Dark Photon

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Outline

- Toy model
- Realistic model
- Phenomenology studies
- Accelerator searches
- Cosmo/astro probes

Two mechanisms for dark photon

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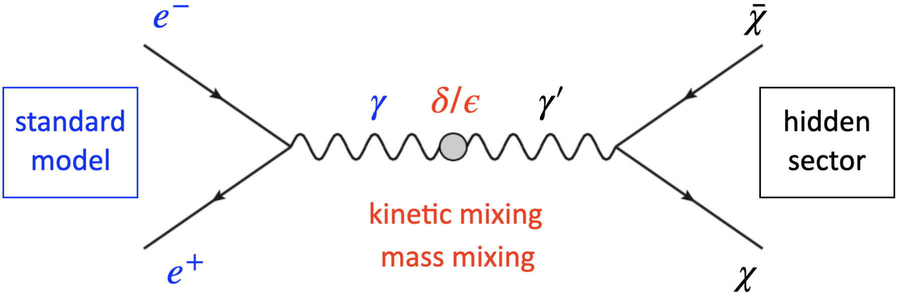
Two mechanisms to generate dark photon (DP):

- Kinetic mixing (KM) [1]
- Mass mixing (MM) [2]

[1] Holdom, PLB 166, 196 (1986); Foot & He, PLB 267, 509 (1991).
[2] Feldman, ZL, Nath, https://arxiv.org/pdf/hep-ph/0702123.pdf

Hypercharge portal



Toy model

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Kinetic mixing between 2 gauge bosons

Consider 2 gauge bosons $A_{1\mu}$ and $A_{2\mu}$ corresponding to two $U(1)$ gauge groups. Consider the following the Lagrangian $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$

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- $J_{1\mu}$ ($J_{2\mu}$) is the current that couples to $A_{1\mu}$ ($A_{2\mu}$). If we identify $A_{1\mu}$ ($A_{2\mu}$) as the gauge boson in the dark (SM) sector, then $J_{1\mu}$ ($J_{2\mu}$) is the dark (SM) sector current.

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Matrix form

Define (omit the Lorentz index)

$$V \equiv \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}, \quad J \equiv \begin{pmatrix} J_1 \\ J_2 \end{pmatrix}, \tag{3}$$

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To correctly interpret the physics, we need to put the kinetic terms in the canonical form, namely transforming K to an identity matrix.

Put the kinetic terms in the canonical form

To put the kinetic energy term in its canonical form, one may use the transformation

$$V^\mu = \begin{pmatrix} A_1^\mu \\ A_2^\mu \end{pmatrix} = G_0 \begin{pmatrix} A'^\mu \\ A^\mu \end{pmatrix} \equiv G_0 E^\mu \tag{5}$$

where the LHS (RHS) is the original (new) basis, and

$$G_0 = \begin{pmatrix} \frac{1}{\sqrt{1-\delta^2}} & 0 \\ \frac{-\delta}{\sqrt{1-\delta^2}} & 1 \end{pmatrix}. \tag{6}$$

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Now we have

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu}. \tag{8}$$

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which has an additional free parameter θ.

Lagrangian under G

With the general transformation $G = G_0 O$, the total Lagrangian $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$ becomes

$$\mathcal{L}_0 = -\frac{1}{4}E_{\mu\nu}E^{\mu\nu} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu},$$

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\mathcal{L}_1 &= J_\mu G E^\mu = J_\mu G_0 O E^\mu \\
&= A'^\mu \left[\frac{\cos\theta}{\sqrt{1-\delta^2}} J_{1\mu} + \left(\sin\theta - \frac{\cos\theta\delta}{\sqrt{1-\delta^2}} \right) J_{2\mu} \right]
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- Kinetic terms are in the canonical form
- Both bosons interact with both currents
- Interactions with current (matter) depend on 2 paras: θ and δ. So one has the freedom (namely θ) to choose the basis.

Asymmetric solutions: case 1

Case 1: $\theta = 0$.

$$\mathcal{L}_1 = A'^{\mu} \left[\frac{1}{\sqrt{1-\delta^2}} J_{1\mu} - \frac{\delta}{\sqrt{1-\delta^2}} J_{2\mu} \right] + A^{\mu} J_{2\mu}. \tag{14}$$

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- Because $A_{1\mu} = \frac{1}{\sqrt{1-\delta^2}}A'^{\mu}$ is the gauge boson in the hidden sector, we can identify A' as the dark photon, which interacts with both the dark current $J_{1\mu}$ and the SM current $J_{2\mu}$.

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- Then A_{μ} is the ordinary photon, which interacts only with the SM current $J_{2\mu}$.

Asymmetric solutions: case 2

Case 2: $\theta = \arctan\left[\delta/\sqrt{1-\delta^2}\right]$

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- Then A_μ is the SM photon.
- A'_μ interacts only with the dark current $J_{1\mu}$.

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- A_μ interacts with both the SM current $J_{2\mu}$ and the dark current $J_{1\mu}$.
- Coupling between A_μ and $J_{1\mu}$ is proportional to the kinetic mixing parameter δ.

Asymmetric solutions: case 2

Case 2: $\theta = \arctan\left[\delta/\sqrt{1-\delta^2}\right]$

$$\begin{pmatrix} A_1^\mu \\ A_2^\mu \end{pmatrix} = V^\mu = GE^\mu = \begin{pmatrix} 1 & -\dfrac{\delta}{\sqrt{1-\delta^2}} \\ 0 & \dfrac{1}{\sqrt{1-\delta^2}} \end{pmatrix} \begin{pmatrix} A'^\mu \\ A^\mu \end{pmatrix} \tag{16}$$

$$\mathcal{L}_1 = A^\mu \left[\frac{1}{\sqrt{1-\delta^2}} J_{2\mu} - \frac{\delta}{\sqrt{1-\delta^2}} J_{1\mu}\right] + A'^\mu J_{1\mu}. \tag{17}$$

- Because $A_{1\mu} = A'_\mu - A_\mu \delta/\sqrt{1-\delta^2}$, we still identify A'_μ as the dark photon.
- Then A_μ is the SM photon.
- A'_μ interacts only with the dark current $J_{1\mu}$.
- A_μ interacts with both the SM current $J_{2\mu}$ and the dark current $J_{1\mu}$.
- Coupling between A_μ and $J_{1\mu}$ is proportional to the kinetic mixing parameter δ. $\Longrightarrow$ hidden matter is millicharged if δ is small.

Mass

So far we have not written down mass terms for the gauge bosons. To make the dark photon massive, mass terms are needed. The general mass terms are

$$\mathcal{L}_{\mathrm{m}} = \frac{1}{2} m_1^2 A_{1\mu} A_1^{\mu} + \frac{1}{2} m_2^2 A_{2\mu} A_2^{\mu} + m_1 m_2 A_{1\mu} A_2^{\mu}. \tag{18}$$

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Write the mass terms in a matrix form:

$$\mathcal{L}_{\rm m} = \frac{1}{2} V_\mu M^2 V^\mu, \quad (19)$$

$$M^2 = \begin{pmatrix} m_1^2 & m_1 m_2 \\ m_1 m_2 & m_2^2 \end{pmatrix} \equiv m_1^2 \begin{pmatrix} 1 & \epsilon \\ \epsilon & \epsilon^2 \end{pmatrix} \quad (20)$$

where $\epsilon \equiv m_2/m_1$.

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Note that the determinant of M^2 is zero so that one of the eigenvalue is zero, which can be identified as the photon mass (this is a must for a successful NP construction); the other (massive) eigenvalue is the dark photon mass-square.

Mass eigenstates: case 3

Diagnolizing the mass matrix M^2 fixes θ: $\theta = \arctan\left[\frac{\epsilon\sqrt{1-\delta^2}}{1-\epsilon\delta}\right]$.

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DP A' and photon A interact with both currents: J_1 (dark) and J_2 (SM).

Millicharge & mixings

Take a closer at the interaction.

$$\mathcal{L}_1 = \frac{1}{\sqrt{1 - 2\delta\epsilon + \epsilon^2}} \left(\frac{\epsilon - \delta}{\sqrt{1-\delta^2}} J_{2\mu} + \frac{1 - \delta\epsilon}{\sqrt{1-\delta^2}} J_{1\mu} \right) A'^{\mu}$$

[4] Recall that millicharge is the electric charge of the dark sector matter, so it is the coupling between the dark sector current $J_{1\mu}$ and the SM photon A^{μ}.

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- If DP is massive, kinetic mixing alone does not lead to millicharged dark matter
- If DP is massive, mass mixing alone generates millicharged dark matter.

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Realistic model

◇ Feldman, ZL, Nath, https://arxiv.org/pdf/hep-ph/0702123.pdf

StkSM

For realistic model, one has to extend the SM, which has the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$.

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We consider the extended electroweak sector with the gauge group $SU(2)_L \times U(1)_Y \times U(1)_X$, where both kinetic mixing and Stueckelberg mass mixing between the 2 $U(1)$'s are present.

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Assume that the SM fields do not carry $U(1)_X$ quantum numbers, and the fields in the hidden sector does not carry quantum numbers of the SM gauge group. The 2 mixings terms are the only connections between the 2 sectors.

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The Standard Model & Higgs

We first review the SM and Higgs.

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The covariant derivative of the Higgs field ϕ in the SM is

$$D_\mu \phi = \left(\partial_\mu - i g_2 A_\mu^a \frac{\sigma^a}{2} - i g_Y B_\mu Y \right) \phi, \tag{24}$$

where σ^a are the Pauli matrices, A_μ^a and B_μ are, respectively, the $SU(2)_L$ and $U(1)_Y$ gauge bosons, and Y is the hypercharge quantum number. For the Higgs doublet, $Y = 1/2$.

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Higgs VEV $\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$.

Neutral gauge boson masses in the SM

The gauge boson masses arise from the $(D_\mu \phi)^\dagger (D^\mu \phi)$ term:

$$\mathcal{L}_{\text{mass}} = \frac{1}{2} \begin{pmatrix} 0 & v \end{pmatrix} \left(g_2 A^a_\mu \frac{\sigma^a}{2} + \frac{1}{2} g_Y B_\mu \right) \left(g_2 A^{b\mu} \frac{\sigma^b}{2} + \frac{1}{2} g_Y B^\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{25}$$

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This then leads to

$$\mathcal{L}_{\text{mass}} = \frac{1}{2} \frac{v^2}{4} \left[g_2^2 \left(A_\mu^1 \right)^2 + g_2^2 \left(A_\mu^2 \right)^2 + \left(-g_2 A_\mu^3 + g_Y B_\mu \right)^2 \right] \tag{26}$$

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Keeping only the neutral gauge bosons, we write the mass terms in the matrix from:

$$\mathcal{L}_{\text{mass}} \supset \frac{1}{2} \frac{v^2}{4} \begin{pmatrix} A^3_\mu & B_\mu \end{pmatrix} \begin{pmatrix} g_2^2 & -g_2 g_y \\ -g_2 g_y & g_y^2 \end{pmatrix} \begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix} . \tag{27}$$

This mass matrix can be diagonalized by the weak mixing angle θ_W where $\tan\theta_W = g_Y / g_2$, leading to a massive Z boson and a massless photon. Note that the determinant of the mass matrix is zero, which ensures the existence of a massless eigenstate.

Diagonlization of the mass matrix of neutral gauge bosons

The orthogonal mass matrix is

$$\begin{pmatrix} B \\ A^3 \end{pmatrix} = O \begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A \\ Z \end{pmatrix} \tag{28}$$

where A is the massless eigenstate (photon) and Z is the massive eigenstate, and $\tan\theta_W = g_Y/g_2$ such that

$$\cos\theta_W = \frac{g_2}{\sqrt{g_2^2 + g_Y^2}}, \quad \sin\theta_W = \frac{g_Y}{\sqrt{g_2^2 + g_Y^2}}, \tag{29}$$

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- photon, $m_A = 0$, $A = c_W B + s_W A^3$
- Z, $m_Z = \frac{v}{2}\sqrt{g_2^2 + g_Y^2}$, $Z = c_W A^3 - s_W B$

Couplings to SM fermions

The neutral current interaction with the SM fermions is given by

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Photon/Z coupling to electron

$$\mathcal{L}_{\rm NC} \supset -\,\bar{e}_L\gamma^\mu \left(g_2\left(s_W A_\mu + c_W Z_\mu\right)\frac{1}{2} + g_Y\left(c_W A_\mu - s_W Z_\mu\right)\frac{1}{2}\right)e_L$$

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& - \bar{e}_R \gamma^\mu \left(g_Y \left(c_W A_\mu - s_W Z_\mu \right) \right) e_R \\
= & - A_\mu \left[\bar{e}_L \gamma^\mu \left(g_2 s_W \frac{1}{2} + g_Y c_W \frac{1}{2} \right) e_L + \bar{e}_R \gamma^\mu \left(g_Y c_W \right) e_R \right]
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= & - A_\mu \left[\bar{e}_L \gamma^\mu \left(g_2 s_W \frac{1}{2} + g_Y c_W \frac{1}{2} \right) e_L + \bar{e}_R \gamma^\mu \left(g_Y c_W \right) e_R \right] \\
& - Z_\mu \left[\bar{e}_L \gamma^\mu \left(g_2 \left(c_W \right) \frac{1}{2} + g_Y \left(-s_W \right) \frac{1}{2} \right) e_L + \bar{e}_R \gamma^\mu \left(g_Y \left(-s_W \right) \right) e_R \right]
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\end{aligned}
$$

Photon/Z coupling to electron

$$
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& - Z_\mu \left[\frac{g_2^2 - g_Y^2}{2\sqrt{g_2^2 + g_Y^2}} \bar{e}_L \gamma^\mu e_L - \frac{g_Y^2}{\sqrt{g_2^2 + g_Y^2}} \bar{e}_R \gamma^\mu e_R \right]
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Thus we find that $Q_e = -1$ and

$$e = \frac{g_2 g_Y}{\sqrt{g_2^2+g_Y^2}}, \quad \text{or} \quad \frac{1}{e^2} = \frac{1}{g_2^2} + \frac{1}{g_Y^2}, \tag{35}$$

Lagrangian of StkSM

The total Lagrangian is

$$\mathcal{L}_{\text{StkSM}} = \mathcal{L}_{\text{SM}} + \Delta\mathcal{L} \tag{36}$$

where

$$\Delta\mathcal{L} \supset -\frac{1}{4}C_{\mu\nu}C^{\mu\nu} - \frac{\delta}{2}C_{\mu\nu}B^{\mu\nu} + \frac{1}{2}(\partial_\mu\sigma + m_1 C_\mu + m_2 B_\mu)^2 + g_X J_X^\mu C_\mu. \tag{37}$$

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Stueckelberg mass terms

The Stueckelberg mass terms

$$\frac{1}{2}(\partial_\mu\sigma + m_1 C_\mu + m_2 B_\mu)^2 \tag{38}$$

are invariant under the $U(1)_X \times U(1)_Y$ gauge transformations. [5]

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Kinetic mixing matrix & mass matrix

The StkSM model has a nondiagonal kinetic matrix (K) and a nondiagonal mass matrix (M^2), and in the unitary gauge in the basis $V^T = (C, B, A^3)$,

$$K = \begin{pmatrix} 1 & \delta & 0 \\ \delta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{41}$$

$$M^2 = \begin{pmatrix} m_1^2 & m_1^2\epsilon & 0 \\ m_1^2\epsilon & m_2^2\epsilon^2 + \frac{1}{4}g_Y^2 v^2 & -\frac{1}{4}g_Y g_2 v^2 \\ 0 & -\frac{1}{4}g_Y g_2 v^2 & +\frac{1}{4}g_2^2 v^2 \end{pmatrix} \tag{42}$$

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Simultaneous diagonalization of the kinetic & mass matrices

A simultaneous diagonalization of the kinetic & mass matrices can be obtained by the transformation $G = G_0 O$, which is a combination of the a $GL(3)$ transformation (G_0) and an orthogonal transformation (O). This allows one to work in the diagonal basis, denoted by E where $E^T = (Z', Z, A)$, through the transformation $V = GE = G_0 O E$.

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The matrix O is then defined by the diagonalization of the mass matrix

$$M_D^2 = O^T (G_0^T M^2 G_0) O. \tag{44}$$

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Mass matrix diagonalization

Thus the matrix to be diagonalized by O is

$$G_0^T M^2 G_0 = \begin{pmatrix} \dfrac{4m_1^2(1-\delta\epsilon)^2 + \delta^2 g_Y^2 v^2}{4\left(1-\delta^2\right)} & \dfrac{4m_1^2\epsilon(1-\delta\epsilon) - \delta g_Y^2 v^2}{4\sqrt{1-\delta^2}} & \dfrac{\delta g_2 g_Y v^2}{4\sqrt{1-\delta^2}} \\ \dfrac{4m_1^2\epsilon(1-\delta\epsilon) - \delta g_Y^2 v^2}{4\sqrt{1-\delta^2}} & m_1^2\epsilon^2 + \dfrac{1}{4} g_Y^2 v^2 & -\dfrac{1}{4} g_2 g_Y v^2 \\ \dfrac{\delta g_2 g_Y v^2}{4\sqrt{1-\delta^2}} & -\dfrac{1}{4} g_2 g_Y v^2 & \dfrac{1}{4} g_2^2 v^2 \end{pmatrix} \tag{45}$$

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- It also has 2 massive modes: Z and Z' (or A').
- We label the additional massive mode as Z' (A') if its mass is larger (smaller) than the Z boson.

The massless mode

It is not difficult to find the eigenvector of the massless mode:

$$A = \frac{1}{N}\begin{pmatrix} -\sqrt{1-\delta^2}g_2\epsilon \\ g_2(1-\delta\epsilon) \\ g_Y \end{pmatrix} \equiv \begin{pmatrix} O_{13} \\ O_{23} \\ O_{33} \end{pmatrix} \tag{46}$$

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The components of the photon eigenvector are the elements of the orthogonal matrix O.

$$V = \begin{pmatrix} C \\ B \\ A^3 \end{pmatrix} \to V = G_0\tilde{V} = G_0\begin{pmatrix} \tilde{C} \\ \tilde{B} \\ A^3 \end{pmatrix} \to V = G_0\tilde{V} = G_0 O E = G_0 O\begin{pmatrix} Z' \\ Z \\ A \end{pmatrix} \tag{48}$$

Neutral current interaction

The neutral current interaction with the visible sector fermions is given by

$$\mathcal{L}_{\rm NC} = \bar{f}_L i\gamma^\mu D_\mu f_L + (L \leftrightarrow R), \tag{49}$$

where D_μ is the covariant derivative with respect to the $SU(2)_L \times U(1)_Y \times U(1)_X$ gauge group.

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Coupling between neutral gauge bosons and SM fermions

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Photon couplings with electrons

$$\mathcal{L}_{\rm NC} \supset \bar{e}_L \gamma^\mu \left(g_2 A^3_\mu \frac{\sigma^3}{2} + g_Y B_\mu Y \right) e_L + (L \leftrightarrow R), \tag{52}$$

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To obtain photon couplings, make the following replacements: [7]

$$B \to (G_0 O)_{23} A = (G_0)_{2a} O_{a3} A = \left[O_{23} - \frac{\delta}{\sqrt{1-\delta^2}} O_{13} \right] A = \frac{g_2}{N} A \tag{53}$$

$$A^3 \to (G_0 O)_{33} A = (G_0)_{3a} O_{a3} A = O_{33} A = \frac{g_Y}{N} A \tag{54}$$

where $N = \sqrt{g_2^2 (1 - 2\delta\epsilon + \epsilon^2) + g_Y^2}$.

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Thus, we have

$$\mathcal{L}_{\rm photon} \supset \frac{g_2 g_Y}{N} A_\mu \left[\bar{e}_L \gamma^\mu \left(\frac{\sigma^3}{2} + Y \right) e_L + (L \leftrightarrow R) \right], \tag{55}$$

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Photon couplings (continued)

We next use $\frac{\sigma^3}{2} = -\frac{1}{2}$ and $Y = -\frac{1}{2}$ for e_L, and $\frac{\sigma^3}{2} = 0$ and $Y = -1$ for e_R to obtain

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[8] Note that g_Y^{SM} is defined such that the relation between e, g_2, and g_Y^{SM} is the same one in the SM.

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Or

$$\frac{1}{e^2} = \frac{1}{g_2^2} + \frac{1 - 2\delta\epsilon + \epsilon^2}{g_Y^2} \equiv \frac{1}{g_2^2} + \frac{1}{(g_Y^{\rm SM})^2} \tag{58}$$

where $g_Y \equiv g_Y^{\rm SM}\sqrt{1 - 2\delta\epsilon + \epsilon^2}$. [8]

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Mass matrix

The mass matrix

$$G_0^T M^2 G_0 = \begin{pmatrix} \frac{4m_1^2(1-\delta\epsilon)^2 + \delta^2 g_Y^2 v^2}{4\left(1-\delta^2\right)} & \frac{4m_1^2\epsilon(1-\delta\epsilon) - \delta g_Y^2 v^2}{4\sqrt{1-\delta^2}} & \frac{\delta g_2 g_Y v^2}{4\sqrt{1-\delta^2}} \\ \frac{4m_1^2\epsilon(1-\delta\epsilon) - \delta g_Y^2 v^2}{4\sqrt{1-\delta^2}} & m_1^2\epsilon^2 + \frac{1}{4}g_Y^2 v^2 & -\frac{1}{4}g_2 g_Y v^2 \\ \frac{\delta g_2 g_Y v^2}{4\sqrt{1-\delta^2}} & -\frac{1}{4}g_2 g_Y v^2 & \frac{1}{4}g_2^2 v^2 \end{pmatrix} \tag{59}$$

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Compute the eigenvalues.

Mass eigenvalues

Three eigenvalues of the mass matrix are (depends on β only)

$$M_A^2 = 0, \quad M_Z^2 = (q-p)/2, \quad M_{Z'}^2 = (q+p)/2, \tag{60}$$

$$p = \sqrt{\left(m_1^2\beta + \frac{(g_Y^{\rm SM})^2\beta + g_2^2}{4}v^2\right)^2 - 4m_1^2\frac{(g_Y^{\rm SM})^2 + g_2^2}{4}v^2\beta}, \tag{61}$$

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A special case: $\epsilon = \delta \Longrightarrow \beta = 1 \Longrightarrow$ (assuming $m_1 > m_Z$)

$$M_Z = \frac{\sqrt{g_2^2 + (g_Y^{\rm SM})^2}}{2}v, \quad M_{Z'} = m_1, \tag{64}$$

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It implies that δ is equivalent to ϵ.

Another orthogonal transformation

To see the equivalence, perform the following orthogonal transformation

$$R = \begin{pmatrix} \sqrt{1-\delta^2} & -\delta & 0 \\ \delta & \sqrt{1-\delta^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{65}$$

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which transforms the mass matrix to

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Note that the mass matrix $\mathcal{M}^2$ looks exactly the same as for the mass matrix (namely M^2) one has if there was just the Stueckelberg mass mixing except that ϵ is replaced by $\bar{\epsilon}$. (Namely compare $\delta = 0$ with $\delta \neq 0$.) See Eq. (42) for the mass matrix M^2.

Mass matrix diagonalization

To diagonalize the mass matrix $\mathcal{M}^2 = R^T G_0^T M^2 G_0 R$ such that $O^T \mathcal{M}^2 O = \mathrm{Diag}(m_{Z'}^2, m_Z^2, 0)$, we use the following parameterization (3 Euler angles)

$$O = \begin{pmatrix} \cos\psi\cos\phi - \sin\theta\sin\phi\sin\psi & \sin\psi\cos\phi + \sin\theta\sin\phi\cos\psi & -\cos\theta\sin\phi \\ \cos\psi\sin\phi + \sin\theta\cos\phi\sin\psi & \sin\psi\sin\phi - \sin\theta\cos\phi\cos\psi & \cos\theta\cos\phi \\ -\cos\theta\sin\psi & \cos\theta\cos\psi & \sin\theta \end{pmatrix} \quad (68)$$

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where the angles are defined so that

$$\tan\theta = \frac{g_Y^{\mathrm{SM}}}{g_2}, \quad \tan\phi = \bar{\epsilon}, \quad \tan 2\psi = \frac{2m_0^2 \sin\theta\bar{\epsilon}}{m_1^2 - m_0^2 + (m_1^2 + m_0^2 - m_W^2)\bar{\epsilon}^2}, \quad (69)$$

and $m_0 = m_Z(\epsilon = \delta) = v\sqrt{g_2^2 + (g_Y^{\mathrm{SM}})^2}/2$, and $m_W = g_2 v/2$.

Neutral current (I)

The neutral current interactions with SM fermions f are

$$\mathcal{L}_{\rm NC} \supset \bar{f}_L \gamma^\mu \left(g_2 A^3_\mu T^3 + g_Y B_\mu Y \right) f_L + (L \to R)$$

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&= g_2 A_\mu^3 \left[T_f^3 \bar{f} \gamma^\mu P_L f \right] + g_Y B_\mu \left[(Q_f - T_f^3) \bar{f} \gamma^\mu P_L f + Q_f \bar{f} \gamma^\mu P_R f \right]
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where $T^3 = \sigma^3/2$. Here T^3_f is only for left-handed fermions; $T^3_f = 0$ for right-handed fermions. In the 3rd line, we have used $Q_f = T^3_f + Y_f$, where Y_f denotes both Y_L and Y_R. The chiral projection operators are $P_{L,R} = \dfrac{1 \mp \gamma_5}{2}$.

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$$J^3_2 = T^3_f \bar{f} \gamma^\mu P_L f = \bar{f} \gamma^\mu \left[\frac{T^3_f}{2} - \gamma_5 \frac{T^3_f}{2} \right] f \tag{71}$$

$$J_Y = \bar{f} \gamma^\mu \left[(Q_f - T^3_f) P_L + Q_f P_R \right] f = \bar{f} \gamma^\mu \left[\left(Q_f - \frac{T^3_f}{2} \right) - \gamma_5 \frac{-T^3_f}{2} \right] f \tag{72}$$

Neutral current (II)

The transformation relating the initial basis and the final diagonal basis is $V = [G_0(\delta)R(\delta)O(\bar{\epsilon})]E$, where $V^T = (C, B, A^3)$, and $E^T = (Z', Z, A_\gamma)$. [9]

[9] Note that there are some hidden dependence in the relation of $g_Y = g_Y^{\rm SM}\sqrt{1 - 2\delta\epsilon + \epsilon^2}$. However, if one uses the SM relation $(g_Y^{\rm SM})^{-2} = e^{-2} - g_2^{-2}$ to find $g_Y^{\rm SM}$, then $g_Y^{\rm SM}$ can be treated as free of NP parameters.

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The neutral current interaction can be written in the form

$$\mathcal{L}_{NC} = J^T S(\bar{\epsilon}, \delta) O(\bar{\epsilon}) E \tag{73}$$

where $J^T = (g_X J_X, g_Y^{\rm SM} J_Y, g_2 J_2^3)$, and S is given by

$$S(\bar{\epsilon}, \delta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{g_Y}{g_Y^{\rm SM}} & 0 \\ 0 & 0 & 1 \end{pmatrix} G_0 R = \begin{pmatrix} 1 & -\frac{\delta}{\sqrt{1-\delta^2}} & 0 \\ 0 & \sqrt{1+\bar{\epsilon}^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{74}$$

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When $J_X = 0$, the neutral current interaction of Eq. (73) has no dependence on δ.

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Neutral current (III)

The neutral current interaction with SM fermions are given by

$$\mathcal{L}_{\rm NC} = g_Y^{\rm SM} J_Y T_{2a} E_a + g_2 J_2^3 T_{3a} E_a \tag{75}$$

where $T = S(\bar{\epsilon}, \delta) O(\bar{\epsilon})$.

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where $T = S(\bar{\epsilon}, \delta) O(\bar{\epsilon})$. It is convenient to write the interaction in the conventional form with the reduced vector & axial vector couplings

$$\mathcal{L}_{\rm NC} = g_Z \bar{f} \gamma^\mu \left[(v'_f - \gamma_5 a'_f) Z'_\mu + (v_f - \gamma_5 a_f) Z_\mu \right] f + e \bar{f} \gamma^\mu Q_f A_\mu f, \tag{76}$$

where $g_Z = \sqrt{g_2^2 + (g_Y^{\rm SM})^2}/2$.

Neutral current (III)

The neutral current interaction with SM fermions are given by

$$\mathcal{L}_{\rm NC} = g_Y^{\rm SM} J_Y T_{2a} E_a + g_2 J_2^3 T_{3a} E_a \tag{75}$$

where $T = S(\bar{\epsilon}, \delta) O(\bar{\epsilon})$. It is convenient to write the interaction in the conventional form with the reduced vector & axial vector couplings

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where $g_Z = \sqrt{g_2^2 + (g_Y^{\rm SM})^2}/2$. Thus, we find

$$\begin{aligned}
v_f &= g_Z^{-1} [(g_2 T_{32} - g_Y^{\rm SM} T_{22}) T_f^3/2 + g_Y^{\rm SM} T_{22} Q_f], \\
a_f &= g_Z^{-1} [(g_2 T_{32} - g_Y^{\rm SM} T_{22}) T_f^3/2], \\
v'_f &= g_Z^{-1} [(g_2 T_{31} - g_Y^{\rm SM} T_{21}) T_f^3/2 + g_Y^{\rm SM} T_{21} Q_f], \\
a'_f &= g_Z^{-1} [(g_2 T_{31} - g_Y^{\rm SM} T_{21}) T_f^3/2].
\end{aligned} \tag{77}$$

Neutral current (IV)

The reduced vector and axial vector couplings (tree level) can be further expressed in terms of the rotation angles:

$$v_f = \cos\psi \left[(1 - \bar{\epsilon}\sin\theta\tan\psi)\, T_f^3 - 2\sin^2\theta\,(1 - \bar{\epsilon}\csc\theta\tan\psi)\, Q_f\right], \tag{78}$$

$$a_f = \cos\psi\,[1 - \bar{\epsilon}\sin\theta\tan\psi]\, T_f^3, \tag{79}$$

$$v'_f = -\cos\psi \left[(\tan\psi + \bar{\epsilon}\sin\theta)\, T_f^3 - 2\sin^2\theta\,(\bar{\epsilon}\csc\theta + \tan\psi)\, Q_f\right], \tag{80}$$

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Because the rotation angles only depend on $\bar{\epsilon}$, we find that the dependencies on δ and ϵ of the vector & axial vector couplings between SM fermions and neutral bosons are only through $\bar{\epsilon}$.

We conclude that kinetic mixing parameter δ and the mass mixing parameter ϵ are degenerate so that only their combination

$$\bar{\epsilon} = \frac{\epsilon - \delta}{\sqrt{1 - \delta^2}} \quad (82)$$

appears in the reduced vector & axial vector couplings of SM fermions.

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where $s_\delta \equiv \dfrac{\delta}{\sqrt{1-\delta^2}}$. Because the only element of S that contains δ is $S_{12} = -s_\delta$, the interaction with hidden current now depends on δ.

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where $s_\delta \equiv \dfrac{\delta}{\sqrt{1-\delta^2}}$. Because the only element of S that contains δ is $S_{12} = -s_\delta$, the interaction with hidden current now depends on δ.

When $J_X \neq 0$, the NC interaction depends on δ, breaking the degeneracy beteen δ and ϵ.

Millicharged dark matter

Consider Dirac fermion χ with $J_X^\mu = \bar{\chi}\gamma^\mu\chi$, the coupling to photon is

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- Millicharged DM can be generated via mass mixing, but not via kinetic mixing.
- This is consistent with the toy model.

Heavy Z' versus light A'

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Recall that the reduced vector and axial vector couplings (tree level) of Z'/A' are

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where the angles are defined so that

$$\tan\theta = \frac{g_Y^{\rm SM}}{g_2}, \qquad \tan\phi = \bar{\epsilon}, \qquad \tan 2\psi = \frac{2m_0^2 \sin\theta\bar{\epsilon}}{m_1^2 - m_0^2 + (m_1^2 + m_0^2 - m_W^2)\bar{\epsilon}^2}, \quad (87)$$

$m_0 = m_Z(\epsilon = \delta) = v\sqrt{g_2^2 + (g_Y^{\rm SM})^2}/2$, and $m_W = g_2 v/2$.

V and A coupling of A' (I)

When $m_1 \ll m_Z = m_0$, we have

$$\tan 2\psi = \frac{2m_0^2 \sin\theta\bar{\epsilon}}{m_1^2 - m_0^2 + (m_1^2 + m_0^2 - m_W^2)\bar{\epsilon}^2}$$

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$$\tan\psi \sim \psi \simeq -\sin\theta\bar{\epsilon}\left[1 + \frac{m_1^2}{m_Z^2}\right] \Longrightarrow \tan\psi + \bar{\epsilon}\sin\theta \simeq -\sin\theta\bar{\epsilon}\frac{m_1^2}{m_Z^2} \tag{89}$$

Note that $a'_f \propto \tan\psi + \bar{\epsilon}\sin\theta$.

V and A coupling of A' (II)

Thus we find that

$$v'_f = -\cos\psi\left[(\tan\psi + \bar{\epsilon}\sin\theta)\,T_f^3 - 2\sin^2\theta\,(\bar{\epsilon}\csc\theta + \tan\psi)\,Q_f\right]$$

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&= \bar{\epsilon}\sin\theta\left[\left(\frac{m_1}{m_Z}\right)^2 T^3_f + 2\cos^2\theta Q_f\right]
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&= \bar\epsilon\left[\left(\sin\theta\frac{m_1^2}{m_Z^2}\right)T^3_f + 2\sin^2\theta\,(\csc\theta - \sin\theta)\,Q_f\right] \\
&= \bar\epsilon\sin\theta\left[\left(\frac{m_1}{m_Z}\right)^2 T^3_f + 2\cos^2\theta Q_f\right] \\
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\end{aligned}
\tag{90}
$$

V and A coupling of A' (II)

Thus we find that

$$
\begin{aligned}
v'_f &= -\cos\psi\left[\left(\tan\psi+\bar{\epsilon}\sin\theta\right)T^3_f - 2\sin^2\theta\left(\bar{\epsilon}\csc\theta+\tan\psi\right)Q_f\right] \\
&\simeq -\left[\left(-\sin\theta\bar{\epsilon}\frac{m_1^2}{m_Z^2}\right)T^3_f - 2\sin^2\theta\left(\bar{\epsilon}\csc\theta+\left(-\sin\theta\bar{\epsilon}\right)\right)Q_f\right] \\
&= \bar{\epsilon}\left[\left(\sin\theta\frac{m_1^2}{m_Z^2}\right)T^3_f + 2\sin^2\theta\left(\csc\theta-\sin\theta\right)Q_f\right] \\
&= \bar{\epsilon}\sin\theta\left[\left(\frac{m_1}{m_Z}\right)^2 T^3_f + 2\cos^2\theta Q_f\right] \\
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$$g_Z \bar{f} \gamma^\mu (v'_f - \gamma_5 a'_f) f A'_\mu \simeq g_Z \bar{\epsilon} (2 \sin\theta) \bar{f} \gamma^\mu \left[\cos^2\theta Q_f + \frac{1-\gamma_5}{2} \left(\frac{m_1}{m_Z} \right)^2 T_f^3 \right] f A'_\mu$$

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- The smaller the dark photon mass, the more photon-like it is.

Phenomenology studies on dark photon

Dark photon mass

Phenomenology studies on dark photon depend on its mass. The dividing line is $\sim$ MeV:

[10] In fact, DP can also decay into a pair of neutrinos, but it is suppressed by $(m_{A'}/m_Z)^4 \leq \mathcal{O}(10^{-20})$.

[11] The decay $A' \to \gamma\gamma$ is forbidden by the Landau-Yang theorem.

[12] However, if the strict definition (vector-like coupling that is proportional to electric charge) is not used, dark photon can refer to any light gauge boson. For example, $U(1)_{B-L}$ boson, $U(1)_{L_i - L_j}$ boson, etc.

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Large dark photon mass introduces both significant axial vector coupling and deviation from the proportionality of the electric charge. [12]

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DP vertex

Both production and decay of DP depends on its SM vertex [13]

$$\bar{\epsilon} e c_W Q_f \bar{f} \gamma^\mu f A'_\mu \equiv \epsilon e Q_f \bar{f} \gamma^\mu f A'_\mu, \tag{92}$$

where the ϵ parameter on the RHS is NOT the mass mixing parameter. Here I redefine the vertex so that it looks similar to that usually used in the literature. So $\epsilon = c_W \frac{\epsilon_{\rm MM} - \delta}{\sqrt{1-\delta^2}}$, where $\epsilon_{\rm MM}$ is the mass mixing parameter. [14] From now on, I will use the new vertex.

[13] DP is just like a massive photon, but with a suppressed coupling to SM fermions: the electric charge Q_f is suppressed by the small parameter ϵ.

[14] The absence of the factor c_W in the literature is due to the fact that people often use the toy model where they mix the C_μ boson with the photon field. In the realistic model, one has to mix the C_μ with the hypercharge boson B_μ; the additional factor c_W is to account for the difference between B_μ and the photon.

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There is also a DP-DM vertex (for Dirac DM with vector coupling & $Q_X = 1$)

$$\sim g_X A'_\mu \bar{\chi} \gamma^\mu \chi \tag{93}$$

which depends on the gauge coupling of the hidden $U(1)_X$.

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DP decays at the tree level

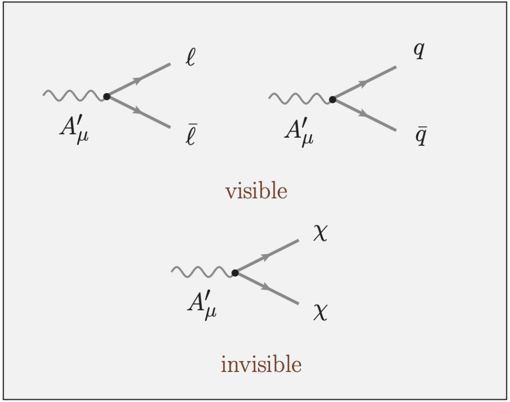

Figure: DP decay. From https://arxiv.org/pdf/2005.01515.pdf.

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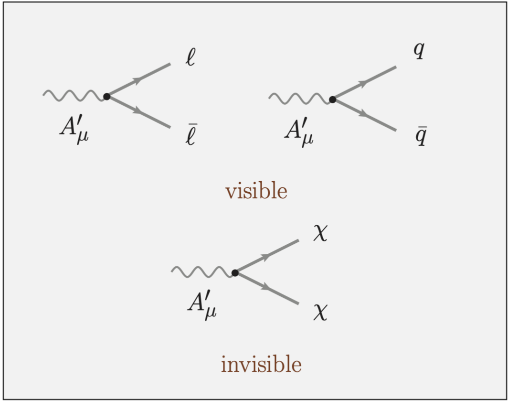

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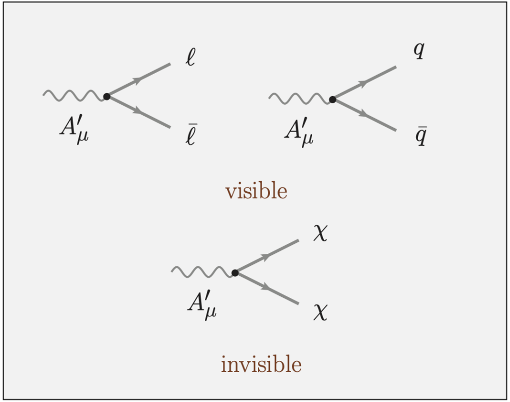

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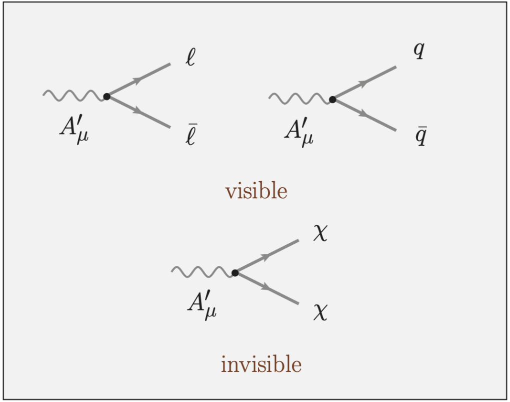

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DP decay width

The dark photon leptonic decay width is

$$\Gamma(A' \to l^+ l^-) = \frac{m_{A'}}{12\pi} \sqrt{1 - 4\frac{m_l^2}{m_{A'}^2}} \left(1 + 2\frac{m_l^2}{m_{A'}^2}\right) (\epsilon e Q_l)^2, \tag{94}$$

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The hadronic decay width can be computed by

$$\Gamma(A' \to \text{hadrons}) = \Gamma(A' \to \mu^+ \mu^-)\, R(m_{A'}^2), \tag{95}$$

where

$$R(m_{A'}^2) \equiv \frac{\sigma(e^+ e^- \to \text{hadrons})}{\sigma(e^+ e^- \to \mu^+ \mu^-)}$$

takes into account the effects of the dark photon mixing with the QCD vector mesons and can be taken from PDG.

DP decays at 1-loop

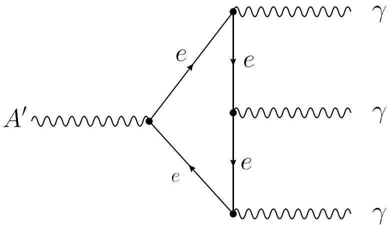

Figure: DP decay to 3 photons via a 1-loop process.

[15] Liu & Miller, https://arxiv.org/pdf/1705.01633.pdf.

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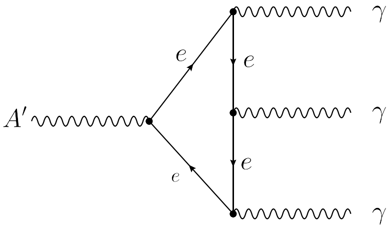

Figure: DP decay to 3 photons via a 1-loop process.

The decay width of $(A' \to 3\gamma)$ [15]

$$\Gamma(A' \to 3\gamma) = \epsilon^2 \frac{\alpha^4}{2^7 3^6 5^2 \pi^3} \frac{m_{A'}^9}{m_e^8} \left[\frac{17}{5} + \frac{67}{42} \frac{m_{A'}^2}{m_e^2} + \frac{128941}{246960} \frac{m_{A'}^4}{m_e^4} + \mathcal{O}\left(\frac{m_{A'}^6}{m_e^6} \right) \right]. \qquad (96)$$

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Visible DP versus invisible DP

DP can be categorized into 2 types:

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Because typically $\epsilon e Q_f \ll g_\chi$, if $m_{A'} > 2m_\chi$, one has $\Gamma(A' \to \bar{\chi}\chi) \gg \Gamma(A' \to \bar{f}f)$, and DP decay predominately into DM final state. $\Longrightarrow$ Invisible DP

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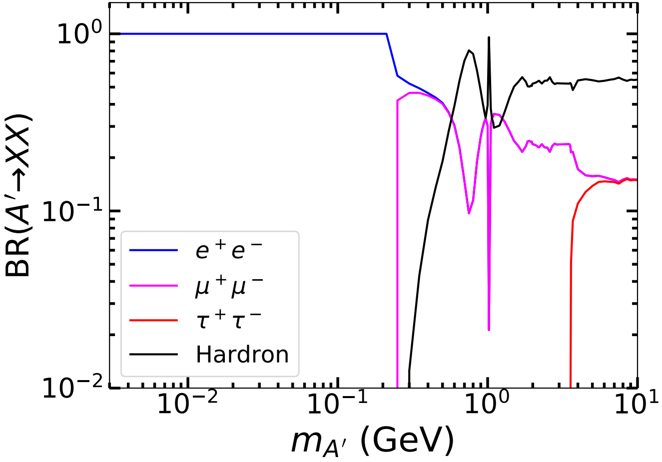
On the other hand, if $m_{A'} < 2m_\chi$, DP can only decay into SM final states. $\Longrightarrow$ Visible DP

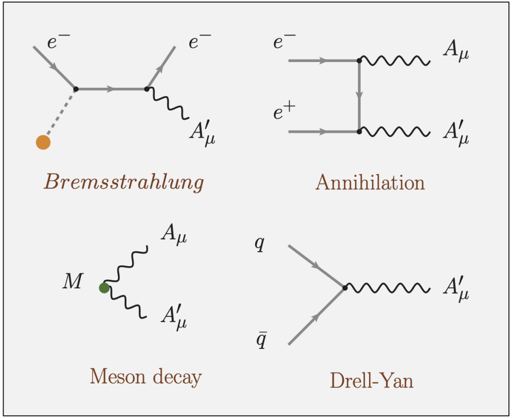
DP decay BR (visible decays only)



BR is independent of ϵ.

Figure: DP decay BR. From https://arxiv.org/pdf/1912.00422.pdf.

DP production



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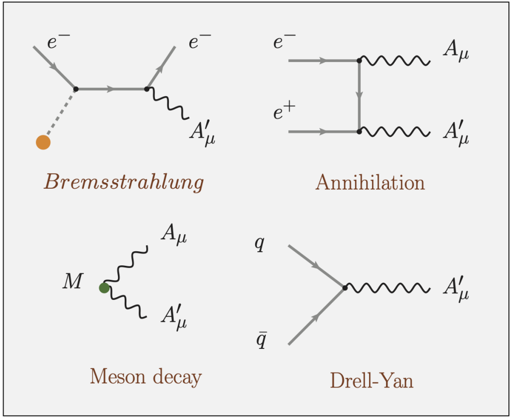

- Bremsstrahlung
  $e^- Z \to e^- Z A'$

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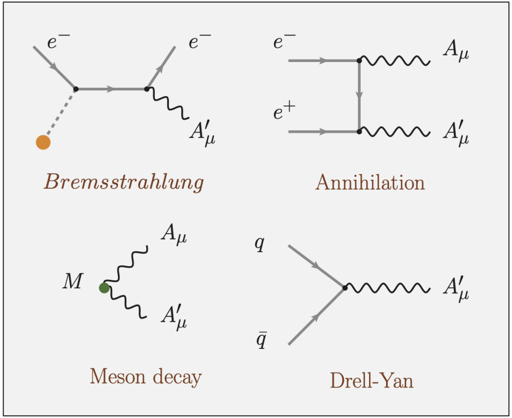

- Bremsstrahlung $e^- Z \to e^- Z A'$
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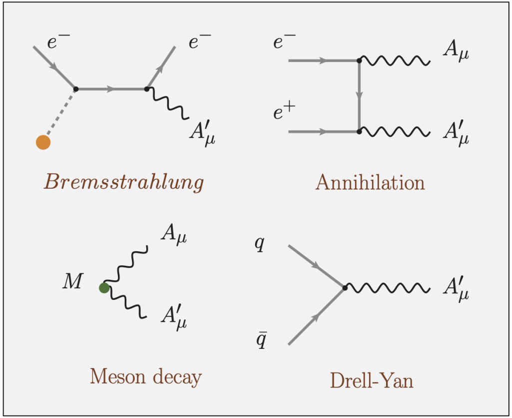

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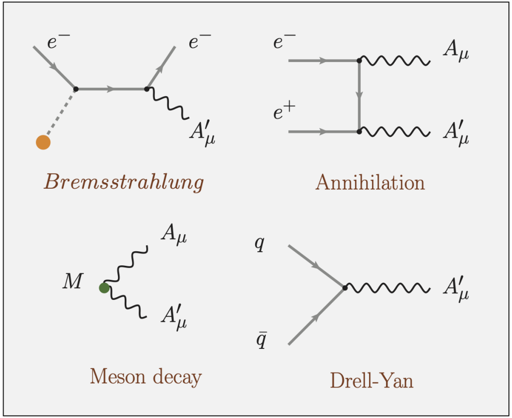

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- Drell-Yan
  $\bar{q} q \to A' \to \bar{f} f (\bar{\chi} \chi)$

Figure: DP production. From https://arxiv.org/pdf/2005.01515.pdf.

Accelerator searches for dark photon

◇ Fabbrichesi, Gabrielli, Lanfranchi, https://arxiv.org/pdf/2005.01515.pdf. (DP review)

DP above MeV (visible)

Two kinds experiments:

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Two kinds experiments:

- colliders

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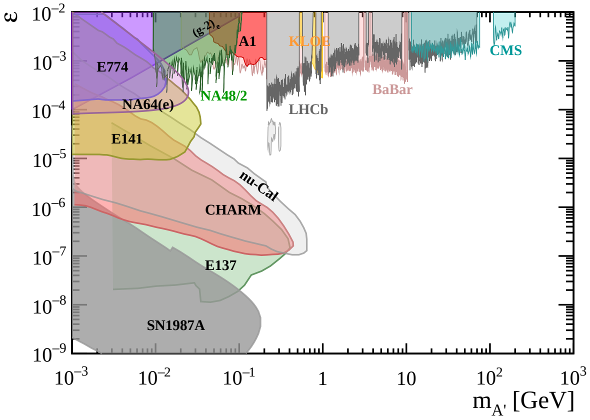
This can be easily understood by looking at the decay distance

$$L = \gamma v \tau_{A'} = \gamma v / \Gamma_{A'} \propto \gamma v \frac{1}{m_{A'} \epsilon^2} \tag{97}$$

where in the last step we have assumed visible decays only.

Existing limits on the massive DP for $m_{A'} > 1$ MeV (visible)



di-lepton searches at experiments at

Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Existing limits on the massive DP for $m_{A'} > 1$ MeV (visible)

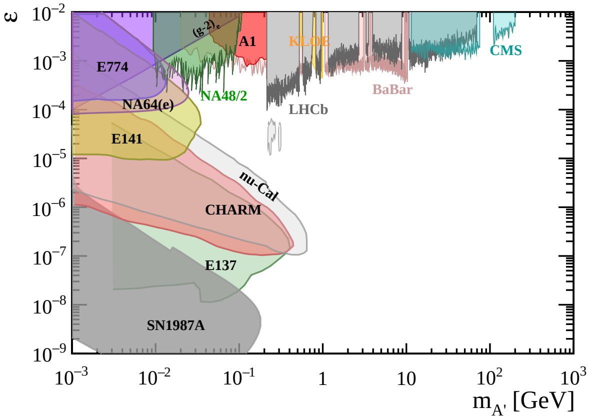
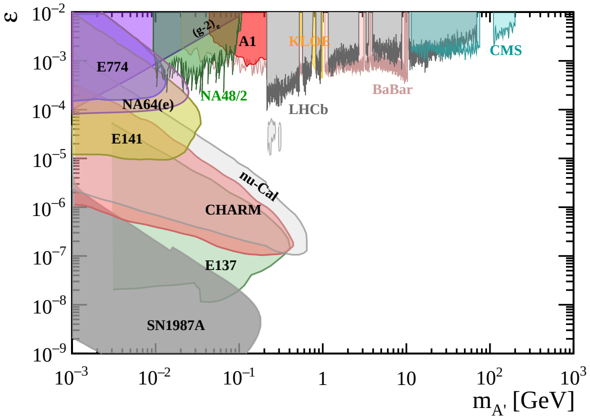


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

di-lepton searches at experiments at

- collider/fixed target: A1, LHCb, CMS, BaBar, KLOE, and NA48/2

Existing limits on the massive DP for $m_{A'} > 1$ MeV (visible)



di-lepton searches at experiments at

- collider/fixed target: A1, LHCb, CMS, BaBar, KLOE, and NA48/2
- old beam dump: E774, E141, E137, ν-Cal, and CHARM.

Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Existing limits on the massive DP for $m_{A'} > 1$ MeV (visible)

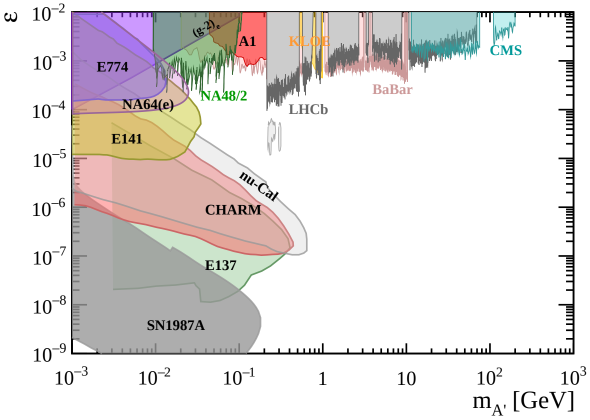


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

di-lepton searches at experiments at

- collider/fixed target: A1, LHCb, CMS, BaBar, KLOE, and NA48/2
- old beam dump: E774, E141, E137, ν-Cal, and CHARM.

Other limits:

Existing limits on the massive DP for $m_{A'} > 1$ MeV (visible)

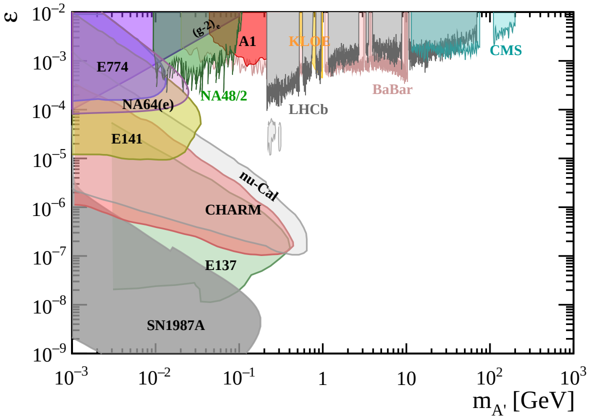


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

di-lepton searches at experiments at

- collider/fixed target: A1, LHCb, CMS, BaBar, KLOE, and NA48/2
- old beam dump: E774, E141, E137, ν-Cal, and CHARM.

Other limits:

- supernovae

Existing limits on the massive DP for $m_{A'} > 1$ MeV (visible)

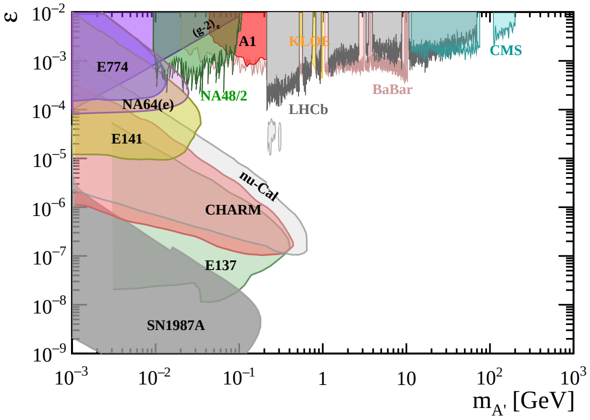


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

di-lepton searches at experiments at

- collider/fixed target: A1, LHCb, CMS, BaBar, KLOE, and NA48/2
- old beam dump: E774, E141, E137, ν-Cal, and CHARM.

Other limits:

- supernovae
- $(g-2)_e$

Projections on the massive DP for $m_{A'} > 1$ MeV (visible)

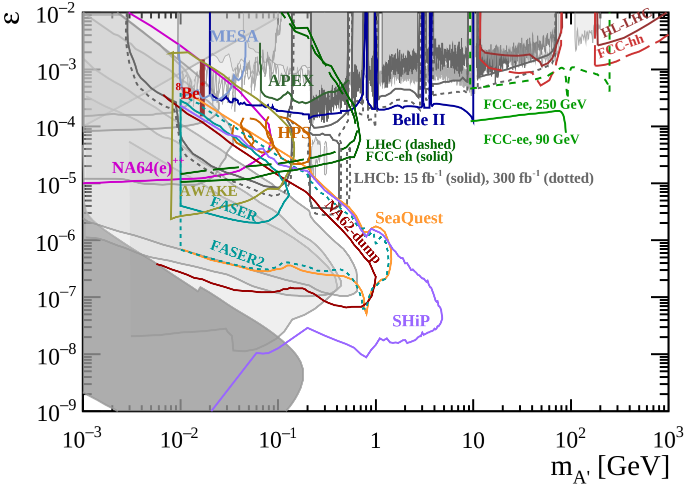
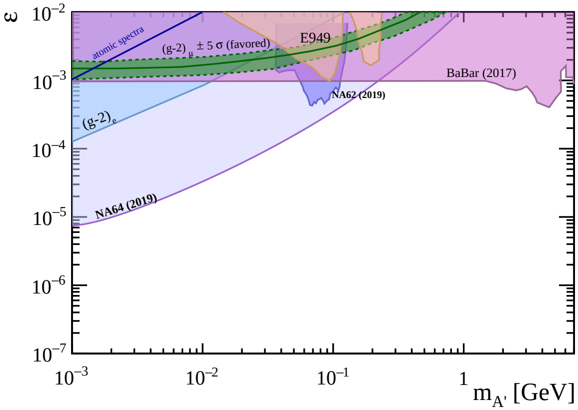


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Existing limits on the massive DP for $m_{A'} > 1$ MeV (invisible)



Invisible limits:

Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Existing limits on the massive DP for $m_{A'} > 1$ MeV (invisible)

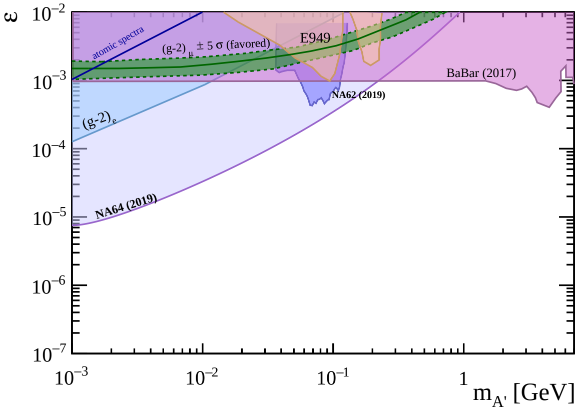


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Invisible limits:

- Kaon decay experiments (E787, E949, NA62)

Existing limits on the massive DP for $m_{A'} > 1$ MeV (invisible)

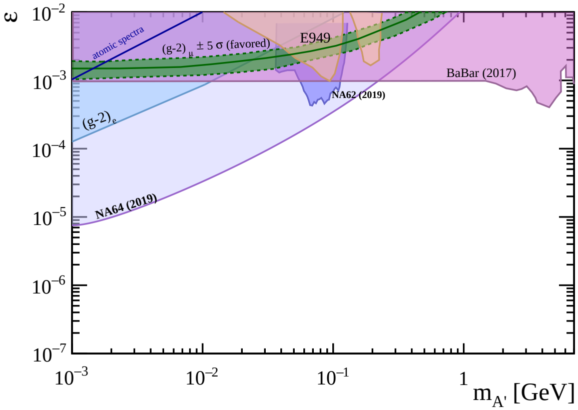


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Invisible limits:

- Kaon decay experiments (E787, E949, NA62)
- BaBar

Existing limits on the massive DP for $m_{A'} > 1$ MeV (invisible)

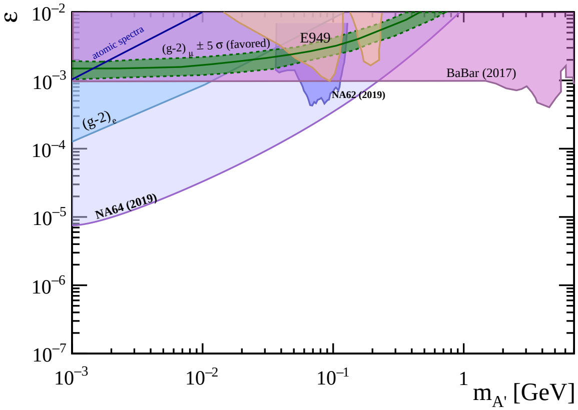


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Invisible limits:

- Kaon decay experiments (E787, E949, NA62)
- BaBar
- NA64(e)

Existing limits on the massive DP for $m_{A'} > 1$ MeV (invisible)

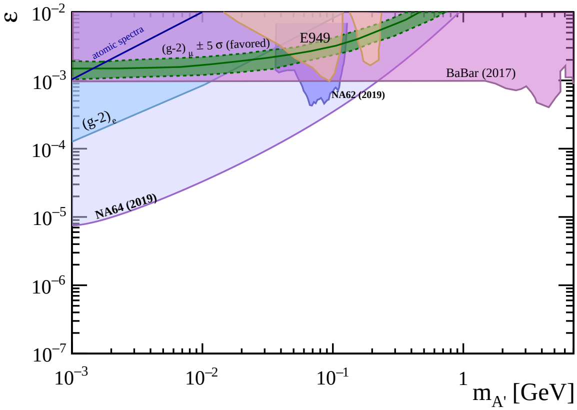


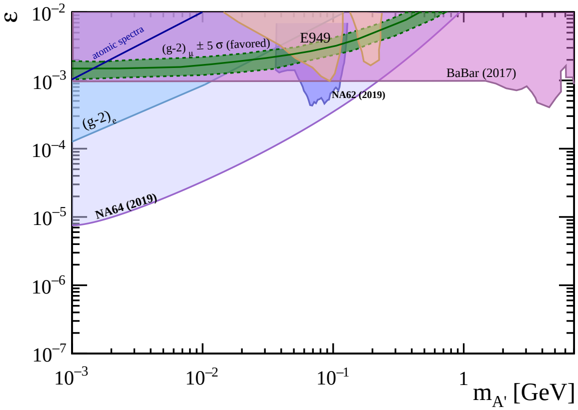
Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Invisible limits:

- Kaon decay experiments (E787, E949, NA62)
- BaBar
- NA64(e)

Others:

Existing limits on the massive DP for $m_{A'} > 1$ MeV (invisible)



Invisible limits:

- Kaon decay experiments (E787, E949, NA62)
- BaBar
- NA64(e)

Others:

- $(g-2)_\mu$

Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Existing limits on the massive DP for $m_{A'} > 1$ MeV (invisible)

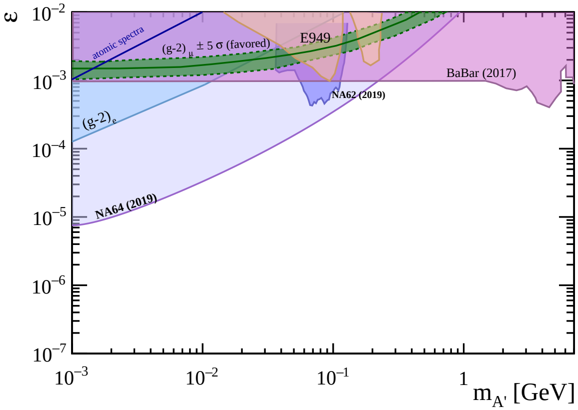


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Invisible limits:

- Kaon decay experiments (E787, E949, NA62)
- BaBar
- NA64(e)

Others:

- $(g-2)_\mu$
- $(g-2)_e$

Projections on the massive DP for $m_{A'} > 1$ MeV (invisible)

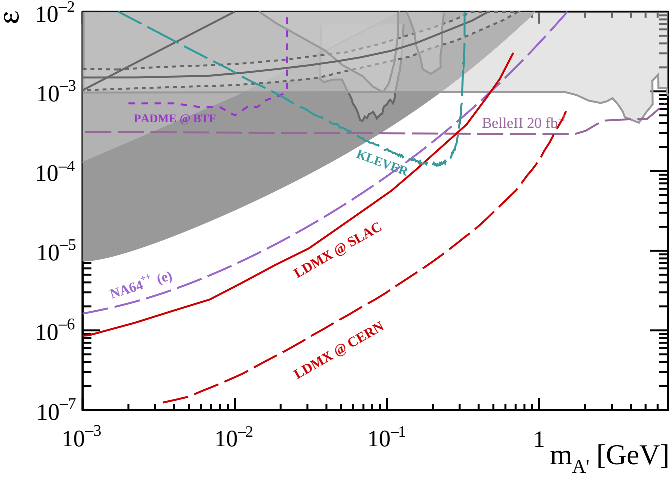


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Astro/cosmo probes to dark photon

⋄ Fabbrichesi, Gabrielli, Lanfranchi, https://arxiv.org/pdf/2005.01515.pdf. (DP review)

Current limits on the massive DP for $m_{A'} < 1$ MeV

Bounds:

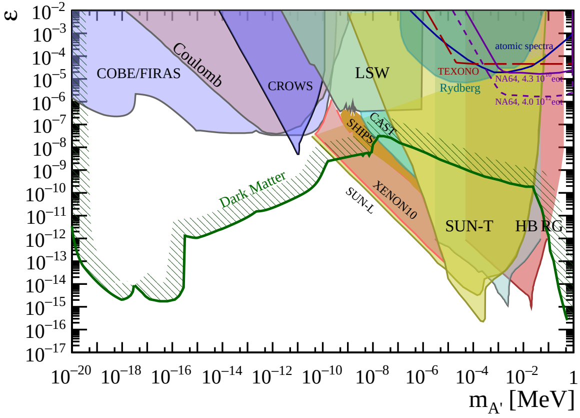


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Current limits on the massive DP for $m_{A'} < 1$ MeV

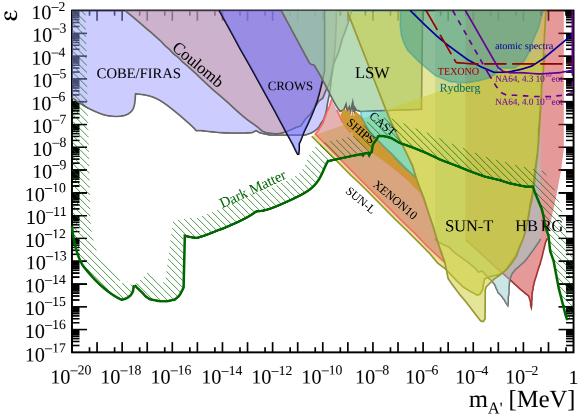


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Bounds:

- CMB: COBE/FIRES

Current limits on the massive DP for $m_{A'} < 1$ MeV

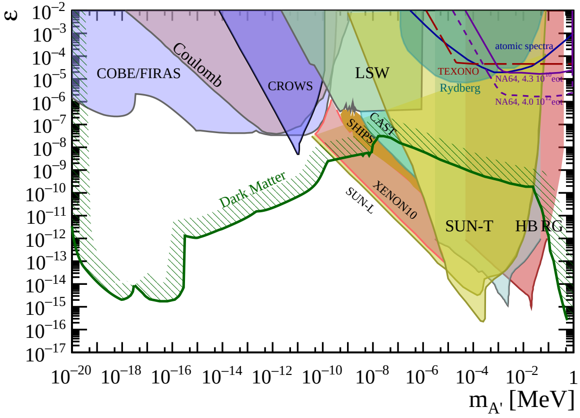


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Bounds:

- CMB: COBE/FIRES
- Coulomb

Current limits on the massive DP for $m_{A'} < 1$ MeV

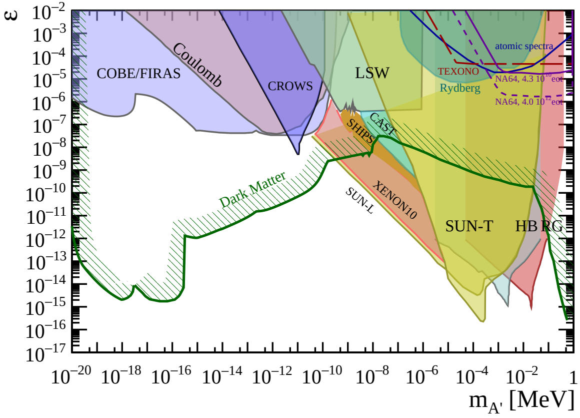


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Bounds:

- CMB: COBE/FIRES
- Coulomb
- Light through a wall (LSW)

Current limits on the massive DP for $m_{A'} < 1$ MeV

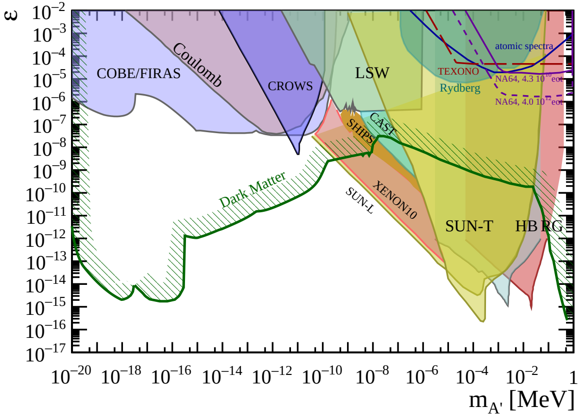


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Bounds:

- CMB: COBE/FIRES
- Coulomb
- Light through a wall (LSW)
- CROWS

Current limits on the massive DP for $m_{A'} < 1$ MeV

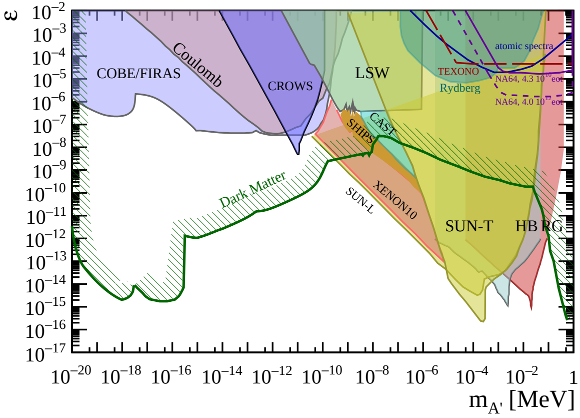


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Bounds:

- CMB: COBE/FIRES
- Coulomb
- Light through a wall (LSW)
- CROWS
- DP from the Sun: CAST, XENON10, SHIPS

Current limits on the massive DP for $m_{A'} < 1$ MeV

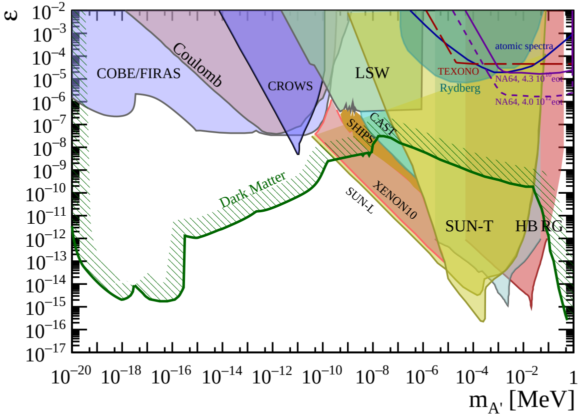


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Bounds:

- CMB: COBE/FIRES
- Coulomb
- Light through a wall (LSW)
- CROWS
- DP from the Sun: CAST, XENON10, SHIPS
- Rydberg

Current limits on the massive DP for $m_{A'} < 1$ MeV

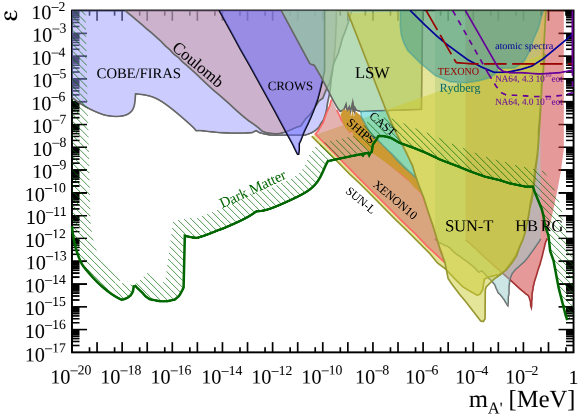


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Bounds:

- CMB: COBE/FIRES
- Coulomb
- Light through a wall (LSW)
- CROWS
- DP from the Sun: CAST, XENON10, SHIPS
- Rydberg
- Nuclear reactor: TEXONO

Current limits on the massive DP for $m_{A'} < 1$ MeV

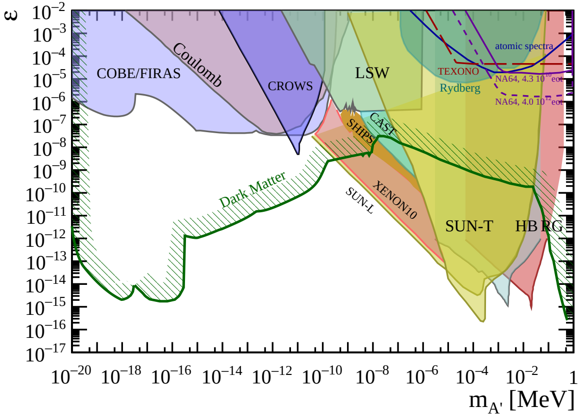


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Bounds:

- CMB: COBE/FIRES
- Coulomb
- Light through a wall (LSW)
- CROWS
- DP from the Sun: CAST, XENON10, SHIPS
- Rydberg
- Nuclear reactor: TEXONO
- Stellar: solar lifetime (SUN-T and SUN-L), red giants (RG), horizontal branches (HB)

Current limits on the massive DP for $m_{A'} < 1$ MeV

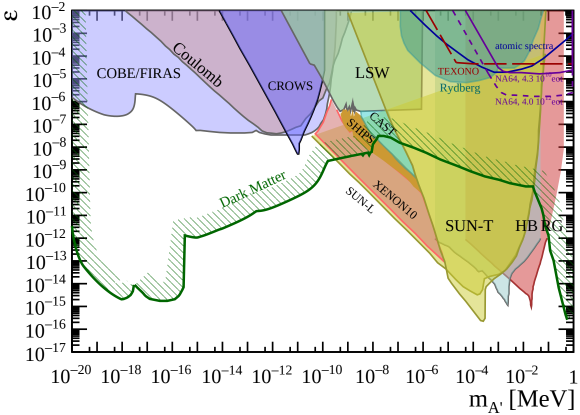


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Bounds:

- CMB: COBE/FIRES
- Coulomb
- Light through a wall (LSW)
- CROWS
- DP from the Sun: CAST, XENON10, SHIPS
- Rydberg
- Nuclear reactor: TEXONO
- Stellar: solar lifetime (SUN-T and SUN-L), red giants (RG), horizontal branches (HB)
- Supernova (another stellar): above MeV

Current limits on the massive DP for $m_{A'} < 1$ MeV

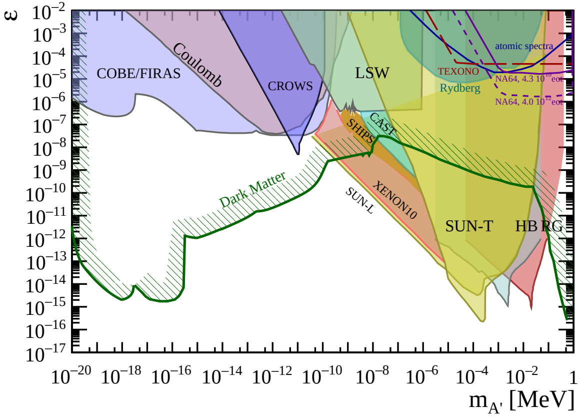


Figure: From https://arxiv.org/pdf/2005.01515.pdf.

Bounds:

- CMB: COBE/FIRES
- Coulomb
- Light through a wall (LSW)
- CROWS
- DP from the Sun: CAST, XENON10, SHIPS
- Rydberg
- Nuclear reactor: TEXONO
- Stellar: solar lifetime (SUN-T and SUN-L), red giants (RG), horizontal branches (HB)
- Supernova (another stellar): above MeV
- DPDM