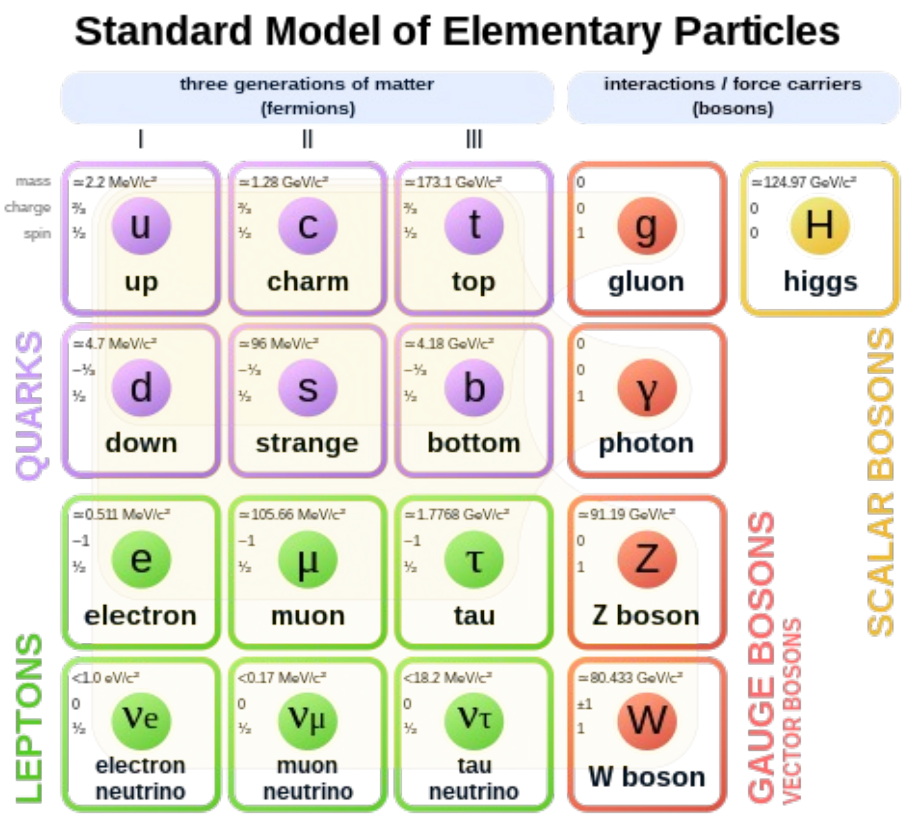
希格斯与宇宙学

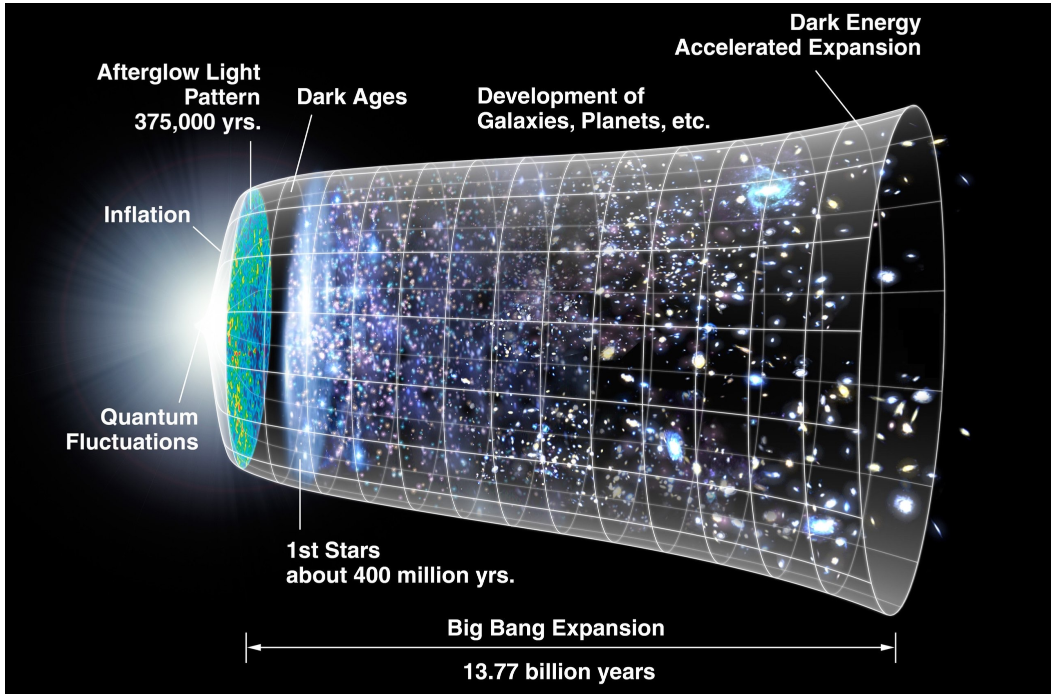
韩成成

中山大学

Standard model



ΛCDM+Inflation



暴胀

Rapid expansion of the universe in the early time

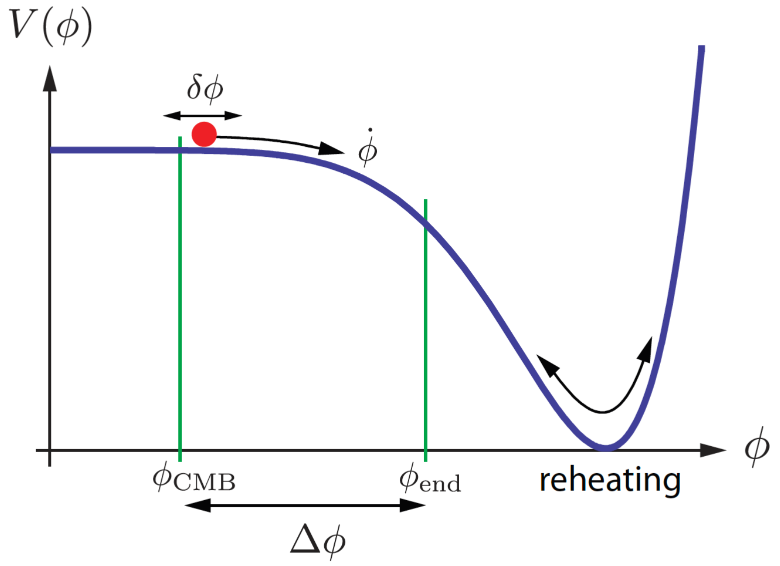
- Flatness problem
- Horizon problem
- Monopole problem?
- Seeding the primordial anisotropies in CMB

暴胀

Assuming a scalar field, with equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$



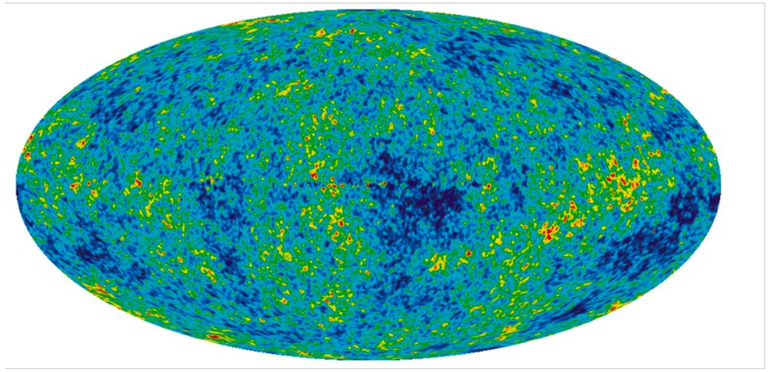
$$H^2 \approx \frac{1}{3}V(\phi) \approx \text{const.}$$

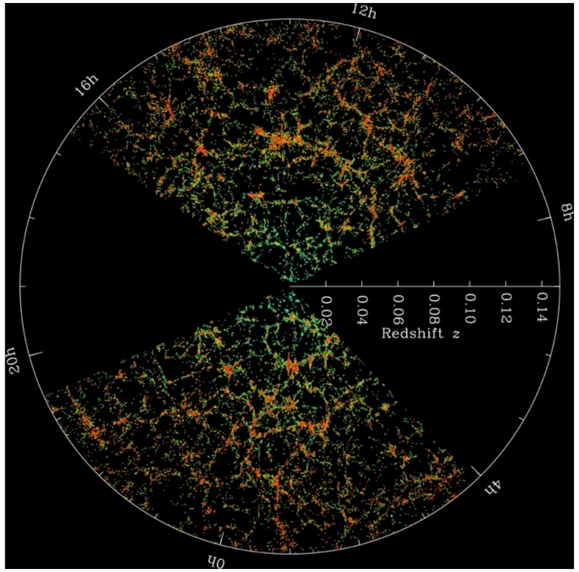
$$\dot{\phi} \approx -\frac{V_{,\phi}}{3H},$$

$$\rightarrow \quad a(t) = a_0 e^{Ht} \qquad Ht \gtrsim 60$$

暴胀

Generating quantum fluctuations(anisotropies in CMB)





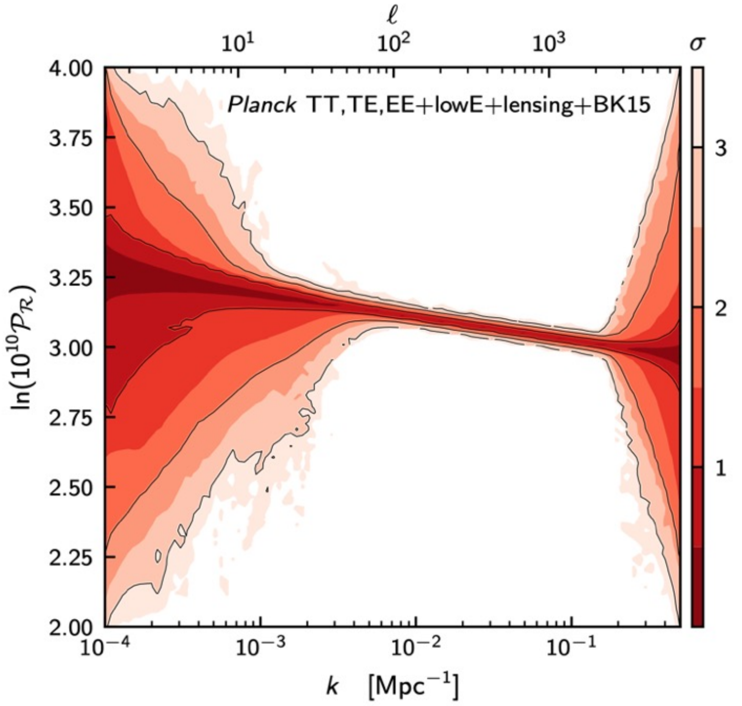
Such small fluctuations finally develops the large structure of our universe

暴胀

$$\epsilon_V(\phi) \equiv \frac{M_{pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2$$

$$\eta_V(\phi) \equiv M_{pl}^2 \frac{V_{,\phi\phi}}{V}$$

$$\epsilon_V, |\eta_V| \ll 1$$



标量扰动

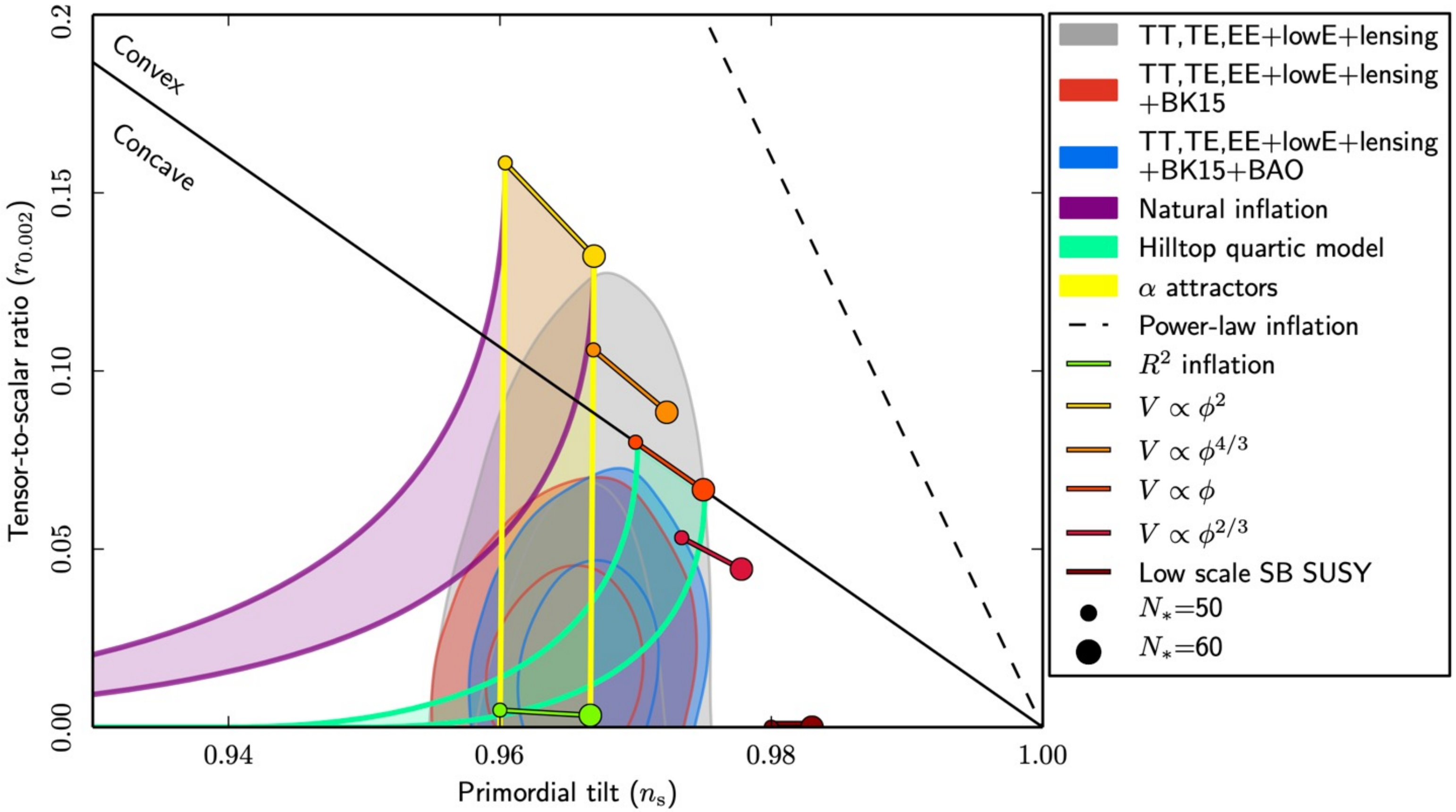
$$\Delta_s^2(k) \approx \frac{1}{24\pi^2}\frac{V}{M_{pl}^4}\frac{1}{\epsilon_V}\bigg|_{k=aH} \qquad n_s - 1 \equiv \frac{d\ln\Delta_s^2}{d\ln k} = 2\eta_V - 6\epsilon_V$$

张量扰动

$$\Delta_t^2(k) \approx \frac{2}{3\pi^2}\frac{V}{M_{pl}^4}\bigg|_{k=aH}$$

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon_V$$

暴胀



$$n_s \simeq 0.965 \qquad r \lesssim 0.056$$

希格斯与宇宙学相关问题

- 希格斯与暗能量
- 希格斯真空稳定性问题
- 希格斯与重子不对称性（电弱重子生成机制）
- 希格斯与暴胀

希格斯与暗能量

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$\Phi^\dagger \Phi = \frac{1}{2}(h+v)^2$$

$$V = \lambda v^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4 - \frac{\lambda}{4} v^4$$

观测到的暗能量 $\rho \sim (2 \times 10^{-3}\mathrm{eV})^4$

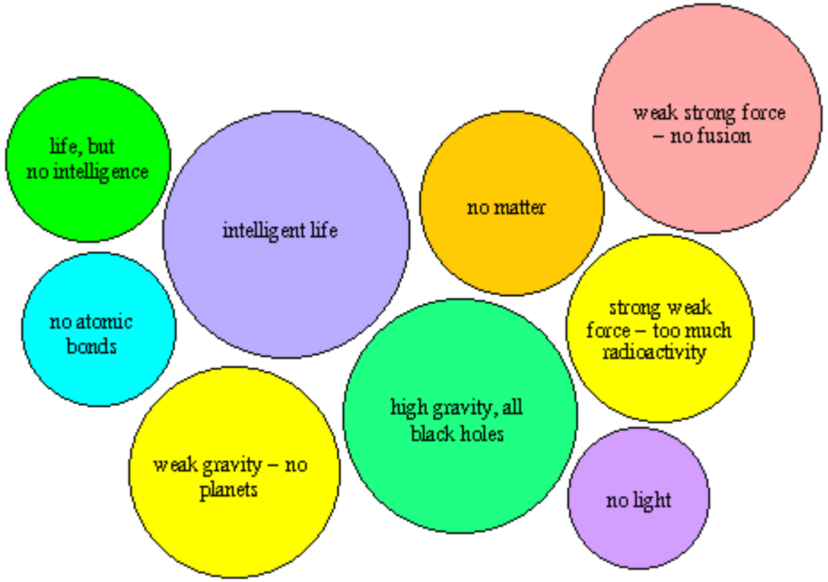
$$\frac{\lambda}{4} v^4 \sim 10^{60} \rho$$

希格斯与暗能量

多宇宙和人择原理

Anthropic Bubbles

one possible solution to the anthropic dilemma is the numerious bubble universes produced by inflation. Each bubble universe may have its own physical constants, which determine the evolution within the bubble



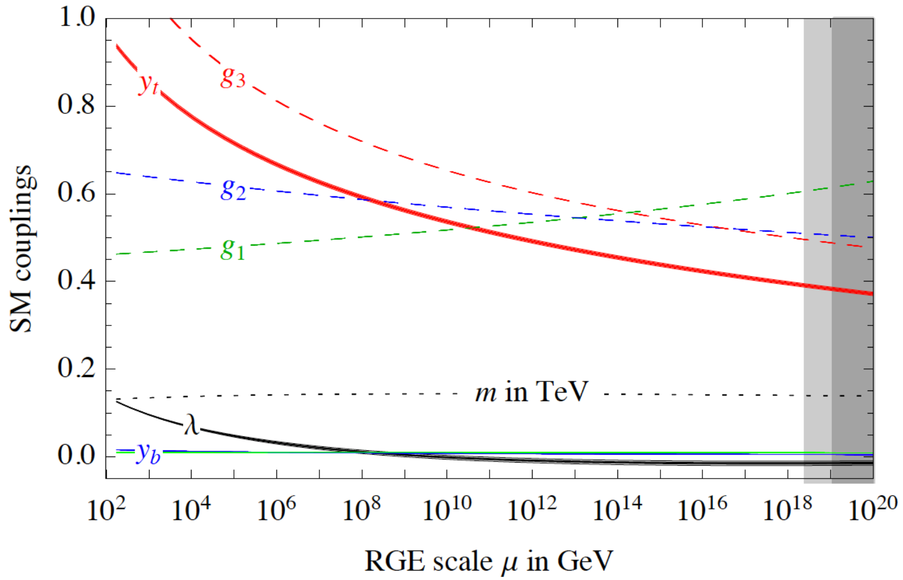
the evolution of intelligent life is extremely sensitive to the initial conditions, but since number of bubble universes is also large, the possibility is finite and our existence is not a big mystery

希格斯真空稳定性

如果把标准模型可以适用到更高的标度，有可能出现真空稳定的问题

$$V(\phi) = \lambda(\phi^\dagger \phi - v^2)^2 \qquad \lambda \sim 0.13 \qquad v \sim 174\ \text{GeV}$$

$$\frac{d\lambda}{d\ln\mu^2} = \frac{1}{(4\pi)^2}\left[\lambda\left(12\lambda + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10}\right) \boxed{- 3y_t^4} + \frac{9g_2^4}{16} + \frac{27g_1^4}{400} + \frac{9g_2^2 g_1^2}{40}\right]$$



y_t: top Yukawa coupling

g_2, g_Y: weak gauge coupling

g_3: strong gauge coupling

希格斯真空稳定性

To study the vacuum structure of the Higgs, RG-improved effective potential:

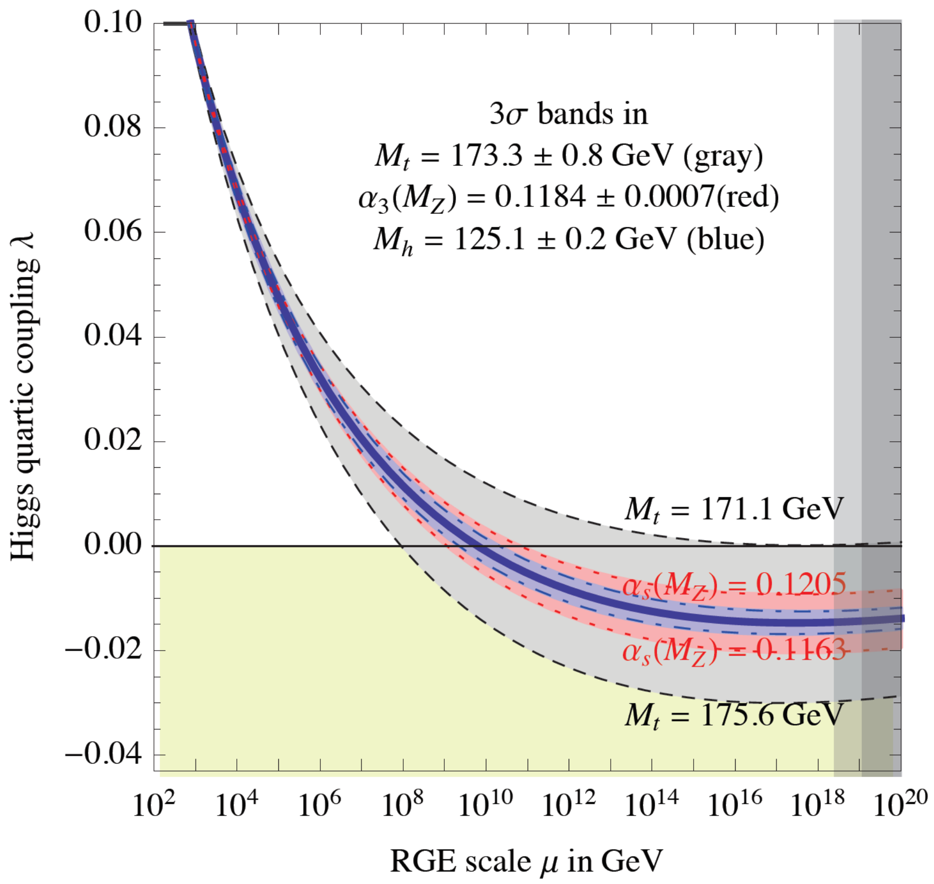
For $h \gg v$

$$V_{\rm eff} = \lambda_{\rm eff}(h)\frac{h^4}{4}$$

$$\lambda_{\rm eff}(h) = e^{4\Gamma(h)}\left[\lambda(\bar\mu = h) + \lambda_{\rm eff}^{(1)}(\bar\mu = h) + \lambda_{\rm eff}^{(2)}(\bar\mu = h)\right]$$

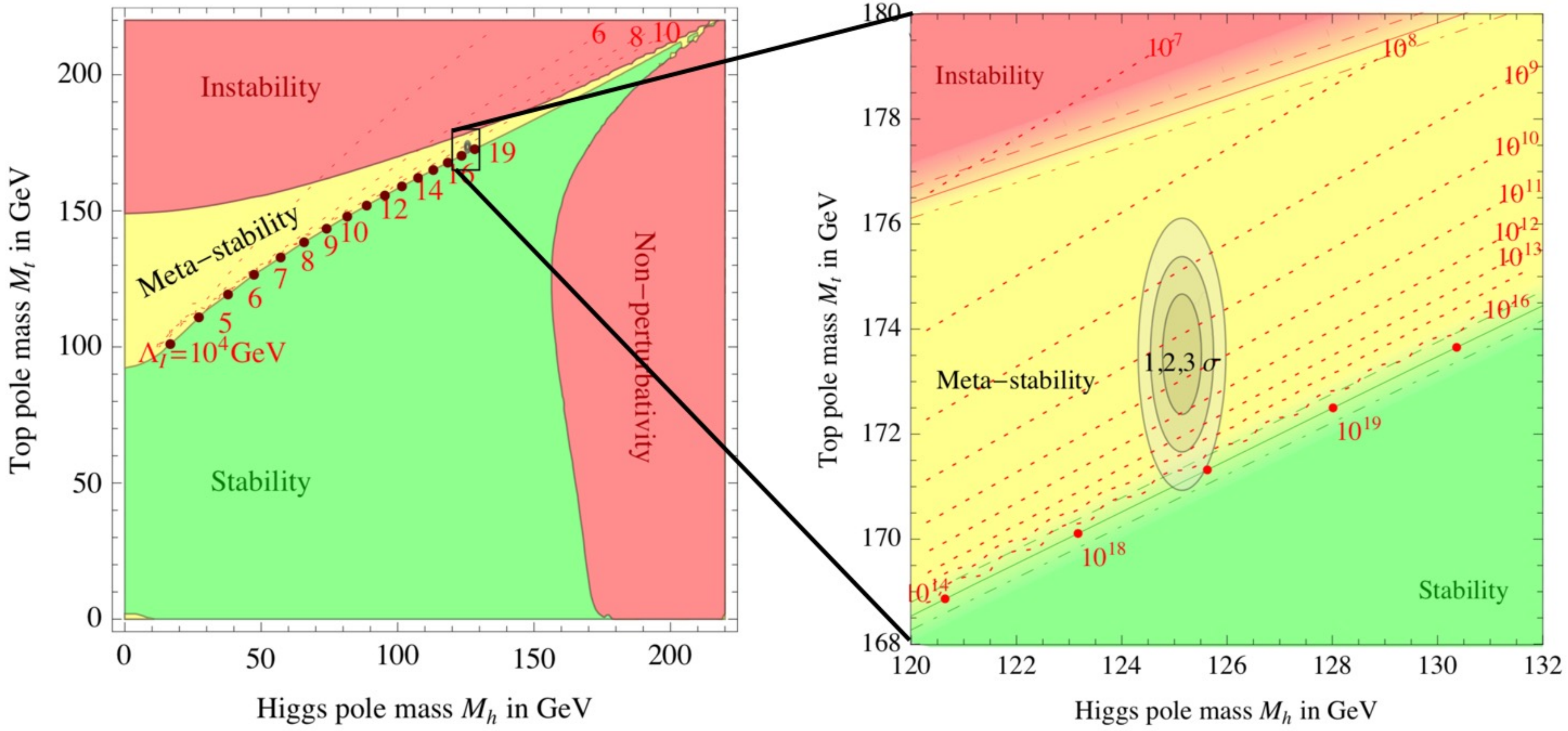
$$\Gamma(h) \equiv \int \gamma(\mu) d\ln\mu$$

γ is the Higgs field anomalous dimension



希格斯真空稳定性

根据希格斯和top夸克质量测量，我们恰好处在亚稳态



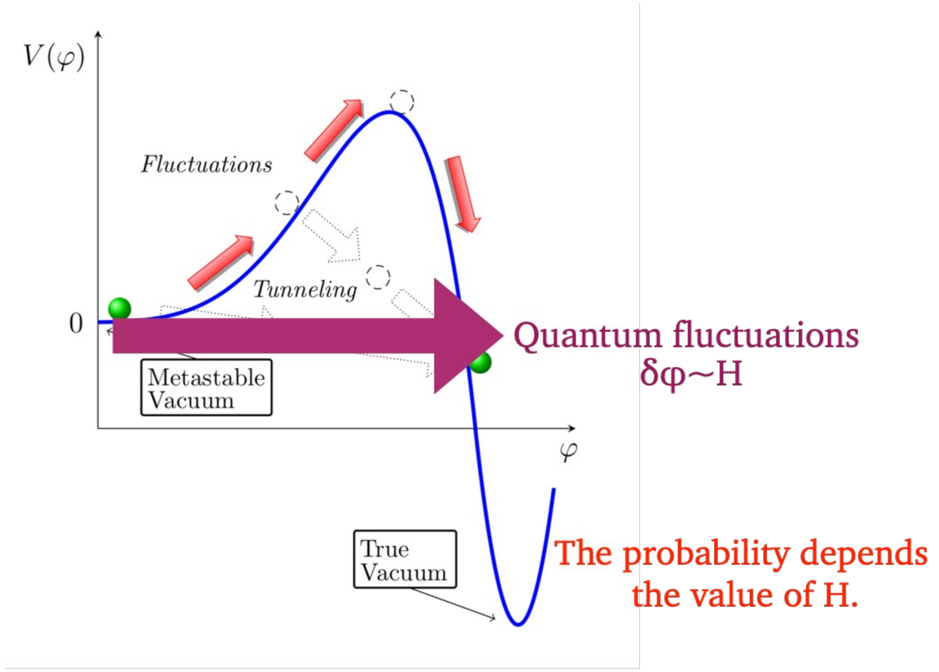
希格斯真空稳定性

CERN理论组主任Gian F. Giudice等发现：这个亚稳态真空在宇宙早期是极不稳定的(暴胀时期)，因此对我们真空存在的合理性提出了质疑，引起理论界广泛关注

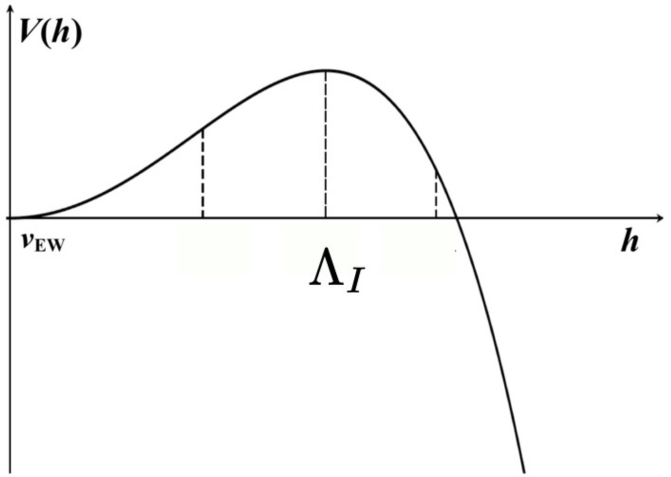
The cosmological Higgstory of the vacuum instability

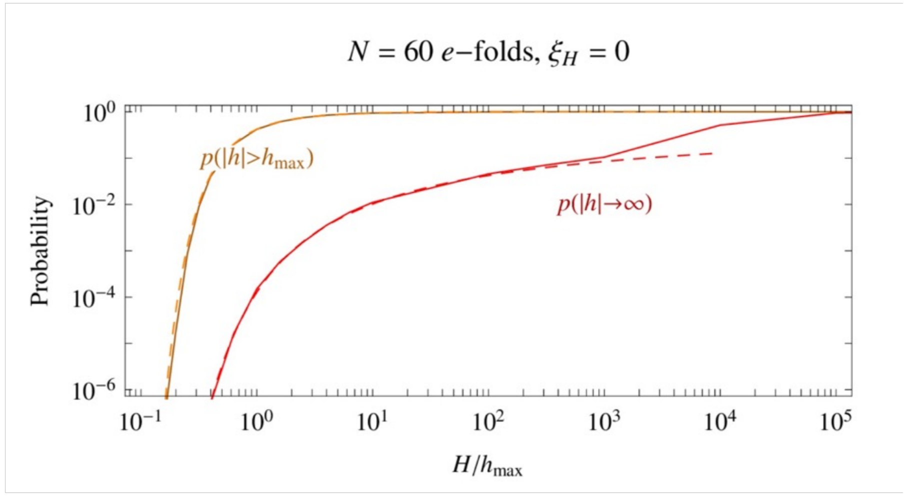
José R. Espinosa[a,b], Gian F. Giudice[c],
Enrico Morgante[d], Antonio Riotto[d], Leonardo Senatore[e],
Alessandro Strumia[f,g], Nikolaos Tetradis[h]

1505.04825(250 citations)



希格斯真空稳定性





$$P(h < \Lambda_I, N_{\rm tot}) = erf(\frac{\sqrt{2}\pi\Lambda_I}{H_{\rm inf}\sqrt{N_{\rm tot}}}) \qquad erf(x) \simeq 1 - \frac{1}{\sqrt{\pi}x}e^{-x^2}$$

Surviving probability $1-(1-P)^{e^{3N_{\rm hor}}}$

$$\text{For } N_{\rm hor} = 60, N_{\rm tot} = 1000, \text{ we need } H_{\rm inf} < 10^{-2}\Lambda_I$$

希格斯真空稳定性

解决方案

- 引入新物理，让希格斯的自耦和在高标度保持为正(例如SUSY)
- 假设暴胀的标度比较低，暴胀时期哈勃常数小于10^8 GeV，对暴胀模型提出了挑战
- 希格斯与暴胀子耦合，暴胀时期希格斯质量很大，贡献一个正的势能

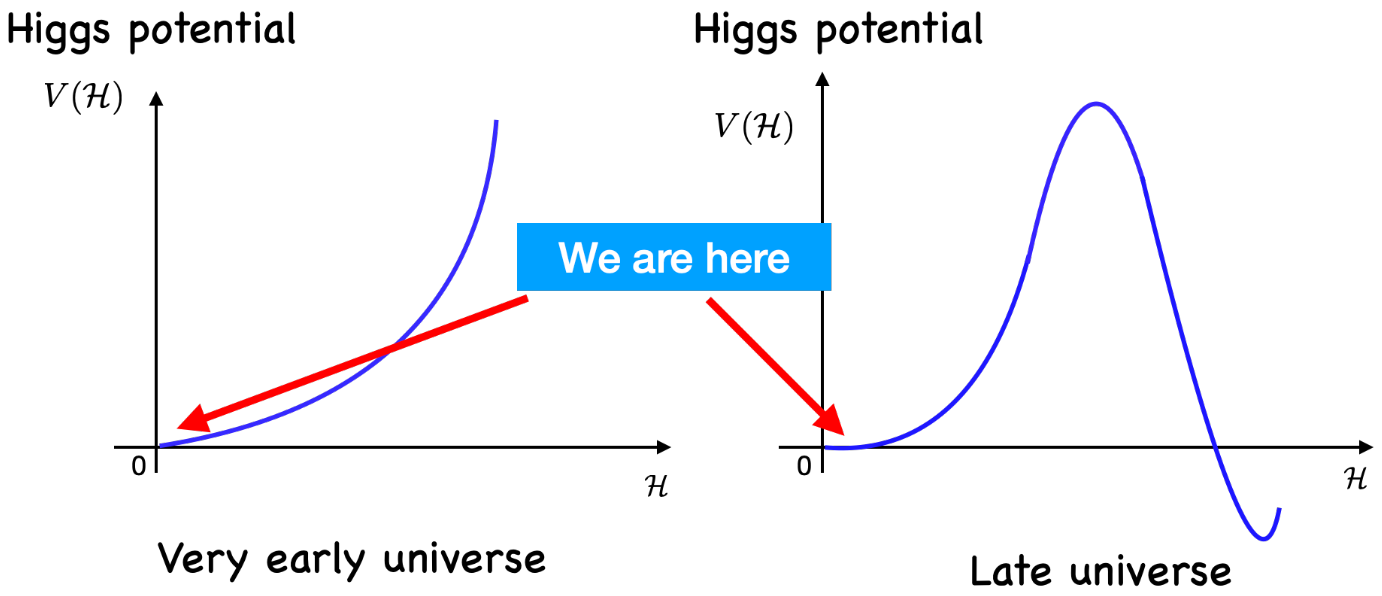
希格斯真空稳定性

我们的工作发现，如果精质暗能量与希格斯耦合在一起，可以改变希格斯在宇宙早期的行为，从而避免我们真空的跃迁

$$V(\phi,\mathcal{H}) = e^{-\xi\phi}\left(\lambda\left(|\mathcal{H}|^2 - v^2\right)^2 + \Lambda\right) \quad (\xi > 0)$$

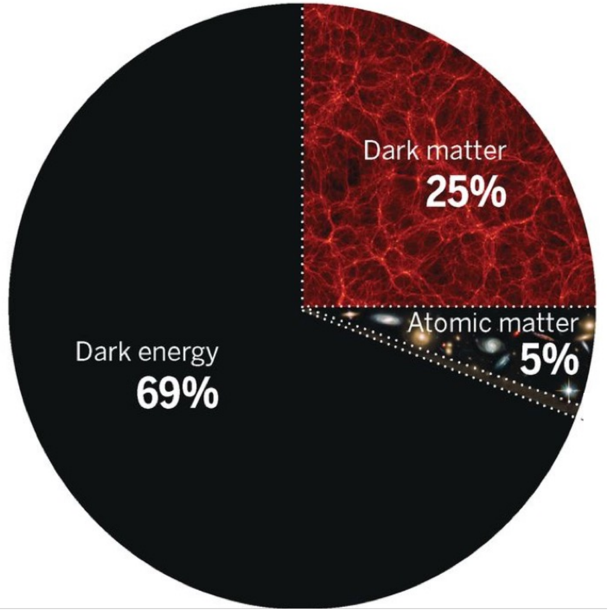
$$\xi > 0.35$$

Phys.Lett.B 791 (2019) 314-318
Quintessence saves Higgs instability
Chengcheng Han [a], Shi Pi [a,*], Misao Sasaki [a,b,c]



重子不对称

What is the Universe made of?



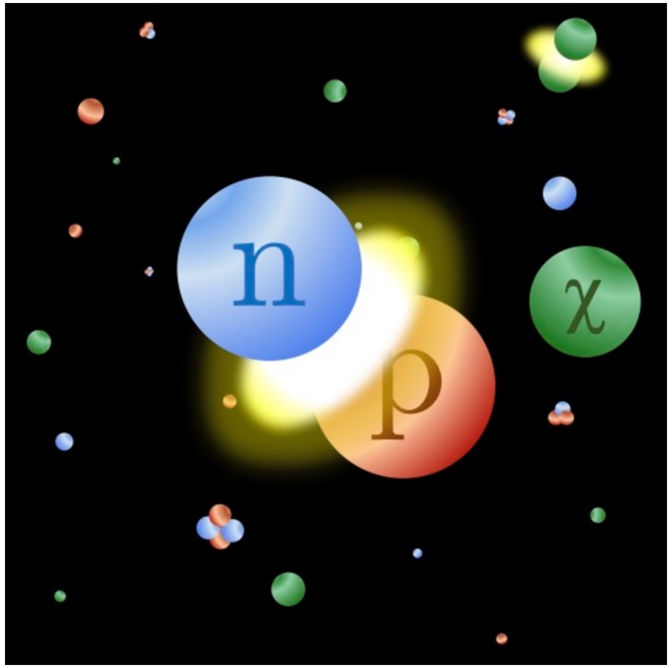
- 暗能量是什么？与粒子物理标准模型有何关联？
- 暗物质是什么？它是不是一种基本粒子？
- 为什么可见物质都是重子，反重子去哪了？

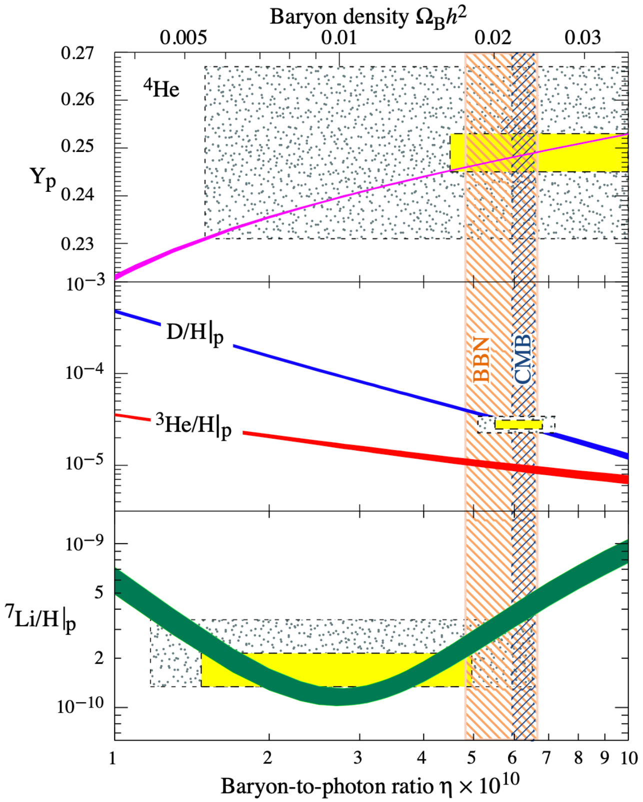
(重子不对称性 or 正反物质不对称性)

原初核合成

原初核合成(BBN)（T~1 MeV, t~3 分钟)，轻核元素的形成，宇宙大爆炸的直接证据

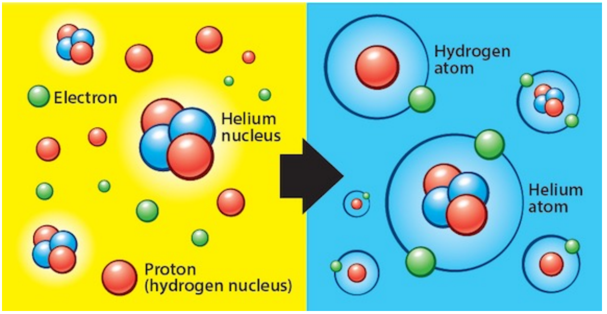
$$\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma} \sim 10^{-10}$$

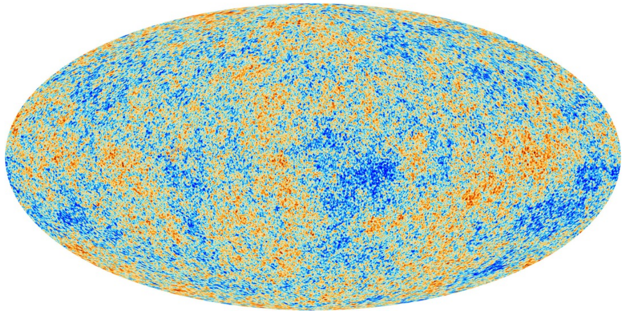


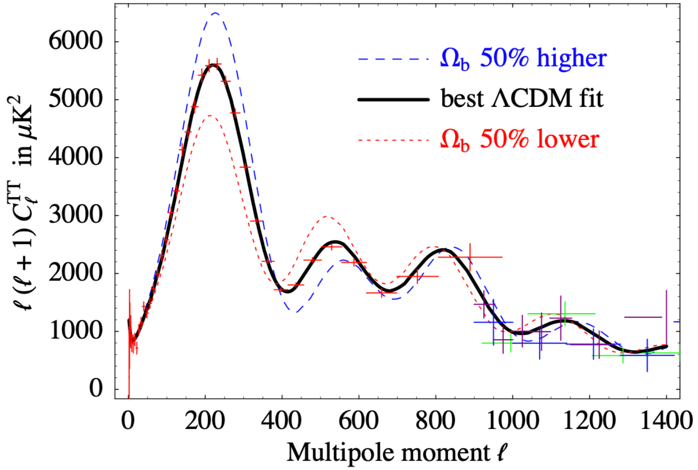


宇宙微波背景辐射

宇宙大爆炸的遗迹：宇宙微波背景辐射(CMB)(T~0.1 eV t~38万年)







Parameter	Plik best fit	Plik [1]	CamSpec [2]	$([2]-[1])/\sigma_1$	Combined
$\Omega_b h^2$	0.022383	0.02237 ± 0.00015	0.02229 ± 0.00015	−0.5	0.02233 ± 0.00015
$\Omega_c h^2$	0.12011	0.1200 ± 0.0012	0.1197 ± 0.0012	−0.3	0.1198 ± 0.0012

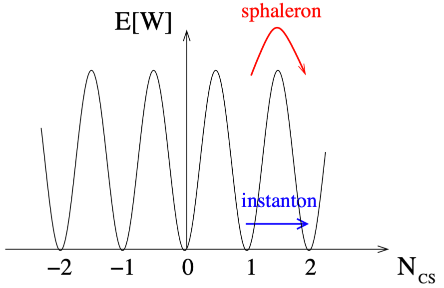
$$\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma} \sim 10^{-10}$$

如何产生重子不对称性?

如果宇宙创生初期就有这个差别，这个差别会在暴胀时期抹平掉

如何从正反物质对称的宇宙演化到正反物质不对称的宇宙？

Sakharov 三条件	标准模型
● 重子数破坏过程	✓
● C 和 CP 破坏	✓✗
● 脱离热平衡	✗



SU(2) vacuum

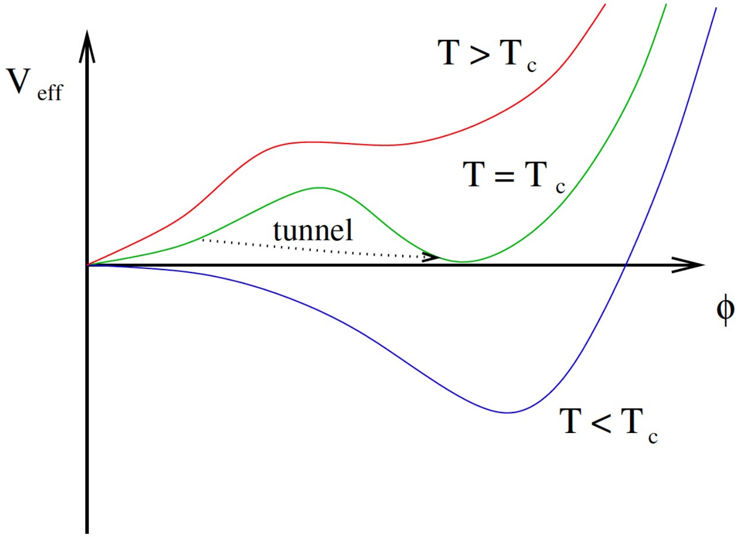
$$\Delta B = \Delta L = -3$$

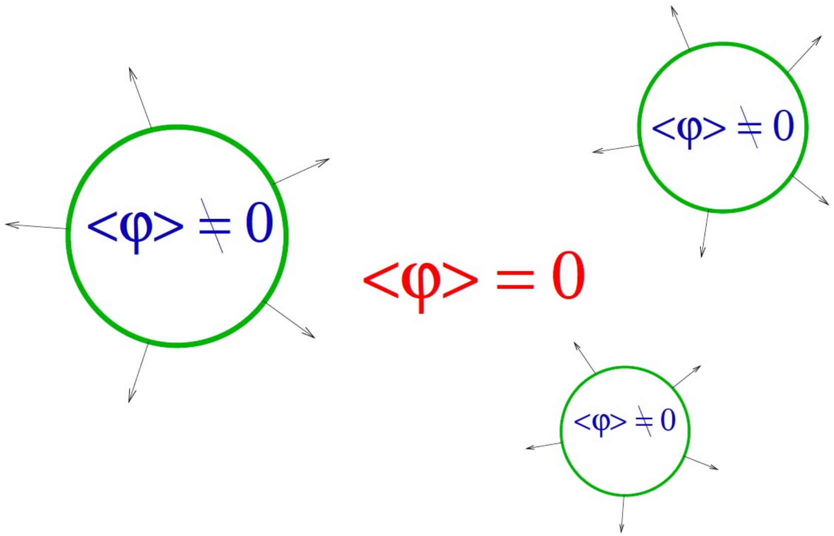
- 无法提供脱离热平衡条件(QCD相变和电弱相变均为 cross over)
- 即使有强一阶相变，夸克部分提供CP破坏太小，不足以解释现在的观测

新的CP破坏源+脱离热平衡条件!

电弱重子生成

电弱相变

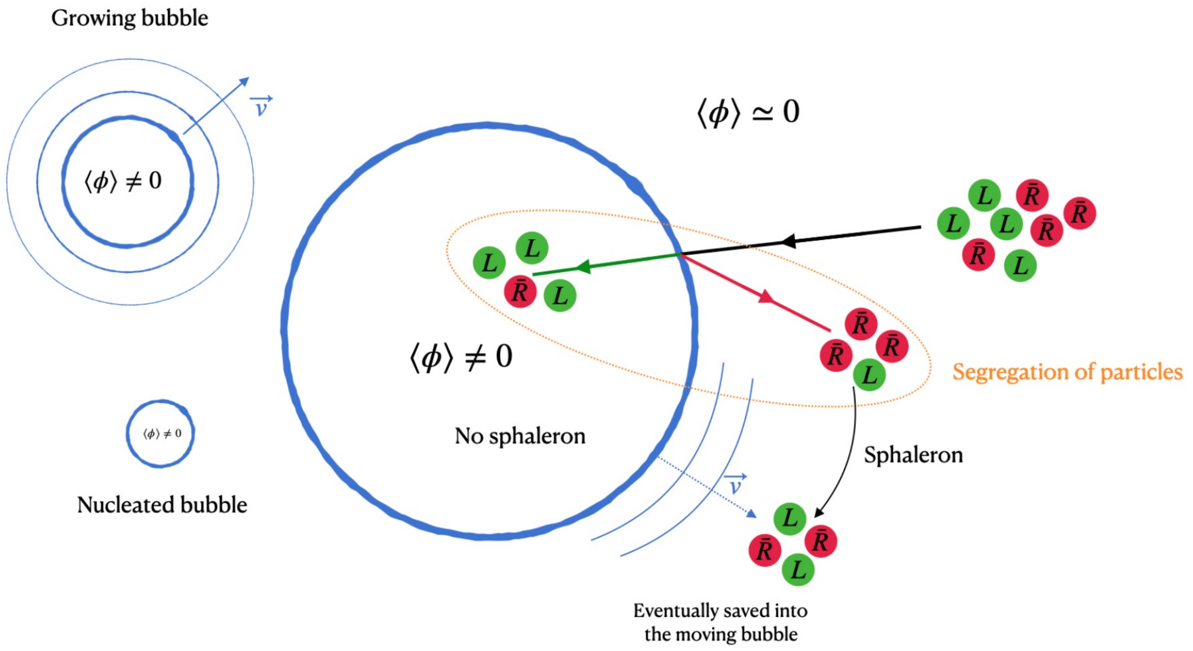




但是Lattice的结果表明，如果希格斯质量小于75 GeV 标准模型才会出现一阶相变

电弱重子生成

实际电弱重子生成机制中，不仅要求一阶相变，这个相变必须是强一阶相变，需要禁戒真空泡里面的sphaleron过程



- 强一阶相变要求希格斯质量小于42 GeV
- 即便如此，标准模型里的夸克部分提供的CP破坏效应太小，不足以产生足够的重子不对称

电弱重子生成

- 在电弱标度增加新的标量粒子(电弱强一阶相变)
- 额外的CP破坏

对撞机限制

电子EDM测量(< 4.1*10^{-30} e.cm)

Is electroweak baryogenesis dead?

James M. Cline[1,2]

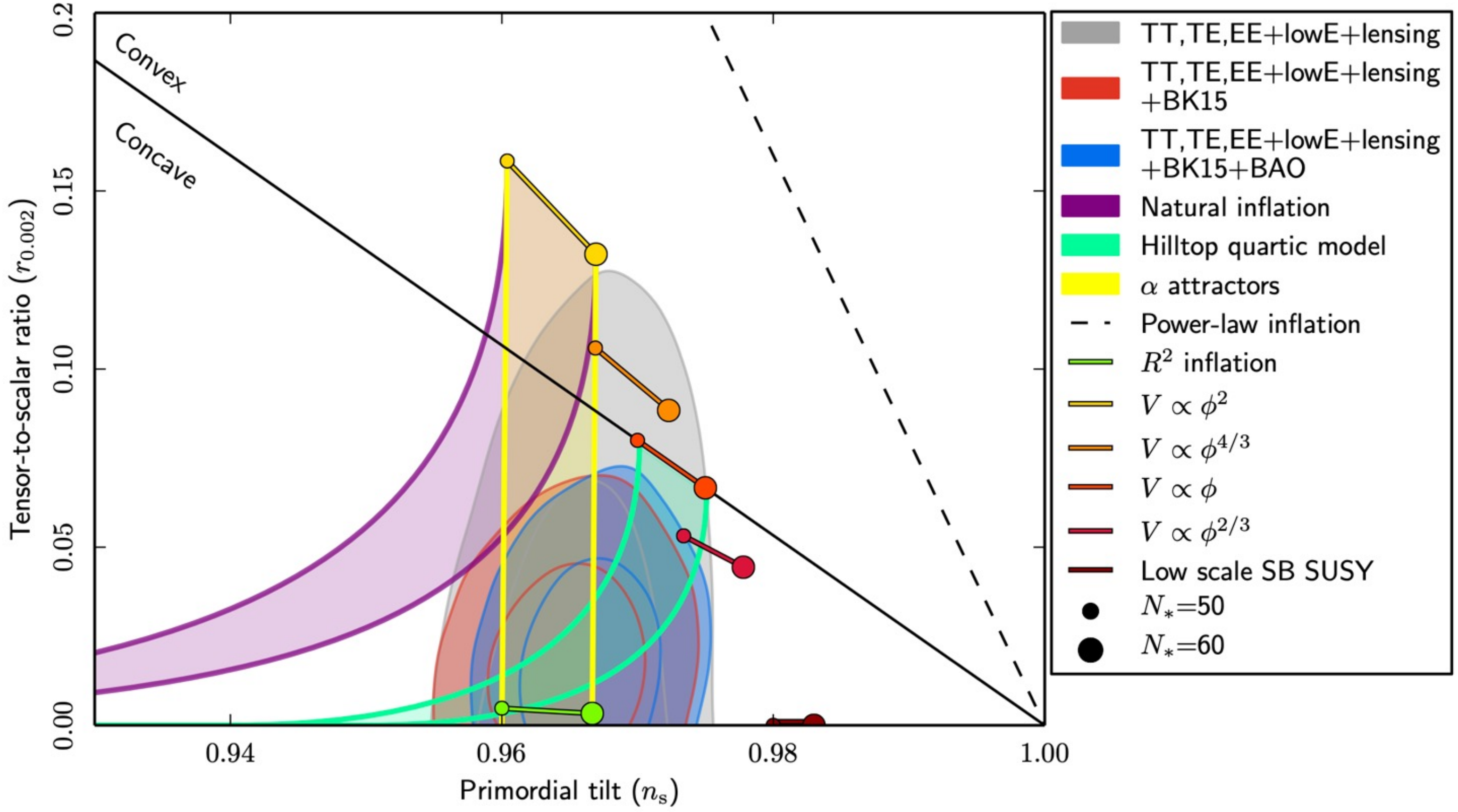
[1]CERN, Theoretical Physics Department, Geneva, Switzerland
[2]Department of Physics, McGill University, 3600 Rue University, Montréal, Québec, Canada H3A 2T8

Challenge in model building

希格斯暴胀

- 希格斯是唯一已知的基本标量粒子
- 暴胀需要标量场来实现
- 希格斯暴胀？(假如高能标希格斯自耦和仍然是正的)

希格斯暴胀



Convex
Concave
Tensor-to-scalar ratio ($r_{0.002}$)
Primordial tilt (n_s)
0.00
0.05
0.10
0.15
0.2
0.94
0.96
0.98
1.00
TT,TE,EE+lowE+lensing
TT,TE,EE+lowE+lensing +BK15
TT,TE,EE+lowE+lensing +BK15+BAO
Natural inflation
Hilltop quartic model
α attractors
Power-law inflation
R^2 inflation
$V \propto \phi^2$
$V \propto \phi^{4/3}$
$V \propto \phi$
$V \propto \phi^{2/3}$
Low scale SB SUSY
N_*=50
N_*=60

希格斯暴胀

Bezrukov and Shaposhnikov, Phys.Lett.B 659 (2008) 703-706

$$S_J = \int d^4x\sqrt{-g_J}\left[\frac{M_P^2}{2}\left(1+\frac{\xi\phi^2}{M_P^2}\right)R_J - \frac{1}{2}|\partial_\mu\phi|^2 - V_J(\phi)\right]$$

Weyl transformation

$$g_{\mu\nu} = \Omega(\phi)^2 g_{J\mu\nu}$$

$$\Omega^2 = 1 + \frac{\xi\phi^2}{M_P^2}$$

希格斯暴胀

$$\frac{d\chi}{d\phi} = \left(\frac{1 + \xi(1 + 6\xi)\phi^2/M_P^2}{(1 + \xi\phi^2/M_P^2)^2}\right)^{1/2}$$

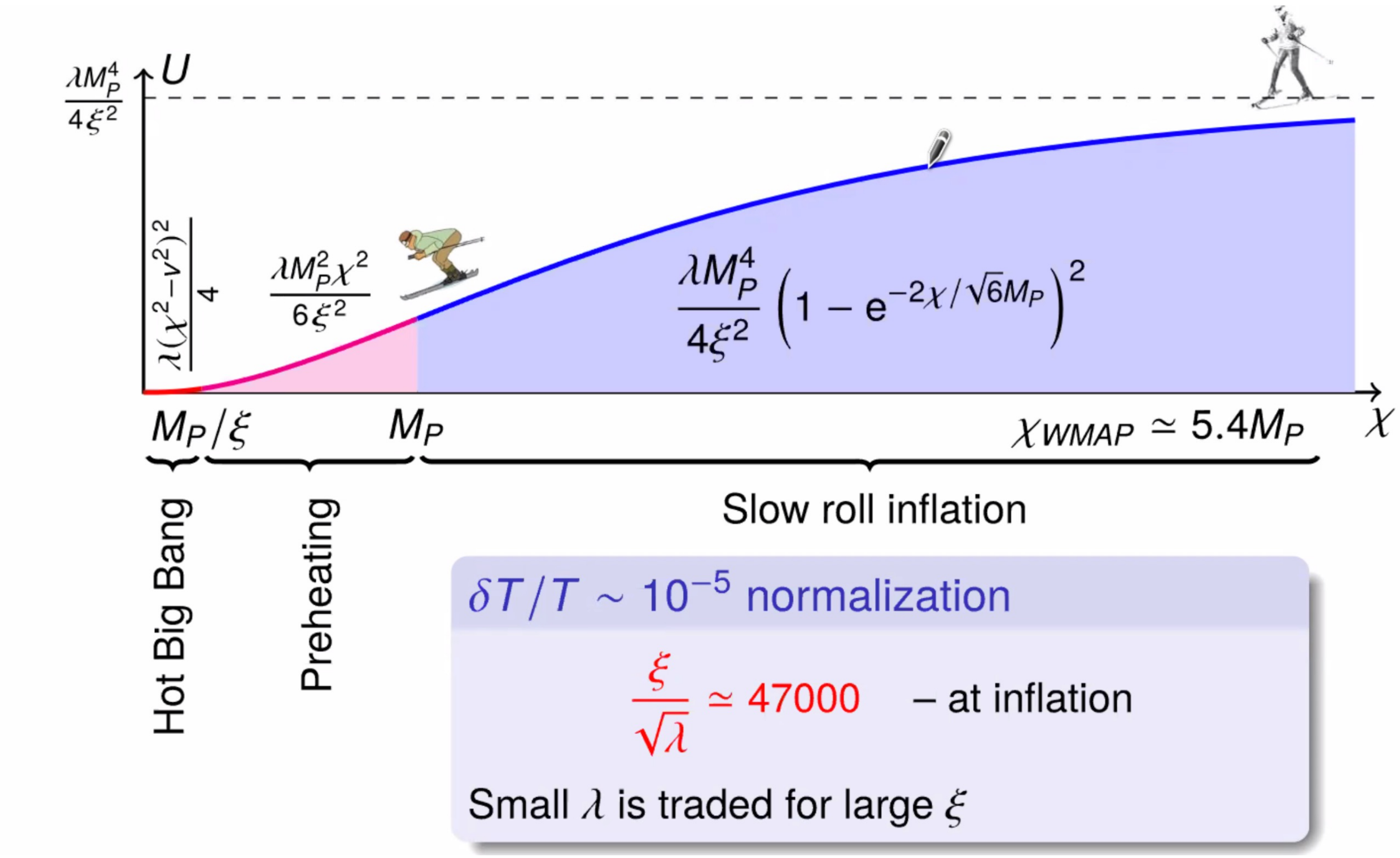
$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi)\right]$$

$$V(\chi) \equiv V_J(\phi(\chi))/\Omega^4(\phi(\chi))$$

$$\Omega^2 = 1 + \frac{\xi\phi^2}{M_P^2}$$

$$V_J = \frac{\lambda}{4}\phi^4 \xrightarrow{\phi \gg M_p/\sqrt{\xi}} V = \frac{\lambda}{4\xi^2}M_p^4$$

希格斯暴胀



$\frac{\lambda M_P^4}{4\xi^2}$
U
$\frac{\lambda(\chi^2-v^2)^2}{4}$
$\frac{\lambda M_P^2\chi^2}{6\xi^2}$
$\frac{\lambda M_P^4}{4\xi^2}\left(1-e^{-2\chi/\sqrt{6}M_P}\right)^2$
M_P/ξ
M_P
$\chi_{WMAP} \simeq 5.4 M_P$
χ
Hot Big Bang
Preheating
Slow roll inflation
$\delta T/T \sim 10^{-5}$ normalization
$\frac{\xi}{\sqrt{\lambda}} \simeq 47000$ – at inflation
Small λ is traded for large ξ

希格斯暴胀与重子生成

PHYSICAL REVIEW LETTERS **128,** 141801 (2022)

Affleck-Dine Leptogenesis from Higgs Inflation

Neil D. Barrie,[1,*] Chengcheng Han,[2,†] and Hitoshi Murayama[3,4,5,‡]

We find that the triplet Higgs of the type-II seesaw mechanism can simultaneously generate the neutrino masses and observed baryon asymmetry while playing a role in inflation. We survey the allowed parameter space and determine that this is possible for triplet masses as low as a TeV, with a preference for a small

Type II Seesaw leptogenesis

JHEP

Neil D. Barrie,[a] Chengcheng Han[b] and Hitoshi Murayama[c,d,e,1]

第二类跷跷板机制

$$H(2,1/2),\ \Delta(3,1),\ L(2,-1/2)$$

$$H=\begin{pmatrix} h^+ \\ h \end{pmatrix},\quad \Delta=\begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

$$\mathcal{L}_{Yukawa}=\mathcal{L}^{\rm SM}_{\rm Yukawa}-\frac{1}{2}y_{ij}\bar{L}^c_i\Delta L_j+h.c.$$

↓

$$\frac{1}{2}y_{ij}\Delta^0\bar{\nu}^c\nu+h.c.$$

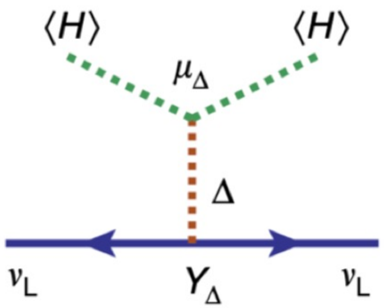
- **Giving neutrino mass matrix with vev of Delta**
- **Delta get a lepton number -2**

第二类跷跷板机制

$$
\begin{aligned}
V(H,\Delta) &= -m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + m_\Delta^2 \mathrm{Tr}(\Delta^\dagger \Delta) + \lambda_1 (H^\dagger H)\mathrm{Tr}(\Delta^\dagger \Delta) \\
&\quad + \lambda_2 (\mathrm{Tr}(\Delta^\dagger \Delta))^2 + \lambda_3 \mathrm{Tr}(\Delta^\dagger \Delta)^2 + \lambda_4 H^\dagger \Delta \Delta^\dagger H \\
&\quad + \left[\mu (H^T i\sigma^2 \Delta^\dagger H) + h.c.\right] + \ldots
\end{aligned}
$$

U(1)L breaking term

$$\langle \Delta^0 \rangle \simeq \frac{\mu v_{\mathrm{EW}}^2}{2 m_\Delta^2}$$



$$M_\nu = \langle H \rangle^2 Y_\Delta \mu_\Delta / M_\Delta^2$$

EW precision measurement

$$\mathcal{O}(1)\ \mathrm{GeV} > |\langle \Delta^0 \rangle| \gtrsim 0.05\ \mathrm{eV}$$

required by neutrino masses

希格斯暴胀

To be consistent with inflation, we add non-minimal couplings(similar to Higgs inflation)

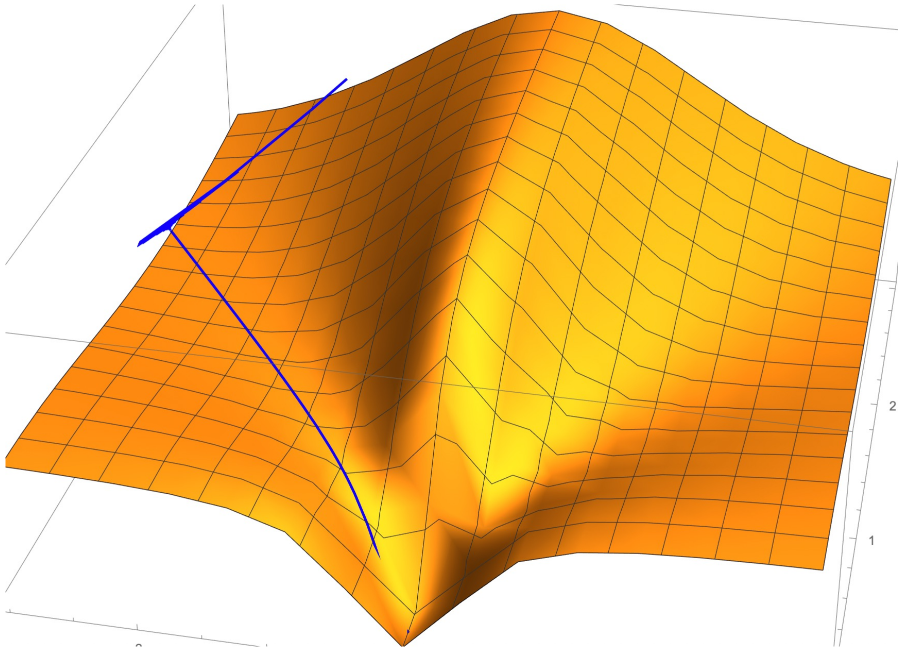
$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}M_P^2 R - f(H,\Delta)R - g^{\mu\nu}(D_\mu H)^\dagger(D_\nu H)$$
$$-g^{\mu\nu}(D_\mu \Delta)^\dagger(D_\nu \Delta) - V(H,\Delta) + \mathcal{L}_{\text{Yukawa}}$$

$$h \equiv \frac{1}{\sqrt{2}}\rho_H e^{i\eta} \qquad \Delta^0 \equiv \frac{1}{\sqrt{2}}\rho_\Delta e^{i\theta}$$

$$F(H,\Delta) = \xi_H |h|^2 + \xi_\Delta |\Delta^0|^2 = \frac{1}{2}\xi_H \rho_H^2 + \frac{1}{2}\xi_\Delta \rho_\Delta^2$$

希格斯暴胀

During inflation(Oleg Lebedev and Hyun Min Lee, arXiv:1105.2284)



$$\frac{\rho_H}{\rho_\Delta} \equiv \tan\alpha = \sqrt{\frac{2\lambda_\Delta \xi_H - \lambda_{H\Delta}\xi_\Delta}{2\lambda_H \xi_\Delta - \lambda_{H\Delta}\xi_H}}$$

$$\rho_H = \varphi \sin\alpha, \;\; \rho_\Delta = \varphi\cos\alpha$$

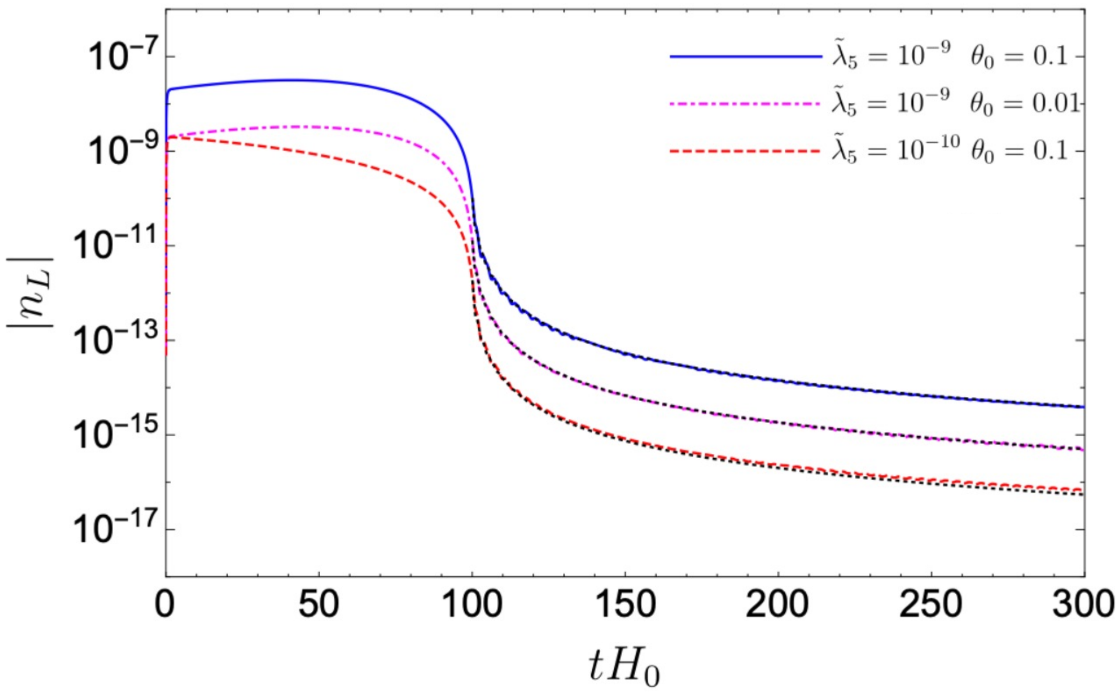
$$\xi \equiv \xi_H \sin^2\alpha + \xi_\Delta \cos^2\alpha$$

Effective a single field inflation

希格斯暴胀

$\xi = 300,\ \lambda = 4.5 \cdot 10^{-5}$

$\chi_0 = 6.0 M_p,\ \dot{\chi}_0 = 0$, and $\theta_0 = 0$



- Lepton number is generated during inflation
- After inflation, Lepton number is conserved

THANK YOU