Probing the quark and gluon OAM at EIC and EicC

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第二届核子三维结构研讨会暨第二届高扭度核子结构研讨会
19th Oct. 2024

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- 质子自旋

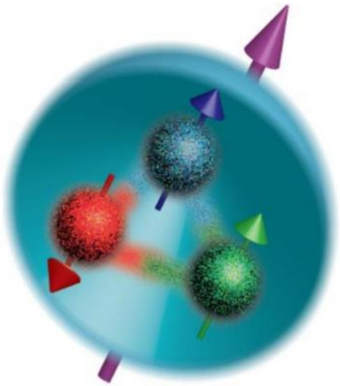
朗德因子:

$$\mu_p = g\mu_B$$

$$g = 2.79$$

Stern, 1933

夸克模型:



Gell-Mann, Zweig, 1960s

质子自旋危机:

EMC, 1988

$$\Delta\Sigma\left(Q^2 = 10.7GeV^2\right) = 0.060 \pm 0.047 \pm 0.069$$

■ 质子自旋

Jaffe-Manohar分解

无穷大坐标系!

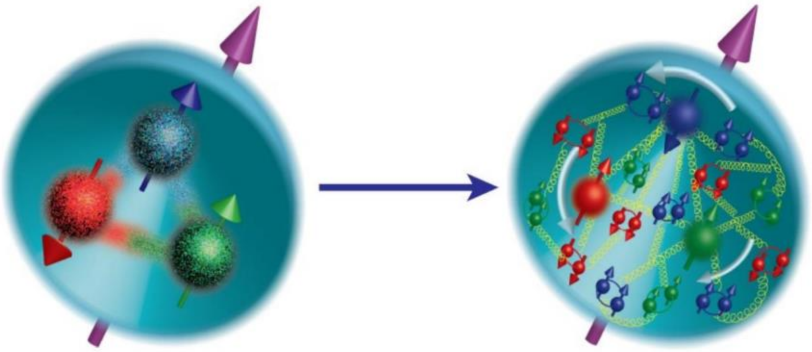
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + l_q + \Delta g + l_g$$

Jaffe, Manohar, 1990

$\Delta\Sigma \approx 0.3$

$\Delta g \approx 0.2$

Via quark and gluon helicity distribution



- 质子自旋

季氏求和规则

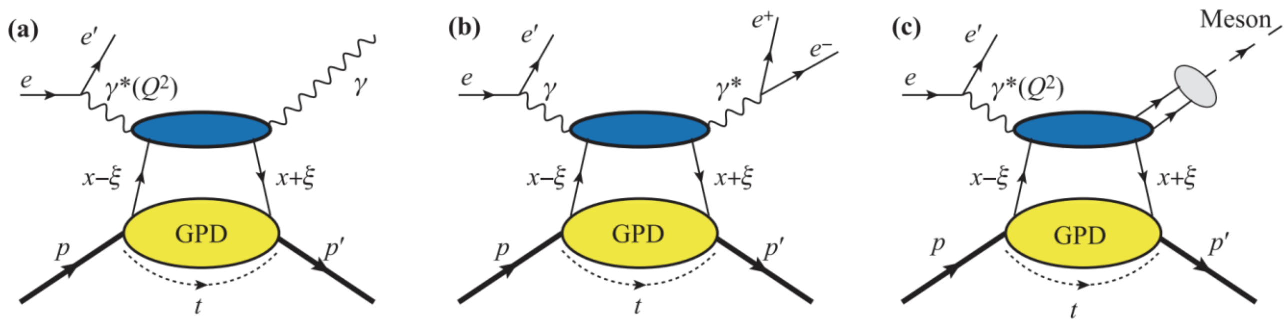
$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

Xiangdong Ji, 1996

$$J_{q,g} = \frac{1}{2}\int_{-1}^{1} dx x[\boxed{H_{q,g}(x,\xi,0)} + \boxed{E_{q,g}(x,\xi,0)}]$$

GPD函数，DVCS，DEMP！

EicC白皮书：



- Wigner函数

量子力学中

$$< A >= \int dx \int dk\, A(x,k) W(x,k)$$

Wigner, 1932

Wigner函数

直观定义:

$$L_z^{q/g} = \int dx \int d^2k d^2b\, (\mathrm{b}_\perp \times k_\perp)_z W(x, b_\perp, k_\perp)$$

Belitsky, Ji, Yuan, 2003

Wigner 函数 $\xrightarrow{\text{傅里叶变换}}$ 广义横动量依赖的部分子分布函数GTMD

$$xL_{g,q}(x,\xi) = -\int d^2k_\perp \frac{k_\perp^2}{M^2} \boxed{F_{1,4}^{g,q}(x,k_\perp,\xi,\Delta_\perp = 0)}$$

Lorce, Pasquini, 2011

$$L_{g,q} = \int_0^1 dx\, L_{g,q}(x, \xi = 0)$$

Wigner算符

$$\widehat{W}^{[\Gamma]}(\vec{b}_\perp,\vec{k}_\perp,x)\equiv\frac{1}{2}\int\frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\,e^{i(xp^+z^- -\vec{k}_\perp\cdot\vec{z}_\perp)}\,\overline{\psi}(y-\tfrac{z}{2})\Gamma\mathcal{W}\,\psi(y+\tfrac{z}{2})\big|_{z^+=0}$$

Wigner函数

$$\Delta = p - p'$$

$$\rho^{[\Gamma]}(\vec{b}_\perp,\vec{k}_\perp,x,\vec{S})\equiv\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,\langle p^+,\tfrac{\vec{\Delta}_\perp}{2},\vec{S}|\widehat{W}^{[\Gamma]}(\vec{b}_\perp,\vec{k}_\perp,x)|p^+,-\tfrac{\vec{\Delta}_\perp}{2},\vec{S}\rangle$$

傅里叶变换:

$$\rho^{[\Gamma]}(\vec{b}_\perp,\vec{k}_\perp,x,\vec{S})=\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,e^{-i\boxed{\vec{\Delta}_\perp\cdot\vec{b}_\perp}}W^{[\Gamma]}(\vec{\Delta}_\perp,\vec{k}_\perp,x,\vec{S})$$

GTMD decomposition

$b_\perp \times k_\perp$

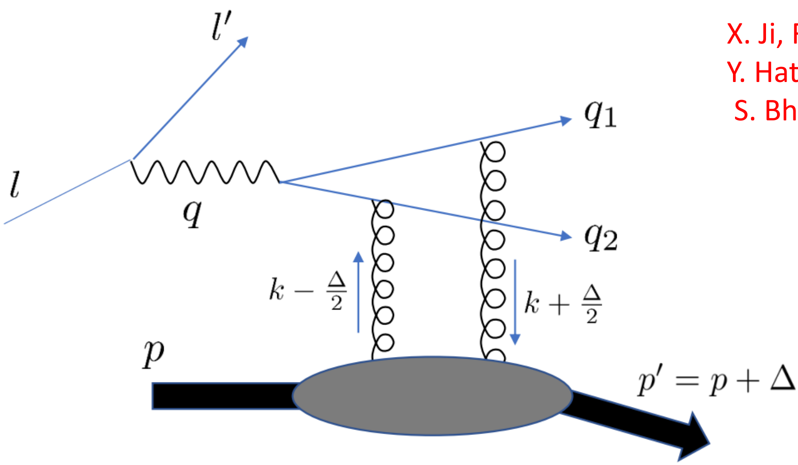
Lorce, Pasquini, 2011

$$W^{q\,[\gamma^+]}_{\lambda,\lambda'}=\frac{1}{2M}\,\bar{u}(p',\lambda')\left[F^q_{1,1}+\frac{i\sigma^{i+}k^i_\perp}{P^+}F^q_{1,2}+\frac{i\,\sigma^{i+}\Delta^i_\perp}{P^+}F^q_{1,3}+\frac{i\sigma^{ij}k^i_\perp\Delta^j_\perp}{M^2}F^q_{1,4}\right]u(p,\lambda)$$

$$W^{q\,[\gamma^+\gamma_5]}_{\lambda,\lambda'}=\frac{1}{2M}\,\bar{u}(p',\lambda')\left[-\frac{i\varepsilon^{ij}_\perp k^i_\perp\Delta^j_\perp}{M^2}G^q_{1,1}+\frac{i\sigma^{i+}\gamma_5 k^i_\perp}{P^+}G^q_{1,2}+\frac{i\sigma^{i+}\gamma_5\Delta^i_\perp}{P^+}G^q_{1,3}+i\sigma^{+-}\gamma_5\,G^q_{1,4}\right]u(p,\lambda)$$

■ 实验探测方案

Gluon $F_{1,4}$



X. Ji, F. Yuan, and Y. Zhao, 2017;
Y. Hatta, Y. Nakagawa, F. Yuan, Y. Zhao, and B. Xiao, 2017;
S. Bhattacharya, R. Boussarie, and Y. Hatta 2022

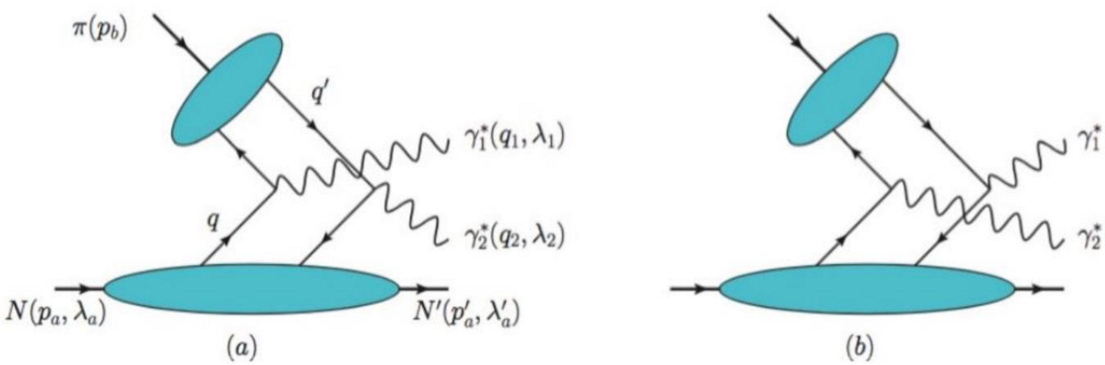
Also spin-orbit correlation $G_{1,1}$ recently!

S. Bhattacharya, R. Boussarie, and Y. Hatta 2024

Quark $F_{1,4}$

$\propto \alpha_e^2$

Only ERBL region



S. Bhattacharya, A. Metz, J. Zhou 2017

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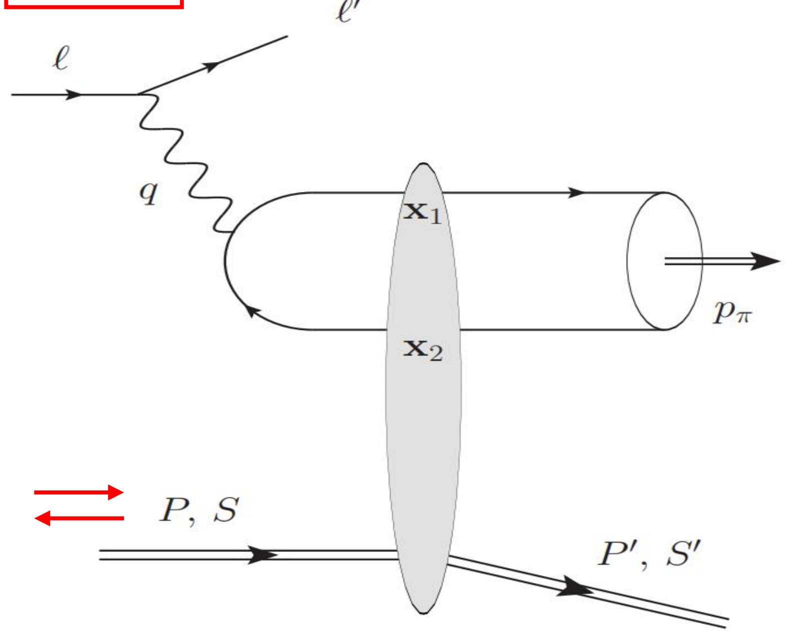
Probing quark OAM via exclusive π^0 production

- $e(l) + \boxed{p(p,\lambda)} \rightarrow \pi^0(l_\pi) + e(l') + p(p',\lambda')$



Unpolarized electron and longitudinally polarized proton

Kinematics:

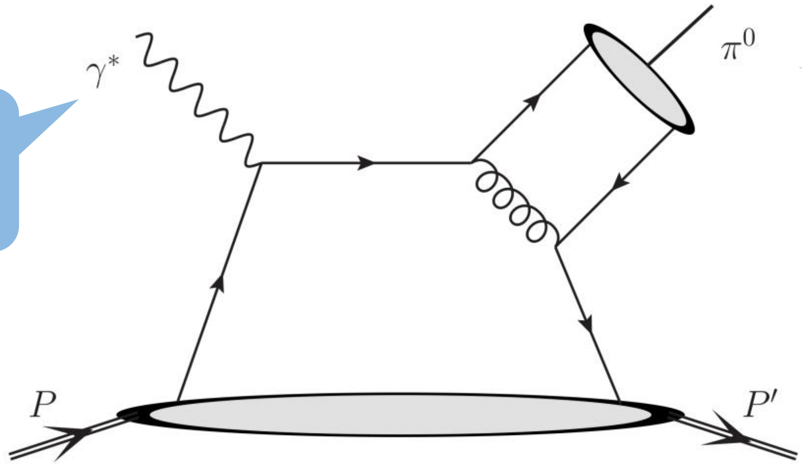
$$Q^2 = -q^2 = -(l - l')^2$$
$$t = (p' - p)^2$$
$$x_B = \frac{Q^2}{2p \cdot q}$$

$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+}$$
$$y = \frac{p \cdot q}{p \cdot l}$$
$$\Delta = p' - p$$

Probing quark OAM via exclusive π^0 production

- $e(l) + p(p,\lambda) \to \pi^0(l_\pi) + e(l') + p(p',\lambda')$

γ_T^* dominated!
Spin-1



Spin-0

Helicity flipped!

$$\frac{d\sigma_T}{dt} = \frac{1}{2\kappa}\left(|M_{0-,++}|^2 + 2|M_{0+,++}|^2\right)$$

$$M_{0-,++} = \frac{e_0}{Q}\sqrt{1-\xi^2}\langle H_T\rangle$$

$$M_{0+,++} = -\frac{e_0}{Q}\frac{\sqrt{-t'}}{4m}\langle \bar{E}_T\rangle.$$

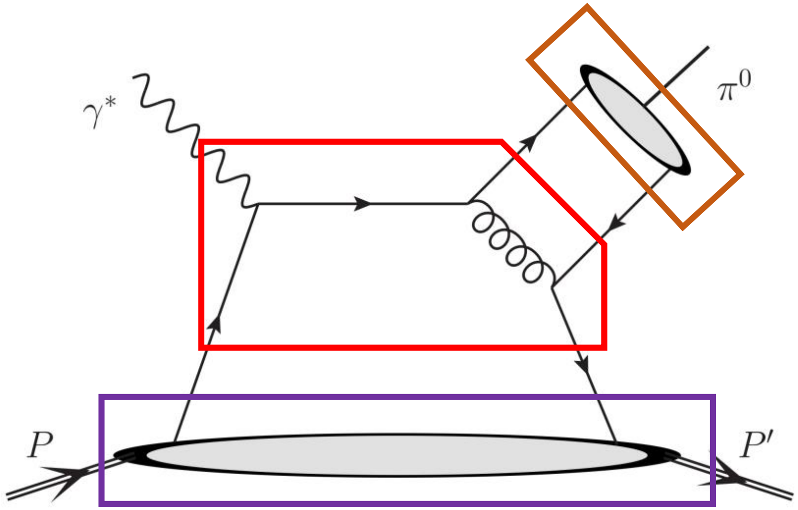
Chiral odd GPDs

Twist-3 pion Distribution Amplitude

L. Frankfurt, P. Pobylitsa, M. Polyakov, and M. Strikman, 1999
S. Goloskokov, Y. p. Xie, X. r. Chen, 2022

Probing quark OAM via exclusive π^0 production

- 散射振幅



$$A \propto \int dx \int d^2k_\perp H(x,\xi,z,k_\perp,\Delta_\perp) f^q(x,\xi,k_\perp\Delta_\perp) \int dz\, \phi_\pi(z)$$

Hard part
Twist-2+twist3+……

Soft part from proton
Twist-2

Pion Distribution Amplitude
Twist-2+twist-3+……

In this work, we used twist-2 pion DA !

Probing quark OAM via exclusive π^0 production

- 散射振幅

Twist-2 GTMDs

Twist-2 pion DA

$$A \propto \int dx \int d^2k_\perp H(x,\xi,z,k_\perp,\Delta_\perp) f^q(x,\xi,k_\perp\Delta_\perp) \int dz\, \phi_\pi(z)$$

$$H(k_\perp,\Delta_\perp) = H(k_\perp=0,\Delta_\perp=0) + \frac{H(k_\perp,\Delta_\perp=0)}{\partial k_\perp^\mu}\Big|_{k_\perp=0} k_\perp^\mu + \frac{H(k_\perp=0,\Delta_\perp)}{\partial \Delta_\perp^\mu}\Big|_{\Delta_\perp=0} \Delta_\perp^\mu$$

Leading twist vanishes

$A \propto \int dk_\perp k_\perp^2$ GTMDs

$A \propto$ GPDs

Leading contribution comes from twist-3!

- 散射振幅

$$\mathcal{M}_1=\frac{g_s^2 e f_\pi}{2\sqrt{2}}\frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}}\delta_{\lambda\lambda'}\frac{\epsilon_\perp\times\Delta_\perp}{Q^2}\{\mathcal{F}_{1,1}+\mathcal{G}_{1,1}\}$$

$$\mathcal{M}_2=\frac{g_s^2 e f_\pi}{2\sqrt{2}}\frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}}\delta_{\lambda,-\lambda'}\frac{M\epsilon_\perp\cdot S_\perp}{Q^2}\{\mathcal{F}_{1,2}+\mathcal{G}_{1,2}\} \qquad S_\perp^\mu = (0^+,0^-,-i,\lambda)$$

$$\mathcal{M}_4=\frac{ig_s^2 e f_\pi}{2\sqrt{2}}\frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}}\lambda\delta_{\lambda\lambda'}\frac{\epsilon_\perp\cdot\Delta_\perp}{Q^2}\{\mathcal{F}_{1,4}+\mathcal{G}_{1,4}\}$$

$$\mathcal{F}_{1,1} = \int_{-1}^{1} dx \frac{x^2\int d^2k_\perp F_{1,1}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times\int_0^1 dz\frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}$$

$$\mathcal{G}_{1,1} = \int_{-1}^{1} dx\int_0^1 dz\frac{\phi_\pi(z)(x^2+2x^2z+\xi^2)}{z^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times\int d^2k_\perp\frac{k_\perp^2}{M^2}G_{1,1}^{u+d}(x,\xi,\Delta_\perp,k_\perp)$$

$$\mathcal{F}_{1,2} = \int_{-1}^{1} dx x\frac{\xi(1-\xi^2)\int d^2k_\perp k_\perp^2 F_{1,2}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times\int_0^1 dz\frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}$$

$$\mathcal{G}_{1,2} = \int_{-1}^{1} dx\int_0^1 dz\frac{\phi_\pi(z)(x^2+2x^2z+\xi^2)(1-\xi^2)}{z^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times\int d^2k_\perp\frac{k_\perp^2}{M^2}G_{1,2}^{u+d}(x,\xi,\Delta_\perp,k_\perp)$$

$$\mathcal{F}_{1,4} = \int_{-1}^{1} dx\frac{x\xi\int d^2k_\perp k_\perp^2 F_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times\int_0^1 dz\frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}$$

$$\mathcal{G}_{1,4} = \int_{-1}^{1} dx\int_0^1 dz\frac{x(4\xi^2z+\xi^2-2x^2z+x^2)}{z^2\xi(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2}\phi_\pi(z) \times\int d^2k_\perp G_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp)$$

■ 散射截面

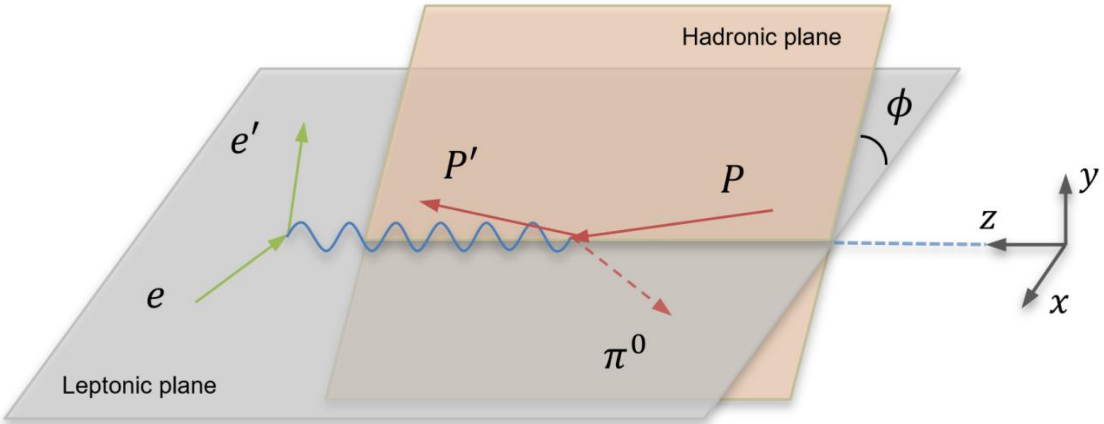
$$\frac{d\sigma_T}{dt dQ^2 dx_B d\phi} = \frac{(N_c^2-1)^2\alpha_{em}^2\alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)} [1+(1-y)^2] \Bigg\{ \Bigg[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2 + 2\frac{M^2}{\Delta_\perp^2}|\mathcal{F}_{1,2}+\mathcal{G}_{1,2}|^2 \Bigg]$$
$$+\cos(2\phi) a \left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2 \right] + \lambda \sin(2\phi)\, 2a\, \mathrm{Re}\left[(i\boxed{\mathcal{F}_{1,4}} + i\mathcal{G}_{1,4})(\mathcal{F}^*_{1,1}+\mathcal{G}^*_{1,1}) \right] \Bigg\}$$

Proton helicity
Polarization dependent!

Distinguished experimental signature!

$$\phi = \phi_{l_\perp} - \phi_{\Delta_\perp}$$

$$a = \frac{2(1-y)}{1+(1-y)^2}$$



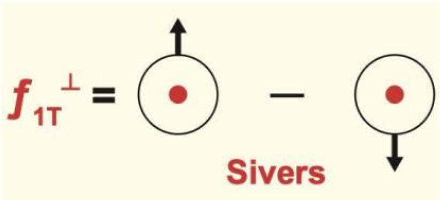
Probing quark OAM via exclusive π^0 production

- 散射截面

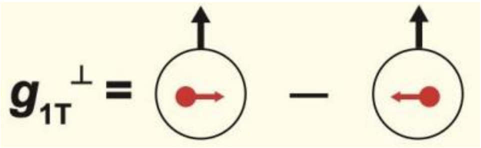
$$\frac{d\sigma_T}{dt\,dQ^2\,dx_B\,d\phi}=\frac{(N_c^2-1)^2\alpha_{em}^2\alpha_s^2 f_\pi^2\xi^3\Delta_\perp^2}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)}\left[1+(1-y)^2\right]\Big\{\Big[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2+2\frac{M^2}{\Delta_\perp^2}|\mathcal{F}_{1,2}+\mathcal{G}_{1,2}|^2\Big]$$
$$+\cos(2\phi)a\left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2\right]+\lambda\sin(2\phi)\,2a\,\mathrm{Re}\left[(i\mathcal{F}_{1,4}+i\mathcal{G}_{1,4})\left(\mathcal{F}_{1,1}^*+\mathcal{G}_{1,1}^*\right)\right]\Big\}$$

Survive at $\Delta_\perp\to 0$

$$\mathrm{Im}[F_{1,2}]|_{\Delta_\perp=0}=-f_{1T}^\perp$$



$$\mathrm{Re}[G_{1,2}]|_{\Delta_\perp=0}=g_{1T}$$



Thus, we can probe spin dependent TMDs through an unpolarized target!

■ 数值计算

$$\mathcal{F}_{1,4} = \int_{-1}^{1} dx \frac{x\xi \int d^2k_\perp k_\perp^2 F_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}$$

GTMDs, GPDs,
Discontinuity at $x=\xi$

DA ϕ_π
Singularity at endpoint

$$\frac{1}{x-\xi+i\epsilon} \to \frac{1}{x-\xi-\langle p_\perp^2\rangle/Q^2+i\epsilon}$$
$$\frac{1}{x+\xi-i\epsilon} \to \frac{1}{x+\xi-\langle p_\perp^2\rangle/Q^2-i\epsilon}$$

$$\int_0^1 dz \to \int_{\langle p_\perp^2\rangle/Q^2}^{1-\langle p_\perp^2\rangle/Q^2} dz$$

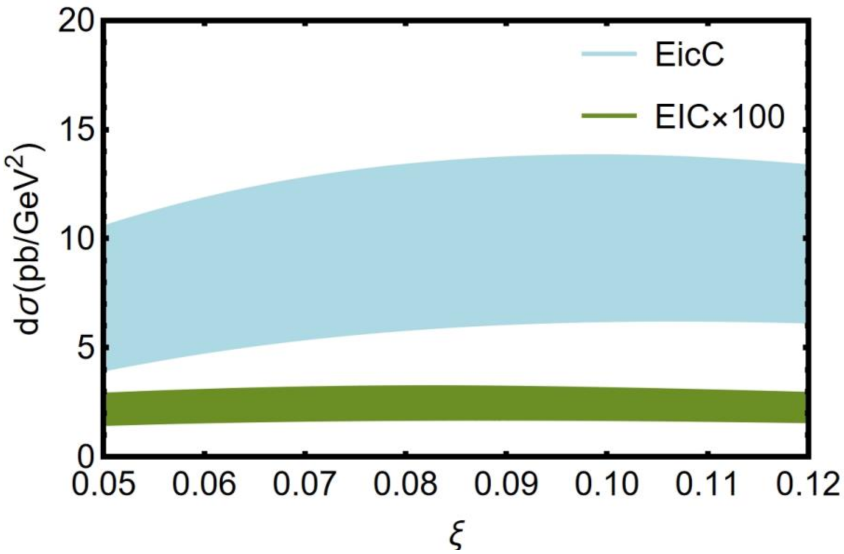
S. V. Goloskokov and P. Kroll, 2005

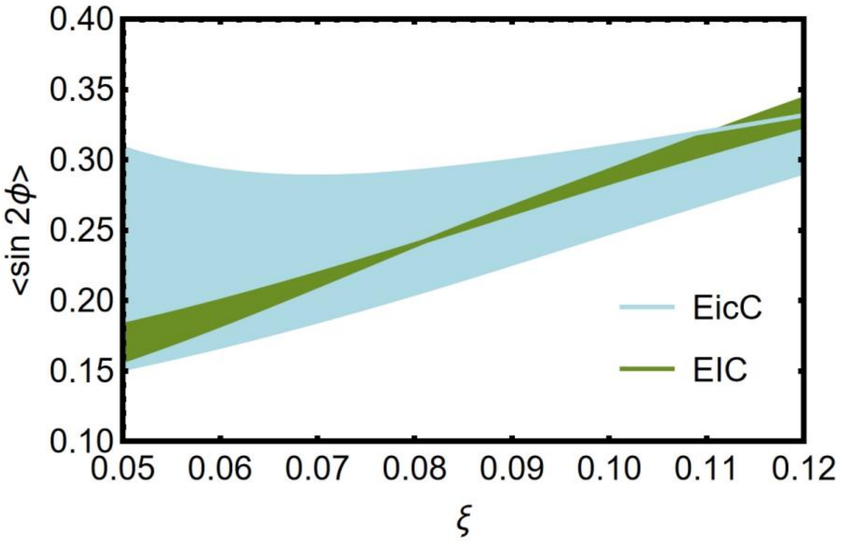
I. V. Anikin, O. V. Teryaev, 2003

Probing quark OAM via exclusive π^0 production

■ 数值计算

	$\sqrt{s_{ep}}$(GeV)	$Q^2(\mathrm{GeV}^2)$
EIC	100	10
EicC	16	3





$$\langle \sin(2\phi) \rangle = \frac{\int \frac{d\Delta\sigma}{d\mathcal{P}.\mathcal{S}.} \sin(2\phi)\ d\mathcal{P}.\mathcal{S}.}{\int \frac{d\sigma}{d\mathcal{P}.\mathcal{S}.} d\mathcal{P}.\mathcal{S}.}$$

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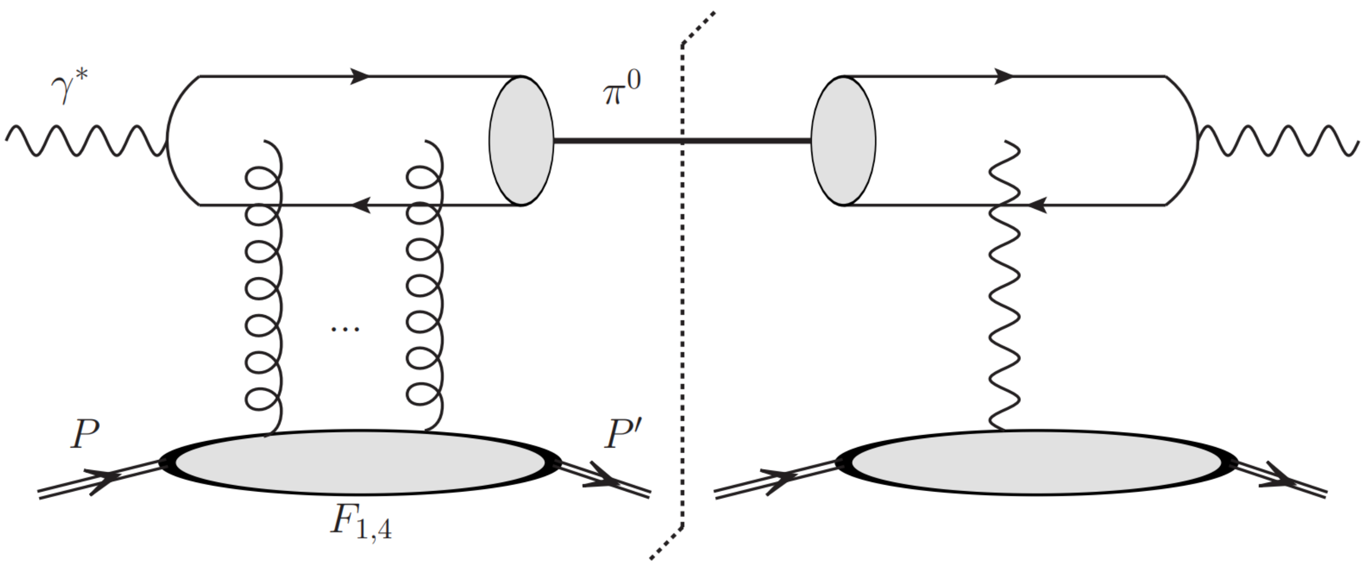
Constraining gluon OAM via exclusive π^0 production

- $e(l) + p(p, \lambda) \rightarrow \pi^0(l_\pi) + e(l') + p(p', \lambda')$

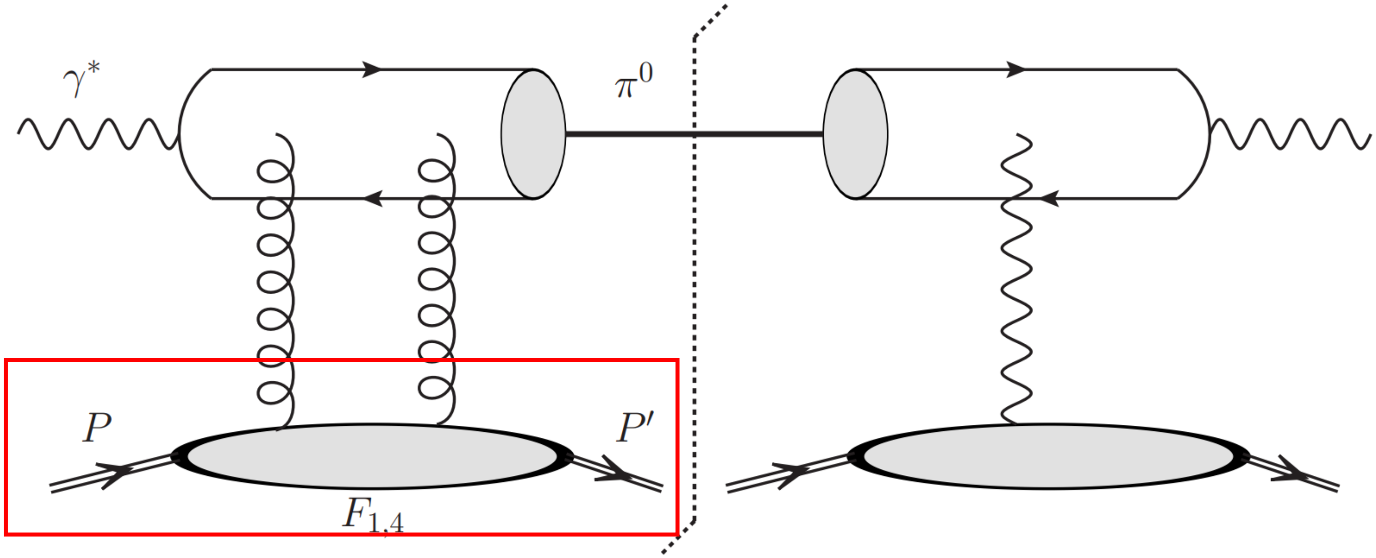
$\xi \ll 1$ ➡ Gluon channel

$t \approx 0$ ➡ Primakoff process



Using interference term to detect gluon $F_{1,4}$!

- Gluon channel



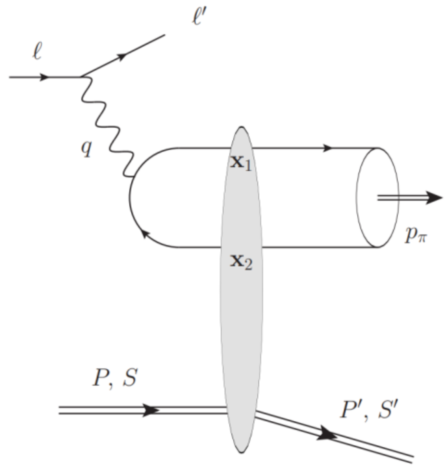
$$\mathcal{M}_L \propto \int d^2k_\perp \left\{ H(x,\xi,k_\perp)\Big|_{k_\perp=0} + \frac{\partial H(x,\xi,k_\perp)}{\partial k_\perp^\mu}\Big|_{k_\perp=0} k_\perp^\mu + \dots \right\} k_\perp \times \Delta_\perp \lambda \delta_{\lambda,\lambda'} F_{1,4}^g$$

$$\mathcal{M}_L \propto \int d^2k_\perp (\epsilon_\perp^{\gamma^*} \times k_\perp)(k_\perp \times \Delta_\perp)\lambda\delta_{\lambda,\lambda'} F_{1,4}^g \propto \boxed{(\epsilon_\perp^{\gamma^*} \cdot \Delta_\perp)} \lambda \delta_{\lambda,\lambda'} \int d^2k_\perp \frac{1}{2} k_\perp^2 F_{1,4}^g$$

$$\mathcal{M}_R^* \propto \boxed{\epsilon_\perp^{\gamma^*} \times \Delta_\perp}$$

Constraining gluon OAM via exclusive π^0 production

■ 非极化截面



$$\frac{d\sigma^{odderon}}{dtdQ^2dx_B} \approx \frac{\pi^5\alpha_{em}^2\alpha_s^2 f_\pi^2}{8x_B N_c^2 M^2 Q^6}\left[1-y+\frac{y^2}{2}\right]\left[\int_0^1 dz \frac{\phi_\pi(z)}{z(1-z)}\int d^2k_\perp \frac{k_\perp^2 x \boxed{f_{1T}^{\perp g}(x,k_\perp^2)}}{k_\perp^2+z(1-z)Q^2}\right]^2$$

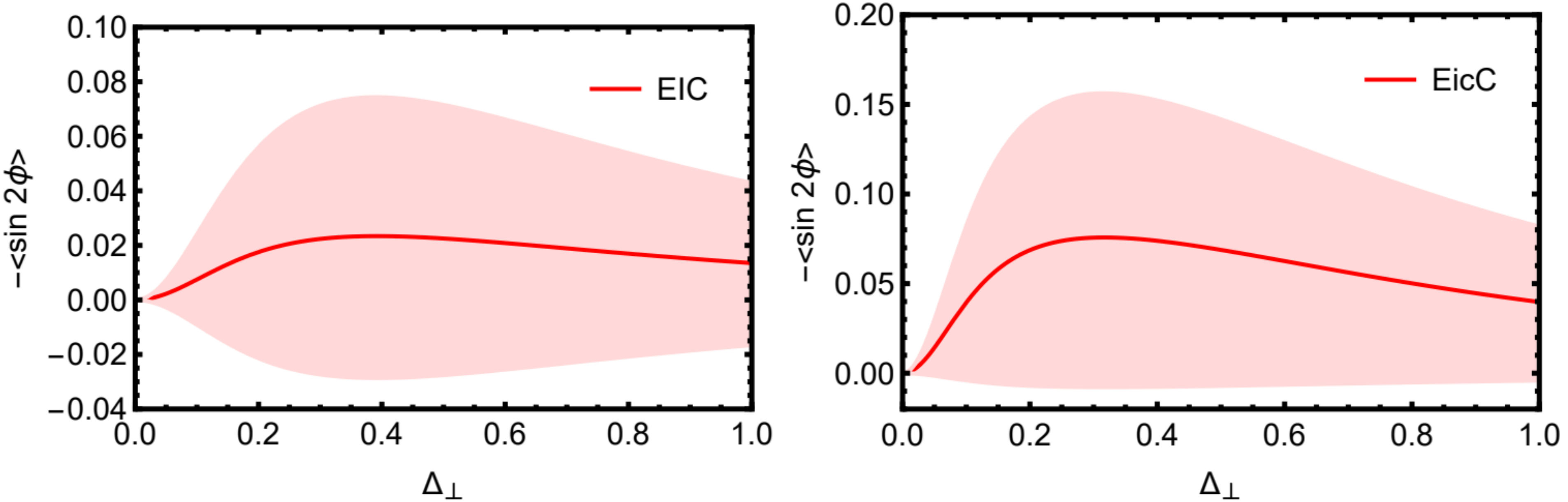
Boussarie, Hatta, Szymanowski, Wallon, 2019

Primakoff过程：

$$\frac{d\sigma^{Pri}}{dtdQ^2dx_B} \approx \frac{\alpha_{em}^4(2\pi)[1+(1-y)^2]f_\pi^2}{x_B Q^6\Delta_\perp^2}\frac{1-\xi}{1+\xi}\mathcal{F}^2(t)\left[\int_0^1 \frac{dz}{6z(1-z)}\phi_\pi(z)\right]^2$$

- Gluon channel

$$\langle \sin(2\phi) \rangle = \frac{\int \frac{d\Delta\sigma}{d\mathcal{P}.\mathcal{S}.} \sin(2\phi)\, d\mathcal{P}.\mathcal{S}.}{\int \frac{d\sigma}{d\mathcal{P}.\mathcal{S}.} d\mathcal{P}.\mathcal{S}.}$$



EIC: $y = 0.02,\ Q^2 = 10\,\mathrm{GeV}^2,\ \sqrt{S_{ep}} = 100\ \mathrm{GeV}$

EicC: $y = 0.5,\ Q^2 = 3\ \mathrm{GeV}^2,\ \sqrt{S_{ep}} = 16\ \mathrm{GeV}$

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Summary

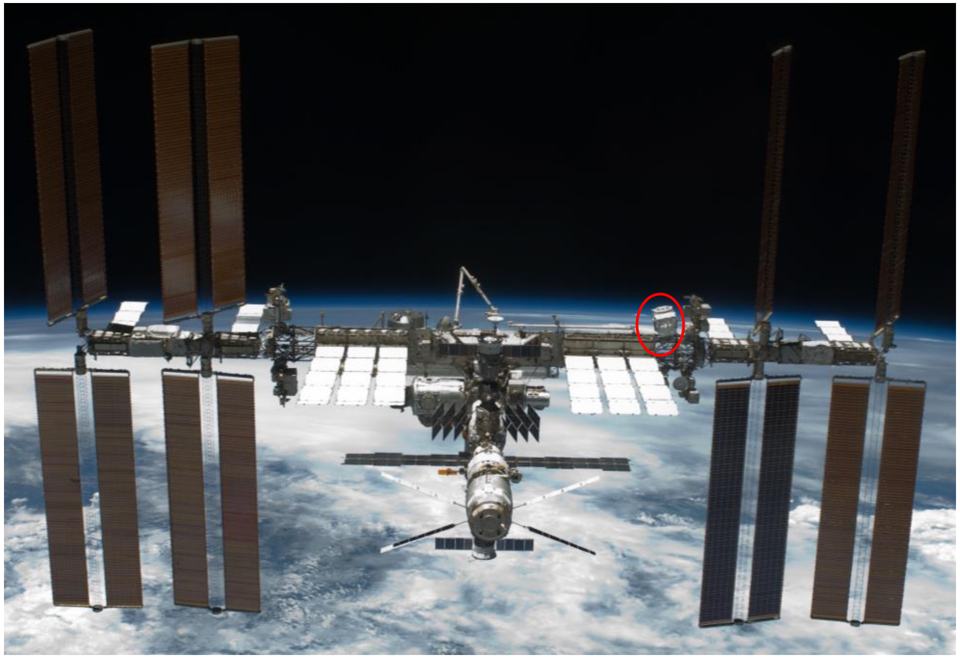
- We propose extracting the quark OAM by measuring the azimuthal angular correlation $\sin 2\phi$ in exclusive π^0 production at EIC and EicC.

- This azimuthal asymmetry is not a power correction, as both the unpolarized and longitudinal polarization-dependent cross sections contribute at twist-3.

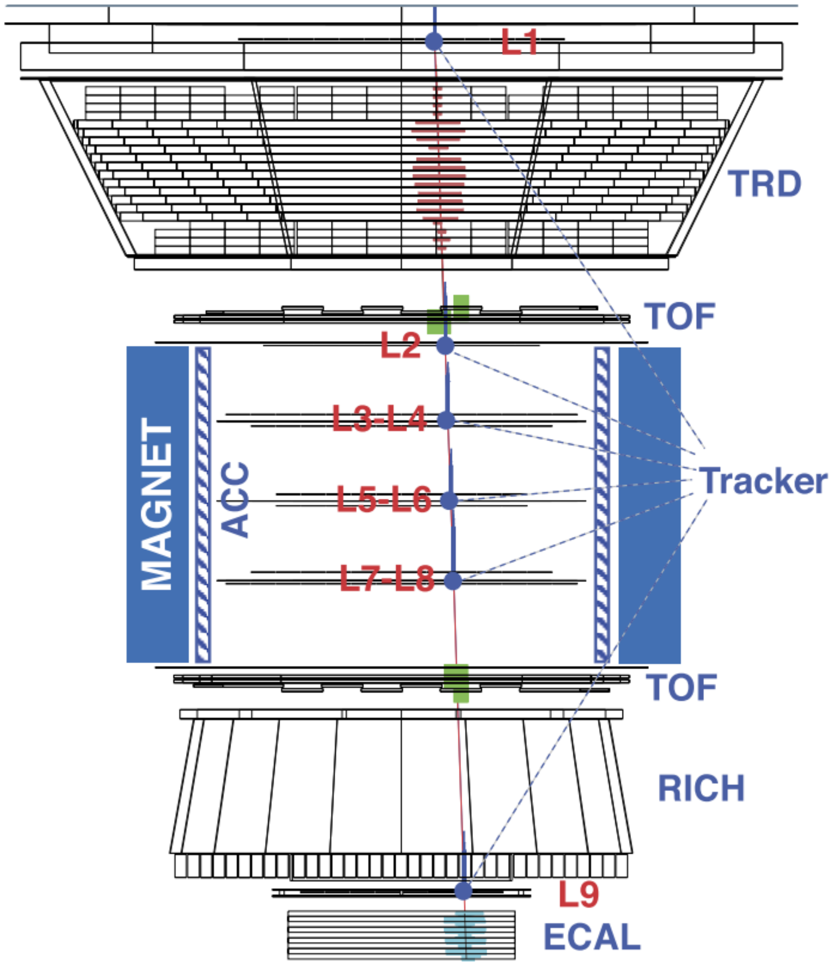
- We propose constraining the gluon OAM in this process by the same azimuthal angular correlation.

Outlook

- AMS

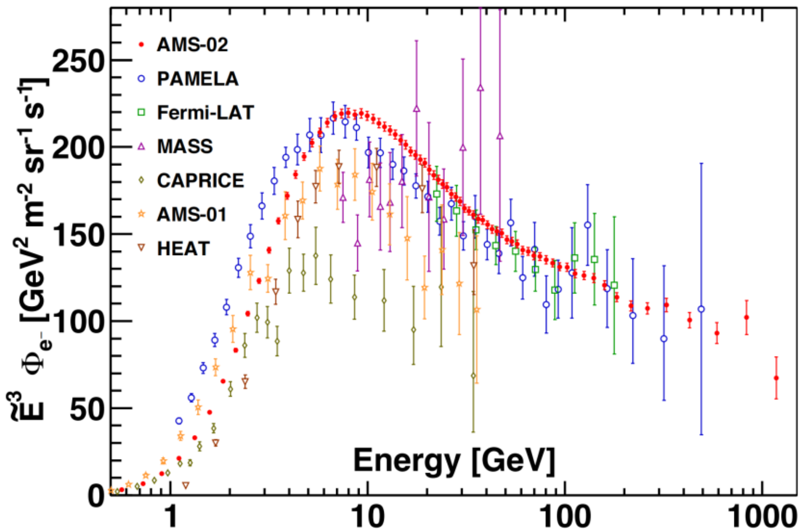


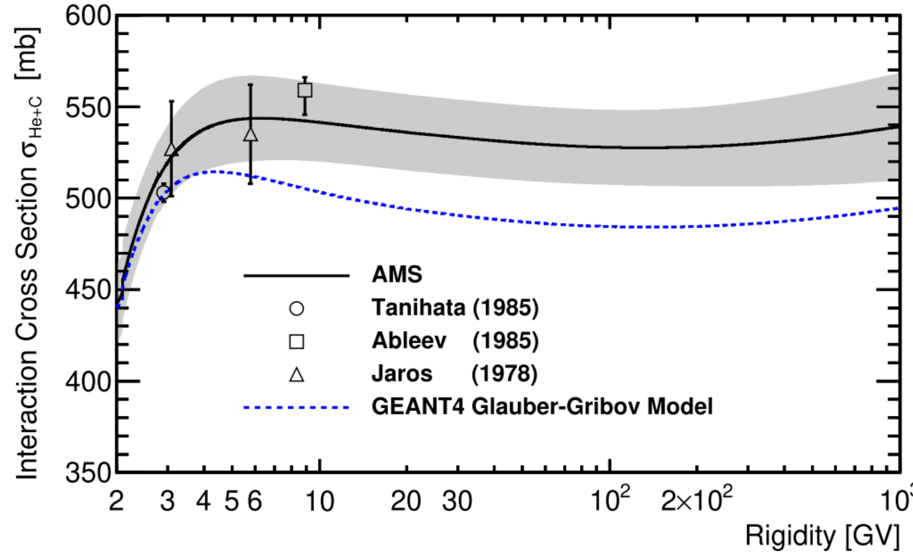
AMS detector on ISS

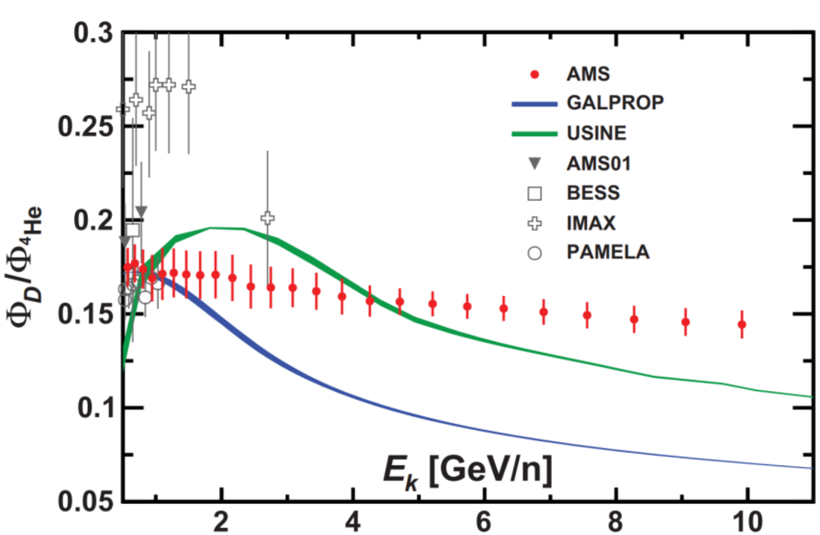


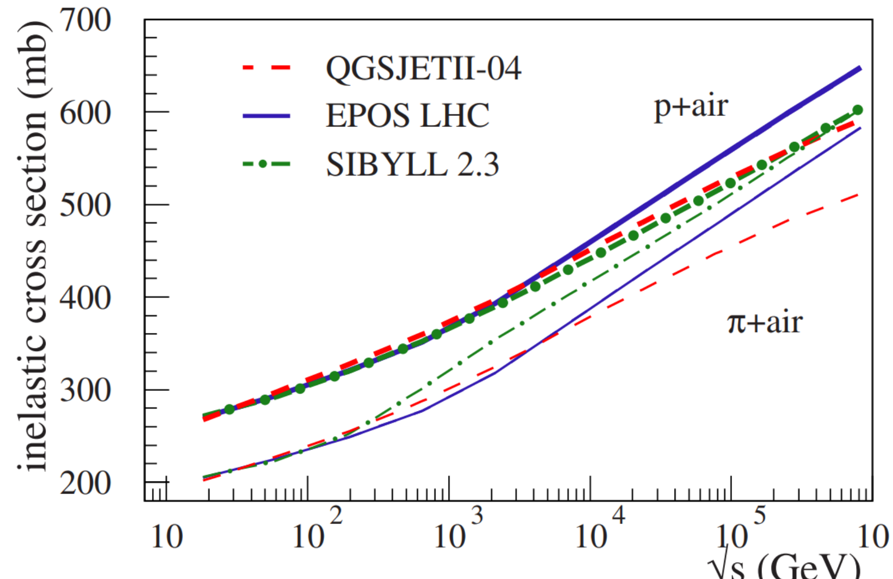
电荷Z，能量E（动量P）

Outlook



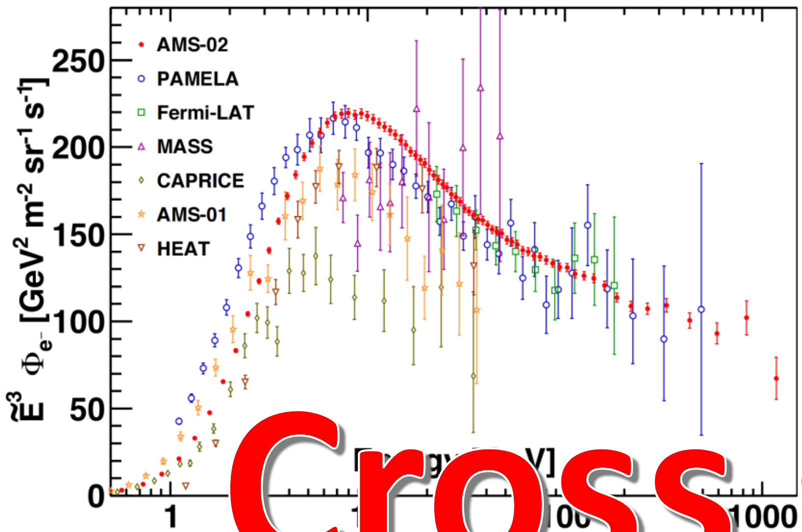


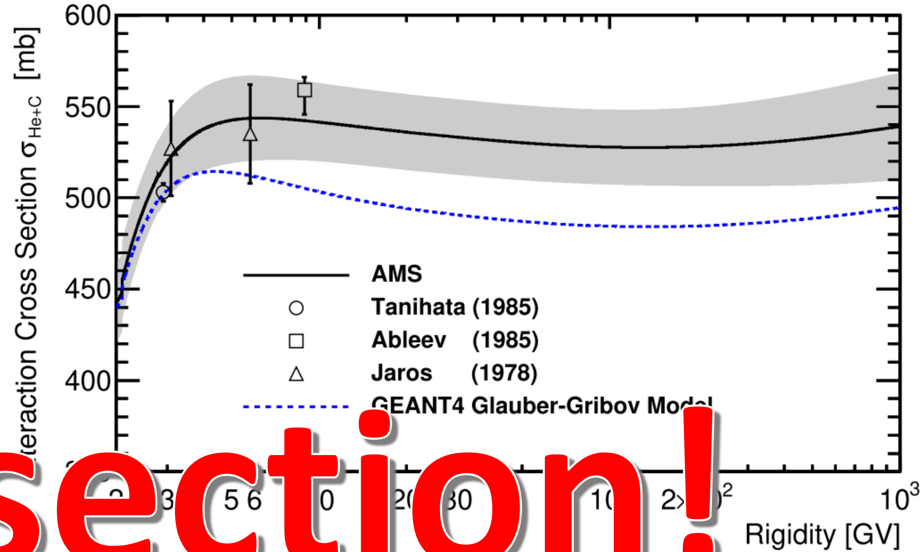




Same case for DPMJET-III!

Outlook





Cross section!