Berezinskii-Kosterlitz-Thouless Phase Transition and Superfluidity In Two Dimensions

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2016-12-03

"For the greatest benefit to mankind"

Alfred Nobel

The Royal Swedish Academy of Sciences has decided to award the

2016 NOBEL PRIZE IN PHYSICS

David J. Thouless
F. Duncan M. Haldane
J. Michael Kosterlitz

"for theoretical discoveries of topological phase transitions and topological phases of matter"

Nobelprize.org

2016 Nobel Prize in Physics

Popular Information

Title: **Strange Phenomena in Matter's flatland**

"*Together, they (Thouless and Kosterlitz) took on the problem of phase transitions in the flatlands (the former out of curiosity, the latter out of ignorance, they themselves claim).*"

"*...an entirely new understanding of phase transitions, which is regarded as one of the twentieth century's most important discoveries in the theory of condensed matter physics.*"—BKT phase transition

"*The wonderful thing...is that it can be used for different types of materials—the KT transition is universal...The theory...also confirmed experimentally.*"

2016 Nobel Prize in Physics

Advanced Information

Title: **Topological Phase Transitions and Topological Phases of Matter**

"In 1972 J. Michael Kosterlitz and David J. Thouless identified a completely new type of phase transition in two-dimensional systems where topological defects play a crucial role [35, 36]. Their theory applied to certain kinds of magnets and to superconducting and superfluid films, and has also been very important for understanding the quantum theory of one-dimensional systems at very low temperatures."

"This insight has provided an important link between statistical mechanics, quantum many-body physics and high-energy physics, and these fields now share a large body of theoretical techniques and results."

Goal of this lecture is to address

(1) What is the Berezinskii-Kosterlitz-Thouless phase transition?

(2) What is the superfluidity?

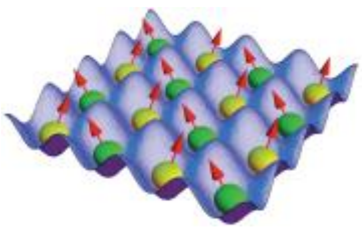
Outline

- Phase transition in high-dimensional XY model
  - ✓ Ginzburg-Landau mean-field theory
- Monte Carlo simulation in three and two dimensions
- Berezinskii-Kosterlitz-Thouless phase transition
  - ✓ Wegener's model (Gaussian model)
  - ✓ Topological excitation and BKT phase transition
  - ✓ Nelson-Kosterlitz relation (universal jump in superfluid density)
  - ✓ Renormalization group analysis
- What is superfluidity
  - ✓ Hallmarks of superfluidity
  - ✓ Two questions—on its relation to BEC/excitation spectrum

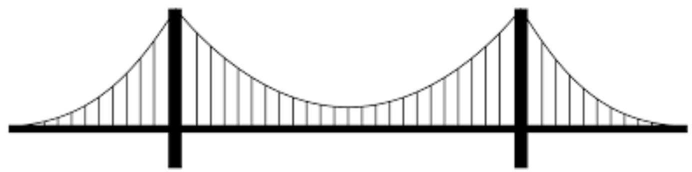
Phase Transition in High-Dimensional XY Model

Emergent Phenomena

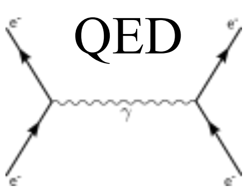
Emergent matter state



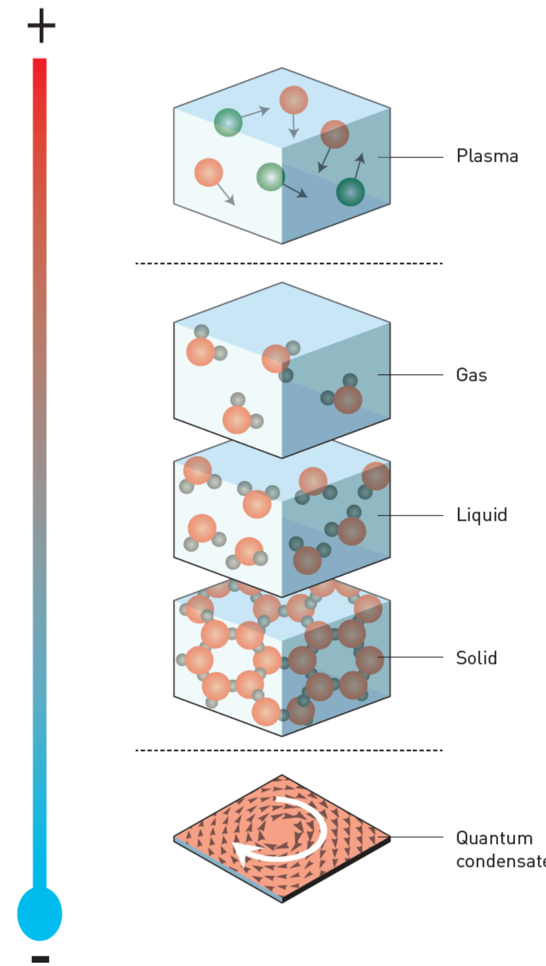
10^{23} particles



Statistical Mechanics
Condensed Matter Physics



few particles

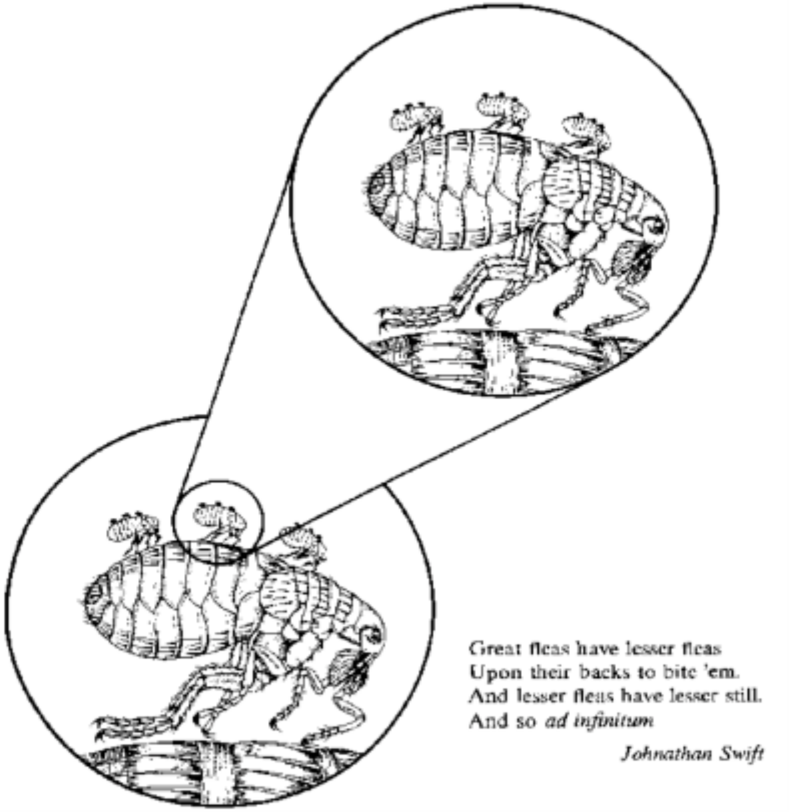


More is different! Each hierarchical level of science requires its own fundamental principles for advancement.

—— P. W. Anderson

Emergent Phenomena

Emergent scale invariance at 2nd-order phase transition



Universal properties!

Universality of Criticality. Critical physics can be captured by a few parameters—symmetry of the **order parameter**, interaction range, and spatial dimensions

Statistical Mechanics

Statistical mechanics is to understand emergent (quantum) macroscopic matter state from microscopic interactions. Given a many-body Hamiltonian, the core task is to calculate partition sum

$$Z = \mathrm{Tr}e^{-H/k_b T} = \sum_i p_i$$

Macroscopic matter state

- gas, liquid, solid

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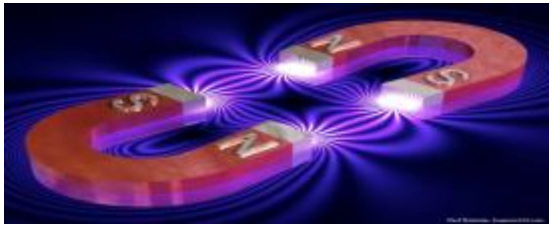
Macroscopic matter state

- gas, liquid, solid
- magnetism
  (ferromagnet, antiferromagnet…)



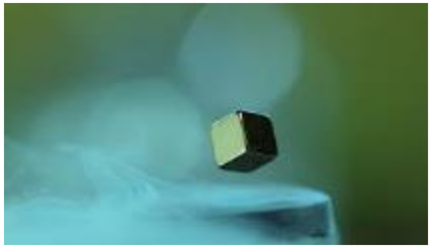
Statistical Mechanics

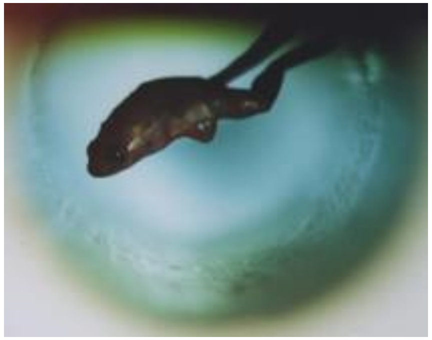
Statistical mechanics is to understand emergent (quantum) macroscopic matter state from microscopic interactions. Given a many-body Hamiltonian, the core task is to calculate partition sum

$$Z = \mathrm{Tr} e^{-H/k_b T} = \sum_i p_i$$

Macroscopic matter state

- gas, liquid, solid
- magnetism
  (ferromagnet, antiferromagnet…)
- superfluidity, superconductivity…
- …





Statistical Mechanics

Which macroscopic matter state?

Free energy

$$F = -k_B T \ln Z$$

k_B — Boltzmann constant

Thermodynamic relation

$$F = E - TS$$

E — internal energy

$S = k_B \ln \Omega$ — entropy

Ω — number of microscopic configurations

Thermodynamic law

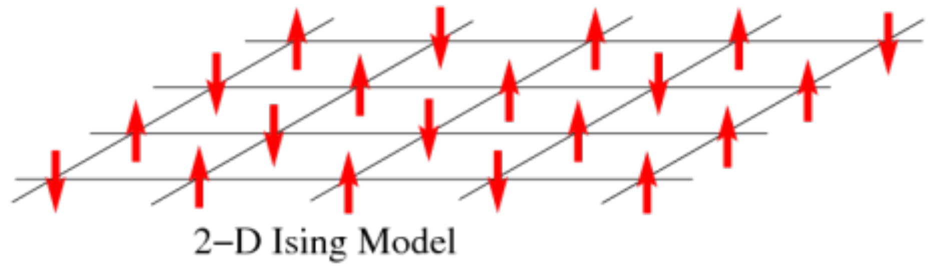
> *The stable matter state at a given temperature T is determined by the minima of the free energy*

Energy-Entropy Competition/Balance

- At high T, "large entropy" dominates—gas, liquid, plasma
- At low T, "low energy" dominates—crystal, superfluid

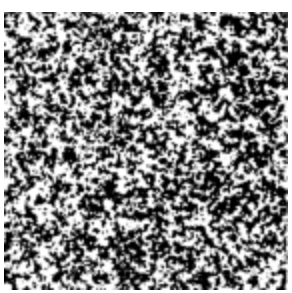
Ising Model

The Ising model (1920)



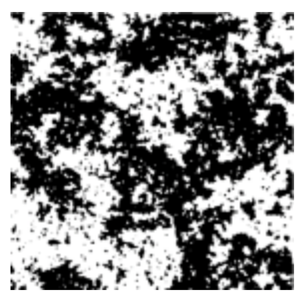
$$H = -\sum_{\langle ij \rangle} \sigma_i \sigma_j \quad (\sigma = \pm 1)$$

- Spontaneous symmetry breaking



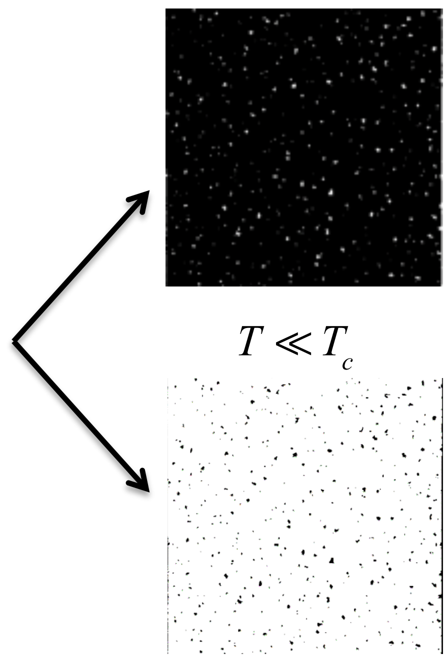
$T \gg T_c$

Paramagnet



$T \approx T_c$

Paramagnet



$T \ll T_c$

Ferromagnet: an *ordered* state spontaneously breaking Z_2 symmetry

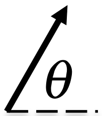
"Spontaneous symmetry breaking" is one of the most **profound** concepts in statistical mechanics and condensed-matter physics.

XY Model

The XY model

$$H = -\sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j = -\sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

$$\vec{S} = (\cos\theta, \sin\theta)$$



✓ *U(1) symmetry: Hamiltonian remains unchanged if all the spins are rotated by fixed angle θ*

Partition sum

$$Z = \int_{[-\pi,\pi]^\Lambda} \prod_{j\in\Lambda} d\theta_j \; e^{-\beta H(\mathbf{s})}$$

✓ Local order *parameter*: $\vec{M} = \frac{1}{V}\sum_i \vec{S}_i$

- At T=0, all spins point to the same direction — ferromagnet
- At T=infinite, each spin points to a random direction — paramagnet

Question

Is there a critical temperature $T_c>0$ between ferromagnet and paramagnet?

XY Model

Ginzburg-Landau mean-field theory

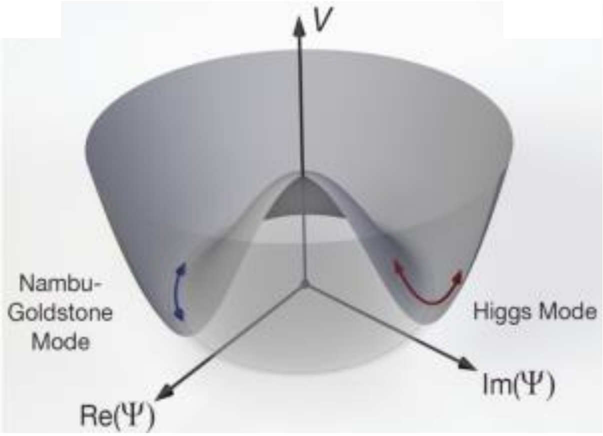
Free energy density

$$V(n_0) = -\frac{\mu}{2}|\psi|^2 + \frac{\lambda}{2}|\psi|^4 \qquad \left(n_0 = |\psi|^2\right)$$

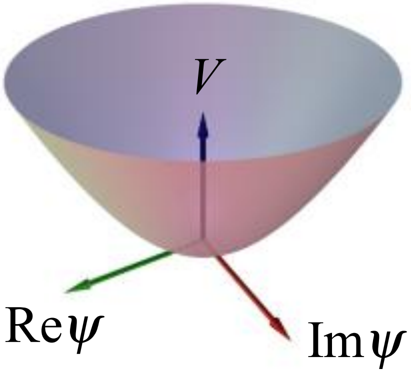
- ✓ *Order parameter is just a complex number* $\psi \equiv \overrightarrow{M}$. *In the context of superfluidity,* n_0 *is the density of* ***Bose-Einstein condensation***
- ✓ μ(T) *is chemical potential;* $\lambda>0$ *is for interaction*

Solution

μ(T)>0, |Ψ|>0, ferromagnet



μ(T)<0, Ψ=0, paramagnet



T_c

XY Model

Mean-field (MF) theory of the XY model

- Ignore fluctuations that are crucial in low-dimensional systems
- For $d>d_c=4$, MF critical exponents are **correct**
  MF critical point are **wrong**
- For $d=3$, MF critical exponents are **correct**
  MF critical point are **wrong**
- For d=2, the **topological BKT** phase transition is
  beyond the Ginzburg-Landau mean-field scenario

Complex field model of the XY model

$$H_\psi = -t \sum_{\langle \mathbf{ik} \rangle} \left(\psi_{\mathbf{i}}^* \psi_{\mathbf{k}} + \mathrm{c.c.} \right) + \frac{U}{2} \sum_{\mathbf{i}} |\psi_{\mathbf{i}}|^4 - \mu \sum_{\mathbf{i}} |\psi_{\mathbf{i}}|^2$$

Research tools

- ✓ Analytical calculation (1D)
- ✓ Renormalization group analysis
- ✓ Monte Carlo simulation

Monte Carlo Simulation
in Three and Two dimensions

Monte Carlo Simulation

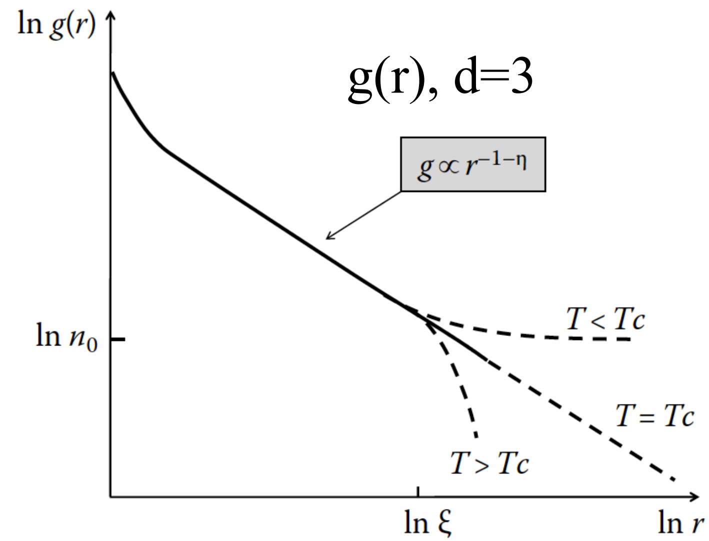
Schematic behavior

Two-point correlation function g(r)

- d=3

$$g(r) \equiv \left\langle \vec{S}_0 \cdot \vec{S}_r \right\rangle = \begin{cases} c_1 e^{-r/\xi} & T > T_c \\ n_0 & T < T_c \end{cases}$$

n_0 —BEC density



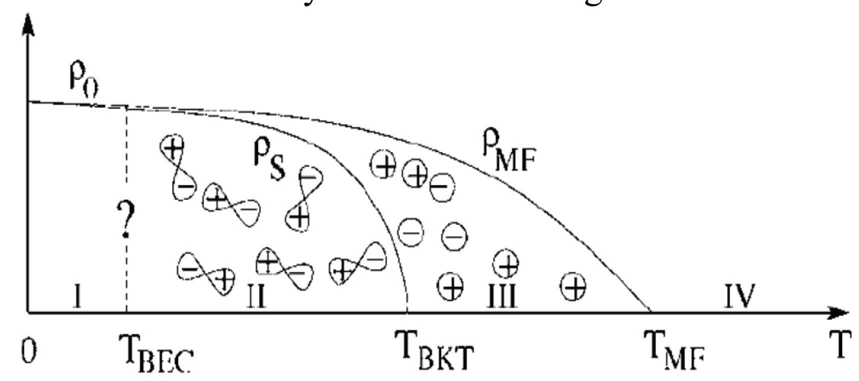
- d=2

$$g(r) = \begin{cases} c_1 e^{-r/\xi} & T > T_c \\ c_2 r^{-\eta(T)} & T \le T_c \end{cases}$$

✓ No BEC in 2D $g(r \to \infty) = 0$

No spontaneous symmetry breaking for 2D XY model

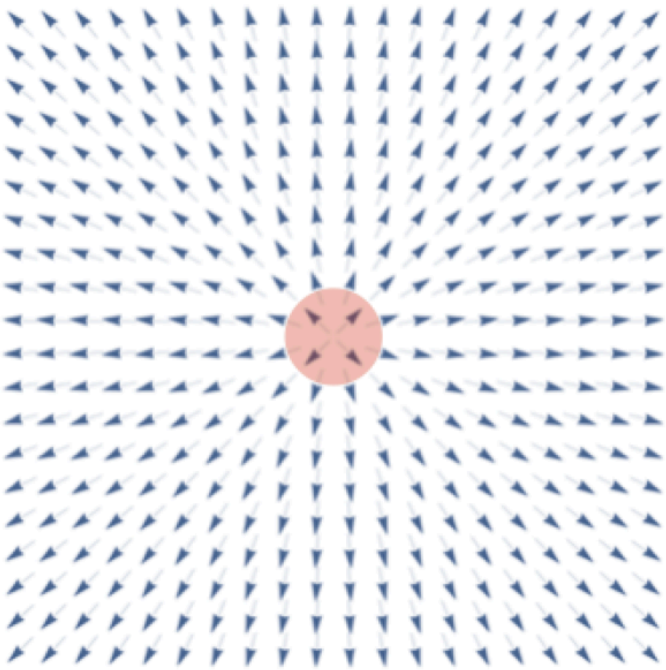
Schematic phase diagram of a 2D trapped weakly interaction Bose gas



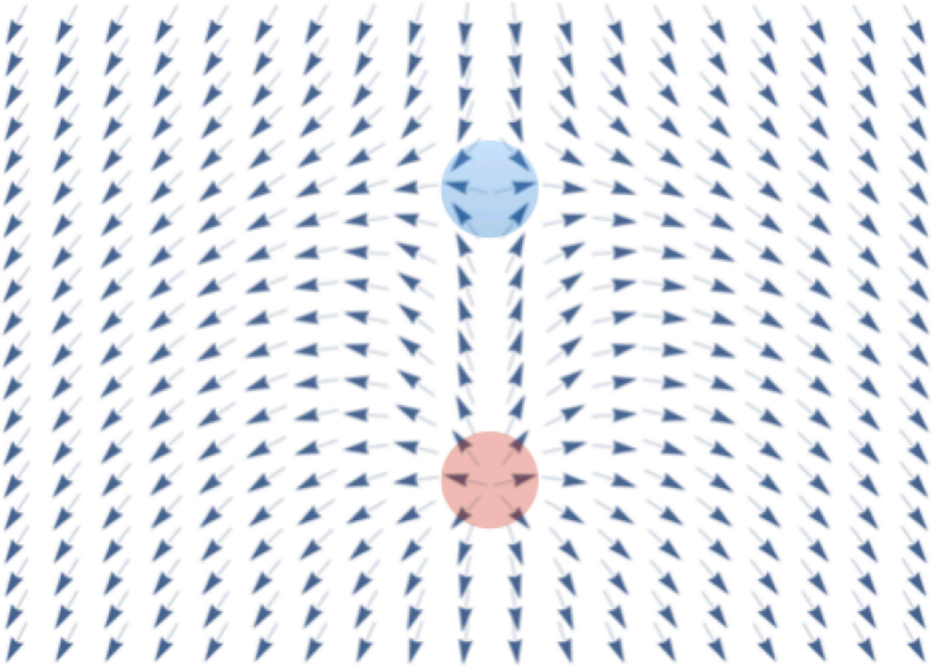
Monte Carlo Simulation

Topologically distinct configurations

Vorticity $$v = \frac{1}{2\pi}\oint_C d\vec{r}\cdot\vec{\nabla}\theta(\vec{r})$$



$v = +1$



$v = 0$

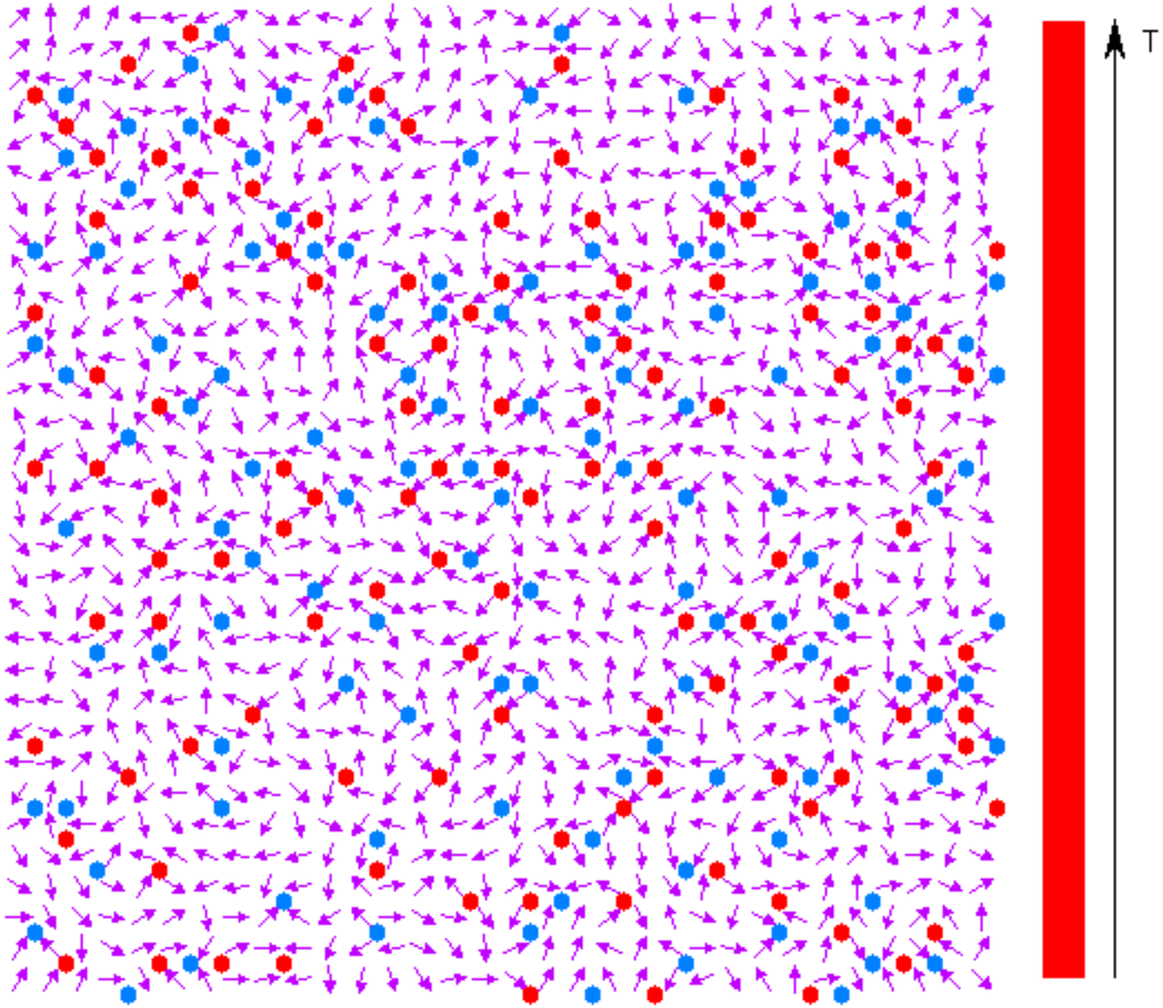
"Topologically distinct"— two configurations cannot be transformed into each other by a continuous rotation of the spins.

Monte Carlo Simulation

Illustration of MC simulation

$v = +1$

$v = -1$



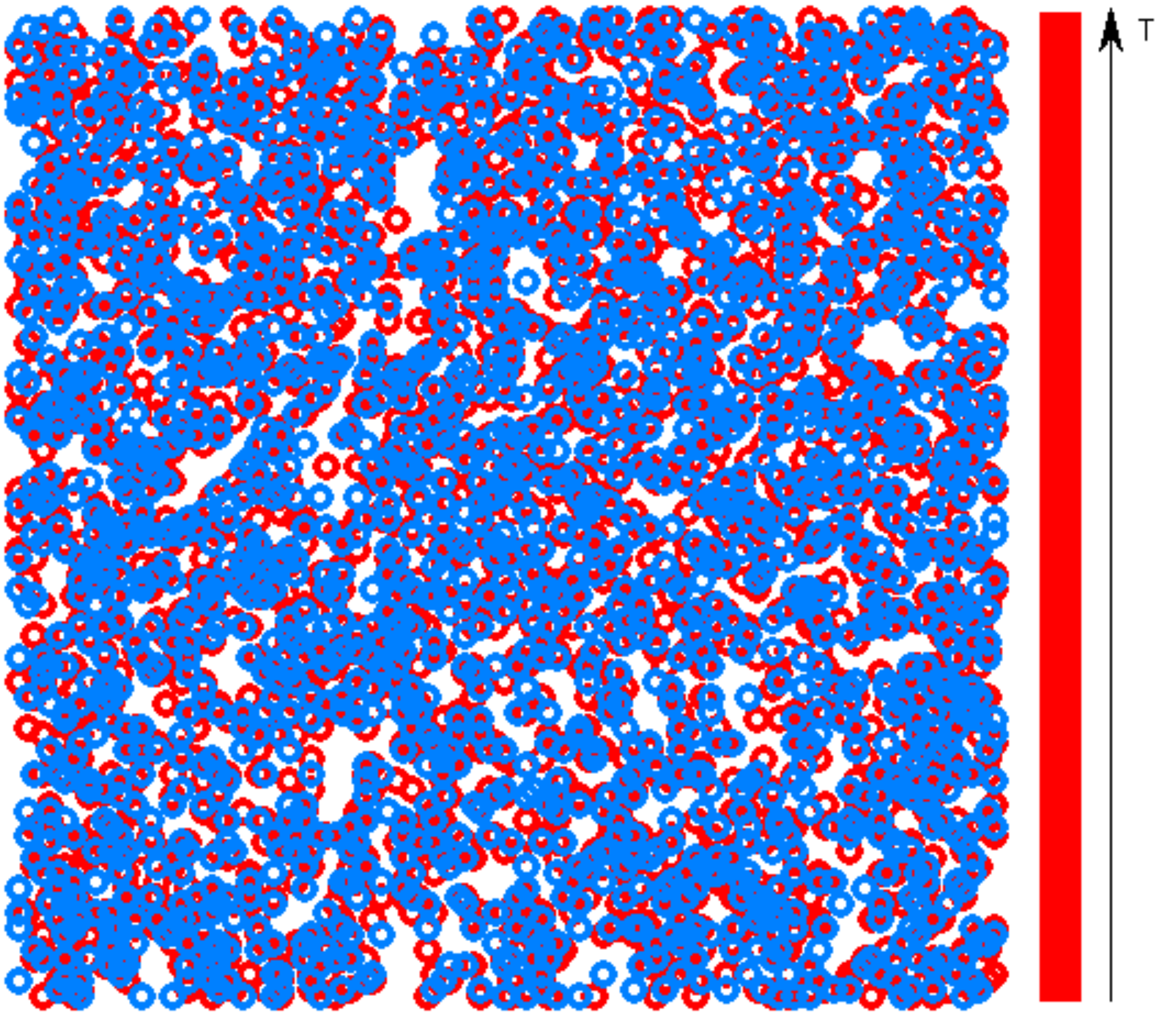
$d = 2, L = 32$

Monte Carlo Simulation

Illustration of MC simulation

● $v = +1$

● $v = -1$

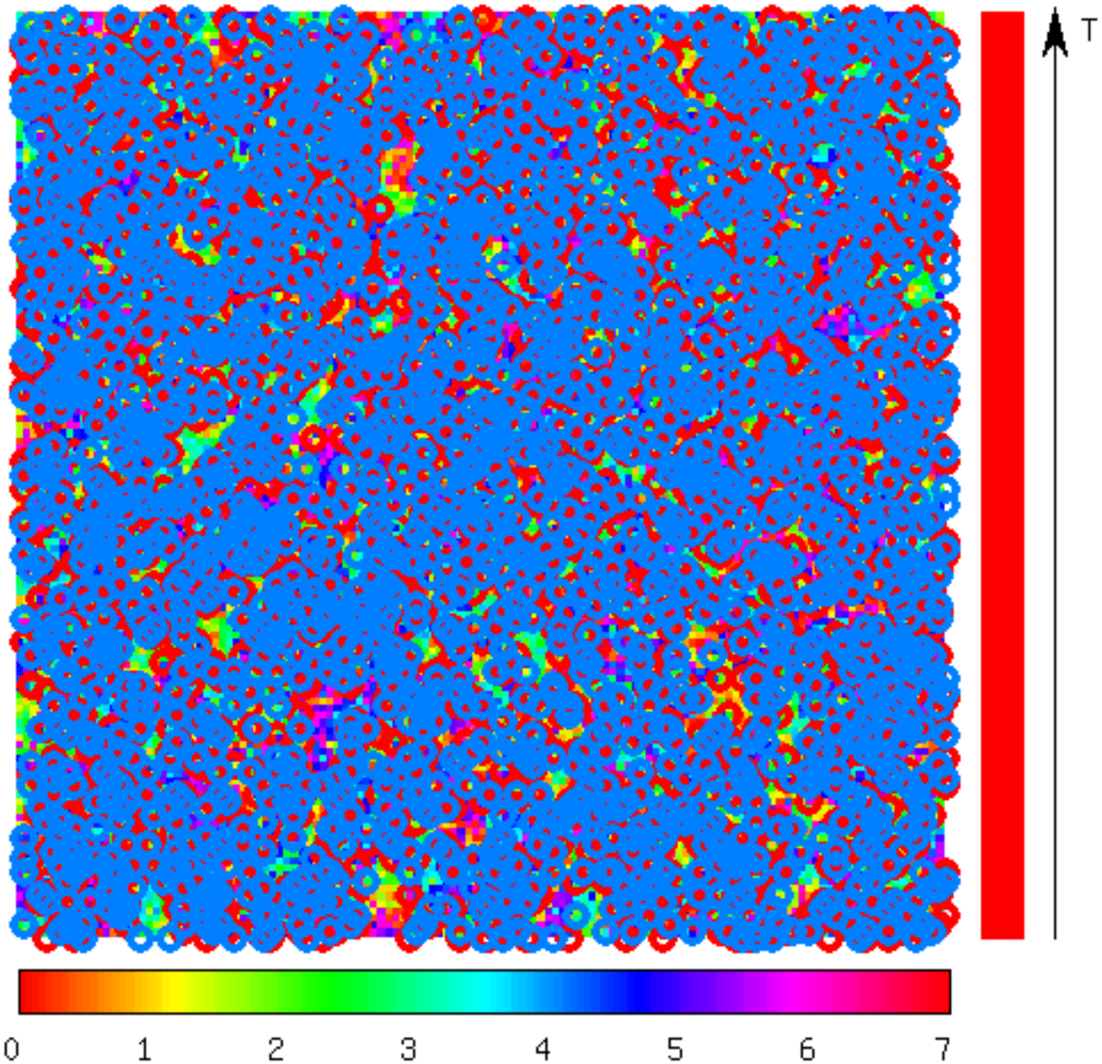


$d = 2, L = 128$

Monte Carlo Simulation

Illustration of MC simulation

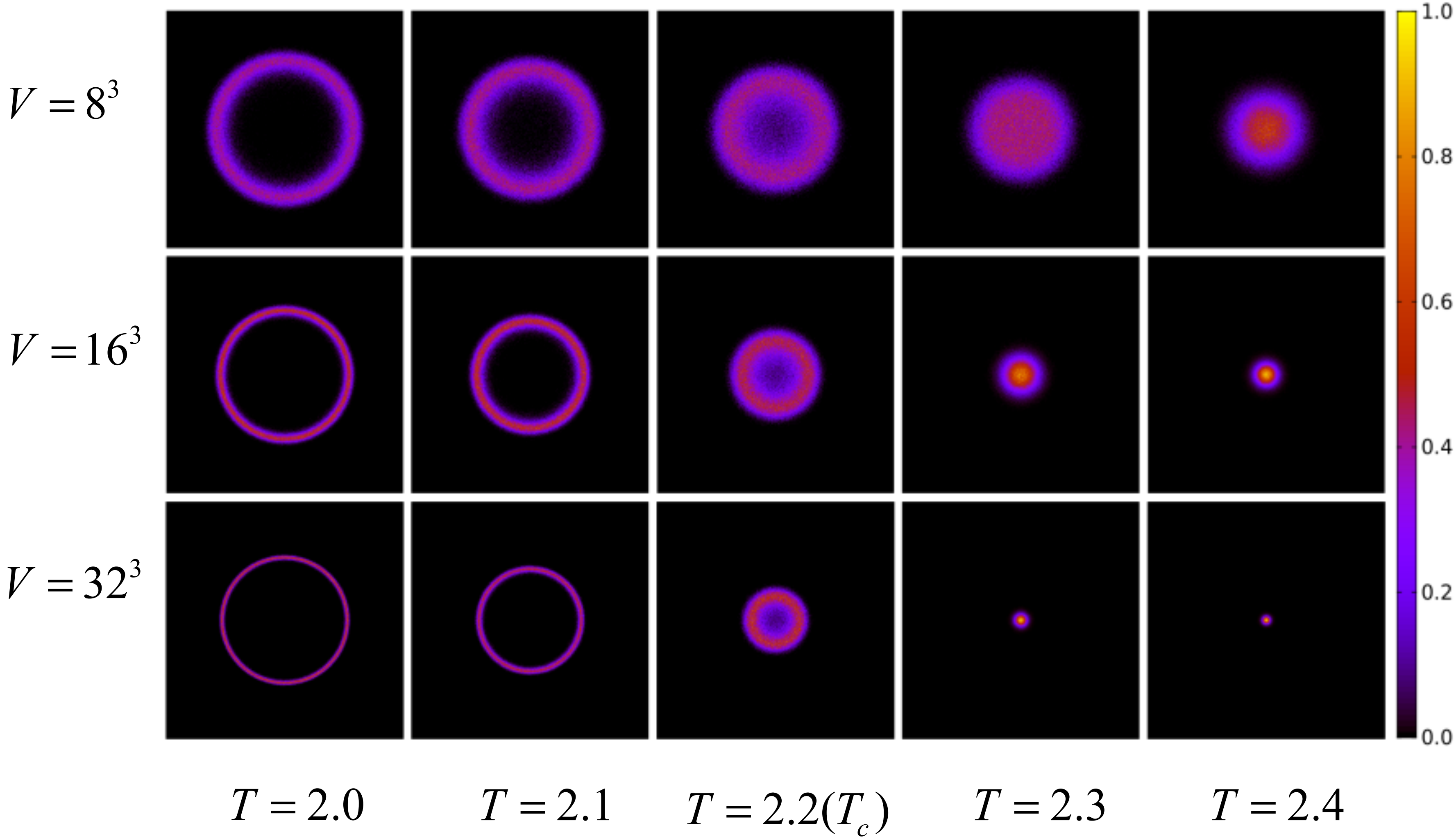
- $v = +1$
- $v = -1$



$d = 2, L = 128$

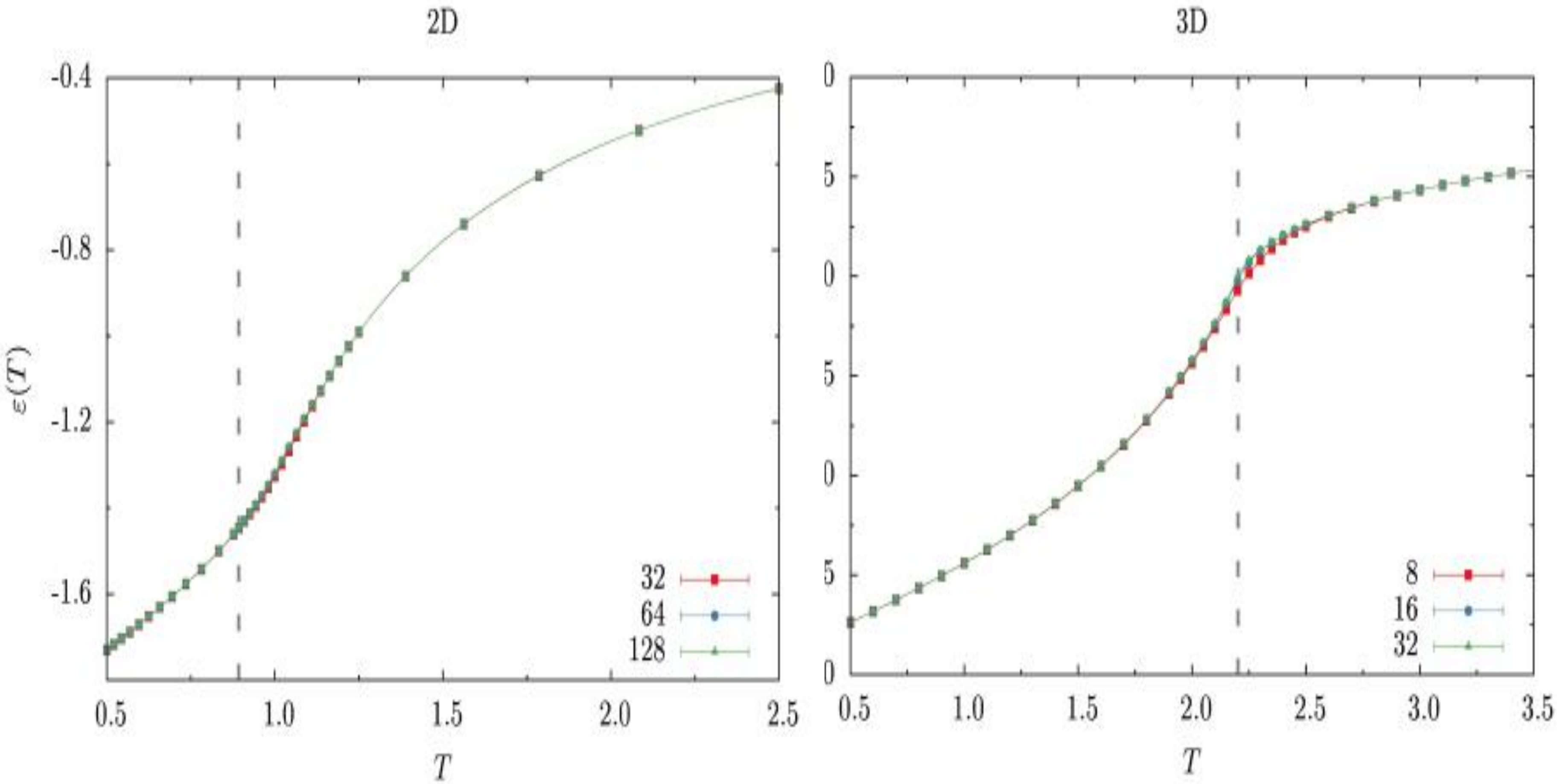
Results of 2D and 3D XY model

Probability distribution of magnetization in 3D



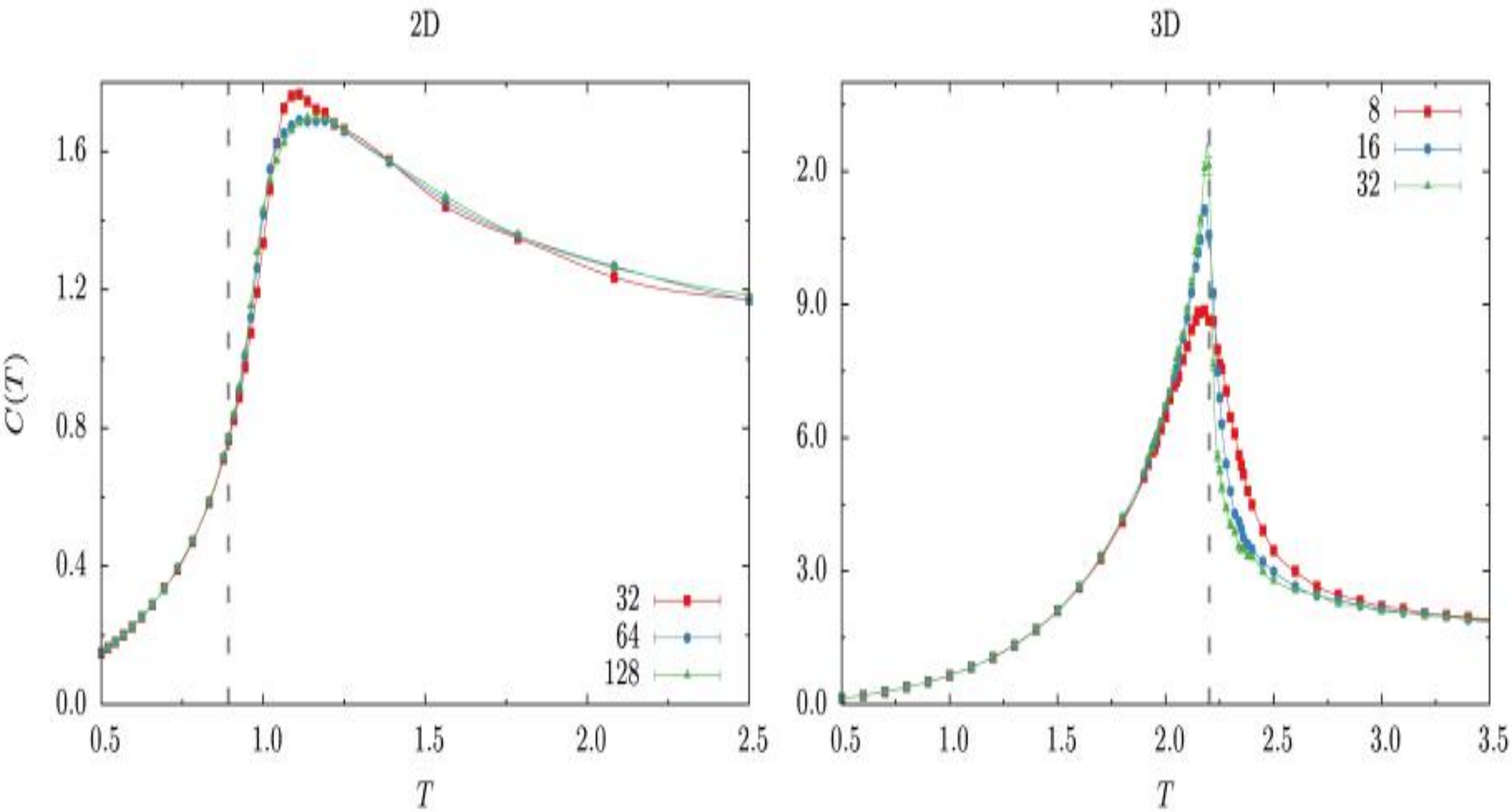
Results of 2D and 3D XY model

Energy density $\varepsilon(T) = \frac{1}{V}\langle U_{total}(T)\rangle$



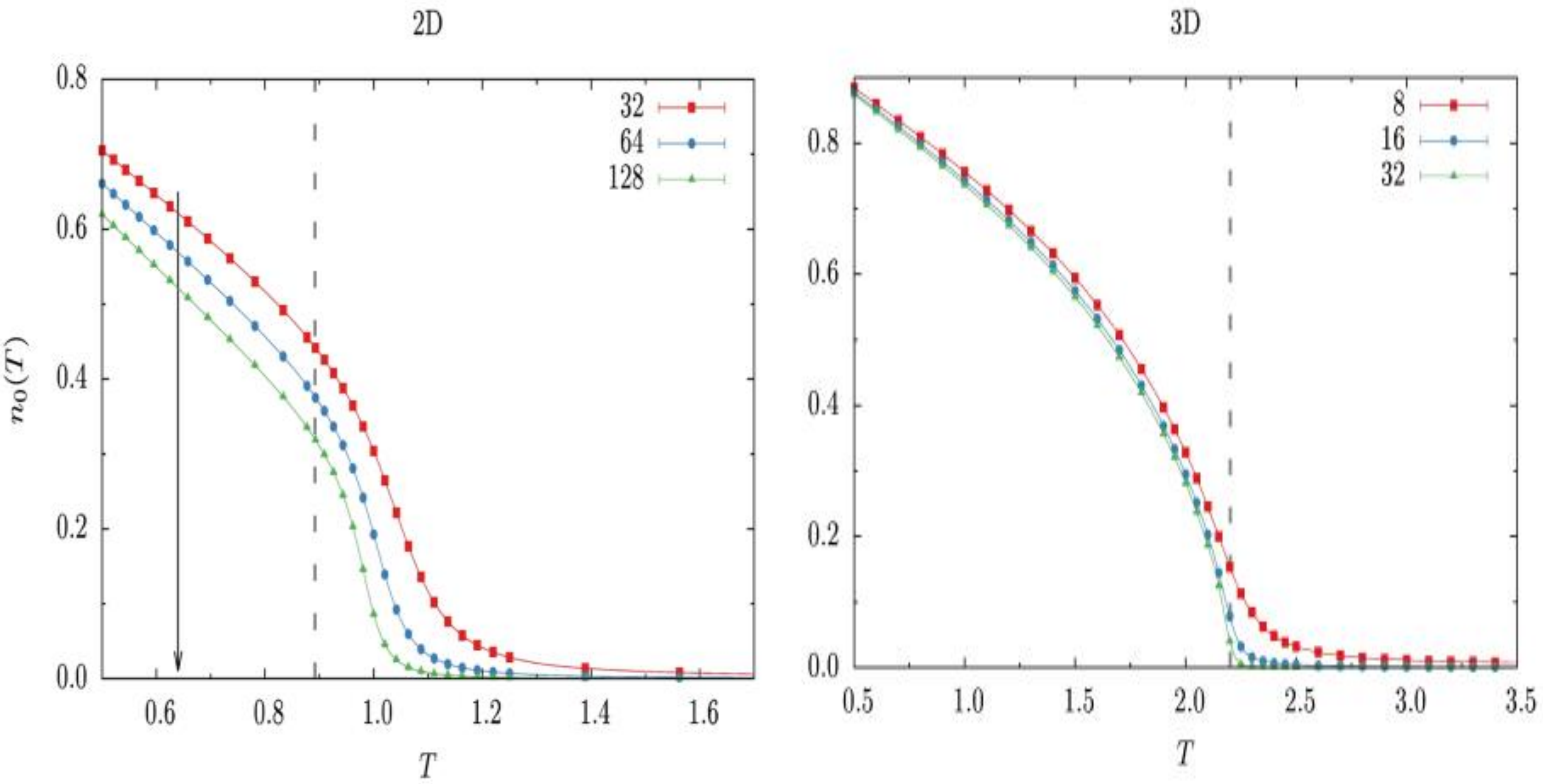
Results of 2D and 3D XY model

Specific heat $C(T) = \dfrac{\partial \varepsilon(T)}{\partial T}$



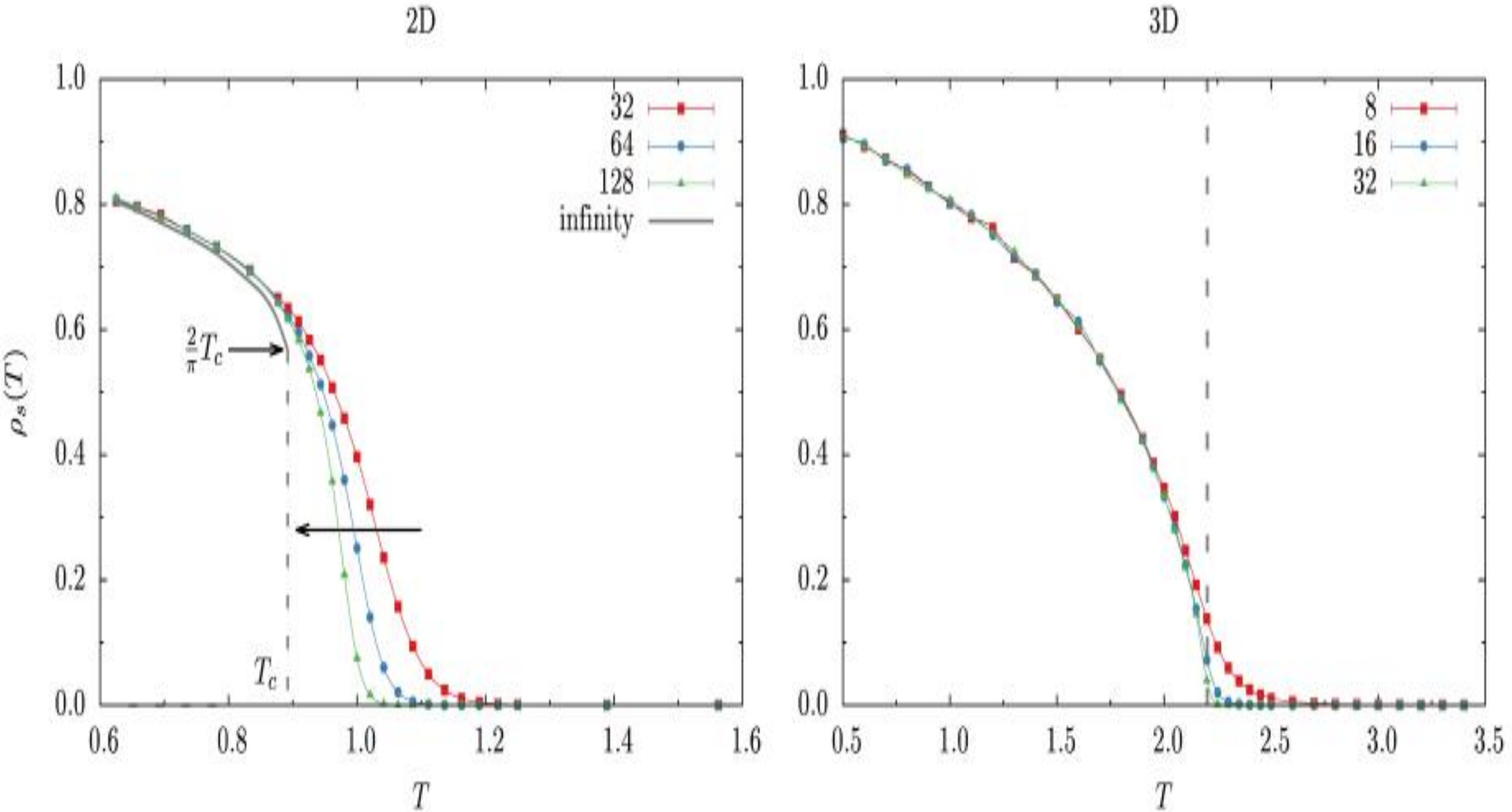
Results of 2D and 3D XY model

BEC density $n_0(T) = \frac{1}{V^2} \left\langle M^2_{total}(T) \right\rangle$



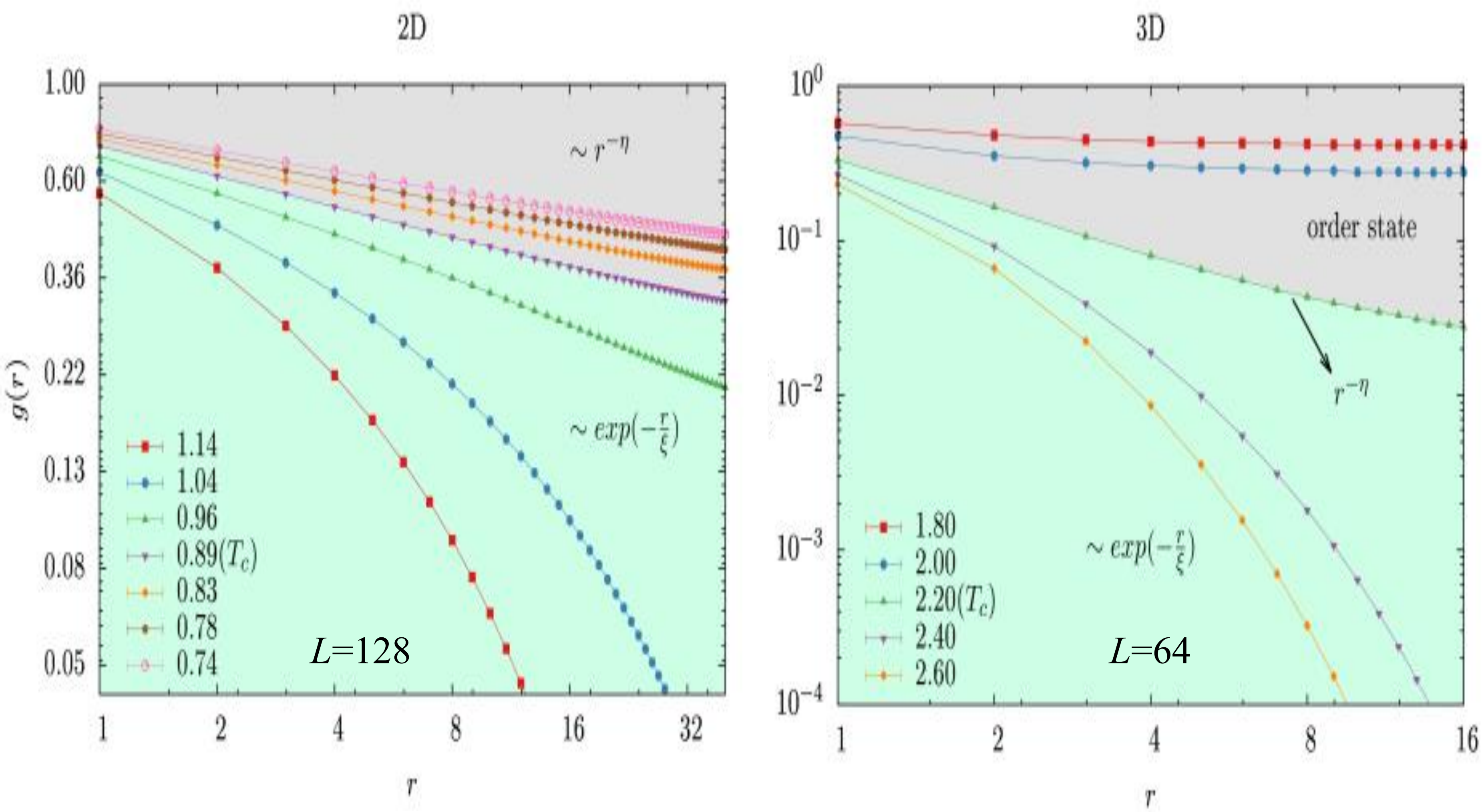
Results of 2D and 3D XY model

Superfluid density ρ_s



Results of 2D and 3D XY model

Two-point correlation function $g(r)=\left\langle e^{i(\theta_0-\theta_r)}\right\rangle$



Berezinskii-Kosterlitz-Thouless Phase Transition in the Two-Dimensional XY Model

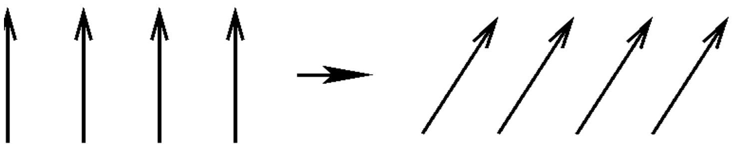
Wegner's model

Wegner's model (Gaussian model)

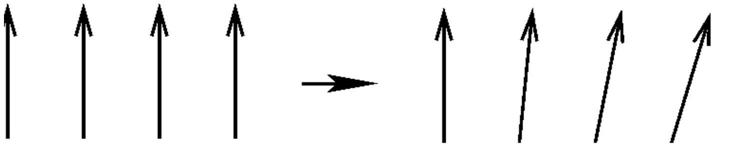
- **Hamiltonian of XY model**

$$H = -J\sum_{\langle ij\rangle}\cos(\theta_i - \theta_j)$$

✓ spin-wave excitation (gapless)



zero wave number, zero energy cost



small wave number, small energy cost

- **Wegner's model** (1967)

Taylor expand till 2nd order &

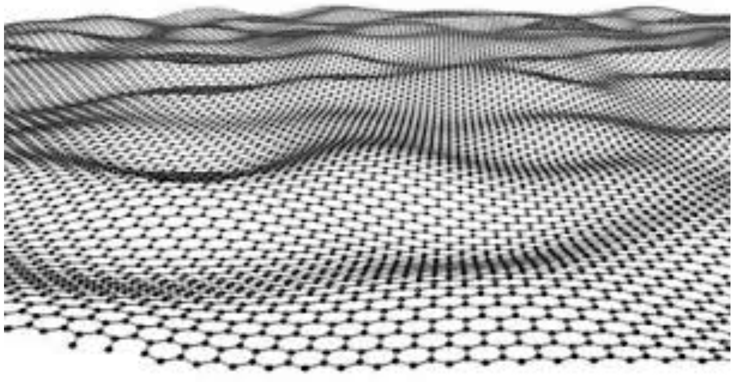
ignore 2π-periodicity

$$H = E_0 - \frac{J}{2}\sum_{\langle ij\rangle}(\varphi_i - \varphi_j)^2 \qquad \varphi \in (-\infty, +\infty)$$

Lattice Fourier transform

$$H = E_0 - \frac{J}{2}\sum_{\mathbf{k}}\varepsilon_k |\varphi_{\mathbf{k}}|^2 \qquad \varepsilon_k = \frac{k^2}{2}$$

✓ ideal gas (in momentum space)



Wegner's model

Spin-Ordering in a Planar Classical Heisenberg Model

FRANZ WEGNER
Max-Planck-Institut für Physik und Astrophysik, München, Germany

Received July 26, 1967

We consider a *D*-dimensional system of classical spins rotating in a plane and interacting via a Heisenberg coupling. The spin-correlation function $g_D(r)$ is calculated for large distances r in a low-temperature approximation (taking into account short-range order):

$$g_1(r) = \exp(-C_1 Tr),$$

$$g_2(r) \sim r^{-C_2 T},$$

$$\lim_{r\to\infty} g_3(r) = \exp(-C_3 T).$$

- ✓ No long-range order for $D \leq 2$
- ✓ Correlation function algebraically decays for any $T \neq 0$ in 2D — *algebraic* order or *quasi-long-range* order

Wegner's model

Mermin-Wegner theorem

In one and two dimensions, continuous symmetries cannot be spontaneously broken at finite temperature in systems with short-range interactions

Direct consequences

✓ No BEC for $T \neq 0$ in 2D and 1D
Yes in 2D harmonic trap

✓ No crystal for $T \neq 0$ in 2D and 1D
Yes for $T=0$ in 2D
No for $T=0$ in 1D (quantum fluctuation)

✓ ……

Explaination

It cost very little energy to induce long-range fluctuations which have enormous entropy and are free-energy-preferred

BKT phase transition in 2D

Topological excitation and BKT transition

Can U(1) nature (θ period) be ignored?

- Energy of a single vortex

$$E_{\mathrm{V}} = \pi J \ln \frac{L}{a}$$ ✓ costive — logarithm of L

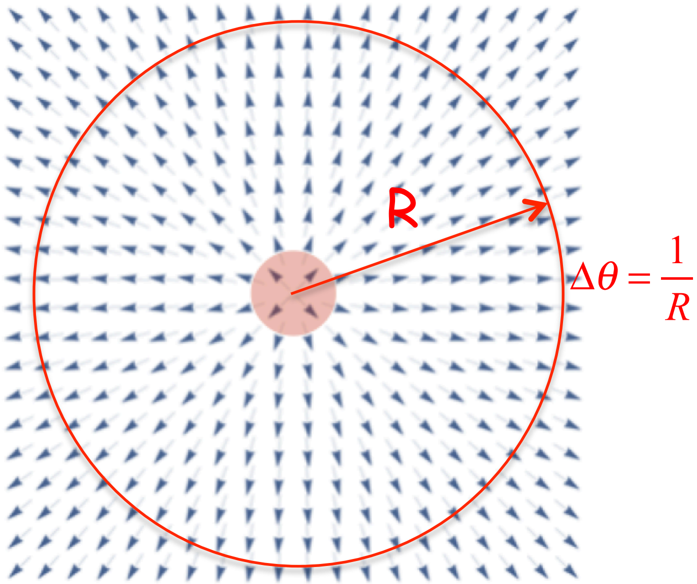
- Entropy of a vortex

$$S_{\mathrm{V}} = 2k_{\mathrm{B}} \ln \frac{L}{a}$$ ✓ also logarithm of L

- Free energy $F=E-TS$ (energy-entropy balance)

$$\Delta F = \left(\pi J - 2k_{\mathrm{B}}T\right) \ln \frac{L}{a} \quad \Rightarrow \quad k_{\mathrm{B}}T_c = \frac{\pi J}{2}$$

Vortex: a topological defect

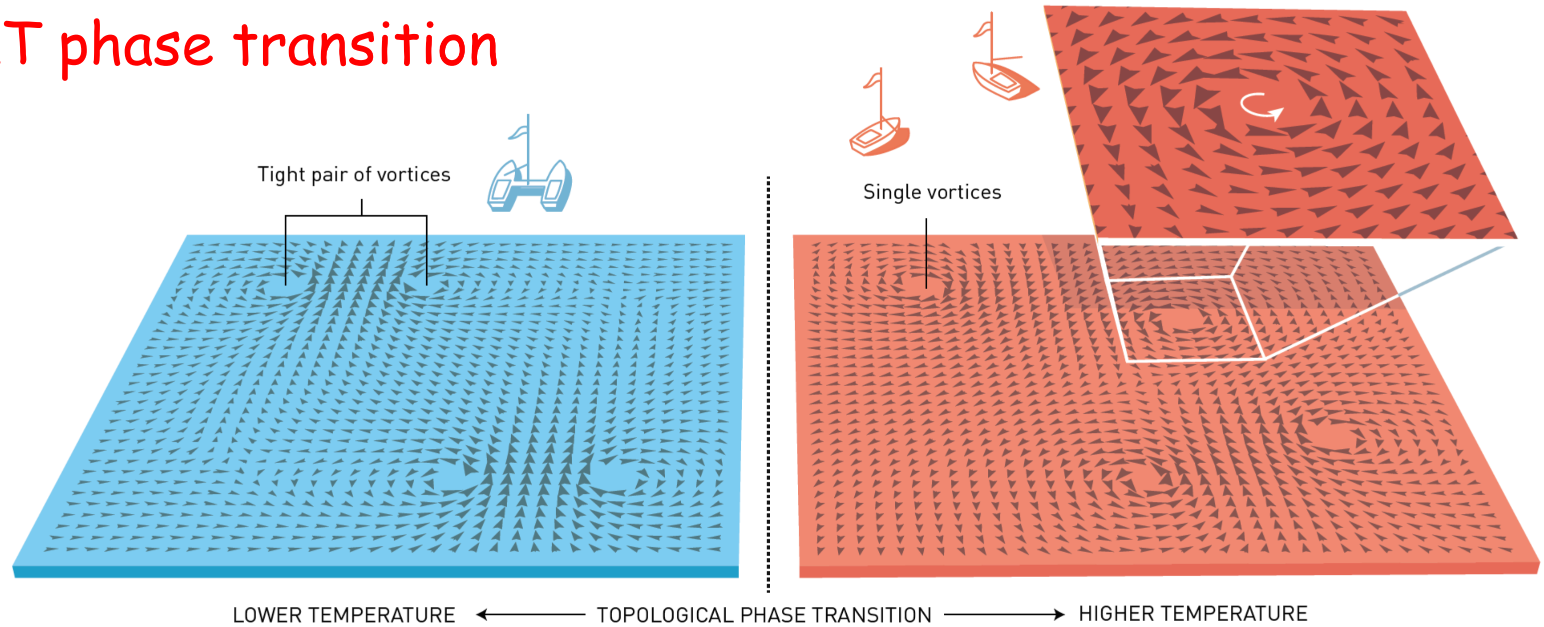


BKT phase transition is "one of *the 20th century's most important discoveries in the theory of condensed matter physics*"

—popular science background for Nobel Prize in Physics, 2016

BKT phase transition in 2D

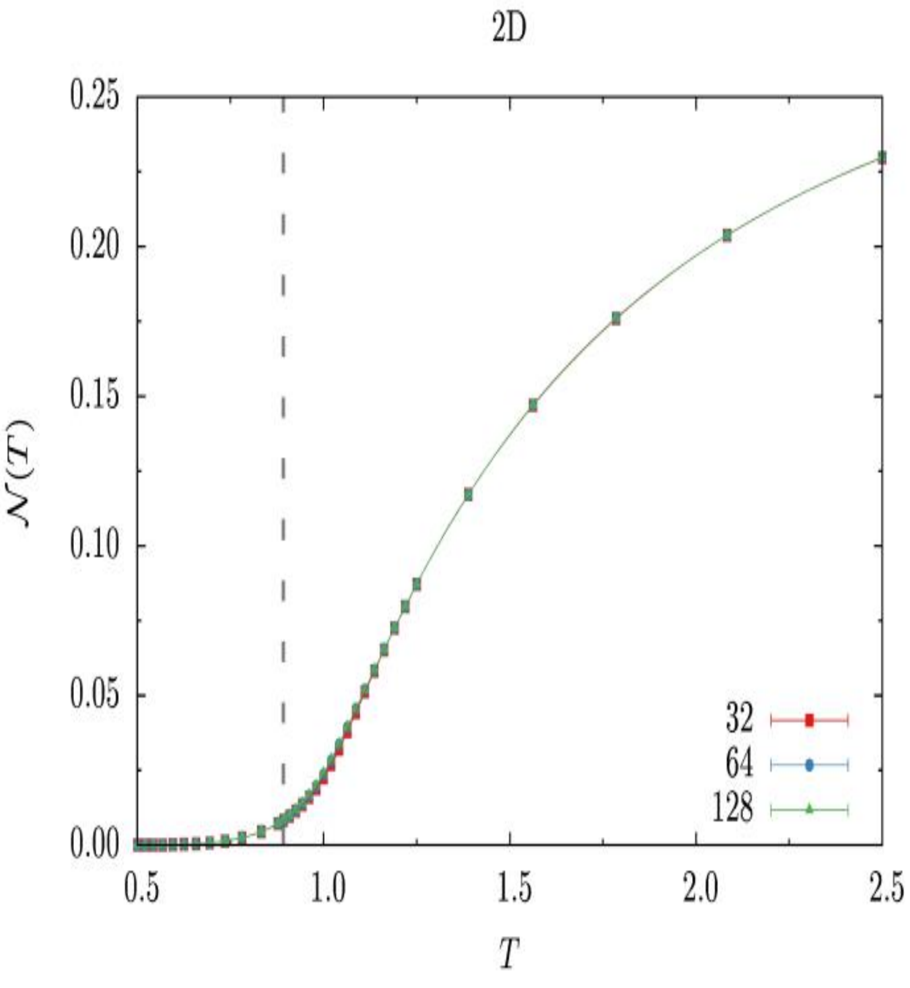
BKT phase transition

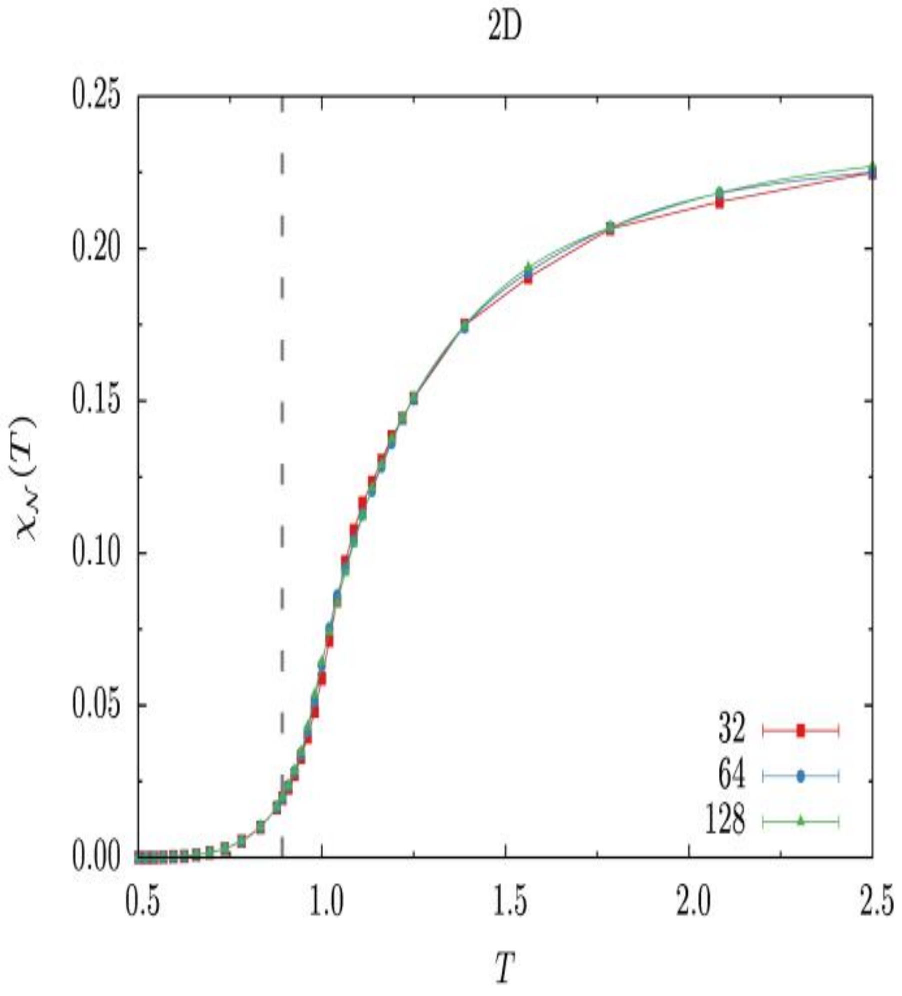


- ✓ Energy of vortex-anti-vortex pair $E = 2\pi J \ln \frac{r}{a}$
- ✓ for $T<T_c$
  - vortex pairs are bound (in order of lattice spacing)
  - spin-wave excitations lead to "*algebraic/quasi-long-range order*"
  - bound vortex pairs lead to renormalization of coupling K that determines critical exponent for the algebraic decay of correlation function

BKT phase transition in 2D

Vortex number per site N and its fluctuation X_N in 2D XY model



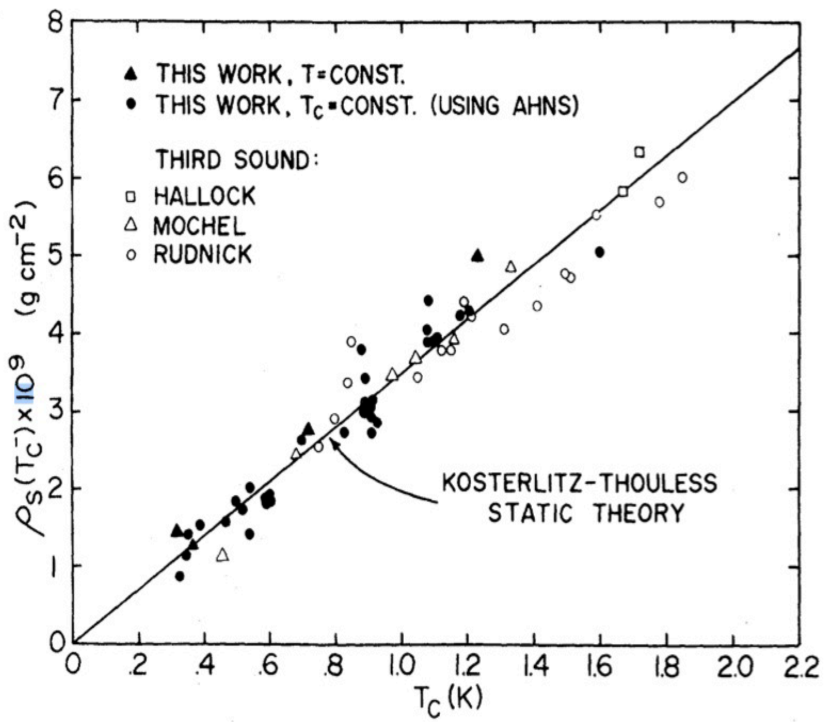


BKT phase transition in 2D

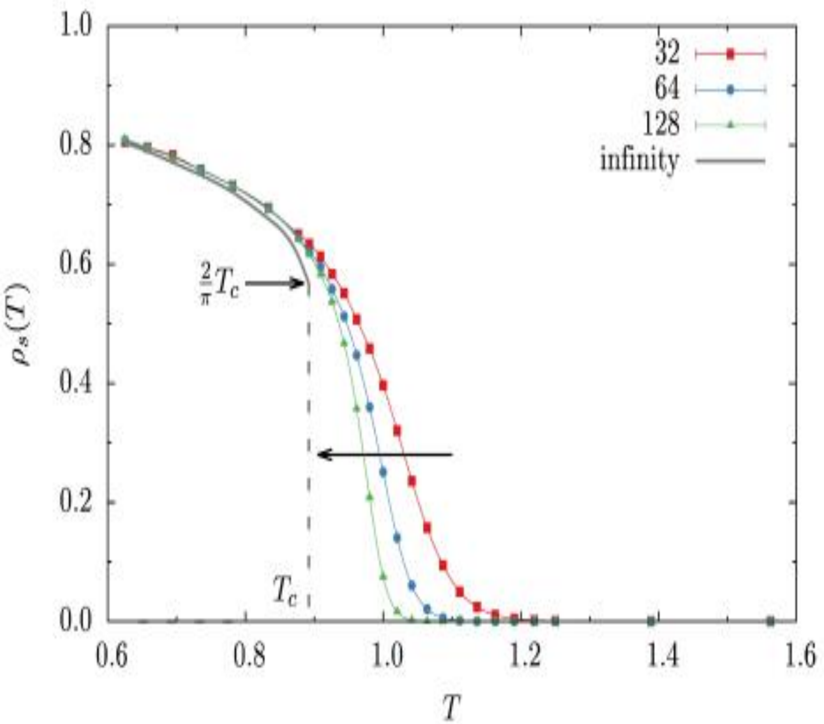
Nelson-Kosterlitz relation

Universal jump in superfluid density at T_c

$$\rho(T_c) = T_c \frac{2}{\pi} \frac{m^2 k_B}{\hbar^2}$$



^{4}He experiment

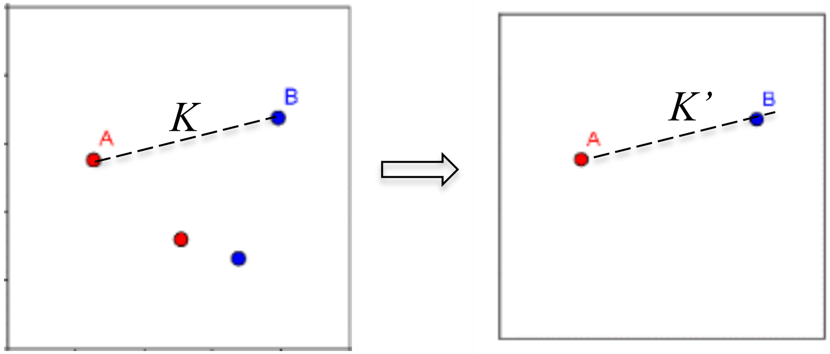


Computer experiment

BKT phase transition in 2D

Renormalization group analysis

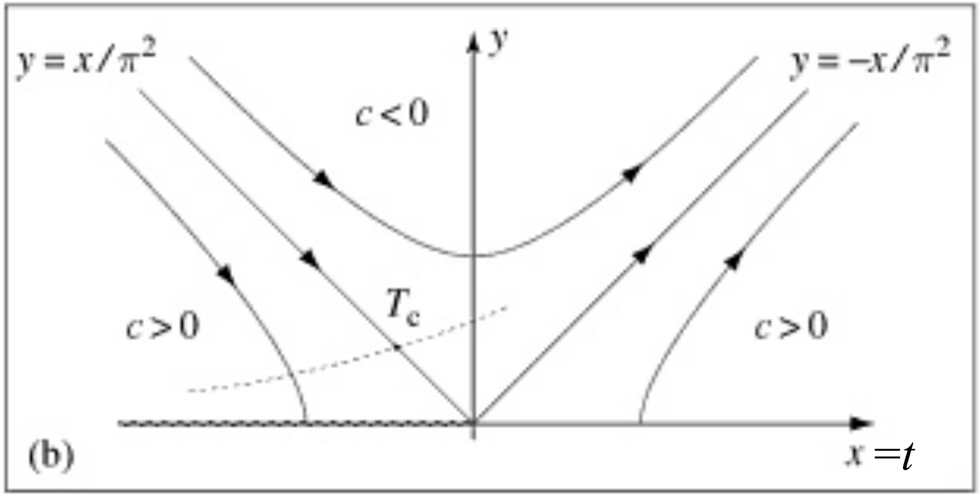
Basic idea: *coarse-grain* and integrate out degrees of freedom for vortex-antivortex pairs of *short* distance



RG equation:

$$\frac{dt}{d\ell} = 4\pi^3 y^2$$

$$\frac{dy}{d\ell} = 4ty/\pi$$



y — fugacity of vortex

t — deviation ~$(T\text{-}T_c)$

$b = e^{\ell} \approx 1 + \ell$ — RG scale

- ✓ y=0 accounts for the "vacuum" states free of vortices, but with various Gaussian fluctuation modes
- ✓ the vacuum is stable for t<0 (superfluid) and unstable for t>0 (normal fluid)

What is Superfluidity

Hallmarks of Superfludity

In ^{4}He-II	In the BEC gas
• Zero-temperature effects	
1. Frictionless flow through narrow pores	(?)
2. Hess-Fairbank effect	✓
3. Persistent currents	(?)
4. Quantized circulation (vortices)	✓
6. Josephson effect	✓
7. ……	

• Finie-temperature (thermomechanical) effects

1. Superconductivity of heat/Counterflow
2. Fountain effect
3. Escaping from a Dewar
4. ……

Hallmarks of Superfludity

Frictionless flow below T_λ

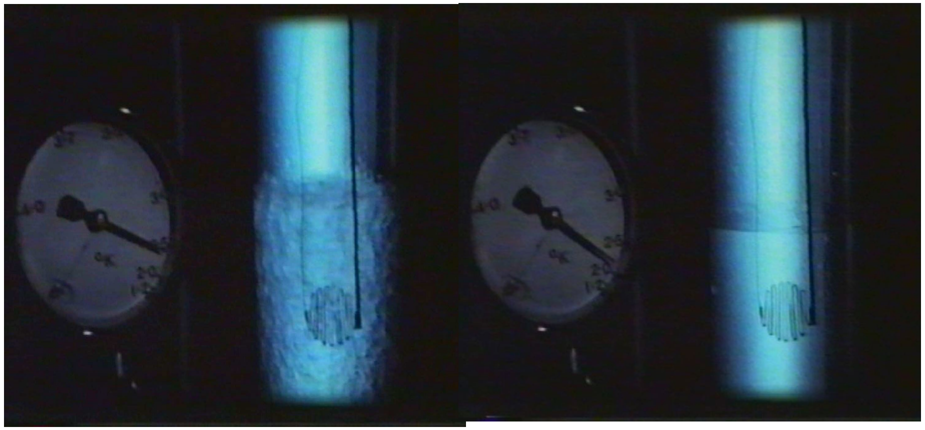
- 1908, ^{4}He liquefied (Onnes)
- 1911, "supraconductivity" of mercury (Onnes)
  ^{4}He stops boiling below certain temperature (Onnes)
- 1922, peculiarity of specific heat (Dana, not published)
- 1927-1937, phase transition (T_λ=2.17K); heat-capacity anomaly, huge reduction of viscosity; "Supra-heat-conductivity" (Keesom, Misner, Allen…)
- 1938, Discovery of superfluidity (Kapitza; Allen and Misener)
- 1938, fountain effect (Allen and Jones)

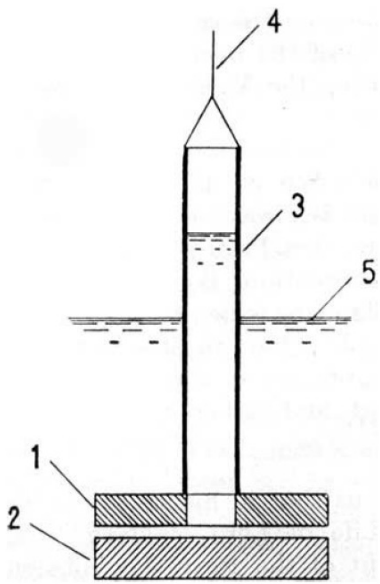
It took 20 years to discover "superfluidity"

Hallmarks of Superfludity

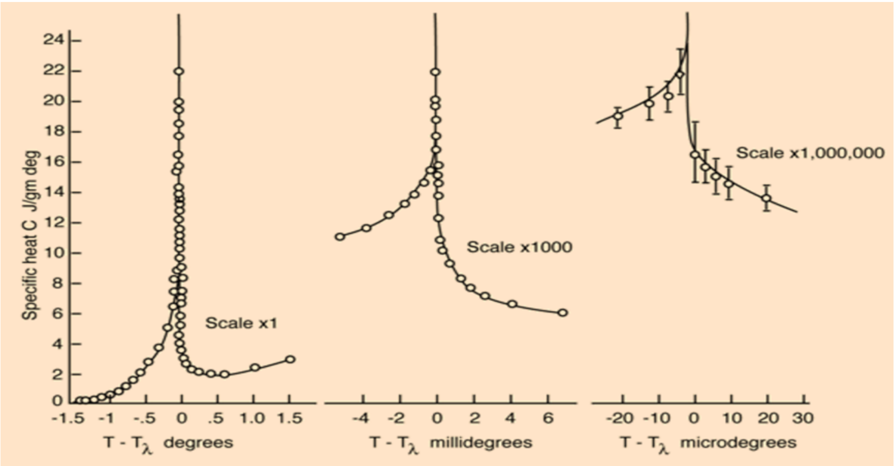
Frictionless flow below T_λ

Liquid ^{4}He stops boiling < 2.3K.
J.C. McLennan *et al*. (1932)





The helium below the λ-point enters a special state which might be called a 'superfluid'
——P. Kapitsa (1937)

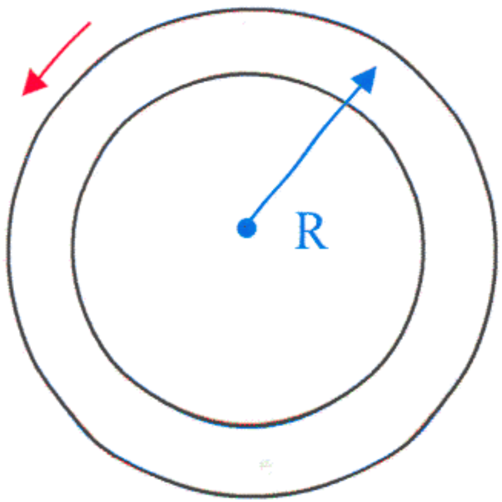


λ-transition of Helium 4.
W.H. Keesom and A.P. Keesom (1935)

Hallmarks of Superfludity

Hess-Fairbank effect and Persistent current

Define $\omega_c = \dfrac{\hbar}{mR^2} \equiv$ quantum unit of rotation (10^{-4} Hz for 1cm)

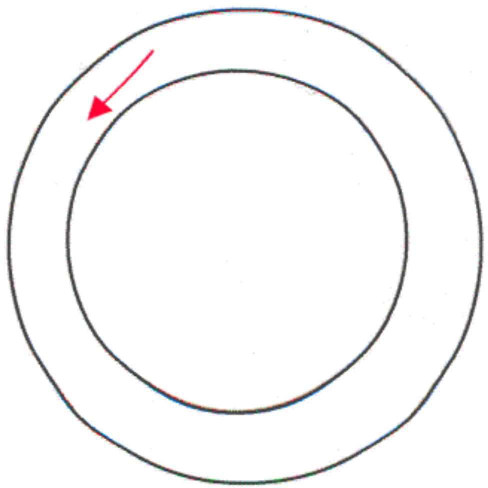


Hess-Fairbank effect

Wall rotates with $\omega \le \omega_c$,

liquid **stationary**

Equilibrium Effect



Persistent current

Wall at **rest**,

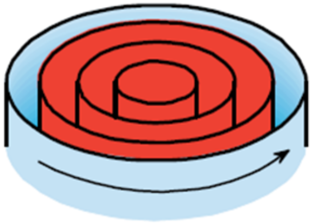
Liquid rotates with $\omega \gg \omega_c$,

Metastable Effect

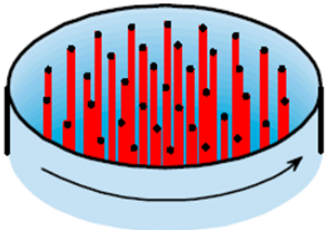
Hallmarks of Superfludity

Quantized vortices

✓ Theoretical predictions of vortices in superfluid (1948, 1955), and superconductor (Abrikosov 1957)



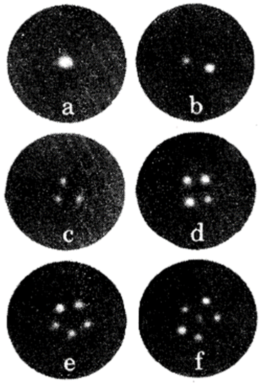
Onsager 1948, Landau–Lifshitz 1955



Onsager 1949, Feynman 1955

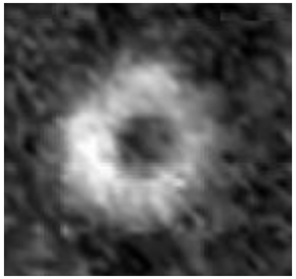
$$\oint_C \vec{v}_s \cdot d\vec{l} = 2\pi\gamma \times \text{integer}$$

$$\gamma = \hbar / m$$

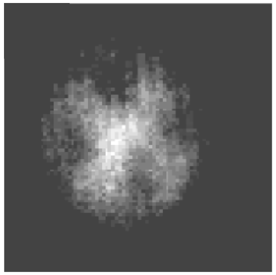


Varmchuk *et al*. 1979

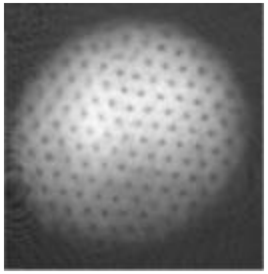
Vortices in ultracold atomic gases



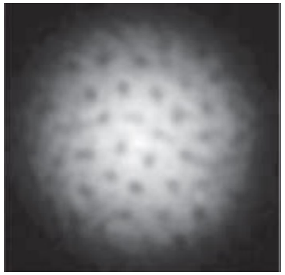
JILA, 1999



ENS, 2000



MIT, 2001



MIT, 2005

Hallmarks of Superfludity

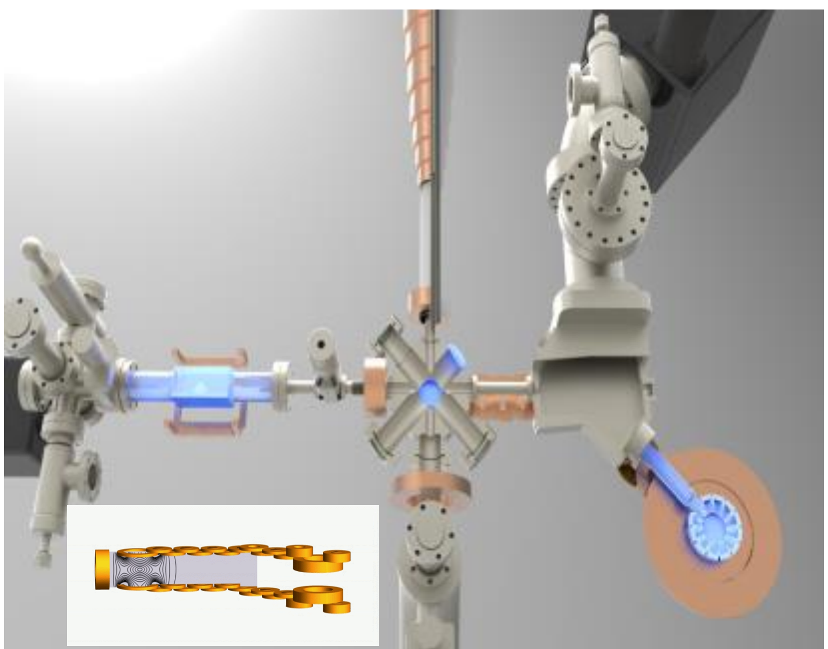
Quantized vortices

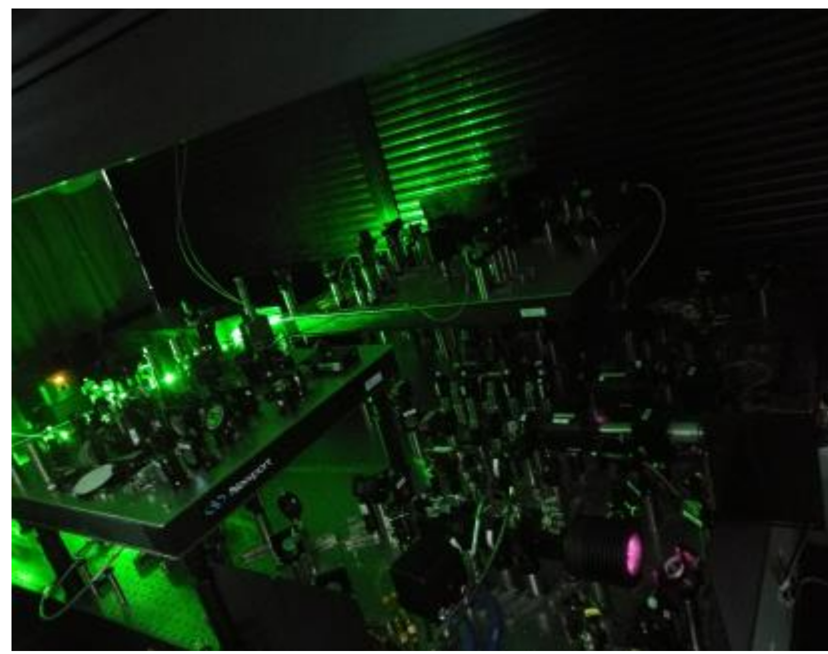
- ✓ USTC: ^{6}Li-^{41}K supercluid mixture

PRL **117,** 145301 (2016) PHYSICAL REVIEW LETTERS week ending 30 SEPTEMBER 2016

Observation of Coupled Vortex Lattices in a Mass-Imbalance Bose and Fermi Superfluid Mixture

Xing-Can Yao,[1,2,3,4] Hao-Ze Chen,[1,2,3] Yu-Ping Wu,[1,2,3] Xiang-Pei Liu,[1,2,3] Xiao-Qiong Wang,[1,2,3] Xiao Jiang,[1,2,3] Youjin Deng,[1,2,3] Yu-Ao Chen,[1,2,3] and Jian-Wei Pan[1,2,3,4]

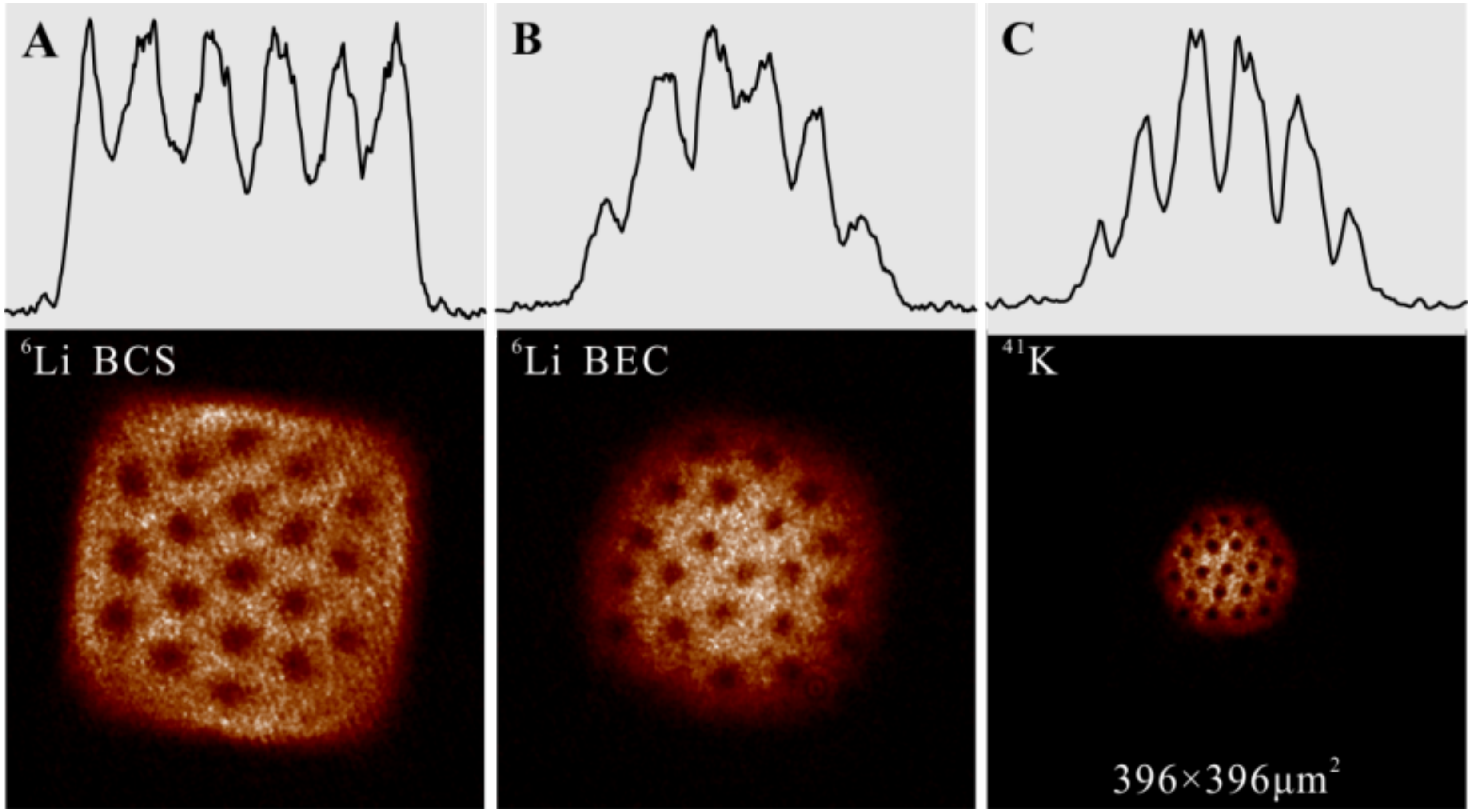




Hallmarks of Superfludity

Quantized vortices

- ✓ USTC: ^{6}Li-^{41}K supercluid mixture

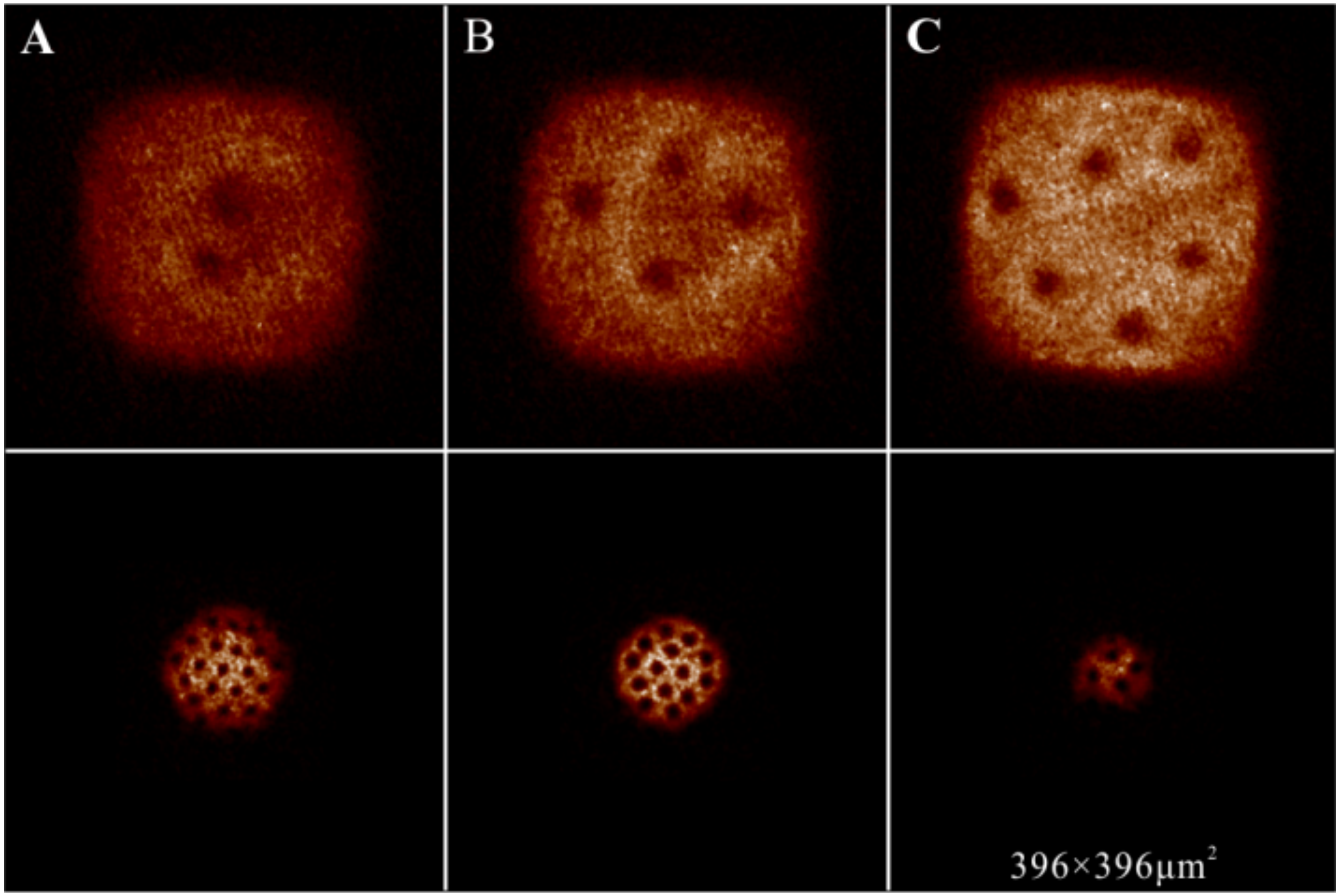


Vortex lattice in single-species superfluid

Hallmarks of Superfludity

Quantized vortices

- ✓ USTC: ^{6}Li-^{41}K supercluid mixture



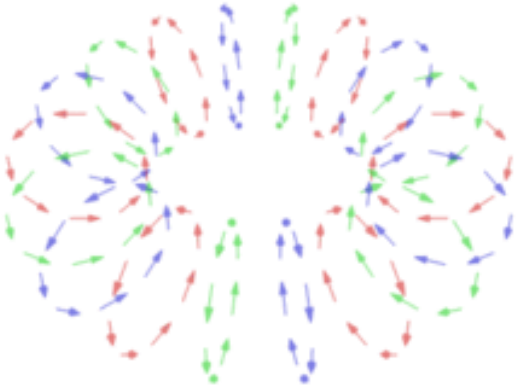
Vortex lattice in two-species superfluid

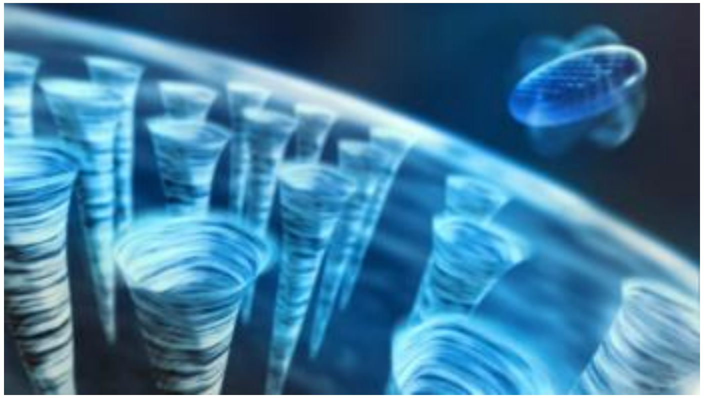
Hallmarks of Superfludity

Vortex ring in 3D and its metastability

A sidedish (superlink) — Crazy pool vortex

- ✓ In 3D, vortices have to form a closed ring or an open chain ending at the surface.
- ✓ Vortices, vortex-rings and persistent currents are metastable due to angular momentum.
- ✓ Unlike classical analogue, quantum vortex has quantized angular momentum





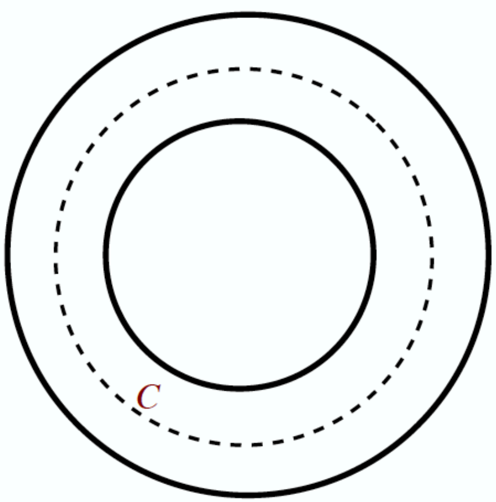
What is Superfludity?

Mordern view of superfluidity: emergent constant of motion

Superfluid is a natural low-T state of a classical complex-valued matter field, arising from an **emergent** constant of motion—i.e., **topological** order

Complex matter field: $\psi(\vec{r}) = |\psi(\vec{r})| e^{i\Phi(\vec{r})}$

Superfluid velocity field: $\vec{v}_s = \gamma \nabla\Phi \quad (\gamma = \hbar/m)$



Emergent constant of motion: **quantized** circulation

$$\oint_C \vec{v}_s \cdot d\vec{l} = \gamma \oint_C d\vec{l} \cdot \nabla\Phi = 2\pi\gamma \times \text{integer}$$

- ✓ leads to superfluidity
- ✓ can be destroyed *only* by **topological** defects (vortices)

Back to Einstein at 1924-1925

Bose-Einstein statistics for an ideal bosonic system

- Single-particle **phase space (μ-space)**: (x, y, z, k_x, k_y, k_z)

  Phase-space **cell**: $\delta x \delta y \delta z \delta k_x \delta k_y \delta k_z = h^3$

- **Occupation number** in each phase-space cell

$$f = \frac{1}{e^{\beta(\varepsilon-\mu)} - 1}$$

— for photons/phonon: $\varepsilon = ck = \hbar\omega, \mu = 0$

— for massive bosons: $\varepsilon = k^2 / 2m, \mu \leq 0$

- Bose-Einstein condensation

$T < T_c : \ f_{k=0} / N > 0$

A **macroscopic** number of atoms **condense** into the quantum state of the lowest momentum.

Thermal wavelength $\lambda_T \gg d$, BEC is a matter field $\psi(\vec{r}) = |\psi(\vec{r})| e^{i\Phi(\vec{r})}$

- For low T, one has $\mu \to 0^-$ and $f \gg 1$ for small-k motions, the classical matter-field description is valid; No BEC (macroscopic occupation) or superfluidity (topological order) is requested.

Back to Einstein at 1924-1925

Quantentheorie des einatomigen idealen Gases.

Von A. Einstein.

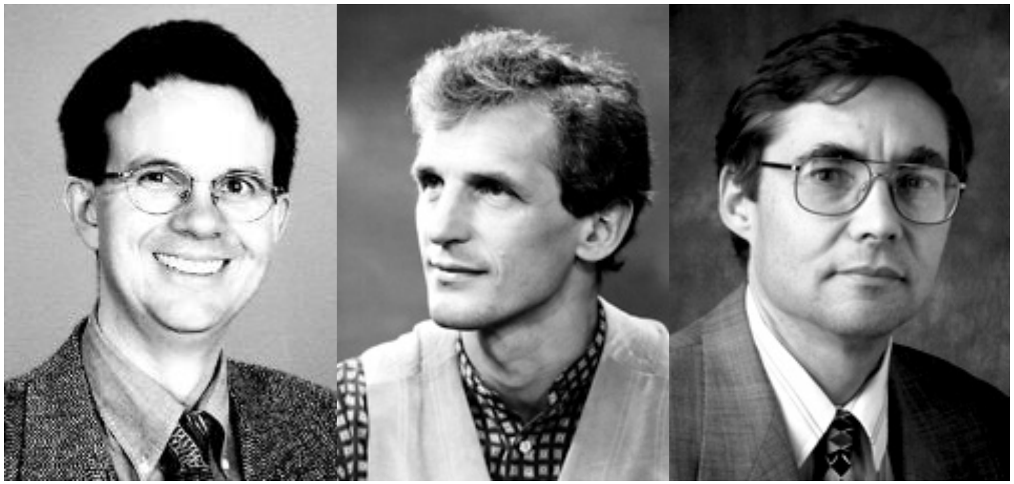
Eine von willkürlichen Ansätzen freie Quantentheorie des einatomigen idealen Gases existiert bis heute noch nicht. Diese Lücke soll im folgenden ausgefüllt werden auf Grund einer neuen, von Hrn. D. Bose erdachten Betrachtungsweise, auf welche dieser Autor eine höchst beachtenswerte Ableitung der Planckschen Strahlungsformel gegründet hat[1].

"*One can assign a scalar* **wave** *field to such a gas… It looks like there would be an* **undulatory field** *associated with each phenomenon of motion, just like the optical undulatory field is associated with the motion of light quanta.*"

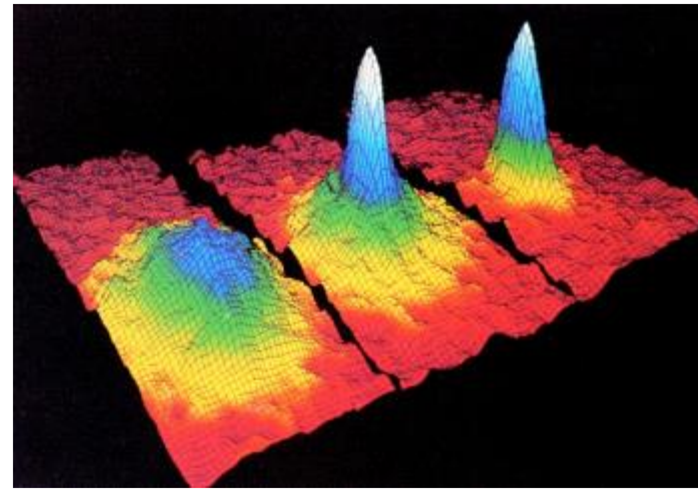
- ✓ Einstein's classical-field idea for quantum gas was forgotten for tens of years, probably due to the advent of rigorous quantum mechanics

Back to Einstein at 1924-1925

2001 Nobel Prize in Physics



E.A. Cornell W. Ketterle C.E. Wieman



^{87}Rb

"for the achievement of **Bose-Einstein condensation** *in dilute gases of alkali atoms, and for early* **fundamental studies** *of the properties of the condensates"*

- ✓ [BEC cartoon](#)

What is Superfludity?

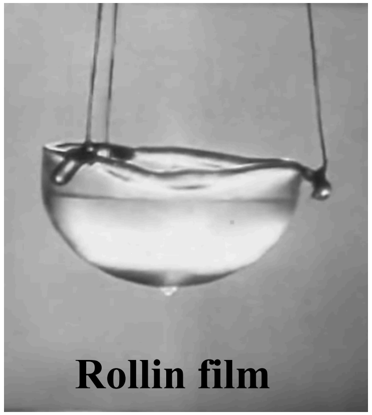
Relation of superfluidity to BEC

- 1938, London argues that Einstein's condensation does exist and suggests that the lambda-transition is related to BEC.
- 1938, Tisza introduces the two-fluid concept, with the conjecture that the superfluid component is nothing but BEC.
- 1955, Penrose introduces the concept of "off-diagonal long-range order"

$$\rho(\mathbf{r}_1,\mathbf{r}_2) = \left\langle \psi^\dagger(\mathbf{r}_2)\,\psi(\mathbf{r}_1) \right\rangle \to \psi_0^*(\mathbf{r}_2)\,\psi_0(\mathbf{r}_1) \quad \text{at} \quad \left|\mathbf{r}_1 - \mathbf{r}_2\right| \to \infty$$

"Paradox" for 2D finite-temperature system,

- ✓ Mermin-Wegner theorem states that no long-range order occurs
- ✓ Experiments show no doubt for the existence of superfluidity



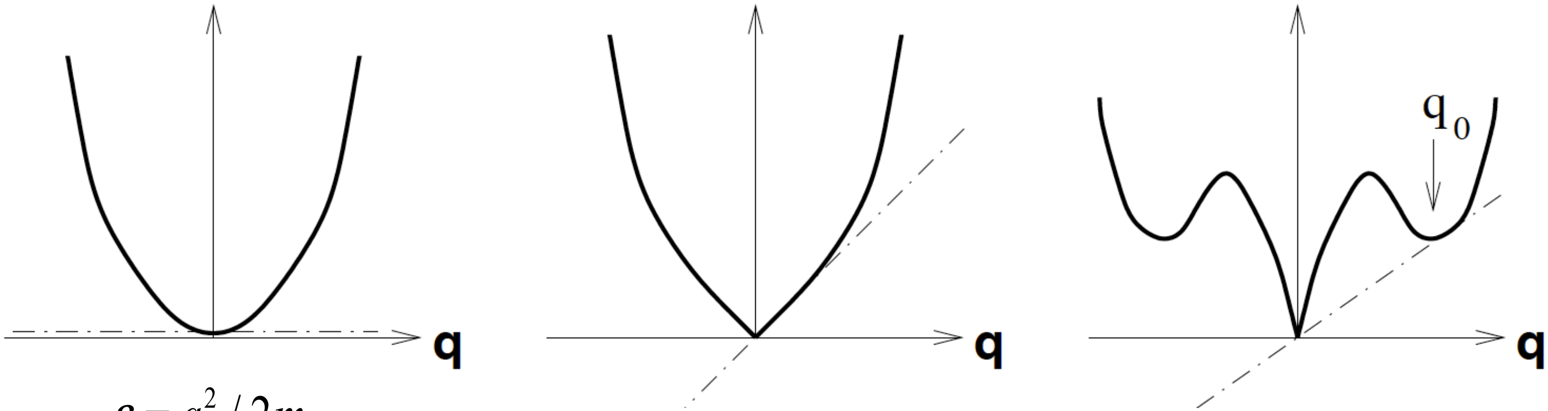
BEC is a sufficient but not a necessary condition for superfluidity

What is Superfludity?

Relation of superfluidity to elementary excitation spectrum

- **1941, Landau's critical velocity**

  dispersion ε(**q**):



$\varepsilon = q^2/2m$ — free particle in ideal BEC

$\varepsilon = cq$ — phonon in dilute quantum gas

phonon/roton in ^{4}He liquid

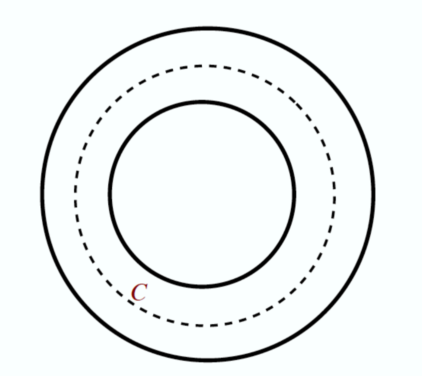
- not sufficient: finite-T superfluidity
- not necessary: ^{3}He-A, "supersolidity"

Landau's criterion for superfluidity is instructive, however, it is **neither** a sufficient **nor** a necessary condition

What is Superfludity?

Superfluidity: emergent topological order

$$\oint_C \vec{v}_s \cdot d\vec{l} = \gamma \oint_C d\vec{l} \cdot \nabla\Phi = 2\pi\gamma \times \text{integer}$$

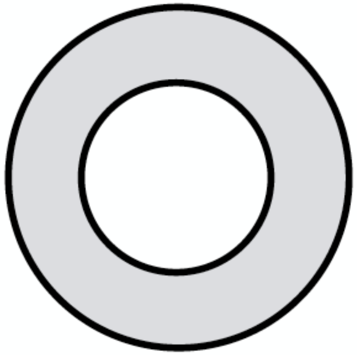


- ✓ BKT phase transition in 2D
  (Berezinskii 1971, Kosterlitz-Thouless 1972, Nelson Kosterlitz 1976)

- ✓ direct experimental evidence of topological order in 2D
  (Telschow and Hallock, 1976)

The value of the persistent current in an annulus is changed by many orders of magnitude—by changing the thickness of the superfluid 4He film (as a response to the chemical potential shared by the film with the vapor in the bulk).

In contrast to a dramatic change in the value of the net persistent current, the velocity of the superflow stays intact.



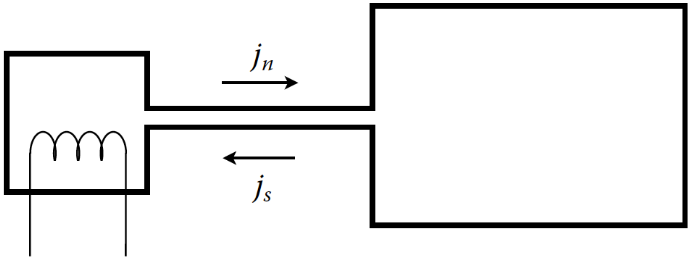
What is Superfludity?

Thermomechanical effect of superfluidity

Basic equation

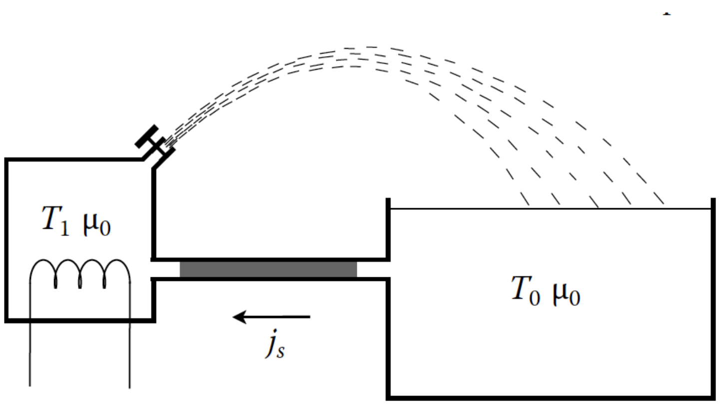
$$\dot{\mathbf{v}}_s = -\nabla\tilde{\mu} \qquad (v_n = 0) \qquad \tilde{\mu} = \mu / m$$

- ✓ Super-heat-conductivity
- ✓ Fountain effect
- ✓ Escaping from a Dewar



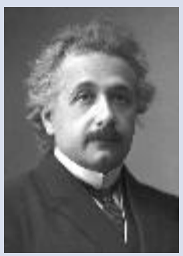
Counterflow: $T\uparrow,\ \mu(T)\downarrow$

- The superfluid component is of zero entropy
- Heat is transported mechanically by flow of normal component



Fountain effect: $T\uparrow,\ \mu(T)\downarrow$

Nobel Prizes in Physics related to Superfluidity

1913		Heike Kamerlingh Onnes	 Netherlands	for his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium.
1921		Albert Einstein	 Germany	for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect
1962		Lev Davidovich Landau	 Soviet Union	for his pioneering theories for condensed matter, especially liquid helium

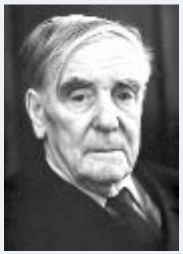
Nobel Prizes in Physics related to Superfluidity

1972		John Bardeen	 USA	for their jointly developed theory of superconductivity, usually called the BCS-theory
		Leon N. Cooper	 USA	
		J. Robert Schrieffer	 USA	

Nobel Prizes in Physics related to Superfluidity

1973		Leo Esaki	 Japan	for their experimental discoveries regarding tunneling phenomena in semiconductors and superconductors, respectively
		Ivar Giaever	  Norway, USA	
		Brian David Josephson	 UK	for his theoretical predictions of the properties of a supercurrent through a tunnel barrier, in particular those phenomena which are generally known as the Josephson effects

Nobel Prizes in Physics related to Superfluidity

1978		Pyotr Leonidovich Kapitsa	 Soviet Union	for his basic inventions and discoveries in the area of low-temperature physics
1987		J. Georg Bednorz	 Germany	for their important break-through in the discovery of superconductivity in ceramic materials
		K. Alexander Müller	 Switzerland	

Nobel Prizes in Physics related to Superfluidity

1996		David M. Lee	 USA	for their discovery of superfluidity in helium-3
		Douglas D. Osheroff	 USA	
		Robert C. Richardson	 USA	

Nobel Prizes in Physics related to Superfluidity

2003		Alexei A. Abrikosov	  Russia, USA	for pioneering contributions to the theory of superconductors and superfluids
		Vitaly L. Ginsburg	 Soviet Union	
		Anthony J. Leggett	 UK	

Nobel Prizes in Physics related to Superfluidity

2016		David J. Thouless	 UK	for theoretical discoveries of topological phase transitions and topological phases of matter
		F. Duncan M. Haldane	 USA	
		J. Michael Kosterlitz	 USA	

Summary

Take-home message

- Superfluid is the property of a complex-valued matter field, which is inherited by quantum bosonic systems
- BKT phase transition is driven by topological defects—vortices

Lecture note is prepared together with

Chun-Jiong Huang
（黄春炯）

Ran Zhao
（赵然）

Thank You

Results of 2D and 3D XY model

Magnetic susceptibility $\chi(T) = \left.\frac{\partial M(T)}{\partial h}\right|_{h=0}$