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Pion Boer-Mulders function using a contact interaction

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Mass in nature

- The character of mass and its consequences in the Standard Model (SM) is a key question in modern physics
- When considering the origin of mass in the SM thoughts typically turn to explicit mass generation through the Higgs mechanism
- Notion behind Higgs mechanism for mass generation was introduced more than fifty years ago; and it became an essential piece of the SM
- 2012 – Discovery of something possessing all anticipated properties of the Higgs boson
  SM became complete
- Nobel Prize in physics awarded to Englert and Higgs
  "... for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles ... "

NOT the complete story. Far from it …

Emergence of hadron mass

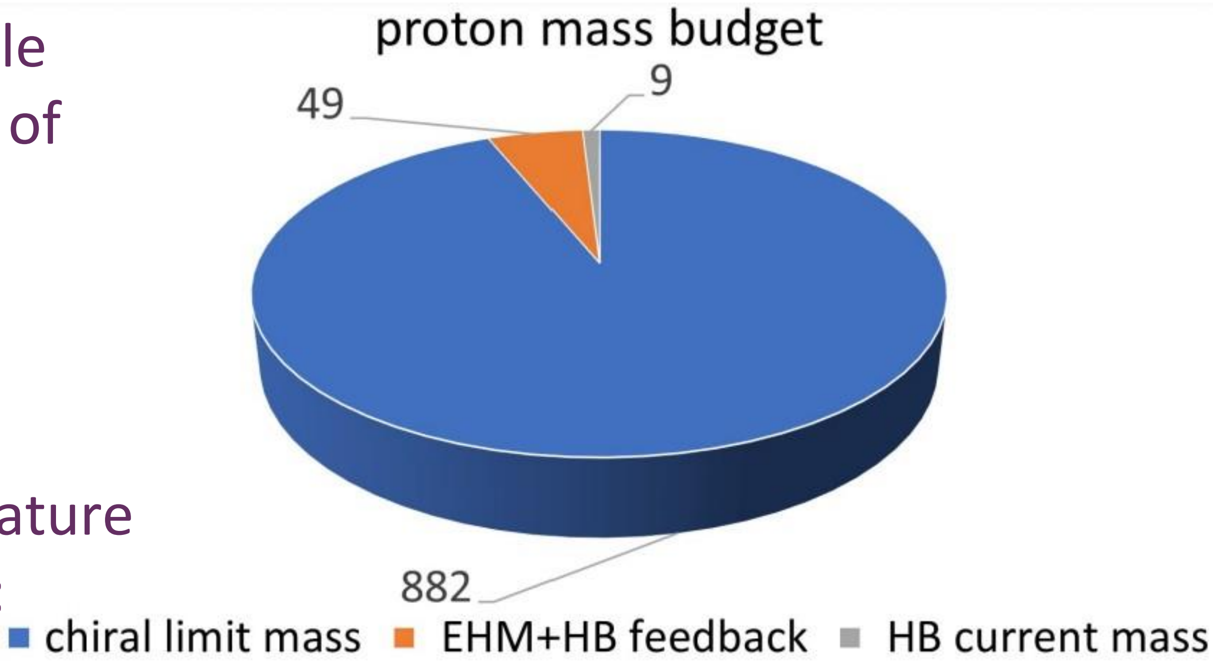
- The Higgs boson only gives mass to very simple particles. It is alone responsible for just $\sim 1\%$ of the visible mass in the Universe
- Proton mass budget

  Only 9 MeV/939 MeV is directly from Higgs

- Evidently, there is another phenomenon in Nature that is extremely effective in producing mass:

  Emergent Hadron Mass (EHM)

  - Alone, it produces 94% of the proton's mass
  - Remaining 5% is generated by constructive interference between EHM and Higgs-boson



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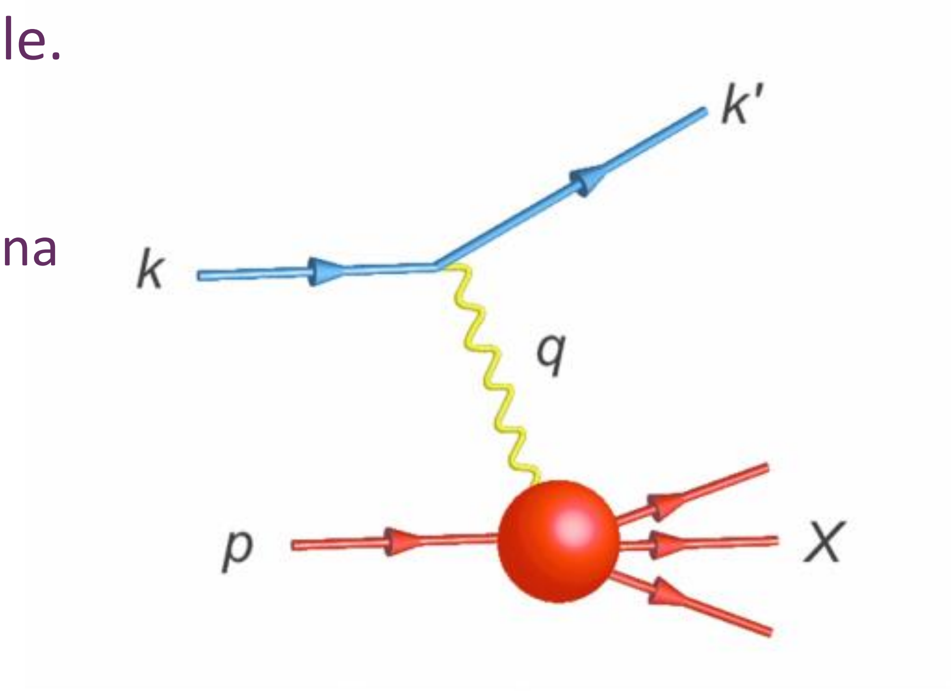
Prog. Part. Nucl. Phys. 120 (2021) 103883

Parton distribution function

- EHM is expressed in EVERY strong interaction observable.
- Challenge to theory =

  Elucidate all observable consequences of these phenomena and highlight the paths to measuring them.

- Parton distribution functions (DFs) = preeminent source of hadron structure information.
- Experiments interpretable in terms of DFs have long been a priority.



  Hadron physics has focused on one dimensional (1D) imaging of hadrons in the past 50 years.

- Facilities have given high priority to experiments that can yield data that may be used to draw three-dimensional (3D) images of hadrons.

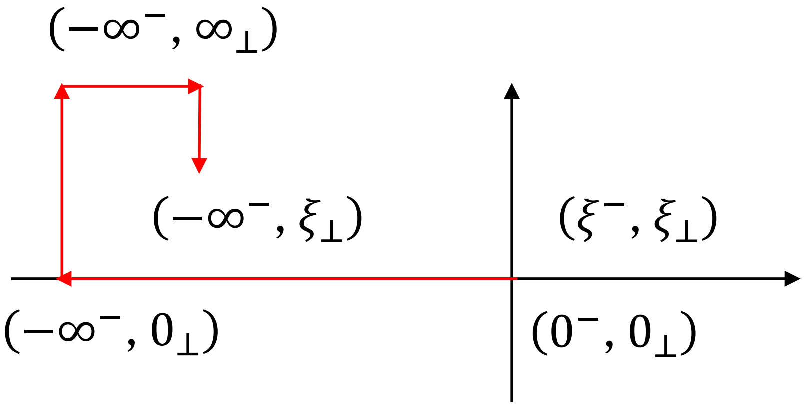
  JLAb, CERN, Eic, EIcC, etc

Gauge invariant correlation function

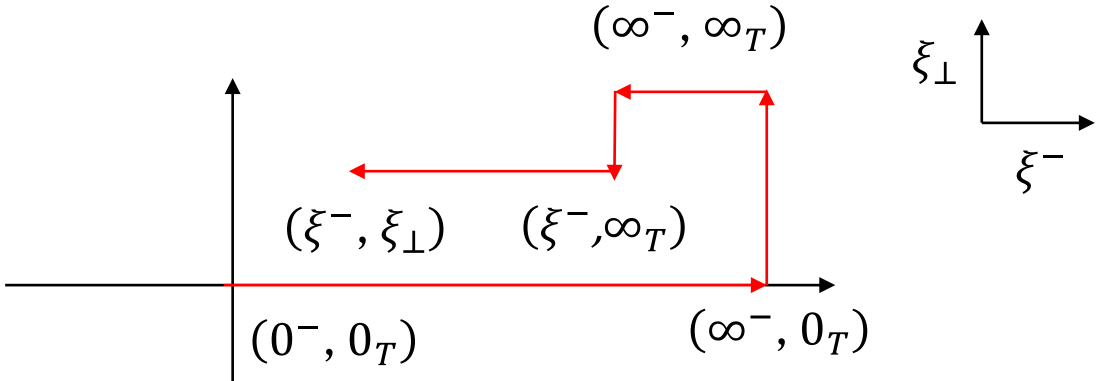
- The gauge-invariant quark-quark correlation function is defined as

$$\Phi_{ij}(k;P,S) = \int d^4\xi e^{ik\cdot\xi}\langle PS|\bar{\psi}_j(0)U_{[0,\xi]}\psi_i(\xi)|PS\rangle$$

- Path ordered exponentials



Initial state interaction in Drell-Yan



Final state interaction in SIDIS

- ✓ Initial/final state interaction to past/future in Drell-Yan/SIDIS process.
- ✓ The light-cone disappears in light cone gauge; the transverse vanishes in Feynman gauge.

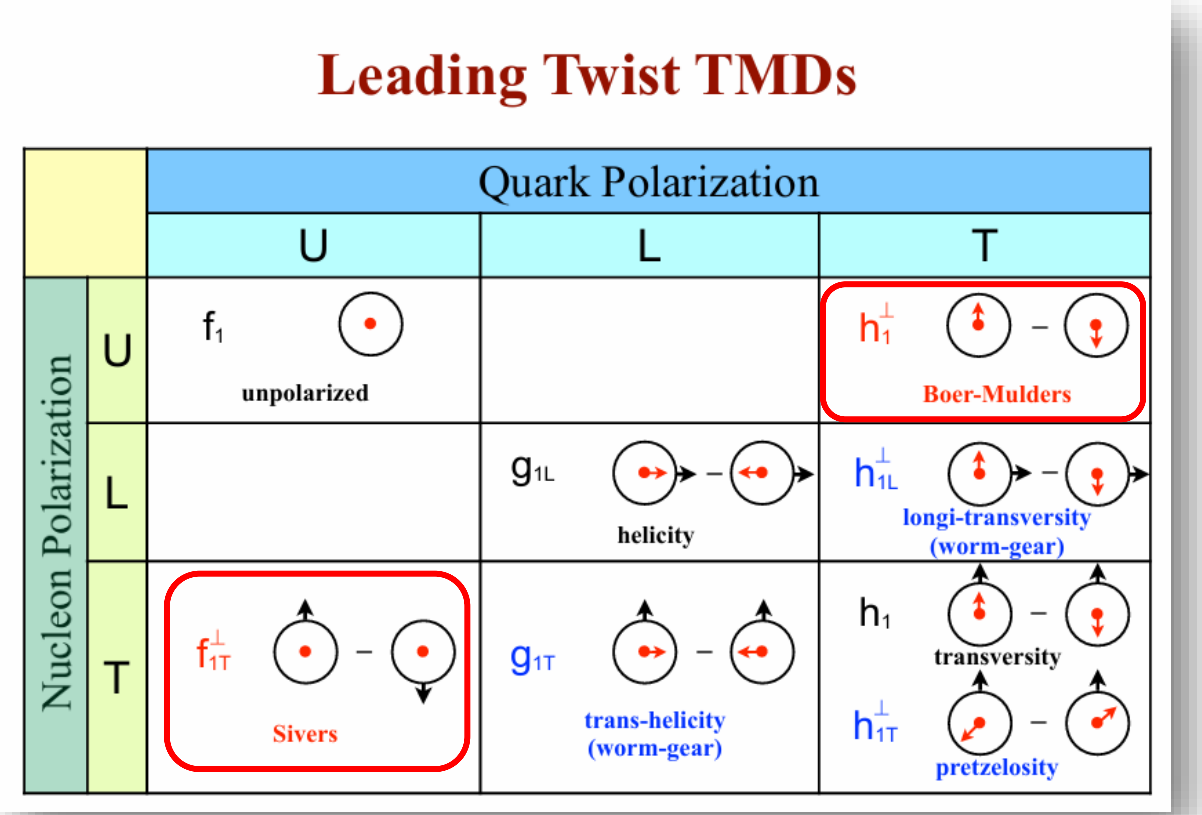
Leading twist TMDs

- Decompose the correlation function on a basis of Dirac structure
  - ✓ Hermiticity, parity invariance, and time reversal invariance

    $f_1(x), g_1(x), h_1(x)$
  - ✓ Considering quark transverse momentum
    - Hermiticity, parity invariance, and time reversal invariance

      $f_1(x,k_\perp), g_{1L}(x,k_\perp), g_{1T}(x,k_\perp), h_1(x,k_\perp), h_{1L}(x,k_\perp), h_{1T}(x,k_\perp)$
    - Without time reversal invariance

      $f_{1T}^\perp(x,k_\perp), h_1^\perp(x,k_\perp)$ (Naive) T-odd TMDs



Pion leading twist TMDs

- The most direct expression of EHM in the SM
  - ✓ Massless in the chiral limit (Nambu-Goldstone boson)
  - ✓ Simplest bound state
- The leading twist pion TMDs

$$\Phi^{[i\gamma^+]}(x, \boldsymbol{k}_\perp) = f_1(x, \boldsymbol{k}_\perp)$$

$$\Phi^{[i\sigma^{i+}\gamma^5]}(x, \boldsymbol{k}_\perp) = -\frac{\varepsilon_T^{ij}\boldsymbol{k}_{\perp j}}{M} h_1^\perp(x, \boldsymbol{k}_\perp)$$

- f_1 and $h_1^\perp$ are constrained by positivity bound relation

$$\left|\boldsymbol{k}_T h_1^\perp(x, \boldsymbol{k}_\perp^2)\right|/M \leq f_1(x, \boldsymbol{k}_T)$$

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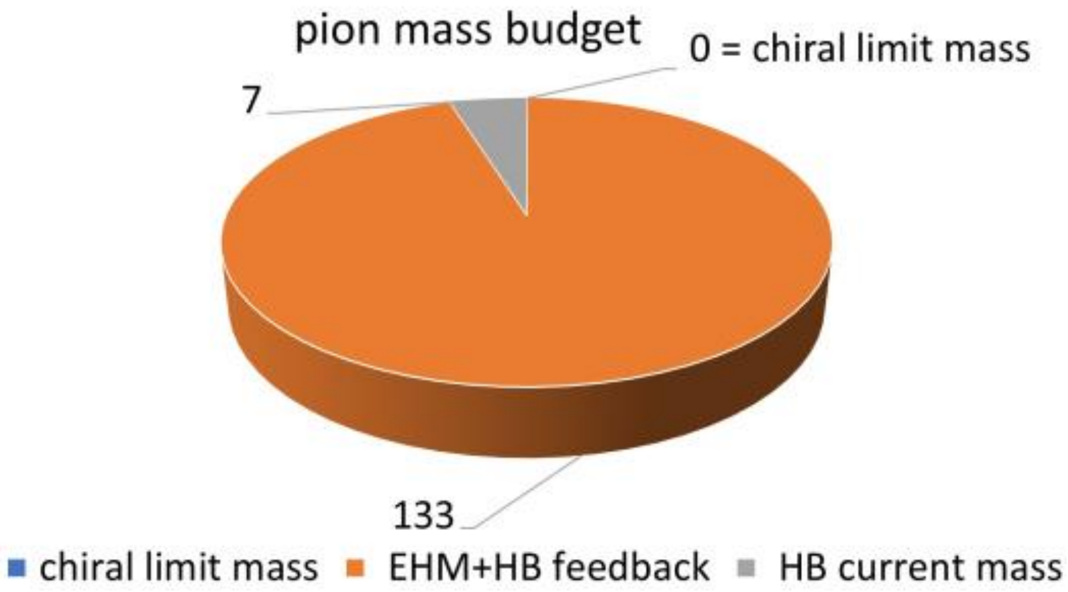


$$\frac{\varepsilon_T^{ij}\boldsymbol{k}_{Tj}}{m_\pi} h_1^\perp(x, \boldsymbol{k}_T) \to \frac{\varepsilon_T^{ij}\boldsymbol{k}_{Tj}}{M} h_1^\perp(x, \boldsymbol{k}_T)$$

- $h_1^\perp$ can not vanish in chiral limit

Number density interpretation

- The distribution of transversely polarized quarks in an unpolarized hadron

$$f_{q^\uparrow/p}(x,\boldsymbol{k}_\perp)=\frac{1}{2}\left[f_1^q(x,\boldsymbol{k}_\perp^2)-\frac{(\widehat{\boldsymbol{P}}\times\boldsymbol{k}_\perp)\cdot\boldsymbol{S}_q}{M}h_1^\perp(x,\boldsymbol{k}_\perp^2)\right]$$

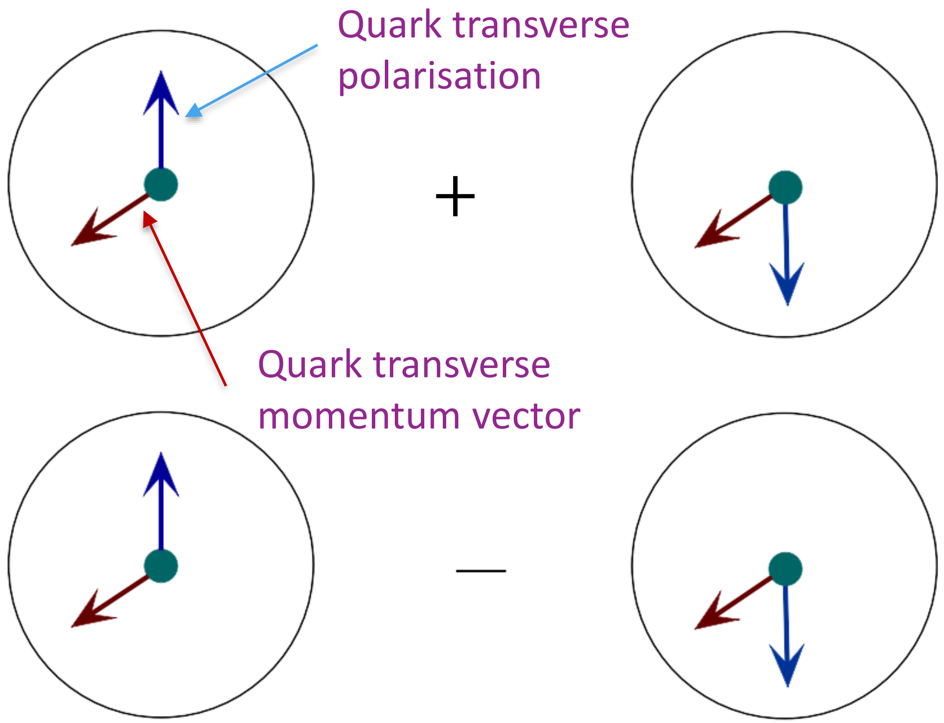
A. Bacchetta, U. DAlesio, M. Diehl, C. A. Miller, Phys. Rev. D 70 (2004) 117504

- Unpolarized distribution function

$$f_{q^\uparrow/N}(x,\boldsymbol{k}_\perp)+f_{q^\downarrow/N}(x,\boldsymbol{k}_\perp)=f_1^q(x,\boldsymbol{k}_\perp^2)$$

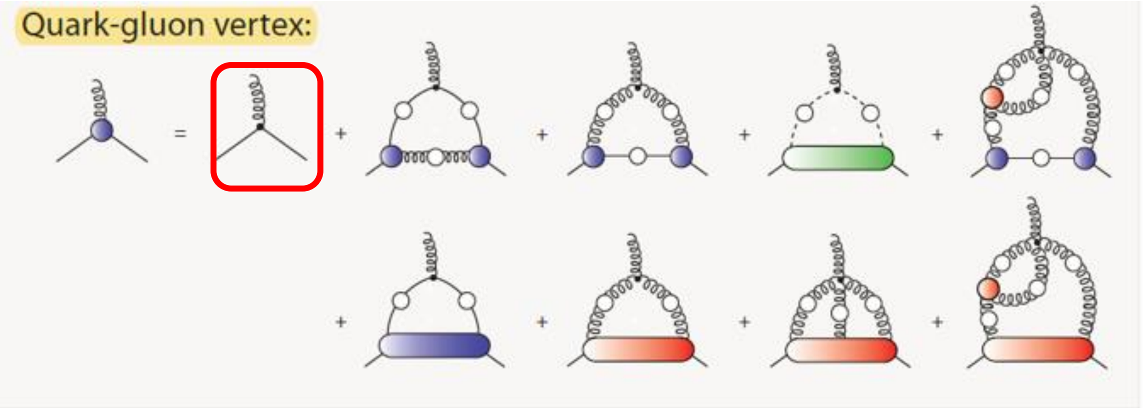
- The Boer-Mulders function arises from the correlation of the quark spin and the quark transverse momentum

$$f_{q^\uparrow/N}(x,\boldsymbol{k}_\perp)-f_{q^\downarrow/N}(x,\boldsymbol{k}_\perp)=-\frac{|\boldsymbol{k}_T|h_1^\perp(x,\boldsymbol{k}_\perp^2)}{M}\sin(\phi_{k_\perp}-\phi_S)$$



Dyson-Schwinger equation

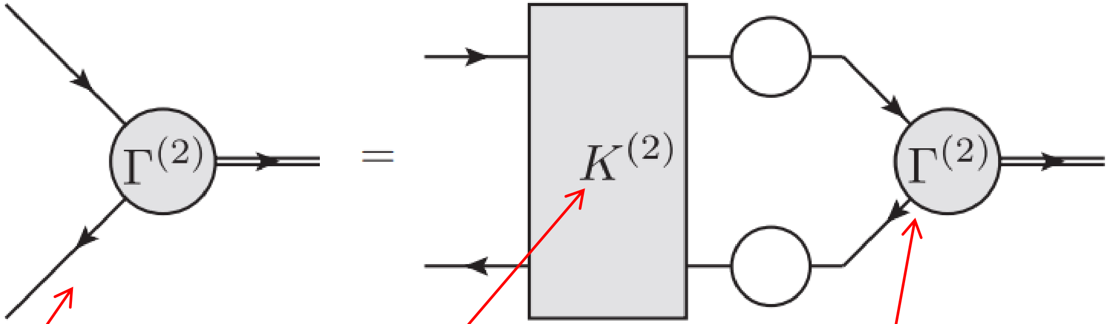
- Continuum Schwinger function methods
  - ✓ nonperturbative
  - ✓ symmetry-preserving
- Owing to the infinite coupling feature, it is necessary to truncate the Dyson-Schwinger equations at a certain level for practical calculations
  - ✓ Rainbow-ladder truncation $\Gamma_\nu^a(q,p) = \frac{\lambda^a}{2}\gamma_\nu$



- The dressed quark propagator (Gap equation)

$$S(p)^{-1} = i\gamma\cdot p + m + \int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2}\gamma_\mu S(q) \frac{\lambda^a}{2}\Gamma_\nu(q,p)$$

- The two body system is described by Bethe-Salpeter equation



$$\Gamma(k;P) = -\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q)\gamma_\mu \frac{\lambda^a}{2} S(q+P)\Gamma(q;P)S(q)\frac{\lambda^a}{2}\gamma_\nu$$

Contact interaction

- Contact interaction (SCI)

$$g^2 D_{\mu\nu}(p-q) = \delta_{\mu\nu} \frac{4\pi\alpha_{IR}}{m_G^2}$$

where $m_G \approx 0.5\text{GeV}$ is a gluon mass-scale and the $\alpha_{IR} \approx \pi$ is the zero-momentum value of a running-coupling constant in QCD.

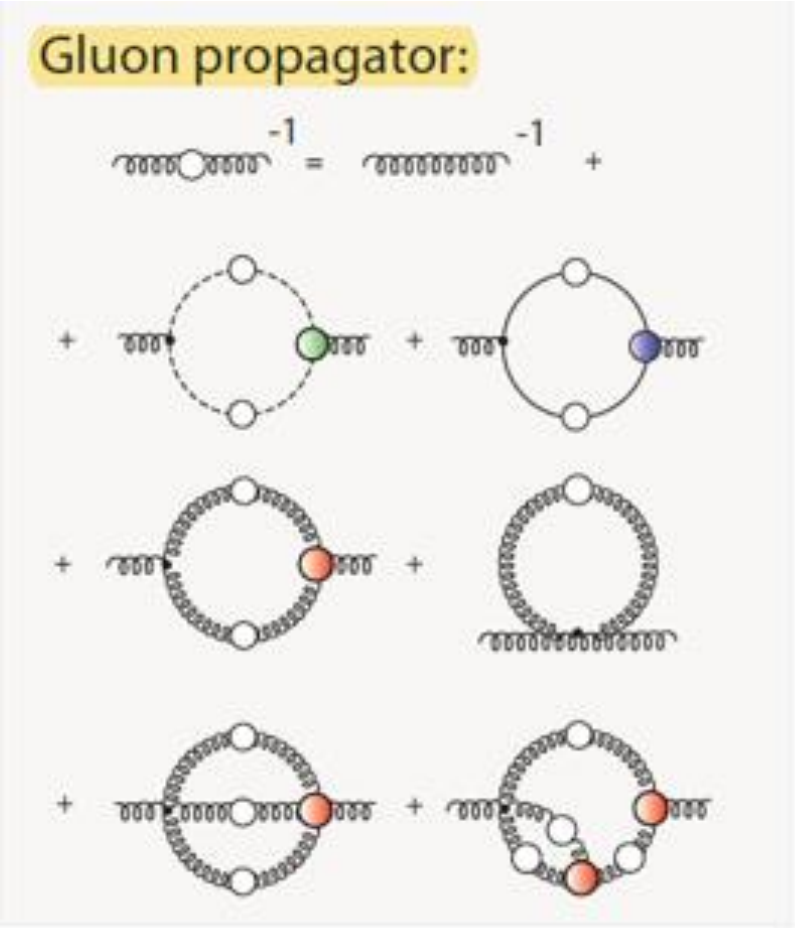
- Gap equation

$$S(p)^{-1} = i\gamma \cdot p + M$$

- Bethe-Salpeter amplitude

$$\Gamma_P(Q) = \gamma_5 \left[iE_P(Q) + \frac{1}{2M_{fg}} \gamma \cdot Q F_P(Q) \right]$$

- ✓ algebraic simplicity
- ✓ Parameter free
- ✓ applicable to a wide variety of systems and processes
- ✓ potential for revealing insights that connect and explain numerous phenomena
- ✓ capacity to serve as a means of checking the validity of algorithms employed in calculations that depend upon high performance computing



Unpolarized distribution function

➢ Ignoring the gauge link in correlation function

$$f_{1\pi}(x,\boldsymbol{k}_\perp^2) = \frac{N_c}{2\pi^3}\left[E_\pi(E_\pi - 2F_\pi)\bar{C}_2(\omega_1) + 3(E_\pi - 2F_\pi)^2 x(1-x)M_\pi^2\bar{C}_3(\omega_1)\right]$$

J.-L. Zhang, et al. Eur. Phys. J. C 81 (2021).

✓ The result at hadron scale (valence degrees-of-freedom carries all pion properties)

✓ The light-front momentum sum rule is saturated by the pion's valence degrees-of-freedom

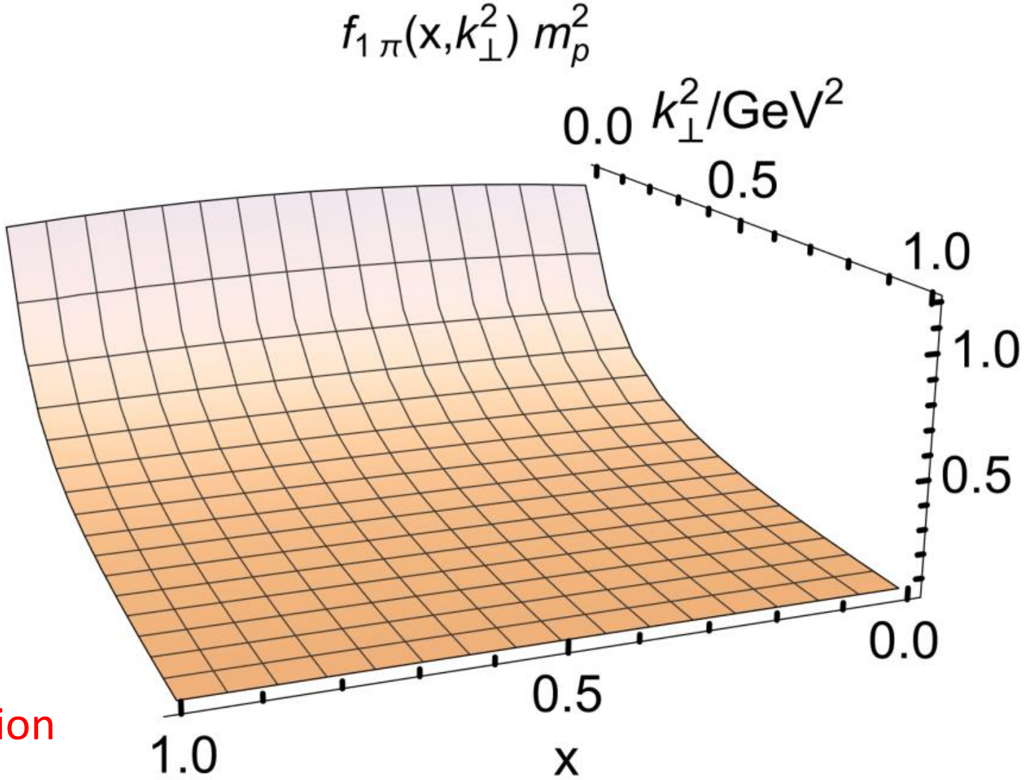
$$\int_0^1 dx \int d^2k_\perp\, 2x f_{1\pi}(x,\boldsymbol{k}_\perp^2) = 1$$

✓ Symmetric around $x = 1/2$. The $k_\perp^2$ profile is almost independent of x

$m_\pi^2/M^2 \ll 1$

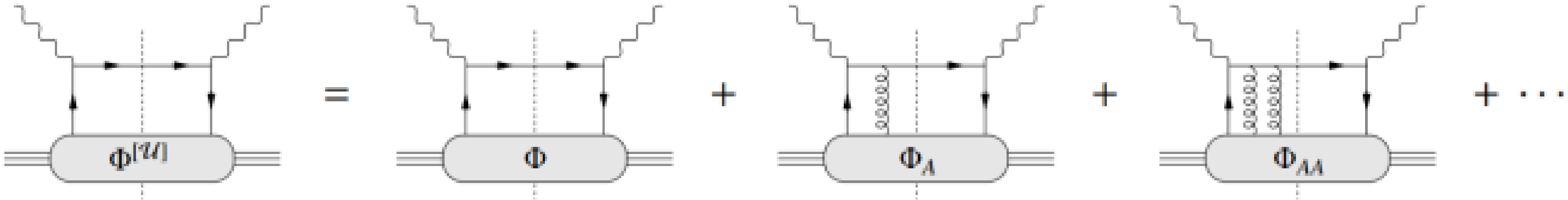
✓ $f_{1\pi}(x,\boldsymbol{k}_\perp^2) \neq 0$ on $x \simeq 0{,}1$ at any finite $k_\perp^2$

artefact of the momentum independent quark + quark interaction



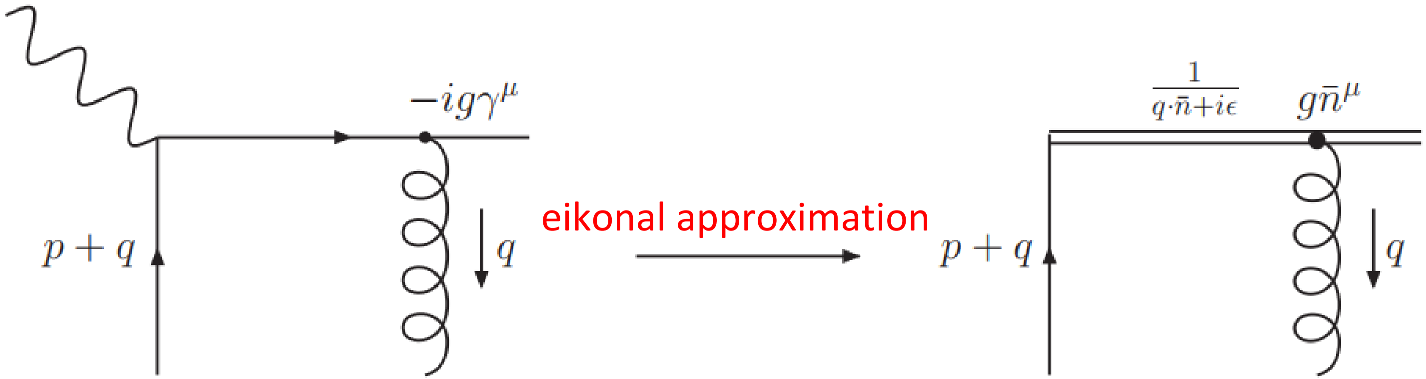
Eikonal approximation

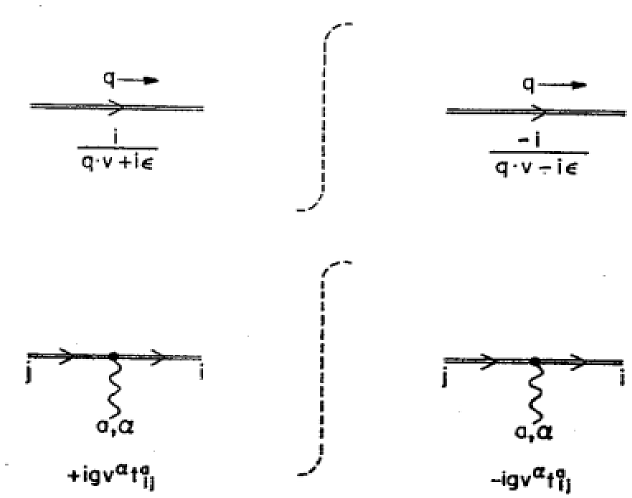
- Non-zero BM function requires valence quark involved in the forward scattering event subsequently/ initially interacts with the spectator via (multiple) gluon exchanges



C. J. Bomhof, Other thesis, Vrije U., 2007.

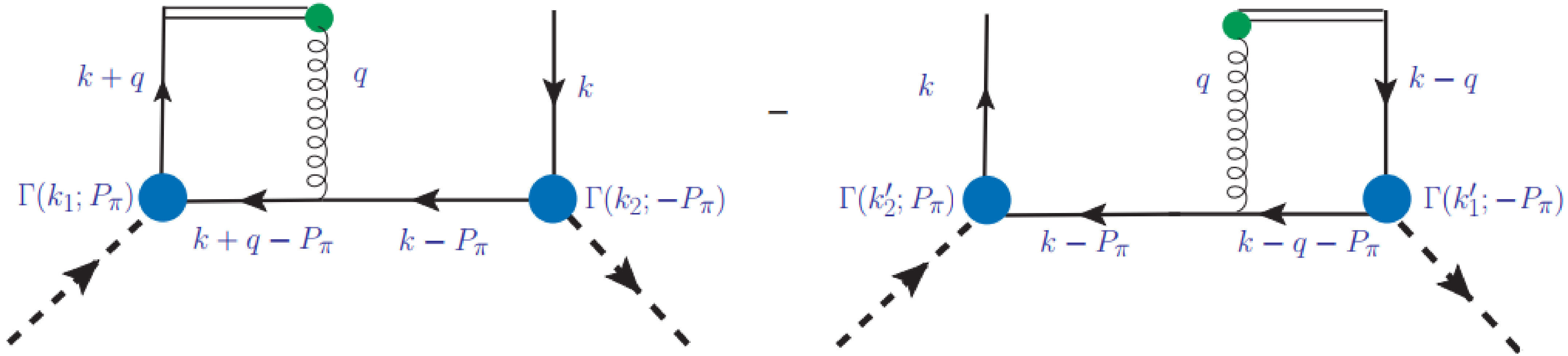
- ✓ Gluon interactions with the struck quark can be approximated by the double line propagator after eikonal approximation





J. C. Collins, D. E. Soper, G. F. Sterman, Adv. Ser. Direct. High Energy Phys. 5 (1989) 1–91

Diagrammatic calculation of BMF



- The relative negative sign expresses the sign-change between initial- and final- state eikonal-quark interactions
- The second diagram can be mapped into the first by time reversal invariance

$$h_1^{\perp}(x,\boldsymbol{k}_{\perp})\frac{\vec{k}_{\perp\alpha}}{M} = N_c tr_D \int \frac{d^4q}{(2\pi)^4}\frac{dk_3dk_4}{(2\pi)^4}\delta_n^x(k)S(k-P_\pi)\Gamma(-P_\pi)S(k)\sigma_{\alpha+}\frac{[-gn_\mu]}{n\cdot q}S(k+q)\Gamma(P_\pi)S(k+q-P_\pi)[-g\gamma_\nu]D_{\mu\nu}$$

- The on shell condition of double line is represented by $1/n\cdot q \to \mp\pi\delta(n\cdot q)$

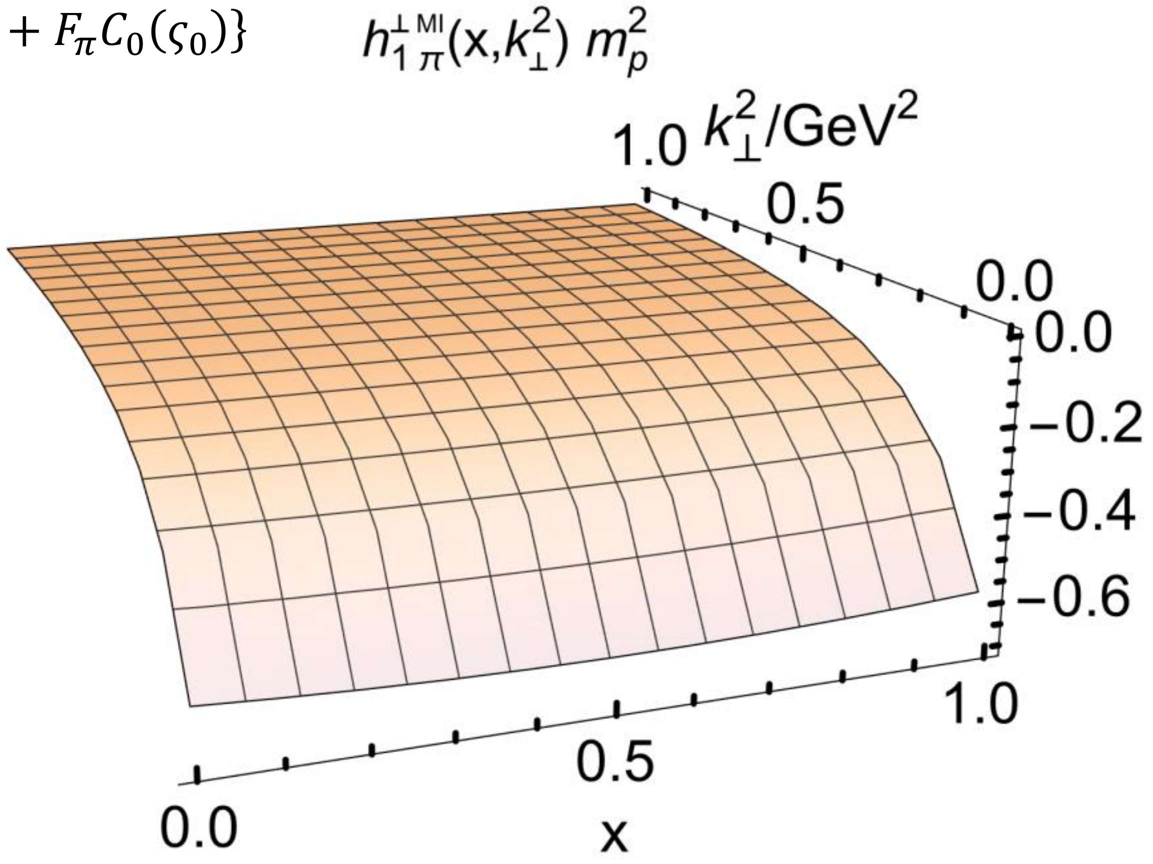
$$h_1^{\perp}(x,\boldsymbol{k}_{\perp})_{SIDIS} = -h_1^{\perp}(x,\boldsymbol{k}_{\perp})_{DY}$$

SCI gauge link

- Symmetry preserving contact interaction

$$g^2 D_{\mu\nu}(q) = \delta_{\mu\nu} \frac{4\pi\alpha_{IR}}{m_G^2}$$

$$h_1^{\perp}(x, \boldsymbol{k}_{\perp}^2) = -\frac{\alpha_{IR}}{m_G^2}\frac{N_c}{4\pi^3}\mathcal{N}_{EF}\frac{\bar{C}_2(\varsigma)}{\varsigma}\{[(E_\pi - F_\pi)M^2 - x(1-x)m_\pi^2 F_\pi]\bar{C}_1(\varsigma_0) + F_\pi C_0(\varsigma_0)\}$$



- ✓ Result at hadron scale
- ✓ Non-zero in the chiral limit
- ✓ Only non-zero because of the interaction between the eikonalized quark and the spectator
- ✓ The magnitude of the effect reflects the scale of EHM $\varsigma = \boldsymbol{k}_{\perp}^2 + M^2 - x(1-x)m_\pi^2$
- ✓ The similar profile compared to $f_{1\pi}$

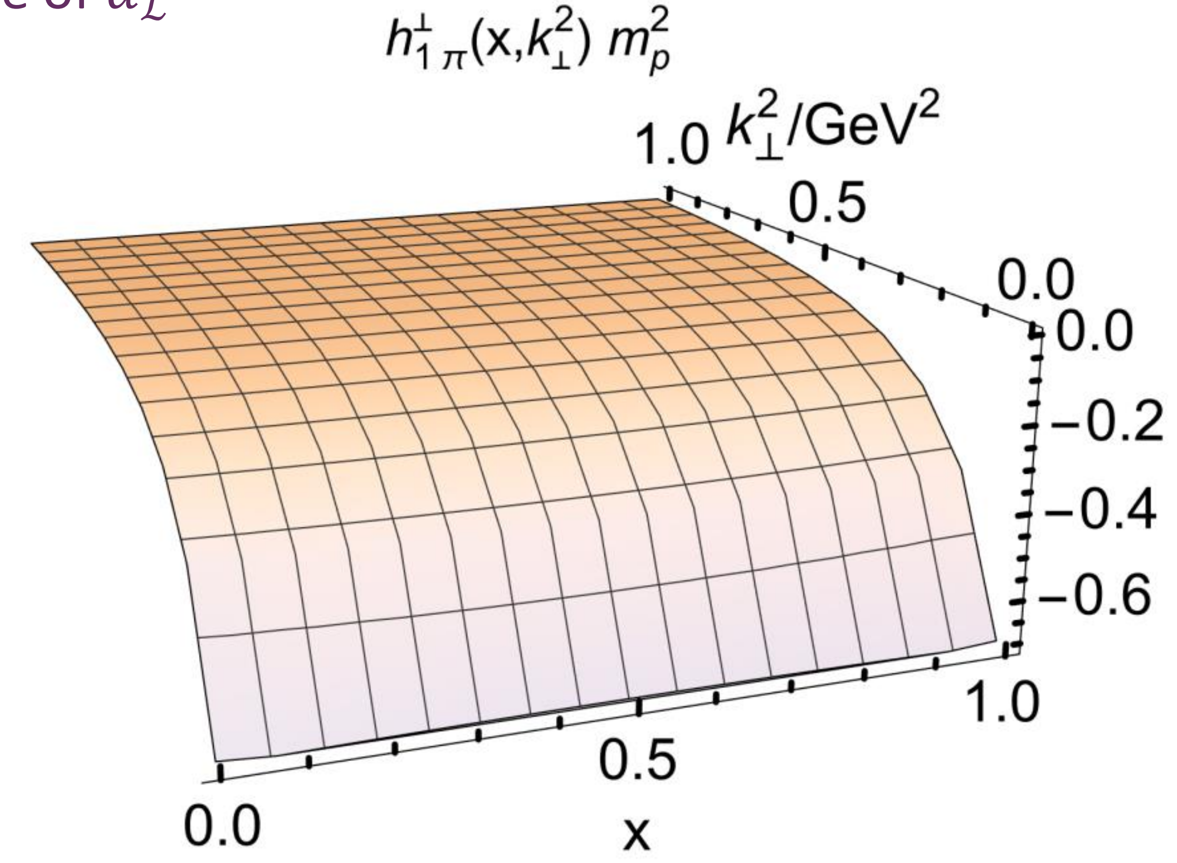
- Practically undamped

Momentum-dependent link completion

$$g^2 D_{\mu\nu}(q) = \delta_{\mu\nu} \frac{4\pi\alpha_{\mathcal{L}}}{q^2 + m_G^2}$$

- More realistic interaction
  - ✓ It is SCI form at infrared momenta when $\alpha_{\mathcal{L}} = \alpha_{IR}$
  - ✓ Provides damping in the ultraviolet for any value of $\alpha_{\mathcal{L}}$

$$h_{1\pi}^{\perp} = -\alpha_{\mathcal{L}} \frac{N_c}{4\pi^3} \mathcal{N}_{EF} \frac{\bar{C}_2(\varsigma)}{\varsigma} \int_0^1 dv \left\{ 2(1-v)(E_\pi M^2 - F_\pi[M^2 + x(x-1)m_\pi^2 + vk_\perp^2]) \frac{\bar{C}_2(\tilde{\varsigma})}{\tilde{\varsigma}} + F_\pi \bar{C}_1(\tilde{\varsigma}) \right\}$$



- Other model calculations also considered a momentum-dependent gauge-link

  M. Ahmady, et, al. Phys. Rev. D, 100 (2019) 5, 054005

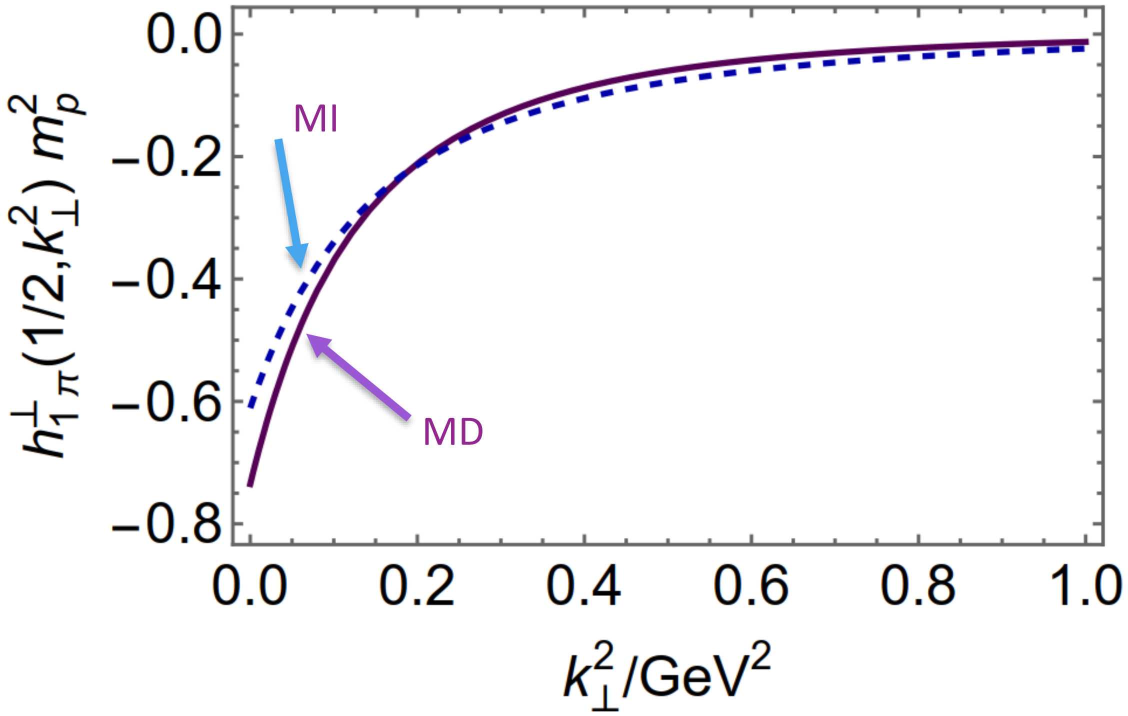
  Wei Kou, et, al. Phys. Rev. D 108 (2023) 3, 036021

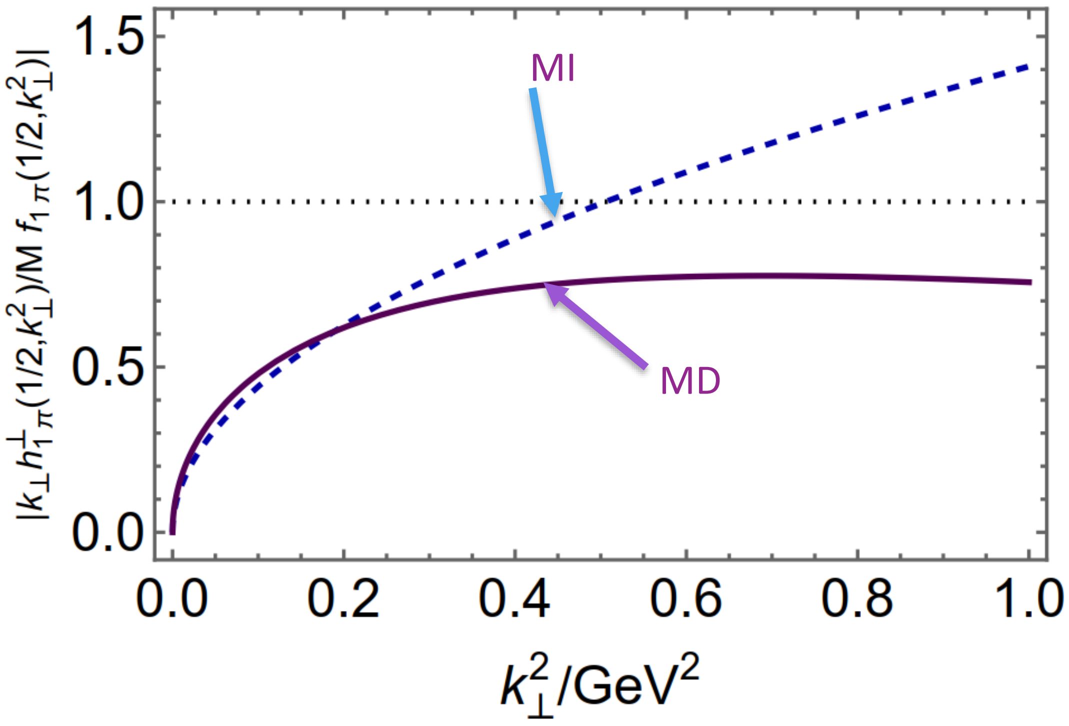
  - ✓ A mismatch between the bound state kernel and the gauge link completion

Comparison of two gauge links

$$\left|\boldsymbol{k}_T h_1^{\perp}(x,\boldsymbol{k}_\perp^2)\right|/(M f_1(x,\boldsymbol{k}_T)) \leq 1$$

- BMF decays quicker with the momentum-dependent gauge link interaction





- The positivity bound is violated in SCI gauge link
  - ✓ provide greater support to the gauge link contribution than it do to the usual quark loop

Z. Lu, B.-Q. Ma, Phys. Rev. D 70 (2004) 094044

Z. Lu, B.-Q. Ma, Phys. Lett. B 615 (2005) 200–206

B. Pasquini, P. Schweitzer, Phys. Rev. D 90 (2014) 014050

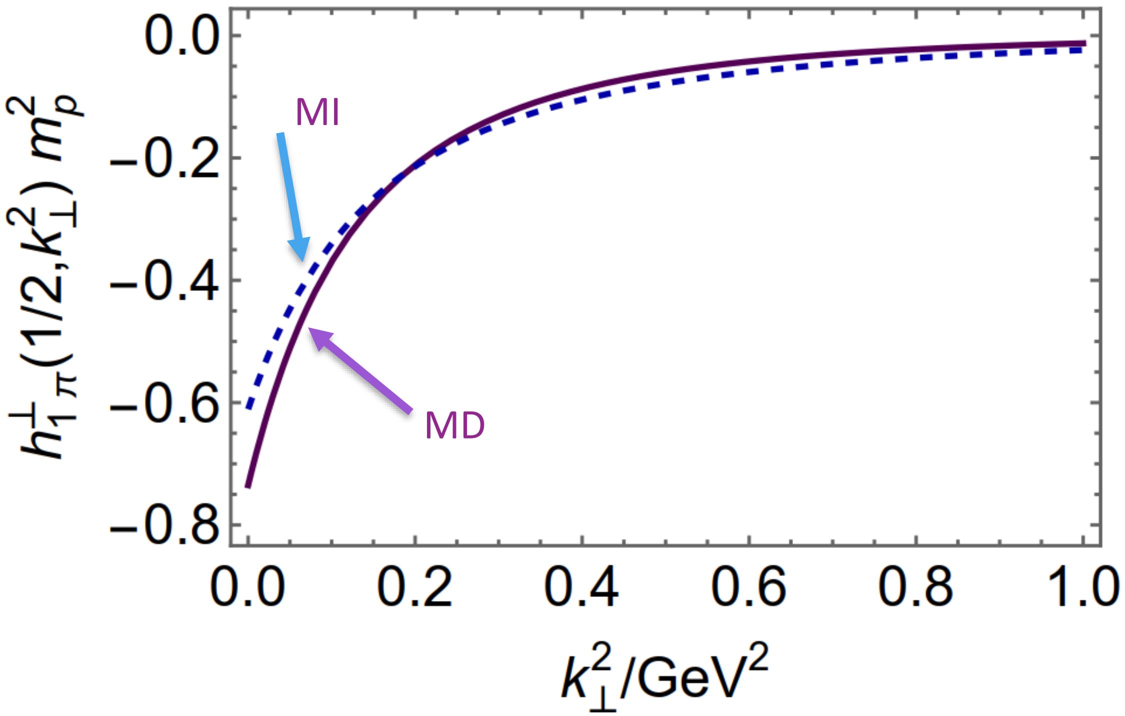
Z. Wang, X. Wang, Z. Lu, Phys. Rev. D 95 (2017) 094004

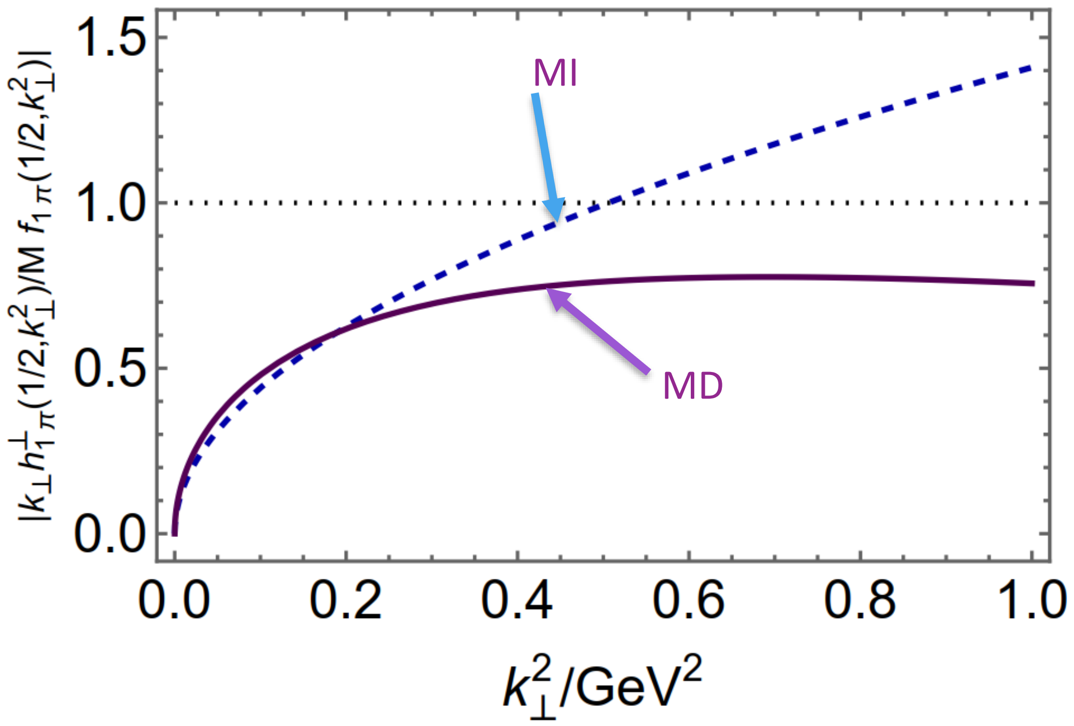
M. Ahmady, et, al. Phys. Rev. D, 100 (2019) 5, 054005

Wei Kou, et, al. Phys. Rev. D 108 (2023) 3, 036021

Comparison of two gauge links

$$\left|\boldsymbol{k}_T h_1^{\perp}(x,\boldsymbol{k}_{\perp}^2)\right|/(M f_1(x,\boldsymbol{k}_T)) \le 1$$

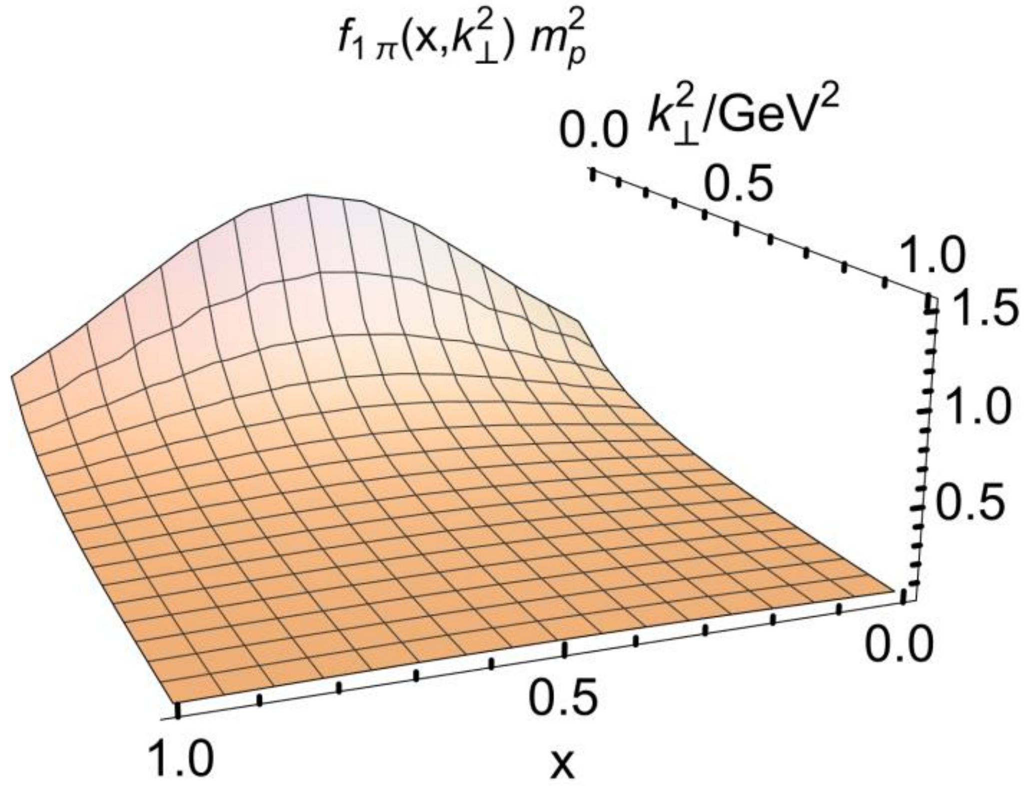


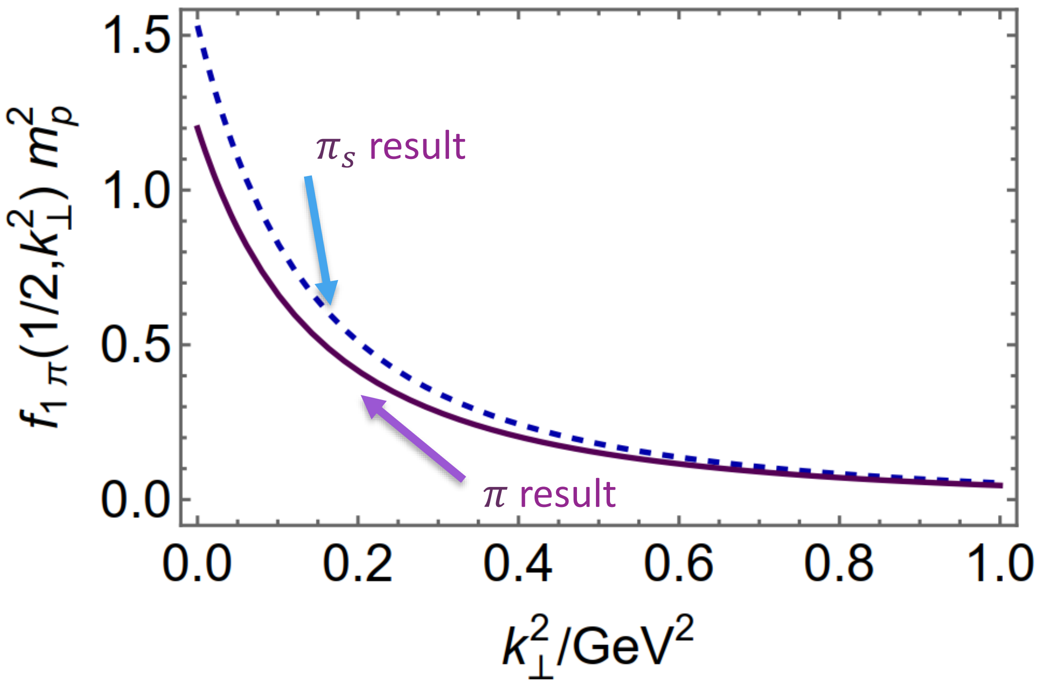


- The positivity bound is satisfied in momentum-dependent gauge link
  - ✓ quark and gluon propagators used throughout possess an ultraviolet momentum dependence that matches QCD expectations
  - ✓ the support of the usual quark loop is not curtailed without at least commensurate and consistent suppression of the gauge link range

Current mass dependence

- Considering a (fictitious) pion composed of a quark s and antiquark $\bar{s}$

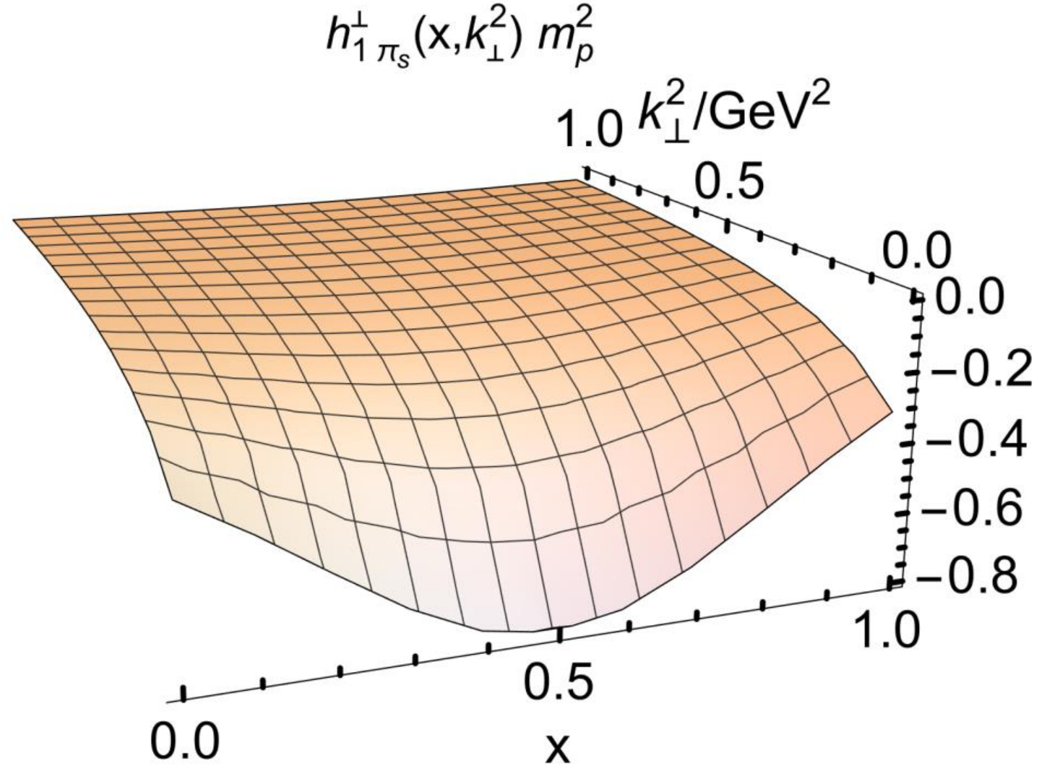


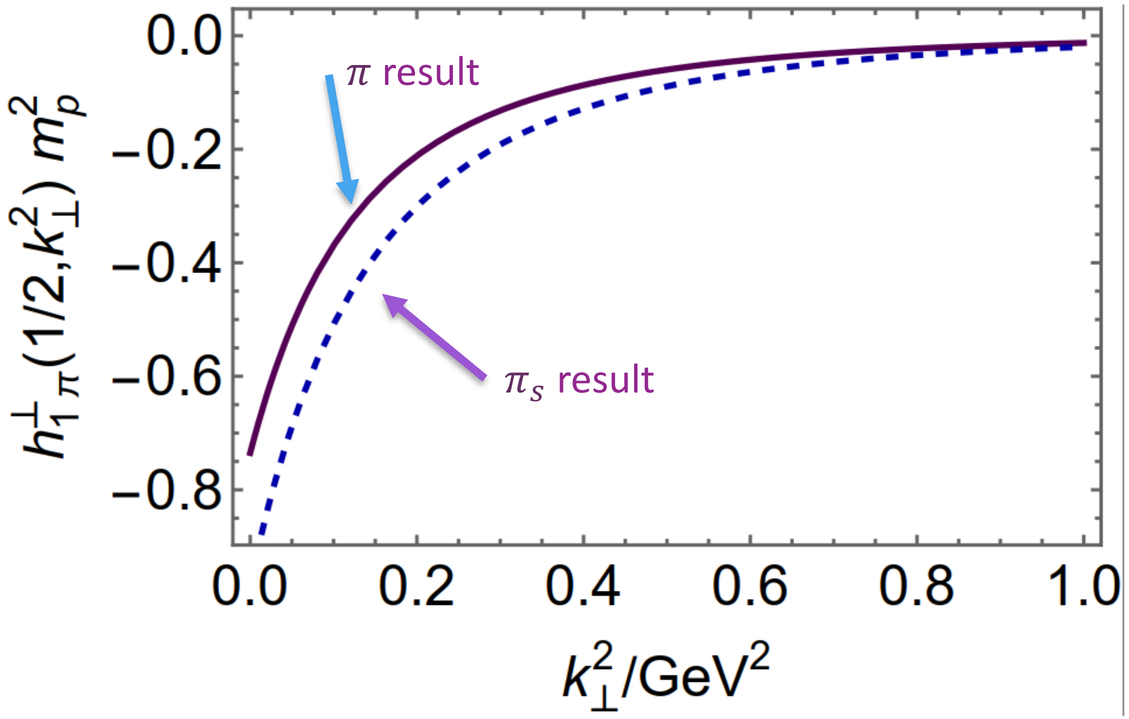


- ✓ The x profile is far less dilated, with greater concertation around $x = 1/2$
- ✓ The increased magnitude of the π_s at its global maximum with respect to that of the π
- ✓ More rapid decrease with increasing $k_\perp^2$ on the infrared domain

Current mass dependence

➢ Considering a (fictitious) pion composed of a quark s and antiquark $\bar{s}$





✓ The response is qualitatively equivalent to that of f_1

Pion Light Front Wave Function

- The pion light front wave function associated with the leading term in the operator expansion of the Fock space is the pion's RL LFWF
- Projecting the SCI Bethe-Salpeter wave function onto the light-front

$$\Psi(x,\boldsymbol{k}_\perp;\lambda_1,\lambda_2)=\psi(x,k_\perp^2)S_{\lambda_1,\lambda_2}(x,\boldsymbol{k}_\perp)$$

$$\psi(x,k_\perp^2)=\frac{\sqrt{2N_c}}{k_\perp^2+M^2-x(1-x)m_\pi^2}$$

$$S_{\lambda_1,\lambda_2}(x,\boldsymbol{k}_\perp)=\begin{bmatrix}S_{\uparrow\uparrow} & S_{\uparrow\downarrow}\\ S_{\downarrow\uparrow} & S_{\downarrow\downarrow}\end{bmatrix}$$

$$S_{\uparrow\uparrow}=-\mathcal{N}_{EF}(k_1-ik_2)$$

$$S_{\uparrow\downarrow}=\frac{E_\pi M^2+F_\pi[(k_\perp^2-M^2)-x(1-x)m_\pi^2]}{M}$$

$$S_{\downarrow\downarrow}=S_{\uparrow\uparrow}^*,S_{\downarrow\uparrow}=-S_{\downarrow\uparrow}$$

- The correlation function can be expressed by LFWFs

$$\Phi^{[g]}(x,\boldsymbol{k}_\perp)=\sum_{\lambda_1,\lambda_1',\lambda_2}\int\frac{d^2k_\perp'}{16\pi^3\sqrt{n\cdot kn\cdot k'}}\mathcal{G}(x,\boldsymbol{k}_\perp,\boldsymbol{k}_\perp')\psi(x,\boldsymbol{k}_\perp'^2)\psi(x,k_\perp^2)\left[S_{\lambda_1',\lambda_2}(x,\boldsymbol{k}_\perp')\right]^\dagger S_{\lambda_1,\lambda_2}(x,\boldsymbol{k}_\perp)\bar{u}(k',\lambda_1')\frac{g}{2}u(k,\lambda_1)$$

Pion TMDs in Contact Interaction

- Unpolarized distribution function

$$f_1(x,\boldsymbol{k}_\perp^2) = \frac{N_c}{2\pi^3}[E_\pi(E_\pi - 2F_\pi)\bar{\mathcal{C}}_2(\omega_1) + 3(E_\pi - 2F_\pi)^2 x(1-x)M_\pi^2\bar{\mathcal{C}}_3(\omega_1)]$$

- Boer-Mulders function

  - ✓ SCI gauge link

$$h_1^\perp(x,\boldsymbol{k}_\perp^2) = -\frac{\alpha_{IR}}{m_G^2}\frac{N_c}{4\pi^3}\mathcal{N}_{EF}\frac{\bar{\mathcal{C}}_2(\varsigma)}{\varsigma}\{[(E_\pi - F_\pi)M^2 - x(1-x)m_\pi^2 F_\pi]\bar{\mathcal{C}}_1(\varsigma_0) + F_\pi\mathcal{C}_0(\varsigma_0)\}$$

  - ✓ Momentum dependent gauge link completion

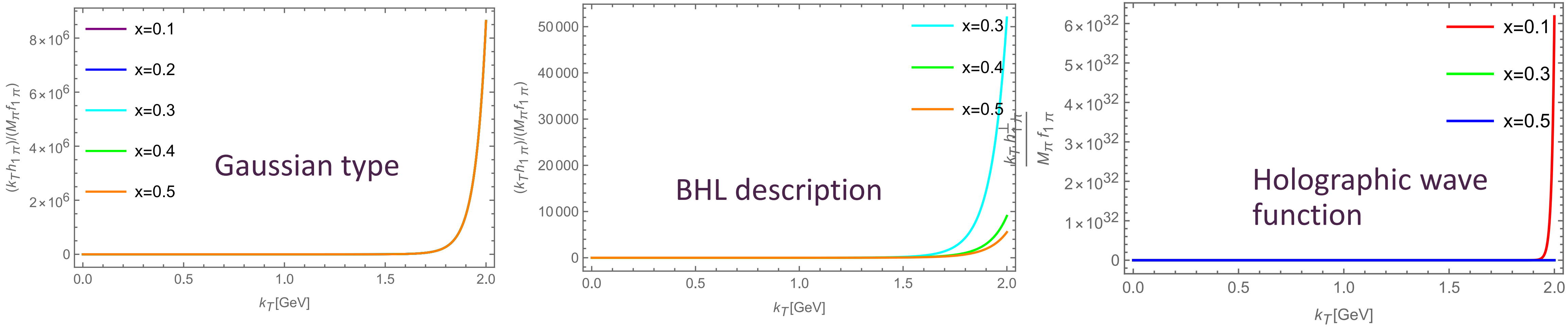
$$h_{1\pi}^\perp = -\alpha_{\mathcal{L}}\frac{N_c}{4\pi^3}\mathcal{N}_{EF}\frac{\bar{\mathcal{C}}_2(\varsigma)}{\varsigma}\int_0^1 dv\left\{2(1-v)(E_\pi M^2 - F_\pi[M^2 + x(x-1)m_\pi^2 + vk_\perp^2])\frac{\bar{\mathcal{C}}_2(\tilde{\varsigma})}{\tilde{\varsigma}} + F_\pi\bar{\mathcal{C}}_1(\tilde{\varsigma})\right\}$$

Positivity bound

$$\psi(x,k_\perp^2)=\frac{\sqrt{2N_c}}{[k_\perp^2+M^2-x(1-x)m_\pi^2]^2}$$

$$g^2D_{\mu\nu}(q)=\delta_{\mu\nu}\frac{4\pi\alpha_{\mathcal{L}}}{q^2+m_G^2}$$

- ➢ Considering the other form of LFWFs
  - ✓ Positivity bound is violated. Support in the quark loop is too compact in comparison with the range of the gauge link
- ➢ Similar violations appear when using other momentum components of LFWFs



- ✓ Positivity can be restored by replacing the re-scattering kernel by some more rapidly damping form
- ✓ Further weakens the connection between any such model and QCD

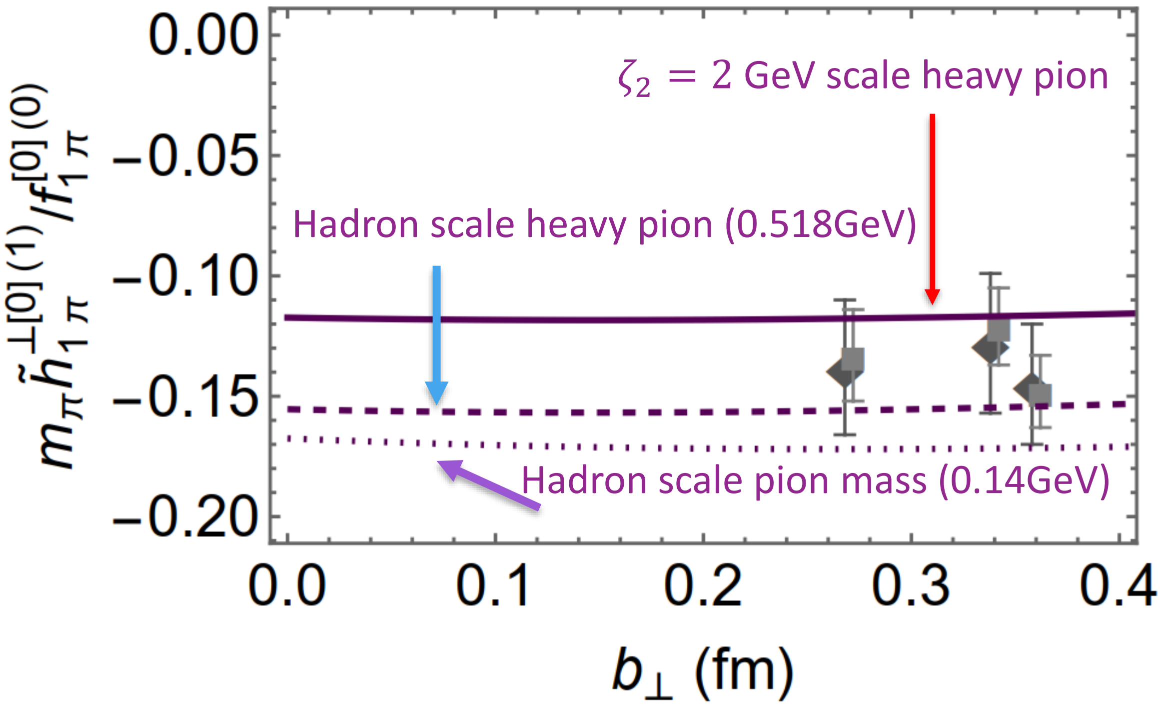
Generalized Boer-Mulders shift

- The so called "generalized Boer-Mulders shift" is defined in an appropriate way to be safely evaluated on the lattice

$$\langle k_y \rangle_{UT}(b_T^2;\zeta_2) \equiv m_\pi \frac{\tilde{h}_1^{\perp[0](1)}(b_T^2;\zeta_2)}{\tilde{f}_1^{[0](0)}(b_T^2;\zeta_2)}$$

  - ✓ The denominator is unity, independent of the resolving scale
  - ✓ The numerator is directly related to the first $k_\perp$ moment of BM function

$$H_0(\zeta_2^2) = H_0(\zeta_1^2)\left[\frac{\alpha_{LO}(\zeta_2^2)}{\alpha_{LO}(\zeta_1^2)}\right]^{\frac{1}{4}(C_F-2\mathcal{S})\gamma_m}$$



- The magnitude of the shift decreases slowly with increasing meson mass
- The impact of evolution is noticeable, but not dramatic

Summary

- The two nonzero pion TMDs have been predicted by using a symmetry preserving contact interaction.
- The two models between spectator and eikonalized quark have been considered to generate non-zero BM function. One is SCI based; and the other involves momentum-dependent gluon exchange.
  - ✓ The positivity constraint is satisfied in momentum dependent gauge link completion.
- The consistent results and analyses can be obtained by using light front wave functions.
- The SCI-based prediction for the generalized Boer-Mulders shift is consistent with existing results obtained using Lattice-regularized QCD.

Thankyou