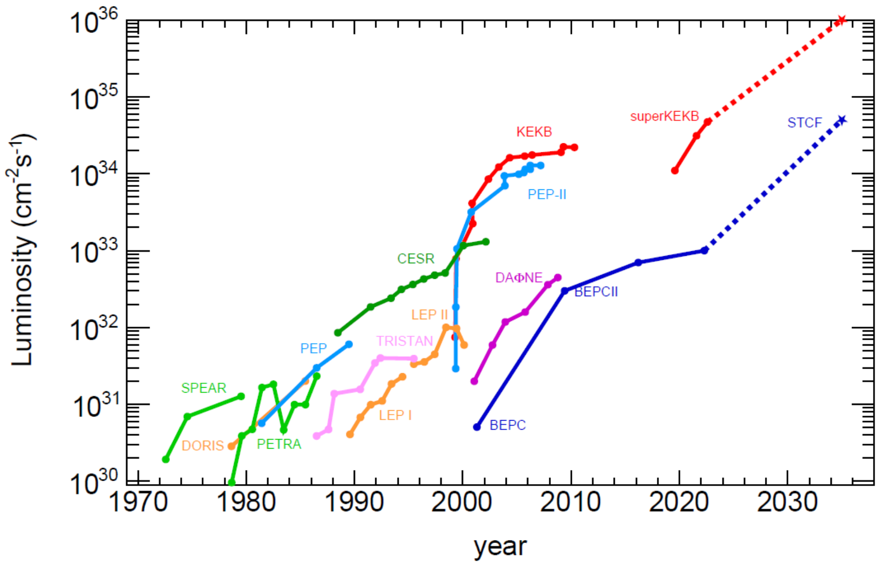
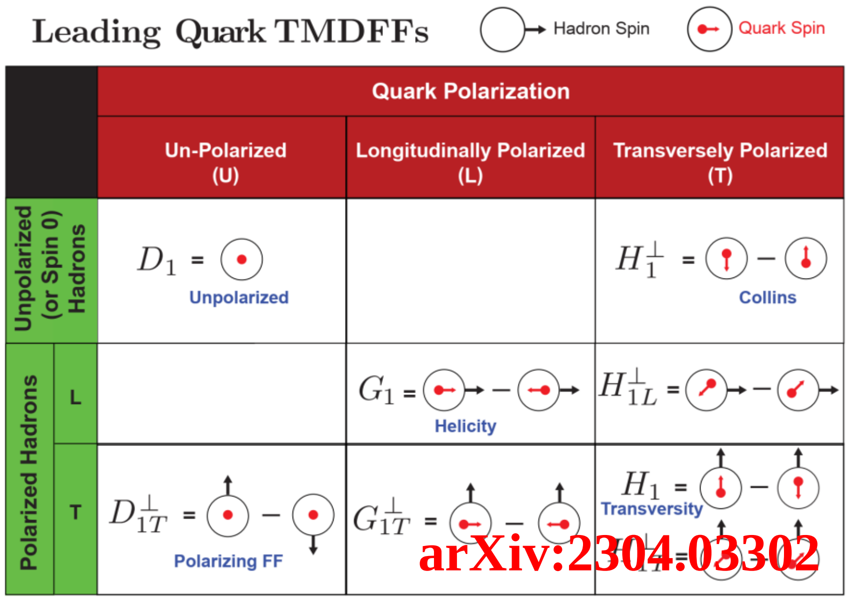
Fragmentation function at low energy e^+e^-

鄢文标(中国科学技术大学)

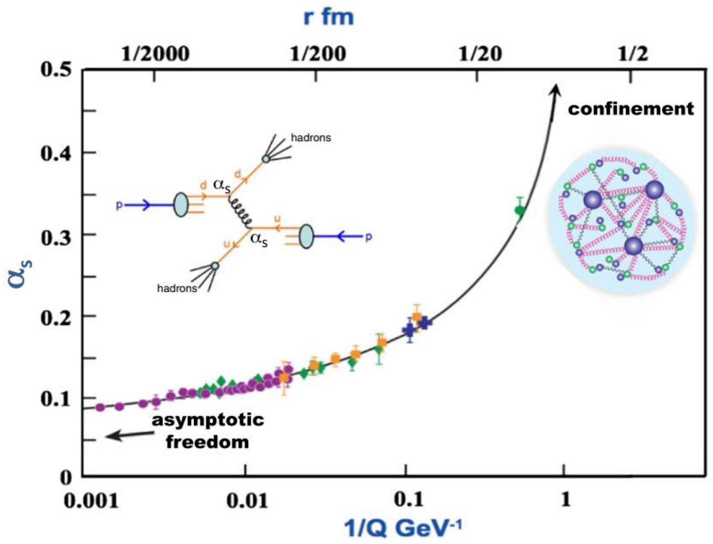




arXiv:2304.03302

第二届核子三维结构研讨会暨第二届高扭度核子结构研讨会，2024.10.18，青岛

QCD: Asymptotic freedom & Confinement



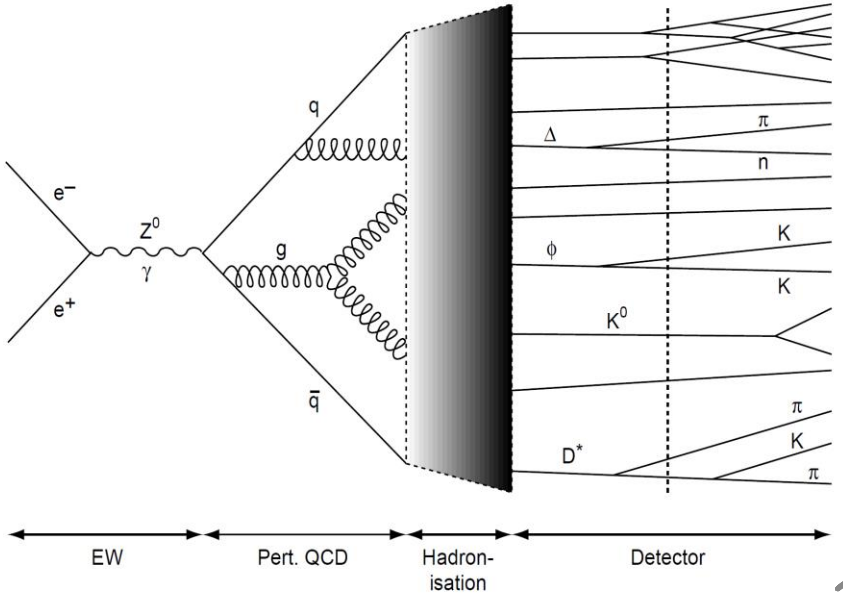
Observation of Fractional Charge of (1/3)*e* on Matter

George S. LaRue, James D. Phillips, and William M. Fairbank
Stanford University, Stanford, California 94035
(Received 24 November 1980)

PRL 46 967 (1981)

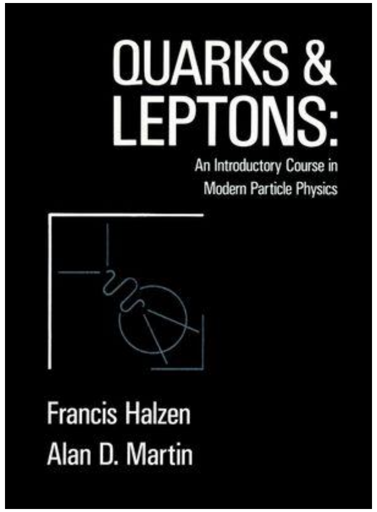
Measurements on niobium spheres which show unambiguously the existence of fractional charges of $\frac{1}{3}e$ are reported. Charge changes of $\frac{1}{3}e$ on particular spheres when they contact other surfaces are continually observed. Of 21 new measurements, four charges of $+\frac{1}{3}e$, four of $-\frac{1}{3}e$, and thirteen of zero are found. Extensive measurements and critical analyses have assured us that the background forces are either negligible or have been measured and taken fully into account.

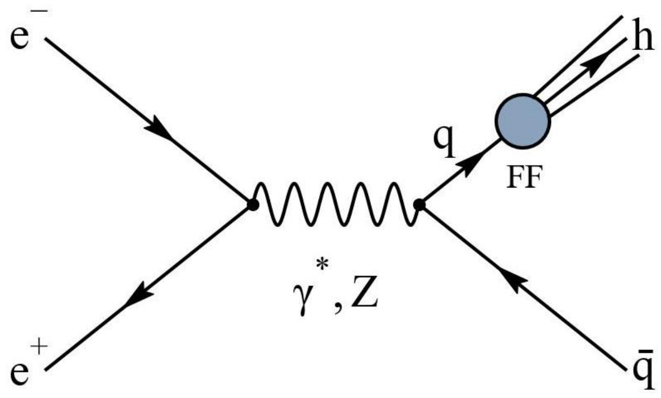
- **QCD coupling constant $\alpha_s(Q)$**
  - ✓ **High Q: asymptotic freedom, perturbative QCD**
  - ✓ **Low Q: non-perturbative QCD**
- **Confinement: partons do not exist as free particles, but are always confined in hadron**
- **Essence of confinement ? Why & how ?**
  - ✓ **Hadronization models & Fragmentation function**
  - ✓ **LPHD: Local Parton Hadron Duality hypothesis**

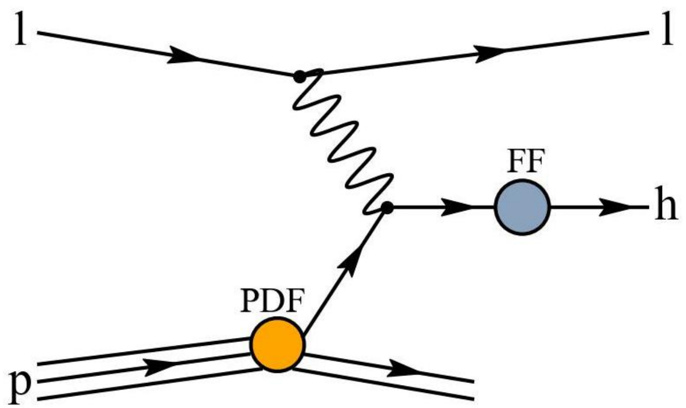


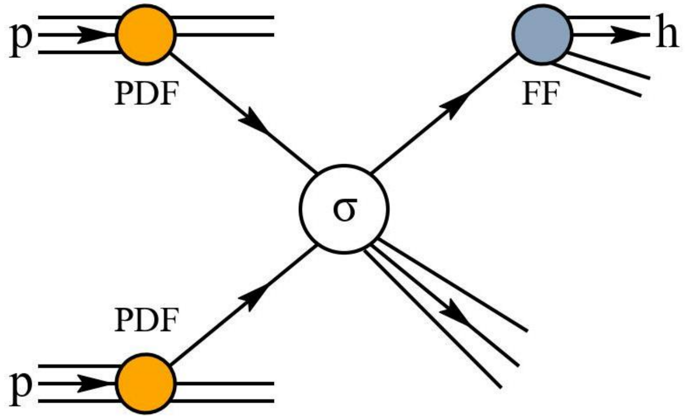
Fragmentation function: integrated $D_1^h(z)$

- Fragmentation function $D_q^h(z)$: probability that hadron h is found in the debris of a parton carrying a fraction z of parton's energy
- Consequence of confinement
- FF: QCD first principle (NOT YET)
  - ✓ FF evolution function: DGLAP
  - ✓ Fitting: parametrization & experimental data
  - ✓ Universality: e^+e^-, DIS, pp, $p\bar{p}$ data
- FFs contribute to virtually all processes









FFs with quark/hadron polarization

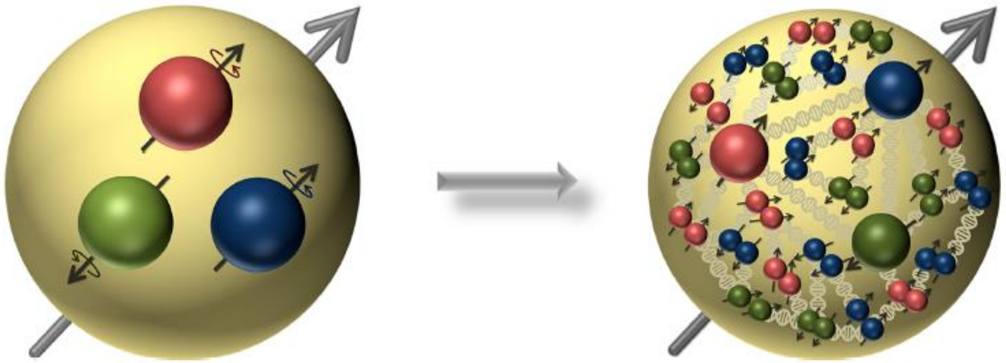
Hadron polarization	Quark polarizatiom @ PPNP 91 136 (2016)		
	Unpolarized	Longitudinally	Transversely
Unpolarized	D_1^h		$H_1^{\perp h}$
Longitudinally		G_1^h	$H_{1L}^{\perp h}$
Transversely	$D_{1T}^{\perp h}$	G_{1T}^h	H_1^h $H_{1T}^{\perp h}$

PLB396 (1993) 161

$$D_{hq^\uparrow}(z,P_{h\perp}) = D_1^q(z,P_{h\perp}^2) + H_1^{\perp q}(z,P_{h\perp}^2)\frac{(\hat{\mathbf{k}}\times\mathbf{P}_{h\perp})\cdot\mathbf{S}_q}{zM_h}$$

- Theoretically many more, in particular with polarized hadrons in the final state and transverse momentum dependence (TMD)

FFs for EIC & EicC

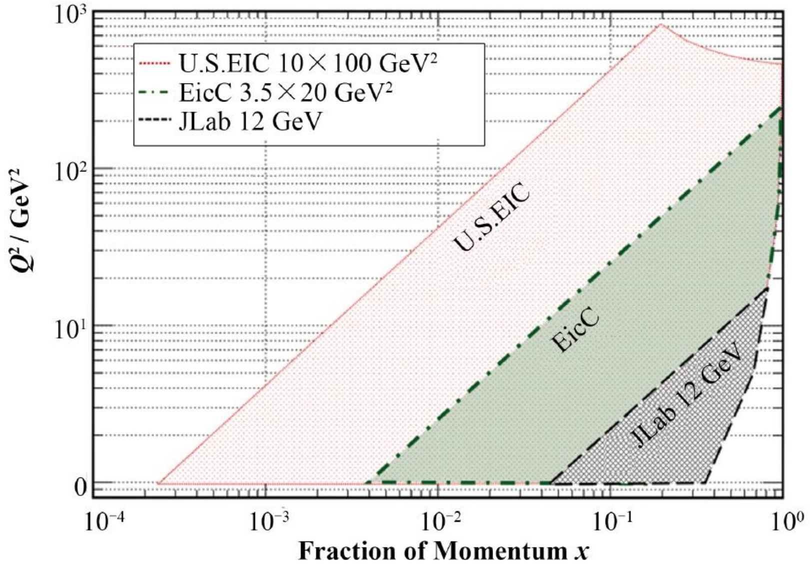


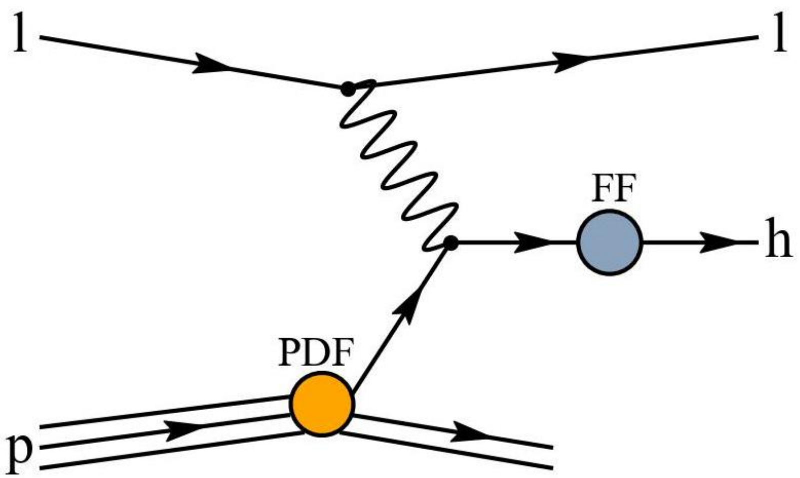
Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

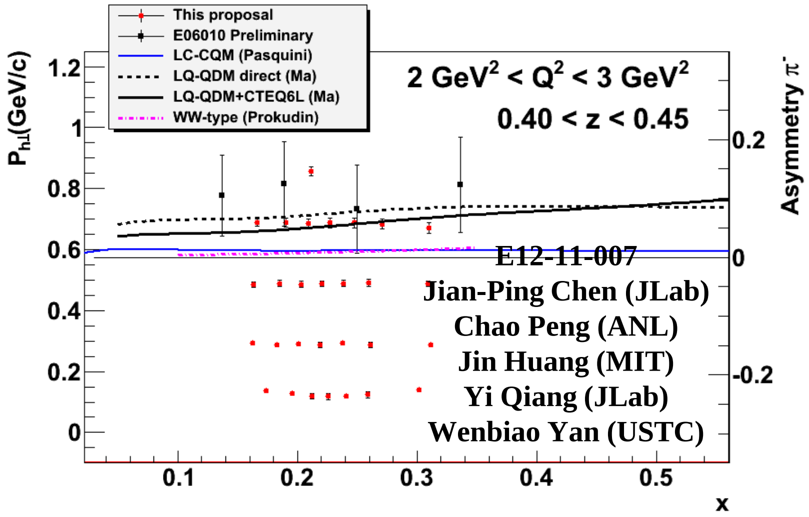
arXiv:2304.03302

TMD Handbook

Renaud Boussarie[1], Matthias Burkardt[2], Martha Constantinou[3], William Detmold[4], Markus Ebert[4,5], Michael Engelhardt[2], Sean Fleming[6], Leonard Gamberg[7], Xiangdong Ji[8], Zhong-Bo Kang[9], Christopher Lee[10], Keh-Fei Liu[11], Simonetta Liuti[12], Thomas Mehen[13], Andreas Metz[3], John Negele[4], Daniel Pitonyak[14], Alexei Prokudin[7,16], Jian-Wei Qiu[16,17], Abha Rajan[12,18], Marc Schlegel[2,19], Phiala Shanahan[4], Peter Schweitzer[20], Iain W. Stewart[4], Andrey Tarasov[21,22], Raju Venugopalan[18], Ivan Vitev[10], Feng Yuan[23], Yong Zhao[24,4,18]

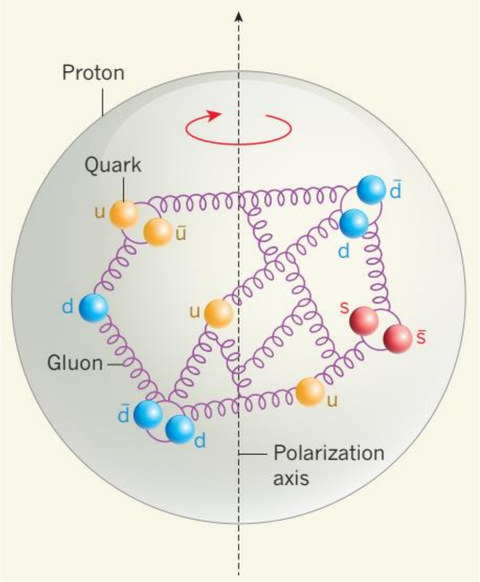


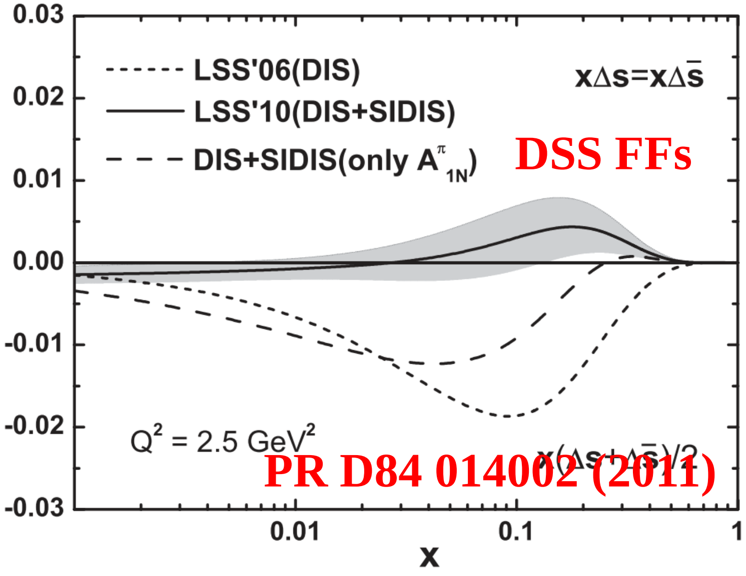


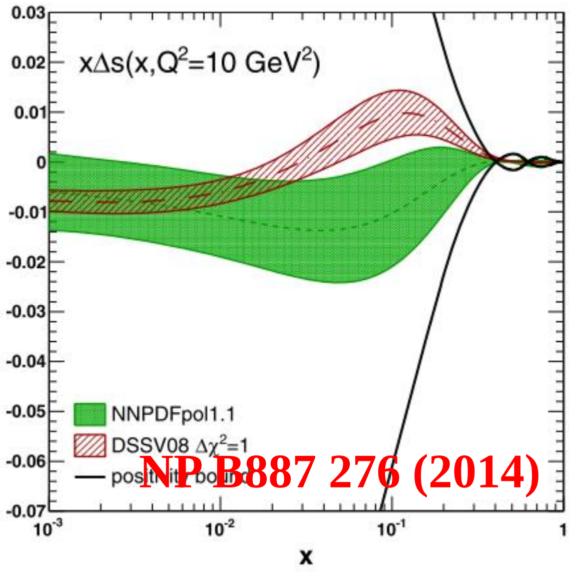


Precise knowledge of FFs will be crucial

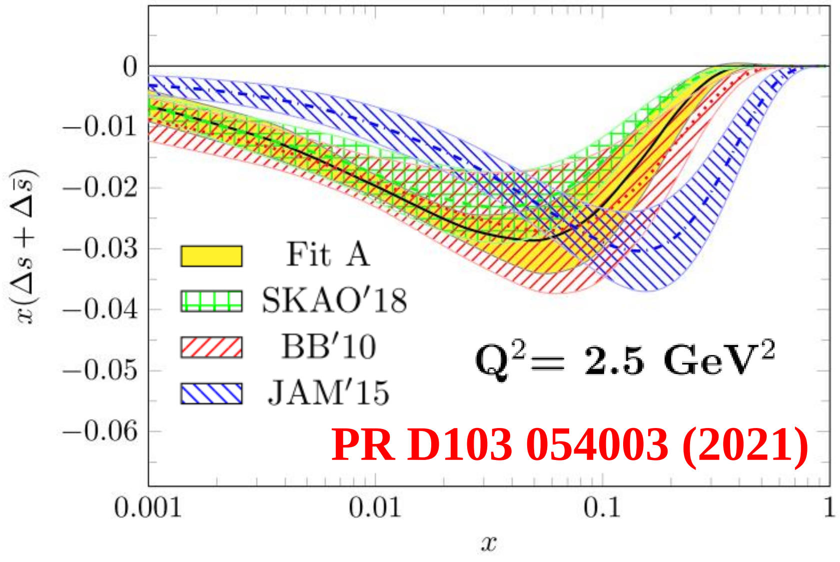
Strange quark polarization puzzle







- **Strange quark density function: $\Delta s(x)+\Delta\bar{s}(x)$**
  - ✓ **Inclusive DIS: only proton PDF**
    - a. **negative for all values of x**
  - ✓ **Semi-inclusive DIS: proton PDF & kaon FF**
    - a. **DSS FFs: positive for most of measured x**
    - b. **NNPDF FF & JAM FF : negative**
- **Reliable FFs knowledge ? Need more efforts**



Global data fit on unpolarized FF

R.D. Field, R.P. Feynman, Phys.Rev.D 15, 2590 1977
J.F. Owens, E. Reya. M. Gluck, Phys.Rev.D 18, 1501 1978
R. Baier, J. Engels and B. Petersson, Z.Phys.C 2, 265 1979
M. Anselmino, P. Kroll E. leader, Z.Phys.C 18,307 1983
…

"model estimates consistent with data"

LO groundbreaking

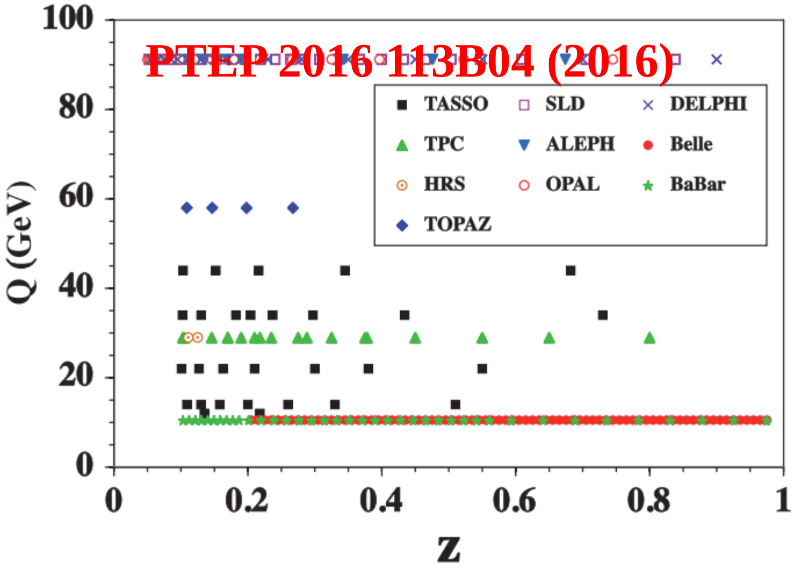
P. Chiappetta et al. , Nucl.Phys.B 412, 3 1994	π^0	CGGRW94
J. Binneweis. B. Kniehl, G. Kramer, Z.Phys.C 65, 471 1995	$\pi^{\pm}, K^{\pm}$	BKK95
J. Binneweis. B. Kniehl, G. Kramer, Phys.Rev.D 52, 4947 1995	$\pi^{\pm}, K^{\pm}$ LEP	
J. Binneweis. B. Kniehl, G. Kramer, Phys.Rev.D 53, 3573 1996	K^0	
D. de Florian. M. Stratmann, W. Vogelsang, Phys.Rev.D 57, 5811 1998	Λ	DSV97
L. Bourhis et al. Eur.Phys.J.C 19, 89 2001	$h^{\pm}$	BFGW00
B. Kniehl G. Kramer, B. Potter, Nucl.Phys.B 582, 514 2000	$\pi^{\pm}, K^{\pm}, p/\bar{p}$	KKP00
S. Kretzer, Phys.Rev.D 62, 054001 2000	Flavor tagging	KRE00
S. Albino, B. Kniehl, G. Kramer, Nucl.Phys.B 725 2005	OPAL tagging	AKK95
M. Hirai et al., Phys.Rev.D 75, 094009 2007	uncertainties	HKNS07
… heavy flavors, hadron mass effects, resummations, …		

NLO e^+e^- paradigm

D. de Florian, R.S., M. Stratmann , Phys.Rev.D 75, 114010 2007	$e^+e^-, pp, SIDIS$	DSS07
S. Albino, B. Kniehl, G. Kramer, Nucl.Phys.B 803, 42 2008	e^+e^-, pp	AKK08
R.S., M. Stratmann, P. Zurita, Phys.Rev.D 81, 054001 2010	nFFs	SSZ10
C. Aidala, et al. Phys.Rev.D 83, 034002 2011	η	AESS11
E. Leader, A.V. Sidorov, D. Stamenov, arXiv:1312.5200	$SIDIS$ only	LSS13
M. Soleymaninia et al., Phys.Rev.D 88, 054019 2013	$e^+e^-, pSIDIS$	SKMNA13
D. de Florian et al. , Phys.Rev.D 91, 4035 2015, D 95 094019 2017	$\pi^{\pm}, K^{\pm}$ update	DSS14/17
E. Leader, A.V. Sidorov, D. Stamenov, Phys.Rev.D 94, 096001 2016	$SIDIS$ only	LSS15
V. Bertone, et al., EPJC 77,516 2017	$h^{\pm}, e^+e^-$ only	NNFF1.0
N. Sato, et al., Phys.Rev.D 101, 074020 2020	$e^+e^-, SIDIS$	JAM19
R. A. Khalek, et al., Phys.Lett.B 834, 137456 2022	$e^+e^-, SIDIS$	MAPFF1.0

Global paradigm

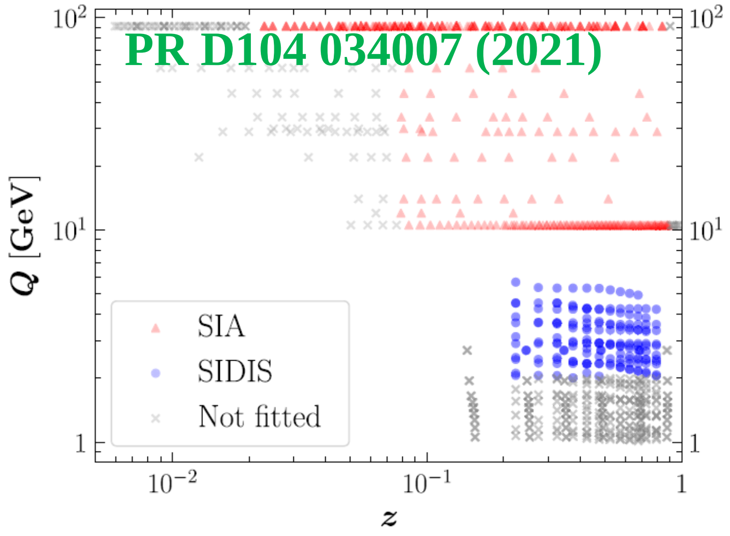
Used data set @ FFs fitting



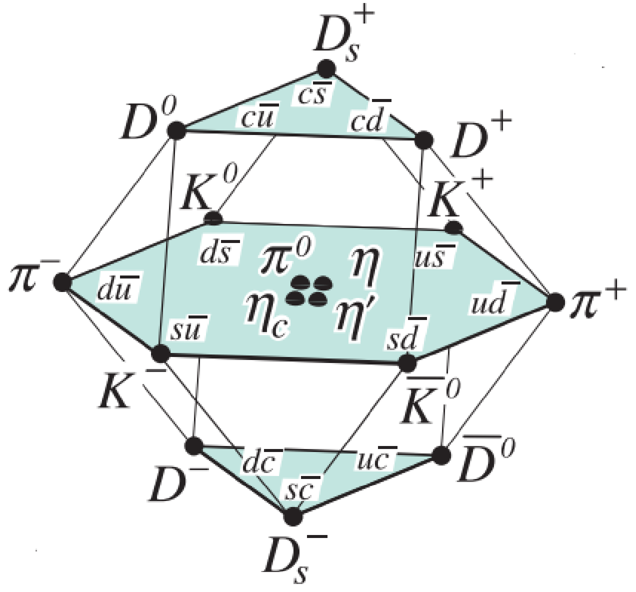
PPNP 91 136 (2016)

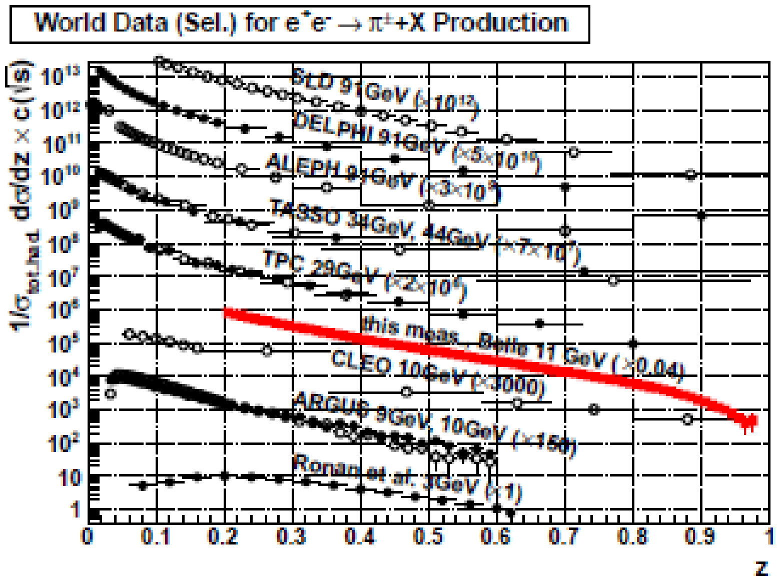
Experiment	Process	$\mathcal{L}[pb^{-1}]$	$Q^2[GeV^2]$	Final states
TPC [288–291]	e^+e^-	140	2[illegible]	$\pi^\pm, K^\pm, p/\bar{p}$
TASSO				
[292–294]	e^+e^-	34	34,44	$\pi^\pm, K^\pm, p/\bar{p}, K_S^0, \Lambda/\bar{\Lambda}$
[295–298]				
SLD [299,300]	e^+e^-	20	M_Z	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
ALEPH [301,302]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
DELPHI [303–306]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
OPAL [307–310]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
H1 [311–313]	$e+p$	500	$27.5(e)+920(p)$	$h^\pm, K_0^S$
ZEUS [314–316]	$e+p$	500	$27.5(e)+920(p)$	$h^\pm$
BELLE [317,318]	e^+e^-	10^6	Near 10.58	$\pi^\pm, K^\pm, p/\bar{p}$
BaBar [319,320]	e^+e^-	$557 \cdot 10^3$	Near 10.58	$\pi^\pm, K^\pm, \eta, p/\bar{p}$
HERMES [321,322]	$e+p(d)$	272 (p) 329 (d)	27.6 fixed target	$\pi^{\pm,0}, K^\pm$
COMPASS [323]	$\mu+p(d)$	775	160 GeV fixed target	$h^\pm$
PHENIX		16×10^{-3}	62.4	
[324–326]	pp	2.5	200	$\pi^{\pm,0}, \eta$
[327–329]		128	510	
STAR				
[330–332]	pp	8	200	$\pi^{0,\pm}, \eta, p/\bar{p}, K_S^0, \Lambda/\bar{\Lambda}$
[333–335]				
ALICE [336]	pp	6×10^{-3}	7×10^3	π^0, η
TOPAZ [337]	e^+e^-	278	52–61.4	$\pi^\pm, K^\pm, K_S^0,$
CDF [338,339]	$p+\bar{p}$	n/a	630 (1800)	$h^\pm, K_S^0 \Lambda^0$

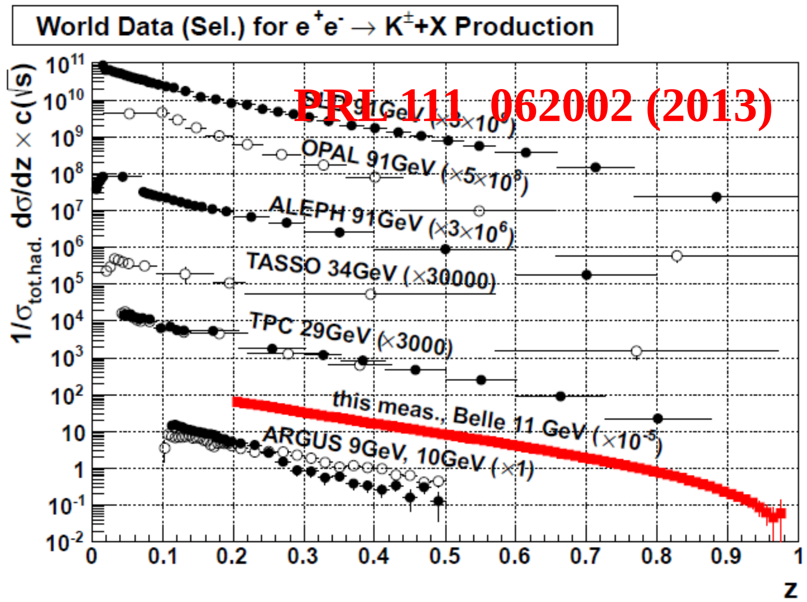
- **Updated HKNS FFs @ 2016**
  - ✓ only e^+e^- ($\sqrt{s} > 10$ GeV) data sets
- **MAPFF1.0 FFs @ 2021**
  - ✓ e^+e^- ($\sqrt{s} > 10$ GeV) and SIDIS data sets
- **Data set at $\sqrt{s} < 10$ GeV e^+e^- collision ?**



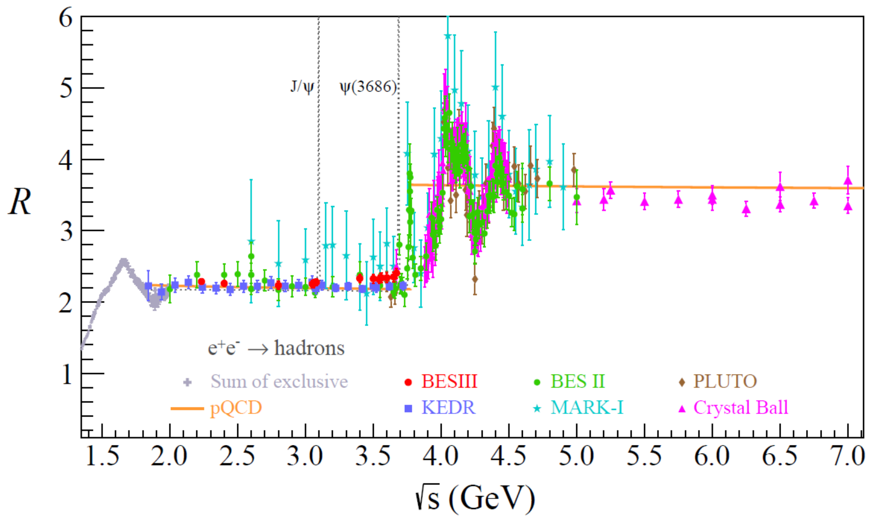
World π & K data on e⁺e⁻



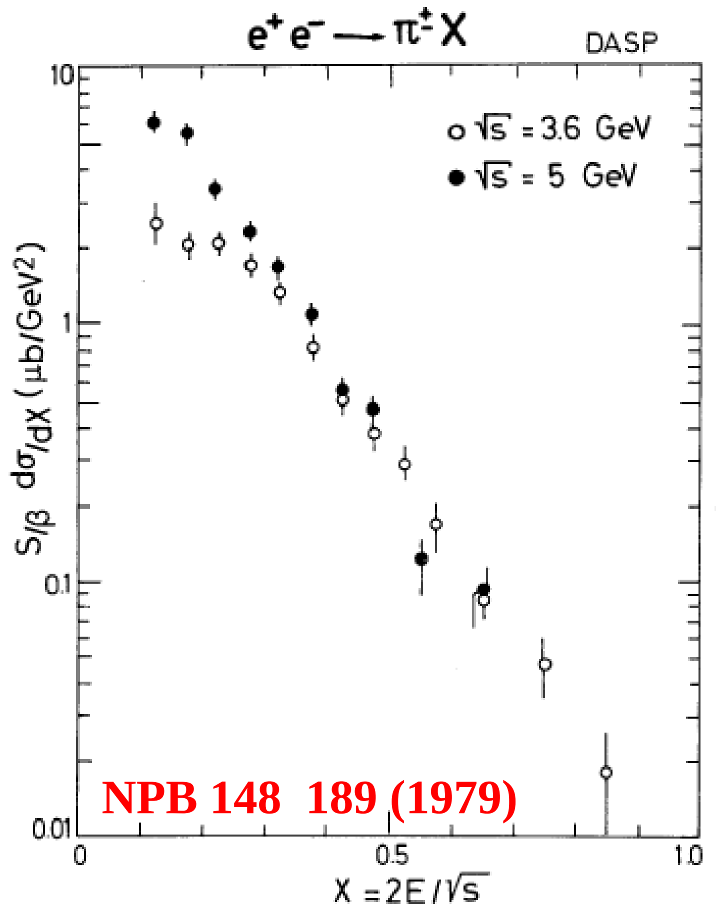




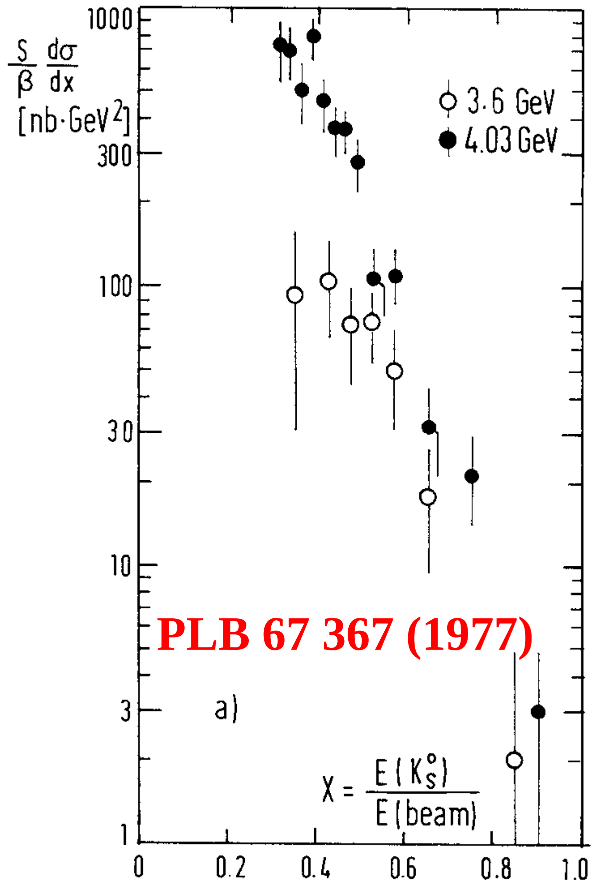
- **Precision data includes charged π, K**
- **Data sets at $\sqrt{s}$ < 10 GeV e⁺e⁻ collision ?**
  - ✓ **high z data sets ?**
- **R scan data @ BESIII: ~10 pb⁻¹ @ each $\sqrt{s}$**
- **Scan data @ STCF: precise measurement**



World π & K data on e^+e^-

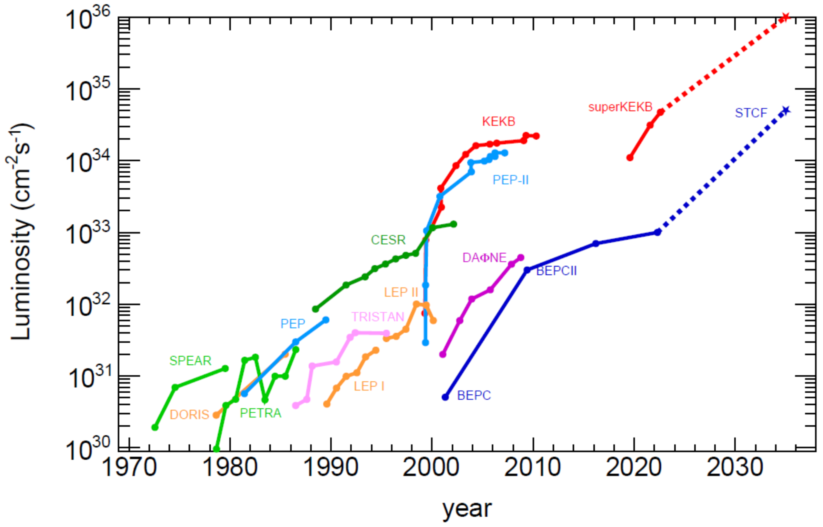


NPB 148 189 (1979)



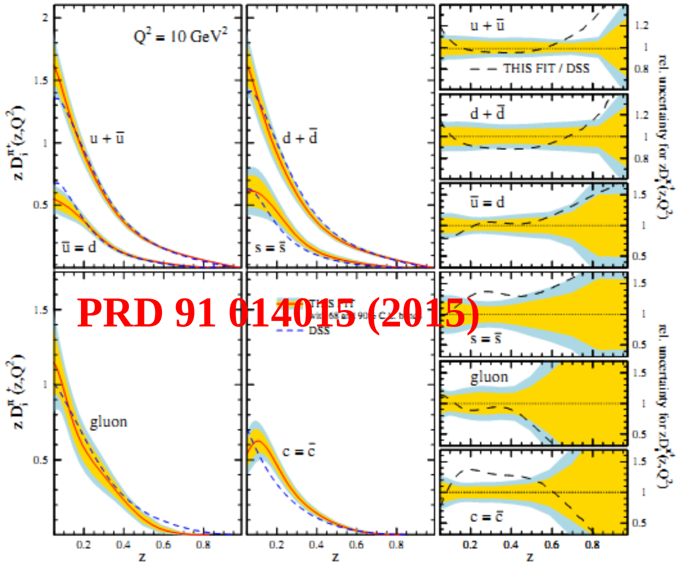
PLB 67 367 (1977)

- **Charged π @ DASP**
  - ✓ **about 45 years ago**
  - ✓ **stat. uncertainty: 18%**
- **K_S^0 @ PLUTO**
  - ✓ **about 47 years ago**
  - ✓ **stat. uncertainty: 18-41%**
- **Precision data ? TMD FFs ?**

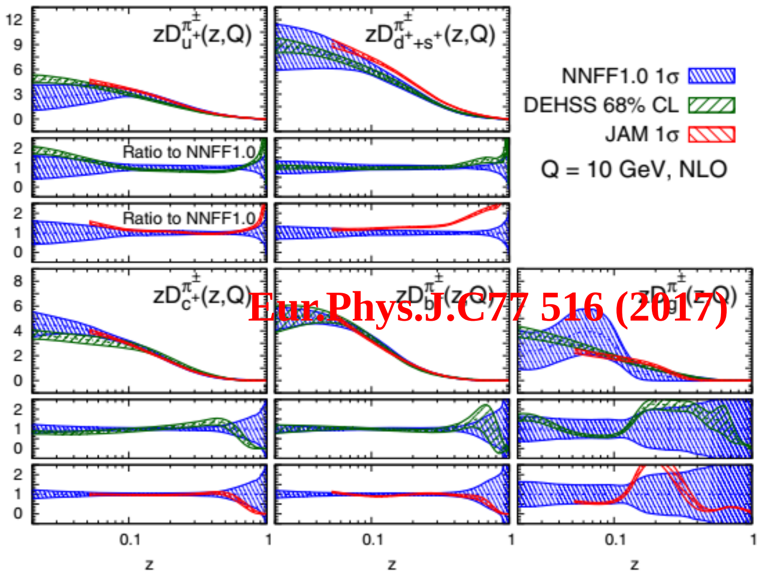


BESIII R scan data ⇒ Precision measurement ?

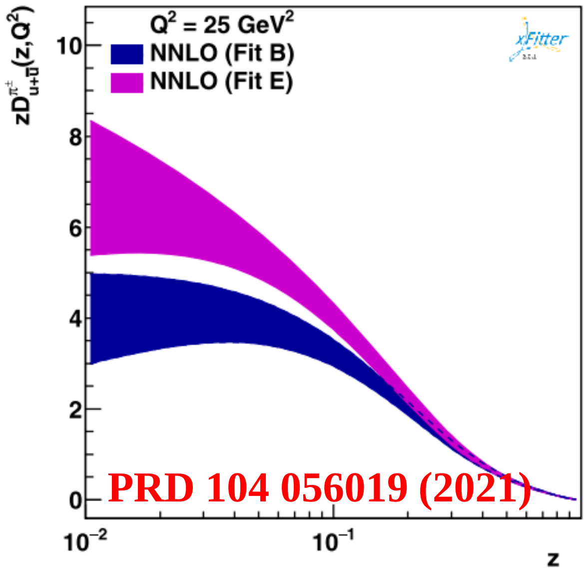
Pion FF: Best known FF



PRD 91 014015 (2015)

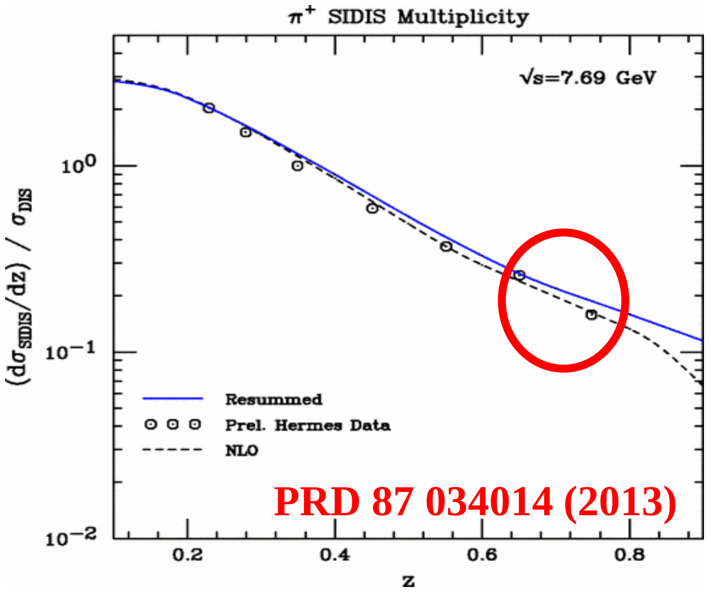


Eur.Phys.J.C77 516 (2017)



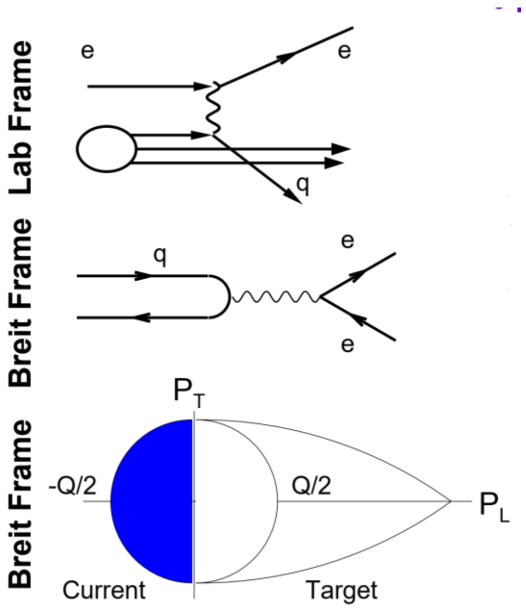
PRD 104 056019 (2021)

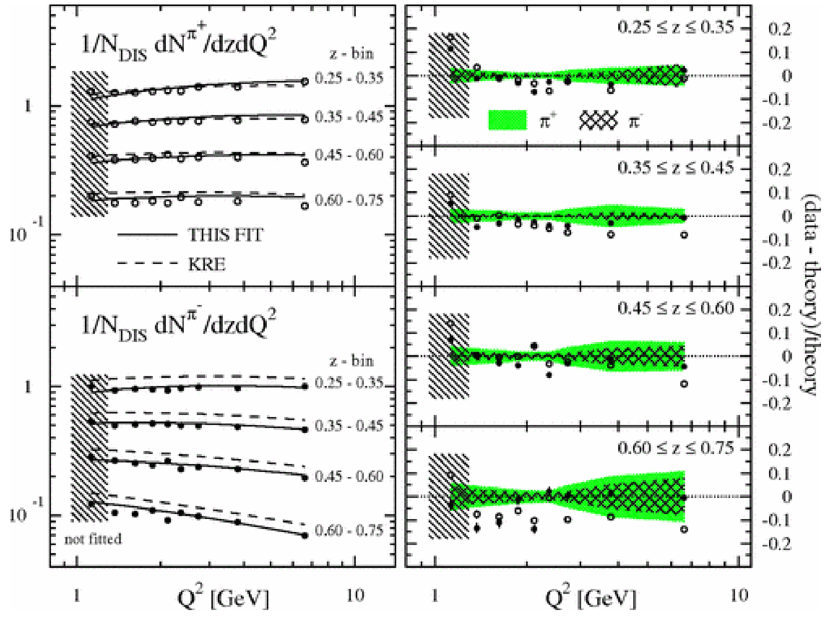
- For z≥0.8, uncertainty rapidly increase because of the lack of experimental data
- Xfitter: experimental data at $\sqrt{s} > 10$ GeV e^+e^-
  - ✓ Low $\sqrt{s}$ e^+e^- data ?
- Large z re-summation
  - ✓ High z data ?

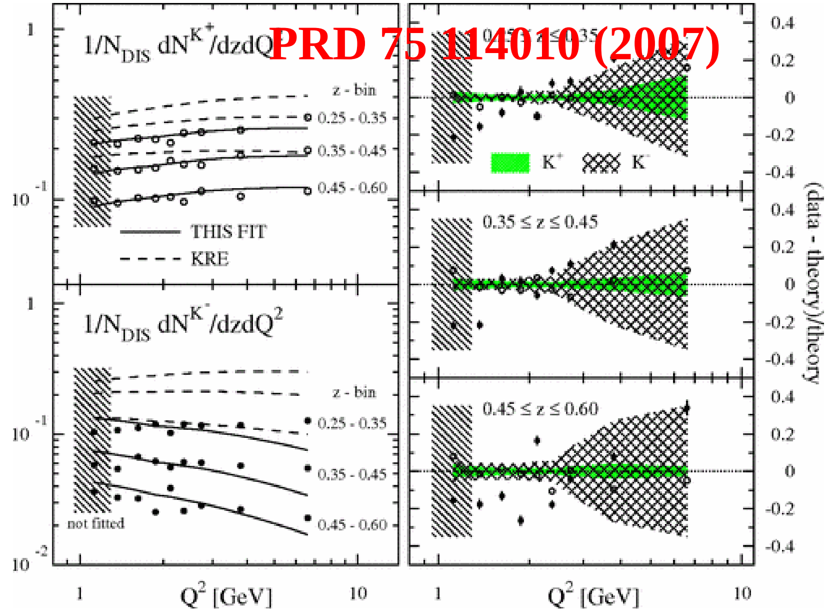


PRD 87 034014 (2013)

Pion FFs

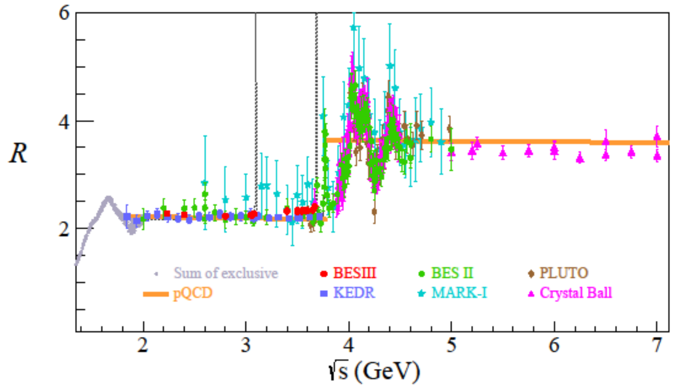




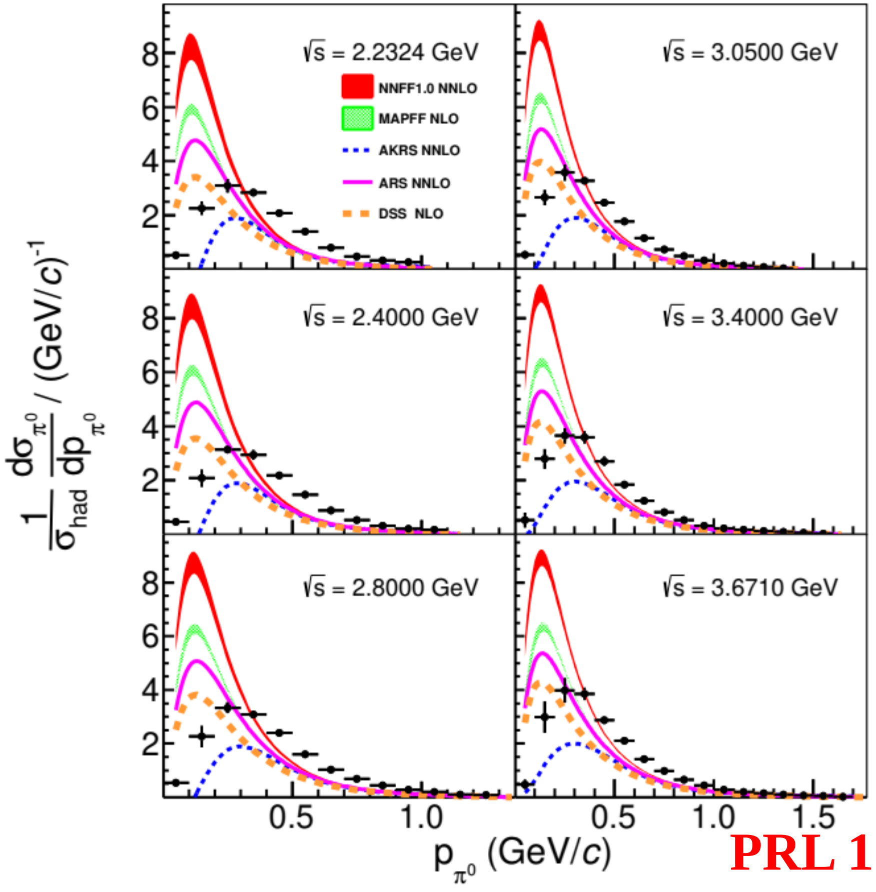


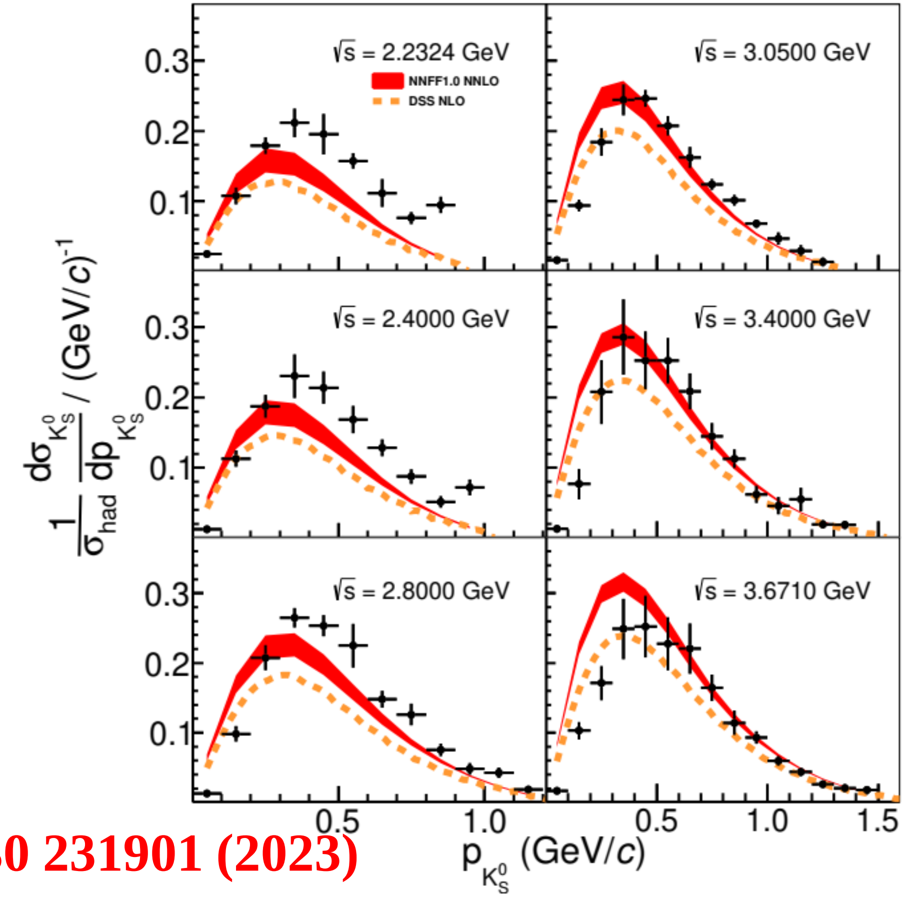
PRD 75 114010 (2007)

- **DIS @ Breit frame**
  - ✓ **Incoming quark scatters off photon and returns along same axis**
- **Current region of Breit frame is analogous to e^+e^-**
  - ✓ **Born level: DIS Q = e^+e^- $\sqrt{s}$**
- **DSS FFs: HERMES ep, pion data at 10% level**
- **e^+e^- data at low energy $\sqrt{s}$**
  - ✓ **FFs packages could describe e^+e^- data at ?% level**



$e^+e^- \to \pi^0/K_S^0 + X$ @ BESIII

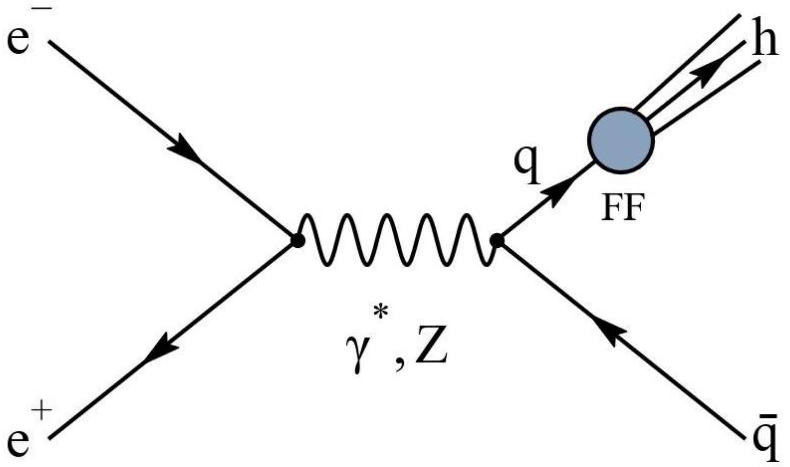


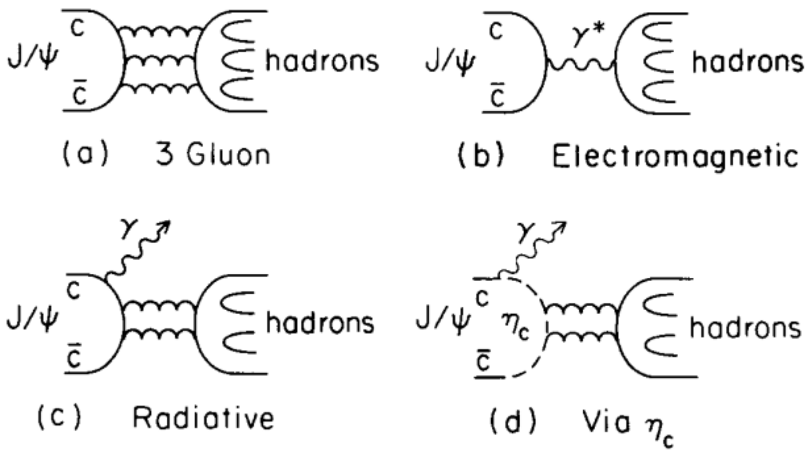


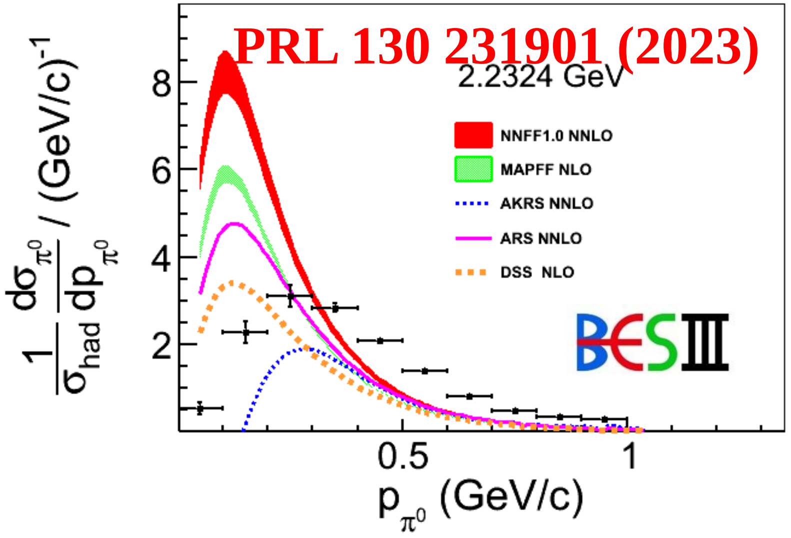
PRL 130 231901 (2023)

- inclusive π^0 production: surprise
- inclusive K_S^0 production: not so bad

$e^+e^- \to \pi^0/K_S^0$ + X @ BESIII

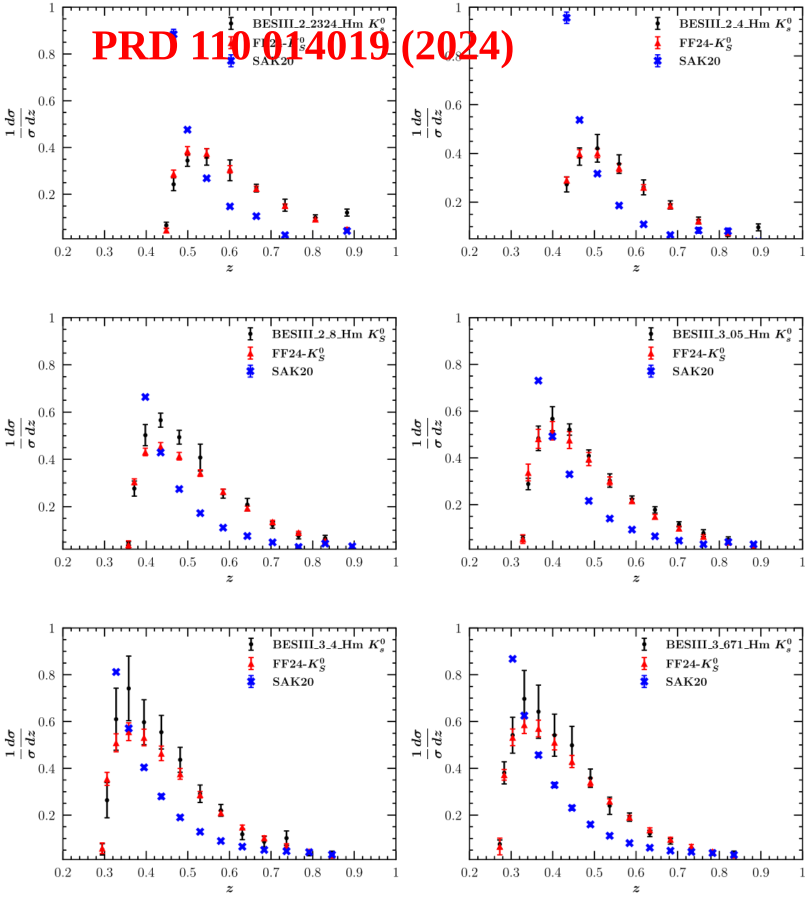


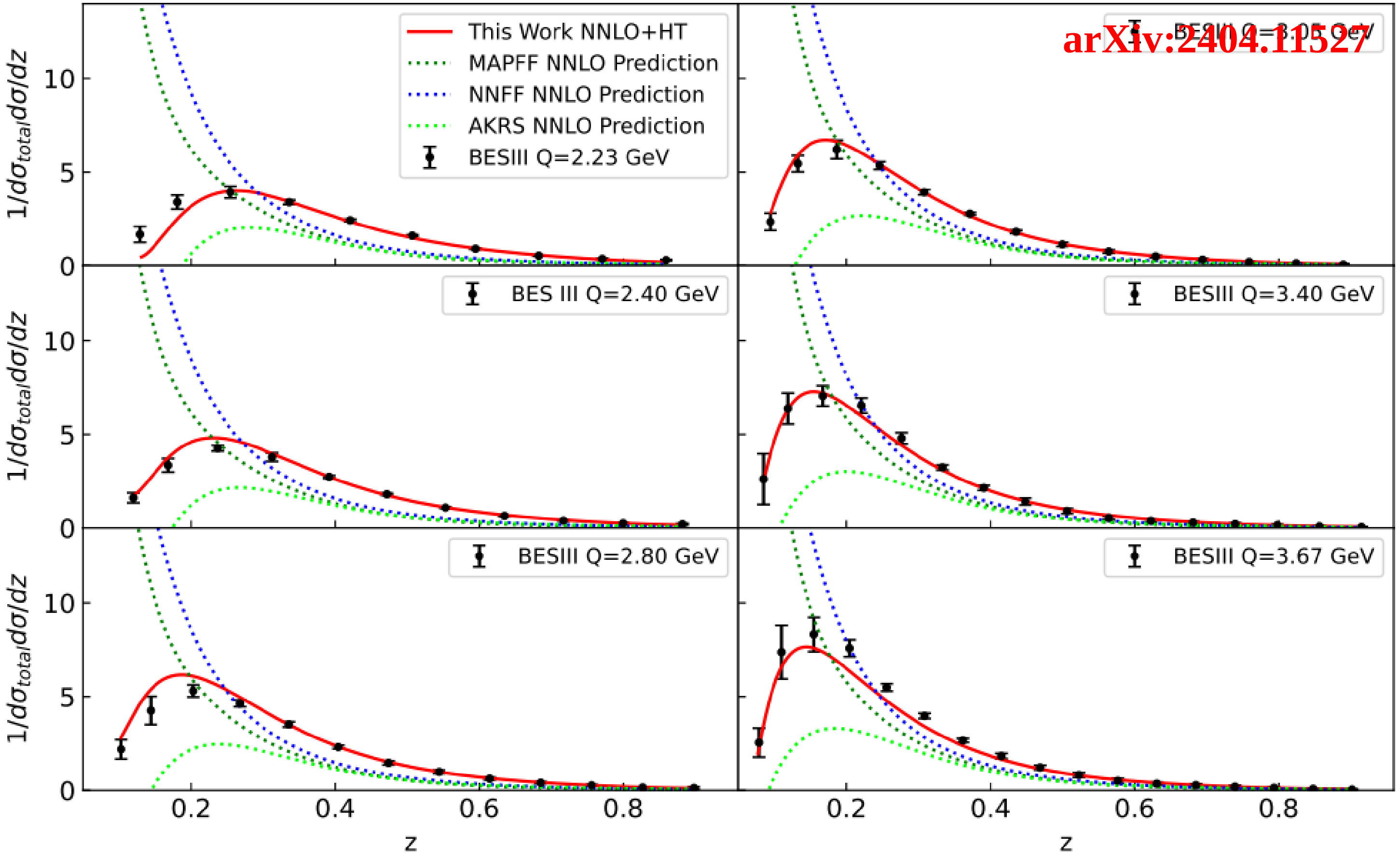




- **From theory side:**
  - ✓ **Fit with BESIII data, hadron mass effect, higher twist contribution, and so on**
  - ✓ **Factorization breaking**
- **From experimental side**
  - ✓ **Primary hadron vs from resonance decay**
  ⟹ measure $e^+ e^- \to \rho(\omega, \phi)$+ X, and so on
  - ✓ **Contribution of vector states ρ^*, ω^* and ϕ^***
  ⟹ $e^+ e^- \to \rho^*/\omega^*/\phi^* \to h + X$

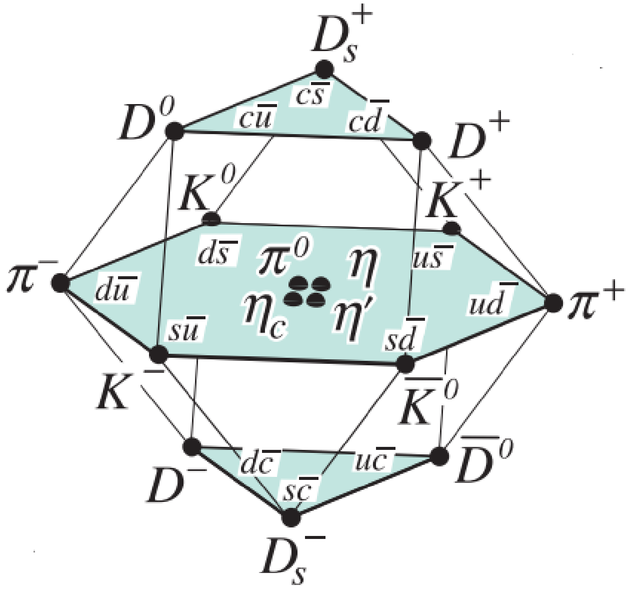
Impact of BESIII $e^+e^- \to \pi^0/K_S^0 + X$

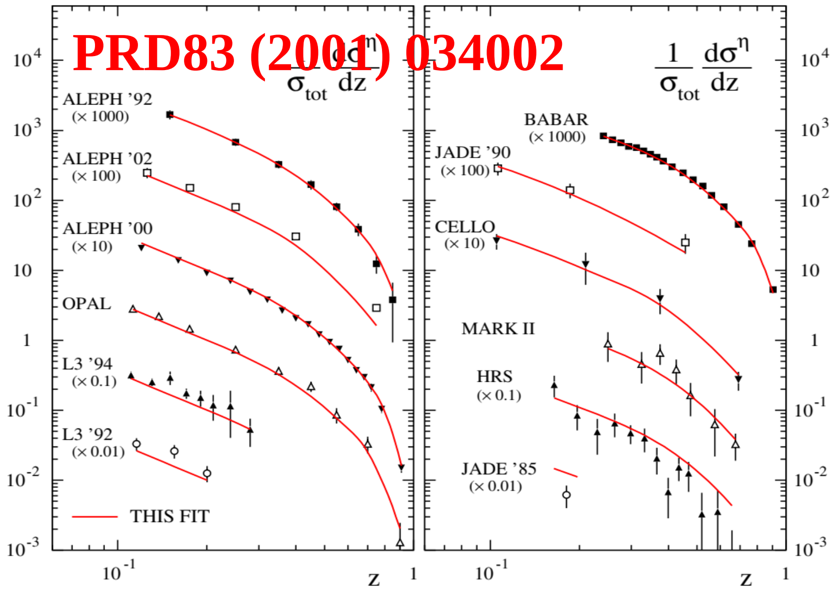


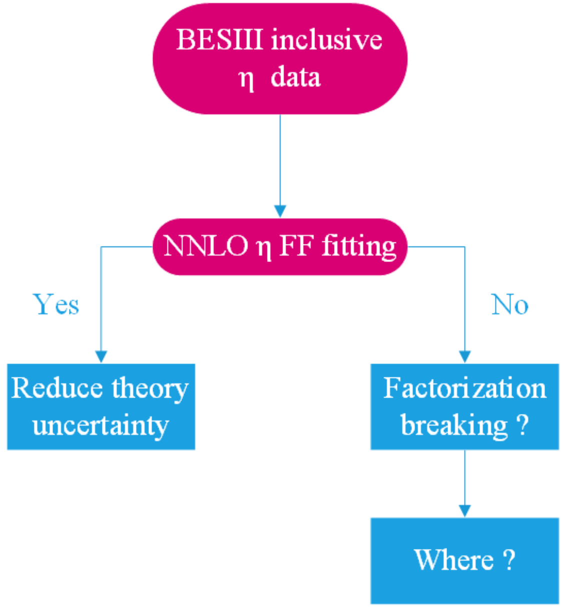


- **PRD 110 014019 (2024): NNLO & hadron mass correction for K_S**
- **arXiv:2404.11527: NNLO & higher twist contribution for π^0**

η FF

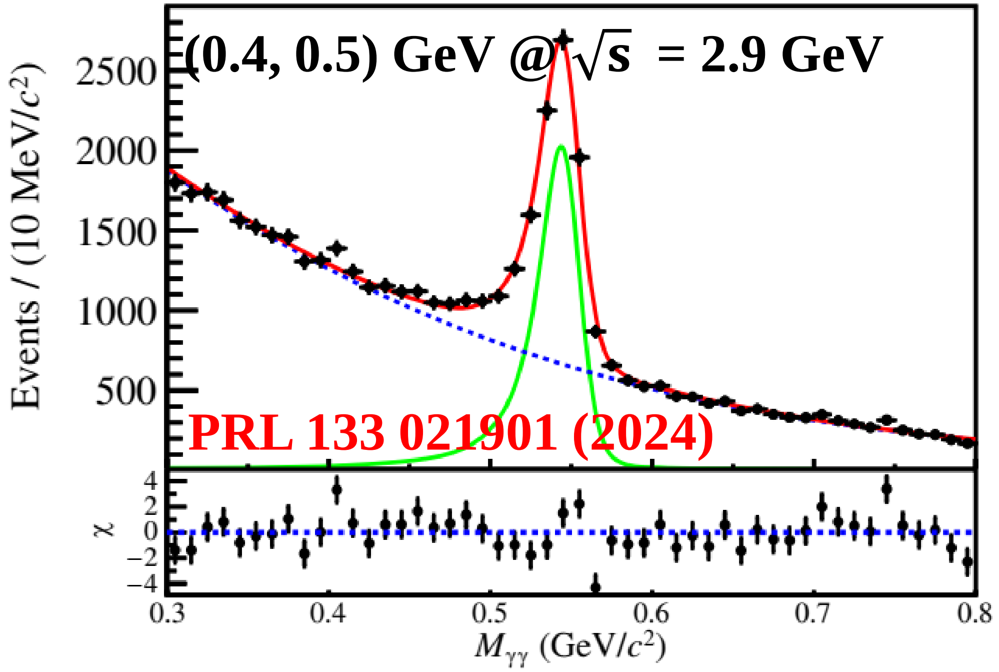


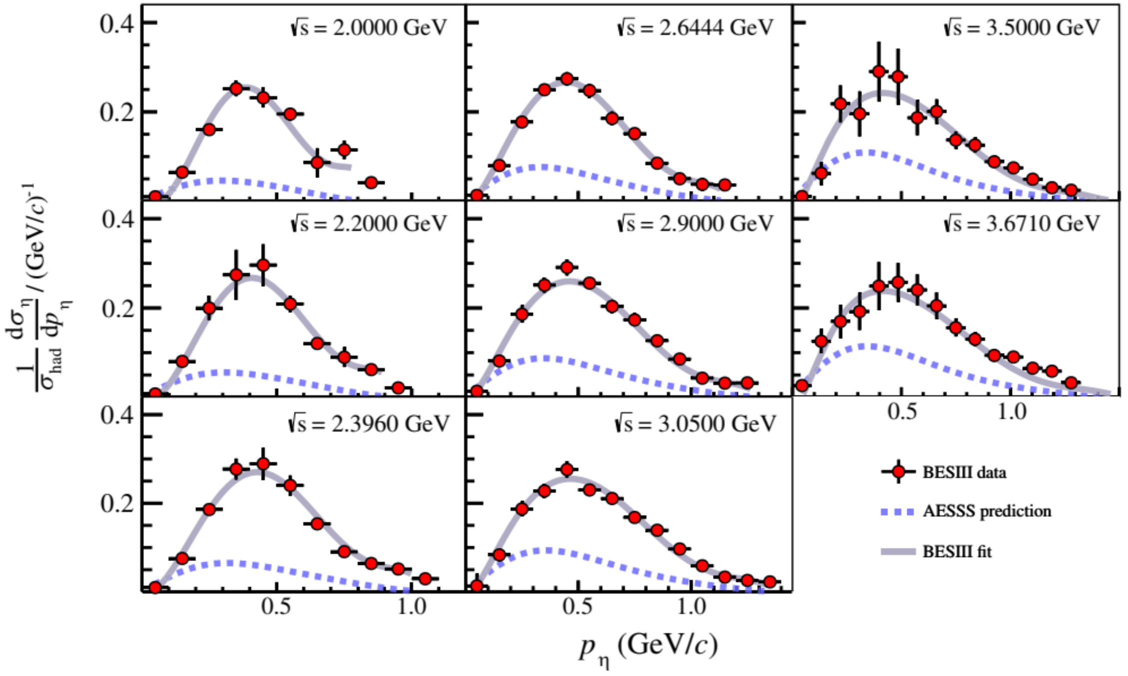




- η: a Goldstone boson due to spontaneous breaking of QCD chiral symmetry
- η FF @ NLO: data at $\sqrt{s}$ > 10 GeV e^+e^- collision
  - ✓ Missing theory uncertainty
- Theory improvement:
  - ✓ NNLO accuracy, hadron mass correction & higher twist contribution
- BESIII results and its possible impact ?

$e^+e^- \to \eta + X$ @ BESIII

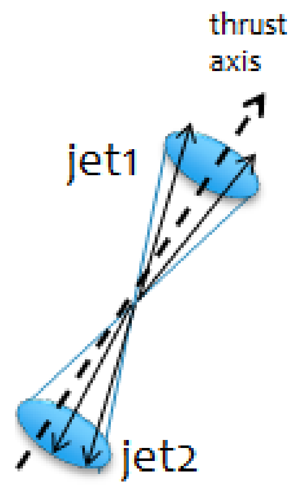


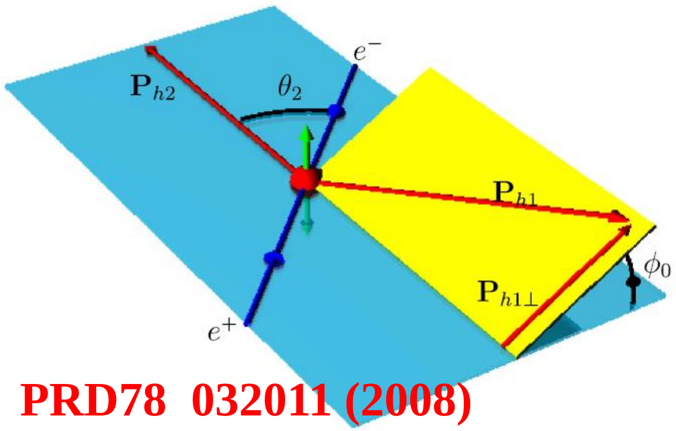


- PR D83 (2001) 034002 prediction vs. BESIII data: tension !
- BESIII fit: detail @ arXiv:2404.11527
  - ✓ $\sqrt{s}$ > 10 GeV e^+e^- data + BESIII data
  - ✓ NNLO accuracy, hadron mass correction & higher twist contributions

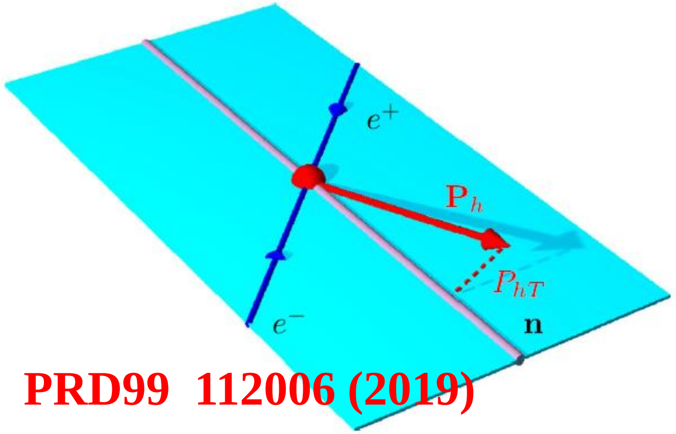
TMD FFs: $D_1^h(z) \Rightarrow D_1^h(z, p_T)$

TMD FF $D_1(z, k_T)$		
$e^+e^- \to h_a h_b X$	$\sum_q e_q^2 D_1^{h_a/q}(z_a, k_{aT}) \otimes D_1^{h_b/\bar{q}}(z_b, k_{bT}) + \{q \leftrightarrow \bar{q}\}$	back-to-back production of hadron pair
$e^+e^- \to (h, \text{jet/thrust axis})X$	$\sum_q e_q^2 D_1^{h/q}(z, k_T)$	can access z, k_T

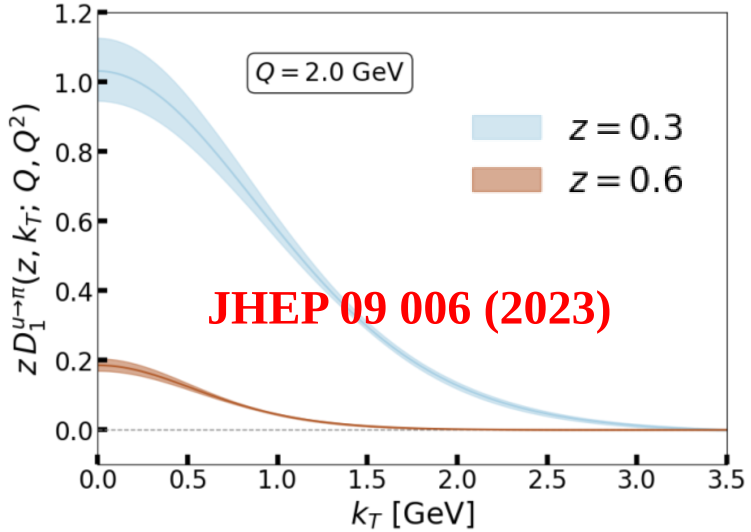




PRD78 032011 (2008)



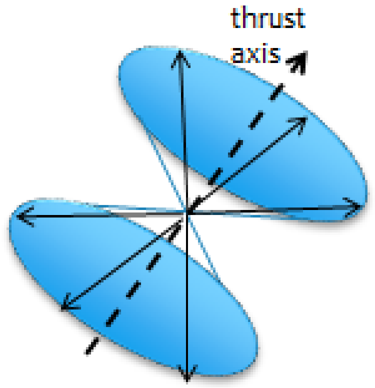
PRD99 112006 (2019)

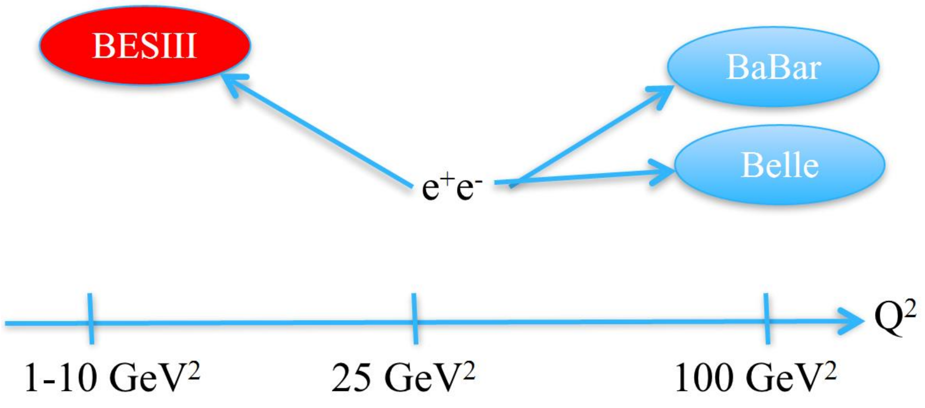


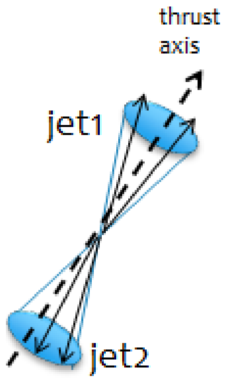
JHEP 09 006 (2023)

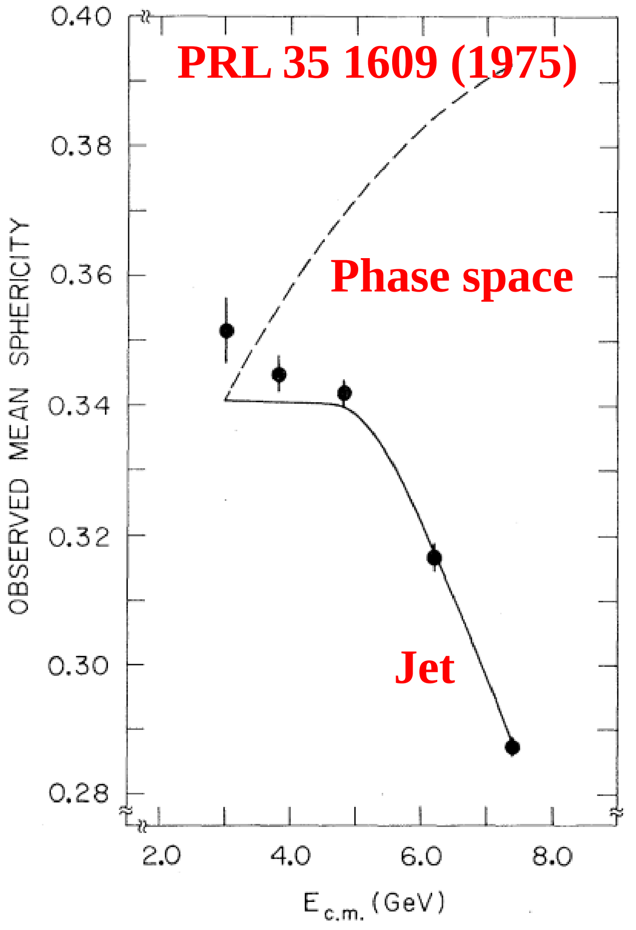
- **Traditional 2-hadron FF**
  - ✓ **Use transverse momentum between two hadron**
- **Single-hadron FF with Thrust or jet axis**
  - ✓ **Need $q\bar{q}$ axis (quark or jet axis)**

TMD FFs: $D_1^h(z) \Rightarrow D_1^h(z, p_T)$

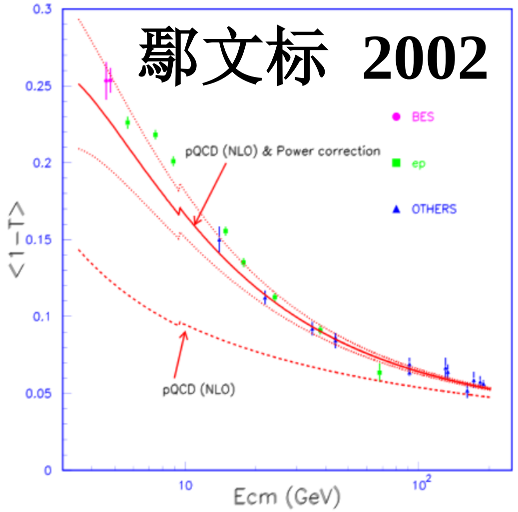








- **Jet structure at BESIII & STCF**
  - ✓ **can reconstruct thrust axis correctly ?**
- **Phase space model vs. Jet model**
  - ✓ **$\sqrt{s} > 5$ GeV ?**
- **At higher $\sqrt{s}$: jet @ [5, 7] GeV ?**
  - ✓ **Evidence for jet structure**
  - ✓ **Events with large thrust value ?**



Spin density matrix of vector meson

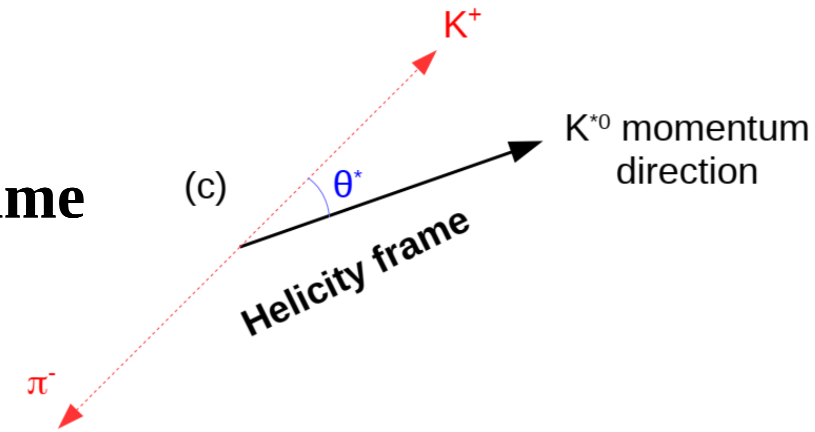
- **The spin state of a vector state is described by 3×3 spin density matrix**
  - ✓ **ρ_{mm}: probability to be in |s; s_z = m> state**

$$\begin{pmatrix} \rho_{-1,-1} & \rho_{-1,0} & \rho_{-1,1} \\ \rho^*_{-1,0} & \rho_{00} & \rho_{01} \\ \rho^*_{-1,1} & \rho^*_{01} & \rho_{11} \end{pmatrix}$$

- **The polarization vector is related with some elements of spin density matrix**

$$\begin{aligned} \vec{\mathcal{P}} &= [\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3] \\ &= \left[\sqrt{2}\mathrm{Re}(\rho_{-10} + \rho_{01}), \sqrt{2}\mathrm{Im}(\rho_{-10} + \rho_{01}), (\rho_{11} - \rho_{-1-1})\right] \end{aligned}$$

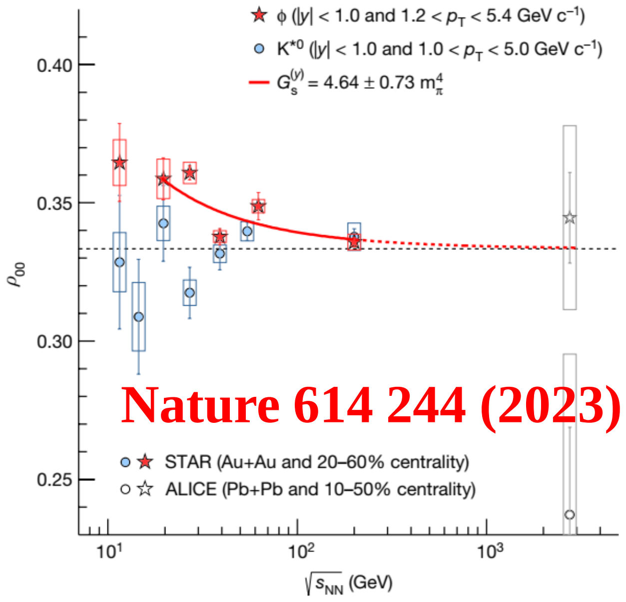
- **Angular of decay particle (kaon) @ K*(892) helicity frame**
  - ✓ **extract some elements, e.g. ρ_{00}**

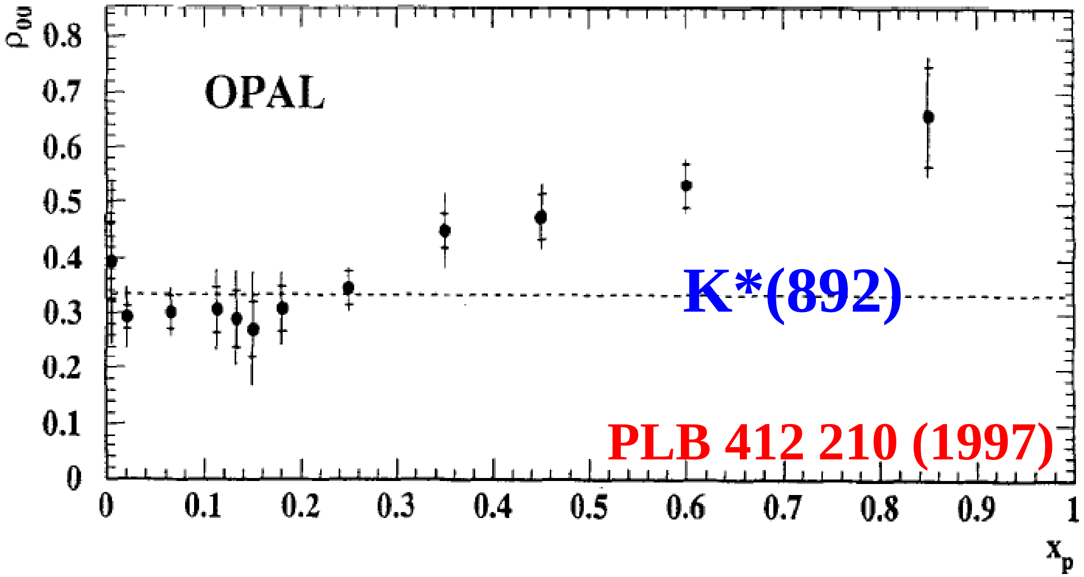


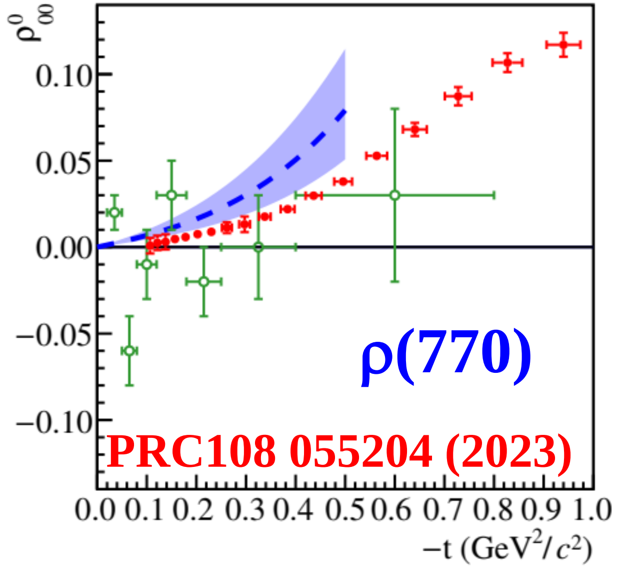
- **vector meson are polarized or not by comparing of ρ_{00} and 1/3**
  - ✓ **$\rho_{00} \neq 1/3$: spin alignment**
  - ✓ **$\rho_{00} = 1/3$: no polarization**
- **The production mechanism for the spin alignment of vector mesons with unpolarized beams**

$$\begin{aligned} W(\cos\theta^*, \phi^*) = (3/4\pi) & \left[\frac{1}{2}(1 - \rho_{00}) + \frac{1}{2}(3\rho_{00} - 1)\cos^2\theta^*\right. \\ & - \mathrm{Re}\,\rho_{1-1}\ \sin^2\theta^* \cos 2\phi^* - \frac{1}{\sqrt{2}}\mathrm{Re}\,(\rho_{10} - \rho_{0-1}) \sin 2\theta^* \cos\phi^* \\ & \left. + \mathrm{Im}\,\rho_{1-1}\ \sin^2\theta^* \sin 2\phi^* + \frac{1}{\sqrt{2}}\mathrm{Im}\,(\rho_{10} - \rho_{0-1}) \sin 2\theta^* \sin\phi^*\right] \end{aligned}$$

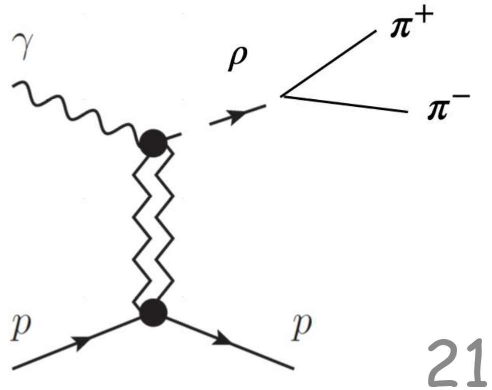
ρ_{00} of vector meson







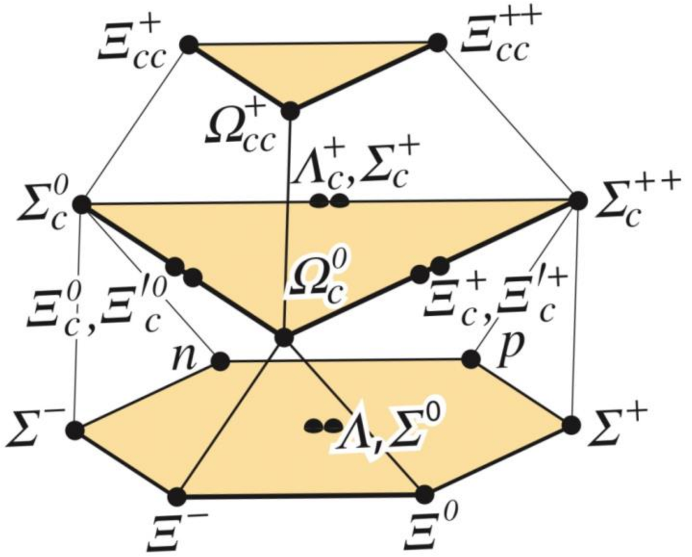
- Heavy ion collision: contribution from QGP & fragmentation
  - ✓ STAR: for phi unexpectedly large than 1/3 (with respect to reaction plane)
- e^+e^- collision: contribution from fragmentation, Z^0 energy
  - ✓ x_p < 0.3, consistent with 1/3; x_p > 0.3, larger than 1/3
- Photoproduction: ρ_{00} @ GlueX
- BESIII: e^+e^- collision, fragmentation, γ^* dominant
  - ✓ ρ_{00} = 1/3: yes or no ?

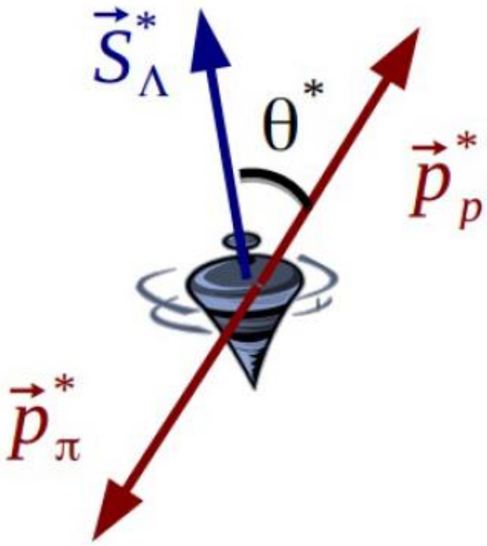


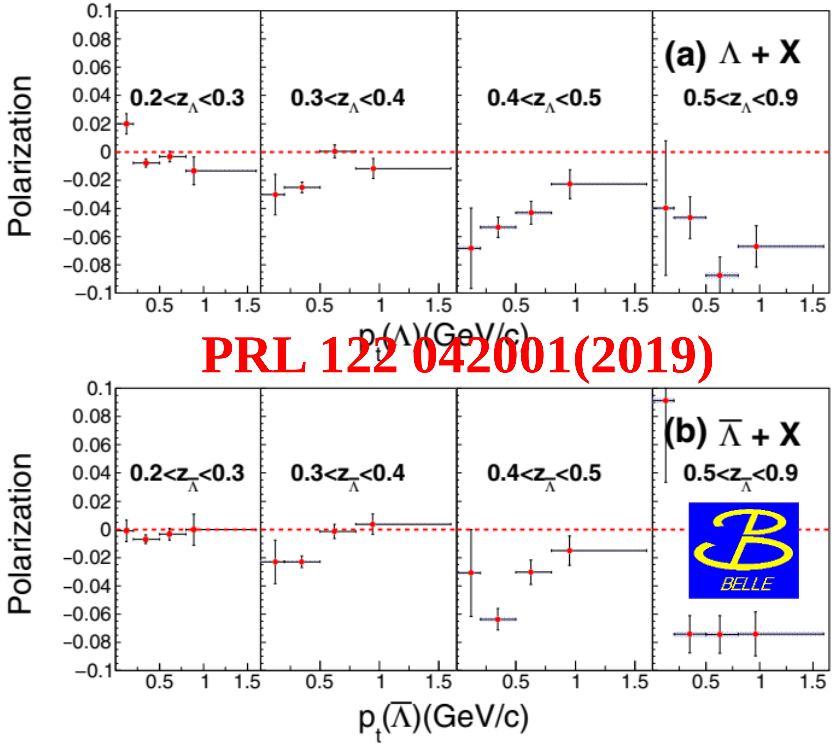
Polarization vector with hyperon

- Polarization vector of hyperon with parity violating decay
- Daughter proton preferentially decays in direction of Λ's spin (opposite for $\bar{\Lambda}$)
  - ✓ Decay parameter α
  - ✓ Polarization vector $\overrightarrow{P_H}$

$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}(1 + \alpha_H |\mathcal{P}_H| \cos\theta^*)$$





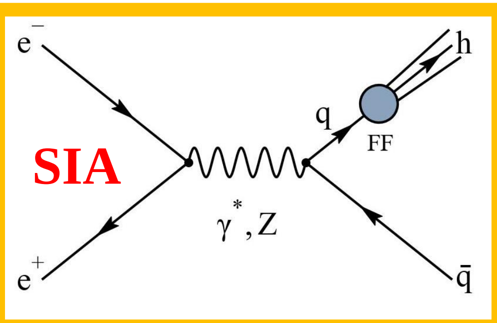


PRL 122 042001(2019)

Summary

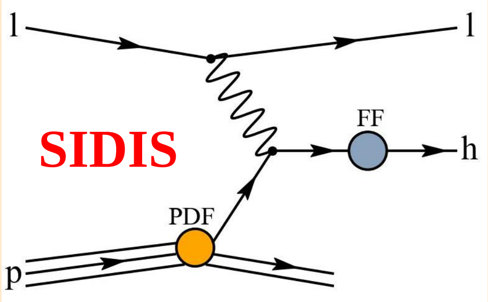
- The knowledge of FFs is an important ingredient in our understanding of non-perturbative QCD dynamics.
- Inclusive π^0 & K_s production @ BESIII: Large discrepancy with theory calculation, need more study on FFs at low energy e^+e^- collision. PRL 130 231901 (2023)
- Inclusive η production @ BESIII: fit with NNLO accuracy, hadron mass correction & higher twist contributions, could describe BESIII data. PRL 133 021901 (2024)
- e^+e^- annihilation provide the cleanest environment to measure FFs.

Access FFs with QCD factorization theorem



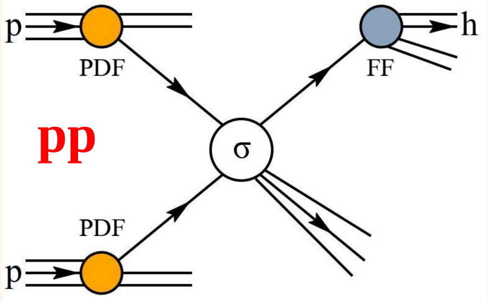
e^+e^-: $\sigma = \sum_q \sigma(e^+e^- \rightarrow q\bar{q}) \otimes FF$

- **No PDFs necessary**
- **Calculations know at NNLO**
- **Flavor structure not directly accessible**



SIDIS: $\sigma = \sum_q PDF \otimes \sigma(eq \rightarrow e'q') \otimes FF$

- **Depend on unpolarized PDFs**
- **Flavor structure directly accessible**
- **FFs and PDFs**

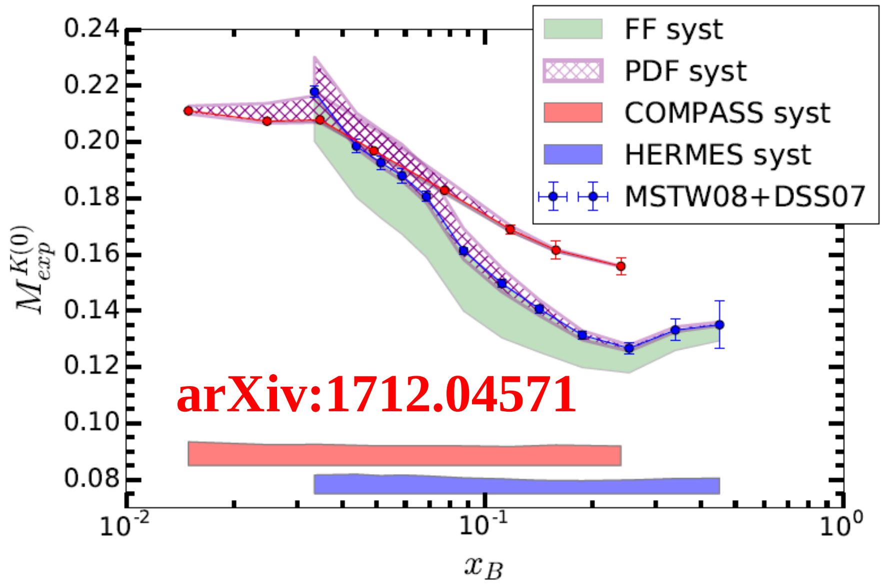


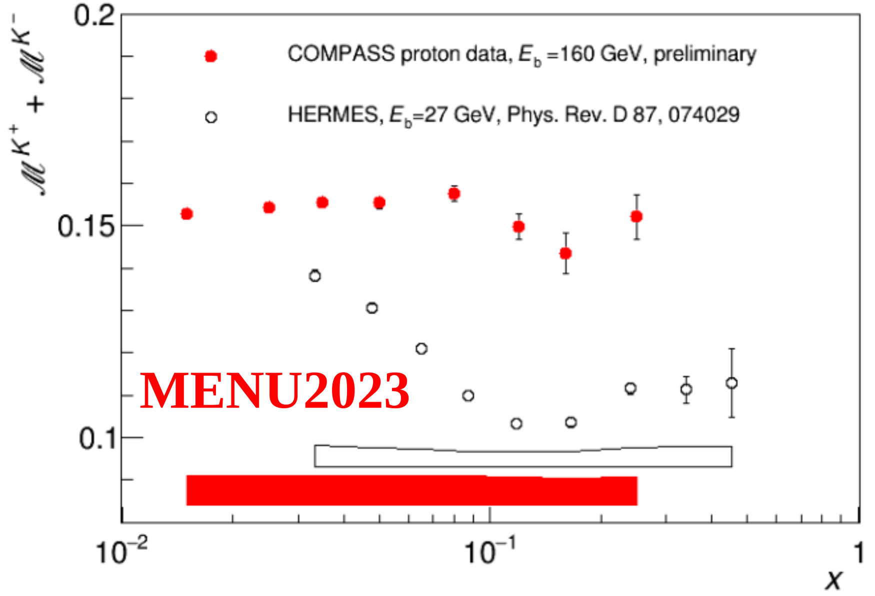
pp: $\sigma = \sum_q PDF \otimes PDF \otimes \sigma(q_1 q_1 \rightarrow q_1' q_2') \otimes FF$

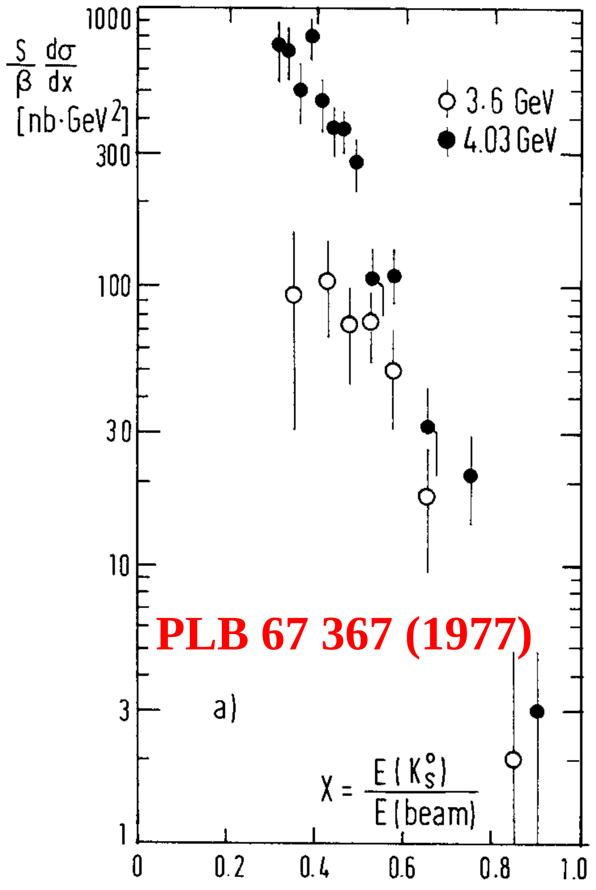
- **Depend on unpolarized PDFs**
- **Leading access to gluon FF**
- **Parton momenta not directly known**

- **SIA @ e^+e^-: the cleanest input for FFs fitting**

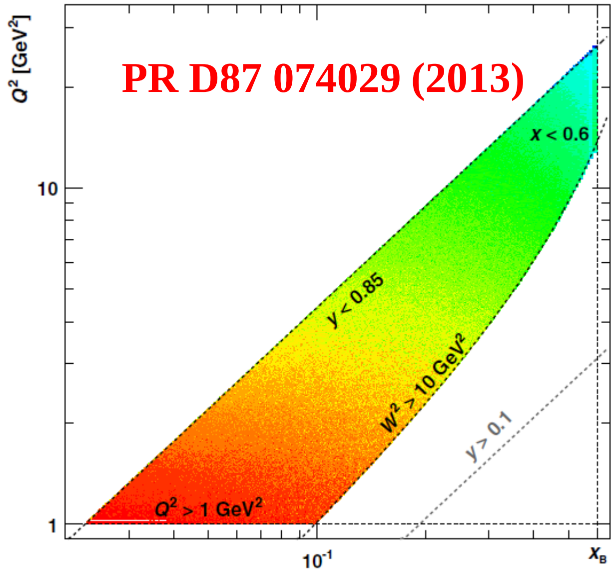
Kaon multiplicity HERMES & COMPASS





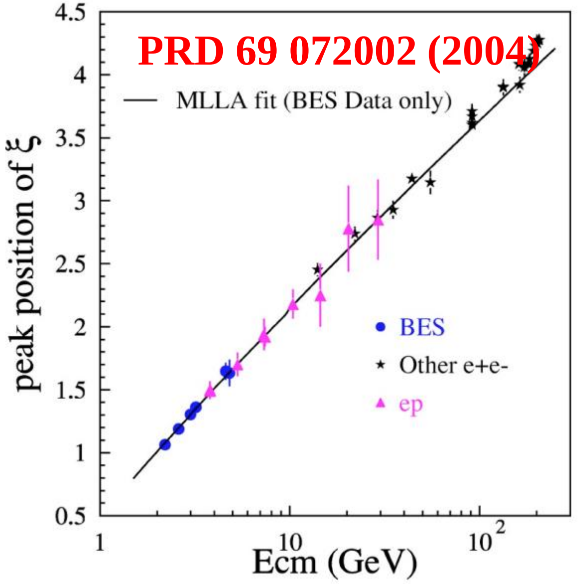


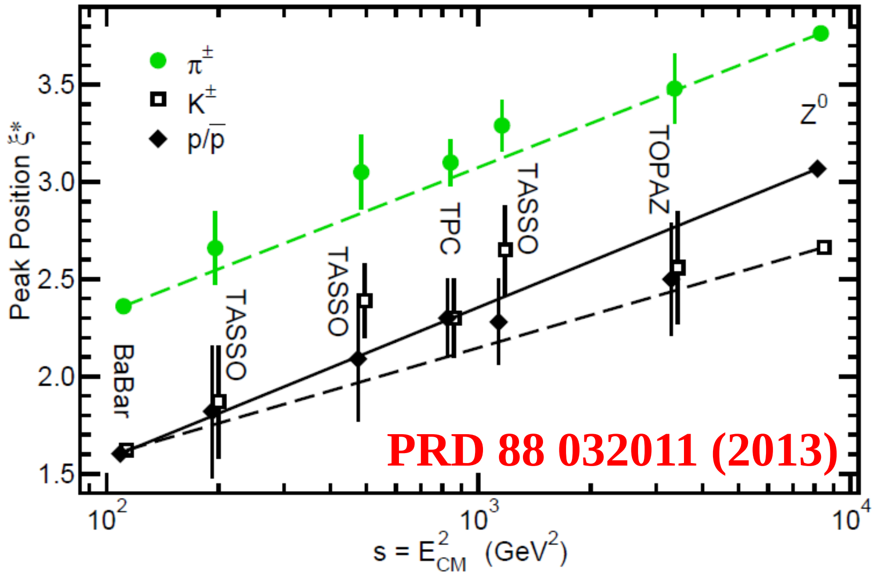
- Hermes data vs. Compass data
  - ✓ Large discrepancies
  - ✓ Kinematic & binning issues
  - ✓ Hadron mass effect
- $e^+e^- \to K + X$ @ few GeV e^+e^- ?
  - ✓ Stat. uncertainty: 18-41%
  - ✓ Precision data ? Not yet

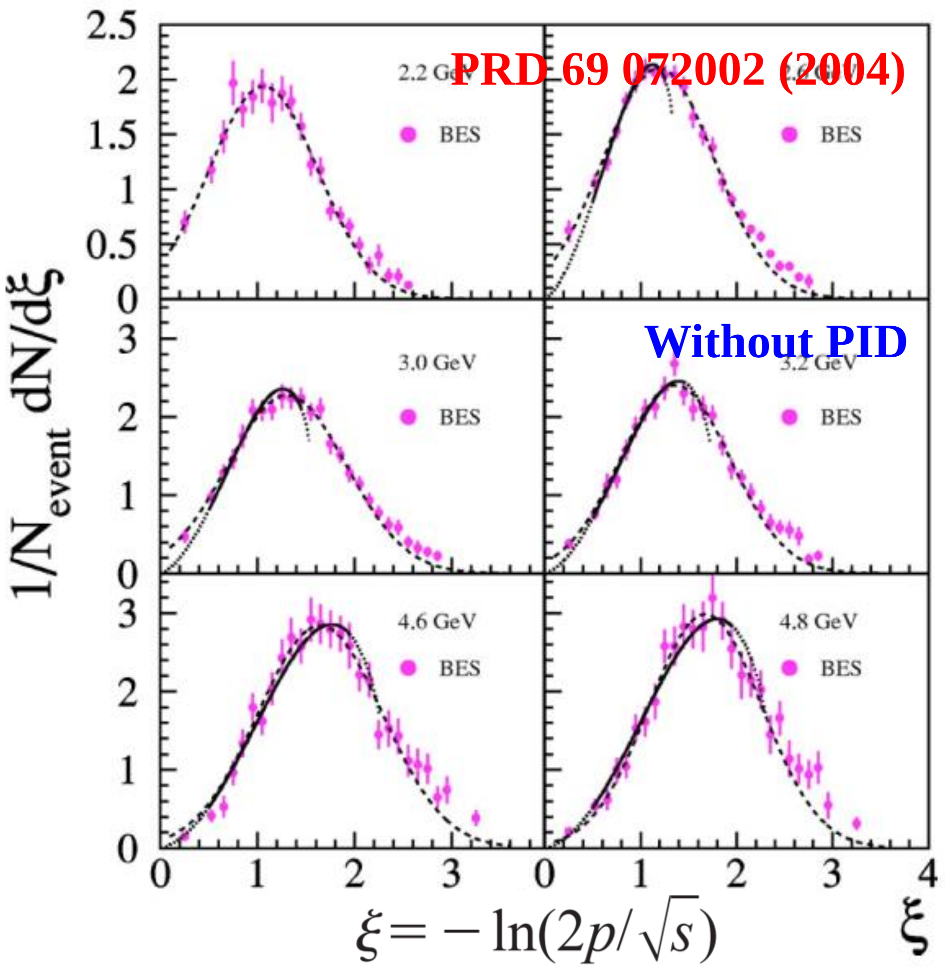


Inclusive hadron: MLLA & LPHD

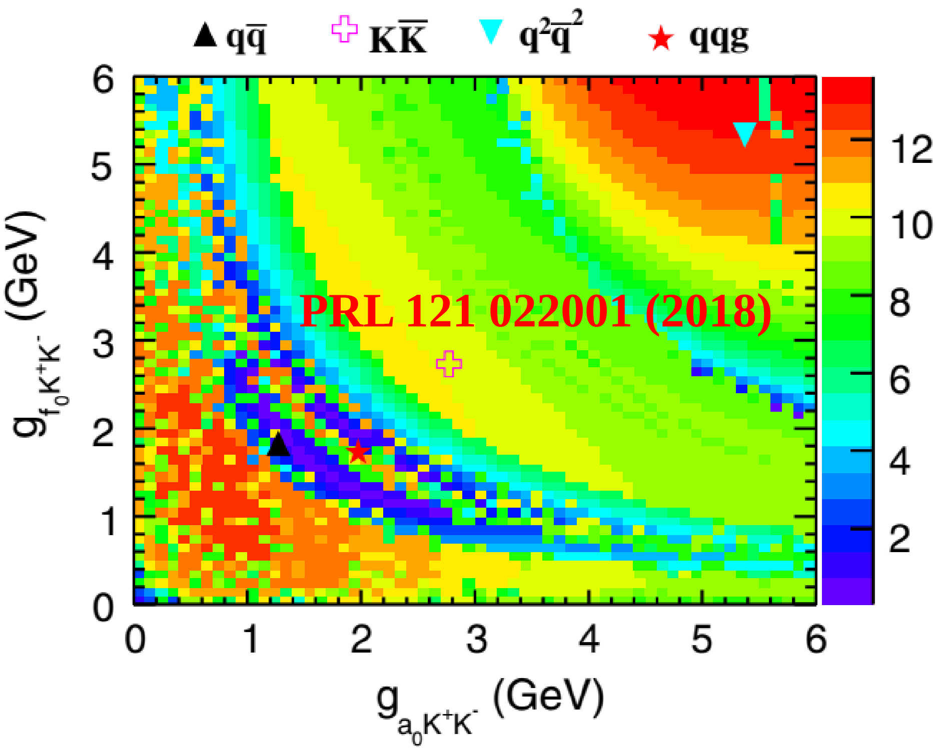
- **MLLA: Modified Leading Log Approximation**
  - ✓ **calculate distribution at partonic level**
  - ✓ **test for re-summation**
- **LPHD: Local Parton Hadronic Duality**
- **Fitted line by BES data could describe high energy e^+e^- data and ep data at 5% level**
- **Inclusive identified hadron at BESIII**

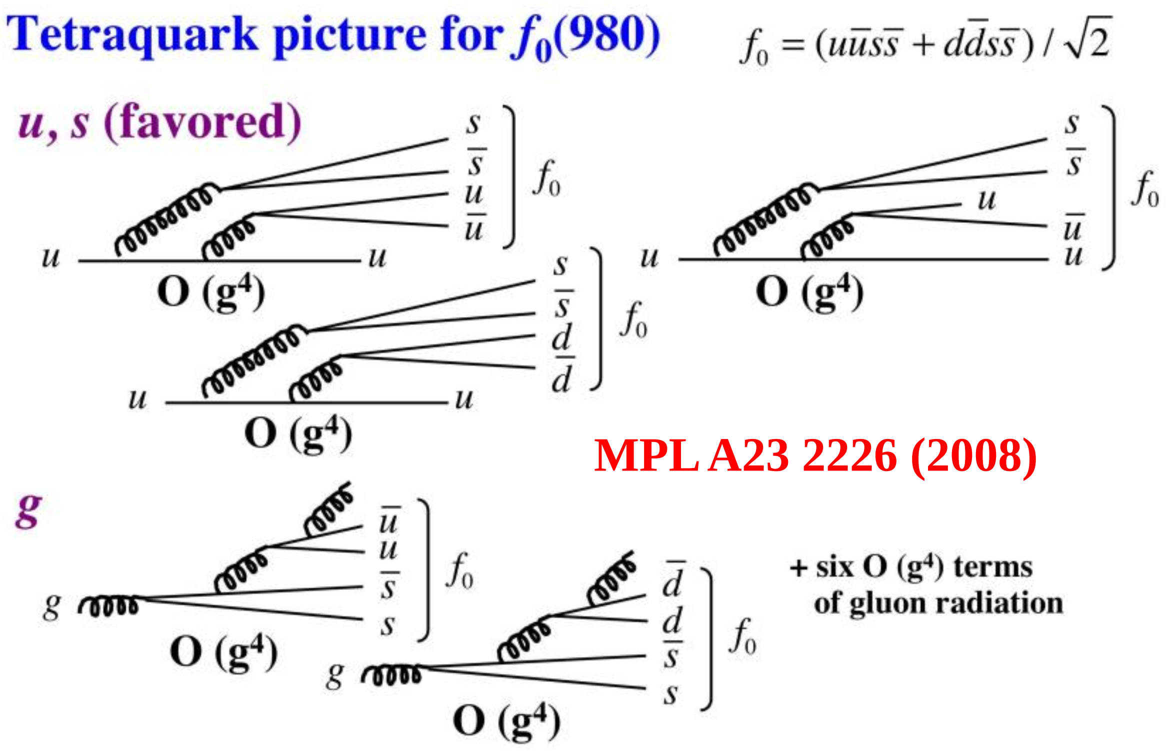






Nature of $f_0(980)$





- Nature of $a_0(980)$ & $f_0(980)$
  - ✓ molecule, tetraquark, hybrid ?
- $a_0(980)$-$f_0(980)$ mixing @ BESIII
- Inclusive $f_0(980)$ production

Table 1. Possible $f_0(980)$ configurations and their relations to the second moments and the peak positions for the fragmentation functions of $f_0(980)$.

Type	Configuration	Second moments	Peak positions
Strange $q\bar{q}$	$s\bar{s}$	$M_u < M_s \lesssim M_g$	$z_u^{max} < z_s^{max}$
Tetraquark (or $K\bar{K}$)	$(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})/\sqrt{2}$	$M_u \sim M_s \lesssim M_g$	$z_u^{max} \sim z_s^{max}$