TMD wave functions for pion and soft functions at one-loop in LaMET

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JHEP 09 (2022) 04&PRD109 (2024) 9, L091503

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2024.10.19

第二届核子三维结构研讨会暨第二届高扭度核子结构研讨会 青岛 山东大学

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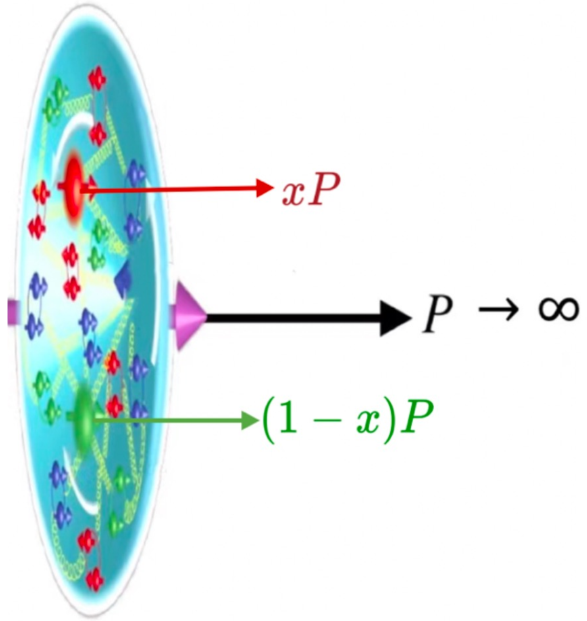
目 录

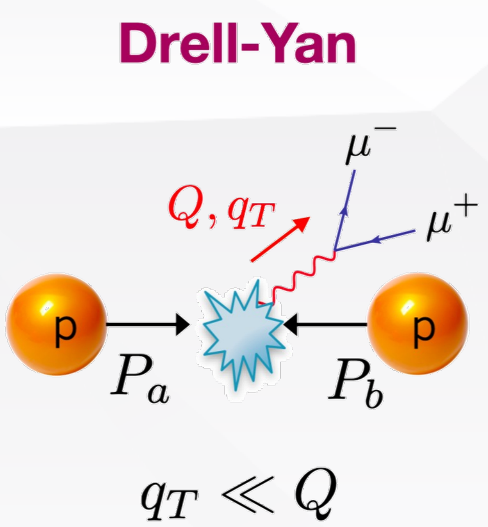
- 费曼 50多年前提出部分子模型，人们通过大量高能实验数据拟合获取了强子结构信息。
- 强子波函数是描述强子中所有部分子动量分布的物理量，反映了强子内部结构。
- 强相互作用基本理论计算强子波函数尤其是横向动量依赖波函数长期以来进展缓慢。



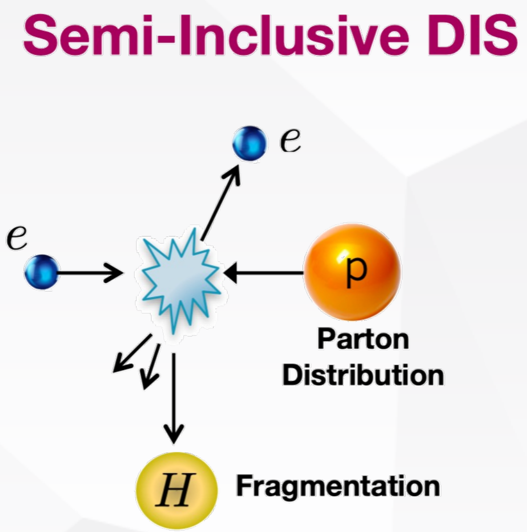
强子波函数缺乏第一性原理的结果！

Transverse-Momentum-Dependent (TMD)

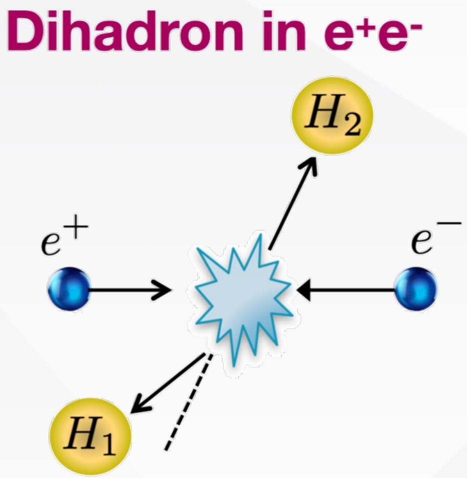
TMD processes:



LHC, FermiLab, RHIC, …



HERMES, COMPASS, JLab, EIC, …



BESIII, Babar, Belle, …

$$\sigma \sim f_{q/P}(x, k_T)\, f_{q/P}(x, k_T)$$

$$\sigma \sim f_{q/P}(x, k_T)\, D_{h/q}(x, k_T)$$

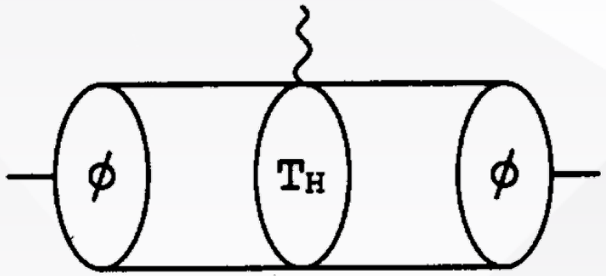
$$\sigma \sim D_{h_1/q}(x, k_T)\, D_{h_2/q}(x, k_T)$$

Transverse-Momentum-Dependent (TMD)

TMD processes:

$$F_\pi(Q^2) = \int_0^1 dx_1\, dx_2 \int d^2\mathbf{k}_{T_1}\, d^2\mathbf{k}_{T_2}\, \psi(x_2, \mathbf{k}_{T_2}, P_2) \times T_H(x_1, x_2, Q, \mathbf{K}_{T_i})\, \psi(x_1, \mathbf{k}_{T_1}, P_1).$$

H.-N. Li, G. Sterman / The pion form factor



G.Peter Lepage, Stanley J. Brodsky, Phys.Rev.D 22 (1980) 2157, 4000+ citations
G.Peter Lepage, Stanley J. Brodsky, Phys.Lett.B 87 (1979) 359-365, 1500+ citations
Stanley J. Brodsky, Hans-Christian Pauli, Stephen S. Pinsky, Phys.Rept. 301 (1998) 299-486, 1500+ citations
H.~n.~Li and G.~F.~Sterman, Nucl. Phys. B 381, 129-140 (1992), 500+ citations
R. Jakob and P.Kroll, Phys. Lett. B315, 463(1993);
N. G. Stefanis, et. al. , Phys. Lett. B449, 299(1999);
...

TMDPDFs/TMDWFs 是非常重要的输入参数!

SJTU

介子波函数

理解重夸克衰变的强项相互作用

$B \to \pi\,\pi$ Phys. Rev. Lett. 83, 1914 (1999) 1452 citations in INSPIRE
$B \to \pi\,K$ Nucl. Phys. B 606, 245 (2001) 1205 citations in INSPIRE
$B \to \pi\,D$ Phys. Rev. D 69, 112002 (2004) 409 citations in INSPIRE

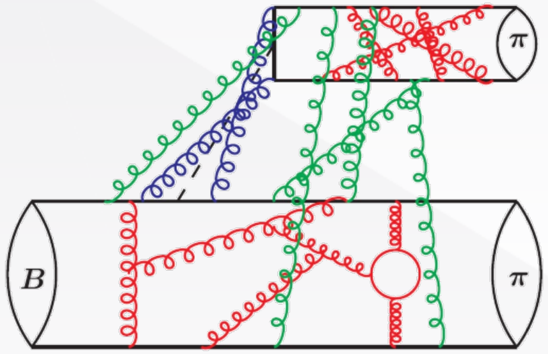
精确测量标准模型参数: Vub

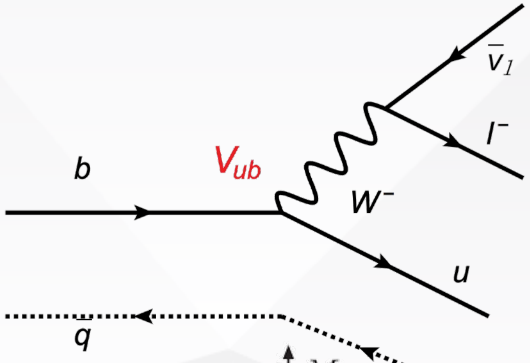
$B \to \pi\,\ell\,\nu$ Phys. Lett. B 633, 61 (2006) 221 citations in INSPIRE

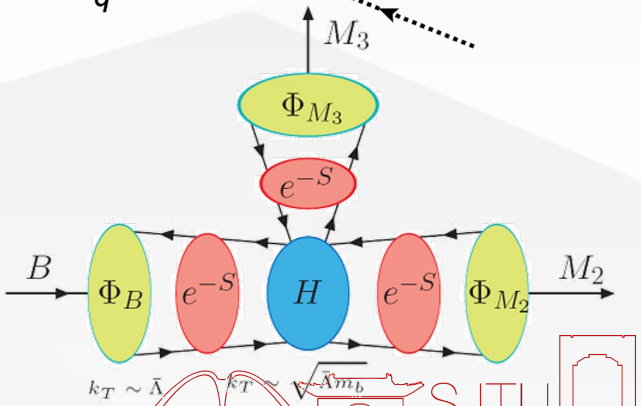
精确测量CP破坏参数：A_{CP}

$A_{CP}(B^+ \to K^+\pi^0)$ $A_{CP}(B^+ \to K^*(892)^+\pi^0)$

$A_{CP}(B^+ \to p\bar{\Lambda}\pi^0)$ $A_{CP}(B^+ \to \rho^+\pi^0)$ $A_{CP}(B^+ \to \pi^+\pi^0)$







介子波函数

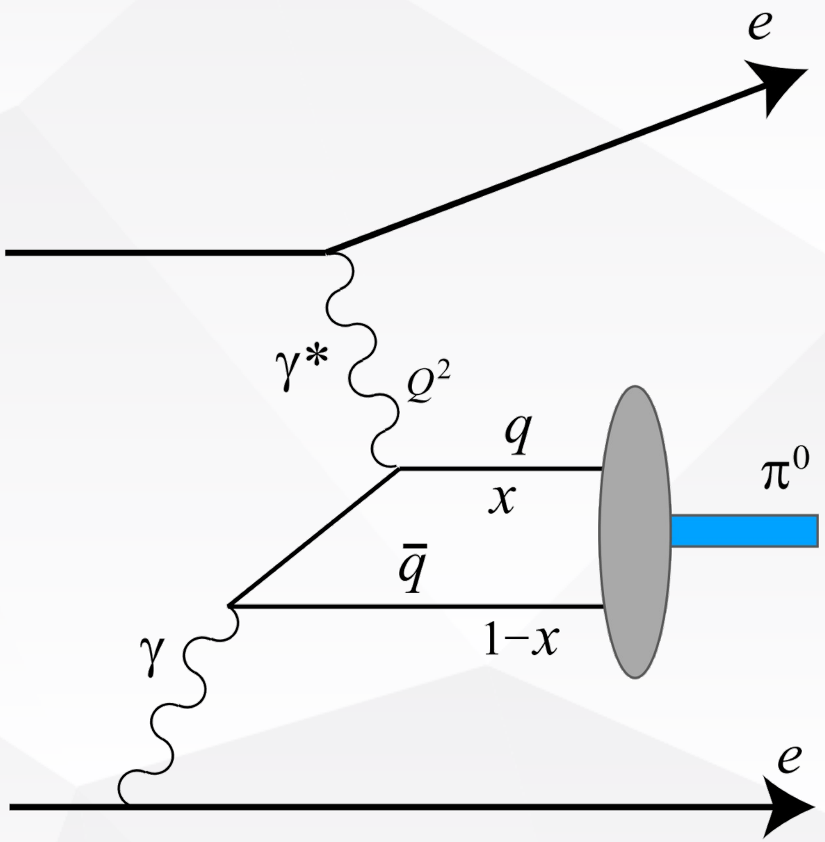
Chernyak, V. L. et al., Meson Wave Functions and SU(3) Symmetry Breaking, Nucl. Phys. B 204, 477 (1982).

CLEO Collaboration, Measurements of the meson - photon transition form-factors of light pseudoscalar mesons at large momentum transfer, PRD, 57 (1998) 33-54 .

BaBar Collaboration, Measurement of the $\gamma\gamma^* \to \pi^0$ transition form factor, PRD, 80 (2009) 052002.

Belle Collaboration, Measurement of $\gamma\gamma^* \to \pi^0$ transition form factor at Belle, PRD, 86 (2012) 092007.

…



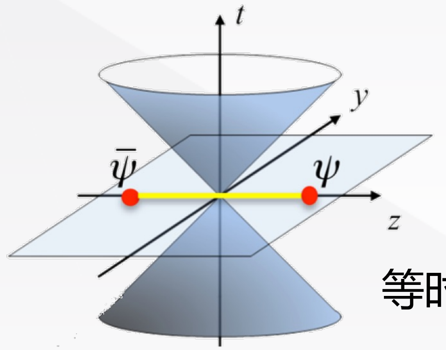
1. 波函数介绍

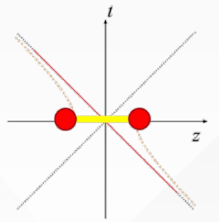
- TMDWF是描述强子中组分动量分布的重要物理量，反映了强子内部的非微扰结构。
- 在跃迁形状因子、B介子弱衰变等过程中发挥着至关重要的作用，这对于检验标准模型(SM)和寻找超越SM的新物理具有重要价值。
- 目前以及未来高能对撞机上高精度测量，强烈要求QCD提高这些非微扰波函数的准确性。
- 大动量有效理论在格点QCD上构造可直接计算的欧氏时空非定域强子算符矩阵元(准分布)，通过匹配的方法得到光锥关联的物理量。

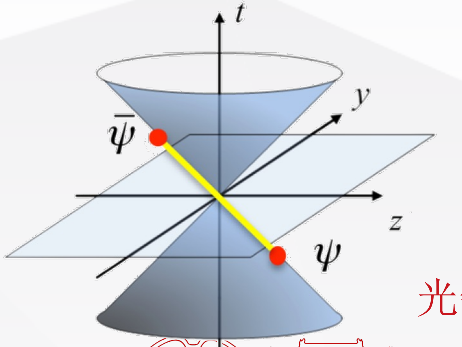
大动量有效理论

- 大动量有效理论(large-momentum effective theory, LaMET)从第一性原理出发，提供了从格点 QCD 计算强子波函数的方法。

- 构造等时强子准波函数(quasi wave function, quasi-WF)，利用微扰可算的匹配系数，通过匹配的方法得到光锥关联的光锥波函数(light-front wave function, LFWF)。



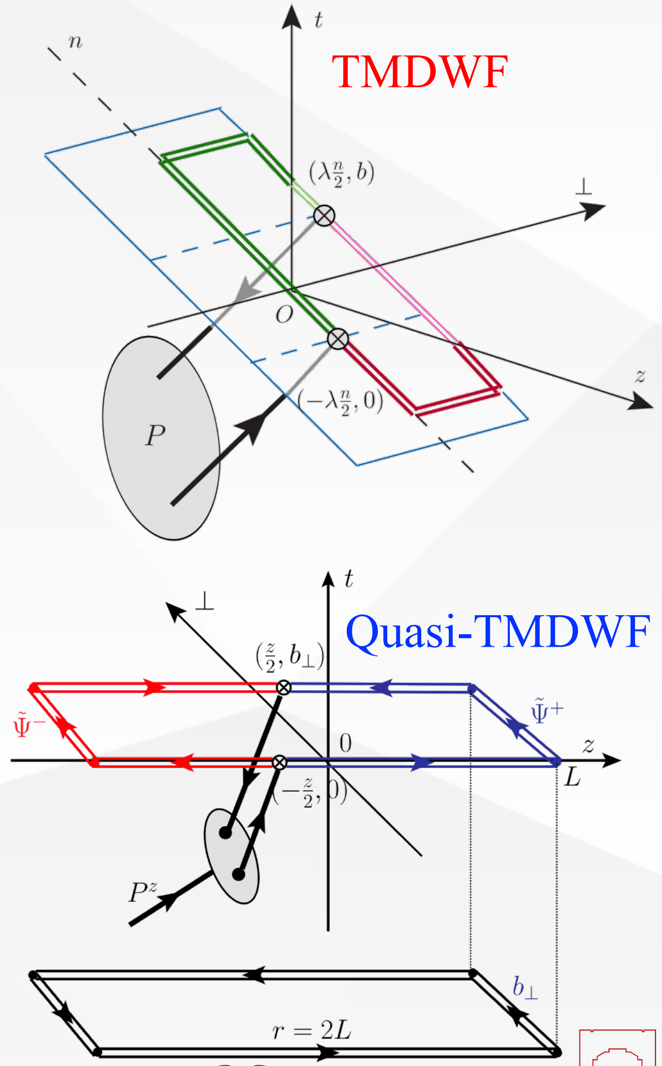




2.TMD波函数与LaMET匹配

$$\psi^{\pm}\left(x, b_{\perp}, \mu, \delta^{-}\right)=\frac{1}{-i f_{\pi} n \cdot P} \int \frac{d(\lambda n \cdot P)}{2 \pi} e^{-i\left(x-\frac{1}{2}\right) \lambda n \cdot P}$$
$$\times\left\langle 0\left|\bar{\Psi}_{n}^{\pm}(\lambda n / 2+\boxed{b}) \gamma^{+} \gamma^{5} \Psi_{n}^{\pm}(-\lambda n / 2)\right| P\right\rangle\Big|_{\delta^{-}}$$

$$\tilde{\psi}^{\pm}\left(x, b_{\perp}, \mu, \zeta^{z}\right)=\lim _{L \rightarrow \infty} \frac{1}{-i f_{\pi} n_{z} \cdot P} \int \frac{d \lambda n_{z} \cdot P}{2 \pi} e^{-i\left(x-\frac{1}{2}\right) \lambda n_{z} \cdot P}$$
$$\times\left\langle 0\left|\bar{\Psi}_{\mp n_{z}}\left(\frac{\lambda n_{z}}{2}+\boxed{b}\right) \gamma^{z} \gamma^{5} \Psi_{\mp n_{z}}\left(-\frac{\lambda n_{z}}{2}\right)\right| P\right\rangle$$

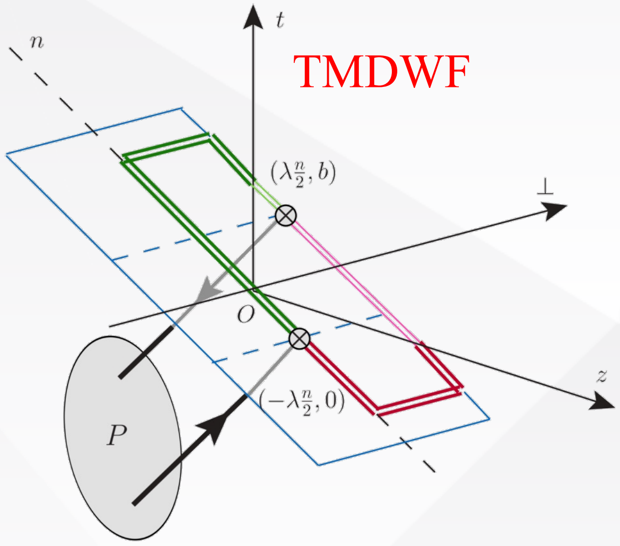


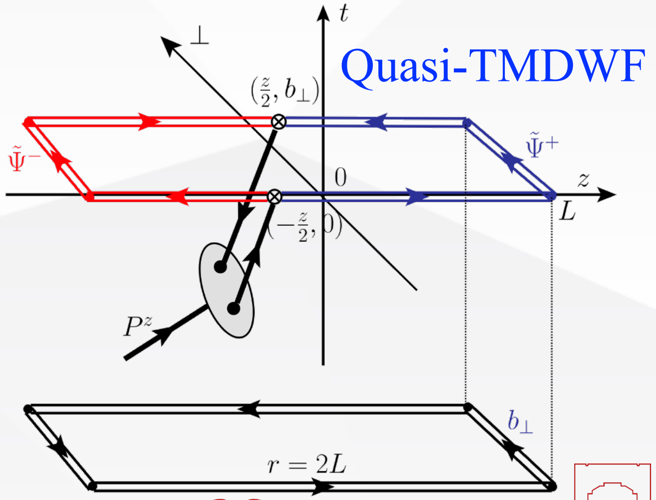
2.TMD波函数与LaMET匹配

$$\psi^{\pm}\left(x, b_{\perp}, \mu, \delta^{-}\right)=\frac{1}{-i f_{\pi} n \cdot P} \int \frac{d(\lambda n \cdot P)}{2 \pi} e^{-i\left(x-\frac{1}{2}\right) \lambda n \cdot P}$$
$$\times\left\langle 0\left|\bar{\Psi}_{n}^{\pm}(\lambda n / 2+b) \gamma^{+} \gamma^{5} \Psi_{n}^{\pm}(-\lambda n / 2)\right| P\right\rangle\Big|_{\delta^{-}}$$

Normalization factor: $\langle 0|\overline{\psi}(0) \gamma^{\mu} \gamma^{5} \psi(0)| \pi\rangle=-i f_{\pi} P^{\mu}$

$$\tilde{\psi}^{\pm}\left(x, b_{\perp}, \mu, \zeta^{z}\right)=\lim _{L \rightarrow \infty} \frac{1}{-i f_{\pi} n_{z} \cdot P} \int \frac{d \lambda n_{z} \cdot P}{2 \pi} e^{-i\left(x-\frac{1}{2}\right) \lambda n_{z} \cdot P}$$
$$\times\left\langle 0\left|\bar{\Psi}_{\mp n_{z}}\left(\frac{\lambda n_{z}}{2}+b\right) \gamma^{z} \gamma^{5} \Psi_{\mp n_{z}}\left(-\frac{\lambda n_{z}}{2}\right)\right| P\right\rangle$$





2.TMD波函数与LaMET匹配

Matching kernel

$$\tilde{\Psi}^{\pm}_{\bar{q}q}\left(x,b_\perp,\mu,\zeta^z\right) S_r^{\frac{1}{2}}\left(b_\perp,\mu\right) = H_1^{\pm}\left(\zeta^z,\bar{\zeta}^z,\mu\right) e^{\frac{1}{2}\ln\frac{\mp\zeta^z+i0}{\zeta} K_1(b_\perp,\mu)} \Psi^{\pm}_{\bar{q}q}\left(x,b_\perp,\mu,\zeta\right) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\zeta_z},\frac{M^2}{(P^z)^2},\frac{1}{b_\perp^2\zeta_z}\right)$$

Quasi-TMDWF Intrinsic soft function Collins-Soper kernel **TMDWF**

$$2\zeta\frac{d}{d\zeta}\ln\Psi^{\pm}_{\bar{q}q}\left(x,b_\perp,\mu,\zeta\right) = K_1\left(b_\perp,\mu\right)$$

Collins-Soper kernel

Phys.Rev.D 106 (2022) 3, 034509

Matching kernel(匹配核)是红外不敏感的量，不依赖于算符定义中的外态。

2.TMD波函数与LaMET匹配：一圈结果

$$\psi^{\pm}_{\bar{q}q}\left(x, b_\perp, \mu, \delta^-\right) = \frac{1}{2P^+}\int \frac{d(\lambda P^+)}{2\pi} e^{-i(x-\frac{1}{2})P^+\lambda}$$

$$\times \left\langle 0 \left| \overline{\Psi}^{\pm}_n (\lambda n/2 + b)\, \gamma^+\gamma^5 \Psi^{\pm}_n(-\lambda n/2) \right| q\bar{q} \right\rangle |_{\delta^-},$$

$$\left\langle 0 \left| \overline{\psi}_{\bar{q}}(0)\, \gamma^+\gamma^5 \psi_q(0) \right| q\bar{q} \right\rangle |_{\text{tree}} = 2P^+$$

tree： $\psi^{\pm(0)}_{\bar{q}q} = \delta(x - x_0)$

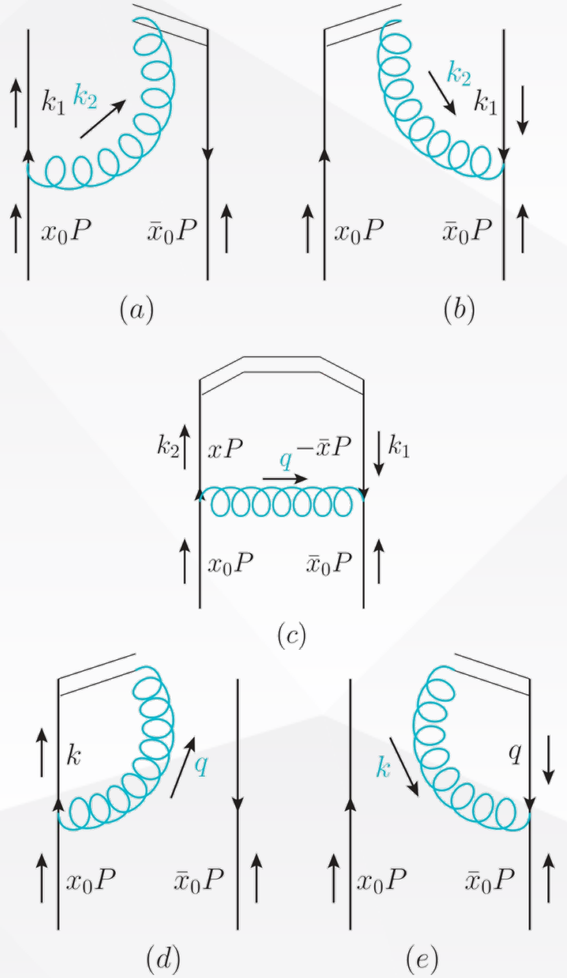


Diagram of wave function. Self-energies of external lines are not shown.

2.TMD波函数与LaMET匹配：一圈结果

Light-front TMDWFs:

$$\psi^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \delta^-) = \delta(x - x_0) + \frac{\alpha_s C_F}{2\pi}\left[f(x, x_0, b_\perp, \mu)\right]_+$$

$$+ \frac{\alpha_s C_F}{2\pi}\delta(x - x_0)\left[L_b\left(\frac{3}{2} + \ln\frac{-\delta^{-2} \mp i0}{4\bar{x}xP^{+2}}\right) + \frac{1}{2}\right],$$

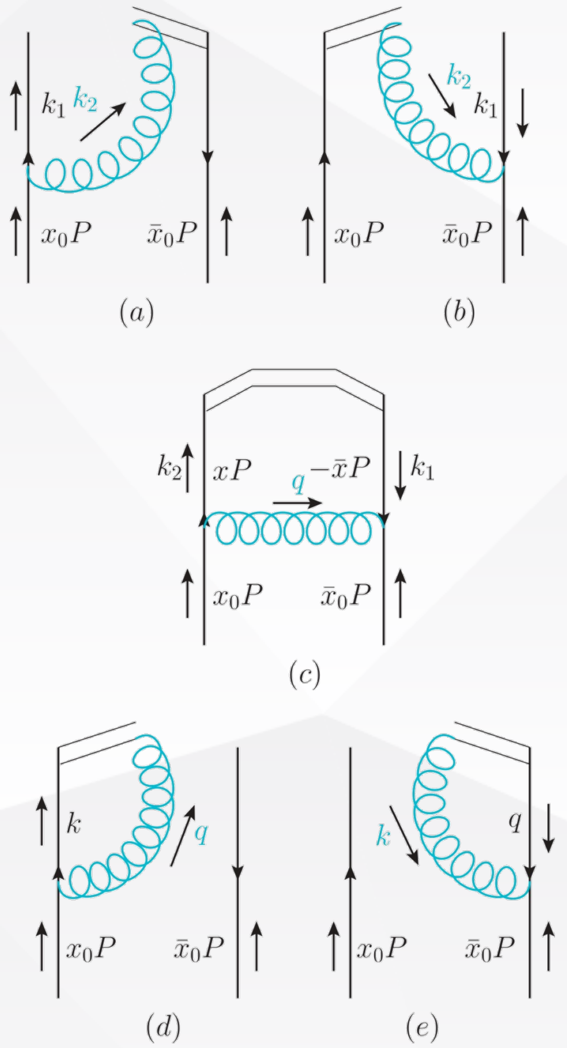
Rapidity divergence!

$$f(x, x_0, b_\perp, \mu) = \left[\left(\frac{x}{x_0(x - x_0)} - \frac{x}{x_0}\right)\left(\frac{1}{\epsilon_{\rm IR}} + L_b\right) + \frac{x}{x_0}\right]\theta(x_0 - x) + \{x \to 1 - x, x_0 \to 1 - x_0\}.$$

$$L_b = \ln\frac{\mu^2 b_\perp^2}{4e^{-2\gamma_E}}$$

$$\Psi^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \zeta) = \lim_{\delta^- \to 0}\frac{\psi^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \delta^-)}{\sqrt{S^{\pm}(b_\perp, \mu, \delta^- e^{2y_n}, \delta^-)}}$$

X. Ji, et. al., Large-momentum effective theory, Rev. Mod. Phys. 93 (2021) 035005.



2.TMD波函数与LaMET匹配：一圈结果

$$S^{\pm}(b_\perp,\mu,\delta^+,\delta^-) = \frac{1}{N_c}\mathrm{tr}\langle 0|\mathcal{T}W_{\bar n}^{-\dagger}(b_\perp)|_{\delta^+}W_n^{\pm}(b_\perp)|_{\delta^-}$$
$$\times W_n^{\pm\dagger}(0)|_{\delta^-}W_{\bar n}^{-}(0)|_{\delta^+}|0\rangle.$$

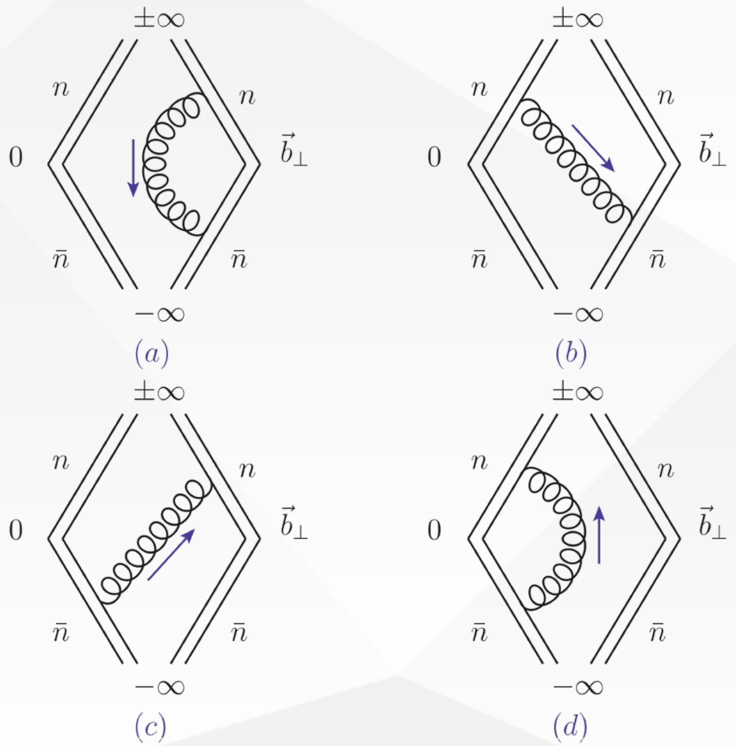
$$\begin{aligned} S^{(a)\pm} &= S^{(d)\pm} \\ &= -\mu_0^{2\epsilon} ig^2 C_F \int \frac{d^dq}{(2\pi)^d}\frac{\bar n^\mu}{q^-+i\frac{\delta^+}{2}}\frac{n_\mu}{q^+\pm i\frac{\delta^-}{2}}\frac{1}{q^2+i\epsilon} \\ &= \frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\epsilon_{UV}^2}+\frac{2}{\epsilon_{UV}}\ln\frac{\mp\delta^-\delta^+-i0}{2\mu^2}\right. \\ &\quad \left.-\ln^2\left(\frac{\mp\delta^-\delta^+-i0}{2\mu^2}\right)-\frac{\pi^2}{2}\right], \end{aligned}$$

$$\begin{aligned} S^{(b)\pm} &= S^{(c)\pm} \\ &= \mu_0^{2\epsilon} ig^2 C_F \int \frac{d^dq}{(2\pi)^d}\frac{\bar n^\mu}{q^-+i\frac{\delta^+}{2}}\frac{n_\mu}{q^+\pm i\frac{\delta^-}{2}}\frac{e^{-iq\cdot b}}{q^2+i\epsilon} \\ &= \frac{\alpha_s C_F}{4\pi}\left[L_b^2+2L_b\ln\frac{\mp\delta^-\delta^+-i0}{2\mu^2}\right. \\ &\quad \left.+\ln^2\left(\frac{\mp\delta^-\delta^+-i0}{2\mu^2}\right)+\frac{2\pi^2}{3}\right]. \end{aligned}$$

$$\mu = \mu_0 e^{(\ln(4\pi)-\gamma_E)/2}$$

$$S^{\pm}(b_\perp,\mu,\delta^+,\delta^-) = 1+\frac{\alpha_s C_F}{2\pi}\left(L_b^2 + 2L_b\ln\frac{\mp\delta^-\delta^+-i0}{2\mu^2}+\frac{\pi^2}{6}\right)$$

M.G. Echevarría, et. al., Phys. Lett. B 726 (2013) 795.
M.G. Echevarria , et. al., JHEP 07 (2012) 002.



One-loop diagrams for the soft funtion. Diagram (a)(d) give the virtual diagram, and diagram (b)(c) give the real diagram.

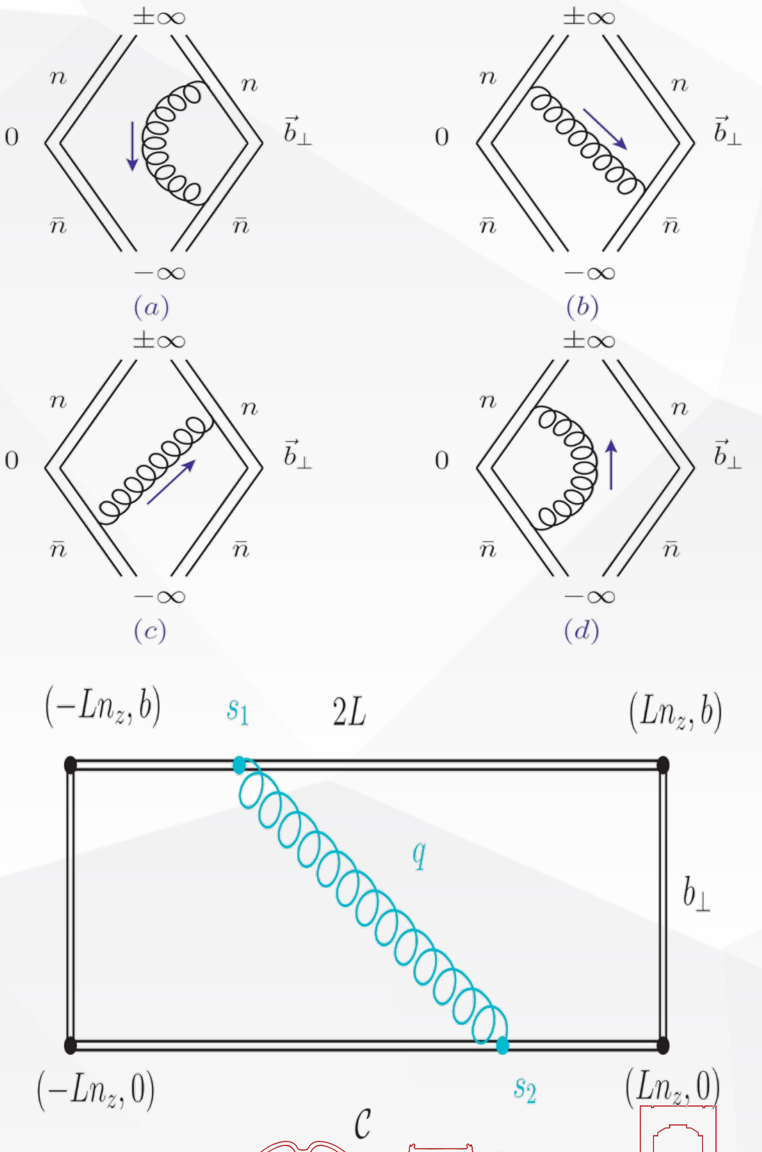
2.TMD波函数与LaMET匹配：重整化

$$\Psi^{\pm}_{\bar{q}q}(x,b_\perp,\mu,\zeta)=\lim_{\delta^-\to 0}\frac{\psi^{\pm}_{\bar{q}q}(x,b_\perp,\mu,\delta^-)}{\sqrt{S^{\pm}(b_\perp,\mu,\delta^- e^{2y_n},\delta^-)}}$$

$$S^{\pm}(b_\perp,\mu,\delta^+,\delta^-)=\frac{1}{N_c}\mathrm{tr}\langle 0|\mathcal{T}W_{\bar{n}}^{-\dagger}(b_\perp)|_{\delta^+}W_n^{\pm}(b_\perp)|_{\delta^-}W_n^{\pm\dagger}(0)|_{\delta^-}W_{\bar{n}}^{-}(0)|_{\delta^+}|0\rangle$$

$$\tilde{\Psi}^{\pm}_{\bar{q}q}(x,b_\perp,\mu,\zeta)=\lim_{L\to\infty}\frac{\tilde{\psi}^{\pm}_{\bar{q}q}(x,b_\perp,\mu,\zeta^z)}{\sqrt{Z_E(2L,b_\perp,\mu)}}$$

$$Z_E(2L,b_\perp,\mu)=\frac{1}{N_c}\mathrm{tr}\langle 0|\mathcal{T}W(\mathcal{C})|0\rangle$$



2.TMD波函数与LaMET匹配：一圈结果

$$\Psi^{\pm}_{\bar{q}q}(x,b_\perp,\mu,\zeta)=\delta(x-x_0)+\frac{\alpha_s C_F}{2\pi}[f(x,x_0,b_\perp,\mu)]_+$$
$$+\frac{\alpha_s C_F}{2\pi}\delta(x-x_0)\left\{-\frac{L_b^2}{2}+L_b\left(\frac{3}{2}+\ln\frac{\mu^2}{\pm\sqrt{\zeta\bar{\zeta}-i0}}\right)+\frac{1}{2}-\frac{\pi^2}{12}\right\},$$

where $\bar{\zeta} = 2(\bar{x}P^+)^2 e^{2y_n}$. $\zeta = 2(xP^+)^2 e^{2y_n}$ $\quad e^{2y_n} = \delta^+/\delta^-$

$$f(x,x_0,b_\perp,\mu) = \left[\left(\frac{x}{x_0(x-x_0)}-\frac{x}{x_0}\right)\left(\frac{1}{\epsilon_{\rm IR}}+L_b\right)\right.$$
$$\left.+\frac{x}{x_0}\right]\theta(x_0-x)+\{x\to 1-x, x_0\to 1-x_0\}.$$

$$K_1(b_\perp,\mu) = -\frac{\alpha_s C_F}{\pi}L_b$$

Collins-Soper kernel

X. Ji and Y. Liu, Phys. Rev. D 105 (2022) 076014.

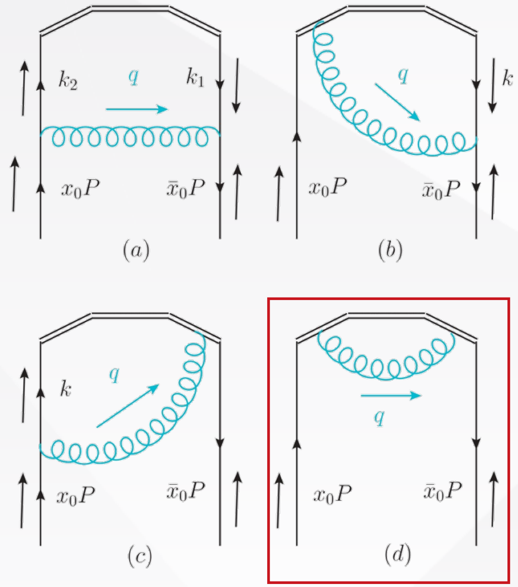
2.TMD波函数与LaMET匹配：一圈结果

$$\tilde{\Psi}^{\pm}_{q\bar{q}}(x, b_\perp, \mu, \zeta^z) = \delta(x - x_0) + \frac{\alpha_s C_F}{2\pi}[f(x, x_0, b_\perp, \mu)]_+ + \frac{\alpha_s C_F}{2\pi}\delta(x - x_0)A^{\pm}\left(x, \mu, \zeta^z, \bar{\zeta}^z\right),$$

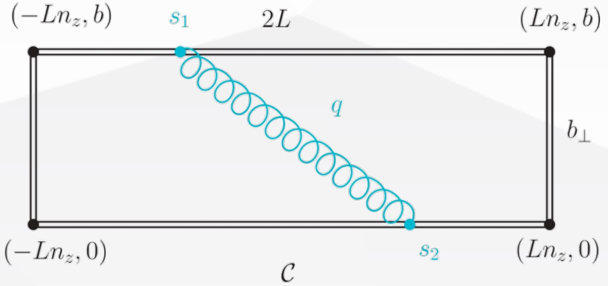
$$\bar{\zeta}^z = (2\bar{x}P \cdot n_z)^2$$

$$A^{\pm}\left(x, \mu, \zeta^z, \bar{\zeta}^z\right) = -\frac{L_b^2}{2} + \frac{5}{2}L_b - \frac{3}{2} - \frac{\pi^2}{2} + \left[-\frac{1}{4}\ln^2\frac{-\zeta^z \pm i0}{\mu^2} + \frac{1}{2}(1 - L_b)\ln\frac{-\zeta^z \pm i0}{\mu^2} + \{\zeta^z \to \bar{\zeta}^z\}\right]$$

X. Ji and Y. Liu, Phys. Rev. D 105 (2022) 076014.



One-loop diagrams for the quasi TMDWF.



One-loop diagrams for the Wilson loop.

3. 四夸克形状因子

$$F(b_\perp, P_1, P_2, \mu) = \frac{\langle P_2 | (\bar\psi_a \Gamma \psi_b)(b)(\bar\psi_c \Gamma' \psi_d)(0) | P_1 \rangle}{f_\pi^2 P_1 \cdot P_2}$$

$$\Gamma = \Gamma' = I,\ \gamma_5 \text{ or } \gamma_\perp \text{ and } \gamma_\perp \gamma_5$$

$$\langle 0 | \bar\psi(0) \gamma^\mu \gamma^5 \psi(0) | P_1 \rangle = -i f_\pi P_1^\mu$$

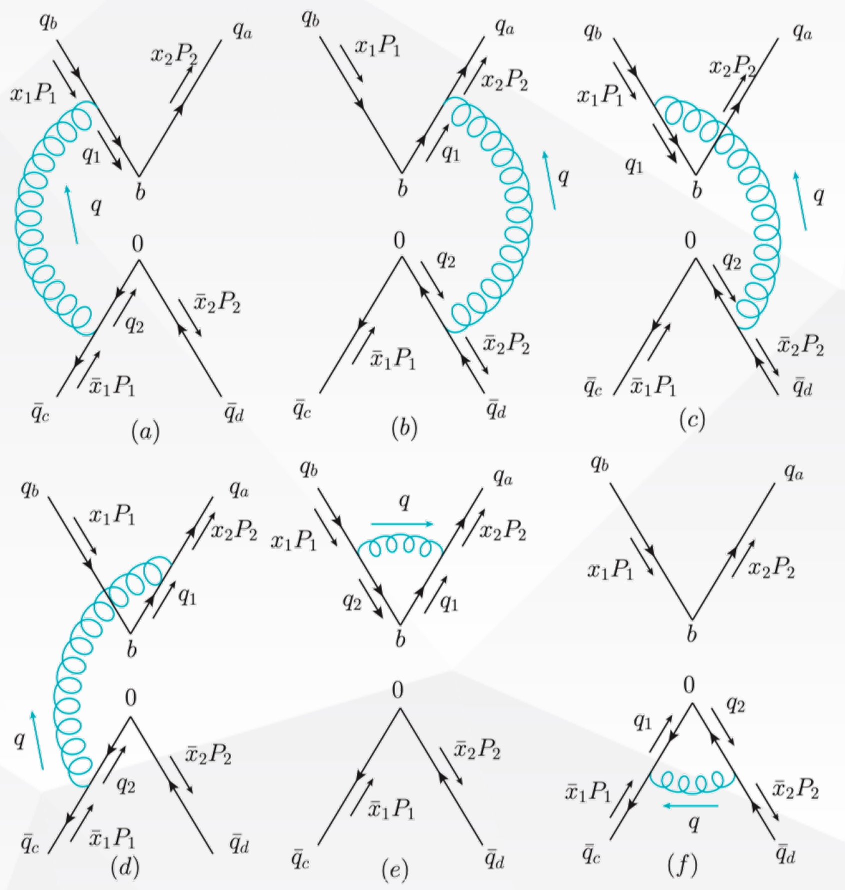
$$\langle P_2 | \bar\psi(0) \gamma_\mu \gamma^5 \psi(0) | 0 \rangle = i f_\pi P_{2\mu}$$

$$\frac{\langle \bar q_d(\bar x_2 P_2) q_a(x_2 P_2) | (\bar\psi_a \Gamma \psi_b)(b)(\bar\psi_c \Gamma \psi_d)(0) | q_b(x_1 P_1) \bar q_c(\bar x_1 P_1) \rangle}{4 P_1 \cdot P_2}$$

$$\left\langle 0 \left| \bar\psi_c \gamma^\mu \gamma^5 \psi_b \right| q_b(x_1 P_1) \bar q_c(\bar x_1 P_1) \right\rangle |_{\text{tree}} = 2 P_1^\mu,$$

$$\left\langle \bar q_d(\bar x_2 P_2) q_a(x_2 P_2) \left| \bar\psi_a \gamma_\mu \gamma^5 \psi_d \right| 0 \right\rangle |_{\text{tree}} = 2 P_{2\mu}.$$

$$P_1^\mu = (P^z, 0, 0, P^z) \text{ and } P_2^\mu = (P^z, 0, 0, -P^z)$$



One-loop Feynman diagrams to the form factor. The quark self-energy corrections are not shown.

3. 四夸克形状因子

$\Gamma = I, \gamma_5$

$$F(b_\perp, P_1, P_2, \mu) = F^0\Bigg\{1 - \frac{\alpha_s C_F}{2\pi}\Bigg[L_b^2 + L_b\Bigg(\ln\frac{4Q^2\bar{Q}^2}{\mu^4} - 3\Bigg) + \frac{1}{2}\ln^2\frac{2Q^2}{\mu^2} + \frac{1}{2}\ln^2\frac{2\bar{Q}^2}{\mu^2} + 1\Bigg]\Bigg\}.$$

$\Gamma = \gamma_\perp,\ \gamma_\perp\gamma_5$

$$F(b_\perp, P_1, P_2, \mu) = F^0\left[1 - \frac{\alpha_s C_F}{2\pi}\left(7 - \frac{3}{2}\ln\frac{Q^2\bar{Q}^2 b_\perp^4}{4e^{-4\gamma_E}} + \frac{1}{2}\ln^2\frac{Q^2 b_\perp^2}{2e^{-2\gamma_E}} + \frac{1}{2}\ln^2\frac{\bar{Q}^2 b_\perp^2}{2e^{-2\gamma_E}}\right)\right].$$

$$F^0 = \begin{cases} \dfrac{1}{4N_c}, & \text{for} \quad \Gamma = I \\ -\dfrac{1}{4N_c}, & \text{for} \quad \Gamma = \gamma_5,\ \gamma_\perp \text{ or } \gamma_\perp\gamma_5. \end{cases}$$

The form factor is an infrared-safe quantity at one-loop order!

3. 四夸克形状因子：因子化

H

$$F(b_\perp, P_1, P_2, \mu) = \int dx_1 dx_2 H_F(Q^2, \bar{Q}^2, \mu^2)$$

C C

$$\times \left[\frac{\psi^{\pm}_{\bar{q}q}(x_2, b_\perp, \mu, \delta'^{+})}{\sqrt{S^{\pm}(b_\perp, \mu, \delta'^{+}, \delta^{-})}}\right]^{\dagger} \left[\frac{\psi^{\pm}_{\bar{q}q}(x_1, b_\perp, \mu, \delta'^{-})}{\sqrt{S^{\pm}(b_\perp, \mu, \delta^{+}, \delta'^{-})}}\right]$$

$$\times \frac{S^{\pm}(b_\perp, \mu, \delta^{+}, \delta^{-})}{\sqrt{S^{\pm}(b_\perp, \mu, \delta'^{+}, \delta^{-}) S^{\pm}(b_\perp, \mu, \delta^{+}, \delta'^{-})}}$$

S

Intrinsic soft function

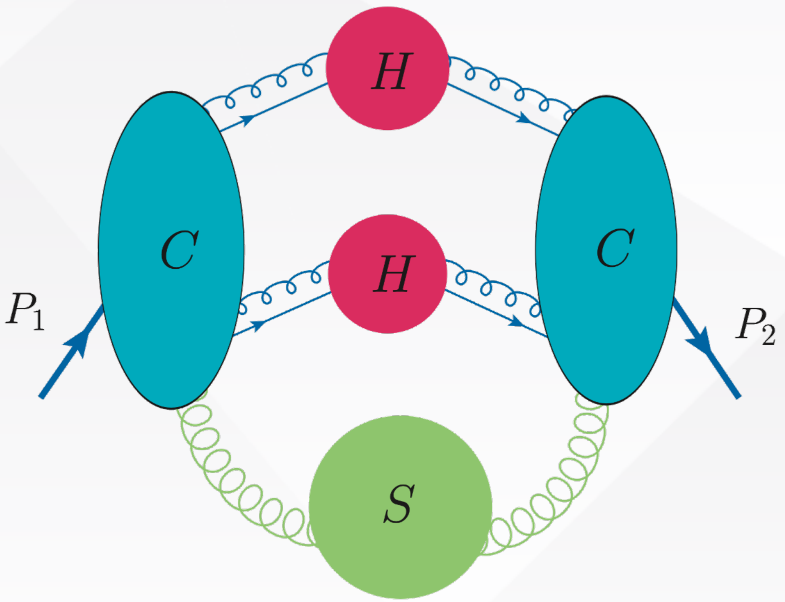
S

$$F(b_\perp, P_1, P_2, \mu) = \int dx_1 dx_2 H(x_1, x_2) S_r(b_\perp, \mu)$$

$$\times \tilde{\Psi}^{\dagger}_{q\bar{q}}(x_2, b_\perp, \mu, \zeta_2^z) \tilde{\Psi}_{q\bar{q}}(x_1, b_\perp, \mu, \zeta_1^z)$$

C C

$$S_r(b_\perp, \mu) = 1 - \frac{\alpha_s C_F}{\pi} L_b$$



The leading-power reduced diagram for the large-momentum form factor of a meson. Two H denote the two hard cores separated in the transverse space by $b_\perp$, C are collinear sub-diagrams and S denotes the soft sub-diagram.

3. 四夸克形状因子：硬函数

For $\Gamma = I$ or $\Gamma = \gamma_5$, we have the hard kernel

$$H_F(Q^2, \bar{Q}^2) = H_F^{(0)}\left[1 + \frac{\alpha_s C_F}{2\pi}\left(-\frac{1}{2}\ln^2\frac{2Q^2}{\mu^2} - \frac{1}{2}\ln^2\frac{2\bar{Q}^2}{\mu^2} + \frac{\pi^2}{6} - 2\right)\right].$$

For $\Gamma = \gamma_\perp$ or $\Gamma = \gamma_\perp\gamma_5$, the hard kernel is calculated as

$$H_F(Q^2, \bar{Q}^2) = H_F^{(0)}\left[1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2}\ln\frac{4Q^2\bar{Q}^2}{\mu^4} - \frac{1}{2}\ln^2\frac{2Q^2}{\mu^2} - \frac{1}{2}\ln^2\frac{2\bar{Q}^2}{\mu^2} + \frac{\pi^2}{6} - 8\right)\right].$$

$$H_F^{(0)} = \begin{cases} \dfrac{1}{4N_c}, & \Gamma = I \\ -\dfrac{1}{4N_c}, & \Gamma = \gamma_5,\ \gamma_\perp \text{ or } \gamma_\perp\gamma_5. \end{cases}$$

3. 四夸克形状因子：硬函数

For $\Gamma = I$ or $\Gamma = \gamma_5$, the matching kernel is then derived as:

$$
\begin{aligned}
H(x_1,x_2) = H^{(0)}\Bigg\{1+\frac{\alpha_s C_F}{8\pi}\Bigg[&4\pi^2+8+\ln^2\left(\frac{-\zeta_1^z\pm i0}{\mu^2}\right)+\ln^2\left(\frac{-\bar{\zeta}_1^z\pm i0}{\mu^2}\right)+\ln^2\left(\frac{-\zeta_2^z\mp i0}{\mu^2}\right)\\
&+\ln^2\left(\frac{-\bar{\zeta}_2^z\mp i0}{\mu^2}\right)-\frac{1}{2}\ln^2\left(\frac{\zeta_1^z\zeta_2^z}{\mu^4}\right)-\frac{1}{2}\ln^2\left(\frac{\bar{\zeta}_1^z\bar{\zeta}_2^z}{\mu^4}\right)-2\ln\frac{\zeta_1^z\zeta_2^z\bar{\zeta}_1^z\bar{\zeta}_2^z}{\mu^8}\Bigg]\Bigg\}\\
= H^{(0)}\Bigg\{1+\frac{\alpha_s C_F}{2\pi}\Bigg[&2+\pi^2+\frac{1}{2}\ln^2\left(-\frac{x_2}{x_1}\mp i0\right)\\
&+\frac{1}{2}\ln^2\left(-\frac{\bar{x}_2}{\bar{x}_1}\mp i0\right)-\ln\frac{16x_1x_2\bar{x}_1\bar{x}_2P^{z4}}{\mu^4}\Bigg]\Bigg\}.
\end{aligned}
$$

For $\Gamma = \gamma_\perp$ or $\Gamma = \gamma_\perp\gamma_5$, we have:

$$
\begin{aligned}
H(x_1,x_2) = H^{(0)}\Bigg\{1+\frac{\alpha_s C_F}{8\pi}\Bigg[&4\pi^2-16+\ln^2\left(\frac{-\zeta_1^z\pm i0}{\mu^2}\right)+\ln^2\left(\frac{-\bar{\zeta}_1^z\pm i0}{\mu^2}\right)+\ln^2\left(\frac{-\zeta_2^z\mp i0}{\mu^2}\right)\\
&+\ln^2\left(\frac{-\bar{\zeta}_2^z\mp i0}{\mu^2}\right)-\frac{1}{2}\ln^2\left(\frac{\zeta_1^z\zeta_2^z}{\mu^4}\right)-\frac{1}{2}\ln^2\left(\frac{\bar{\zeta}_1^z\bar{\zeta}_2^z}{\mu^4}\right)+\ln\frac{\zeta_1^z\zeta_2^z\bar{\zeta}_1^z\bar{\zeta}_2^z}{\mu^8}\Bigg]\Bigg\}\\
= H^{(0)}\Bigg\{1+\frac{\alpha_s C_F}{2\pi}\Bigg[&\pi^2-4+\frac{1}{2}\ln^2\left(-\frac{x_2}{x_1}\mp i0\right)\\
&+\frac{1}{2}\ln^2\left(-\frac{\bar{x}_2}{\bar{x}_1}\mp i0\right)+\frac{1}{2}\ln\frac{16x_1\bar{x}_1x_2\bar{x}_2P^{z4}}{\mu^4}\Bigg]\Bigg\}.
\end{aligned}
$$

SJTU

3. 四夸克形状因子：硬函数与匹配系数

$$H(x_1,x_2)=\frac{H_F(Q^2,\bar{Q}^2,\mu^2)}{\left[H_1^{\pm}\left(\zeta_2^z,\bar{\zeta}_2^z,\mu\right)\right]^{\dagger}\left[H_1^{\pm}\left(\zeta_1^z,\bar{\zeta}_1^z,\mu\right)\right]},$$

where $\zeta_i^z=(2x_iP\cdot n_z)^2$, $\bar{\zeta}_i^z=(2\bar{x}_iP\cdot n_z)^2$, and the condition $\zeta_1^z\zeta_2^z=\zeta_1\zeta_2$ is used.

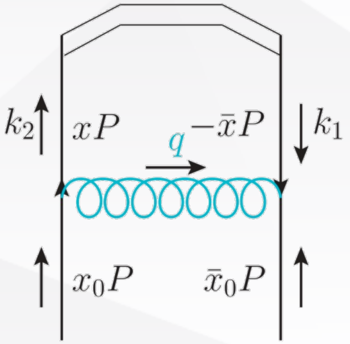
$$H_1^{\pm}\left(\zeta^z,\bar{\zeta}^z,\mu\right)=1+\frac{\alpha_sC_F}{2\pi}\left\{-\frac{5\pi^2}{12}-2+\frac{1}{2}\left[\ln\frac{-\zeta^z\pm i0}{\mu^2}-\frac{1}{2}\ln^2\frac{-\zeta^z\pm i0}{\mu^2}+\{\zeta^z\to\bar{\zeta}^z\}\right]\right\}.$$

3. 四夸克形状因子：因子化

$$
\begin{aligned}
F^{(1,a)} = & \mu_0^{2\epsilon}\frac{ig^2C_F}{4N_cP_1\cdot P_2}\int\frac{d^dq}{(2\pi)^d}e^{-iq\cdot b} \\
&\times\frac{1}{[(q+x_1P_1)^2+i\epsilon][(q-\bar{x}_1P_1)^2+i\epsilon](q^2+i\epsilon)} \\
&\times c_\Gamma\bar{u}_a(x_2P_2)\gamma_\nu\gamma_5 v_d(\bar{x}_2P_2) \\
&\times\bar{v}_c(\bar{x}_1P_1)\gamma^\mu(\not{q}-\bar{x}_1\not{P}_1)\gamma^\nu\gamma_5(\not{q}+x_1\not{P}_1)\gamma_\mu u_b(x_1P_1) \\
= & \mu_0^{2\epsilon}\frac{ig^2C_F}{4P_1\cdot P_2}\int\frac{d^dq}{(2\pi)^d}e^{iq\cdot b} \\
&\times\frac{1}{[(q-x_1P_1)^2+i\epsilon][(q+\bar{x}_1P_1)^2+i\epsilon](q^2+i\epsilon)} \\
&\times(-H_F^{(0)})\bar{u}_a(x_2P_2)\gamma_\nu\gamma_5 v_d(\bar{x}_2P_2) \\
&\times\bar{v}_c(\bar{x}_1P_1)\gamma^\mu(\not{q}+\bar{x}_1\not{P}_1)\gamma^\nu\gamma_5(\not{q}-x_1\not{P}_1)\gamma_\mu u_b(x_1P_1).
\end{aligned}
$$

$$
\begin{aligned}
F^{(1,a)} = & H_F^{(0)}\mu_0^{2\epsilon}\frac{ig^2C_F}{2P_1^+}\int\frac{d^dq}{(2\pi)^d}e^{iq\cdot b} \\
&\times\frac{1}{(q-x_1P_1)^2+i\epsilon][(q+\bar{x}_1P_1)^2+i\epsilon](q^2+i\epsilon)} \\
&\times \\
&\times\bar{v}_c(\bar{x}_1P_1)\gamma^\mu(\not{q}+\bar{x}_1\not{P}_1)\gamma^\nu\gamma_5(x_1\not{P}_1-\not{q})\gamma_\mu u_b(x_1P_1) \\
= & H_F^{(0)}\times\int dx\,\psi_{\bar{q}q}^{(1,c)}(x).
\end{aligned}
$$

Fierz transformation



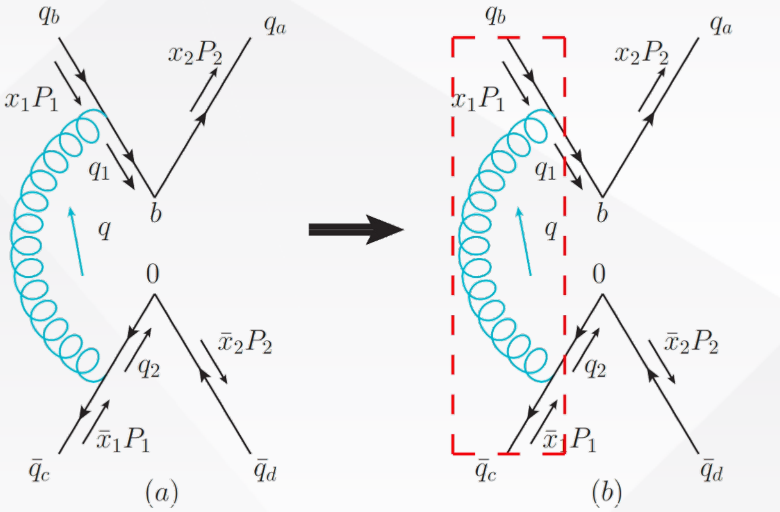


FIG. 6: Factorization of form factor shown in Fig. 5 (a). Only collinear mode contributes in this diagram, while both hard and soft contributions are power suppressed.

$$F^{(1,a)} = H_F^{(0)}\otimes\psi_{\bar{q}q}^{(1,c)}\otimes(\psi_{\bar{q}q}^{(0)})^\dagger\times\left(\frac{1}{S}\right)^{(0)}$$

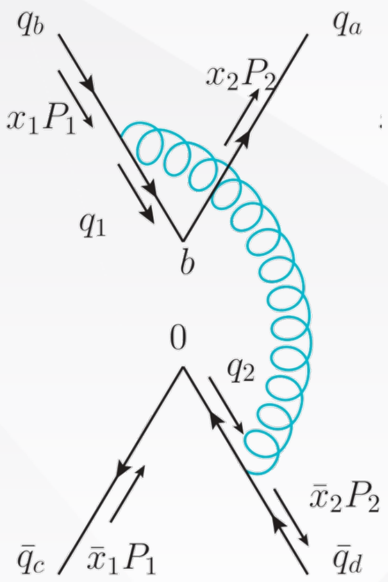
Hard　Collinear　Soft

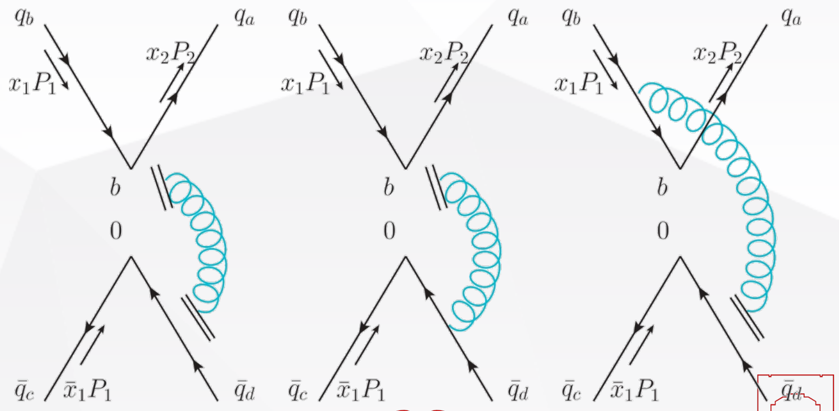
$$F^{(1,b)} = H_F^{(0)}\otimes\psi_{\bar{q}q}^{(0)}\otimes(\psi_{\bar{q}q}^{(1,c)})^\dagger\times\left(\frac{1}{S}\right)^{(0)}$$

3. 四夸克形状因子：因子化

$$F^{(1,c)}|_{soft} = H_F^{(0)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(1,b)}$$

$$F^{(1,c)}|_{collinear} = \begin{cases} H_F^{(0)} \otimes \psi_{\bar{q}q}^{(1,a)}|_{\text{collinear}} \otimes (\psi_{\bar{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(0)} & q\,//\,P_1 \\ H_F^{(0)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(1,a)})^\dagger|_{\text{collinear}} \times \left(\frac{1}{S}\right)^{(0)} & q\,//\,P_2 \end{cases}$$





3. 四夸克形状因子：因子化

$$H_F^{(1,e)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(0)})^{\dagger} \times \left(\frac{1}{S}\right)^{(0)}$$

$$+H_F^{(0)} \otimes \psi_{\bar{q}q}^{(1,d)} \otimes (\psi_{\bar{q}q}^{(0)})^{\dagger} \times \left(\frac{1}{S}\right)^{(0)}$$

$$+H_F^{(0)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(1,d)})^{\dagger} \times \left(\frac{1}{S}\right)^{(0)}$$

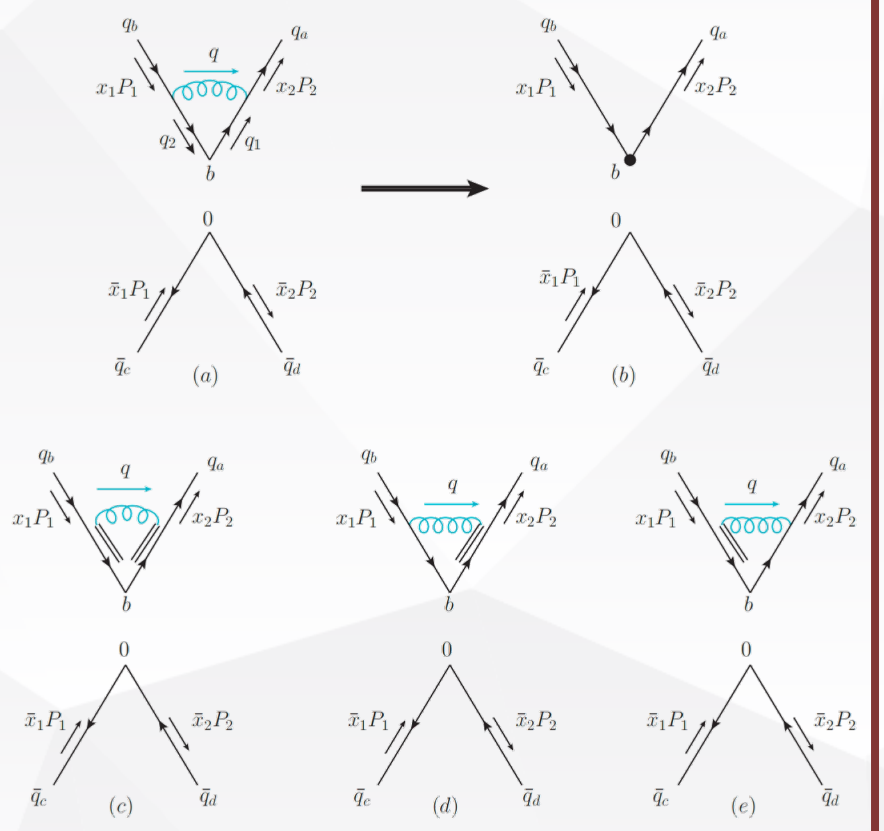
$$+H_F^{(0)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(0)})^{\dagger} \times \left(\frac{1}{S}\right)^{(1,d)}$$

$$H_F(Q^2, \bar{Q}^2) = H^{Sud}(-Q^2) H^{Sud}(-\bar{Q}^2)$$

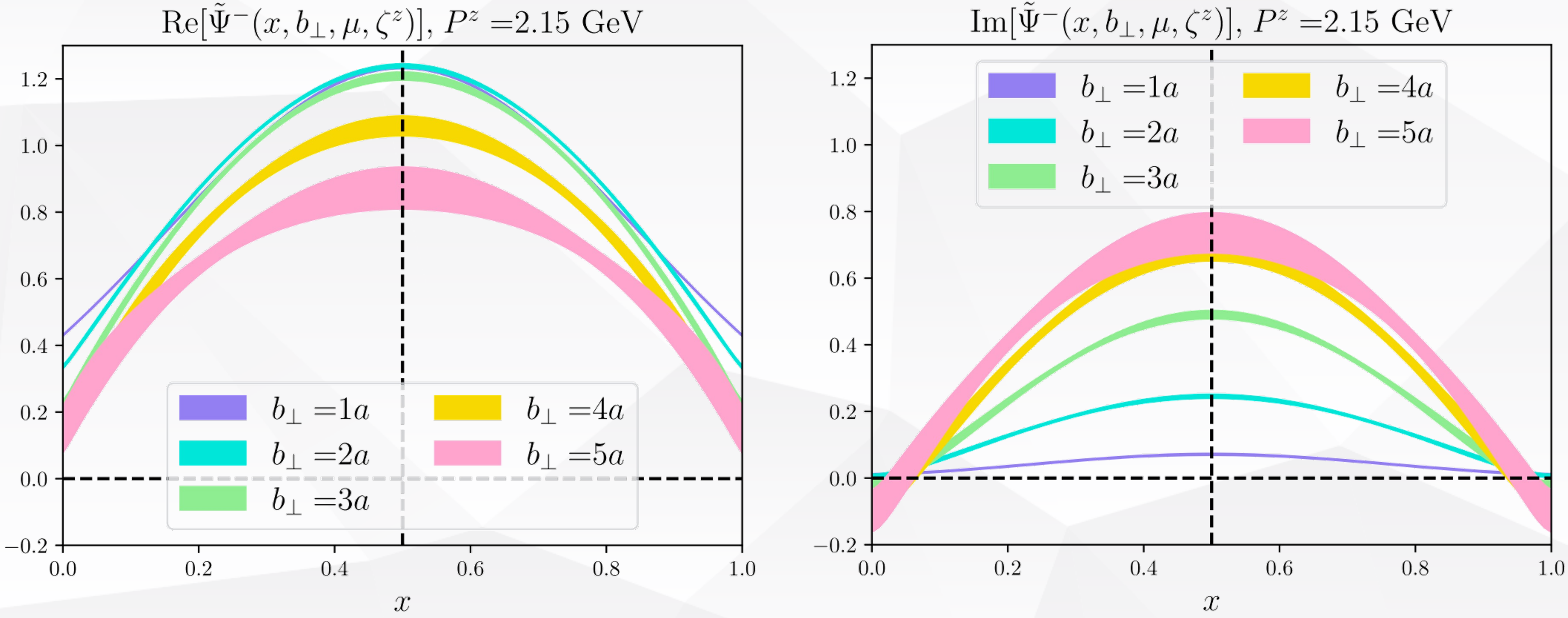
$$Q^2 = x_1 x_2 P_1 \cdot P_2$$

$$\bar{Q}^2 = \bar{x}_1 \bar{x}_2 P_1 \cdot P_2$$

J. Collins and T.C. Rogers, Phys. Rev. D 96 (2017) 054011.

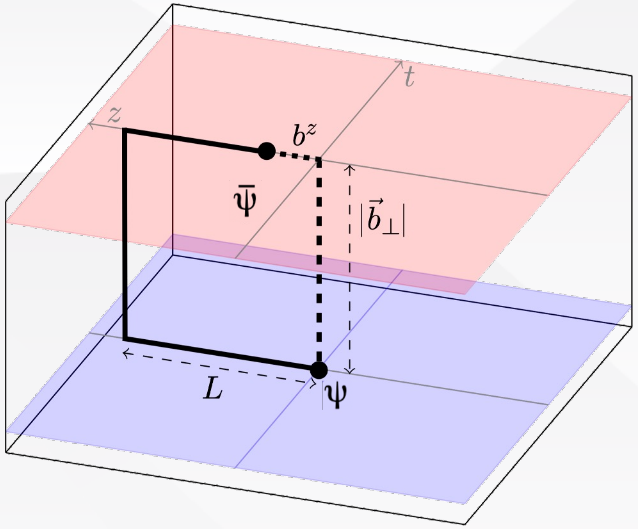


4. 格点结果



The quasi-TMDWF in momentum space, with hadron momentum Pz=2.15 GeV and for the MILC ensemble.

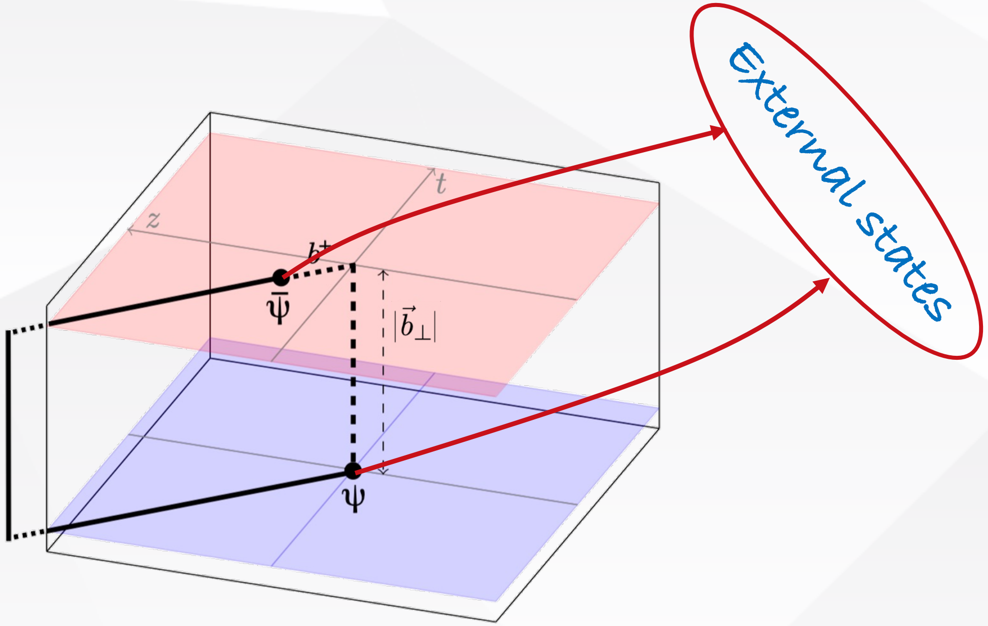
4. 格点结果：LaMET匹配



Lorentz boost

$L \to \infty$

Ji, PLB811(2020); Ebert, JHEP04(2022)



Equal-time correlators,
directly calculable on lattice

Space-like correlators,
NO effective method for directly calculation

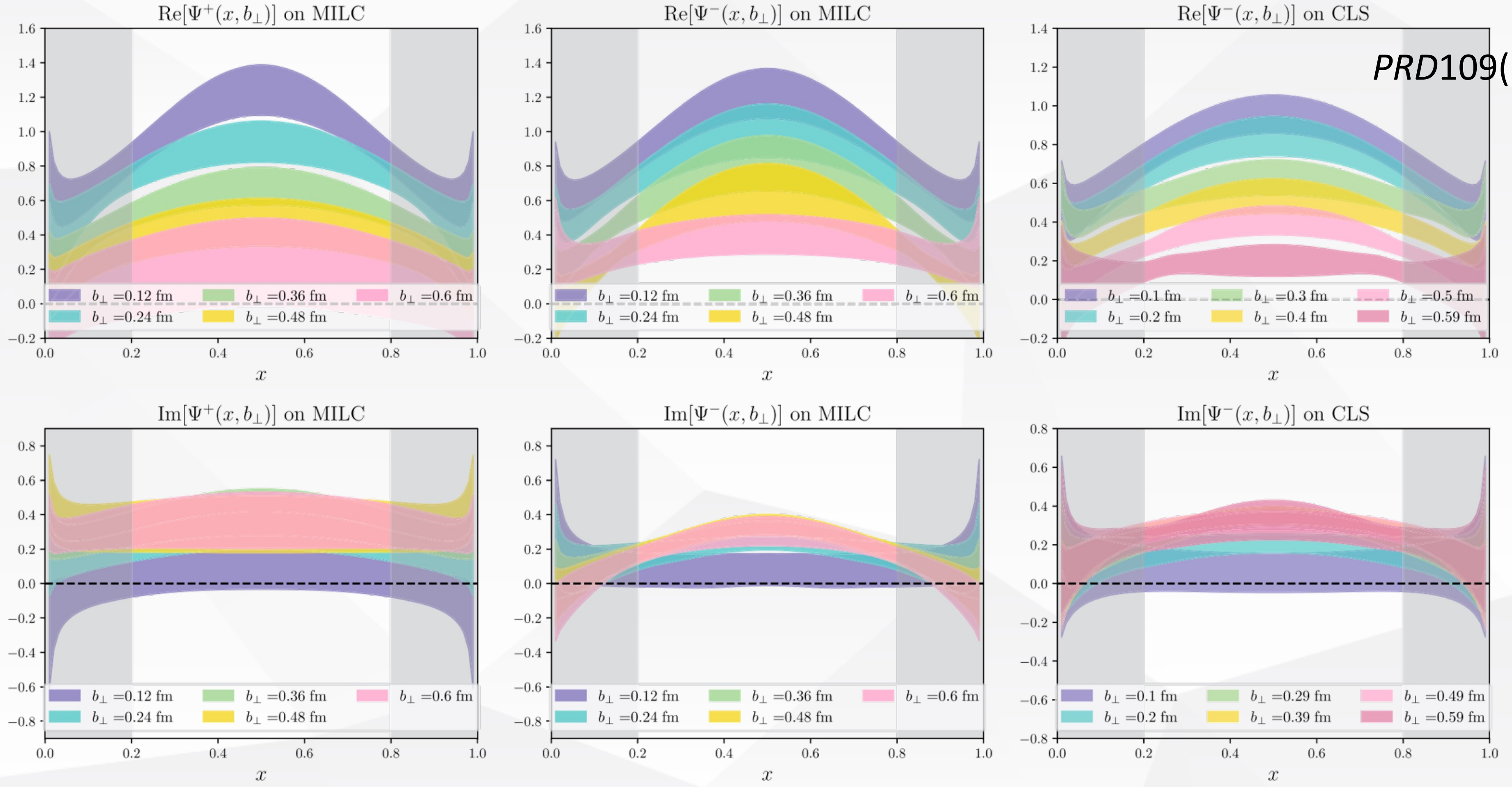
Connected at large-momentum limit

Matching kernel

$$\tilde\Psi^{\pm}_{\bar q q}(x,b_\perp,\mu,\zeta^z)\, S_r^{\frac{1}{2}}(b_\perp,\mu) = H_1^{\pm}(\zeta^z,\bar\zeta^z,\mu)\, e^{\frac{1}{2}\ln\frac{\mp\zeta^z+i0}{\zeta}K_1(b_\perp,\mu)}\, \Psi^{\pm}_{\bar q q}(x,b_\perp,\mu,\zeta) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{\zeta_z},\frac{M^2}{(P^z)^2},\frac{1}{b_\perp^2\zeta_z}\right)$$

Quasi-TMDWF Intrinsic soft function Collins-Soper kernel **TMDWF**

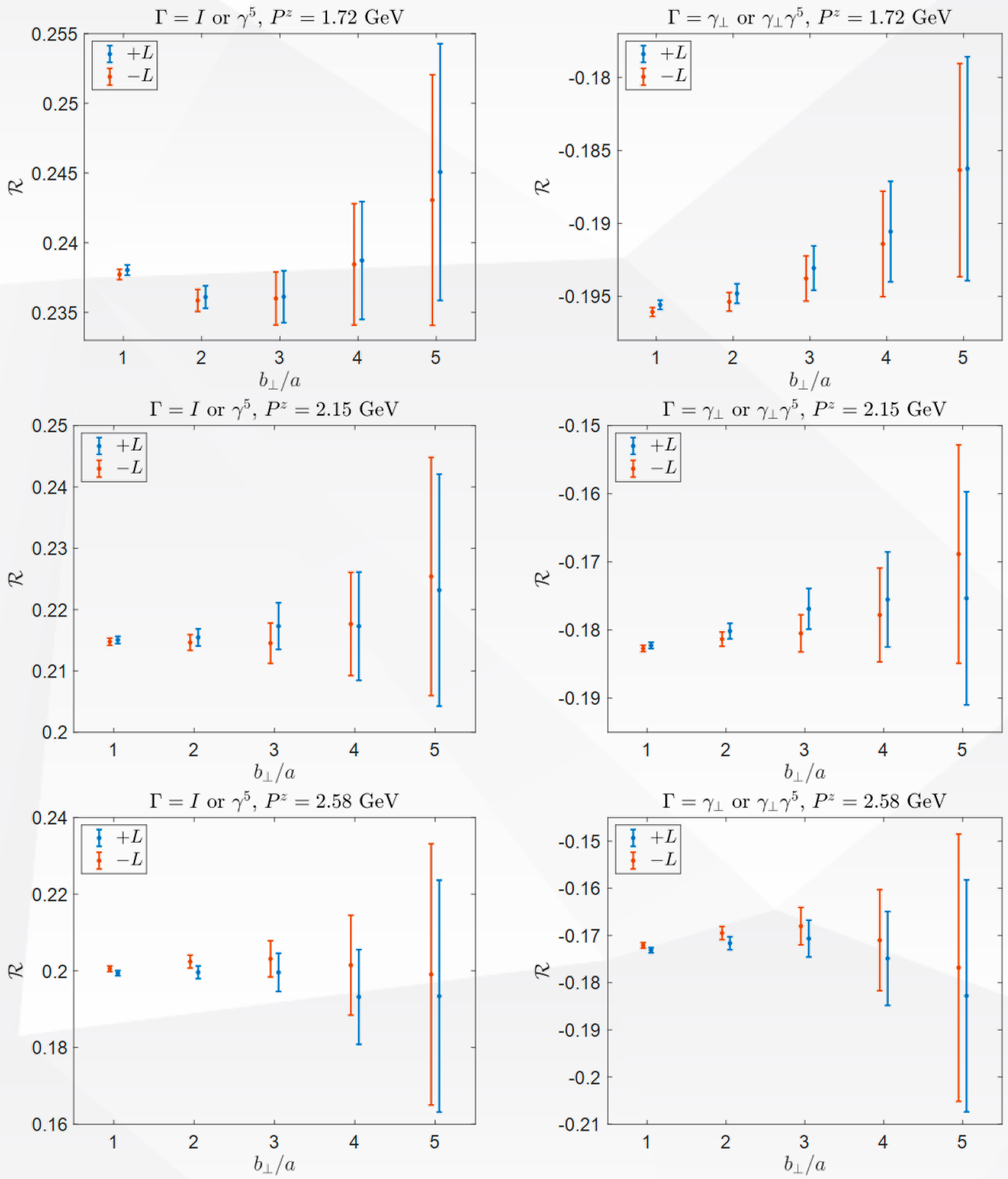
4. 格点结果



*PRD*109(2024) 9,L091503

FIG. 4. The left two parts are for real (upper panel) and imaginary parts (lower panel) of the TMDWF Ψ^+, and the central two correspond to Ψ^- all for the MILC ensemble. The right two parts correspond to Ψ^- and the CLS ensemble. These results approach the infinite P^z limit with $\zeta = (6\text{ GeV})^2$ and $\mu = 2$ GeV.

4. 格点结果



The lattice data on quasi-TMDWFs from LPC.

LPC collaboration, Phys. Rev. D 106 (2022) 034509.

$$S_r(b_\perp,\mu) = \frac{F(b_\perp,P_1,P_2,\mu)}{\mathcal{H}}$$

$a = 0.12\,\mathrm{fm}$

$$\mathcal{H} = \int dx_1 dx_2 H(x_1,x_2) \times \tilde{\Psi}^\dagger(x_2,b_\perp,P^z,\zeta_2^z)\tilde{\Psi}(x_1,b_\perp,P^z,\zeta_1^z)$$

$$\mathcal{R} = \frac{\mathcal{H}_1 - \mathcal{H}_0}{\mathcal{H}_0}$$

5.总结

- 在大动量有效理论下，可以从四夸克形状因子中抽取TMDWF和软函数。
- 证明了形状因子的单圈TMD因子化。
- 软函数的微扰修正依赖于定义形状因子的洛伦兹结构，但对横向分离不太敏感。
- 这些结果将有助于从第一原理中精确提取软函数和TMDWF。

谢谢！