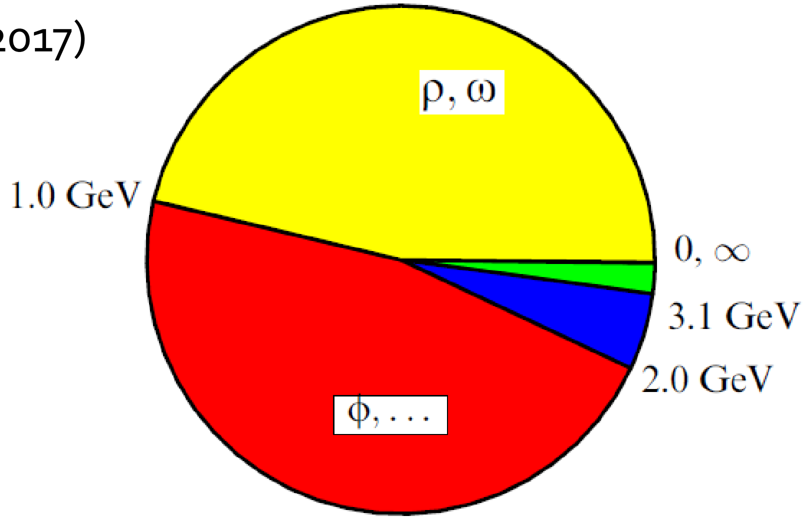
$$a_\mu^{had}(LO) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} R(s) K_\mu(s) \sim \int \frac{R(s)}{s^2} ds$$



FJ(2017)



Contribution to the integral

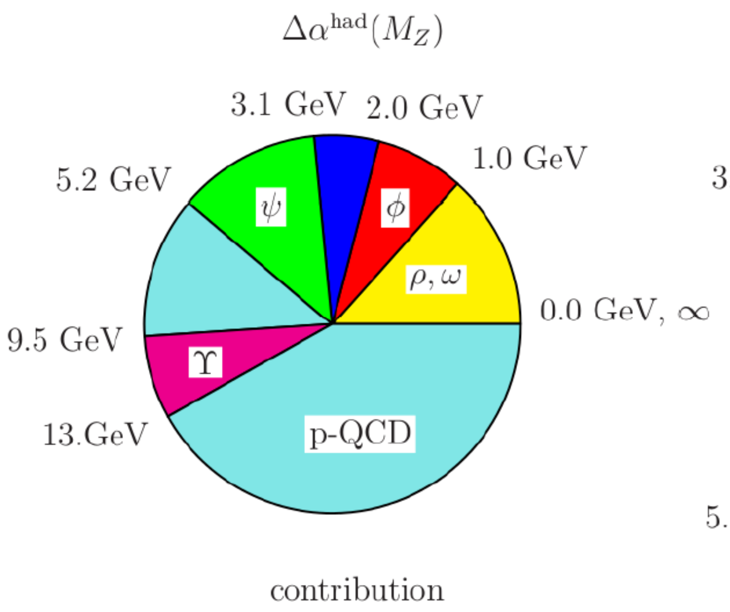


Contribution to the error of integral

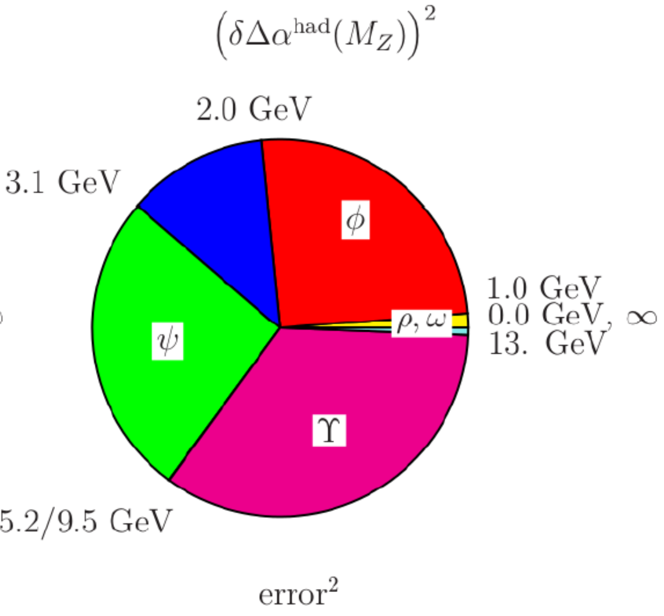
$\Delta\alpha^{(5)}_{had}(M_Z^2)$

$$\Delta\alpha^{(5)}_{had}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\mathrm{Re}\int_{4m_\pi^2}^{\infty}\frac{R(s)ds}{s(s-M_Z^2-i\epsilon)}$$

Important contribution to electroweak fit



Contribution to the integral



Contribution to the error of integral

A. Blondel et al., arXiv:1905.05078

How well do we need to measure R(s)

From the White Paper (Physics Reports 887 (2020) 1):

$$a_\mu^{\mathrm{had}}(LO) = 693.1(4.0) \times 10^{-10}$$

The expected final precision of the Fermilab measurement

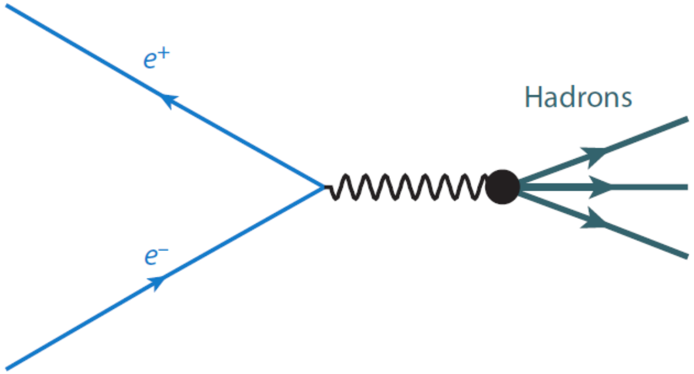
$$\Delta a_\mu = 1.6 \times 10^{-10}$$

We need to know $R(s)$ to 0.23% to match Fermilab precision

Now the hadronic contribution is known to 0.57% (underestimated!)

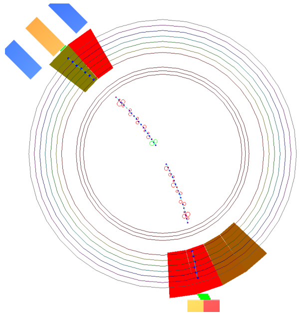
Measurement techniques:

Direct vs ISR

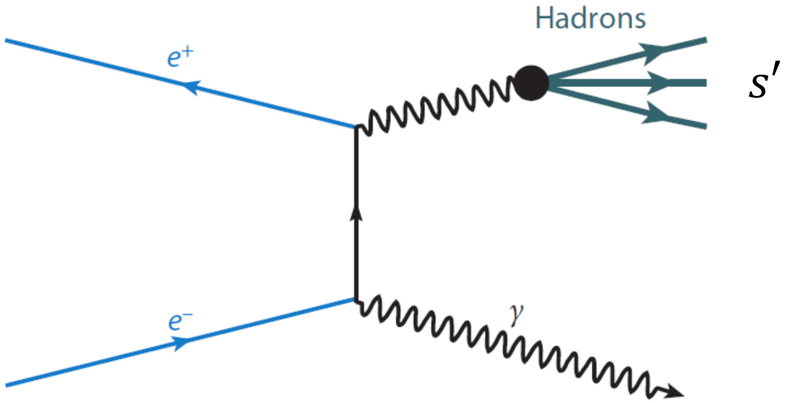


Direct measurement (Energy scan)

At fixed s: $\sigma_{e^+e^-\to H}(s) \sim N_H/L$
Data is taken at different s

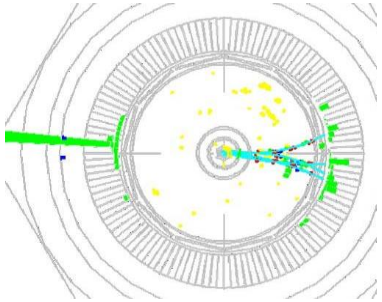


VEPP-2M: CMD-2, SND
VEPP-2000: CMD-3, SND2k



ISR (Initial State Radiation)

$\sigma_{e^+e^-\to H}(s') \sim \frac{dN_{H+\gamma}/ds'}{L\cdot dW/ds'}$
Data is taken at fixed $s > s'$



KLOE, BABAR, BES-III, CLEO

ISR vs energy scan

- Energy scan analysis is generally simpler, but ISR measurements were done with superior detectors
- Before VEPP-2000, ISR measurements had more statistics
- In general, background is higher for ISR measurements
- ISR approach allows for larger detector coverage and smaller model-dependence
- In both approaches the visible cross-section is smeared and we need to unfold it:

Energy scan

The cross-section is smeared by ISR

$$\sigma_{vis}(s) = \int_0^1 dx_1 dx_2 D(x_1, s) D(x_2, s) \sigma_0(x_1 x_2 s)$$

The beam energy is known to high precision ($\sim 10^{-4} - 10^{-3}$)

The “unfolding” is done via radiative corrections

The “response” function is model-dependent, but it does not have unknown pieces

ISR

The cross-section is smeared by detector resolution

$$\frac{d\sigma_{vis}(s, s')}{ds'} = \frac{2s'}{s} W(s, s') \sigma_0(s')$$

The energy of the final state s' is reconstructed from the kinematics.

If the detector response function is known, the unfolding is the robust procedure.

But tails in the response function can lead to large effects.

Exclusive vs inclusive measurement

Detection efficiency is (usually) calculated using MC simulation

- In order to calculated ε, we need to know the energy and angular distributions of final particles (including all correlations)

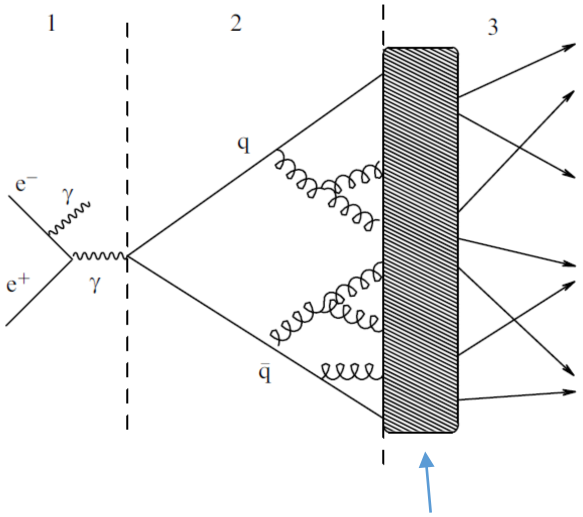
$$\sigma = \frac{N_{obs} - N_{bg}}{\varepsilon \cdot \int \mathcal{L} dt}$$

For high energies, where multiplicity is large enough, there are effective models of hadronization, which describe data reasonably well

At low energy the detection efficiency varies significantly between different final states and different paths of hadronization (intermediate states)

At low energies we have to measure cross section for each possible final state separately and then calculate sum to get R (***exclusive approach***)

At high energy we can measure total cross section directly (***inclusive approach***)



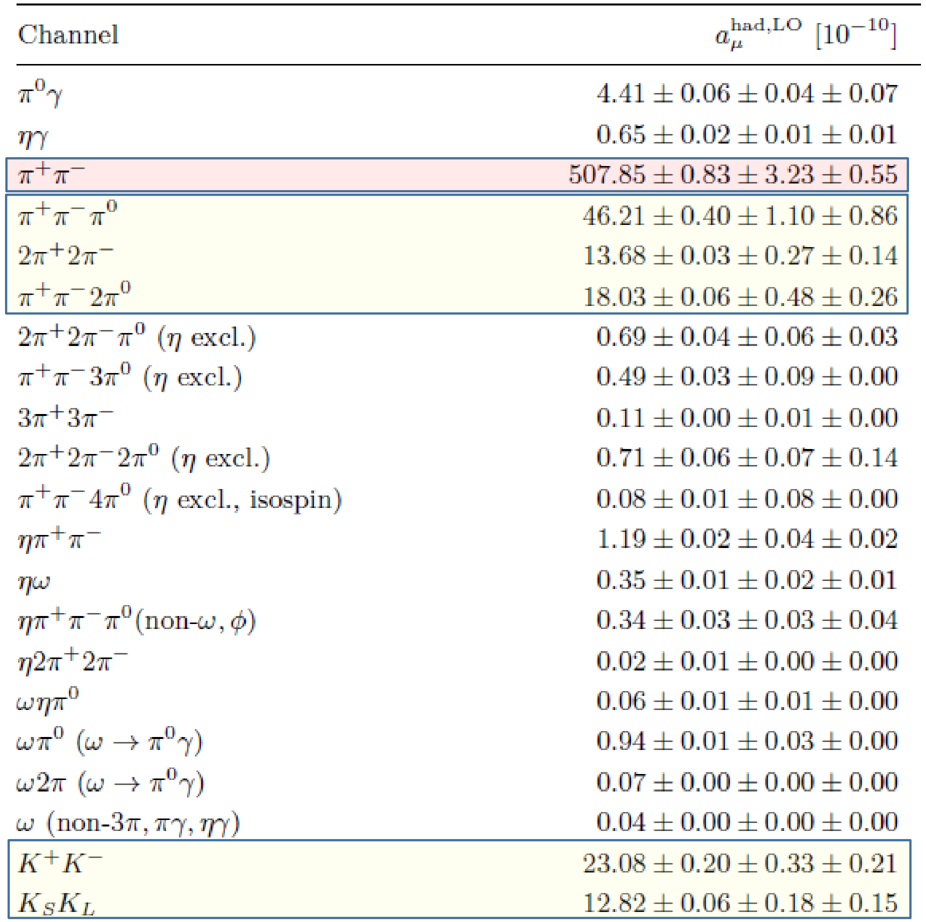
The practical boundary between two approaches in $\sqrt{s} = 2$ GeV.

The $a_\mu^{had}(LO)$ calculation is mostly based on exclusive measurements.

Contribution of exclusive hadronic cross sections to a_μ

In exclusive approach, we calculate a_μ integral for each final state and sum them:

$$a_\mu^{had}(LO) = \sum_{X=\pi^0\gamma,\pi^+\pi^-,\ldots} a_\mu^X(LO) = \sum_X \frac{1}{4\pi^3}\int \sigma^0(e^+e^- \to X)K_\mu(s)ds$$

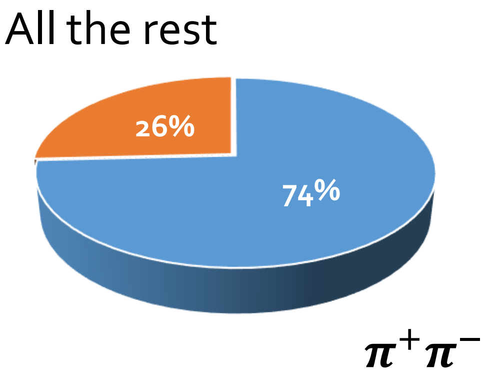


Channel	$a_\mu^{\mathrm{had,LO}}$ $[10^{-10}]$
$\pi^0\gamma$	$4.41 \pm 0.06 \pm 0.04 \pm 0.07$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$
$\pi^+\pi^-$	$507.85 \pm 0.83 \pm 3.23 \pm 0.55$
$\pi^+\pi^-\pi^0$	$46.21 \pm 0.40 \pm 1.10 \pm 0.86$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$
$\omega 2\pi$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi, \pi\gamma, \eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$

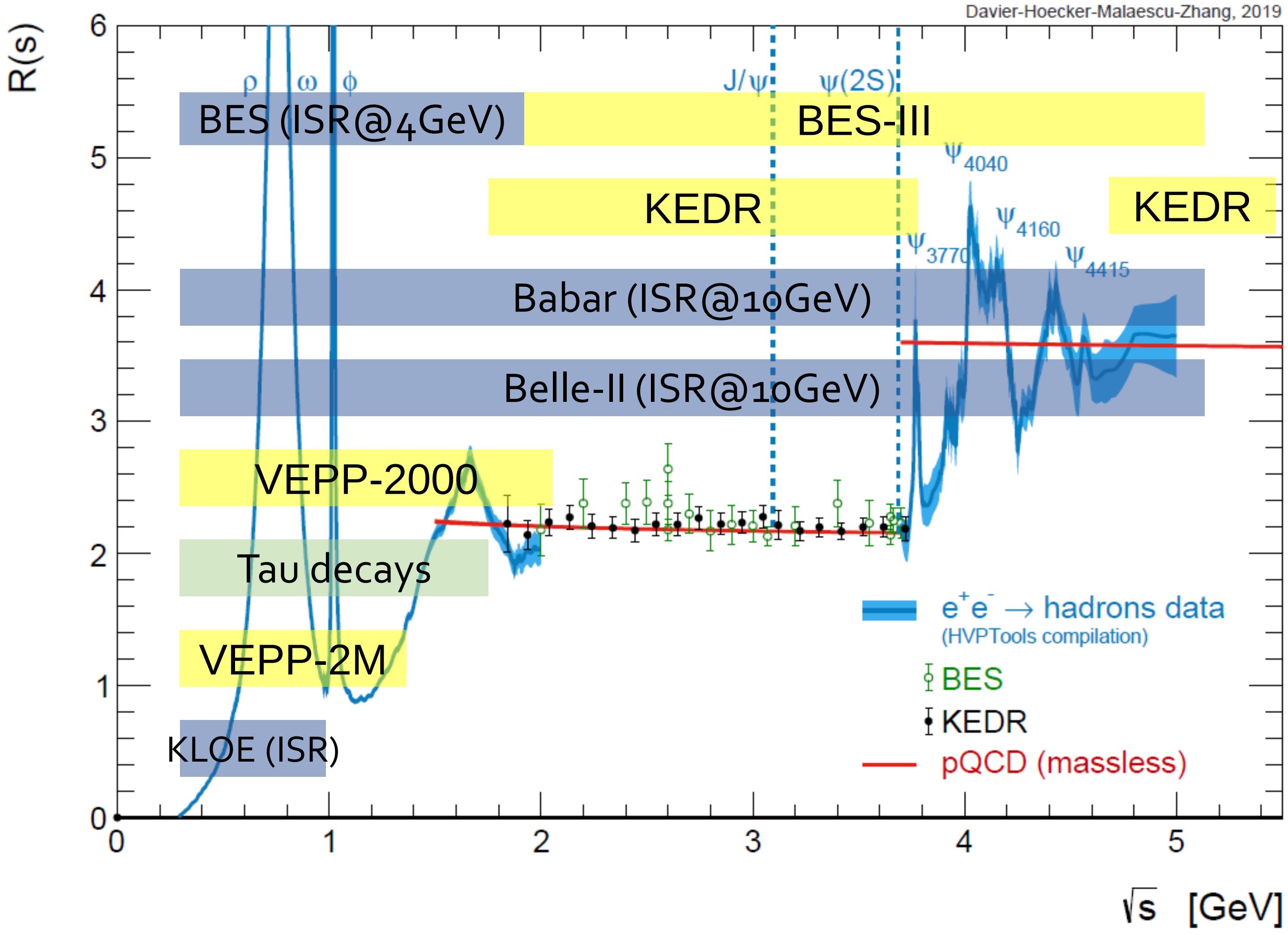
From DHMZ'19

The larger the contribution, the better relative precision is required

$e^+e^- \to \pi^+\pi^-$ is by far the most challenging and has got the most attention (74% of total hadronic contribution!)

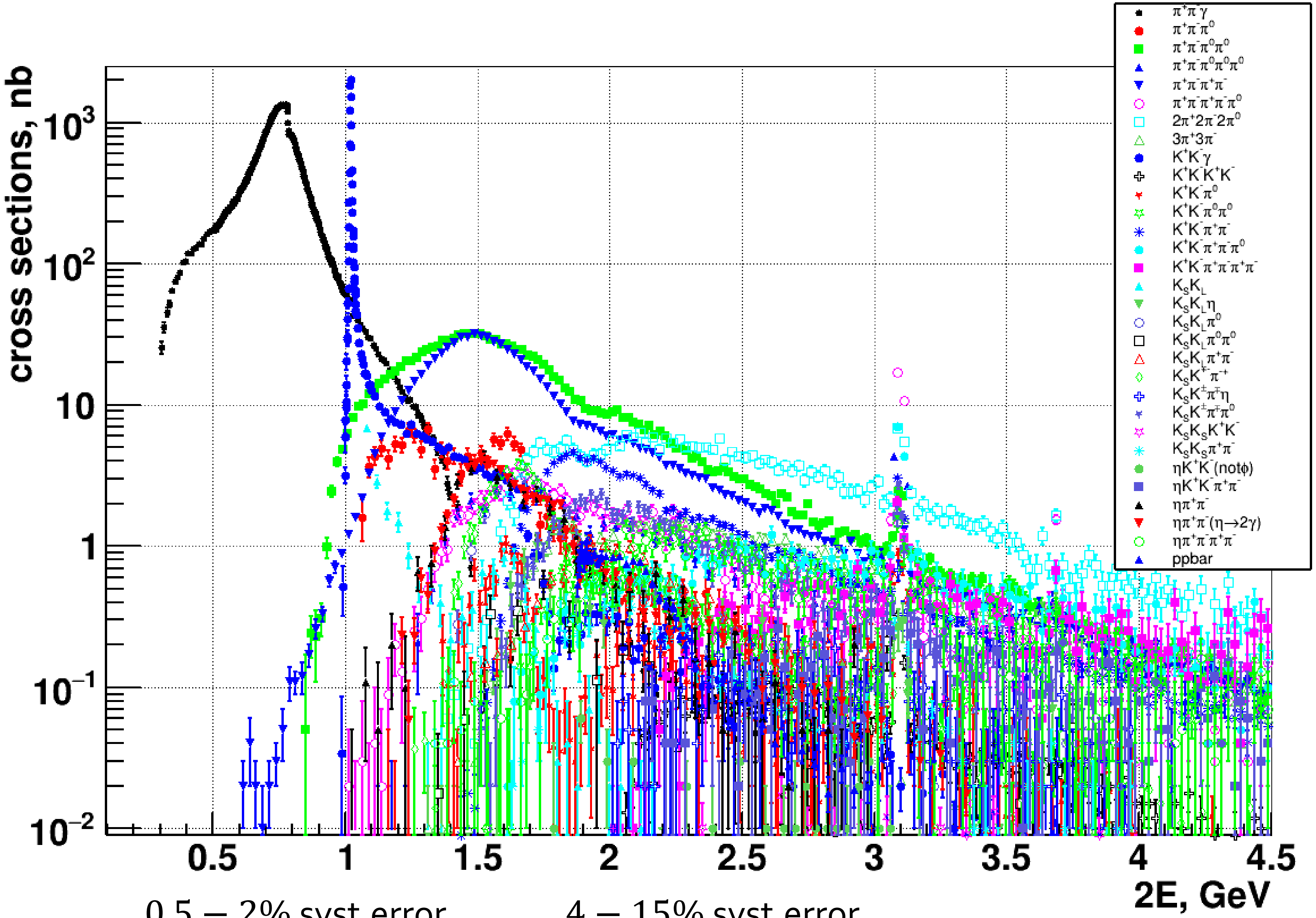


Where the measurements are done



Energy scan ISR Tau decays

BABAR

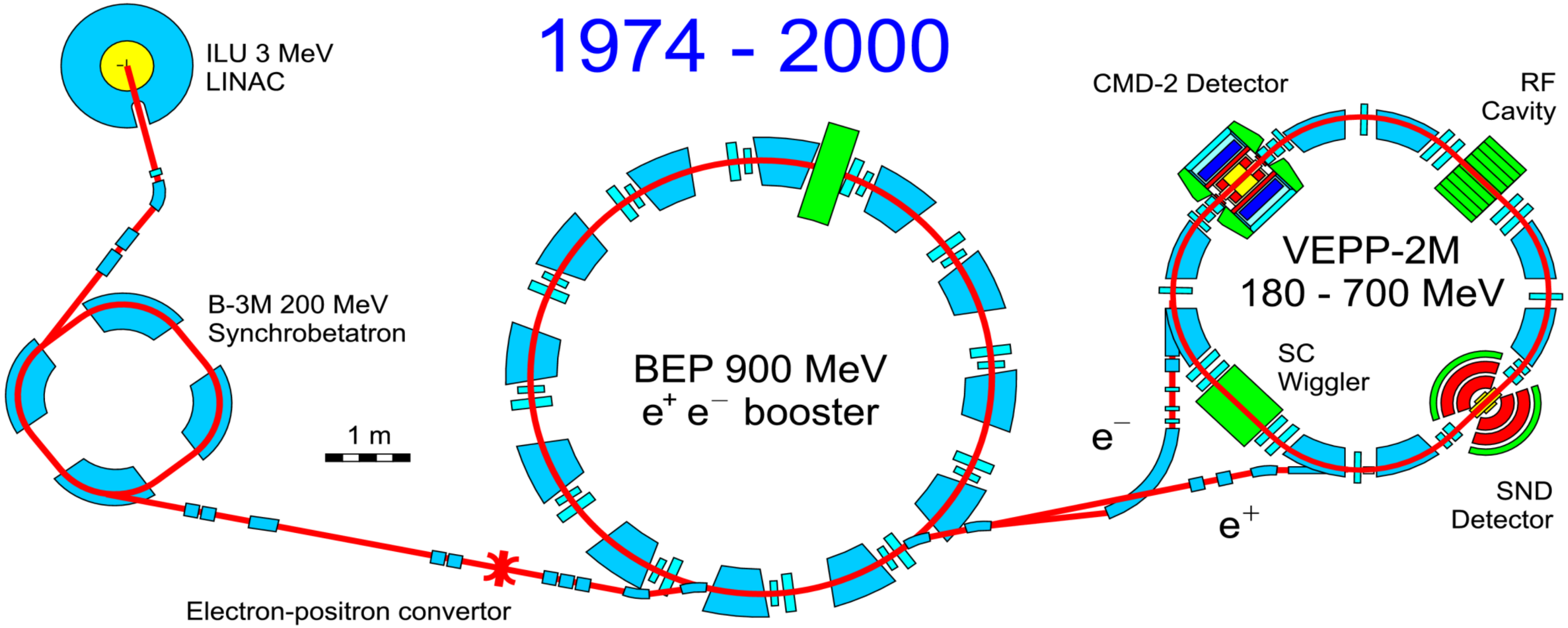


0.5 – 2% syst.error

4 – 15% syst.error

BABAR measurements are mostly tagged

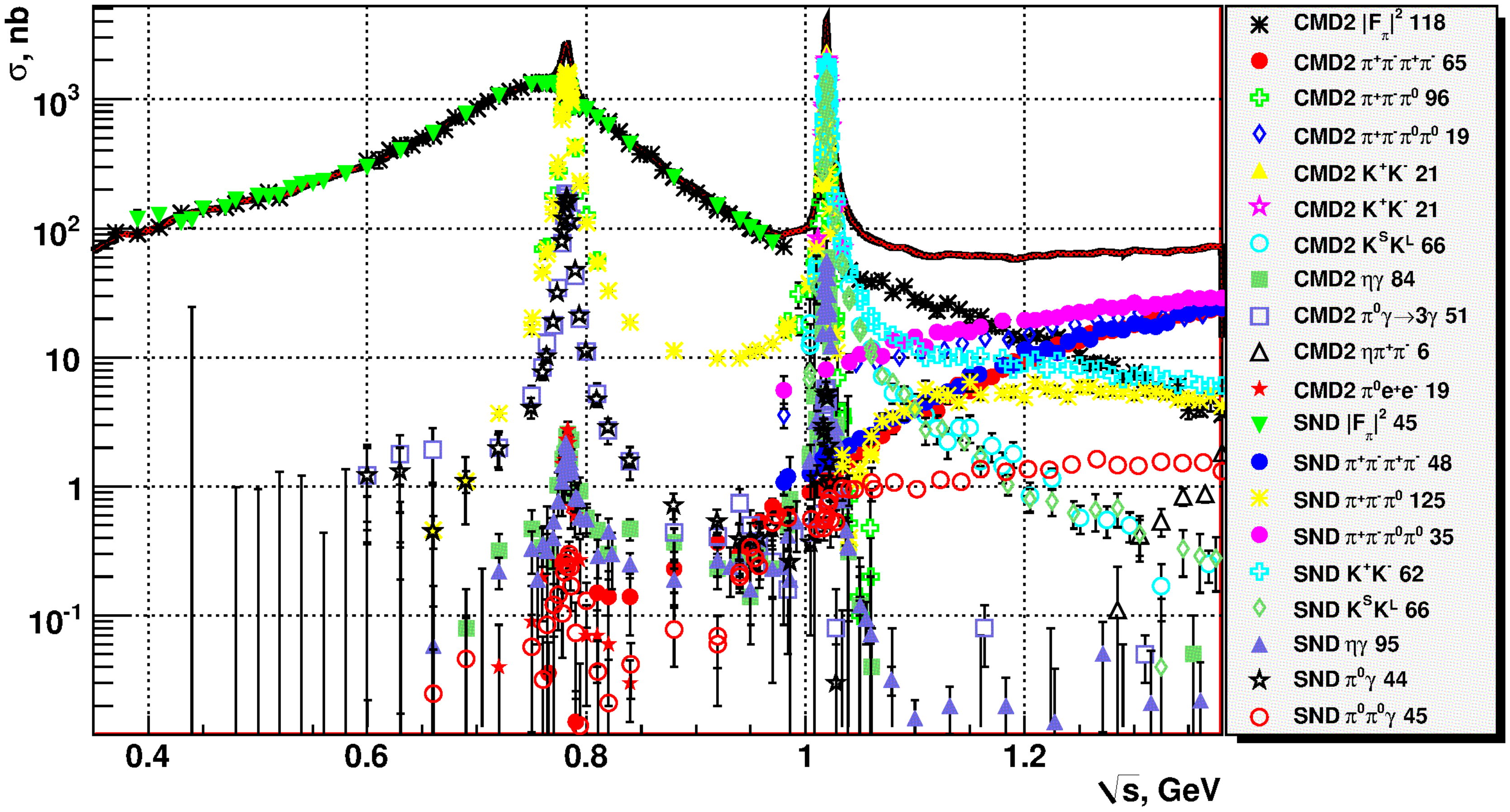
VEPP-2M (1993-2000)



Energy range: 0.36 – 1.4 GeV

Luminosity up to $5*10^{30}$ 1/cm^{2}s

Overview of VEPP-2M measurements

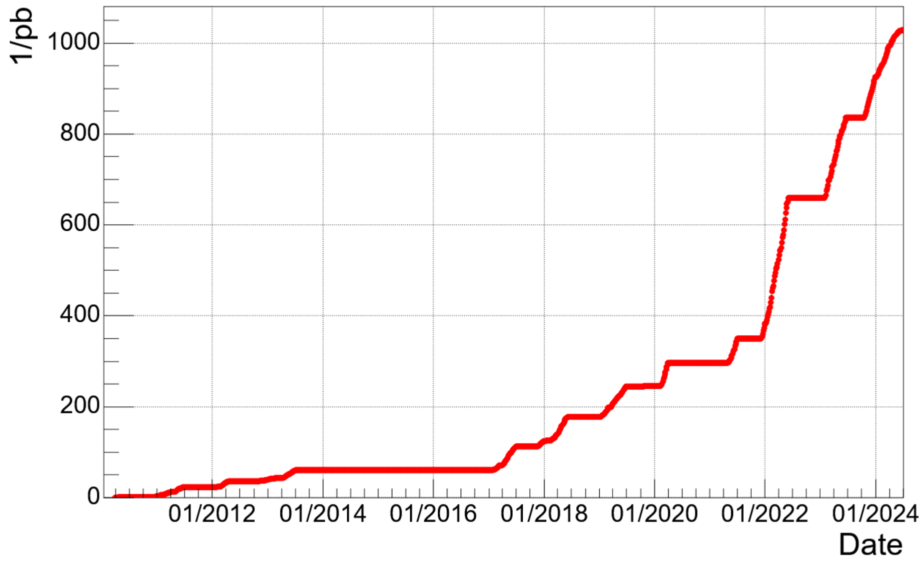


VEPP-2000 (2011-)

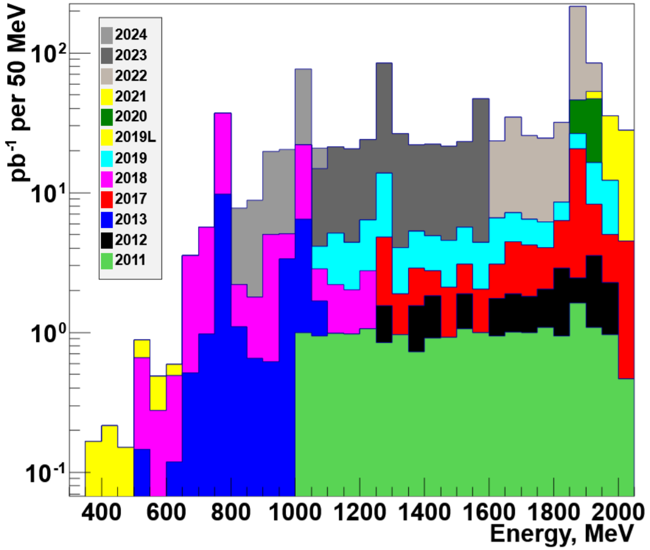
250 m beamline

VEPP-2000

New injection complex

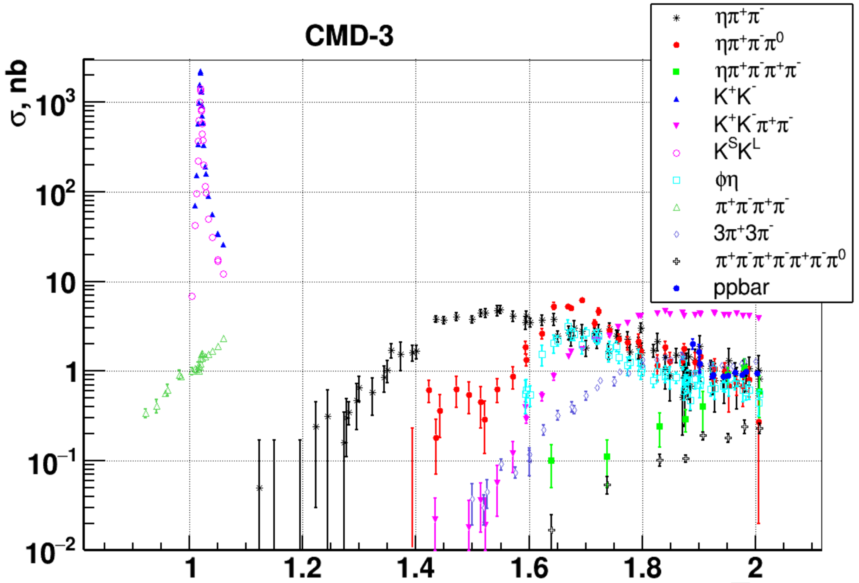


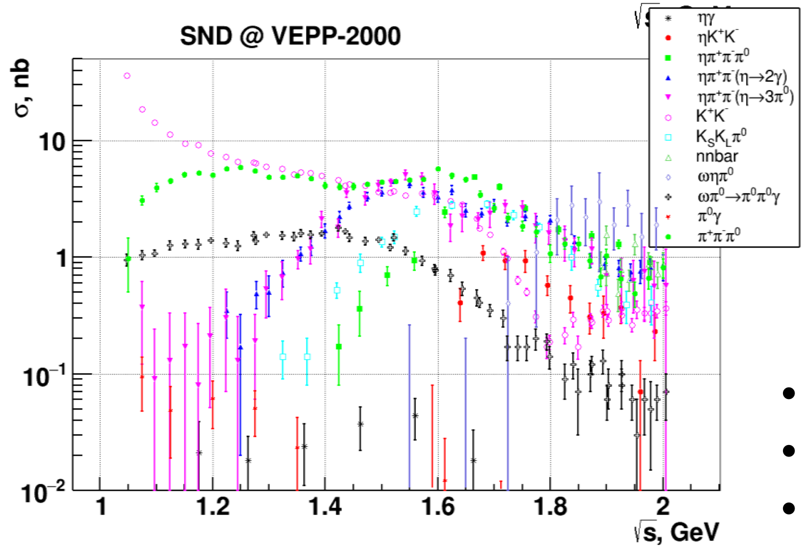
Data taking history



Collected integral by energy

Measurements at VEPP-2000



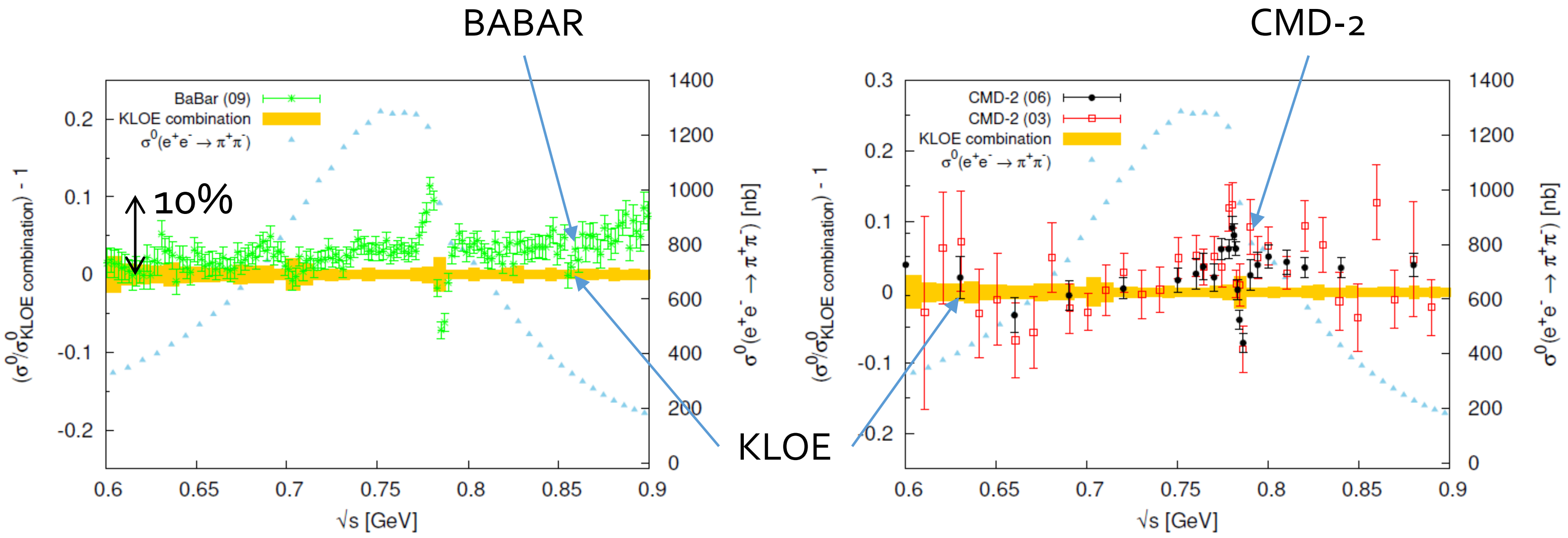


Final states under analysis at CMD-3

Signature	Final states (preliminary, published)
2 charged	$\pi^+\pi^-$, K^+K^-, K_SK_L, $p\bar{p}$
2 charged + γ's	$\pi^+\pi^-\gamma$, $\pi^+\pi^-\pi^0$, $\pi^+\pi^-2\pi^0$, $\pi^+\pi^-3\pi^0$, $\pi^+\pi^-4\pi^0$, $\pi^+\pi^-\eta$, $\pi^+\pi^-\pi^0\eta$, $\pi^+\pi^-2\pi^0\eta$, $K^+K^-\pi^0$, $K^+K^-2\pi^0$, $K^+K^-\eta$, $K_SK_L\pi^0$, $K_SK_L\eta$
4 charged	$2(\pi^+\pi^-)$, $K^+K^-\pi^+\pi^-$, $K_SK^\pm\pi^\mp$
4 charged + γ's	$2(\pi^+\pi^-)\pi^0$, $2\pi^+2\pi^-2\pi^0$, $\pi^+\pi^-\eta$, $\pi^+\pi^-\omega$, $2\pi^+2\pi^-\eta$, $K^+K^-\omega$, $K_SK^\pm\pi^\mp\pi^0$
6 charged	$3(\pi^+\pi^-)$, $K_SK_S\pi^+\pi^-$
6 charged + γ's	$3(\pi^+\pi^-)\pi^0$
Neutral	$\pi^0\gamma$, $2\pi^0\gamma$, $3\pi^0\gamma$, $\eta\gamma$, $\pi^0\eta\gamma$, $2\pi^0\eta\gamma$
Other	$n\bar{n}$, $\pi^0e^+e^-$, ηe^+e^-
Rare decays	η', $D^*(2007)^0$

- More final states compare to VEPP-2M
- 1-2 order of magnitude more data
- The experiments are collecting data

Tensions in $e^+e^- \to \pi^+\pi^-$ data

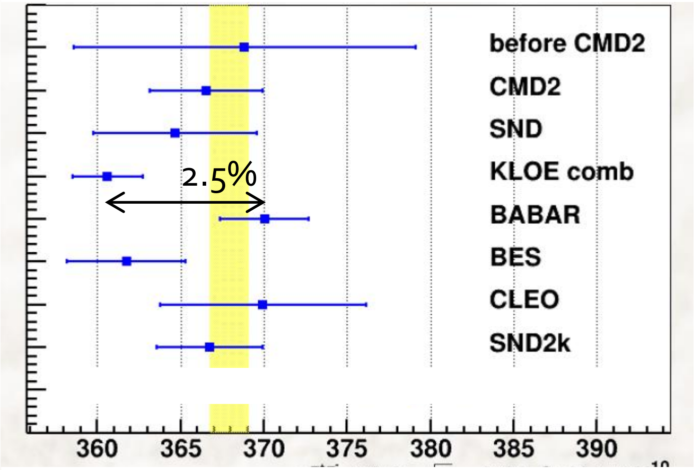


There are few-% discrepancies between various sub-% measurements of $\sigma(e^+e^- \to \pi^+\pi^-)$ *Unexplained*

WP2020: scale factor for $\Delta a_\mu(Had; LO)$

CMD-3 goal: new high statistics low systematics measurement of $\sigma(e^+e^- \to \pi^+\pi^-)$ via energy scan

$a_\mu^{had}(LO; 2\pi, 0.6 < \sqrt{s} < 0.88\ \text{GeV})$



$$\frac{1}{4\pi^3}\int_{0.6}^{0.88} \sigma^0(e^+e^- \to \pi^+\pi^-) K_\mu(s)\, ds$$

CMD-3 measurement of $\sigma(e^+e^- \to \pi^+\pi^-)$ (2023)

Phys.Rev.Lett. 132 (2024) 23, 231903

PHYSICAL REVIEW LETTERS **132,** 231903 (2024)

Editors' Suggestion

Measurement of the Pion Form Factor with CMD-3 Detector and Its Implication to the Hadronic Contribution to Muon $(g-2)$

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(Received 26 September 2023; revised 6 March

Phys.Rev.D 109 (2024) 11, 112002

PHYSICAL REVIEW D **109,** 112002 (2024)

Editors' Suggestion

Measurement of the $e^+e^- \to \pi^+\pi^-$ cross section from threshold to 1.2 GeV with the CMD-3 detector

F. V. Ignatov,[1,2,*] R. R. Akhmetshin,[1,2] A. N. Amirkhanov,[1,2] A. V. Anisenkov,[1,2] V. M. Aulchenko,[1,2] N. S. Bashtovoy,[1] D. E. Berkaev,[1,2] A. E. Bondar,[1,2] A. V. Bragin,[1] S. I. Eidelman,[1,2] D. A. Epifanov,[1,2] L. B. Epshteyn,[1,2,3] A. L. Erofeev,[1,2] G. V. Fedotovich,[1,2] A. O. Gorkovenko,[1,3] F. J. Grancagnolo,[4] A. A. Grebenuk,[1,2] S. S. Gribanov,[1,2] D. N. Grigoriev,[1,2,3] V. L. Ivanov,[1,2] S. V. Karpov,[1] A. S. Kasaev,[1] V. F. Kazanin,[1,2] B. I. Khazin,[1] A. N. Kirpotin,[1] I. A. Koop,[1,2] A. A. Korobov,[1,2] A. N. Kozyrev,[1,3] E. A. Kozyrev,[1,2] P. P. Krokovny,[1,2] A. E. Kuzmenko,[1] A. S. Kuzmin,[1,2] I. B. Logashenko,[1,2] P. A. Lukin,[1,2] A. P. Lysenko,[1] K. Yu. Mikhailov,[1,2] I. V. Obraztsov,[1,2] V. S. Okhapkin,[1] A. V. Otboev,[1] E. A. Perevedentsev,[1,2] Yu. N. Pestov,[1] A. S. Popov,[1,2] G. P. Razuvaev,[1,2] Yu. A. Rogovsky,[1,2] A. A. Ruban,[1] N. M. Ryskulov,[1] A. E. Ryzhenenkov,[1,2] A. V. Semenov,[1,2] A. I. Senchenko,[1] P. Yu. Shatunov,[1] Yu. M. Shatunov,[1] V. E. Shebalin,[1,2] D. N. Shemyakin,[1,2] B. A. Shwartz,[1,2] D. B. Shwartz,[1,2] A. L. Sibidanov,[5] E. P. Solodov,[1,2] A. A. Talyshev,[1,2] M. V. Timoshenko,[1] V. M. Titov,[1] S. S. Tolmachev,[1,2] A. I. Vorobiov,[1] I. M. Zemlyansky,[1] D. S. Zhadan,[1] Yu. M. Zharinov,[1] A. S. Zubakin,[1] and Yu. V. Yudin[1,2]

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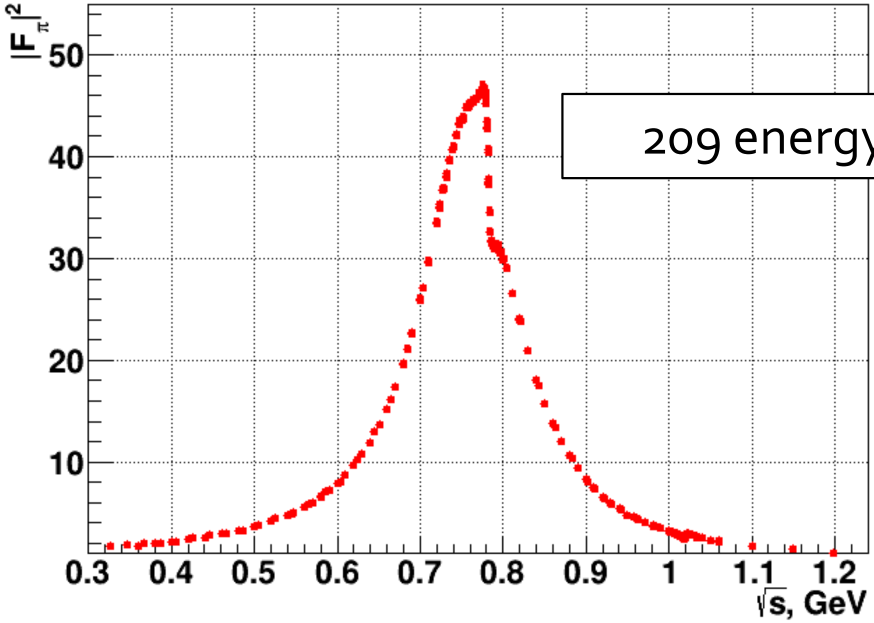
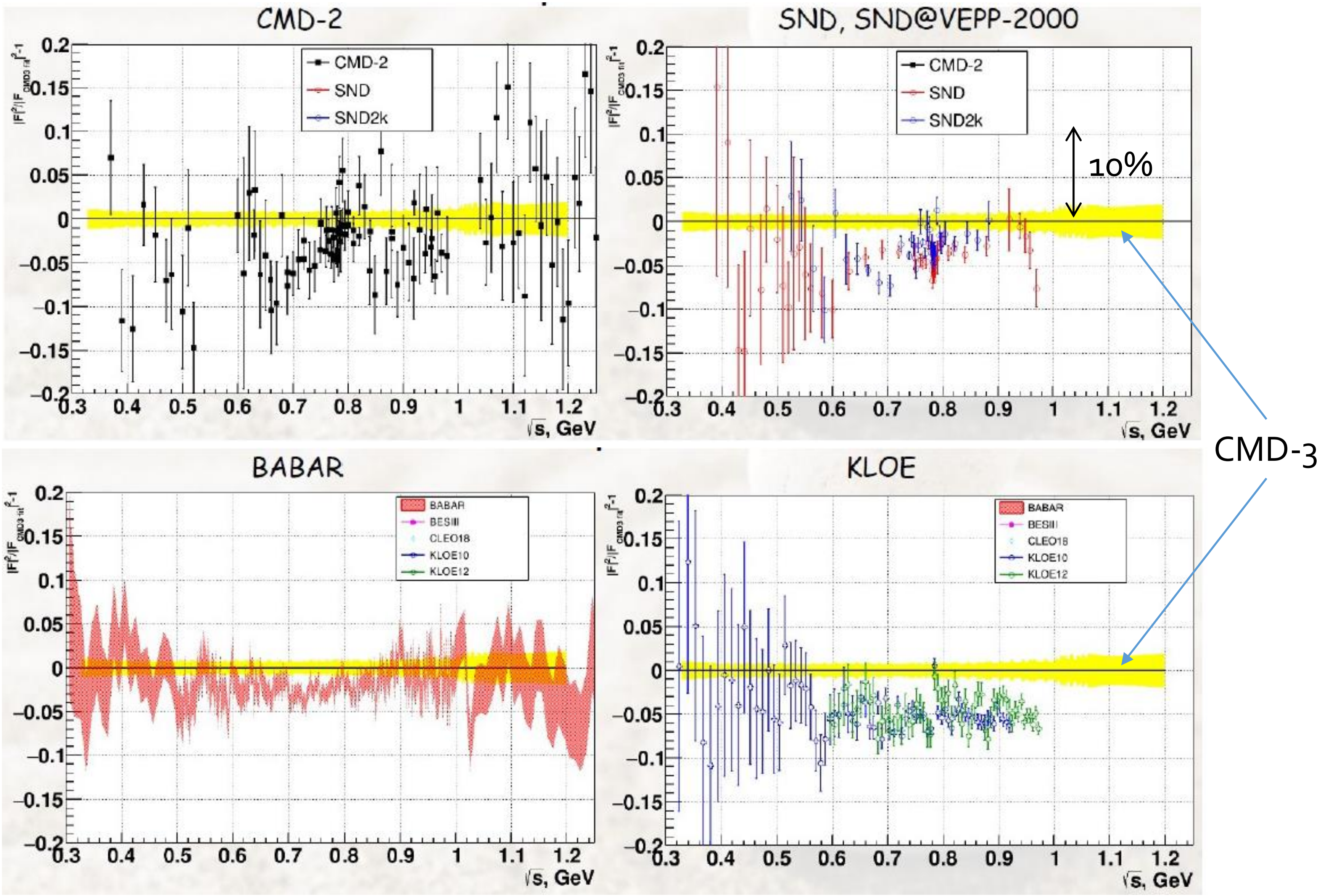
February 2024; published 4 June 2024)



209 energy points

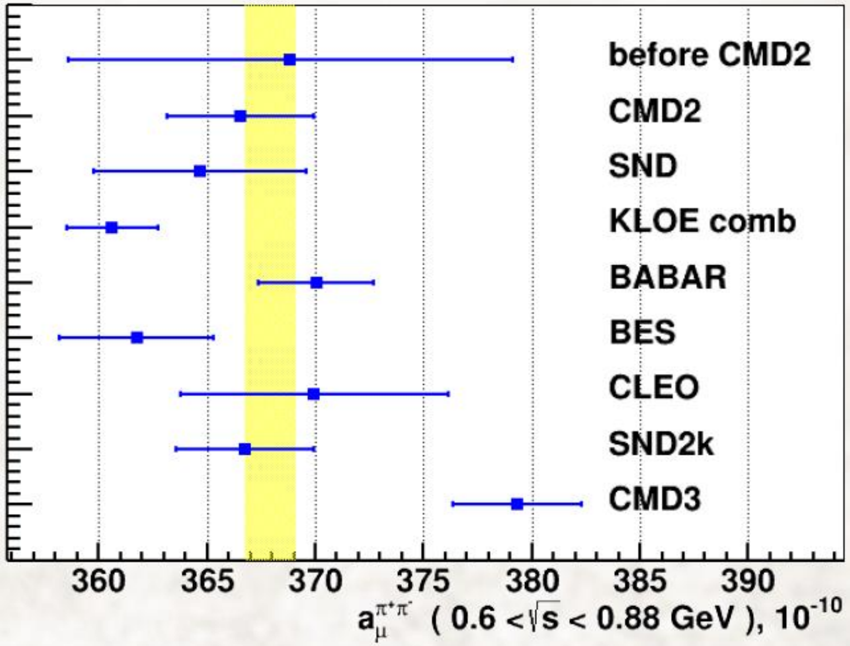
Comparison of CMD-3 to other measurements

CMD-3 is systematically above previous measurements by ~2-5%



$a_\mu(had;LO)$: the status

$$a_\mu^{had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{4m_\pi^2}^{\infty} \frac{\sigma_{e^+e^-\to\gamma^*\to hadrons}(s)K(s)}{s}ds$$



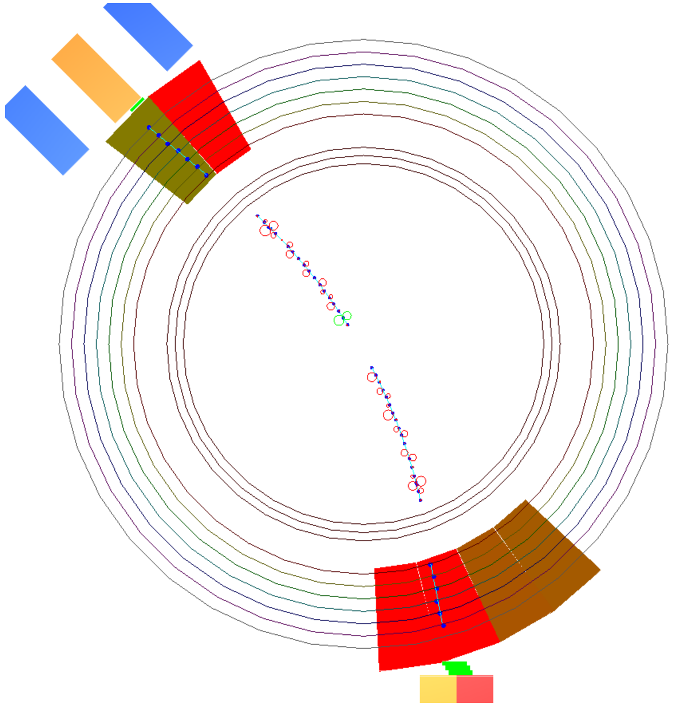
Discrepancies in data "blind" $a_\mu(SM)$

It seems that existing measurements of $e^+e^- \to \pi^+\pi^-$ underestimated systematic uncertainty (at least at some energy range)

CMD-3 simply exaggerated the problem, but it was there already

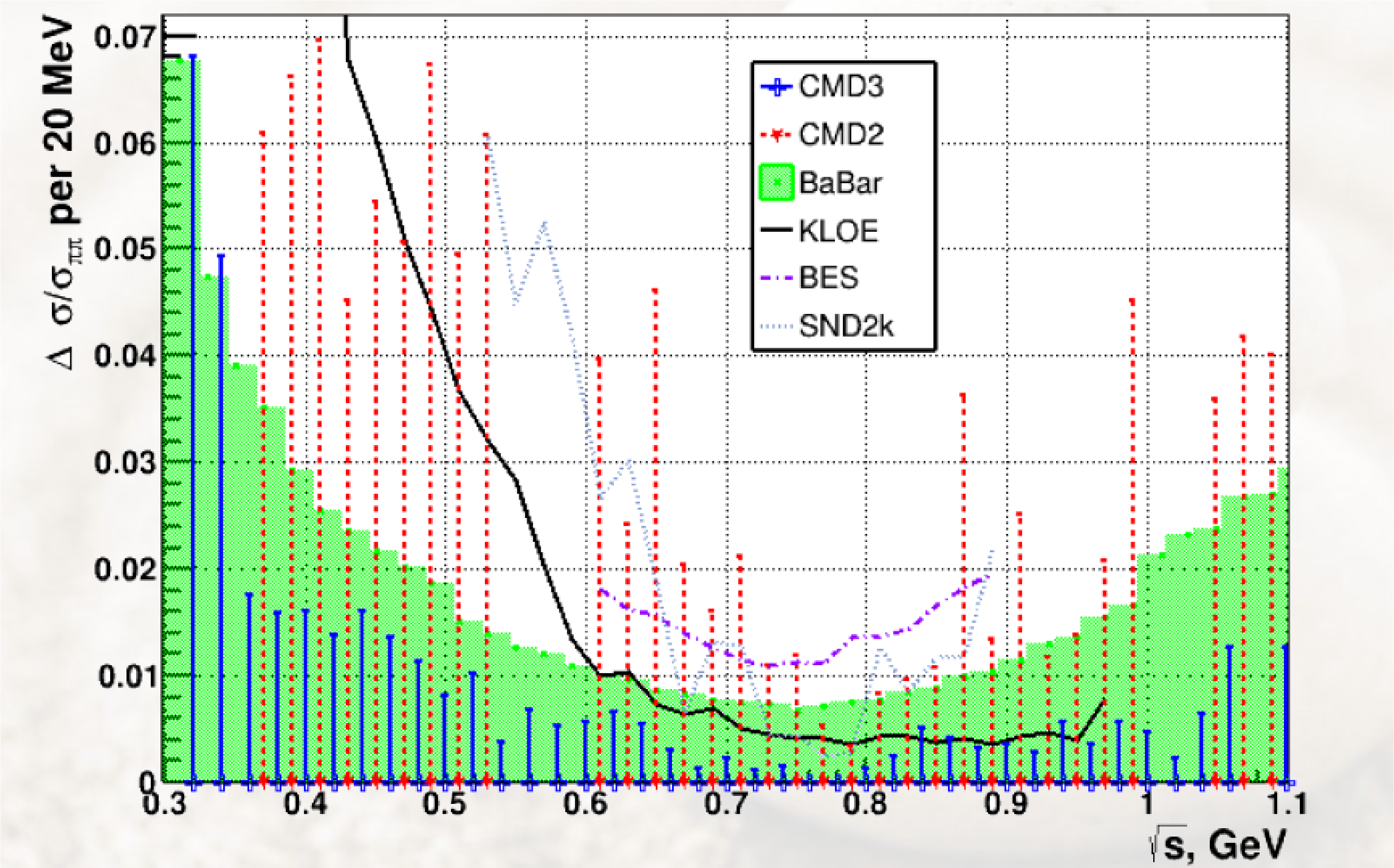
Features of CMD-3 measurement

- World-largest statistics
  - 34 000 000 $e^+e^- \to \pi^+\pi^-$
  - 3 700 000 $e^+e^- \to \mu^+\mu^-$
  - 44 000 000 $e^+e^- \to e^+e^-$
- Many built-in cross checks
  - 3 methods for final states indentification
  - 2 methods for angle measurement
  - Measurement of $\sigma(e^+e^- \to \mu^+\mu^-)$
  - Measurement of charge asymmetry
- Very detailed study of potential systematics



Example of $e^+e^- \to \pi^+\pi^-$ event

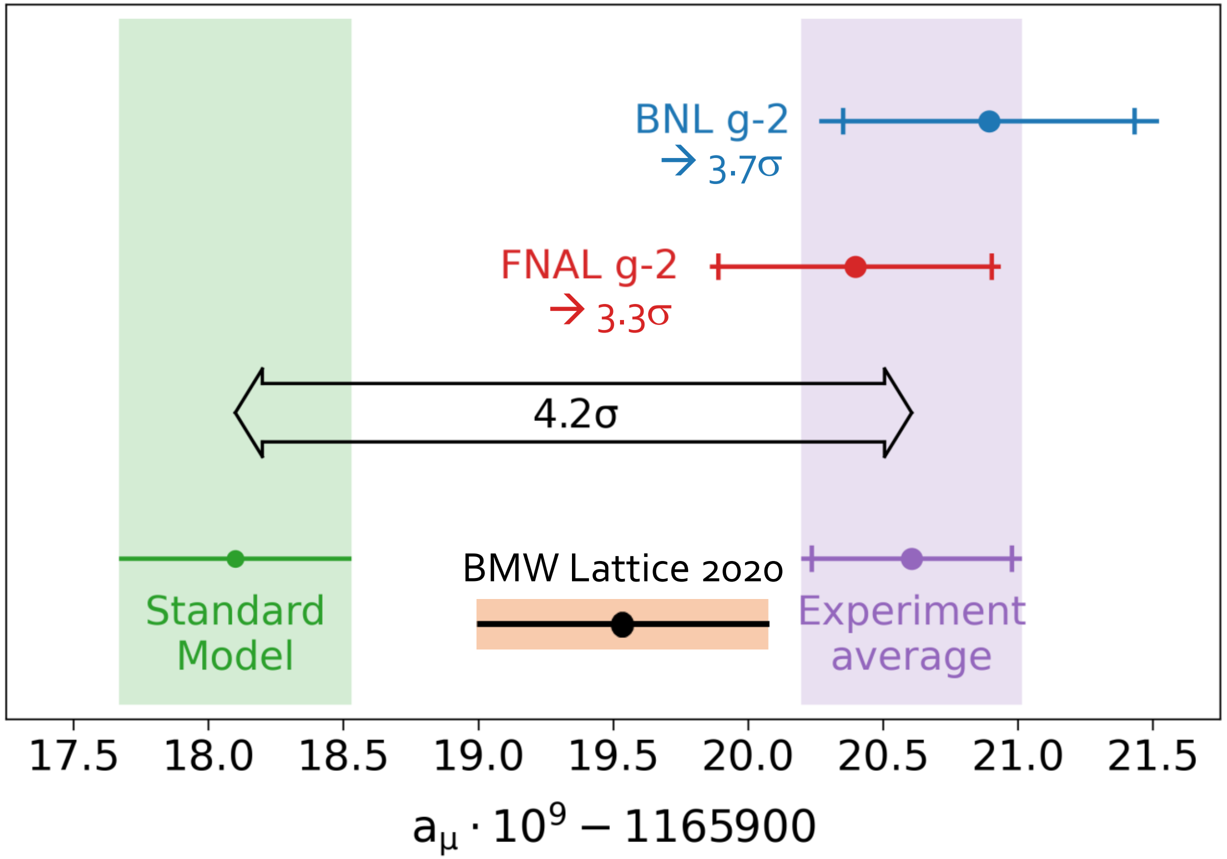
Statistical precision of CMD-3 data



based on L = 61.9 pb^{-1} at $\sqrt{s}$ < 1 GeV (+25.7 pb^{-1}, 1.0-1.2 GeV)

At the beginning of 2023…

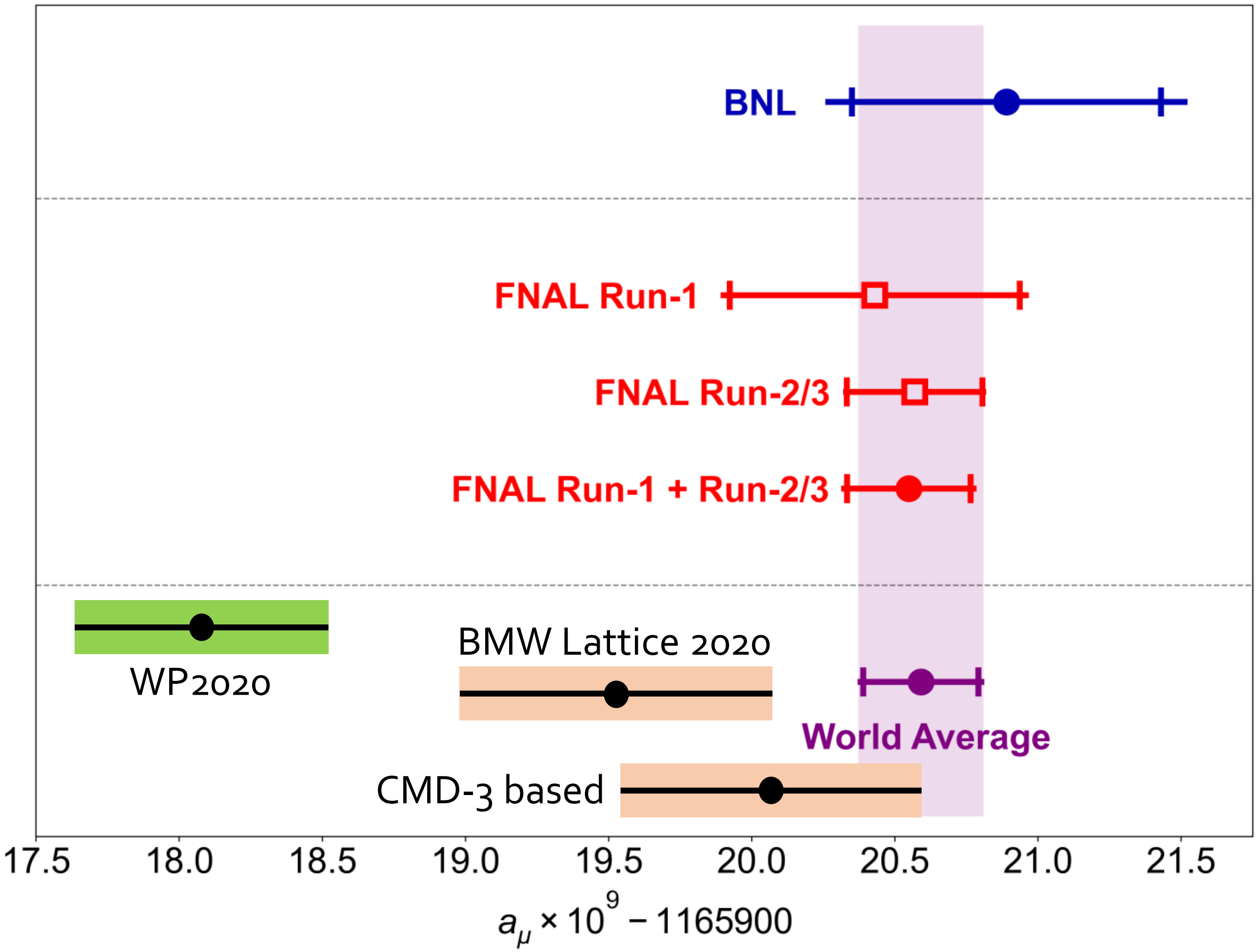
$a_\mu(\text{SM}) = 0.00116591810(43) \rightarrow 368$ ppb



$a_\mu(\text{Exp}) - a_\mu(\text{SM}) = 0.00000000251(59) \rightarrow 4.2\sigma$

Experiment vs SM prediction

End of 2023



At the moment, the SM prediction for a_μ is unclear (due to hadronic contribution)

Is there need for new measurements of hadronic cross sections?

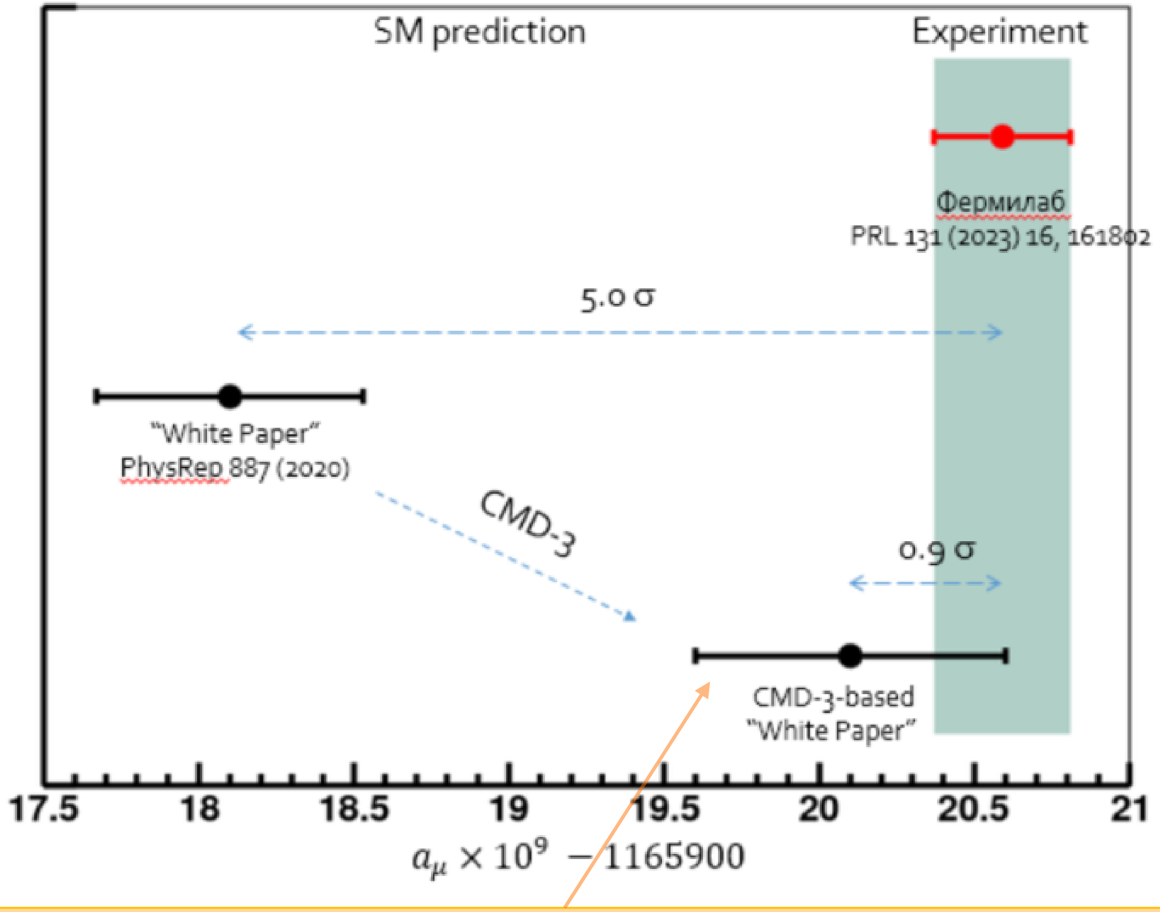
There are significant efforts to understand the discrepancies and to obtain additional new $e^+e^- \to H$ data:

- SND has the same amount of data collected as CMD-3, analysis is in progress
- BABAR is making reanalysis of old data using new approach (angular analysis)
- BELLE-II plans to do ISR measurement of $e^+e^- \to H$ cross sections
- BES-III and KLOE perform analysis of additional data

In order to match FNAL, cross sections need to be measured to ~0.2%
Neither of existing experiment expect to reach such precision – need next generation experiments

Channel	Contribution, $\cdot 10^{10}$ (KNT19)	Relative accuracy, need (now)
$\pi^+\pi^-$	504.23(1.90) (0.4%) ???	0.23% (0.8%)
$\pi^+\pi^-\pi^0$	46.63(94) (2.0%)	1.1% (1.5-3%)
$\pi^+\pi^-\pi^+\pi^-$	13.99(19) (1.4%)	0.8% (2-3%)
$\pi^+\pi^-\pi^0\pi^0$	18.15(74) (4.0%)	2.3% (5%)
K^+K^-	23.00(22) (1.0%)	0.6% (2%)
K_SK_L	13.04(19) (1.5%)	0.7% (2%)
$\boldsymbol{a_\mu(had; LO)}$	**692.8(2.4) (0.35%)**	**0.2%**

Is there need for new measurements of hadronic cross sections?



Quest for next-generation experiments: reduce these error bars
Ultimate goal: Hadron data = Lattice QCD = MuONe

Large statistics

Detector improvements

Improved MC generators for radiative corrections

Inclusive measurements

Inclusive measurements were systematically performed at $\sqrt{s} \gtrsim 2$ GeV

Signal events: one or more hadrons in the final state + any number of extra particles
Cuts on multiplicity, sphericity,…
With or without particle identification

$$\sigma_{\mathrm{mh}}^{\mathrm{obs}}(s) = \frac{N_{\mathrm{mh}} - N_{\mathrm{res.bg}}}{\int \mathcal{L}\,\mathrm{d}t}$$

$$R = \frac{\sigma_{\mathrm{mh}}^{\mathrm{obs}}(s) - \sum \varepsilon_{\mathrm{bg}}(s)\,\sigma_{\mathrm{bg}}(s) - \sum \varepsilon_{\psi}(s)\,\sigma_{\psi}(s)}{\varepsilon(s)\left(1+\delta(s)\right)\sigma_0^{e^+e^-\to\mu^+\mu^-}(s)}$$

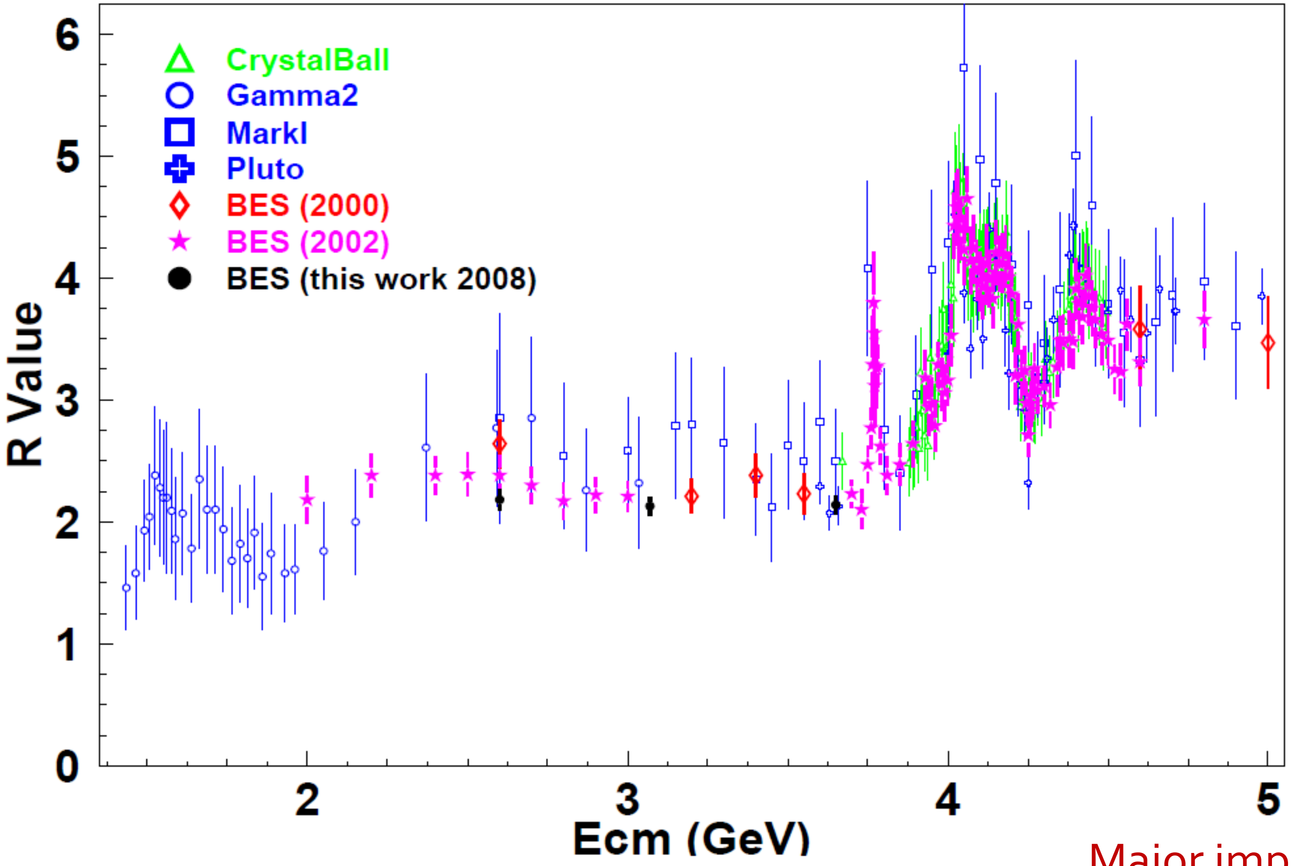
The analysis depends on the same ingredients as the exclusive measurement: event selection, luminosity measurement, calculation of radiative corrections, evaluation of detector efficiency

Key difficulty: to properly model hadronic events for evaluation of efficiencies and radiative corrections. There are dedicated MC generators: JETSET, LUARLW

"Typical" good precision: $\frac{\delta R}{R} \sim 3\%$, best achieved $\sim 2\%$.
Important to have large detection efficiency (now $\sim 75\%$)

BES-II

PRL88(2002)101802
PLB677(2009)239



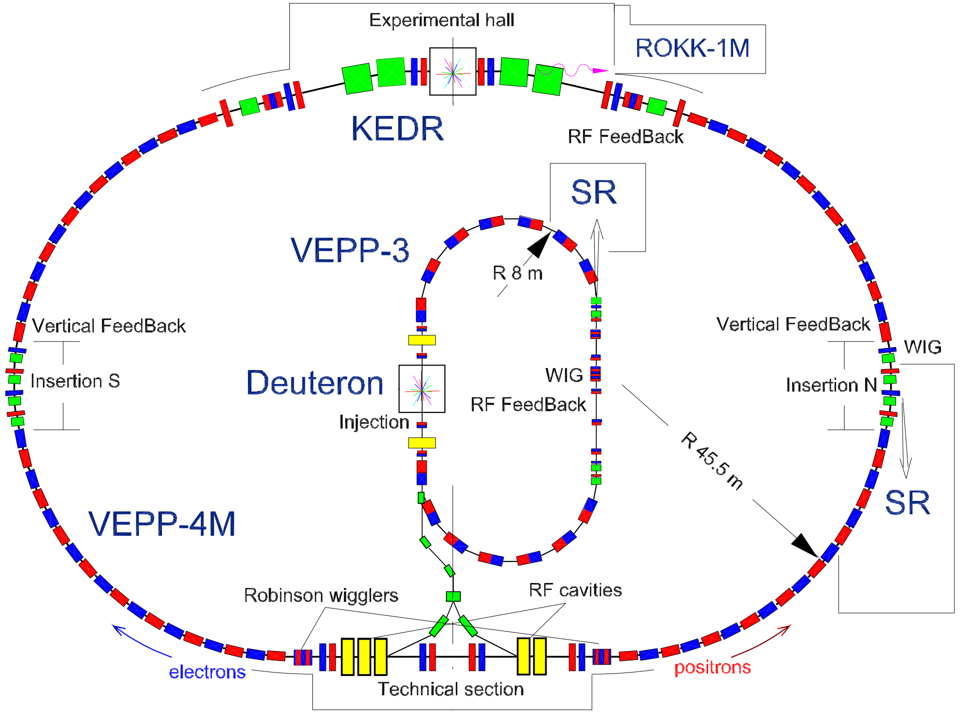
Major improvement!

BES-II performed detailed R(s) scan between 2 and 5 GeV

- 3 – 5% statistical error per point
- 5 – 8% systematical error

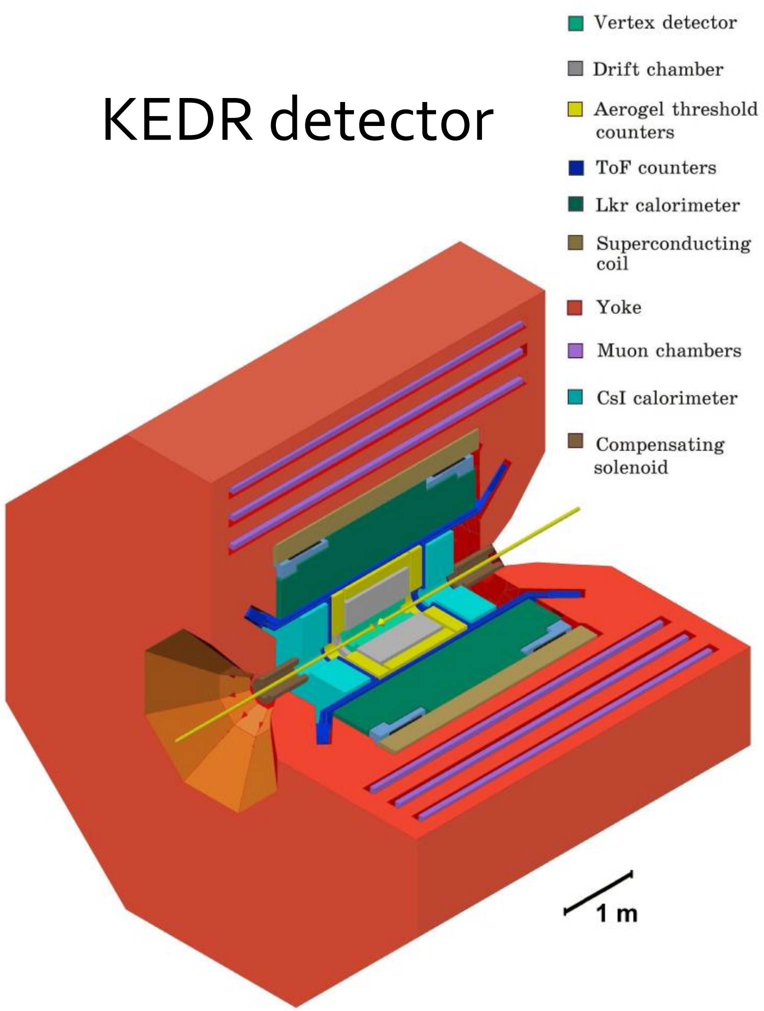
BES-III collected a lot of R(s) data, partly published

KEDR



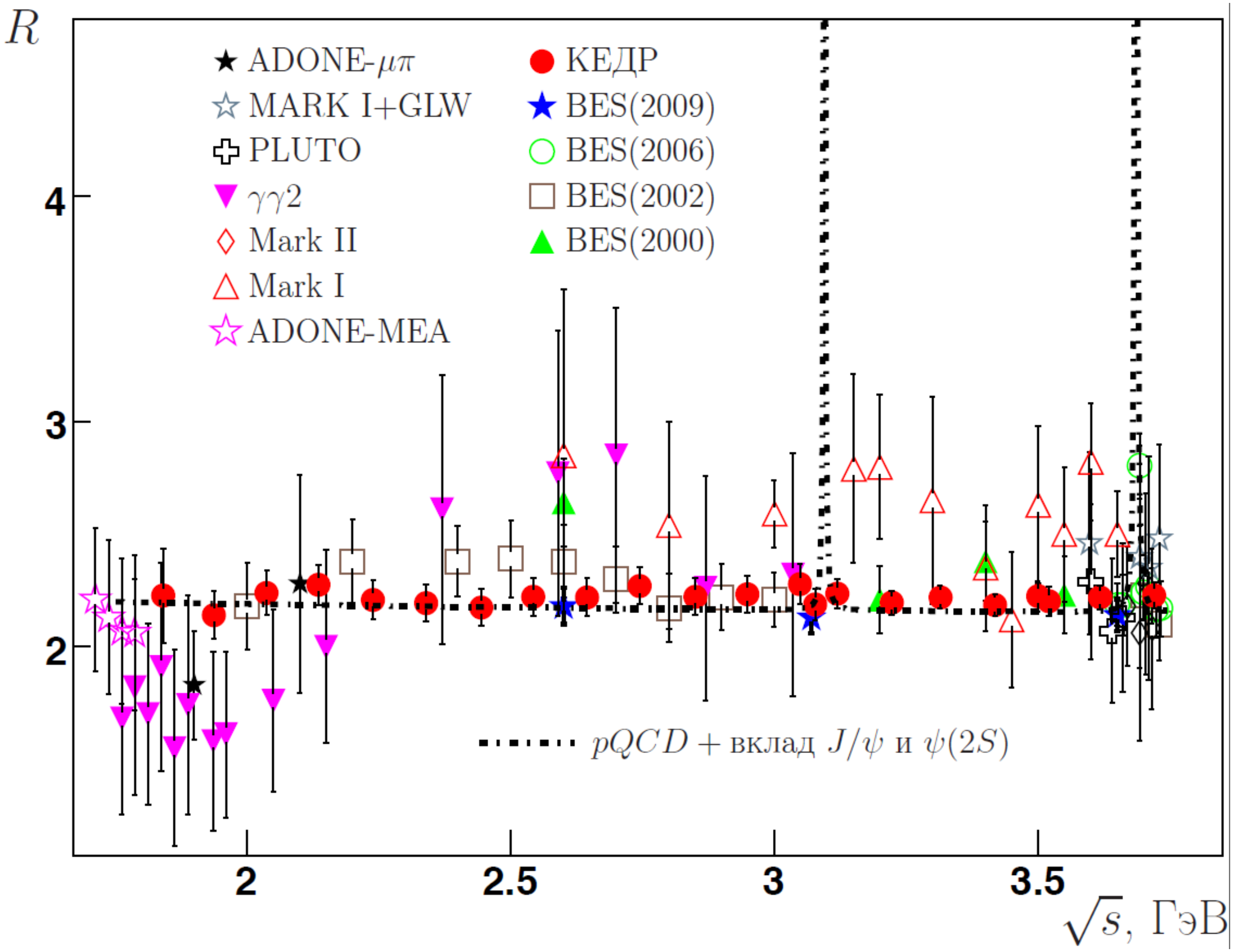
VEPP-4M collider

KEDR detector



Beam energy range 0.925-5.3 GeV
Luminosity $\sim 4 \cdot 10^{31}\ cm^{-2}s^{-1}$
Beam energy is determined to 20-30 keV (using Compton backscattering and resonance depolarization)

KEDR

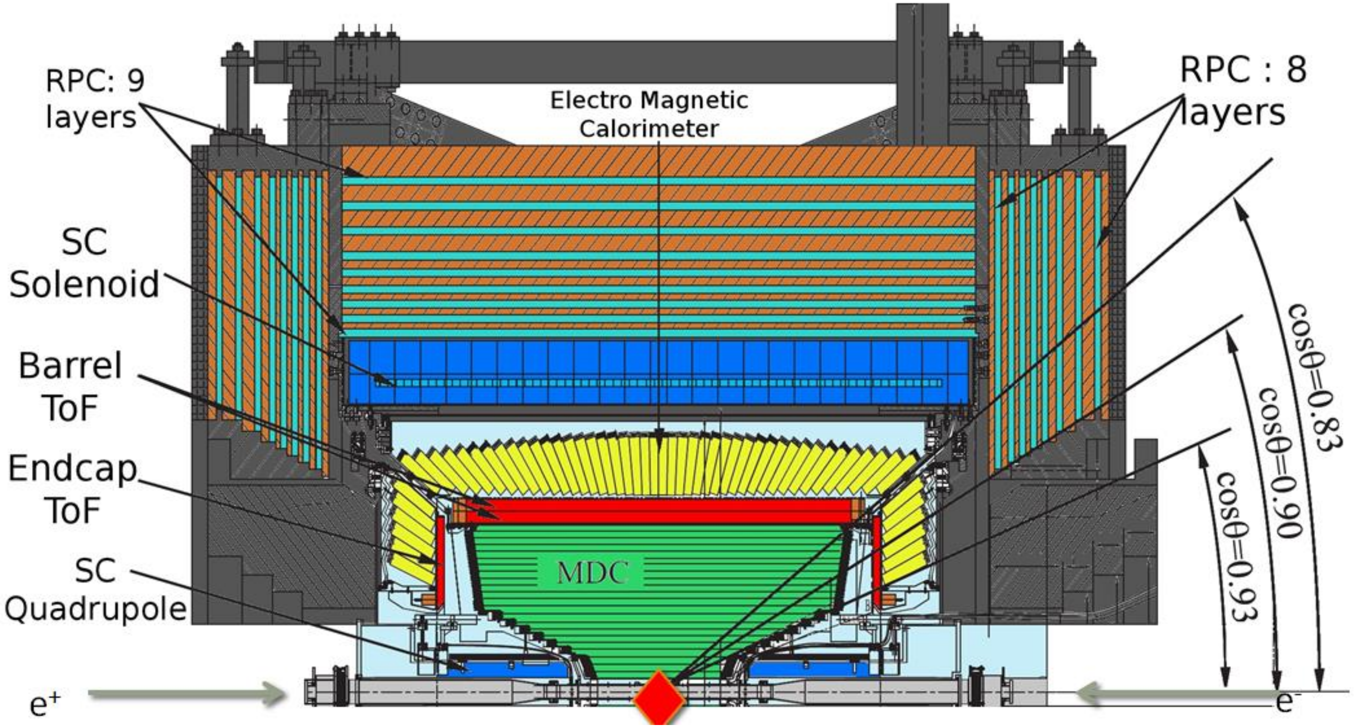


KEDR performed detailed R(s) scan between 1.8 and 3.7 GeV

- $2-3\%$ statistical error per point
- $2-3\%$ systematical error

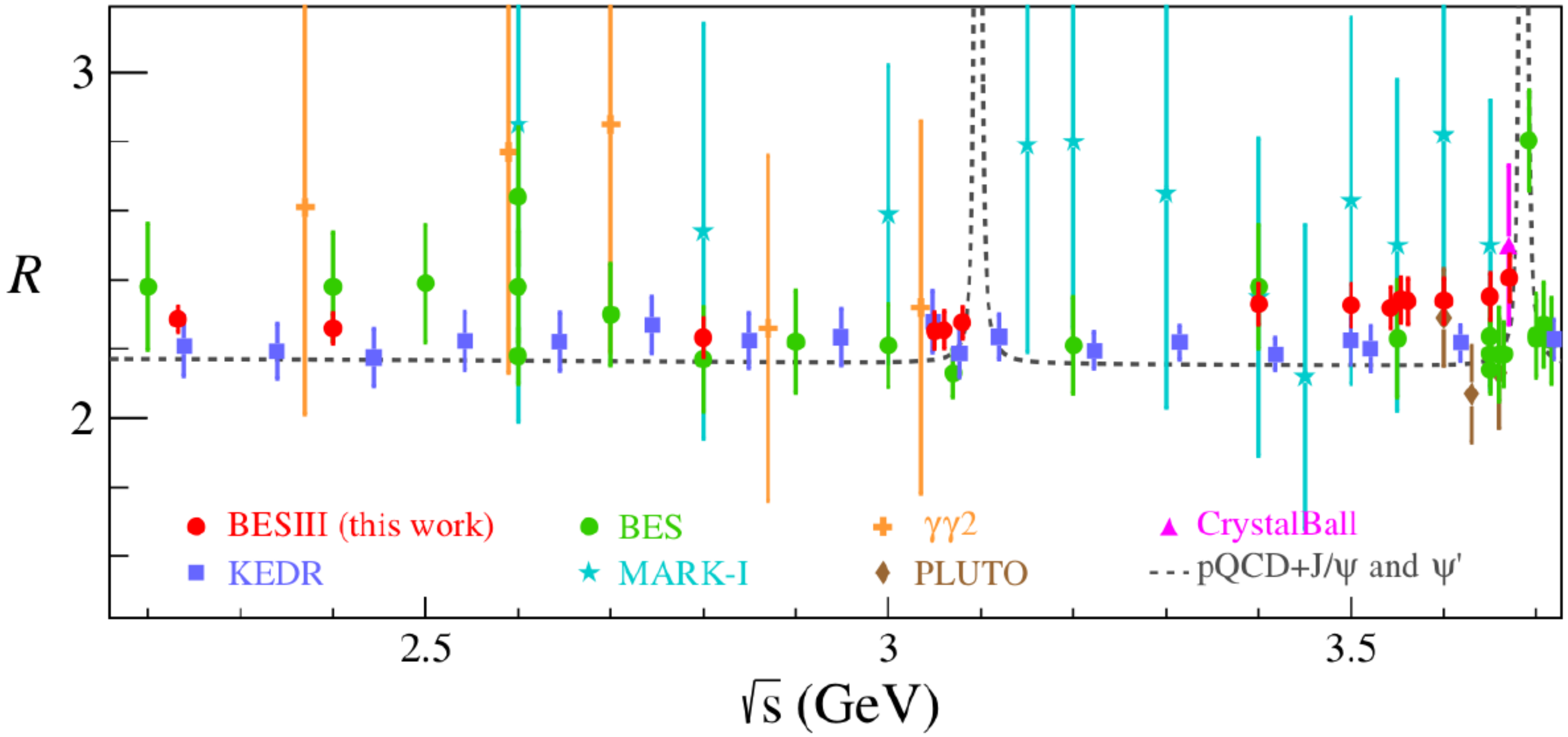
KEDR collected R(s) data between 4.7 and 7.0 GeV (17 points)

BES-III



BEPC-II collider covers c.m.energy range from 2 to 5 GeV
“$c\tau$-factory”

BES-III

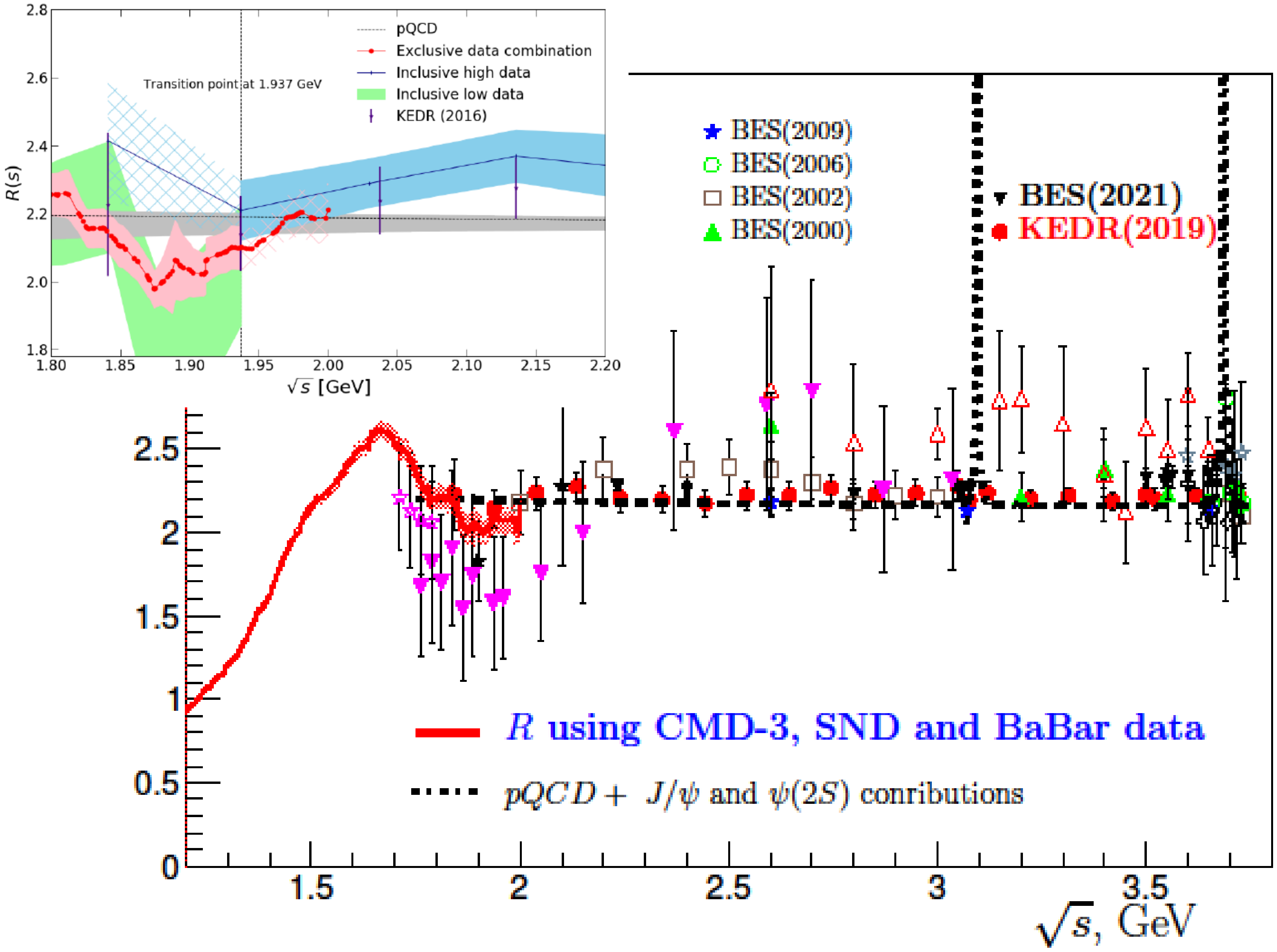


Phys. Rev. Lett. 128, 062004 (2022)

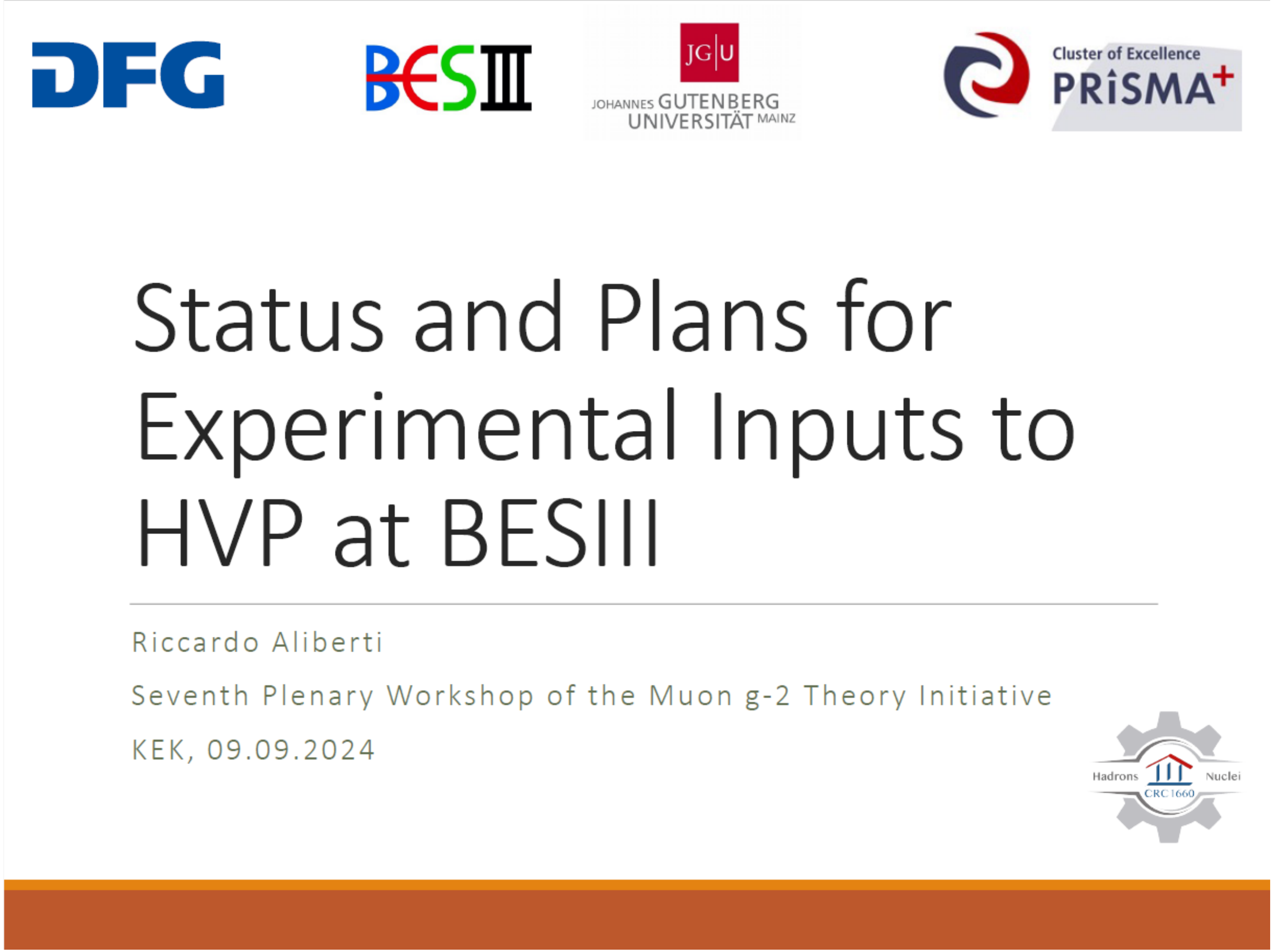
$\text{Syst.error} \approx 1.6 - 2.8\%,\ \text{stat.error} < 0.4\%$

Discrepency with pQCD and KEDR?

Is there agreement between inclusive and exclusive?



BES-III: expected soon



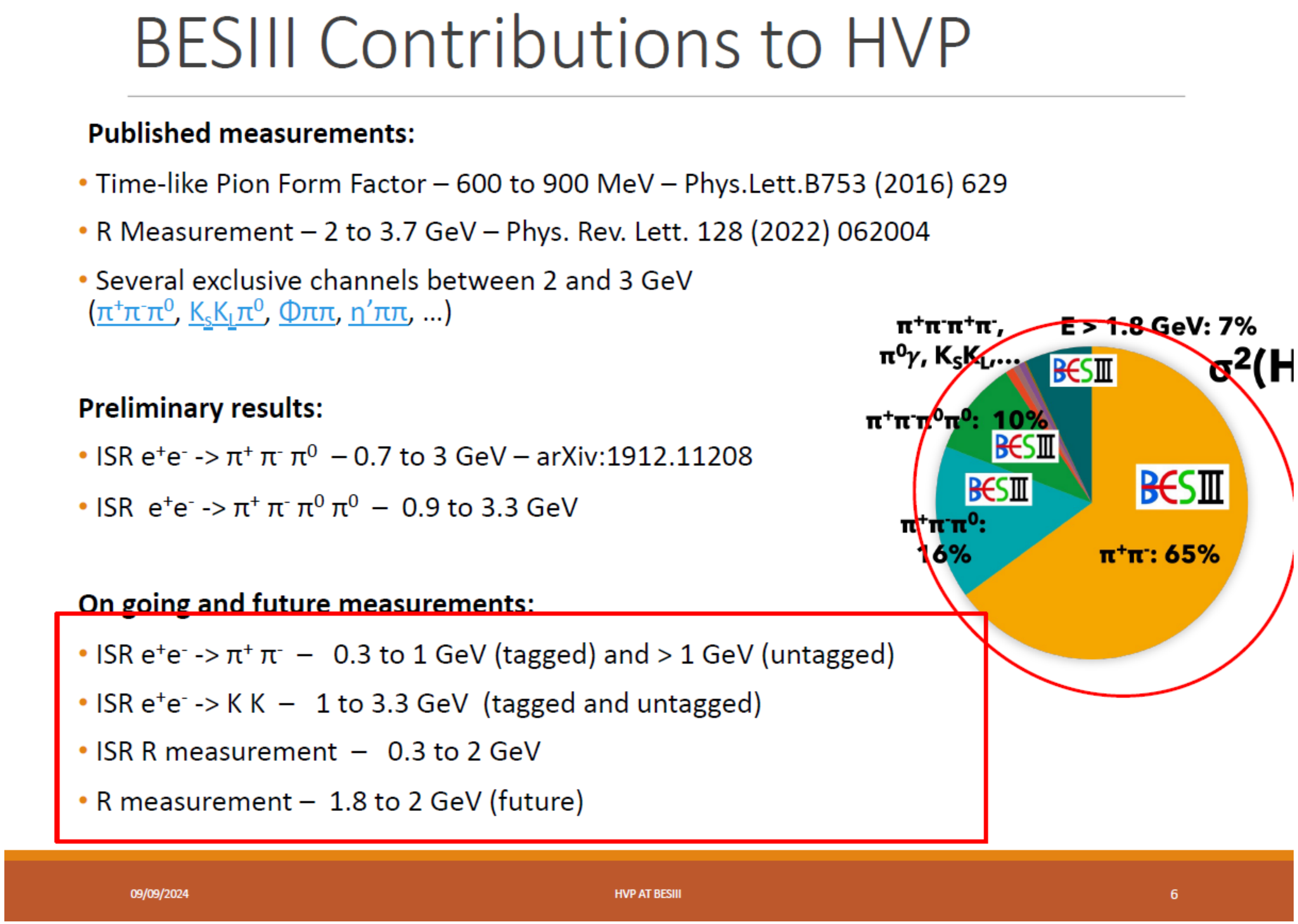
Status and Plans for Experimental Inputs to HVP at BESIII

Riccardo Aliberti

Seventh Plenary Workshop of the Muon g-2 Theory Initiative

KEK, 09.09.2024

BES-III: expected soon



BESIII Contributions to HVP

Published measurements:

- Time-like Pion Form Factor – 600 to 900 MeV – Phys.Lett.B753 (2016) 629
- R Measurement – 2 to 3.7 GeV – Phys. Rev. Lett. 128 (2022) 062004
- Several exclusive channels between 2 and 3 GeV ($\pi^+\pi^-\pi^0$, $K_sK_l\pi^0$, $\Phi\pi\pi$, $\eta'\pi\pi$, …)

Preliminary results:

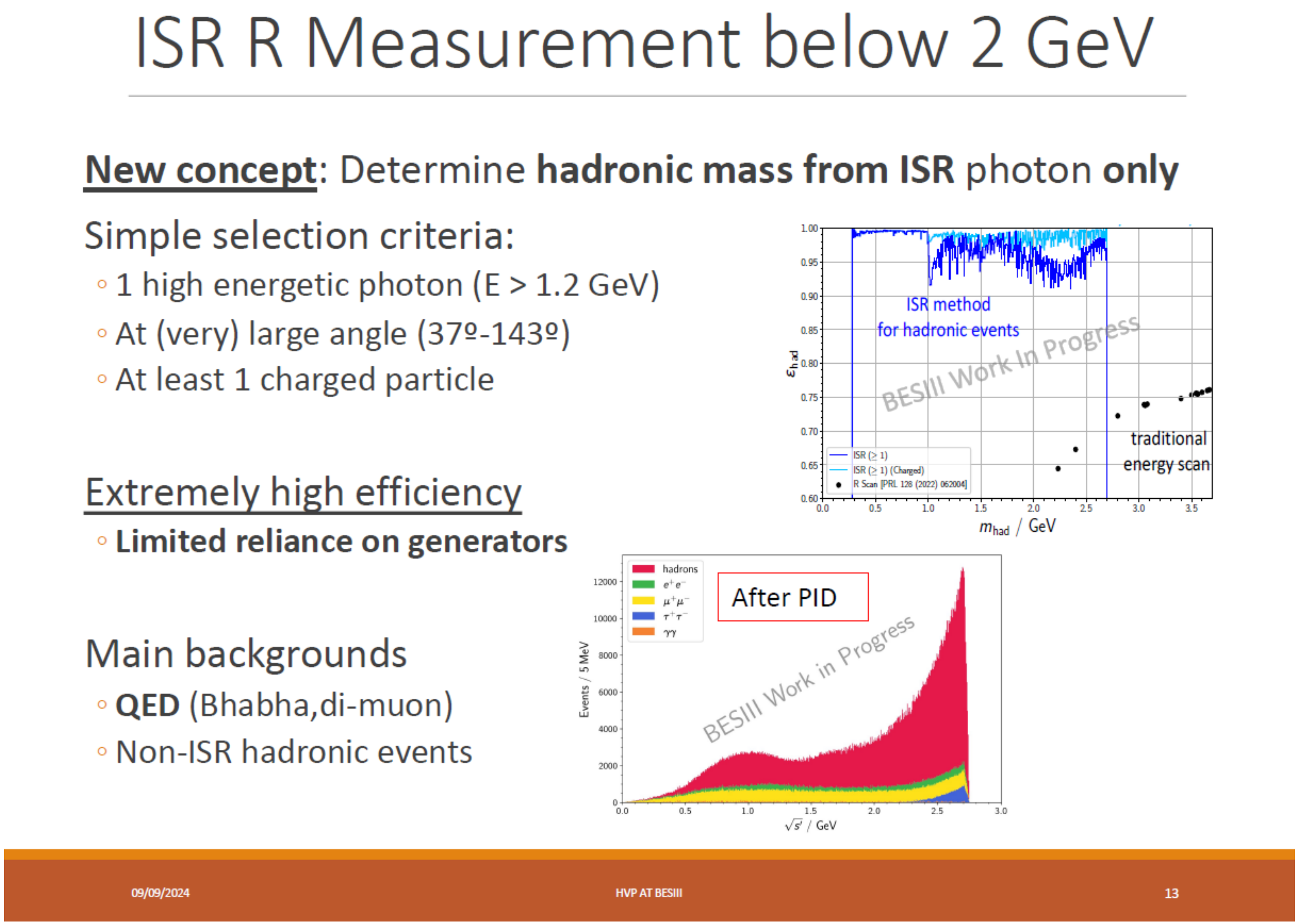
- ISR $e^+e^- \to \pi^+ \pi^- \pi^0$ – 0.7 to 3 GeV – arXiv:1912.11208
- ISR $e^+e^- \to \pi^+ \pi^- \pi^0 \pi^0$ – 0.9 to 3.3 GeV

On going and future measurements:

- ISR $e^+e^- \to \pi^+ \pi^-$ – 0.3 to 1 GeV (tagged) and > 1 GeV (untagged)
- ISR $e^+e^- \to K\ K$ – 1 to 3.3 GeV (tagged and untagged)
- ISR R measurement – 0.3 to 2 GeV
- R measurement – 1.8 to 2 GeV (future)

09/09/2024 HVP AT BESIII 6

BES-III: inclusive R via ISR?



ISR R Measurement below 2 GeV

New concept: Determine **hadronic mass from ISR** photon **only**

Simple selection criteria:

- 1 high energetic photon (E > 1.2 GeV)
- At (very) large angle (37º-143º)
- At least 1 charged particle

Extremely high efficiency

- **Limited reliance on generators**

Main backgrounds

- **QED** (Bhabha,di-muon)
- Non-ISR hadronic events

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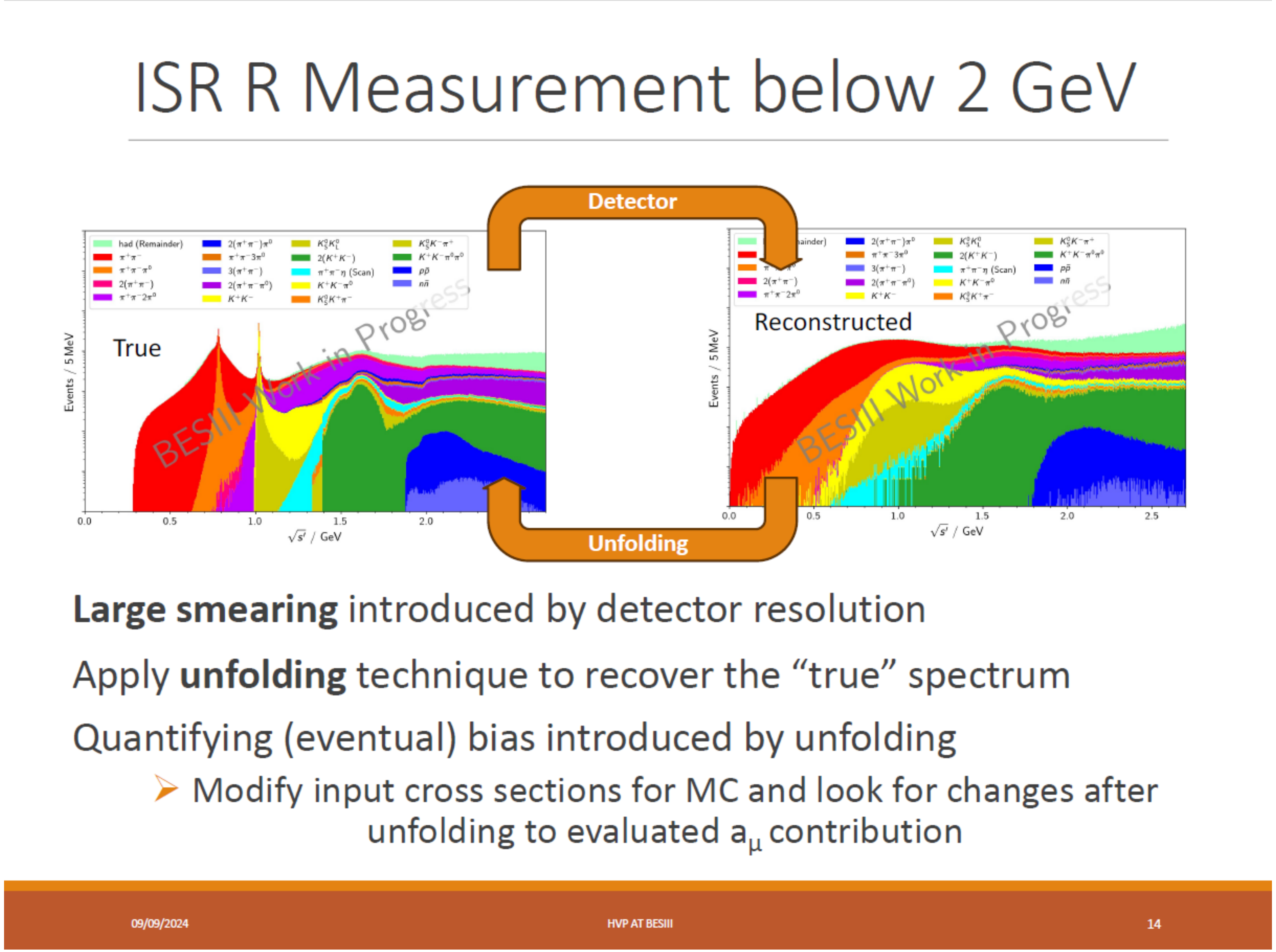
BES-III: inclusive R via ISR?



ISR R Measurement below 2 GeV

Large smearing introduced by detector resolution

Apply **unfolding** technique to recover the “true” spectrum

Quantifying (eventual) bias introduced by unfolding

- Modify input cross sections for MC and look for changes after unfolding to evaluated a_μ contribution

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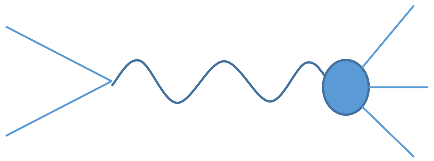
Final remarks

- Measurement of R(s) remains very active field of research
- It is required for number of precise tests, especially for (g-2) of muon
- Precise measurement of R(s), both direct inclusive and ISR exclusive, will remain an important task for future charm-tau factory even in ~10 years
- The huge statistics of the future experiments will be important for reduction of systematic errors:
    - Detailed studies of detector efficiencies
    - Detailed studies of radiative corrections (NNLO and beyond)
    - Possibility to detect γ through conversion
    - …

Backup slides

Radiative corrections



We want to measure $e^+e^- \to H$, but these events are accompanied by similar events where photons are emitted by any of the particles.

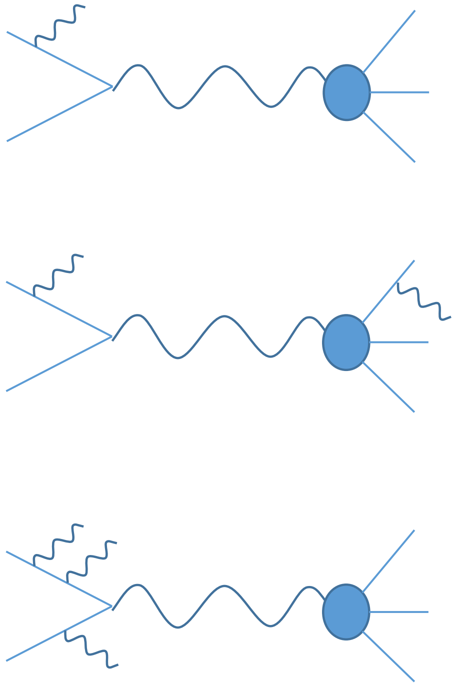
Radiation of high-energy γ is suppresses by α, but radiation of soft photons is enhanced.

Radiation changes both the cross-section and the kinematics of the final state:

$$\sigma = \frac{N_{obs} - N_{bg}}{\varepsilon(\delta) \cdot (1+\delta) \cdot \underset{\uparrow}{\int \mathcal{L} dt}}$$

And we have to calculate radiative corrections to the cross section of monitoring process as well

Radiative processes

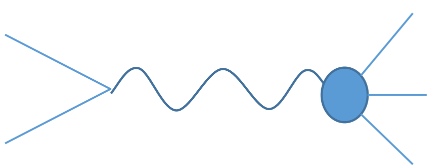


ISR FSR

Initial *Final*

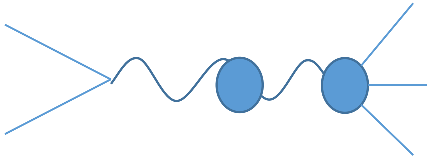
state radiation

Vacuum polarization



$\sigma^0(e^+e^- \to \gamma \to X)$

In a_μ calculation



$\sigma(e^+e^- \to \gamma^* \to X)$

In experiment

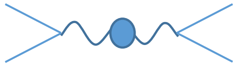
In the calculation of a_μ, we assume the lowest order photon propagator $1/q^2$. But the real propagator includes higher order effects (loop corrections): $1/(q^2 - \Pi(q^2))$. Therefore the measured cross section have to be corrected:

$$\sigma^0(e^+e^- \to X) = \sigma(e^+e^- \to X) \times \frac{|\alpha(s)|^2}{\alpha^2}$$

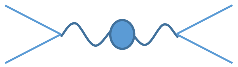
The running fine structure constant is also calculated via dispersion relation based on R(s):

$$\Delta\alpha_{had}(s) = -\frac{\alpha s}{3\pi}\int_0^\infty \frac{R(s')}{s'(s-s'-i0)}ds'$$

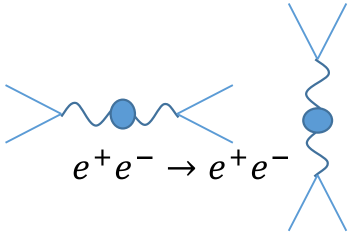
Nice way to avoid this correction is to use $e^+e^- \to \mu^+\mu^-$ for luminosity measurement



$e^+e^- \to X$



$e^+e^- \to \mu^+\mu^-$



$e^+e^- \to e^+e^-$

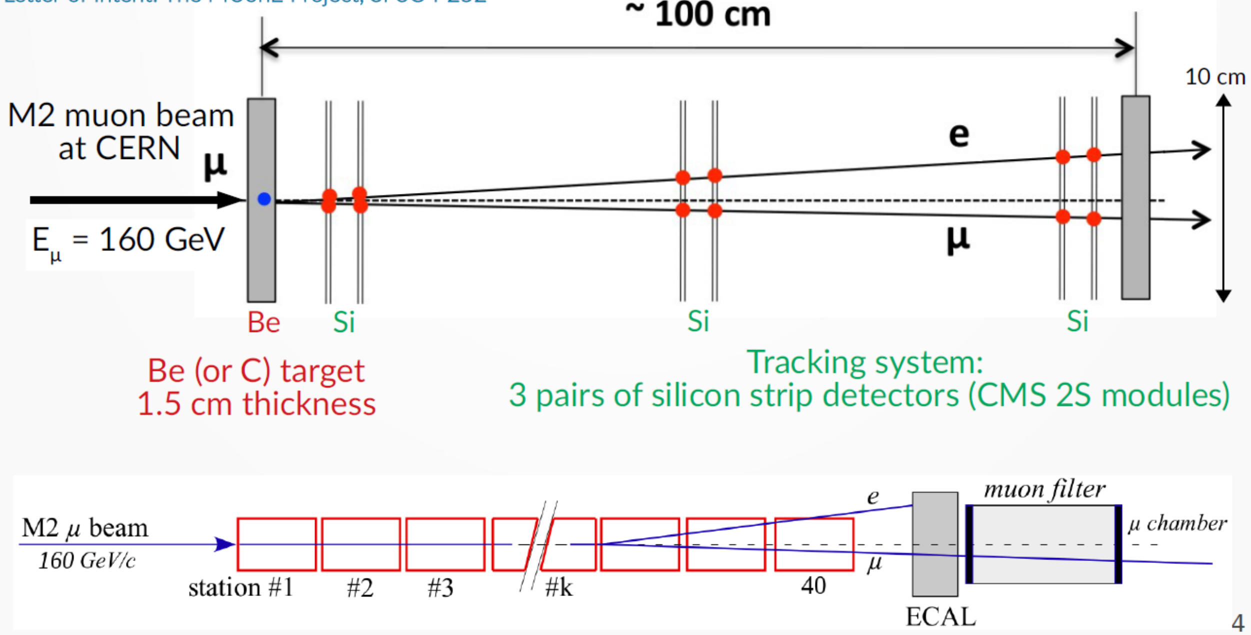
MUonE @CERN

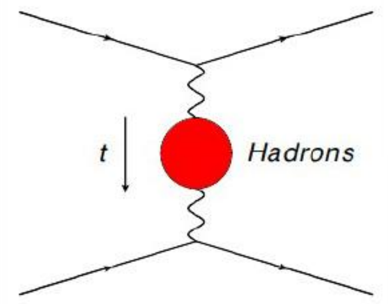
Dedicated experiment to measure hadronic contribution in t-channel.

$$a_\mu^{HLO} = \frac{\alpha_0}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)]$$

Lautrup, Peterman, De Rafael, Phys. Rep. C3 (1972), 193

Letter of Intent: The MUonE Project, SPSC-I-252





Measured: angular distribution of μe scattering; $4 \cdot 10^{12}$ events!

Now: proof-of-concept data taking; final result after LHC LS3 (2029-)