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# 离子阱物理简介

林毅恒

**Spin.ustc.edu.cn**

**2020/10**

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## 林毅恒 简介:



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•2015年美国科罗拉多大学博尔德分校 (CU Boulder) 物理学博士  
离子阱量子信息处理实验研究

•2018年回校，加入杜江峰院士带领的中科院微观磁共振重点实验室。

•正在搭建离子阱量子信息处理实验平台

•远期目标是实验演示多离子的量子模拟和量子计算

•QQ: 326120109

•实验室主页: [spin.ustc.edu.cn](http://spin.ustc.edu.cn)

# 内容

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背景介绍

自组织的离子链

光与离子的相互作用

量子计算和量子模拟

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# 量子信息：比特

信息储存在两个量子态和相位和布局上

$$\Psi = C_0|0\rangle + C_1|1\rangle$$

归一化：  
 $\sum_{i,j\dots} |C_{i,j\dots}|^2 = 1$

两比特纯态需要四个复数完全表示，自由度为6

$$\Psi = C_{00}|00\rangle + C_{01}|01\rangle + C_{10}|10\rangle + C_{11}|11\rangle$$

N个量子比特需要 $2^{N+1} - 2$ 个实数表示

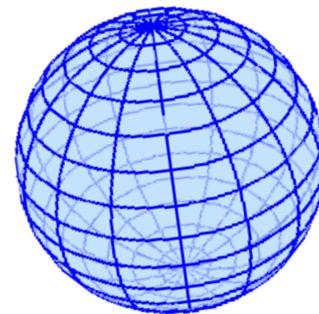
N个经典比特需要N个实数表示

对于一个经典计算机完全模拟量子体系需要指数增长的资源

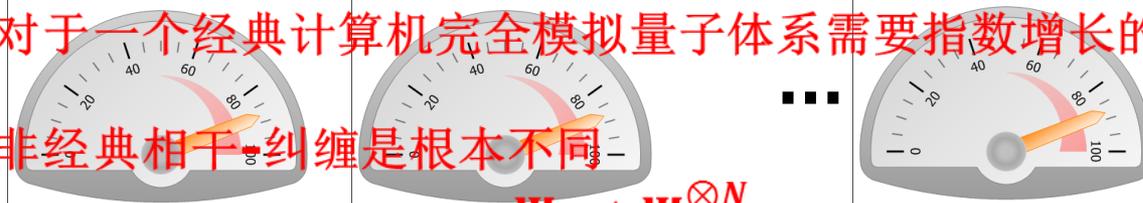
非经典相干-纠缠是根本不同

$$\Psi_N \neq \Psi_i^{\otimes N}$$

$|0\rangle$



$|1\rangle$

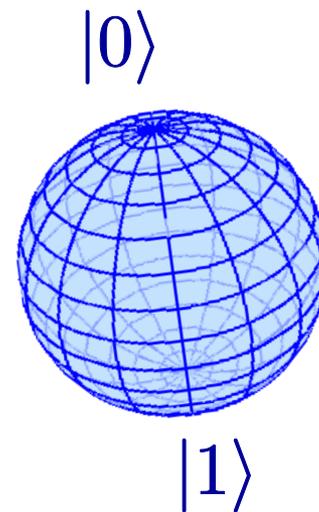


# 量子信息：比特

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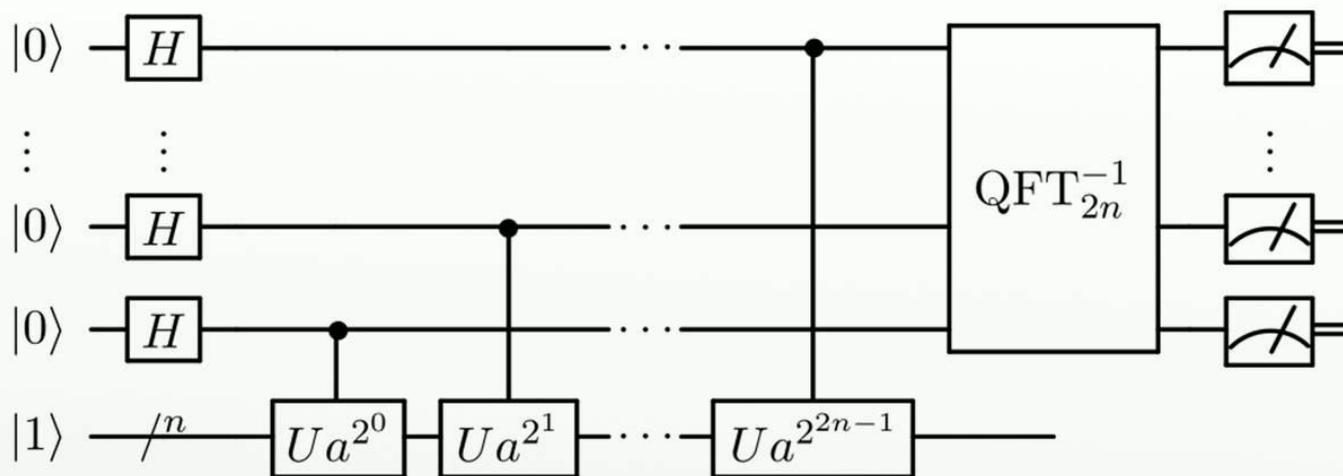
纠缠态例子

$$\Psi = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \neq \Psi_{\text{qubit 1}} \otimes \Psi_{\text{qubit 2}}$$

非经典相干-纠缠很关键

# 典型的量子计算过程

## Shor's algorithm



[https://en.wikipedia.org/wiki/File:Shor's\\_algorithm.svg](https://en.wikipedia.org/wiki/File:Shor's_algorithm.svg)

初态 单比特门 多比特操作

读出

# 量子模拟



Richard P. Feynman

**Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy**

实验得出结果

哈密顿量，待测物理量

待模拟问题

可能应用：求解分子基态，优化问题，人工智能，凝聚态模型.....

."Simulating Physics with Computers", *International Journal of Theoretical Physics*, volume 21, 1982, p. 467-488, at p. 486

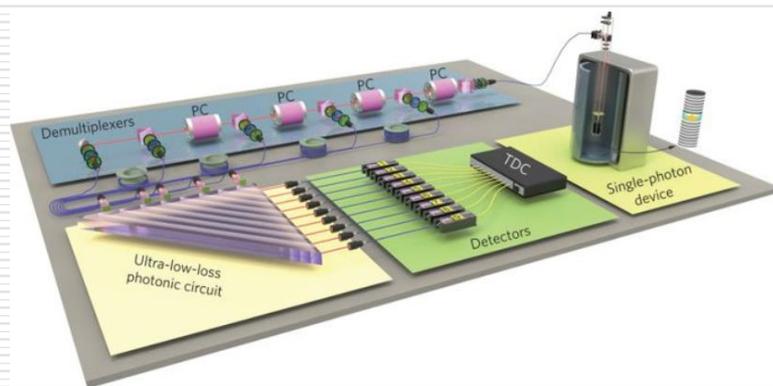
“...we never experiment with just one electron or atom or (small) molecule. In thought experiments, we sometimes assume that we do; this invariably entails ridiculous consequences ....”

我们从没有在一个电子，一个原子或一个（小）分子上做实验。在思想实验里，我们优势假设我们可以这样做；这总是有奇怪的后果.....

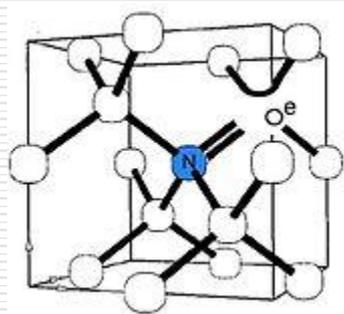
-- 薛定谔，1952



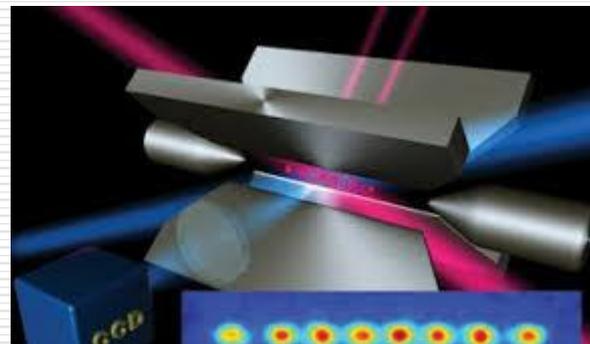
# 物理实现



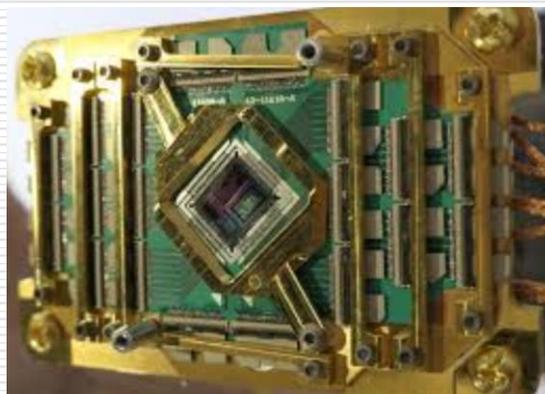
光子



固态量子点



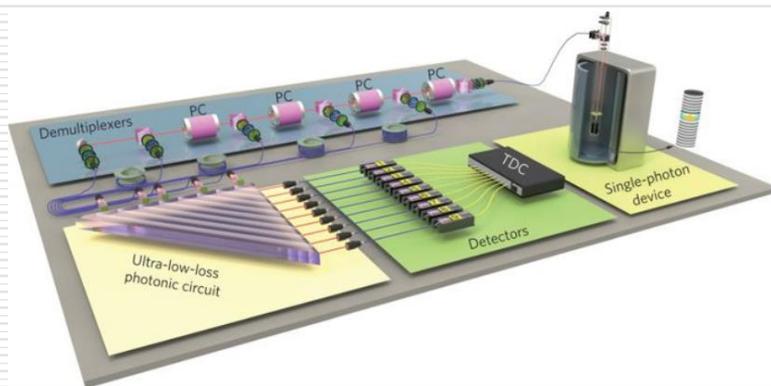
离子阱



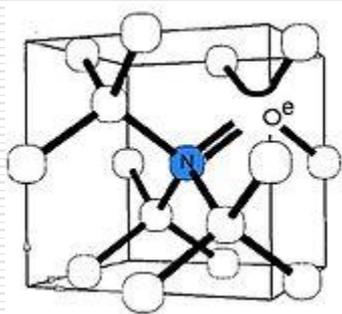
超导量子比特

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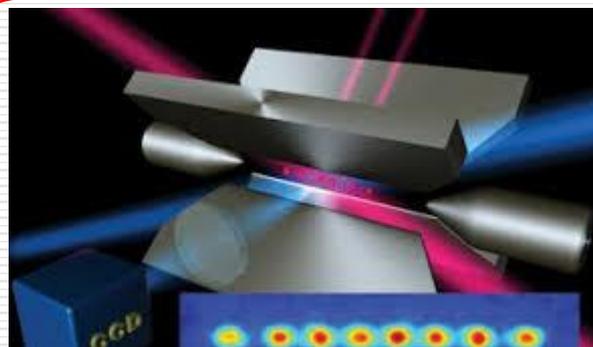
# 物理实现



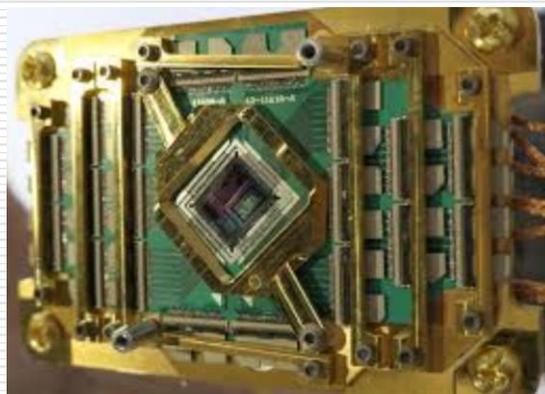
光子



固态量子点



离子阱



超导量子比特

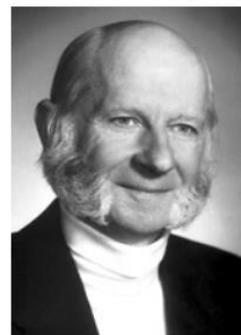
.....

# 历史沿革

□ 带电粒子的电磁束缚 (1960's) - 质量谱仪, 真空泵

□ 单电子束缚 (1973)

□ 单电子g因子精密测量 (1977)



Hans G. Dehmelt



Wolfgang Paul

1989 Nobel Prize

□ 激光冷却Mg离子低于室温 (1978)

□ 单个Ba离子束缚 (1980)

- 单独离子实验演示基本量子物理，发明离子性原子钟（1980's）
- 离子的激光两比特量子门（1995）
- Hg离子光钟精度超过Cs原子钟（2006）；Al离子钟演示10cm高度差的引力变化（2010）
- 两比特门精度 >90%(2003); >99%(2008), >99.9%(2016)

PRL 75, 4714, 1995; NATURE 422, 412, 2003; PRL 97, 020801, 2006;  
Nat.Phys 4, 463, 2008; PRL 117, 060505 (2016); PRL 117, 060504 (2016);  
Science 329, 1630, 2010

"for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems"

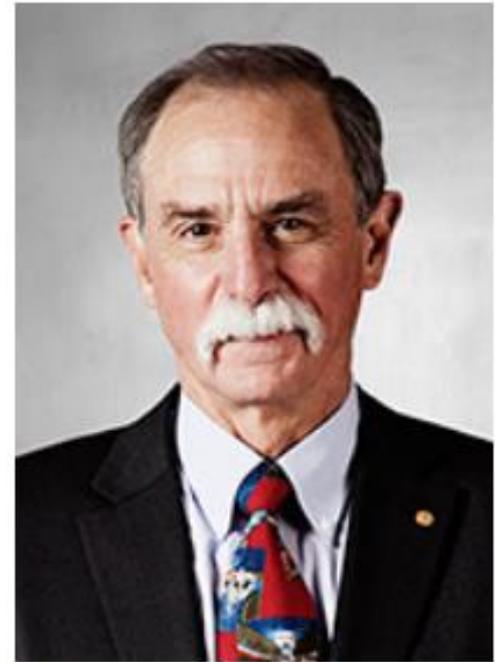


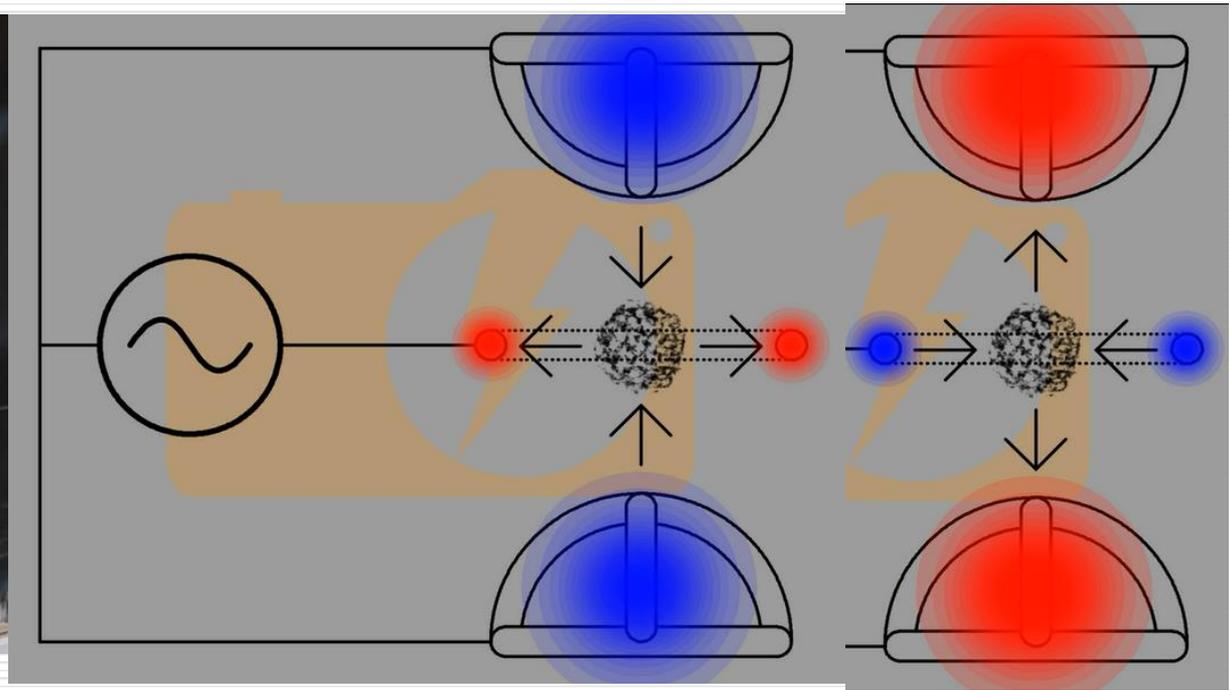
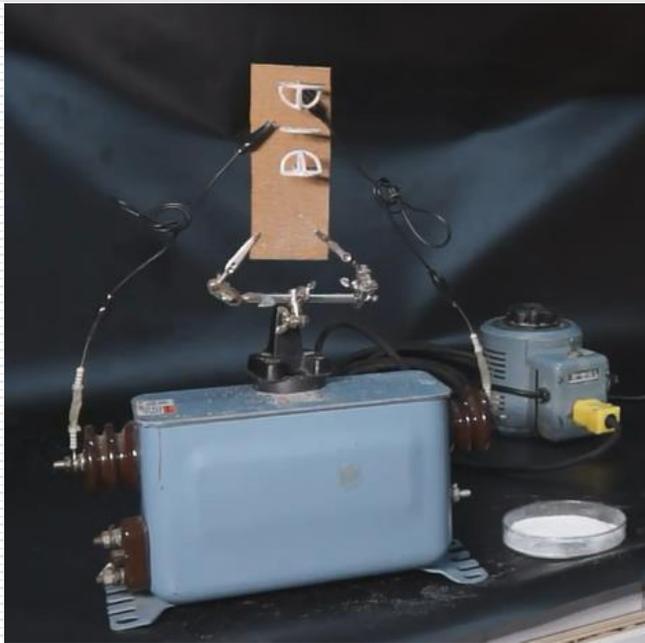
Photo: U. Montan

**David J. Wineland**

**单离子的测量和操控 (2012)**

# 车库实验：束缚带电的颗粒

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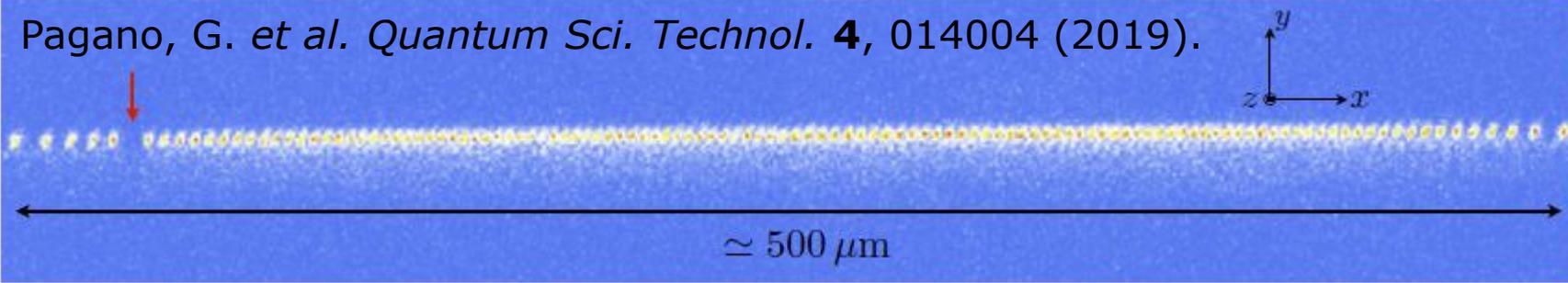
Credit: [youtube.com/watch?v=u\\_OlVz95\\_tw](https://www.youtube.com/watch?v=u_OlVz95_tw)

真空腔

透镜-收集离子  
荧光

电极

Pagano, G. et al. *Quantum Sci. Technol.* **4**, 014004 (2019).



激光

电离的原子-离子



$\sim 5 \mu\text{m}$

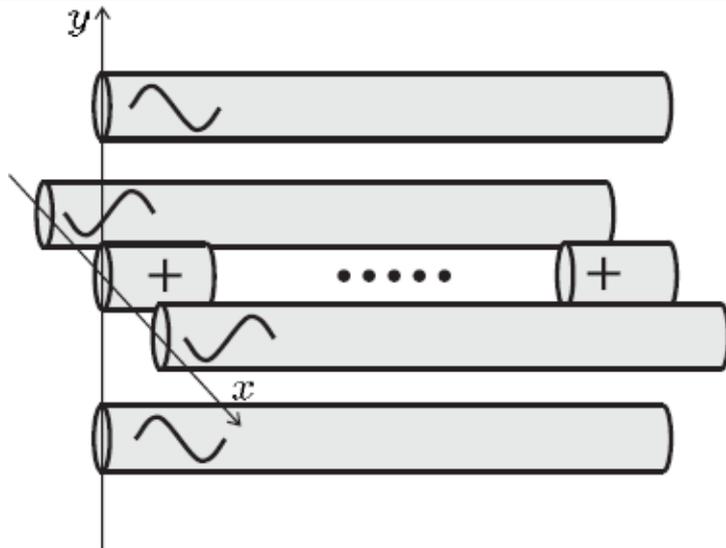
Credit: Monroe group

# 原理

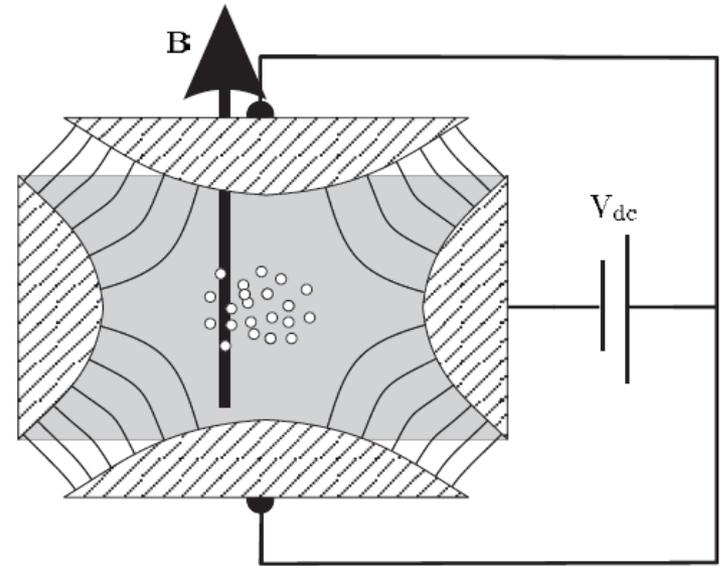
**Earnshaw's theorem:** a point charge acted on by electrostatic forces cannot rest in a stable equilibrium in an electric field.

交变电场

加静磁场



Linear Paul Trap



Penning Trap

# 束缚-运动平衡

通过微运动可以将不稳定平衡点转化成稳定平衡点

— — Mathieu方程



倒立单摆

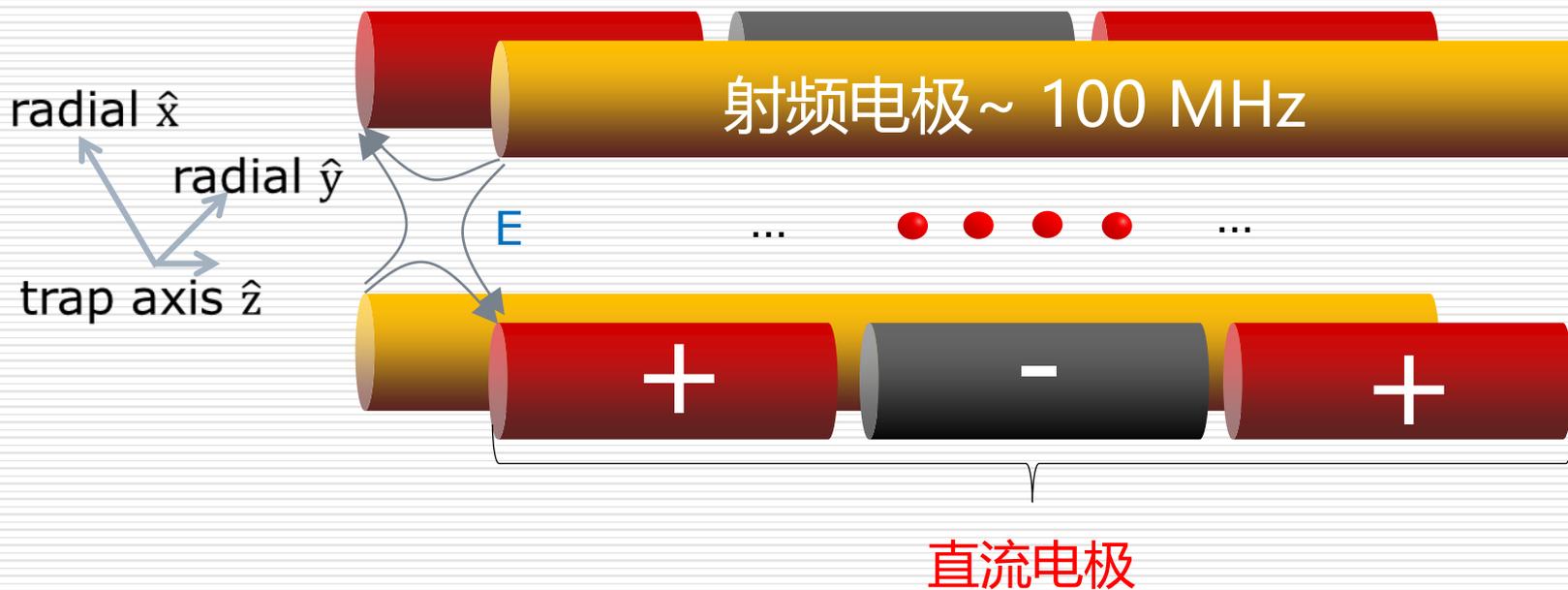


旋转马鞍

<https://www.youtube.com/watch?v=5oGYCxkgnHQ>

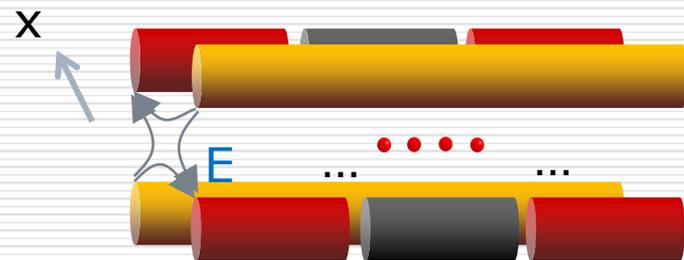
<https://www.youtube.com/watch?v=XTJznUkAmIY>

# 束缚-振动模式



雪茄形状的三维势阱，  
离子最低能量状态为一维链\*

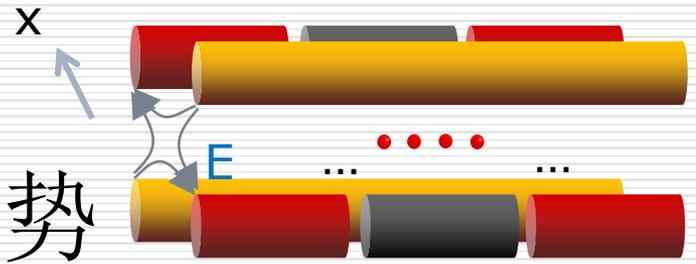
# 轴向束缚



轴向静电势  $qU(z)$

$$qU(z) = \frac{1}{2} m \omega_z z^2, \quad \longrightarrow$$

$$\omega_z = \sqrt{\frac{q}{m} \frac{d^2 U(z)}{dz^2}}$$



# 径向束缚的简化情况-赝势

假设均匀的震荡电场  $E = E_0 \cos \Omega t$ ,

带电粒子的运动速度  $v = \frac{q}{m\Omega} E_0 \sin \Omega t$ , ( $m\dot{v} = qE$ )

动能  $KE = \frac{q^2}{2m\Omega^2} E_0^2 \sin^2 \Omega t$ , ( $KE = \frac{1}{2} m v^2$ )

于是  $KE = \frac{q^2}{4m\Omega^2} E_0^2 (1 - \cos 2\Omega t)$

假设  $E_0$  有微小的空间变化, 在震荡的一周期内平均得到

$$KE(x) = \frac{q^2}{4m\Omega^2} E_0^2(x) \quad \frac{1}{2} m \omega_x^2 x^2 = KE(x) \quad \omega_x = \frac{q}{m\Omega} \sqrt{\frac{d^2 E_0^2(x)}{dx^2}}$$

# 一般的情况-微运动

运动方程整理为Mathieu形式

$$\frac{d^2x}{d\zeta^2} + \left[ a_x + 2q_x \cos(2\zeta) \right] x = 0$$

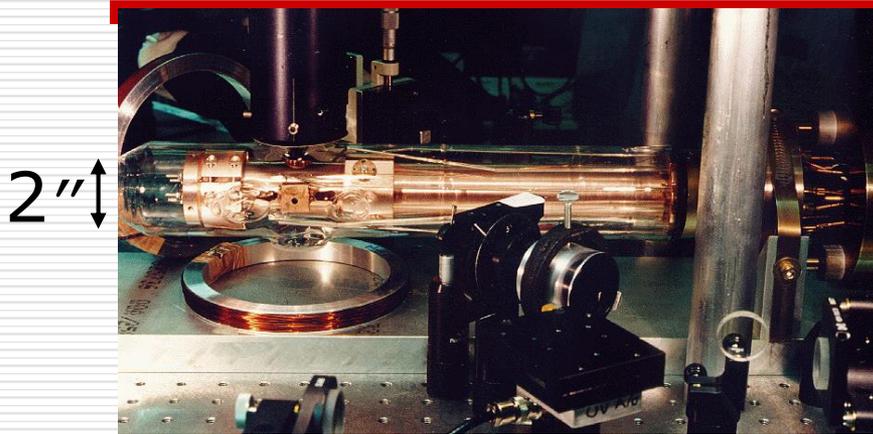
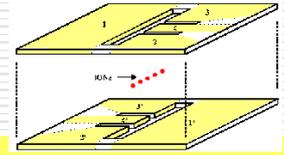
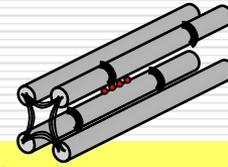
其中  $\zeta = \Omega t/2$ , 一般  $a_x < q_x^2 \ll 1$ , (弹簧系数  $q \frac{dE}{dx} \ll m\Omega^2$ )。近似解可以表示为

$$u_i(t) = A_i \left( \cos(\omega_i t + \phi_i) \left[ 1 + \frac{q_i}{2} \cos(\Omega_T t) + \frac{q_i^2}{32} \cos(2\Omega_T t) \right] + \beta_i \sin(\omega_i t + \phi_i) \sin(\Omega_T t) \right)$$

久期  
振动频率

振动受外加  
交流频率的  
调制-微运动

# Ion Traps - initial micromachining:



典型值:

DC:  $U_0 \approx 10 \text{ V}$

RF:  $V_0 \approx 200 \text{ V}$

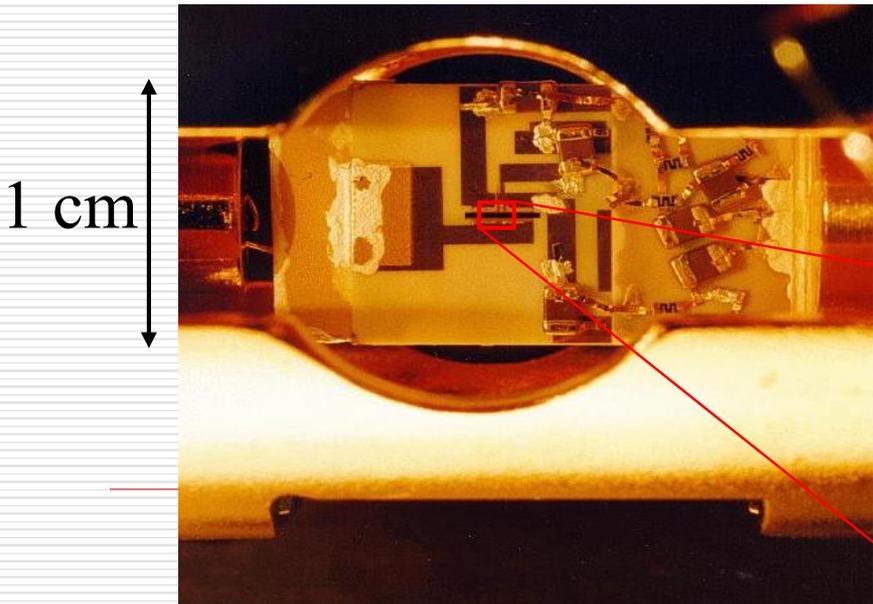
$\Omega \approx 80 \text{ MHz}$

$\omega_x \approx 12 \text{ MHz}$

pressure  $< 2 \times 10^{-11} \text{ torr}$

single ion lifetime:  $\sim h$

At room temperature



0.2 mm



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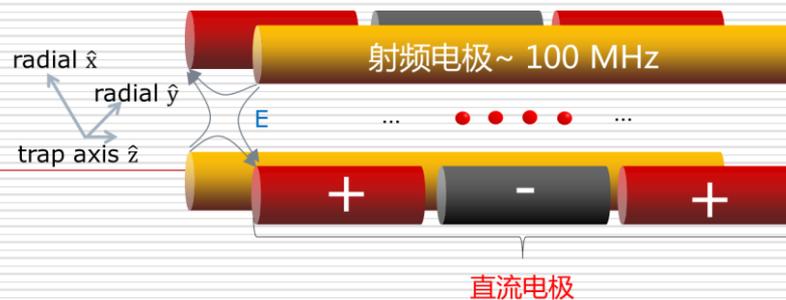
# 束缚-求解振动模式

单离子束缚

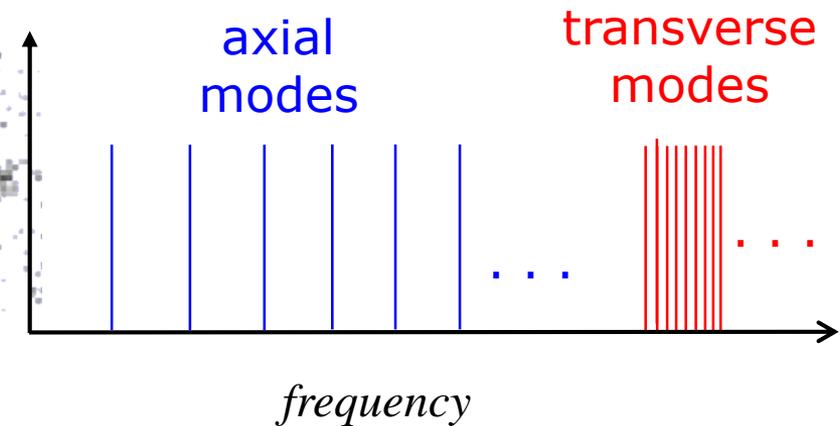
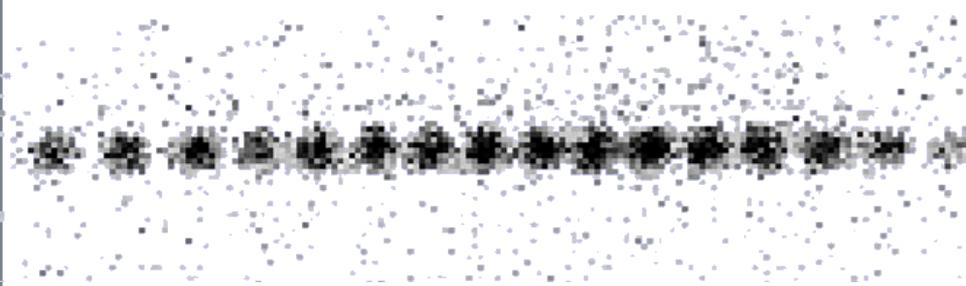
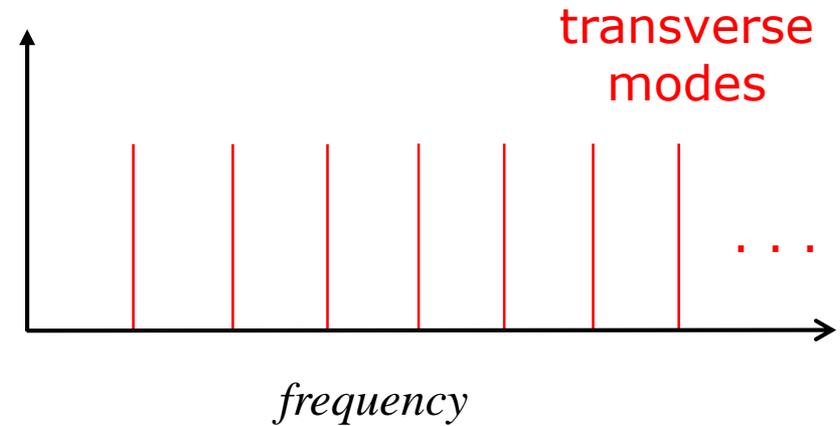
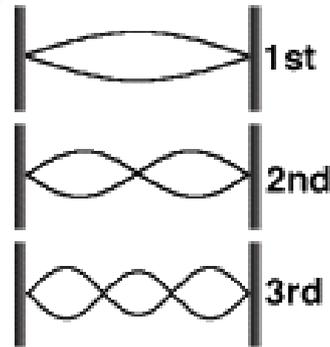
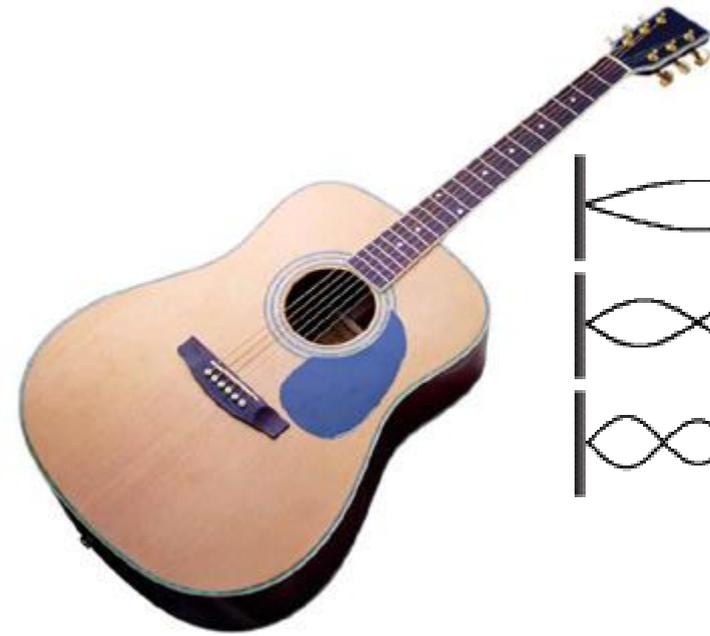
$$V = \sum_i \frac{1}{2} m \omega_z^2 z_i^2 + \sum_i \frac{1}{2} m \omega_x^2 x_i^2 + \sum_i \frac{1}{2} m \omega_y^2 y_i^2 + \sum_{i>j} \frac{e^2}{4\pi\epsilon_0 \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}}$$

离子间相互作用

思路：首先求解平衡位置，再展开求简正模式



# Aside: transverse Modes of an atom chain



S.-L. Zhu et al., Phys. Rev. Lett. 97, 050505 (2006)

A. Serafini et al., New J. Phys. 11, 023007 (2009)

Credit: C. Monroe

# 束縛-振動模式

