Transverse Spin Correlation in Unpolarized Collisions

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Table of Content

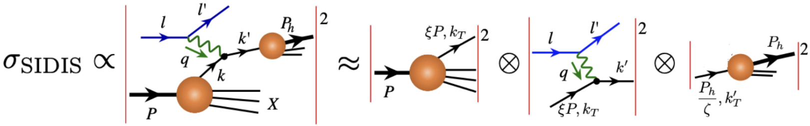


Fig. 1: This figure comes from TMD Handbook.

The SIDIS cross section can be factorized as

$$E' E_h \frac{\mathrm{d}\sigma_{ep\to e' hX}}{\mathrm{d}^3 l' \mathrm{d}^3 P_h} \simeq \hat{\sigma}_{eq\to e' q'} \otimes f_1 \tilde{\otimes} D_{h/q}$$

where the $\tilde{\otimes}$ represents the convolution of the both momentum fraction ξ and k_T or ζ and k'_T in case of FFs.

The definition of correlation function is

$$\Xi_{ij}(p, P_h, S_h) = \sum_X \frac{1}{(2\pi)^4} \int \mathrm{d}^4\xi e^{\mathrm{i}p\cdot\xi} \langle 0|\mathcal{L}(0,\xi)\psi_i(\xi)|P_h, X\rangle\langle P_h, X|\overline{\psi}_j(0)|0\rangle$$

With 16 Dirac-gamma matrix decomposition

$$\begin{aligned}
&\frac{1}{4z}\int \mathrm{d}p^+ \Xi(p, P_h, S_h)\Big|_{p^-=P_h^-/z, p_T} \\
=&\frac{1}{4}\Big[D_1(z,\boldsymbol{p}_T)\not{n}_+ + D_{1T}^\perp(z,\boldsymbol{p}_T)\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu p_T^\rho S_{hT}^\sigma}{M_h} - \lambda G_{1L}(z,\boldsymbol{p}_T)\not{n}_+\gamma_5 - G_{1T}(z,\boldsymbol{p}_T)\frac{(\boldsymbol{p}_T\cdot\boldsymbol{S}_T)}{M_h}\not{n}_+\gamma_5 \\
&-H_{1T}(z,\boldsymbol{p}_T)\mathrm{i}\sigma_{\mu\nu}\gamma_5 S_{hT}^\mu n_+^\nu - \lambda H_{1L}^\perp(z,\boldsymbol{p}_T)\frac{\mathrm{i}\sigma_{\mu\nu}\gamma_5 p_T^\mu n_+^\nu}{M_h} - H_{1T}^\perp(z,\boldsymbol{p}_T)\frac{(\boldsymbol{p}_T\cdot\boldsymbol{S}_T)}{M_h}\frac{\mathrm{i}\sigma_{\mu\nu}\gamma_5 p_T^\mu n_+^\nu}{M_h} \\
&+H_1^\perp(z,\boldsymbol{p}_T)\frac{\sigma_{\mu\nu}p_T^\mu n_+^\nu}{M_h}\Big]
\end{aligned}$$

TMDFFs[TMD Handbook]

Hadron Spin | Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		D_1 = Unpolarized		$H_1^{\perp}$ = − Collins
Polarized Hadrons	L		G_1 = − Helicity	$H_{1L}^{\perp}$ = −
Polarized Hadrons	T	$D_{1T}^{\perp}$ = − Polarizing FF	$G_{1T}^{\perp}$ = −	H_1 = − Transversity $H_{1T}^{\perp}$ = −

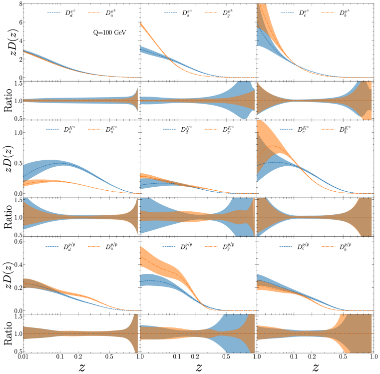
Collinear FFs

Hadron Spin | Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		D_1 = Unpolarized		
Polarized Hadrons	L		G_1 = − Helicity	
Polarized Hadrons	T			H_1 = − Transversity

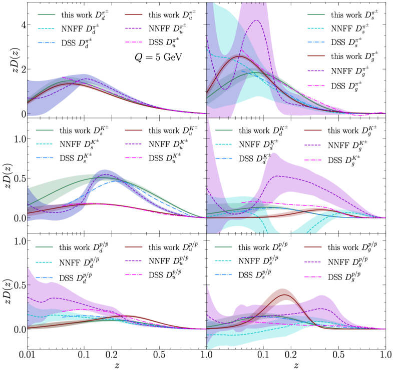
$Q = 100$ GeV

The fragmentation functions have been pushed toward small-z region.



$Q = 5$ GeV

Comparison of NLO fragmentation functions to those from NNFFs and DSS



Gao, Jun and Liu, ChongYang and Shen, XiaoMin and Xing, Hongxi and Zhao, Yuxiang. Phys.Rev.Lett. 132 (2024) 26
Gao, Jun and Liu, ChongYang and Shen, XiaoMin and Xing, Hongxi and Zhao, Yuxiang. 2407.04422 [hep-ph]

Background

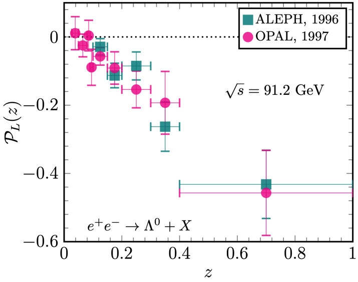
For polarized FFs, such as G_{1L} and H_{1T}:

LEP, RHIC; and HERMES, COMPASS for DIS

$G_{1L} \Longleftarrow$ LEP: e^+e^- collider at the Z_0 pole.

$$\mathcal{P}_L(y,z) = \frac{\sum_q \lambda_q(y)\omega_q(y)G_{1L,q}(z) + \{q \leftrightarrow \bar{q}; y \leftrightarrow (1-y)\}}{\sum_q \omega_q(y)D_{1,q}(z) + \{q \leftrightarrow \bar{q}; y \leftrightarrow (1-y)\}}$$

where $\lambda_q(y) = \Delta\omega_q(y)/\omega_q(y)$ is the helicity of the fragmenting quark.



Transverse spin dependent FFs: H_{1T} ?

[Kai-Bao Chen, Tianbo Liu, Yu-Kun Song, Shu-Yi Wei. Particles 6 (2023) 2, 515-545, Particles 6 (2023) 515-545]
[ALEPH Collaboration. Phys. Lett. B 1996, 374, 319–330.]
[OPAL Collaboration. Eur. Phys. J. C 2, 49–59 (1998)]

How to study transverse spin dependent FFs H_{1T} ?

- Xiangdong Ji.et.al have proposed to measure the longitudinal spin transfer $G_{1L}(z)$ at unpolarized electron-positron colliders utilizing the helicity correlation of back-to-back dihadron at 1994.
- Longitudinal spin transfer (unpolarized e^+e^- and pp; photon-nucleus collisions)
- It can be used to study the effects of jet quenching

[Kun Chen, Gary R. Goldstein, R. L. Jaffe, and Xiangdong Ji. Nucl.Phys.B 445 (1995) 380-398]
[Zhao-Xuan Chen Hui Dong and Shu-Yi Wei. Phys.Rev.D 110 (2024) 5, 056040]
[Hao-Cheng Zhang, Shu-Yi Wei. Phys.Lett.B 839 (2023) 137821]
[Xiaowen Li, Zhao-Xuan Chen, Shanshan, Shu-Yi Wei. Phys.Rev.D 109 (2024) 1, 014035]

Transverse Spin Correlation in e^+e^- Annihilation

The definition of transverse spin correlation is

$$\mathcal{C}_{TT} = \frac{\mathcal{P}(\boldsymbol{n}_T, \boldsymbol{n}_T) + \mathcal{P}(-\boldsymbol{n}_T, -\boldsymbol{n}_T) - \mathcal{P}(\boldsymbol{n}_T, -\boldsymbol{n}_T) - \mathcal{P}(-\boldsymbol{n}_T, \boldsymbol{n}_T)}{\mathcal{P}(\boldsymbol{n}_T, \boldsymbol{n}_T) + \mathcal{P}(-\boldsymbol{n}_T, -\boldsymbol{n}_T) + \mathcal{P}(\boldsymbol{n}_T, -\boldsymbol{n}_T) + \mathcal{P}(-\boldsymbol{n}_T, \boldsymbol{n}_T)}$$

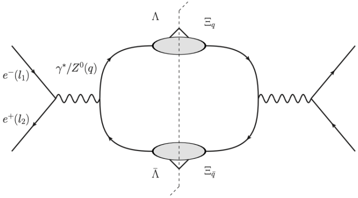
- $\mathcal{P}(\boldsymbol{a}_T, \boldsymbol{b}_T)$ is the probability for hadron being transverse polarized along $\boldsymbol{a}_T$ direction and $\boldsymbol{b}_T$ is anti-hadron.

In the parton model, the LO differential cross section reads

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\mathrm{d}z_1\mathrm{d}z_2} = \frac{2\pi N_c \alpha^2}{Q^4} L_{\mu\nu} \hat{W}^{\mu\nu}$$

The leptonic tensor is

$$L_{\mu\nu} = l_{1\mu} l_{2\nu} + l_{1\nu} l_{2\mu} - g_{\mu\nu} l_1 \cdot l_2$$



The hadronic tensor

$$\hat{W}^{\mu\nu} = \sum_q e_q^2 \mathrm{Tr}\left[2\hat{\Xi}_q(z_1)\gamma^\mu\hat{\Xi}_{\bar{q}}(z_2)\gamma^\nu\right]$$

where the correlation function $\hat{\Xi}_q(z_1)$, $\hat{\Xi}_{\bar{q}(z_2)}$ are

$$\hat{\Xi}_q(z_1) = \frac{1}{4}\not{n}_+ D_{1,q}(z_1) + \frac{1}{8}\left[\not{S}_{T1}, \not{n}_+\right]\gamma_5 H_{1T,q}(z_1)$$
$$\hat{\Xi}_{\bar{q}}(z_2) = \frac{1}{4}\not{n}_+ D_{1,\bar{q}}(z_2) + \frac{1}{8}\left[\not{S}_{T1}, \not{n}_+\right]\gamma_5 H_{1T,\bar{q}}(z_2)$$

Here, $\hat{\Xi}_q(z_1)$, $\hat{\Xi}_{\bar{q}}(z_2)$ is the $\boldsymbol{p}_T$-integrated version of $\Xi_q(z_1, \boldsymbol{p}_T)$, $\Xi_{\bar{q}}(z_2, \boldsymbol{p}_T)$.

Transverse Spin Correlation in e^+e^- Annihilation

The differential cross section containing both electromagnetic and weak interactions is

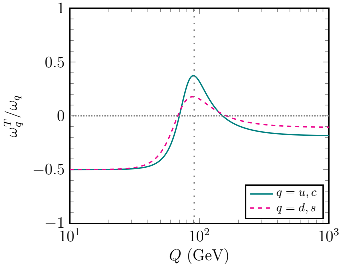
$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\mathrm{d}z_1\mathrm{d}z_2} = \frac{2\pi N_c \alpha^2}{Q^2}\left\{\sum_q \left[\omega_q(y) D_{1,q}(z_1) D_{1,\bar{q}}(z_2) + (\boldsymbol{S}_{T1}\cdot\boldsymbol{S}_{T2})\omega_q^T(y) H_{1T,q}(z_1) H_{1T,\bar{q}}(z_2)\right] + [y \leftrightarrow (1-y), q \leftrightarrow \bar{q}]\right\}$$

- $\omega_q(y) = e_q^2 A(y) + \chi_{\mathrm{int}}^q T_0^q(y) + \chi T_0^q(y)$
- $\omega_q^T(y) = -\{e_q^2 + \chi_{\mathrm{int}}^q c_V^e c_V^q + \chi c_1^e[(c_V^q)^2 - (c_A^q)^2]\} C(y)$
- $T_0^q(y) = c_1^e c_1^q A(y) - c_3^e c_3^q B(y)$; $I_0^q(y) = c_V^e c_V^q A(y) - c_A^e c_A^q B(y)$
- $A(y) = y^2 + (1-y)^2$; $B(y) = 1 - 2y$; $C(y) = 2y(1-y)$
- $\chi = \frac{Q^4}{[(Q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2]\sin^4 2\theta_W}$; $\chi_{\mathrm{int}}^q = -\frac{2e_q Q^2(Q^2 - M_Z^2)}{[(Q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2]\sin^2 2\theta_W}$

The dihadron transverse spin correlation

$$\mathcal{C}_{TT} = \frac{\sum_q \left[\omega_q^T(y) H_{1T,q}(z_1) H_{1T,\bar{q}}(z_2) + \omega_q^T(1-y) H_{1T,\bar{q}}(z_1) H_{1T,q}(z_2)\right]}{\sum_q \left[\omega_q(y) D_{1T,q}(z_1) D_{1T,\bar{q}}(z_2) + \omega_q(1-y) D_{1T,\bar{q}}(z_1) D_{1T,q}(z_2)\right]}$$

- Transverse spin correlation of $q\bar{q}$: $\dfrac{\omega^T}{\omega} = \dfrac{\int_0^1 \mathrm{d}y \omega^T(y)}{\int_0^1 \mathrm{d}y \omega(y)}$
- The electromagnetic vertex: γ^μ
- The weak vertex: $\gamma^\mu(c_V - c_A\gamma_5)$
- Positivity constrain: $|\mathcal{C}_{TT}| \leq \sqrt{1-|\mathcal{P}_L|^2}$, $|\mathcal{P}_L|$ is the magnitude of the helicity of $q/\bar{q}$
- If assume vertex $\gamma^\mu(1 \pm \gamma_5)$, transverse spin correlation reduces to 0



The unpolarized cross section can simply be related to the helicity amplitude[JHEP10(2021)078]

$$\frac{\mathrm{d}\hat{\sigma}_{ab\to cd}}{\mathrm{d}t} = \frac{1}{16\pi s^2} \sum_{\lambda_a\lambda_b\lambda_c\lambda_d} \mathcal{M}_{\lambda_a,\lambda_b}(\lambda_c,\lambda_d)\mathcal{M}^*_{\lambda_a,\lambda_b}(\lambda_c,\lambda_d)$$

The transversely polarized cross section

$$\frac{\mathrm{d}\hat{\sigma}^T_{ab\to cd}}{\mathrm{d}t} = \frac{1}{16\pi s^2} \sum_{\lambda_a\lambda_b\lambda_c\lambda_d} \mathcal{M}_{\lambda_a,\lambda_b}(\lambda_c,\lambda_d)\mathcal{M}^*_{\lambda_a,\lambda_b}(-\lambda_c,-\lambda_d)$$

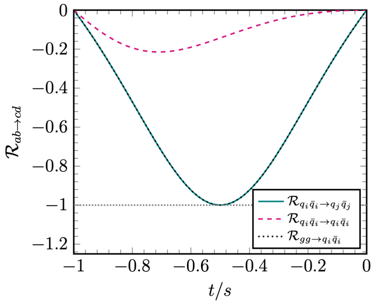
There is a helicity flip in transverse spin correlation!

Only helicity flip parts contribute to transverse spin correlation!

$$\frac{d\sigma_{pp\to\Lambda\bar{\Lambda}}}{dy_1 d^2\boldsymbol{p}_{T1} dy_2 d^2\boldsymbol{p}_{T2}} = \int \frac{dz_1}{z_1}\frac{dz_2}{z_2} \sum_{ab\to cd} x_a f_{1,a}(x_a) x_b f_{2,b}(x_b) \delta^{(2)}\left(\frac{\boldsymbol{p}_{T1}}{z_1} + \frac{\boldsymbol{p}_{T2}}{z_2}\right)$$
$$\times \frac{1}{\pi}\left[\frac{d\hat{\sigma}_{ab\to cd}}{dt} D_{1,c}(z_1) D_{1,d}(z_2) + (\boldsymbol{S}_{T1}\cdot\boldsymbol{S}_{T2}) \frac{d\hat{\sigma}^T_{ab\to cd}}{dt} H_{1T,c}(z) H_{1T,d}(z_2)\right]$$

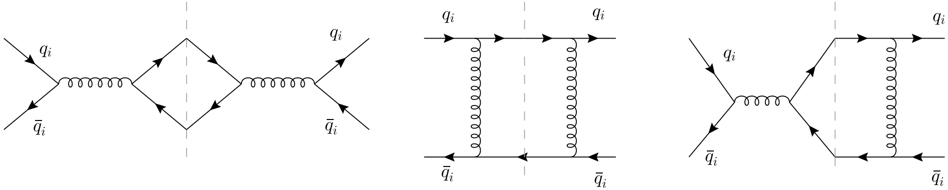
Transversely polarized cross ssection:

- $q_i\bar{q}_i \to q_j\bar{q}_j$: $-\dfrac{2\pi\alpha_s^2}{9s^2}\dfrac{4ut}{s^2}$
- $q_i\bar{q}_i \to q_i\bar{q}_i$: $\dfrac{2\pi\alpha_s^2}{9s^2}\dfrac{4u(s-3t)}{3s^2}$
- $gg \to q_i\bar{q}_i$: $\dfrac{\pi\alpha_s^2}{12s^2}\dfrac{ut-4u^2-4t^2}{s^2}$

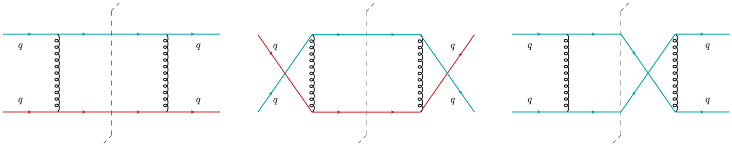


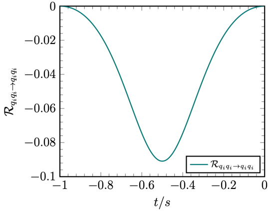
$$\mathcal{R}_{ab\to cd} \equiv d\hat{\sigma}^T_{ab\to cd}/d\hat{\sigma}_{ab\to cd}$$

pp Collision $q_i\overline{q}_i \to q_i\overline{q}_i$



The process $q_i q_i \to q_i q_i$ also have a contribution.



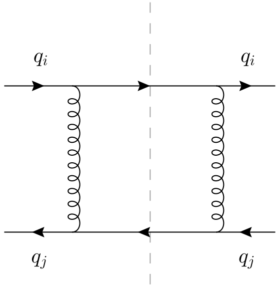


$$\frac{\mathrm{d}\hat{\sigma}^{T}_{q_i q_i \to q_i q_i}}{\mathrm{d}t} = -\frac{2\pi\alpha_s^2}{9s^2}\frac{4}{3}$$

The interference diagrams contribute to the transverse spin correlation, it can be understand through the respect of helicity amplitude flip.

For process $q_i(k_1)q_j(k_2) \to q_i(k_3)q_j(k_4)$, there is no transverse spin correlation because of the fixed initial state.
From another perspective, this is the trace of an odd number of Dirac-gamma matrices, so the transverse spin correlation is zero.

$$\frac{\mathrm{d}\sigma^T}{\mathrm{d}t} \propto \mathrm{tr}\left\{\not{k}_1\gamma^\mu\frac{1}{8}[\not{S}_{T1},\not{k}_3]\gamma_5\gamma^\nu\right\}\mathrm{tr}\left\{\not{k}_2\gamma^\rho\frac{1}{8}[\not{S}_{T1},\not{k}_4]\gamma_5\gamma^\sigma\right\} = 0$$



For UPC process, partonic hard scattering consists of $\gamma g \to q\bar{q}$ and $\gamma q \to qg$.
The transverse spin correlation:

$$\mathcal{C}_{TT} = \frac{\int dP \cdot S \cdot \sum_q x_\gamma f_\gamma(x_\gamma) x_g f_{1,g}(x_g) \frac{1}{\pi} \frac{d\hat{\sigma}^T_{\gamma g \to q\bar{q}}}{dt} H_{1T,q}(z_1) H_{1T,\bar{q}}(z_2)}{\int dP \cdot S \cdot \sum_{b,c,d} x_\gamma f_\gamma(x_\gamma) x_b f_{1,b}(x_b) \frac{1}{\pi} \frac{d\hat{\sigma}_{\gamma b \to cd}}{dt} D_{1,c}(z_1) D_{1,d}(z_2)}$$

Only $\gamma g \to q\bar{q}$ contribute!
The transversely polarized cross section thus reads

$$\frac{d\hat{\sigma}^T_{\gamma g \to q\bar{q}}}{dt} = -\frac{2\pi\alpha\alpha_s e_q^2}{s^2}$$

DGLAP Evolution

DGLAP evolution equation

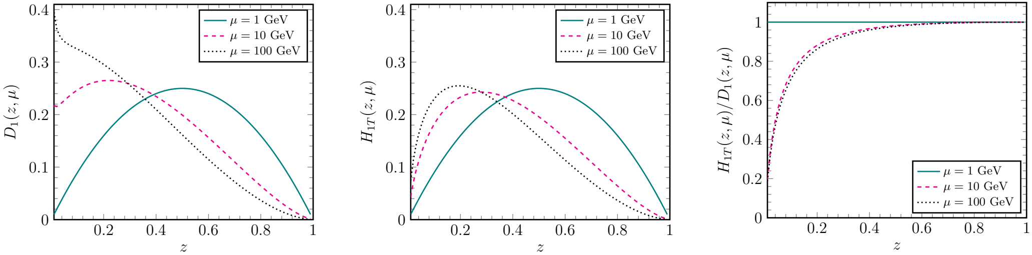
$$\frac{\partial D_{1,q}(z,\mu_f^2)}{\partial \ln \mu_f^2} = \frac{\alpha_s(\mu_f^2)}{2\pi} \int \frac{\mathrm{d}\xi}{\xi} P_{qq}(\xi) D_{1,q}\left(\frac{z}{\xi},\mu_f^2\right)$$
$$\frac{\partial H_{1T,q}(z,\mu_f^2)}{\partial \ln \mu_f^2} = \frac{\alpha_s(\mu_f^2)}{2\pi} \int \frac{\mathrm{d}\xi}{\xi} P_{qq}^T(\xi) H_{1T,q}\left(\frac{z}{\xi},\mu_f^2\right)$$

Splitting function

$$P_{qq}^T(\xi) = P_{qq}(\xi) - C_F(1-\xi) = C_F \frac{1+\xi^2}{(1-\xi)_+} + 2\delta(1-\xi) - C_F(1-\xi)$$

The fragmentation function at initial scale μ_0 is $H_{1T,q}(z,\mu_0^2) = D_{1,q}(z,\mu_0^2) = z(1-z)$.

DGLAP Evolution



We have neglected the gluon contribution to the DGLAP evolution of D_1.

Positivity constrain always be satisfied.

Summary

1. e^+e^- annihilation spin correlation
2. Helicity approach
3. *pp* collision spin correlation
4. Fragmentation function DGLAP evolution

Thank You!