Gluon imaging by lepton-pair azimuthal modulation

in time-like Compton scattering at small-x

Hao-Cheng Zhang (张皓程)
Advised by Dr. Feng Yuan @ LBNL

Taishan College, Shandong University

2024 年 10 月 18 日

Outline

Motivation

- The small-x gluon tomography from DVCS is already studied in Hatta, Xiao, Yuan, 2017 using dipole model.

Gluon Tomography from Deeply Virtual Compton Scattering at Small-x

Yoshitaka Hatta,[1] Bo-Wen Xiao,[2] and Feng Yuan[3]

[1] *Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

[2] *Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China*

[3] *Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*

Motivation

- The small-x gluon tomography from DVCS is already studied in Hatta, Xiao, Yuan, 2017 using dipole model.

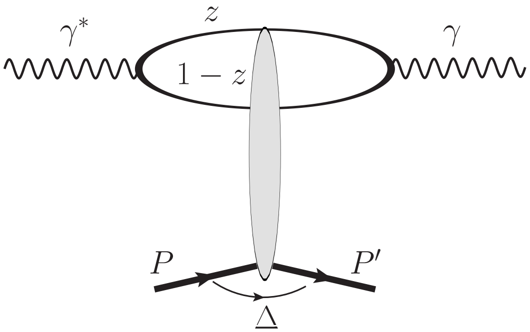

图: DVCS in dipole framework of small-x

Motivation

- The information of small-x proton is encoded in S-matrix

$$S_x\left(b_\perp + (1-z)r_\perp, b_\perp - zr_\perp\right) \equiv \left\langle \frac{1}{N_c} \mathrm{Tr}\left[U\left(b_\perp + (1-z)r_\perp\right) U^\dagger\left(b_\perp - zr_\perp\right)\right]\right\rangle_x$$

.

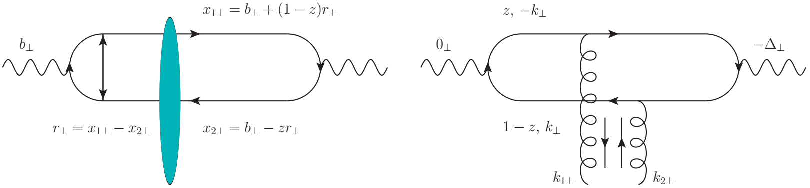

图: DVCS in transverse coordinate and momentum space

- Phenomenologically, we have the Golec-Biernat-Wüsthoff model inspired parametrization as in Mäntysaari, Roy, Salazar, Schenke 2021

$$S_x\left(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp\right) = \exp\left[-\frac{\boldsymbol{r}_\perp^2 Q_{s0}^2}{4} e^{-\boldsymbol{b}_\perp^2/(2B)} C_\phi\left(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp\right)\right],$$

Motivation

We can do Fourier transformation to S-matrix.

$$\text{S-matrix} \quad S\left(b_\perp + (1-z)r_\perp, b_\perp - zr_\perp\right)$$

$$\downarrow r_\perp$$

$$\text{Wigner distribution} \quad W\left(k_\perp, b_\perp\right)$$

$$\swarrow b_\perp \qquad\qquad \searrow \int dk_\perp$$

$$\text{GTMD} \quad F(k_\perp, \Delta_\perp) \qquad \rightarrow \qquad \text{GPD} \quad H_g(b_\perp),\ E_{Tg}(b_\perp), \text{ etc.}$$

$$F_x\left(k_\perp, \Delta_\perp\right) = F_0\left(|k_\perp|, |\Delta_\perp|\right) + 2\cos 2\left(\phi_{k_\perp} - \phi_{\Delta_\perp}\right) F_\epsilon\left(|k_\perp|, |\Delta_\perp|\right) + \cdots,$$

the collinear limit can relate GTMDs to GPDs. The red term is the "elliptic" gluon distribution.

Motivation

- Calculating the cross section, we find (neglecting the Bethe-Heitler contribution)

$$\frac{d\sigma\left(ep\to e'\gamma p'\right)}{dx_B dQ^2 d^2\Delta_\perp} = \frac{\alpha_{em}^3}{\pi x_{Bj} Q^2}\left\{\left(1-y+\frac{y^2}{2}\right)\left(\mathcal{A}_0^2+\mathcal{A}_2^2\right)+2(1-y)\mathcal{A}_0\mathcal{A}_2\cos\left(2\phi_{\Delta l}\right)\right.$$
$$\left.+(2-y)\sqrt{1-y}\left(\mathcal{A}_0+\mathcal{A}_2\right)\mathcal{A}_L\cos\phi_{\Delta l}+(1-y)\mathcal{A}_L^2\right\},$$

where $\mathcal{A}_0$ and $\mathcal{A}_2$ are helicity conserved and helicity-flip amplitudes proposed by Ji-Hoodbhoy,

$$\frac{1}{2}\sum_\lambda \mathcal{A}_T^{\lambda=\lambda'}\left(\Delta_\perp\right)=\mathcal{A}_0,\quad \frac{1}{2}\sum_\lambda \mathcal{A}_T^{\lambda\neq\lambda'}\left(\Delta_\perp\right)=-\mathcal{A}_2\cos 2\phi_{\Delta_\perp},$$

with

$$\mathcal{A}_T^{\lambda,\lambda'}\left(\Delta_\perp\right)=2\int d^2b_\perp e^{ib_\perp\cdot\Delta_\perp}N_c\sum_q\int d^2r_\perp\int_0^1\frac{dz}{4\pi}\Psi_{\gamma^*}^{\lambda}\left(z,r_\perp\right)\Psi_{\gamma}^{\lambda'*}\left(z,r_\perp\right)$$
$$\times\left(1-S\left(b_\perp+(1-z)r_\perp,b_\perp-zr_\perp\right)\right),$$

where $\Psi_{\gamma^*}^{\lambda}\left(z,r_\perp\right)$ and $\Psi_{\gamma}^{\lambda'*}\left(z,r_\perp\right)$ are wavefunctions of virtual photon and photon, showed explicitly in Bartels, Golec-Biernat, Peters, 2003

Motivation

- For the incoming virtual photon, it is given by

$$\Psi^{T\lambda}_{\gamma^*\alpha\beta}(z, r_\perp) = \frac{ie_q}{\pi}\epsilon_q K_1(\epsilon_q |r_\perp|)\begin{cases} \frac{r_\perp\cdot\epsilon_\perp^{(1)}}{|r_\perp|}\left[\delta_{\alpha+}\delta_{\beta+}z - \delta_{\alpha-}\delta_{\beta-}(1-z)\right], & \lambda = 1, \\ \frac{r_\perp\cdot\epsilon_\perp^{(2)}}{|r_\perp|}\left[\delta_{\alpha-}\delta_{\beta-}z - \delta_{\alpha+}\delta_{\beta+}(1-z)\right], & \lambda = 2, \end{cases}$$

$$\Psi^{L}_{\gamma^*\alpha\beta}(z, r_\perp) = \frac{e_q z(1-z)Q}{\pi}K_0(\epsilon_q |r_\perp|)\,\delta_{\alpha\beta},$$

where α and β are the quark and antiquark helicities, e_q is the electric charge of the quark (in units of e) and $\epsilon_q^2 = z(1-z)Q^2$. The quark mass has been neglected. For the outgoing real photon, we have

$$\Psi^{T\lambda}_{\gamma\alpha\beta}(z, r_\perp) = e_q\frac{i}{\pi}\begin{cases} \frac{r_\perp\cdot\epsilon_\perp^{(1)}}{r_\perp^2}\left[\delta_{\alpha+}\delta_{\beta+}z - \delta_{\alpha-}\delta_{\beta-}(1-z)\right], & \lambda = 1, \\ \frac{r_\perp\cdot\epsilon_\perp^{(2)}}{r_\perp^2}\left[\delta_{\alpha-}\delta_{\beta-}z - \delta_{\alpha+}\delta_{\beta+}(1-z)\right], & \lambda = 2. \end{cases}$$

Motivation

- The relationship between the amplitudes and GTMDs is given by

$$\mathcal{A}_0(\Delta_\perp) \approx \sum_\sigma e_q^2 N_c \int_0^1 dz\left[z^2+(1-z)^2\right]\int d^2k_\perp \ln\left[1+\frac{(k_\perp+\delta_\perp)^2}{z(1-z)Q^2}\right] F_0\left(|k_\perp|,|\Delta_\perp|\right),$$

where $\delta_\perp \equiv \frac{1-2z}{2}\Delta_\perp$. In the collinear limit, it can reduce to the situation of GPD

$$\mathcal{A}_0 = \sum_q \frac{e_q^2\alpha_s}{Q^2} xH_g(x,\Delta_\perp)\left[-\frac{1}{\varepsilon}+\ln\frac{Q^2}{\mu^2}-2\right],$$

providing

$$xH_g(x,\Delta_\perp) = \frac{2N_c}{\alpha_s}\int d^2q_\perp q_\perp^2 F_0$$

- As well as,

$$\mathcal{A}_2\left(\Delta_\perp\right) = -8\pi \sum_q e_q^2 N_c \int_0^1 dz z(1-z)$$
$$\times \int_0^\infty q_\perp dq_\perp \left[H_{02}\left(q_\perp, \delta_\perp\right) F_0\left(q_\perp, \Delta_\perp\right) + H_{20}\left(q_\perp, \delta_\perp\right) F_\epsilon\left(q_\perp, \Delta_\perp\right)\right]$$

where

$$H_{02}\left(q_\perp, \delta_\perp\right) \equiv \int_0^\infty dr_\perp \epsilon_q K_1\left(\epsilon_q r_\perp\right) J_0\left(q_\perp r_\perp\right) J_2\left(\delta_\perp r_\perp\right),$$
$$H_{20}\left(q_\perp, \delta_\perp\right) \equiv \int_0^\infty dr_\perp \epsilon_q K_1\left(\epsilon_q r_\perp\right) J_2\left(q_\perp r_\perp\right) \left[J_0\left(\delta_\perp r_\perp\right) + J_4\left(\delta_\perp r_\perp\right)\right].$$

We further take the collinear limit $Q^2 \gg q_\perp^2$ and arrive at

$$\mathcal{A}_2\left(\Delta_\perp\right) = -\sum_q \frac{e_q^2 N_c}{Q^2} \int d^2 q_\perp q_\perp^2 F_\epsilon\left(q_\perp, \Delta_\perp\right) = -\sum_q \frac{e_q^2 \alpha_s \Delta_\perp^2}{4M^2 Q^2} x E_{Tg}\left(x, \Delta_\perp\right),$$

which is consistent with Hoodbhoy, Ji, 1998.

Numerical results for DVCS

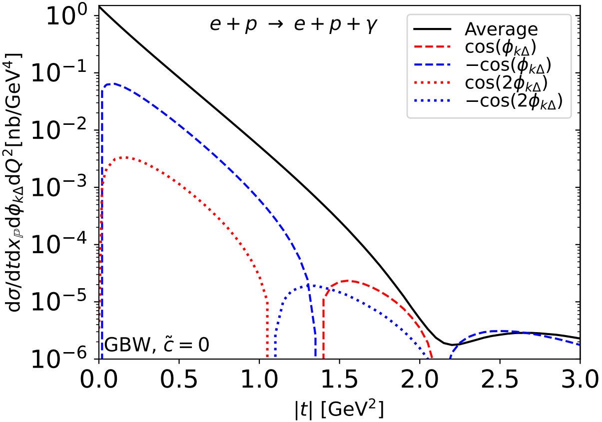

图: Cross section of DVCS

Mantysaari, Roy, Salazar, Schenke, 2021

Switch to TCS

- For TCS, the situation is quite similar. This is almost the inversed process, with two main difference. i) different kinematical variables, ii) virtual photon is time-like, leading to complex amplitudes.

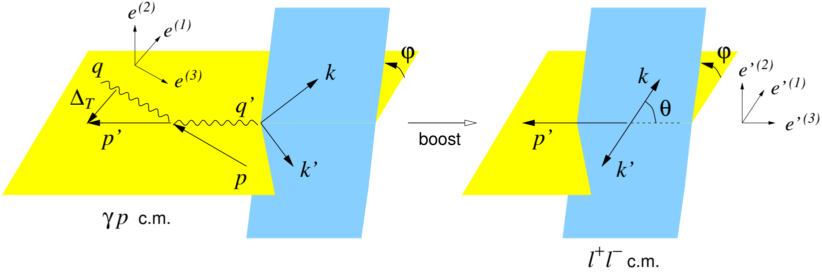

图: Presentation of azimuthal angle in TCS.

Pire, Szymanowski, Wagner, 2009

- The dilepton production from TCS was addressed in ultraperipheral collisions (UPCs) at fixed-target experiment, like AFTER@LHC.

Numerical calculation of dipole amplitudes

$$\mathcal{A}_T^{\lambda,\lambda'}(\Delta_\perp) = 2\int d^2b_\perp e^{ib_\perp\cdot\Delta_\perp} N_c \sum_q \int d^2r_\perp \int_0^1 \frac{dz}{4\pi} \Psi_{\gamma^*}^{\lambda}(z, r_\perp)\, \Psi_{\gamma}^{\lambda' *}(z, r_\perp)$$
$$\times\left(1 - S\left(b_\perp + (1-z)r_\perp, b_\perp - zr_\perp\right)\right),$$

so that,

$$\mathcal{A}_0 = \frac{1}{2}\sum_\lambda \mathcal{A}_T^{\lambda=\lambda'}(\Delta_\perp)$$
$$= \sum_q \frac{e_q^2 N_c}{\pi(2\pi)^2}\int_0^1 dz\left[z^2 + (1-z)^2\right]\int d^2b_\perp e^{i\Delta_\perp\cdot b_\perp}\int \frac{d^2r_\perp}{r_\perp}\epsilon_q K_1(\epsilon_q r_\perp)$$
$$\left[e^{-i\delta_\perp\cdot r_\perp} D_Y(r_\perp, b_\perp)\right],$$

where $\epsilon_q = \sqrt{-z(1-z)M^2}$ and $K_1(\epsilon_q r_\perp)$ are complex comparing with DVCS where they're real, and $D_Y(r_\perp, b_\perp) = 1 - \exp\left(-\frac{r_\perp^2 Q_{s0}^2}{4}e^{-b^2/(2B)}\right)$ is depicted in GBW model.

Numerical results for TCS amplitudes

We can compute $\mathcal{A}_2$ and $\mathcal{A}_L$ similarly. For TCS, it should have a likely form with DVCS. Given $Q_{s0} = 1\mathrm{GeV}^2, B = 4\mathrm{GeV}^{-2}, M = 5\mathrm{GeV}, N_c = 3$

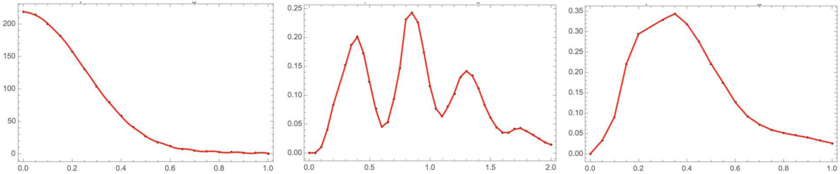

图: TCS helicity amplitudes vs transverse momentum $\Delta_\perp$

Conventions

The hadronic tensor can be decomposed as $\mathcal{M}^{\mu\nu} = \mathcal{M}^{\mu\nu}_{TT} + \mathcal{M}^{\mu\nu}_{TL} + \mathcal{M}^{\mu\nu}_{LL}$, and the lepton tensor $L_{\mu\nu} = 2\left(l_{1\mu} l_{2\nu} + l_{1\nu} l_{2\mu} - g_{\mu\nu} l_1 \cdot l_2\right)$.

- Define two sets of linear independent tensors in transverse plane:

$$g_{\perp}^{\mu\nu} = -g^{\mu\nu} + (\hat{p}^{\mu}\hat{n}^{\nu} + \hat{p}^{\nu}\hat{n}^{\mu})/\hat{p}\cdot\hat{n},$$

$$h_{\perp}^{\mu\nu} = -g_{\perp}^{\mu\nu} - 2\Delta_{\perp}^{\mu}\Delta_{\perp}^{\nu}/\Delta_{\perp}^{2},$$

$$\hat{h}_{\perp}^{\mu\nu} = -g_{\perp}^{\mu\nu} - 2l_{\perp}^{\mu}l_{\perp}^{\nu}/l_{\perp}^{2},$$

  satisfying $\hat{h}_{\perp}^{\mu\nu} h_{\perp\mu\nu} = 2\cos 2\phi$ showing azimuthal angle dependence.
- We can write down

$$\mathcal{A}_{T}^{\mu\nu}(\Delta_{\perp}) = g_{\perp}^{\mu\nu}\mathcal{A}_0(\Delta_{\perp}) + h_{\perp}^{\mu\nu}\mathcal{A}_2(\Delta_{\perp}),$$

$$L_{\mu\nu} = 2\left[(l\cdot l' - l_{\perp}\cdot l'_{\perp})g_{\perp\mu\nu} - l_{\perp}\cdot l'_{\perp}\hat{h}_{\perp\mu\nu}\right].$$

In this case, $\mathcal{M}_{TT}^{\mu\nu} = s^2 g_{\perp\alpha\beta}\mathcal{A}_T^{\mu\alpha}(\mathcal{A}_T^{\nu\beta})^*$, where $s = (q+P)^2 = 2EM_P$,

- After contracting,

$$\mathcal{M}_{TT}^{\mu\nu}L_{\mu\nu} = 4s^2\left[(m^2 - l_\perp^2)\cosh(\eta_1 - \eta_2)(|\mathcal{A}_0|^2 + |\mathcal{A}_2|^2) + l_\perp^2(\mathcal{A}_0\mathcal{A}_2^* + \mathcal{A}_2\mathcal{A}_0^*)\cos 2\phi\right],$$

where η_1 and η_2 are rapidities of electrons. Comparing with DVCS

$$\mathcal{M}_{TT}^{\mu\nu}L_{\mu\nu} \propto \left(1 - y + \frac{y^2}{2}\right)(\mathcal{A}_0^2 + \mathcal{A}_2^2) + (1-y)^2\mathcal{A}_0\mathcal{A}_2\cos(2\phi_{\Delta l}),$$

where $y = q\cdot P/(l\cdot P)$.

Transverse + Longitudinal

In this case, $\mathcal{M}_{TL}^{\mu\nu} = -2s^2 \,\mathrm{Re} \sum_\lambda \epsilon_\mu^{T(\lambda)} \epsilon_{\mu'}^{T(\lambda)*} g_{\perp\alpha\beta} \mathcal{A}_T^{\mu'\alpha} \left(\mathcal{A}_L^{\nu'\beta}\right)^* \epsilon_{\nu'}^L \epsilon_\nu^L$,

- we notice that $\sum_\lambda \epsilon_\mu^{T(\lambda)} \epsilon_{\mu'}^{T(\lambda)*} = g_{\perp\mu\mu'}$ and $\mathcal{A}_L^{\nu'\beta} \epsilon_{\nu'}^L = \frac{\Delta_\perp^\beta}{|\Delta_\perp|} \mathcal{A}_L$, so we can derive

$$\mathcal{M}_{TL}^{\mu\nu} = -2s^2 \mathrm{Re} \left[g_{\perp\mu\mu'} g_{\perp\alpha\beta} \mathcal{A}_T^{\mu'\alpha} \frac{\Delta_\perp^\beta}{|\Delta_\perp|} \epsilon_\nu^L \mathcal{A}_L^* \right],$$

$$= 2s^2 \mathrm{Re} \left[\frac{\Delta_{\perp\mu}}{|\Delta_\perp|} (\mathcal{A}_0 + \mathcal{A}_2) \epsilon_\nu^L \mathcal{A}_L^* \right].$$

Then

$$\mathcal{M}_{TL}^{\mu\nu} L_{\mu\nu} \propto \Delta_{\perp\mu} (\alpha q^\nu + \beta P^\nu + k \Delta_\perp^\nu)(A l_\perp^\mu l_\perp^\nu + B g_\perp^{\mu\nu}) \propto A \cos 2\phi + B,$$

comparing with DVCS

$$\mathcal{M}_{TL}^{\mu\nu} L_{\mu\nu} \propto \Delta_{\perp\mu} (\alpha l^\nu + \beta P^\nu)(A l_\perp^\mu l_\perp^\nu + B g_\perp^{\mu\nu}) \propto A \cos \phi,$$

Transverse + Longitudinal

In this case, $\mathcal{M}_{TL}^{\mu\nu} = -2s^2\,\mathrm{Re}\sum_\lambda \epsilon_\mu^{T(\lambda)}\epsilon_{\mu'}^{T(\lambda)*} g_{\perp\alpha\beta}\mathcal{A}_T^{\mu'\alpha}\left(\mathcal{A}_L^{\nu'\beta}\right)^* \epsilon_{\nu'}^L\epsilon_\nu^L,$

- we notice that $\sum_\lambda \epsilon_\mu^{T(\lambda)}\epsilon_{\mu'}^{T(\lambda)*} = g_{\perp\mu\mu'}$ and $\mathcal{A}_L^{\nu'\beta}\epsilon_{\nu'}^L = \frac{\Delta_\perp^\beta}{|\Delta_\perp|}\mathcal{A}_L$, so we can derive

$$\begin{aligned}\mathcal{M}_{TL}^{\mu\nu} &= -2s^2\mathrm{Re}\left[g_{\perp\mu\mu'}g_{\perp\alpha\beta}\mathcal{A}_T^{\mu'\alpha}\frac{\Delta_\perp^\beta}{|\Delta_\perp|}\epsilon_\nu^L\mathcal{A}_L^*\right], \\ &= 2s^2\mathrm{Re}\left[\frac{\Delta_{\perp\mu}}{|\Delta_\perp|}(\mathcal{A}_0+\mathcal{A}_2)\epsilon_\nu^L\mathcal{A}_L^*\right].\end{aligned}$$

- Finally, we can calculate

$$\begin{aligned}\mathcal{M}_{TL}^{\mu\nu}L_{\mu\nu} &= 4s^2\mathrm{Re}\left[\mathcal{A}_L^*(\mathcal{A}_0+\mathcal{A}_2)\right]\frac{p_z|\Delta_\perp|}{E'M}\left[(l\cdot l' - l_\perp\cdot l'_\perp) - l_\perp\cdot l'_\perp\cos 2\phi\right] \\ &= 4s^2\mathrm{Re}\left[\mathcal{A}_L^*(\mathcal{A}_0+\mathcal{A}_2)\right]\frac{p_z|\Delta_\perp|}{E'M}\left[(m^2 - l_\perp^2)\cosh(y_1-y_2) + l_\perp^2\cos 2\phi\right]\end{aligned}$$

Longitudinal + Longitudinal

In this case,

$$\begin{aligned}\mathcal{M}_{LL}^{\mu\nu} =& s^2 \epsilon_{\mu}^{L*} \epsilon_{\mu'}^{L} g_{\perp\alpha\beta} \mathcal{A}_L^{\mu'\alpha} (\mathcal{A}_L^{\nu'\beta})^* \epsilon_{\nu'}^{L} \epsilon_{\nu}^{L} \\ =& s^2 g_{\perp\alpha\beta} \frac{\Delta_\perp^\alpha \Delta_\perp^\beta}{|\Delta_\perp|^2} \mathcal{A}_L \mathcal{A}_L^* \epsilon_{\mu}^{L*} \epsilon_{\nu}^{L} \\ =& s^2 |\mathcal{A}_L|^2 \epsilon_{\mu}^{L*} \epsilon_{\nu}^{L}.\end{aligned}$$

By the identity $\sum_\lambda \epsilon_\mu^{\lambda *} \epsilon_\nu^{\lambda} = -g^{\mu\nu} + \frac{(q+\Delta)^\mu (q+\Delta)^\nu}{M^2}$, where $(q+\Delta)^\mu$ is momenta of virtual photon. We have,

$$\epsilon_{\mu}^{L*} \epsilon_{\nu}^{L} = \frac{(q+\Delta)^\mu (q+\Delta)^\nu}{M^2} - \frac{(\hat{p}^\mu \hat{n}^\nu + \hat{p}^\nu \hat{n}^\mu)}{\hat{p} \cdot \hat{n}}.$$

So that,

$$\mathcal{M}_{LL}^{\mu\nu} = W^4 |\mathcal{A}_L|^2 \left[\frac{(q+\Delta)^\mu (q+\Delta)^\nu}{M^2} - \frac{(\hat{p}^\mu \hat{n}^\nu + \hat{p}^\nu \hat{n}^\mu)}{\hat{p} \cdot \hat{n}} \right],$$

Longitudinal + Longitudinal

$$
\begin{aligned}
\mathcal{M}_{LL}^{\mu\nu} L_{\mu\nu} &\propto \left[\frac{(q+\Delta)^\mu (q+\Delta)^\nu}{M^2} - \frac{(\hat{p}^\mu \hat{n}^\nu + \hat{p}^\nu \hat{n}^\mu)}{\hat{p}\cdot\hat{n}} \right] (A l_\perp^\mu l_\perp^\nu + B g_\perp^{\mu\nu}) \\
&\propto \Delta_\perp^\mu \Delta_\perp^\nu (A l_\perp^\mu l_\perp^\nu + B g_\perp^{\mu\nu}) \\
&\propto A \cos 2\phi + B
\end{aligned}
$$

Finally,

$$
\mathcal{M}_{LL}^{\mu\nu} L_{\mu\nu} = -W^4 |\mathcal{A}_L|^2 \frac{\Delta_\perp^2}{M^2} \left[(m^2 - l_\perp^2) \cosh(y_1 - y_2) - l_\perp^2 \cos 2\phi \right]
$$

Future and outlook

- Derive the cross section of TCS in forms of Drell-Yan process, W_T, W_L, W_Δ, $W_{\Delta\Delta}$. Check if Lam-Tung relation holds.
- Using more sophisticated CGC EFT framework?
- Thank you!