北京航空航天大學
BEIHANG UNIVERSITY

The Tomography of Nucleon:

Lattice QCD calculation of the unpolarized transverse-momentum-dependent parton distributions

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Oct. 19 @第二届核子三维结构研讨会

Lattice Parton Collaboration

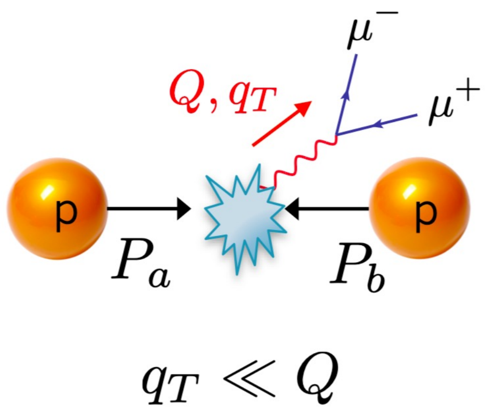
OUTLINE

- Motivation
- Lattice QCD calculation of TMDPDFs
  - Extract TMDPDFs from LaMET
  - Quasi TMDPDF matrix elements and their renormalization
  - From Quasi TMDPDF to physical TMDPDF
  - Numerical results
- Summary and Outlook

TMDPDFs: 3D tomography of the nucleon

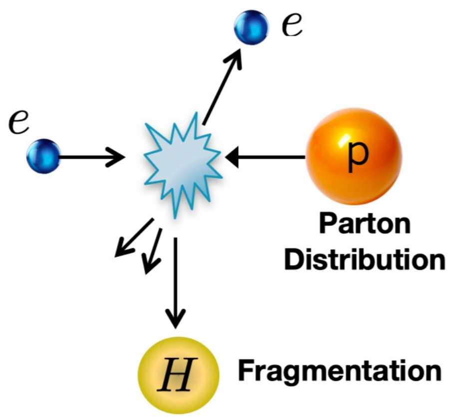
TMD processes:

Drell-Yan



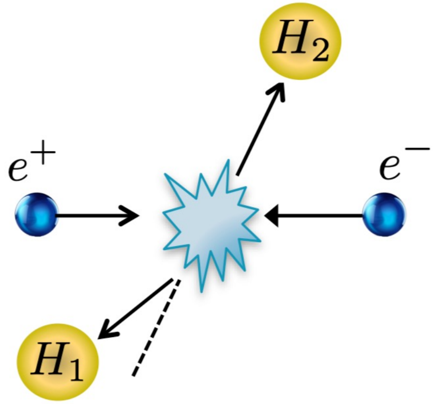
LHC, FermiLab, RHIC, …

Semi-Inclusive DIS



HERMES, COMPASS, JLab, EIC, …

Dihadron in e+e-



BESIII, Babar, Belle, …

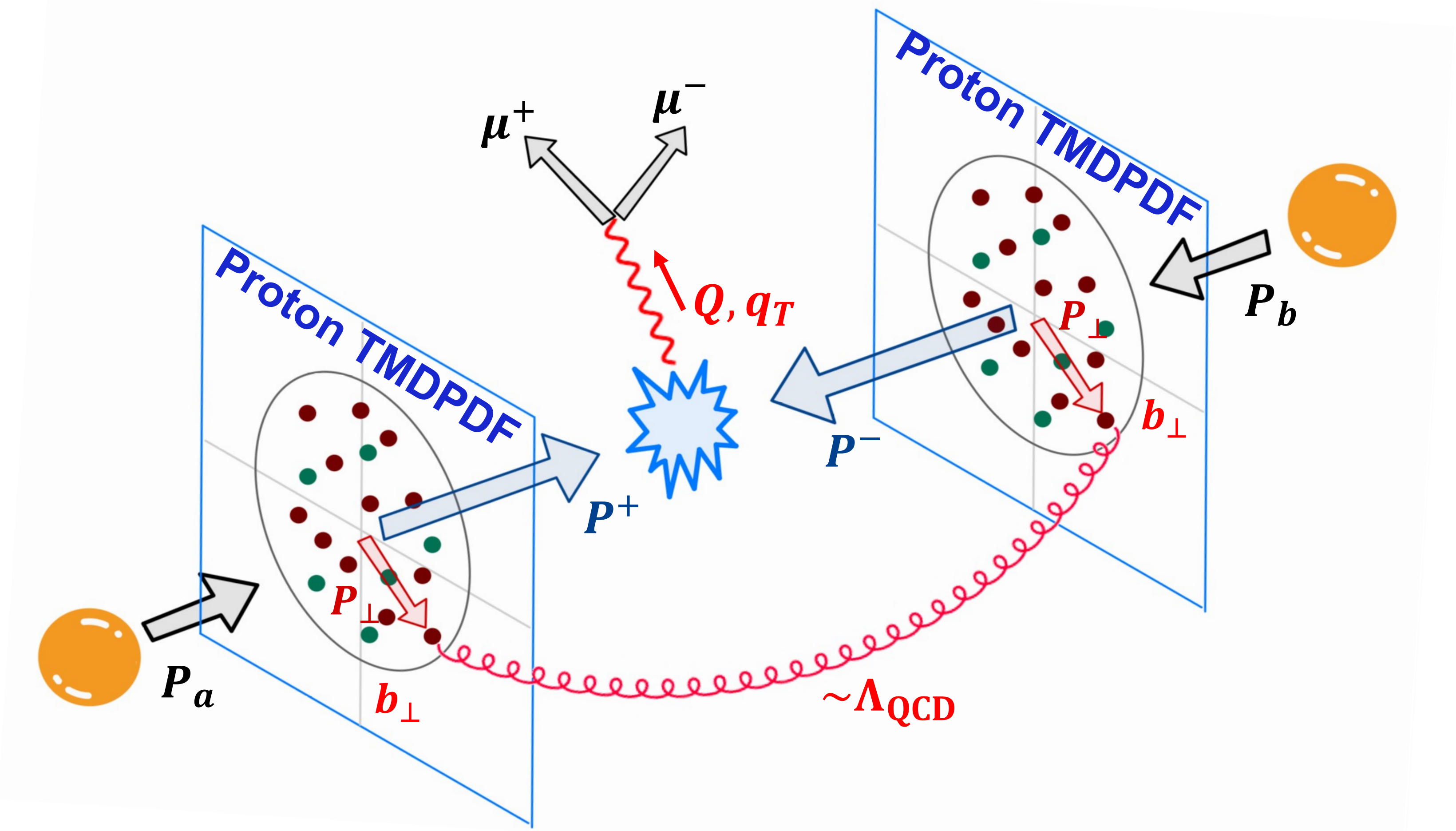
$$\sigma \sim f_{q/P}(x, k_T)\, f_{q/P}(x, k_T)$$

$$\sigma \sim f_{q/P}(x, k_T)\, D_{h/q}(x, k_T)$$

$$\sigma \sim D_{h_1/q}(x, k_T)\, D_{h_2/q}(x, k_T)$$

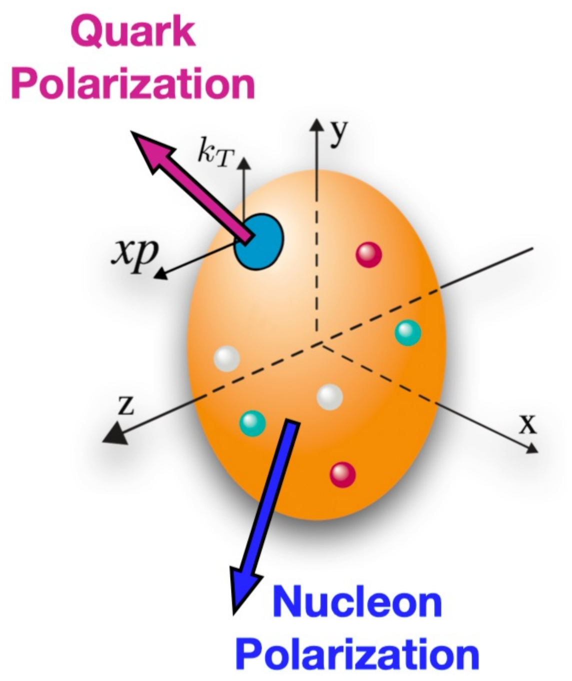
TMDPDFs: 3D tomography of the nucleon

- **Low-q_T region of Drell-Yan Process:**



Revealing the confined motion of partons inside the nucleon

TMDPDFs: 3D tomography of the nucleon



Leading Quark TMDPDFs

Nucleon Spin   Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
	L		g_{1L} = − Helicity	$h_{1L}^\perp$ = − Worm-gear
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity; $h_{1T}^\perp$ = − Pretzelosity

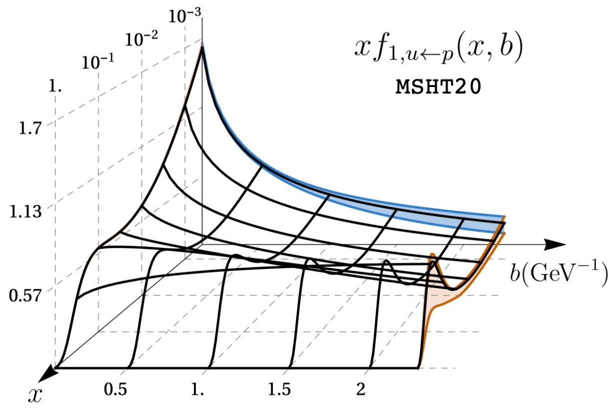
TMD Handbook, TMD Collaboration, 2304.03302

Progress in the study of TMDPDFs

- **Theoretical analysis**
  - **TMD factorization, evolution and resummation:**

    Boussarie et al., TMD handbook, 2304.03302;

    Collins, Foundations of perturbative QCD; ……



u-quark unpolarized TMDPDF, 2201.07114

- **Phenomenological parametrizations and extractions**
  - **Unpolarized:**

    Moos, JHEP05 (2024); Bacchetta, JHEP10 (2022); Bury, JHEP10 (2022);

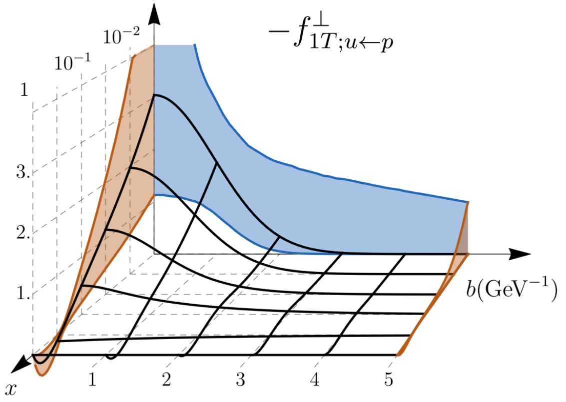
    Scimem, JHEP06 (2020); Bacchetta, JHEP06 (2017); ……

  - **Sivers, Boer-Mulders:**

    Bury, PRL126 (2021), JHEP05 (2021) ; Cammarota, PRD102(2020);

    Zhang, PRD77 (2008), Lu, PRD81 (2010) ; ……

  - **Others: worm-gear, gluon TMDs, ……**



u-quark Sivers function, PRL126 (2021)

➢ Lattice calculations

- **Lorentz-invariant approach:** ratios of Mellin moments

  Hagler, EPL88(2009); Musch, PRD85(2012); Engelhardt, PRD93(2016); Yoon, 1601.05717, PRD96(2017); ……

- **LaMET formalism:**

  ✓ **I: theoretical analysis of matching kernel, soft function, Collins-Soper kernel, ……**

  Rio, PRD108(2023); Ji, JHEP08(2023), RMP93(2021), NPB955(2020), PLB811(2020);

  Ebert, JHEP04(2022); Deng, JHEP09(2022)……

  ✓ **II: lattice calculation of intrinsic soft function, Collins-Soper kernel, beam function, ...**

  LPC, JHEP08(2023), PRL125(2020); Li, PRL128(2022); LPC, PRD106(2022);

  Shanahan, PRD104(2021); Schlemmer, JHEP08(2021); ……

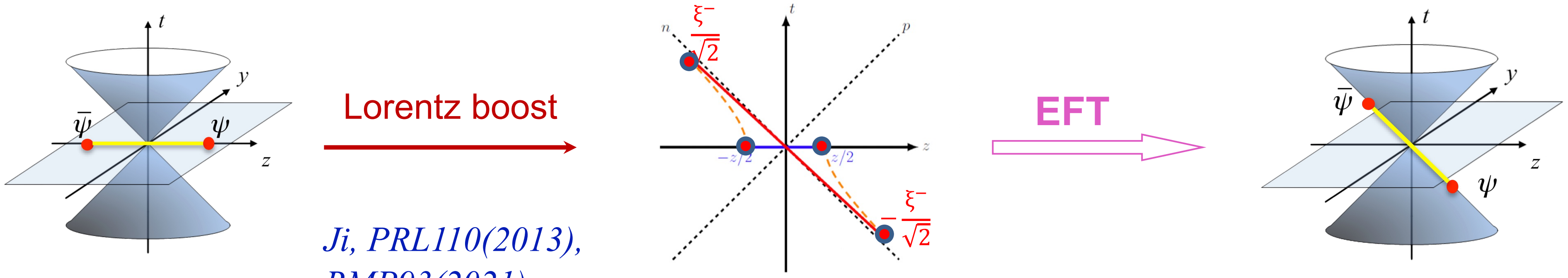
  ✓ **III: Nonperturbative renormalization, resummation, ……**

  Zhang, PLB884(2023); Ji, JHEP08(2023); Su, NPB991(2023); LPC, PRL129(2022); NPB991(2023)……

  ➢ **IV: A real lattice calculation of TMD observable?**

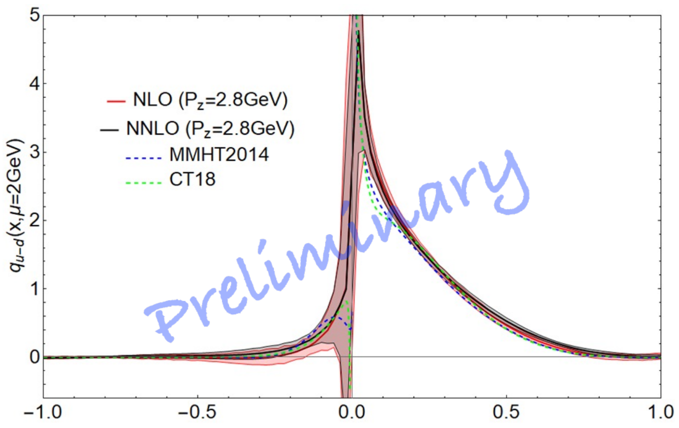
Extracting TMDs in LaMET formalism

- **Large-momentum effective theory: connecting Euclidean lattice and physical observables**

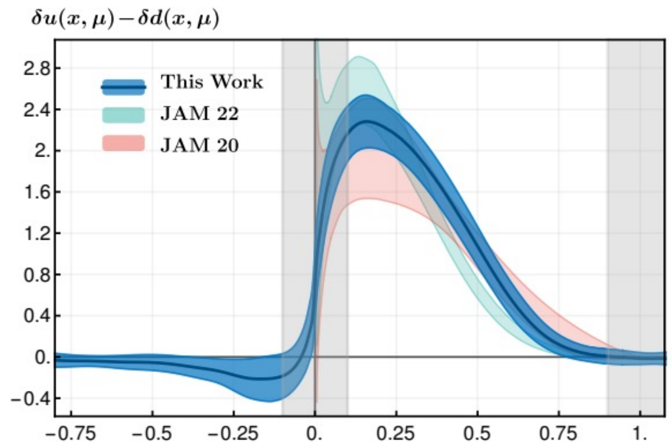


Ji, PRL110(2013), RMP93(2021), …

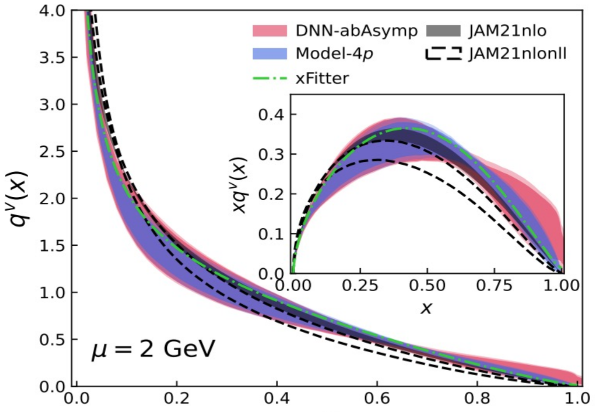
- **Achieved great success in the studies of PDF:**



Proton unpolarized PDF, in preparation

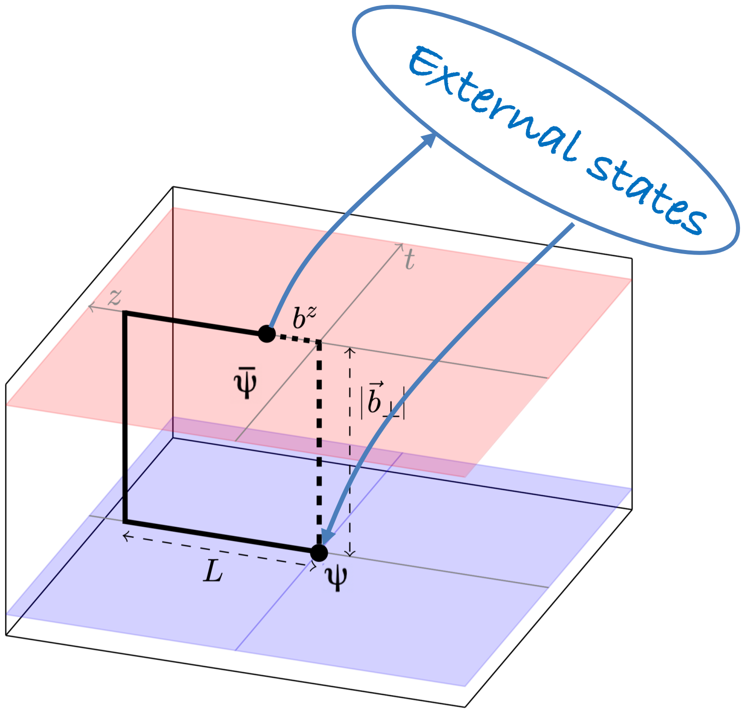


Proton transversity PDF, PRL131(2023)



Pion valance PDF, PRD106(2022)

- **Matching from quasi TMDs to TMDs**



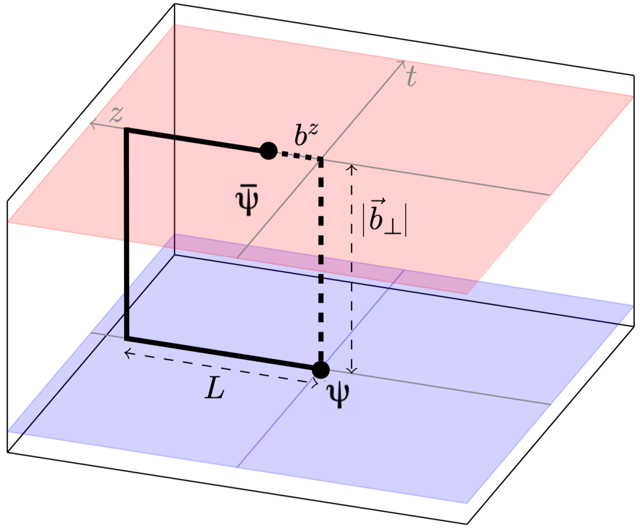
- Hadronic matrix element reduced from equal-time correlators:

$$\tilde{h}^0_\Gamma(z, b_\perp, P^z) = \lim_{L\to\infty} \left\langle P^z \middle| \bar{\psi}(b_\perp \hat{n}_\perp)\Gamma \right.$$
$$\times\, U_\sqsubset(b_\perp \hat{n}_\perp \leftarrow b_\perp \hat{n}_\perp + L\hat{n}_z; b_\perp \hat{n}_\perp + L\hat{n}_z \leftarrow L\hat{n}_z; L\hat{n}_z \leftarrow z\hat{n}_z)$$
$$\left. \times\ \psi(z\hat{n}_z) \middle| P^z \right\rangle$$

Equal-time correlators
with staple-shaped Wilson link,
directly calculable on lattice

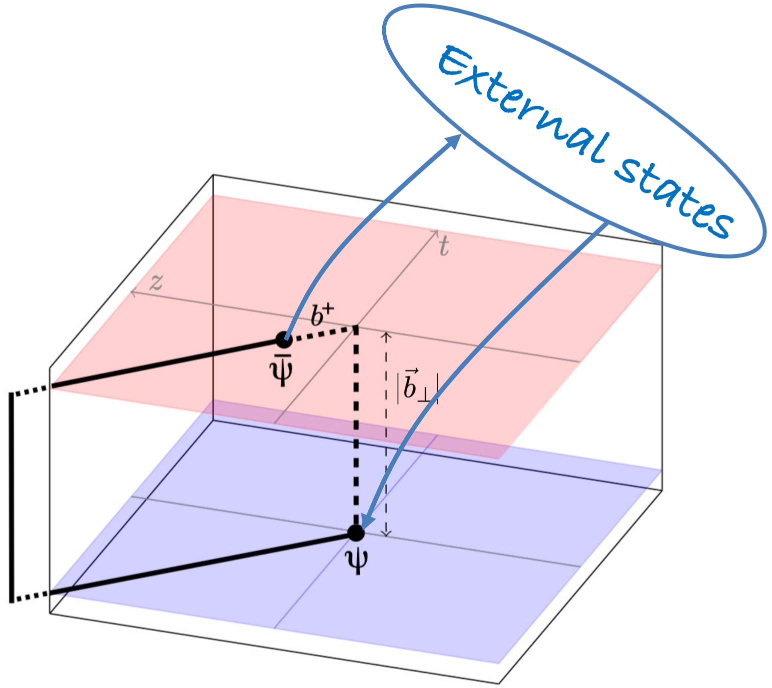
- Subtracted quasi TMDPDFs:

$$\tilde{f}_\Gamma(x, b_\perp, P^z, \mu) \equiv \lim_{\substack{a\to 0\\ L\to\infty}} \int \frac{dz}{2\pi} e^{-iz(xP^z)} \frac{\tilde{h}^0_\Gamma(z, b_\perp, P^z, a, L)}{\sqrt{Z_E(2L+z, b_\perp, a)} Z_O(1/a, \mu, \Gamma)}$$



Lorentz boost

$L \to \infty$



Equal-time correlators,
directly calculable on lattice

Space-like correlators,
NO effective method for directly calculation

Connected at large-momentum limit

Ji, PLB811(2020); Ebert, JHEP04(2022)

Matching kernel

$$\tilde{f}_\Gamma(x, b_\perp, \zeta_z, \mu)\sqrt{S_I(b_\perp, \mu)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right) K(b_\perp, \mu)} f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$

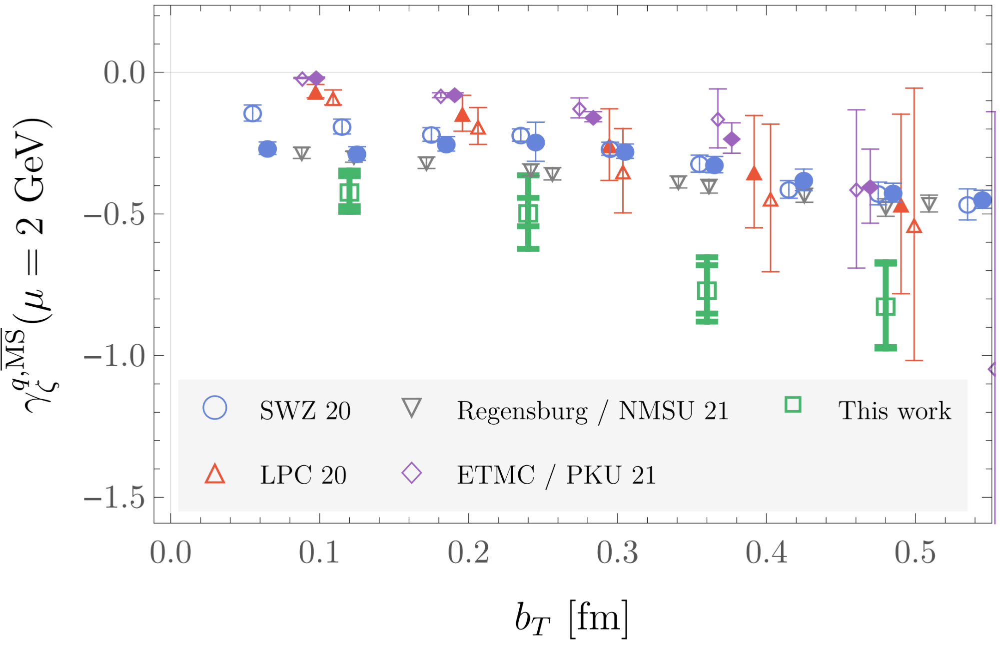
Quasi TMDPDF — Intrinsic soft function — Collins-Soper kernel — Light-cone TMDPDF

Collins–Soper kernel and intrinsic soft function

- **Collins-Soper kernel**

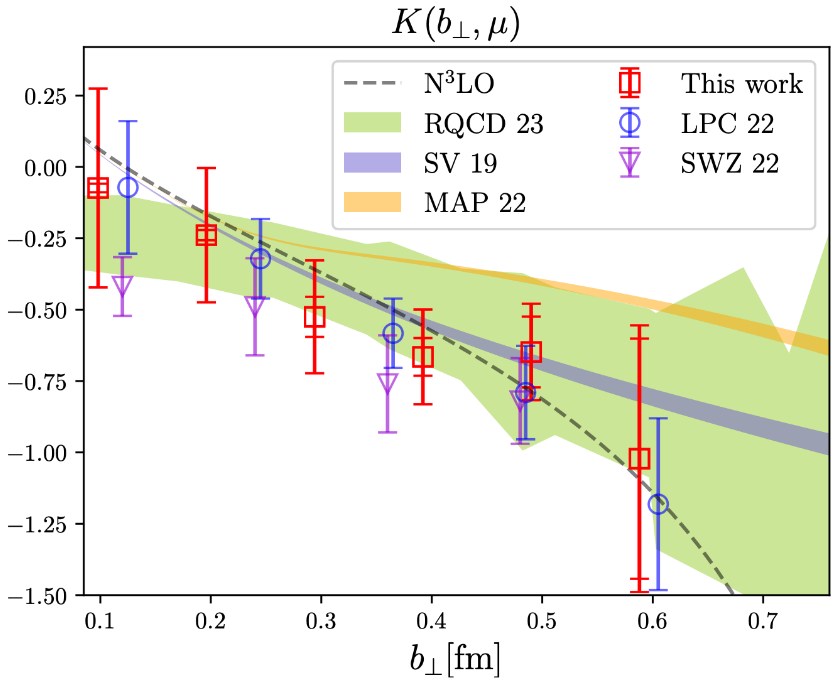
From quasi beam function:

Shanahan, PRD104(2021), PRD102(2020);
Schlemmer, JHEP08(2021); ……



From quasi TMDWF:

Chu, JHEP08(2023), PRD106(2022);
Zhang, PRL125(2020); Li, PRL128(2022); ……



Collins–Soper kernel and intrinsic soft function

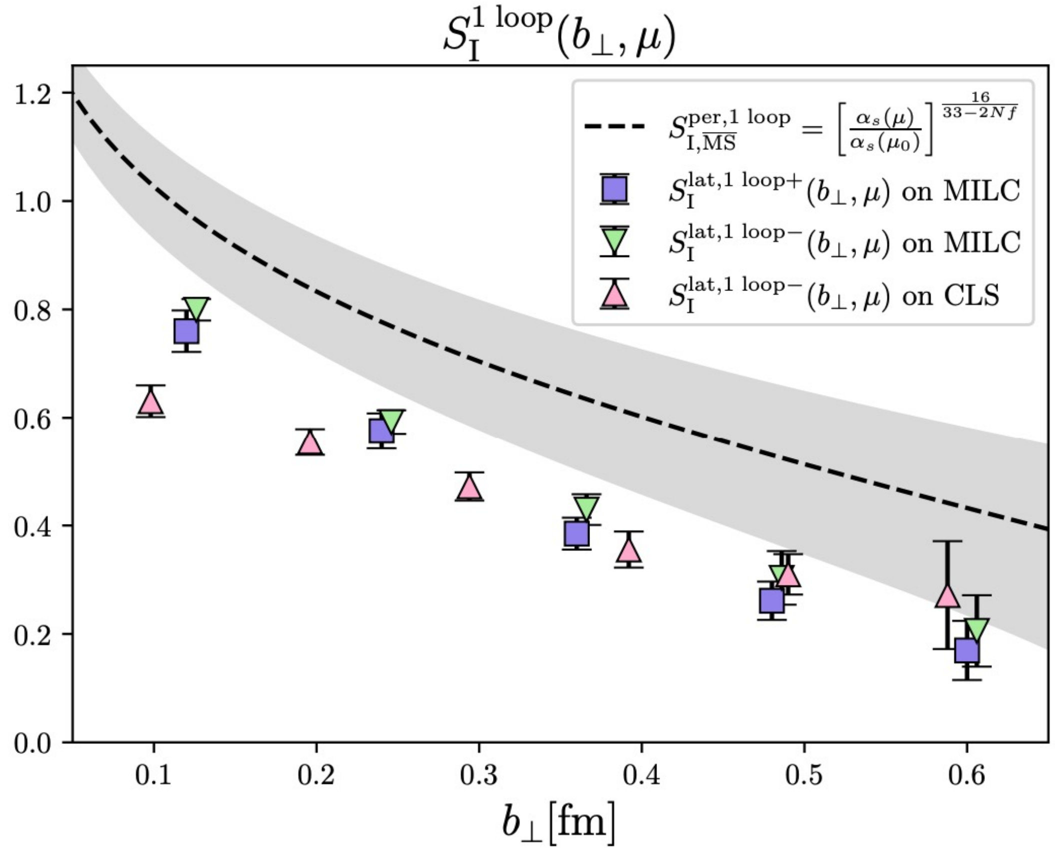
- **Intrinsic/reduced soft function**

From quasi TMDWF + 4-quark matrix element:

Chu, PRD109(2024); Ji, NPB955(2020);

Zhang, PRL125(2020); Li, PRL128(2022);

……



Matching kernel and RG resummation

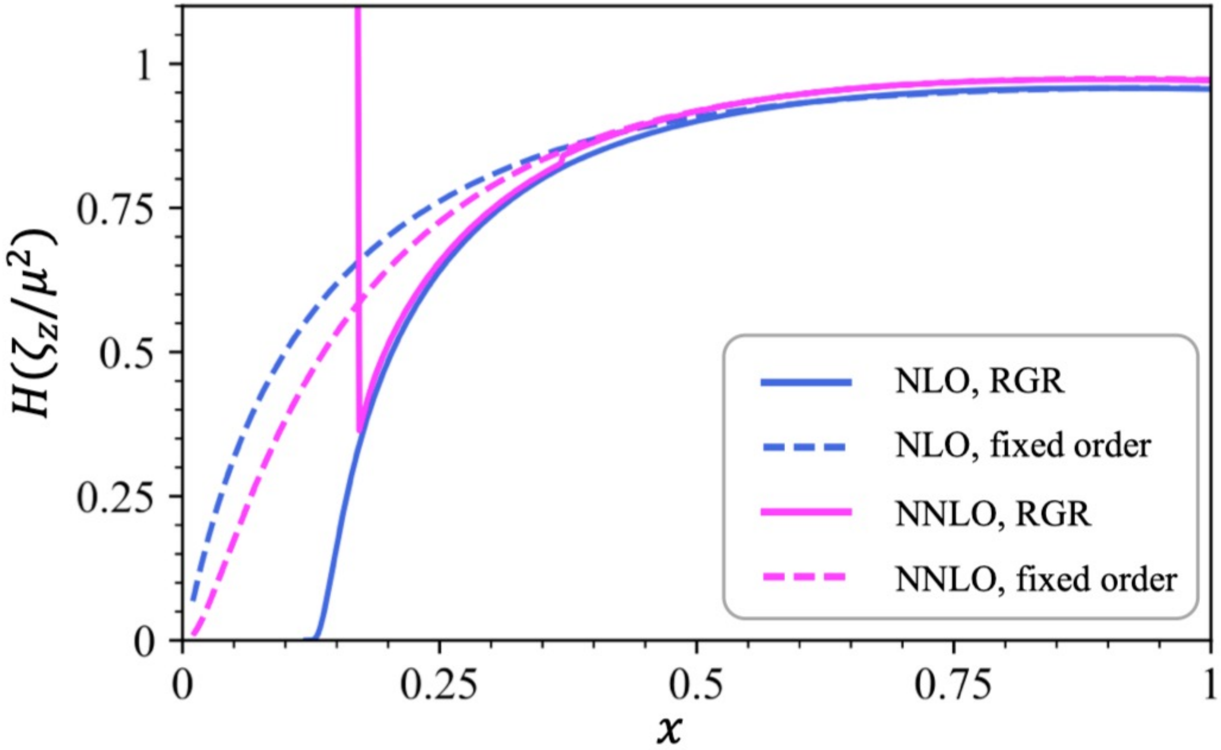
$$\tilde{f}_\Gamma(x, b_\perp, \zeta_z, \mu)\sqrt{S_I(b_\perp, \mu)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right) K(b_\perp, \mu)} f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$

Matching kernel

- **NLO:** *Ji, PLB811(2020); RMP93(2021)*
- **NNLO:** *Río, PRD108(2023); Ji, JHEP08(2023)*

➢ **Fixed order: $\mu = 2$GeV;**

➢ **RGR: RG evolution from lattice scale $\zeta_z = 2xP^z$ to $\overline{\rm MS}$ scale $\mu = 2$GeV.**



Lattice calculation of physical TMDPDF?

Matching kernel ✓

$$\tilde{f}_\Gamma(x, b_\perp, \zeta_z, \mu)\sqrt{S_I(b_\perp, \mu)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right) K(b_\perp, \mu)} f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$

Quasi TMDPDF   Intrinsic soft function ✓   Collins-Soper kernel ✓

⇩

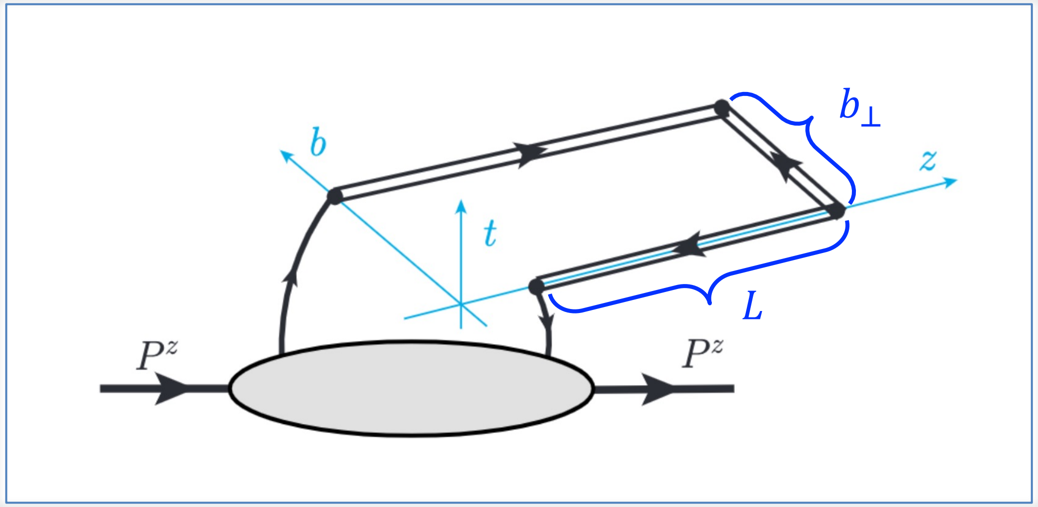
Simulating quasi TMDPDF on a Euclidean lattice:

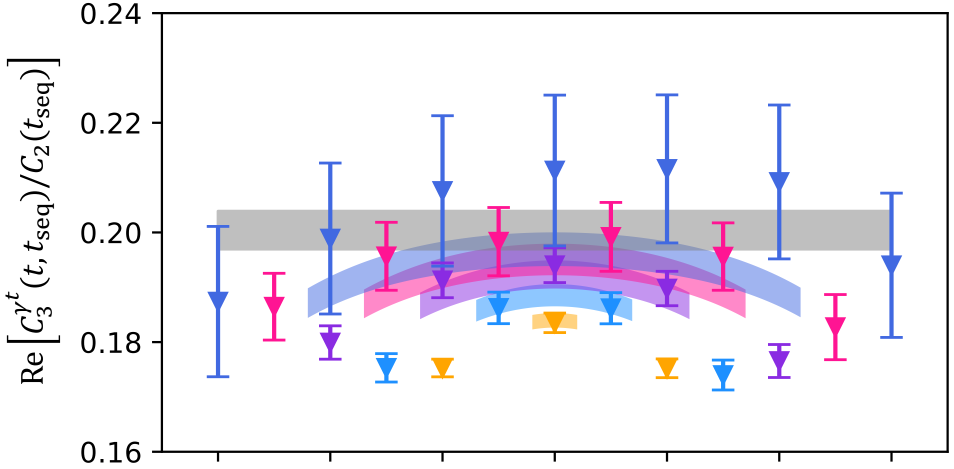
- MILC configuration: $48^3 \times 64$, $a = 0.12\mathrm{fm}$;
- Pion mass: $m_\pi^{\mathrm{sea}} = 130\mathrm{MeV}$, $m_\pi^{\mathrm{val}} = \{310, 220\}\mathrm{MeV}$ ⇒ extrapolate to physical mass
- Large momentum: $P^z = \{1.72,\ 2.15,\ 2.58\}\mathrm{GeV}$ ⇒ extrapolate to infinity
- Saturated length of Wilson link $L = 0.72\mathrm{fm}$;
- $z_{\max} = 1.44\mathrm{fm}$, $b_{\perp\max} = 0.6\mathrm{fm}$.

Quasi TMDPDF matrix element

Bare quasi TMDPDF matrix element

$$\tilde{h}^0_\Gamma(z, b_\perp, P^z) = \lim_{L\to\infty} \left\langle P^z \middle| \bar{\psi}(b_\perp \hat{n}_\perp)\Gamma \; U_\sqsubset(b_\perp \hat{n}_\perp \leftarrow b_\perp \hat{n}_\perp + L\hat{n}_z; b_\perp \hat{n}_\perp + L\hat{n}_z \leftarrow L\hat{n}_z; L\hat{n}_z \leftarrow z\hat{n}_z) \; \psi(z\hat{n}_z) \middle| P^z \right\rangle$$

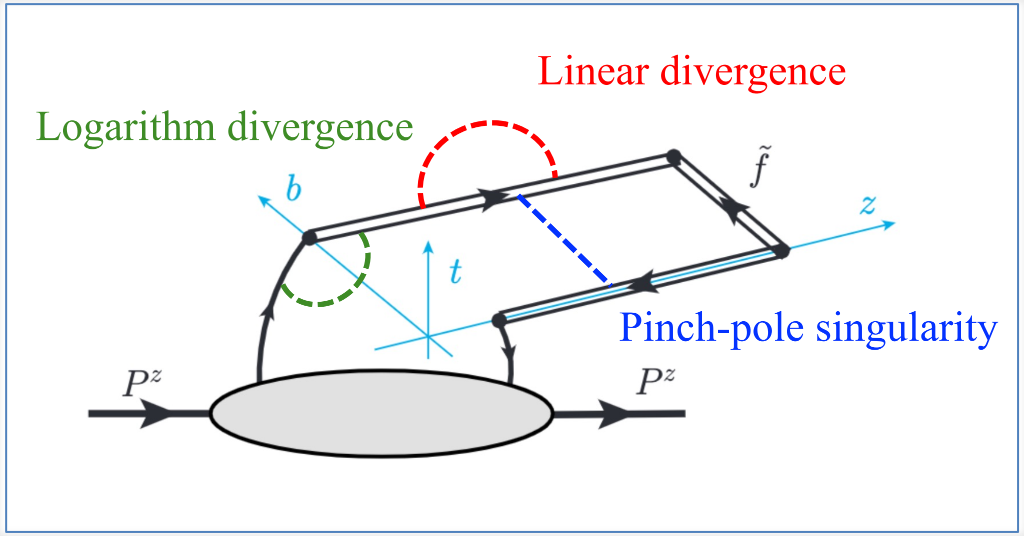




- **Extracted from 3- and 2-point functions**

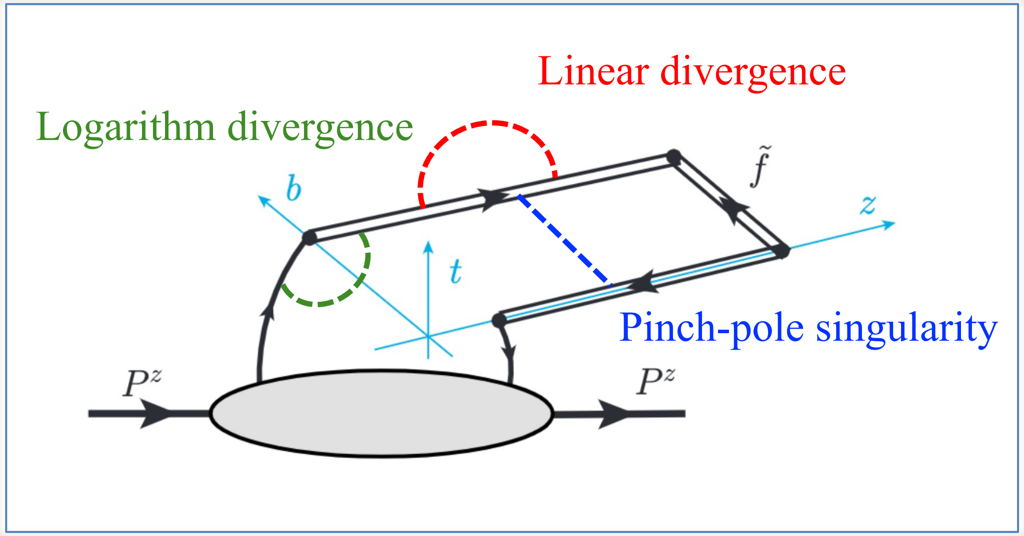
Quasi TMDPDF matrix element and renormalization

1. Divergences in bare quasi TMDPDF

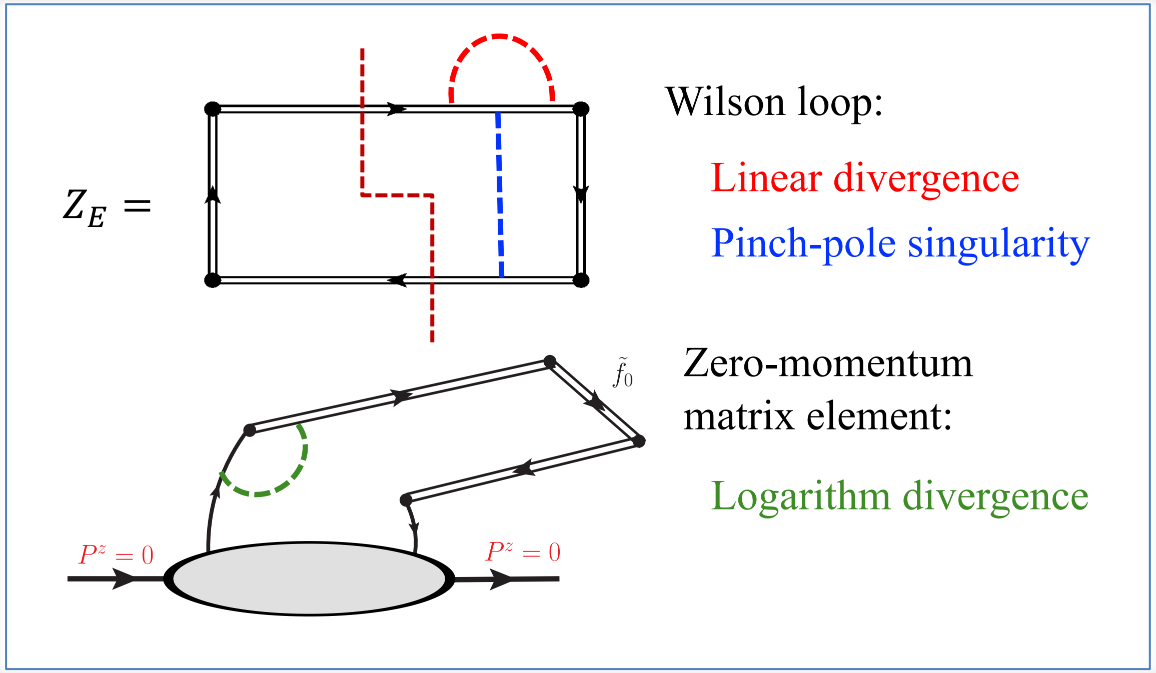


Quasi TMDPDF matrix element and renormalization

1. Divergences in bare quasi TMDPDF



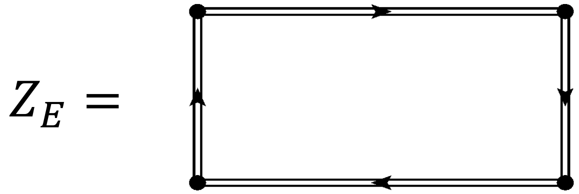
2. Renormalization

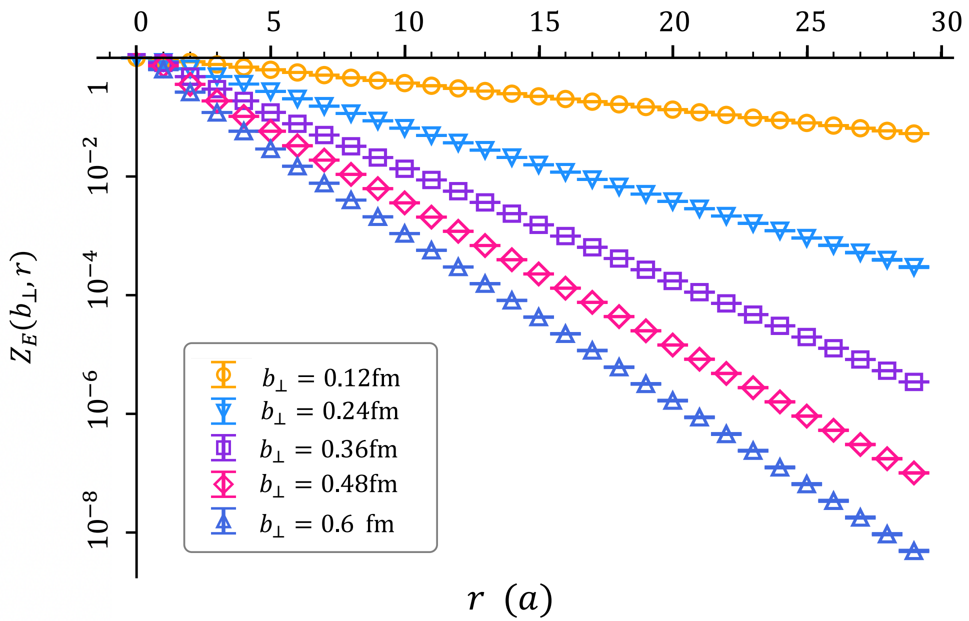


Ji, PRL120(2018), NPB964(2021), PLB257(1991); Zhang, PRD95(2017), NPB939(2019); Ishikawa, PRD96(2017); Green, PRL121(2018); Huo, NPB969(2021); Chen, NPB915(2017); Musch, PRD83(2011); ……

Quasi TMDPDF matrix element and renormalization

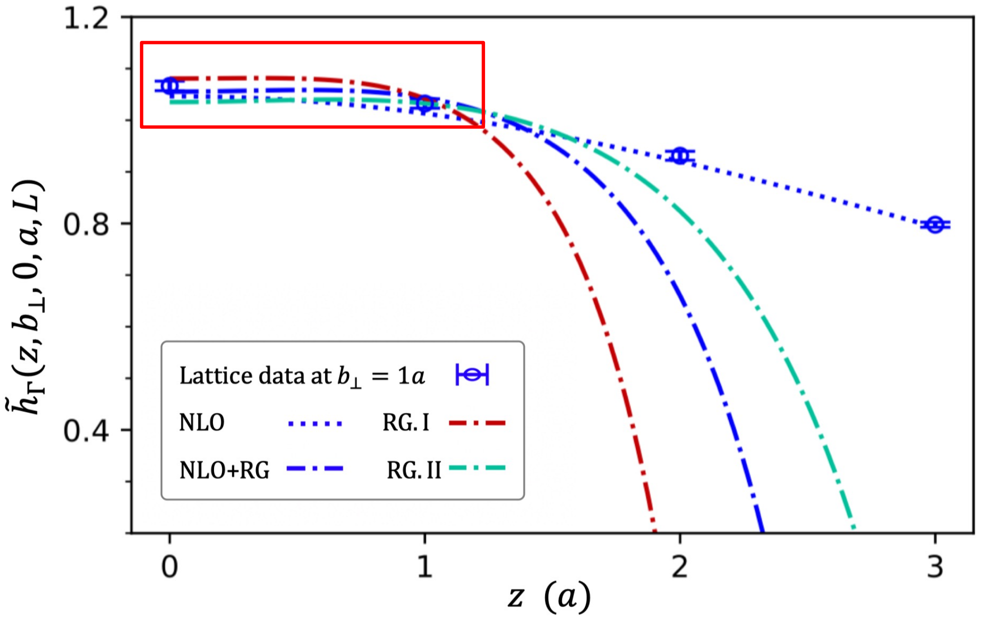
- **Wilson loop**



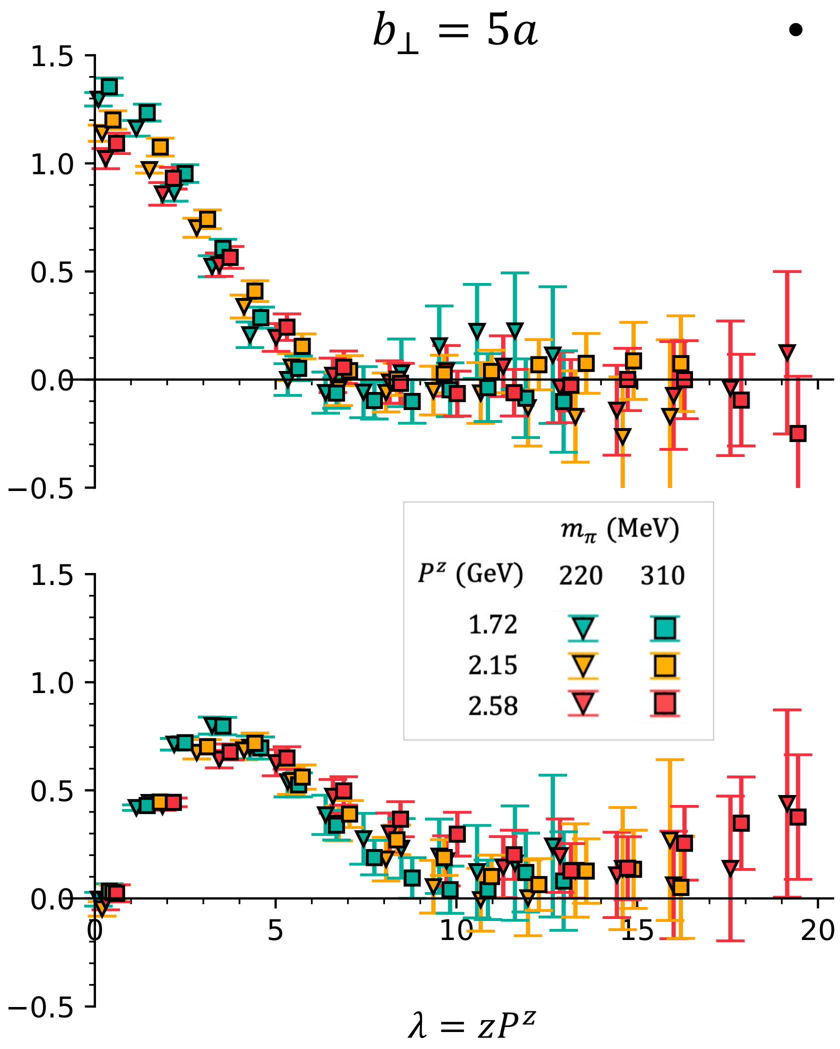


- **Logarithmic divergences factor**

$$Z_O(1/a, \mu, \Gamma) = \lim_{L\to\infty} \frac{\tilde{h}^0_\Gamma(z, b_\perp, 0, a, L)}{\sqrt{Z_E(2L+z, b_\perp, a)}\tilde{h}^{\overline{\rm MS}}_\Gamma(z, b_\perp, \mu)}$$

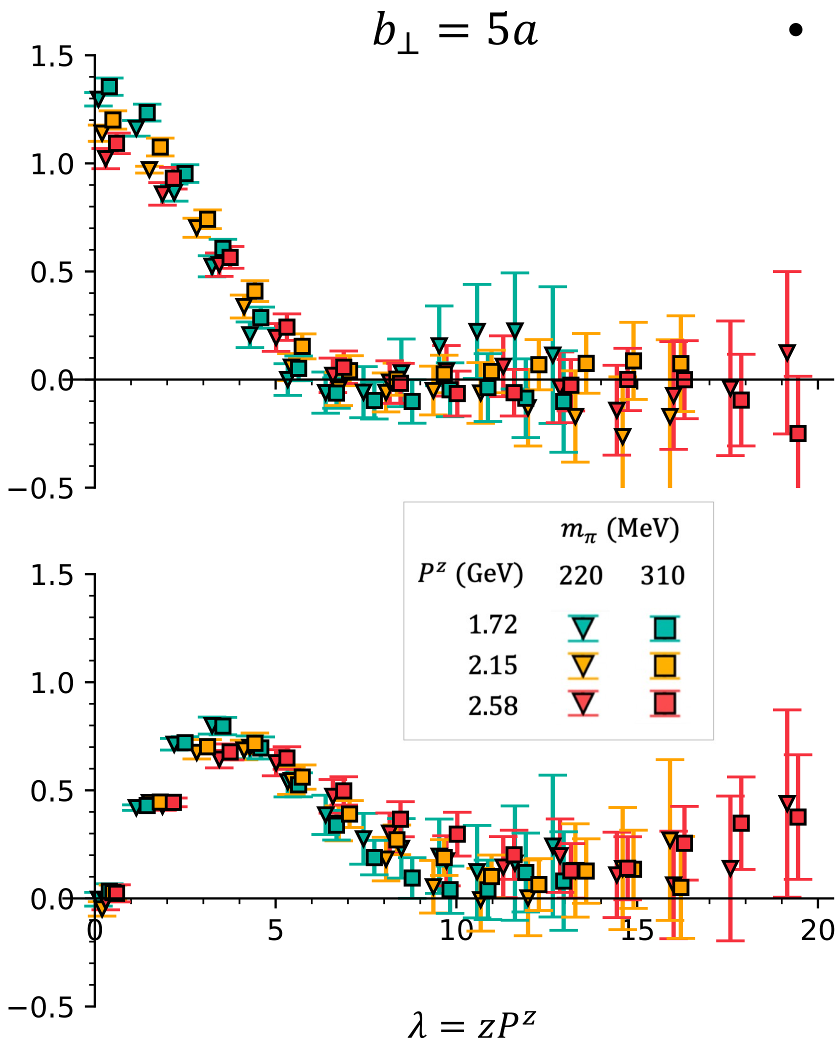


Quasi TMDPDF matrix element and λ extrapolation



- A brute-fore truncation at large λ will lead to **strong oscillation** after FT ⇒ need additional extrapolation….

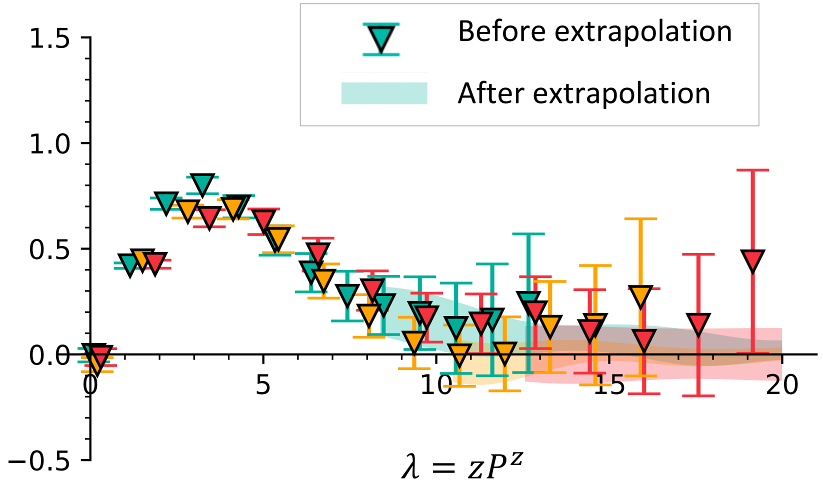
Quasi TMDPDF matrix element and λ extrapolation



- A brute-fore truncation at large λ will lead to **strong oscillation** after FT ⇒ need additional extrapolation….

$$\tilde{h}_{\text{extra}}(\lambda) = \left[\frac{c_1}{(-i\lambda)^{n_1}} + e^{i\lambda}\frac{c_2}{(i\lambda)^{n_2}}\right] e^{-\lambda/\lambda_0}$$

- end point power-law behavior $x^a(1-x)^b$;
- correlation function has a finite correlation length λ_0.



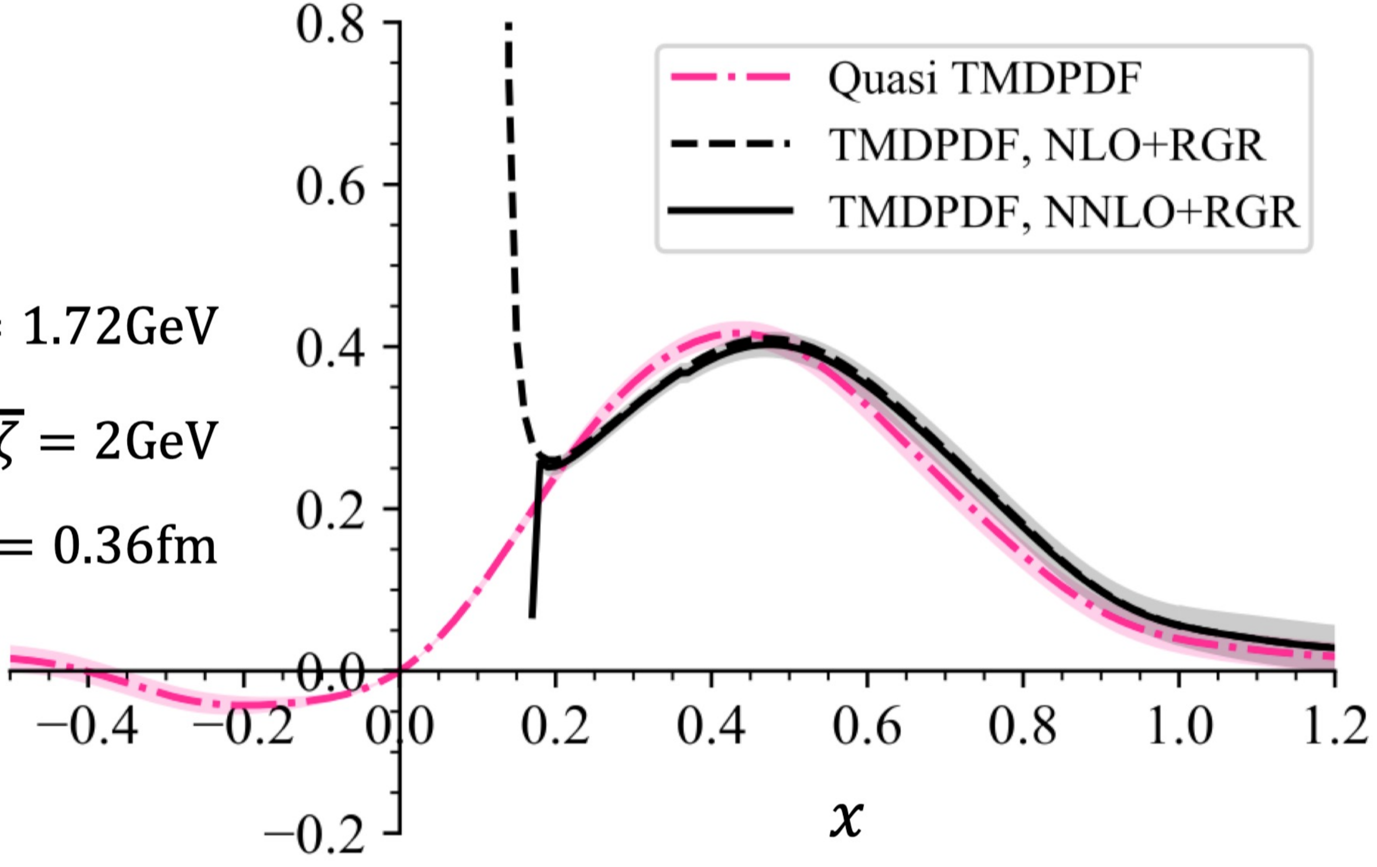
From Quasi TMDPDF to TMDPDF

$x\tilde{f}_{\gamma^t}$ v.s. xf_{γ^t}

$P^z = 1.72\text{GeV}$

$\mu = \sqrt{\zeta} = 2\text{GeV}$

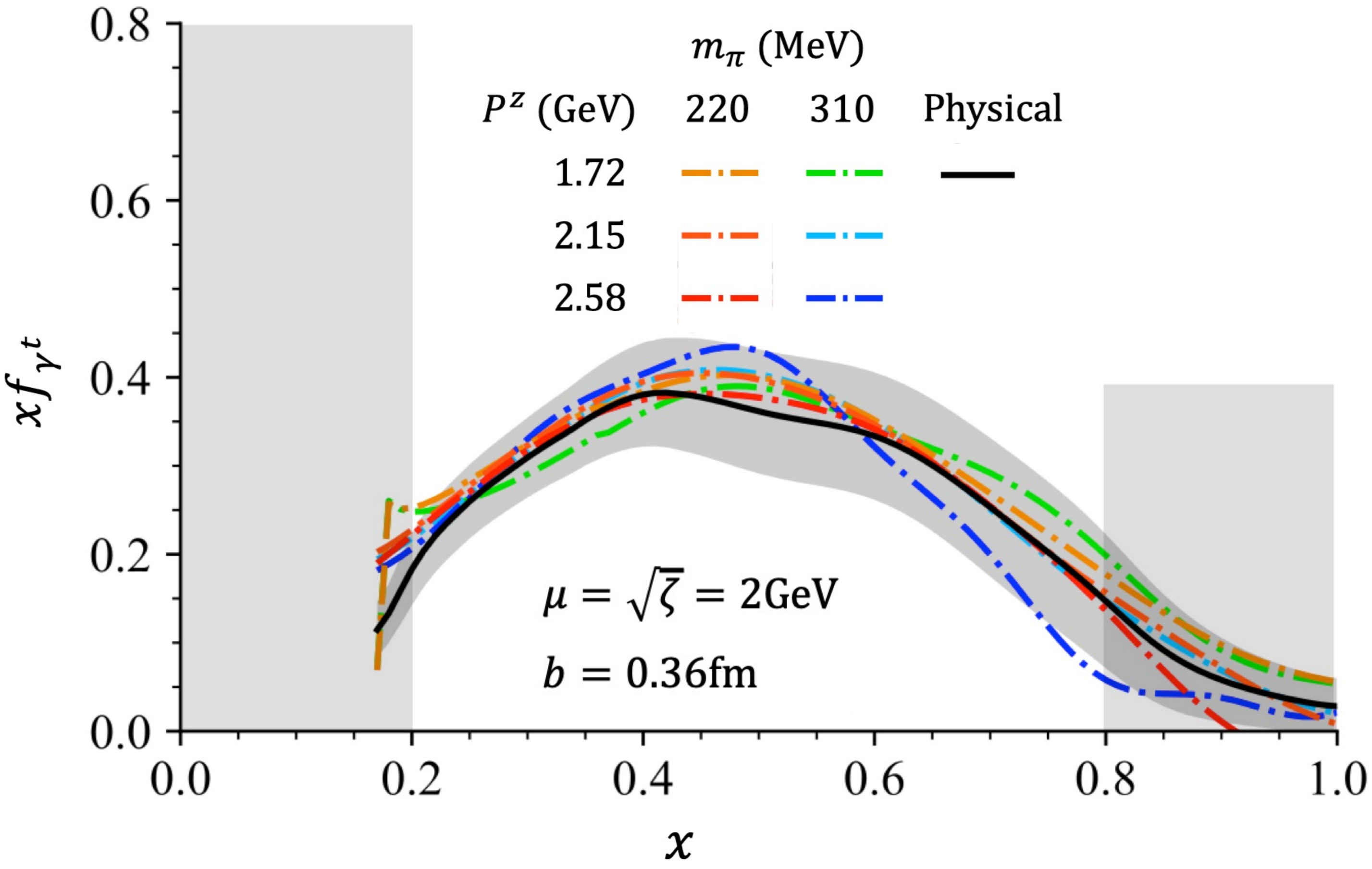
$b = 0.36\text{fm}$



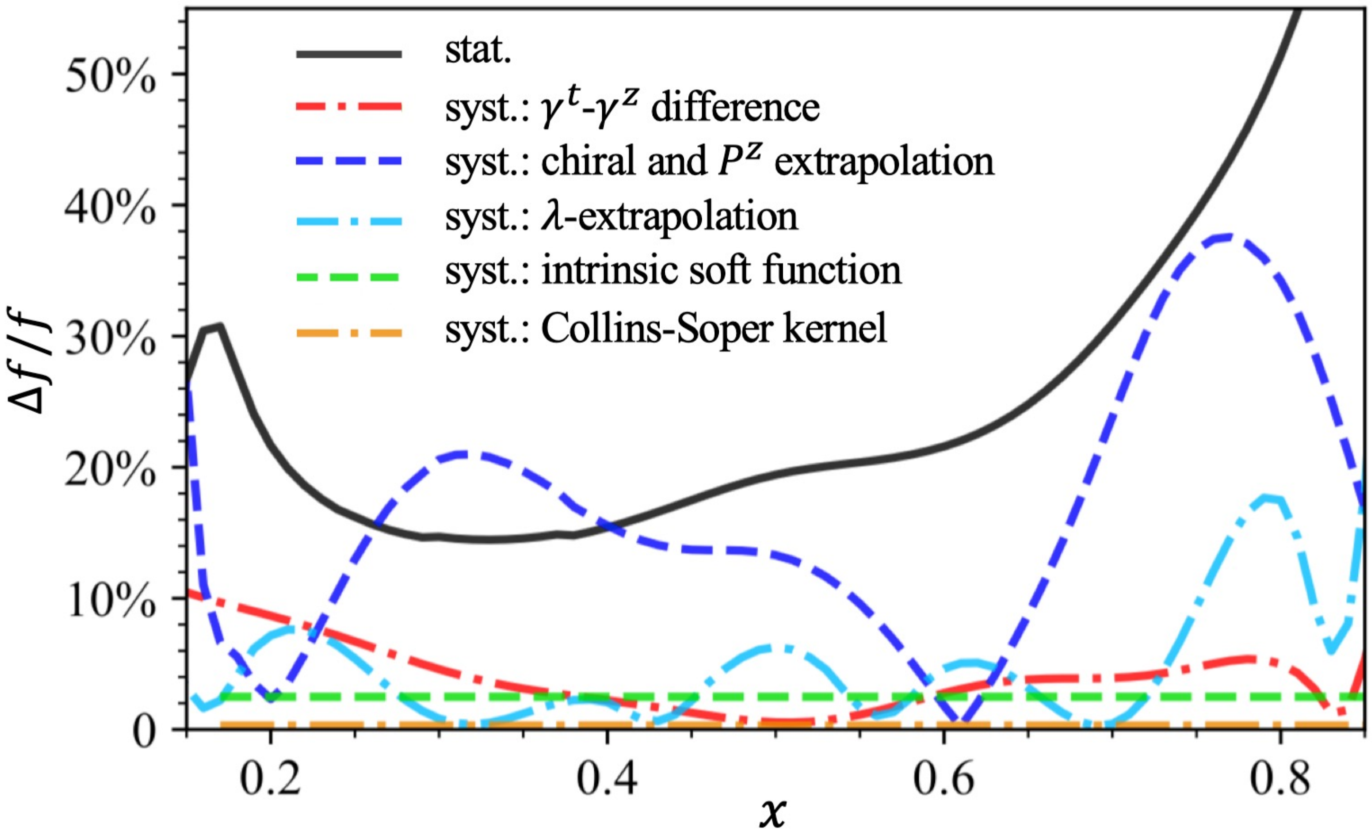
Physical TMDPDF

Chiral and large-P^z joint extrapolation:

$$d_0(m_\pi^2 - m_{\pi,\mathrm{phy}}^2) + \frac{d_1}{(P^z)^2}$$



Error estimation



All errors:

- **Statistical error;**

(1) From difference of γ^t and γ^z

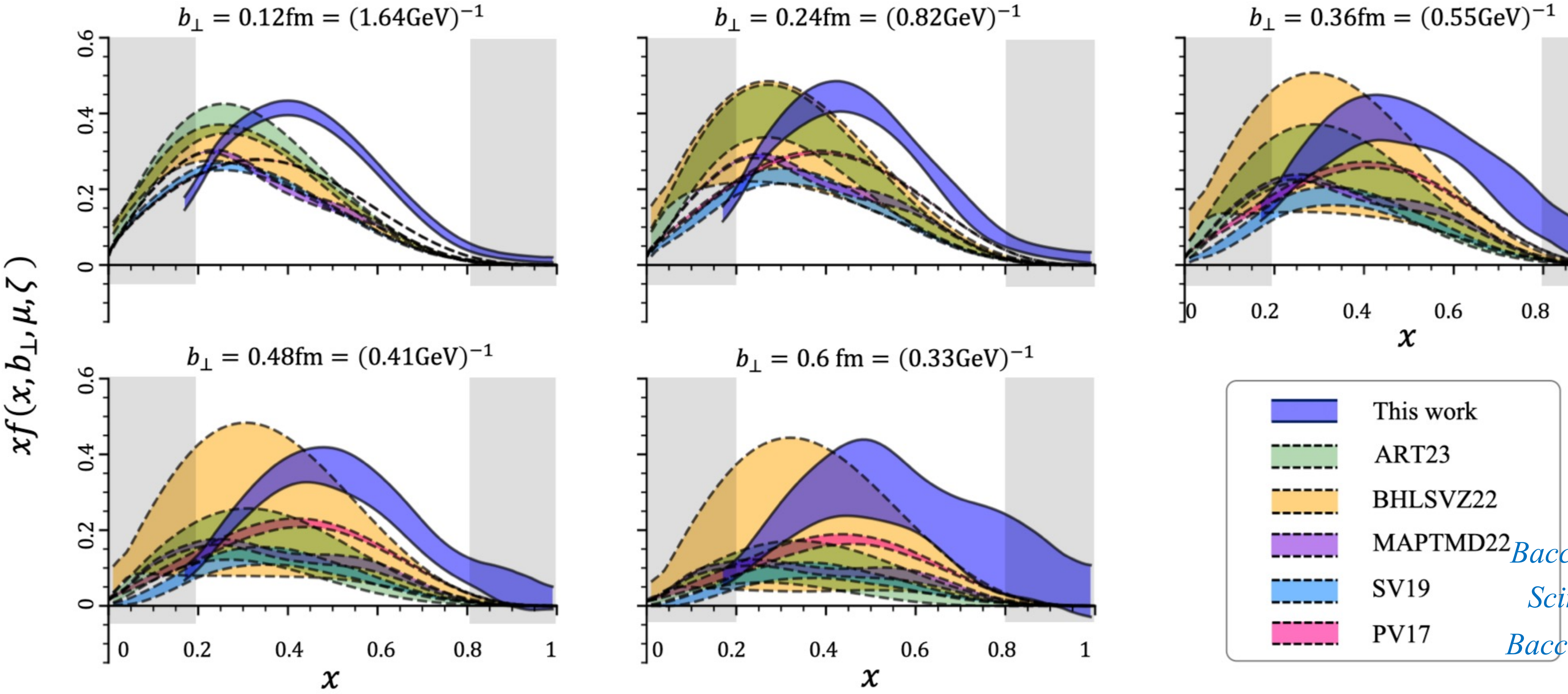
(2) From physical extrapolation

(3) From λ-extrapolation

(4) From soft function

(5) From Collins-Soper kernel

Final results and discussion



Moos, JHEP2024

Bury, JHEP2022

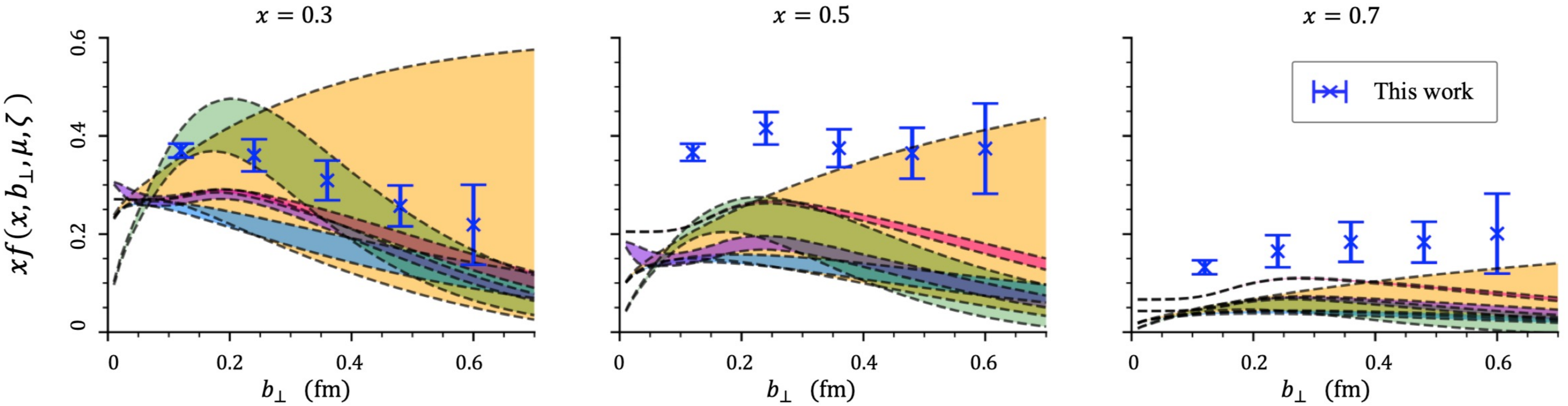
Bacchetta, JHEP2022

Scimemi, JHEP2020

Bacchetta, JHEP2019

Final results and discussion

Compare the $b_\perp$-dependence of lattice and phenomenological results:



Summary and Outlook

We present the lattice QCD calculation of TMDPDF at first attempt:

- ✓ **The state-of-the-art techniques in renormalization and extrapolation on the lattice;**
- ✓ **The latest perturbative kernel up to 2-loop with RG evolution;**
- ✓ **Physical extrapolation include chiral-continuum and infinity momentum;**
- ✓ **Comparable results with phenomenological global fits.**

Summary and Outlook

While there is still much room for further improvement:

- 🤔 **Better control of uncertainties;**
- 🤔 **Continuum extrapolation: more lattice spacings;**
- 🤔 **Larger $b_\perp$ (up to nucleon radius?) to obtain a converge distribution in coordinate space;**
- 🤔 **Theoretical improvements:**

Power correction (small-x region), higher twist effects (operator mixing), ……

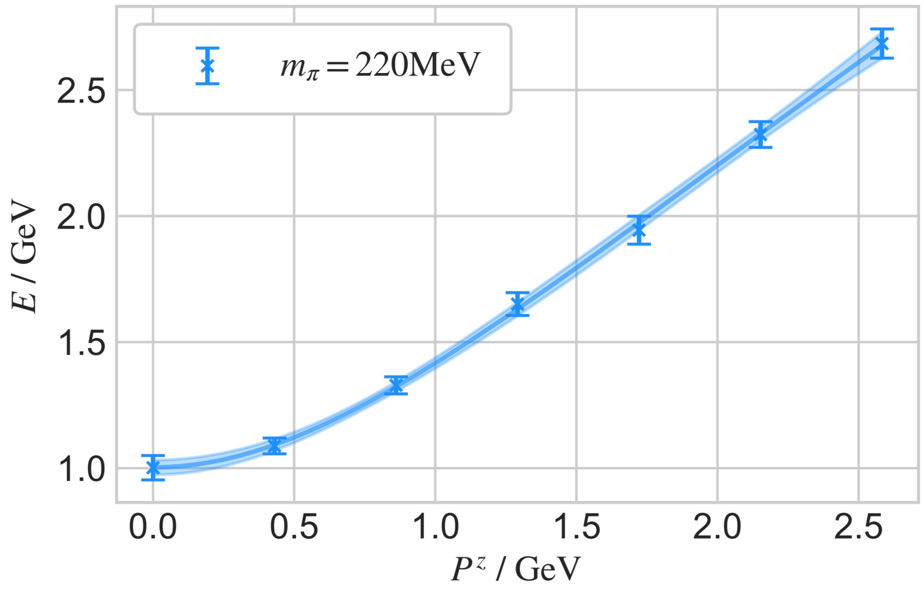
Thank you for your attention!

Backup slides

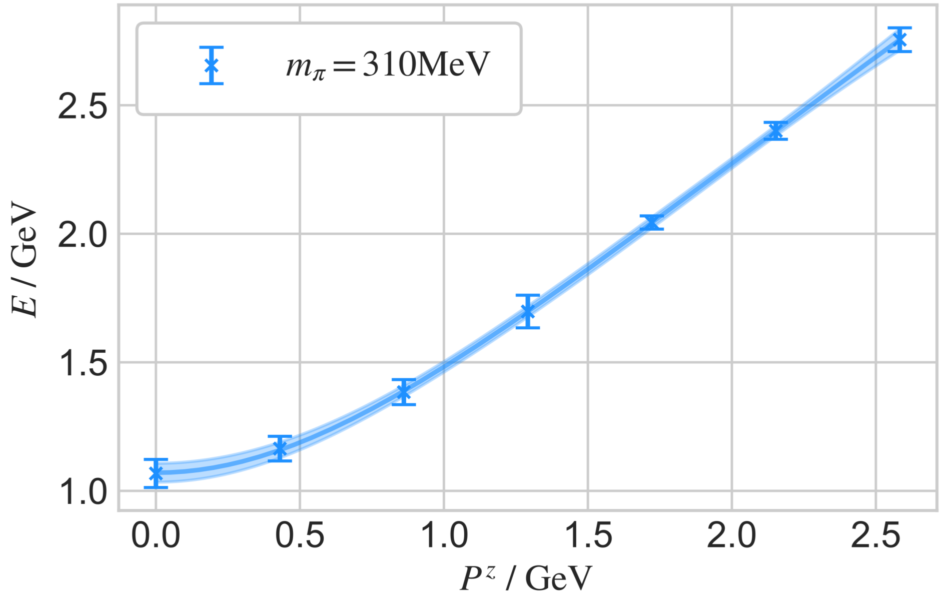
Dispersion relation

$$E = \sqrt{m^2 + c_1(P^z)^2 + c_2(P^z)^4 a^2}$$

$c_1 = 1.014(95),\ c_2 = -0.014(17)$



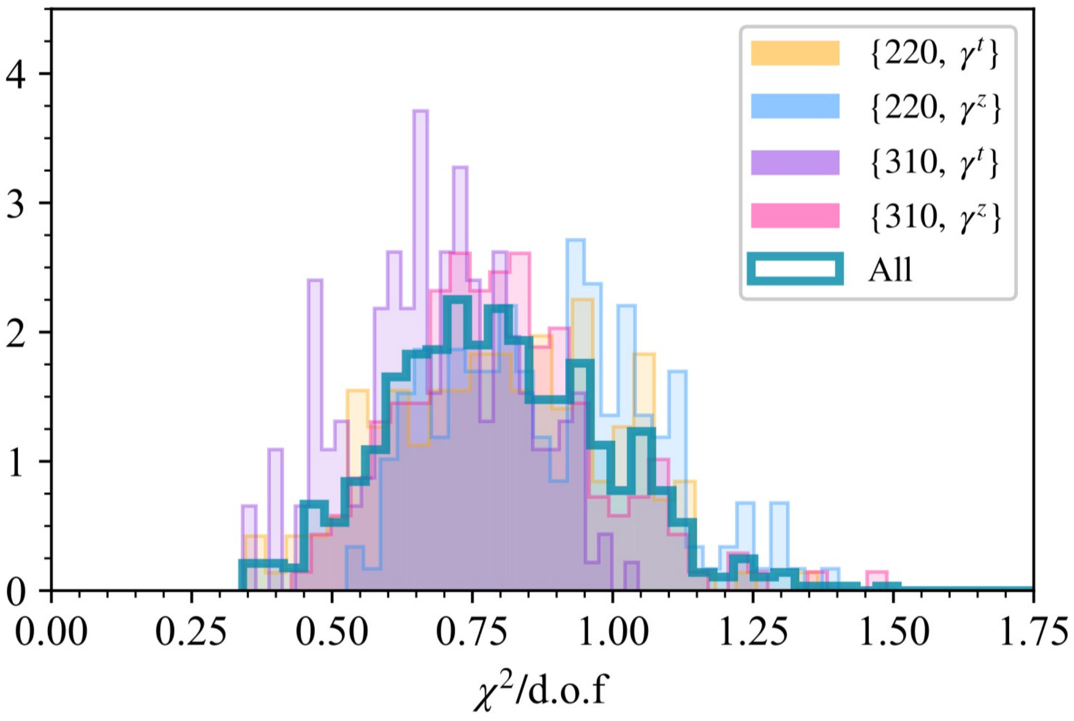
$c_1 = 1.066(80),\ c_2 = -0.015(14)$

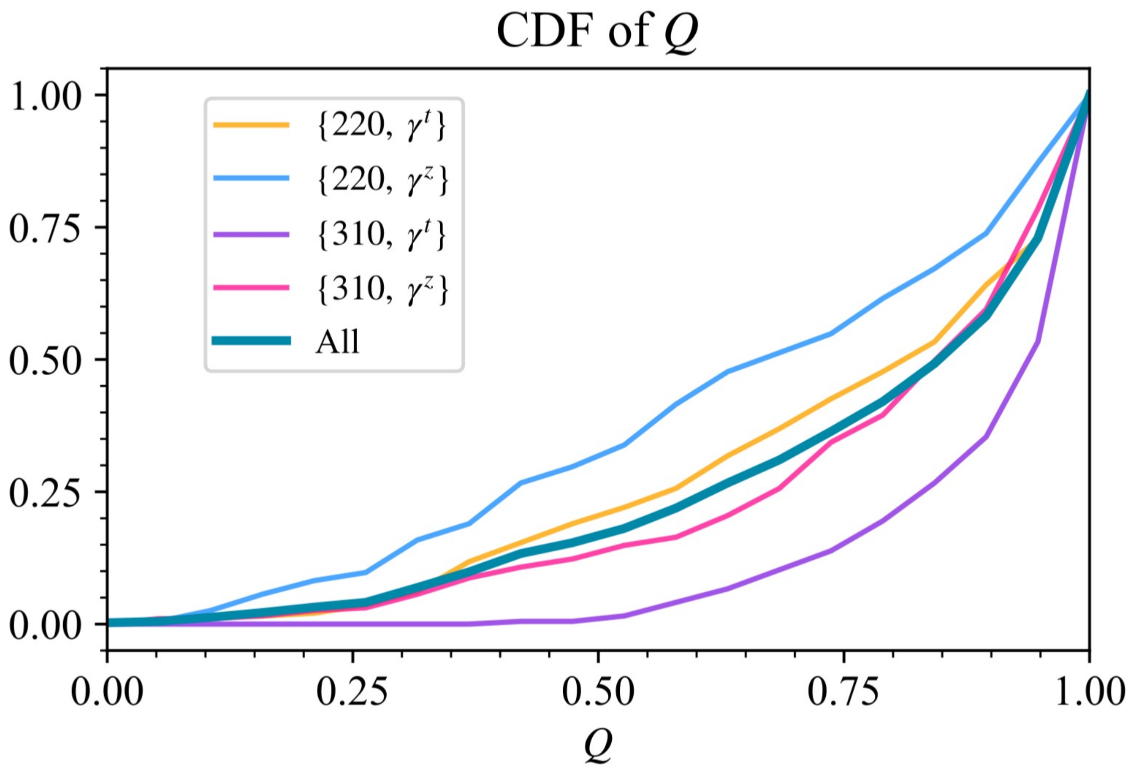


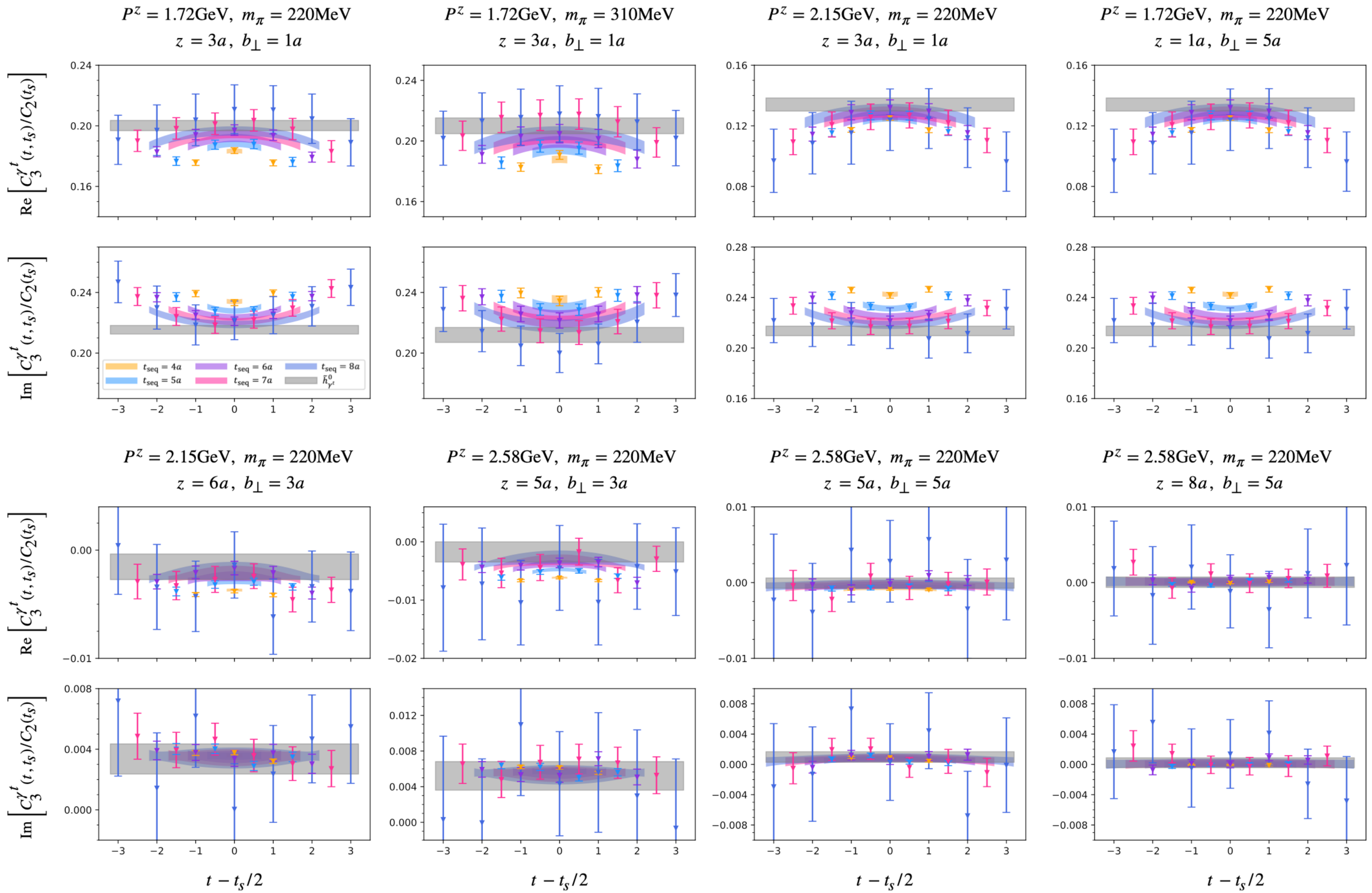
Details of correlated joint fits

Fit quality:

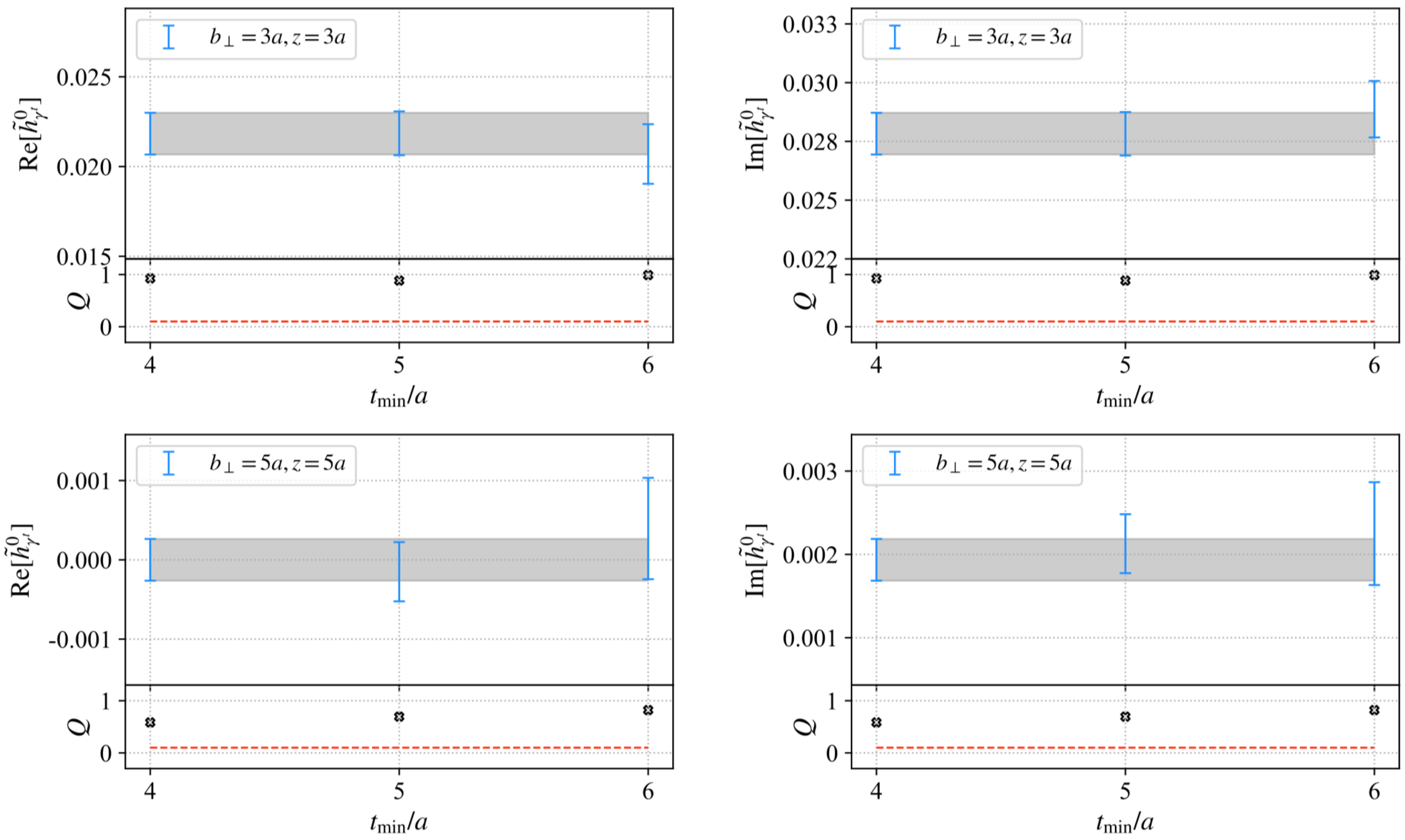
- Utilizing bootstrap resampling to establish correlations among all datasets;
- Employing fully-correlated Bayesian constrained fits to extract ground-state matrix elements.



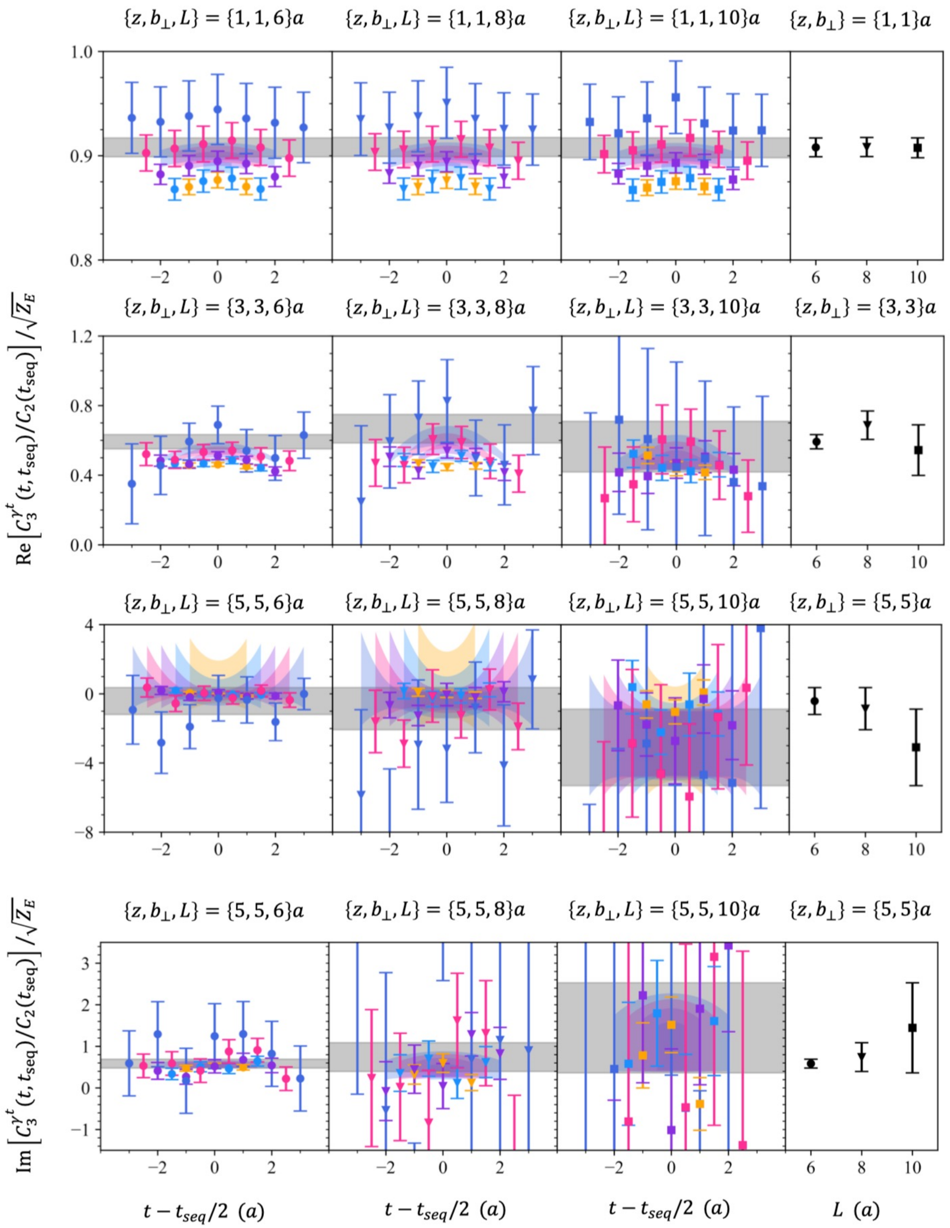



$P^z = 1.72\text{GeV},\ m_\pi = 220\text{MeV}$
$z = 3a,\ b_\perp = 1a$
$P^z = 1.72\text{GeV},\ m_\pi = 310\text{MeV}$
$z = 3a,\ b_\perp = 1a$
$P^z = 2.15\text{GeV},\ m_\pi = 220\text{MeV}$
$z = 3a,\ b_\perp = 1a$
$P^z = 1.72\text{GeV},\ m_\pi = 220\text{MeV}$
$z = 1a,\ b_\perp = 5a$
$\mathrm{Re}\left[C_3^{\gamma^t}(t,t_s)/C_2(t_s)\right]$
$\mathrm{Im}\left[C_3^{\gamma^t}(t,t_s)/C_2(t_s)\right]$
$t_{\rm seq} = 4a$
$t_{\rm seq} = 5a$
$t_{\rm seq} = 6a$
$t_{\rm seq} = 7a$
$t_{\rm seq} = 8a$
$\tilde{h}^0_{\gamma^t}$
$P^z = 2.15\text{GeV},\ m_\pi = 220\text{MeV}$
$z = 6a,\ b_\perp = 3a$
$P^z = 2.58\text{GeV},\ m_\pi = 220\text{MeV}$
$z = 5a,\ b_\perp = 3a$
$P^z = 2.58\text{GeV},\ m_\pi = 220\text{MeV}$
$z = 5a,\ b_\perp = 5a$
$P^z = 2.58\text{GeV},\ m_\pi = 220\text{MeV}$
$z = 8a,\ b_\perp = 5a$
$t - t_s/2$

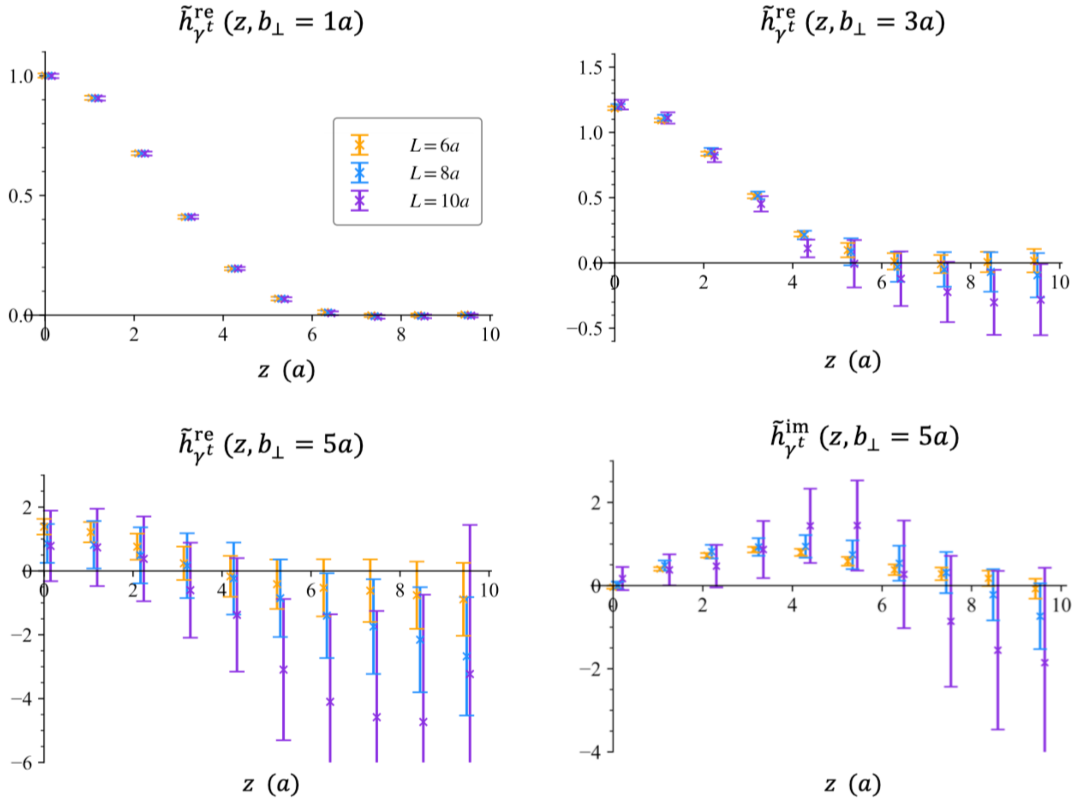
- Stability of the joint fits: $t_{\min}$ dependence of the fit result, which fit range is $[t_{\min}, t_{\max}]$.



L-dependence

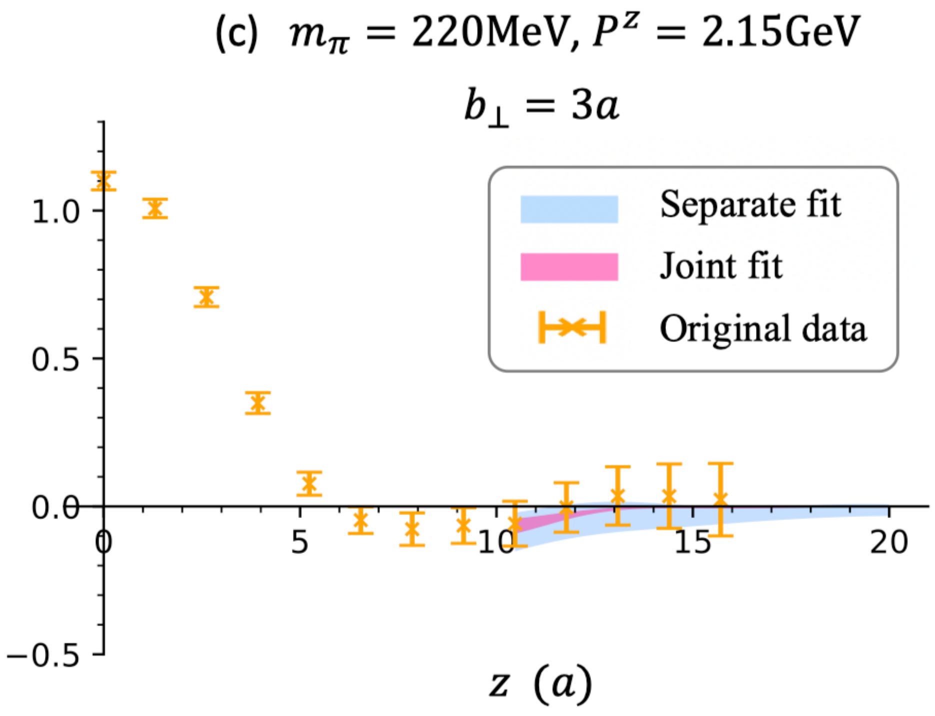


Saturation length of Wilson link:



Factorization of z and $b_\perp$?

$$\tilde{h}_{\text{extra}}(\lambda) = \left[\frac{c_1}{(-i\lambda)^{n_1}} + e^{i\lambda}\frac{c_2}{(i\lambda)^{n_2}}\right] e^{-\lambda/\lambda_0}$$



The power-law behavior and correlation length for each $b_\perp$ should be similar,

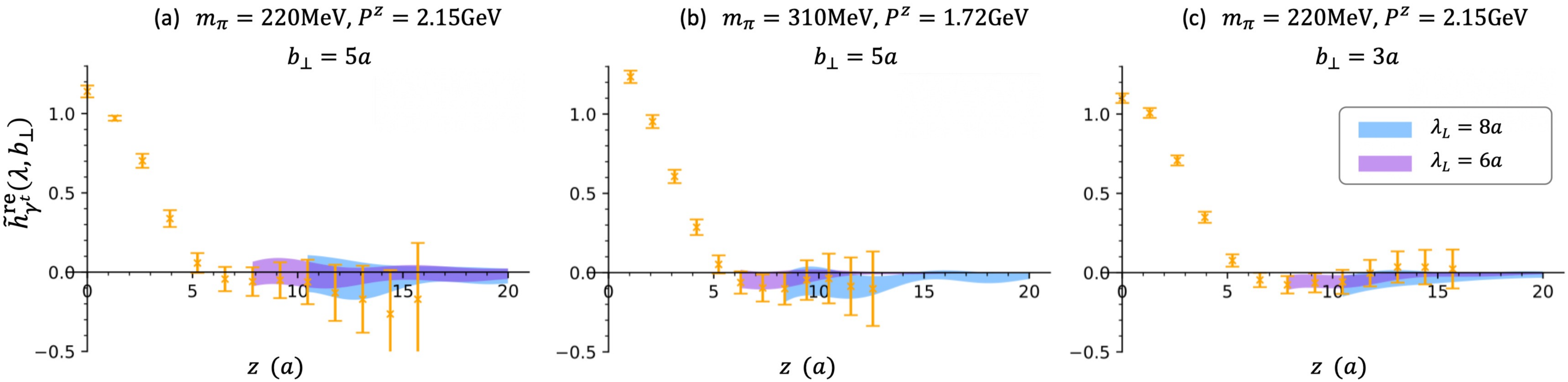
but the joint fit will give a strict limit for large-$b_\perp$ cases:

$b_\perp\ (a)$	1	2	3	4	5	Joint
n_1	0.909(39)	0.943(61)	0.89(10)	0.801(78)	0.84(16)	0.887(28)
n_2	1.31(34)	2.37(68)	1.71(31)	1.55(38)	1.22(44)	1.65(12)
λ	2.63(38)	3.20(80)	2.42(85)	4.3(1.6)	4.4(2.8)	2.53(28)
χ^2/d.o.f.	1.0	1.1	1.3	0.75	0.57	1.2

λ-extrapolation

Systematic uncertainty from fit region $[\lambda_L : \lambda_{\max}]$

$$\tilde{h}_{\text{extra}}(\lambda) = \left[\frac{c_1}{(-i\lambda)^{n_1}} + e^{i\lambda}\frac{c_2}{(i\lambda)^{n_2}}\right] e^{-\lambda/\lambda_0}$$

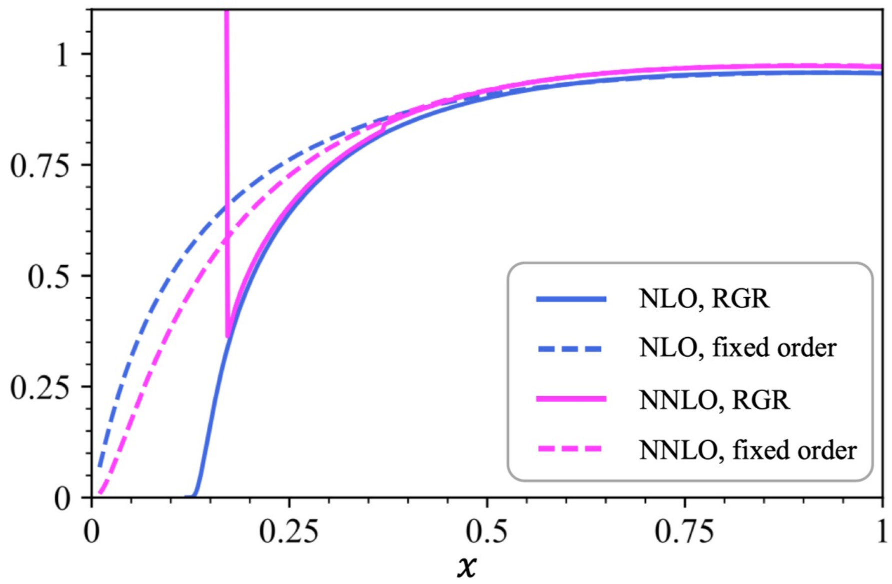


Perturbative matching kernel and RG resummation

- **Fixed-order perturbative results up to the 2-loop level:**

$$h^{(1)}\left(\frac{\zeta_z}{\mu^2}\right) = \frac{\alpha_s C_F}{2\pi}\left(-2+\frac{\pi^2}{12}+\ln\frac{\zeta_z}{\mu^2}-\frac{1}{2}\ln^2\frac{\zeta_z}{\mu^2}\right),$$

$$h^{(2)}\left(\frac{\zeta_z}{\mu^2}\right) = \alpha_s^2\left[c_2-\frac{1}{2}\left(\gamma_C^{(2)}-\beta_0 c_1\right)\ln\frac{\zeta_z}{\mu^2}-\frac{1}{4}\left(\Gamma_{\text{cusp}}^{(2)}-\frac{\beta_0 C_F}{2\pi}\right)\ln^2\frac{\zeta_z}{\mu^2}-\frac{\beta_0 C_F}{24\pi}\ln^3\frac{\zeta_z}{\mu^2}\right]$$



- **RG equation of the matching kernel:**

$$\mu^2\frac{d}{d\mu^2}\ln H\left(\frac{\zeta_z}{\mu^2}\right) = \frac{1}{2}\Gamma_{\text{cusp}}\left(\alpha_s\right)\ln\frac{\zeta_z}{\mu^2}+\frac{\gamma_C\left(\alpha_s\right)}{2},$$

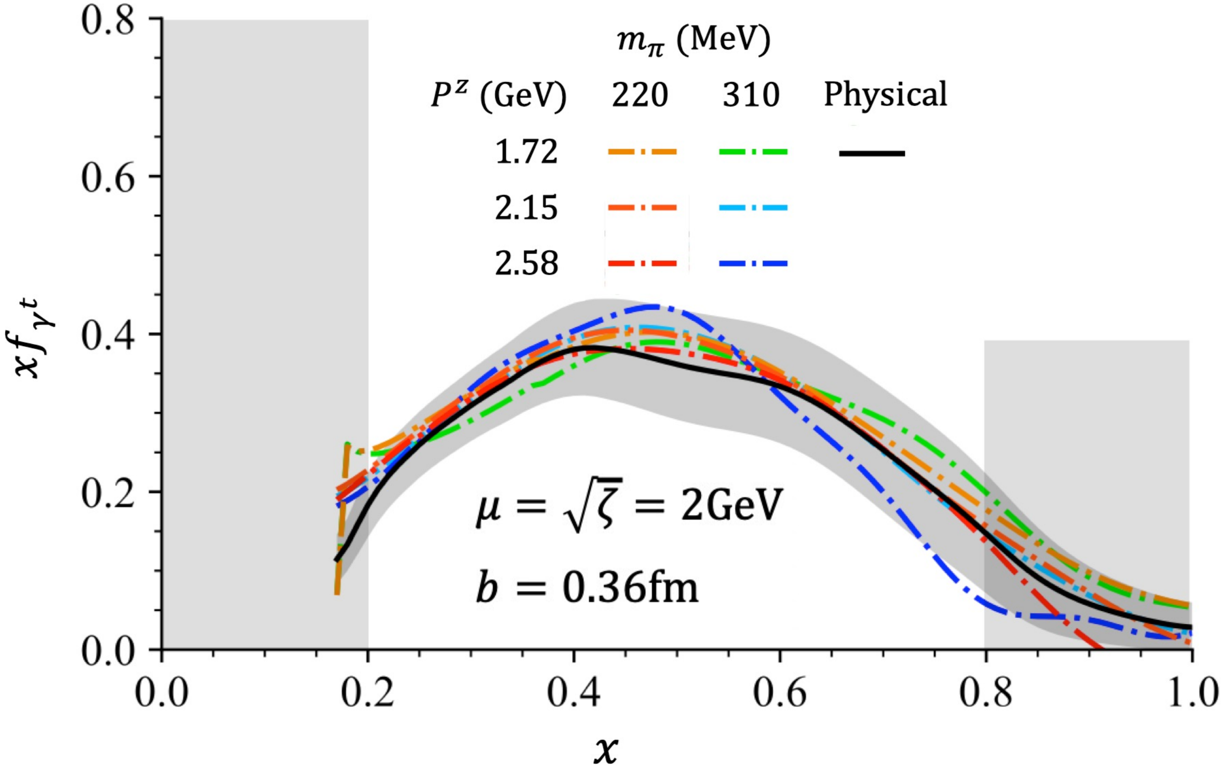
and its solution:

$$H\left(\zeta_z/\mu^2\right) = H\left(\zeta_z/\mu_0^2\right)\exp\left[\int_{\mu_0}^{\mu}\frac{d\mu}{\mu}\left(\Gamma_{\text{cusp}}^{(1)}\ln\frac{\zeta_z}{\mu^2}\alpha_s(\mu)+\gamma_C^{(1)}\alpha_s(\mu)+\Gamma_{\text{cusp}}^{(2)}\ln\frac{\zeta_z}{\mu^2}\alpha_s^2(\mu)\right.\right.$$
$$\left.\left.+\gamma_C^{(2)}\alpha_s^2(\mu)+\Gamma_{\text{cusp}}^{(3)}\ln\frac{\zeta_z}{\mu^2}\alpha_s^3(\mu)+\gamma_C^{(3)}\alpha_s^3(\mu)+\Gamma_{\text{cusp}}^{(4)}\ln\frac{\zeta_z}{\mu^2}\alpha_s^4(\mu)\right)\right].$$

Physical TMDPDF

Chiral and large-P^z joint extrapolation:

$$d_0(m_\pi^2 - m_{\pi,\text{phy}}^2) + \frac{d_1}{(P^z)^2}$$

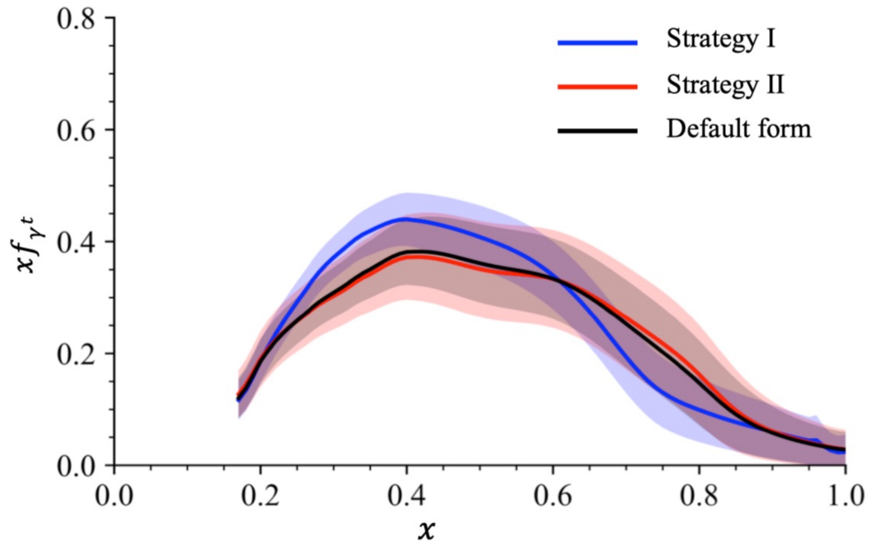


Systematic from chiral extrapolation (strategy I):

$$d_0(m_\pi^2 - m_{\pi,\text{phy}}^2)^2 + \frac{d_1}{(P^z)^2}$$

from large-P^z extrapolation (strategy II):

$$d_0(m_\pi^2 - m_{\pi,\text{phy}}^2) + \frac{d_1}{(P^z)^2} + \frac{d_2}{P^z}$$



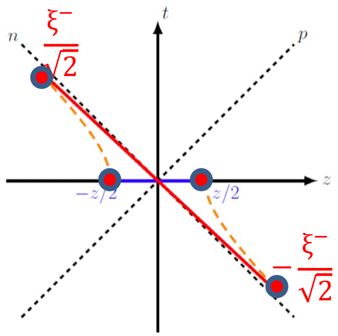
Power correction

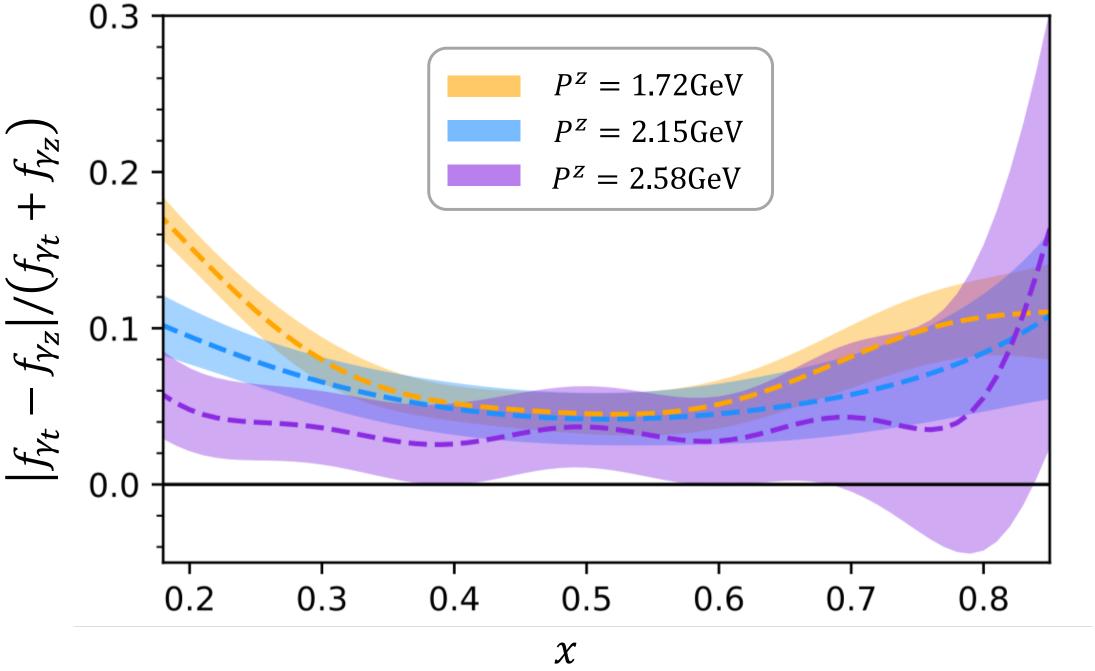
After Lorentz boost:

Leading power | Higher power

$$\bar{\psi}(z)\gamma^t\psi(0) = \frac{1}{2}\bar{\psi}(z)\gamma^+\psi(0) + \frac{1}{2}\bar{\psi}(z)\gamma^-\psi(0)$$

$$\bar{\psi}(z)\gamma^z\psi(0) = \frac{1}{2}\bar{\psi}(z)\gamma^+\psi(0) - \frac{1}{2}\bar{\psi}(z)\gamma^-\psi(0)$$





- **Ratios denote the deviations from light-like correlator with specific P^z;**
- **Ratio becomes smaller with P^z increasing.**

Final results and discussion

🤔 **The unpolarized TMDPDFs seem not converge in $b_\perp$-space?**

Of course not! Perhaps there will be abrupt change at the edge of nucleon

⇒ Need larger $b_\perp$ and more statistics!

🤔 **Lattice discretization and finite-volume systematics are still absent in this preliminary work…**

- **It is a challenging work for calculating the TMDPDF at small lattice spacing**
- **From the previous experience of PDF (*Lin, 2011.14971*), we can roughly estimate that:**

Finite-volume effect is less than 1%;

Discretization effects overall within 2 standard deviations.