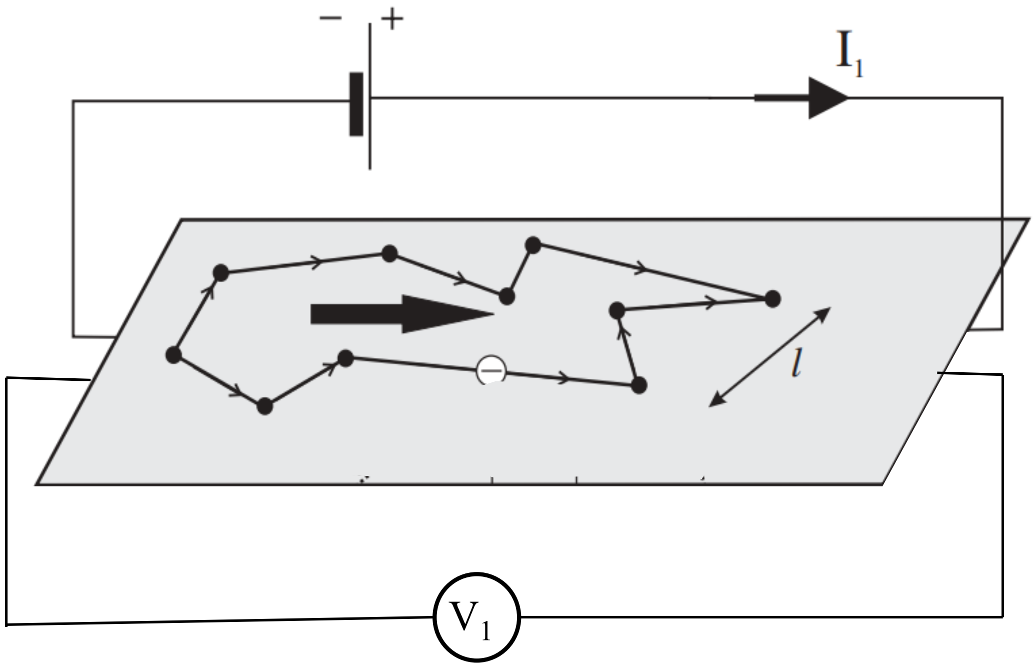
近代物理专题讲座II

层间拖拽输运效应

Changgan Zeng (曾长淦)
http://staff.ustc.edu.cn/~cgzeng
ICQD/Department of Physics
University of Science and Technology of China

Nov. 24, 2022

Transport in solids



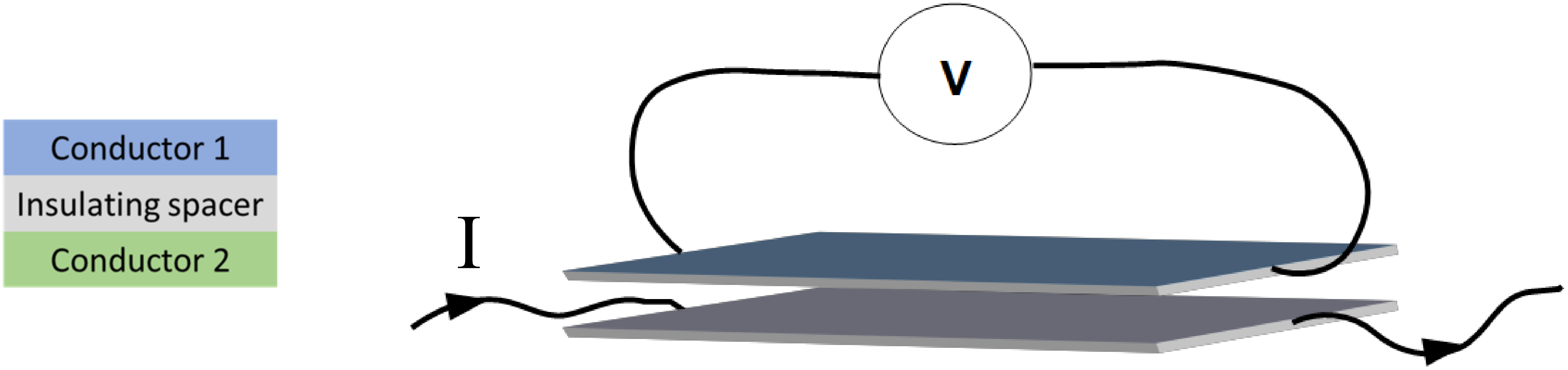
Resistance: $R = V_1 / I_1$

Frictional drag

Drag between two closely spaced but electrically isolated conductors

M.B Pogrebinskii, Sov. Phys. Semicond. 11,372 (1977)

P. M. Price, Physica 117B, 750 (1983)

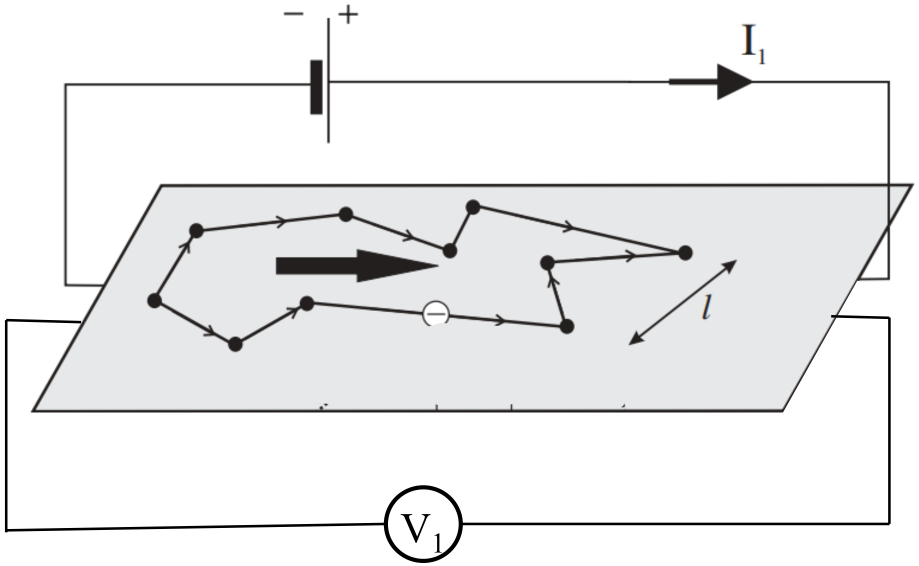


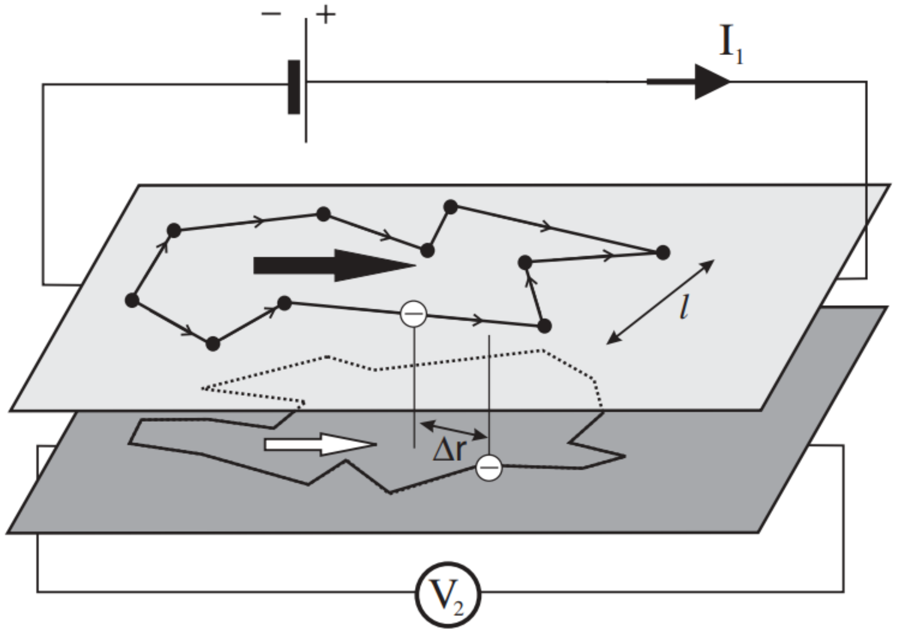
Electric current flowing in one layer (active layer)

↓ **Mutual friction**

An open-circuit voltage (or a closed-circuit current) in the other layer (passive layer)

Comparison





☐ Intra-layer transport

Resistance: $R = V_1/I_1$

Disorder, phonon, etc

Measuring the momentum lost

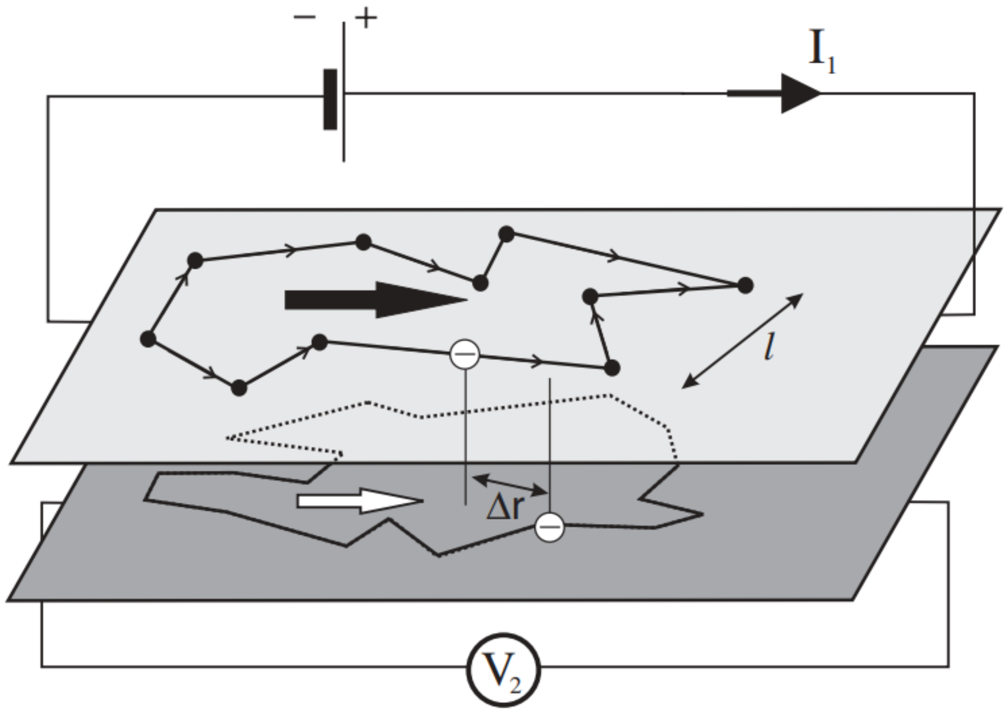
☐ Inter-layer drag effect

Drag resistance: $R_{\text{drag}} = V_2/I_1$

Inter-layer long-range scattering

Measuring the momentum gained

Mechanism: Coulomb scattering



B. N. Narozhny *et al.*, Rev. Mod. Phys. 88, 025003 (2016)

Fermi liquid regime （$E_F >> k_BT$ and $k_Fd >> 1$）

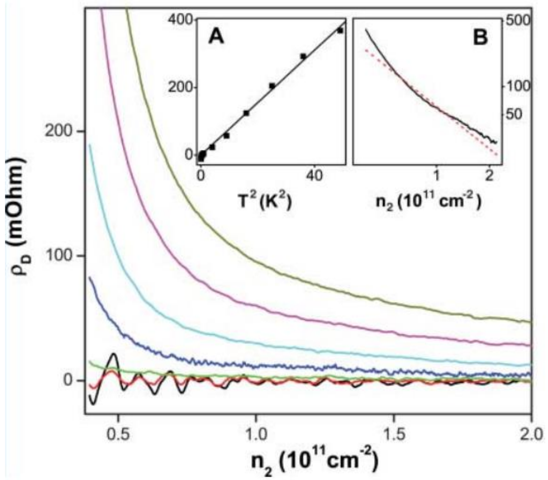
(weak coupling limit)

- Magnitude of R_{drag}

$$R_{\text{drag}} \propto T^2/(n_1{}^{3/2} n_2{}^{3/2} d^4)$$

- Polarity of R_{drag}

electron/hole: **positive**
electron/electron or hole/hole: **negative**



A. S. Price *et al.*, Science 316, 99 (2007)

Interlayer long-range Coulomb interaction between carriers

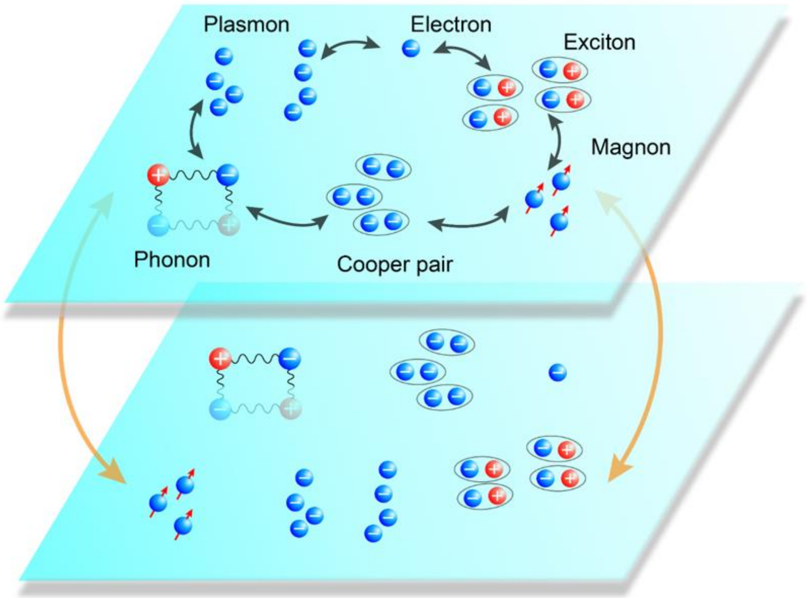
➡ **Inter-layer momentum and energy transfer**

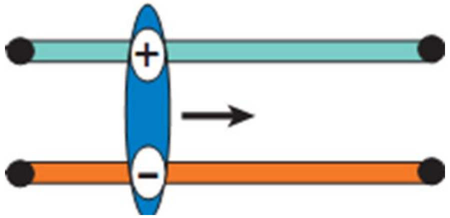
Rich physics in drag effect

Other mechanisms

- Phonon mediation
- Plasmon mediation
- Magnetic interaction (vortices in superconductors)

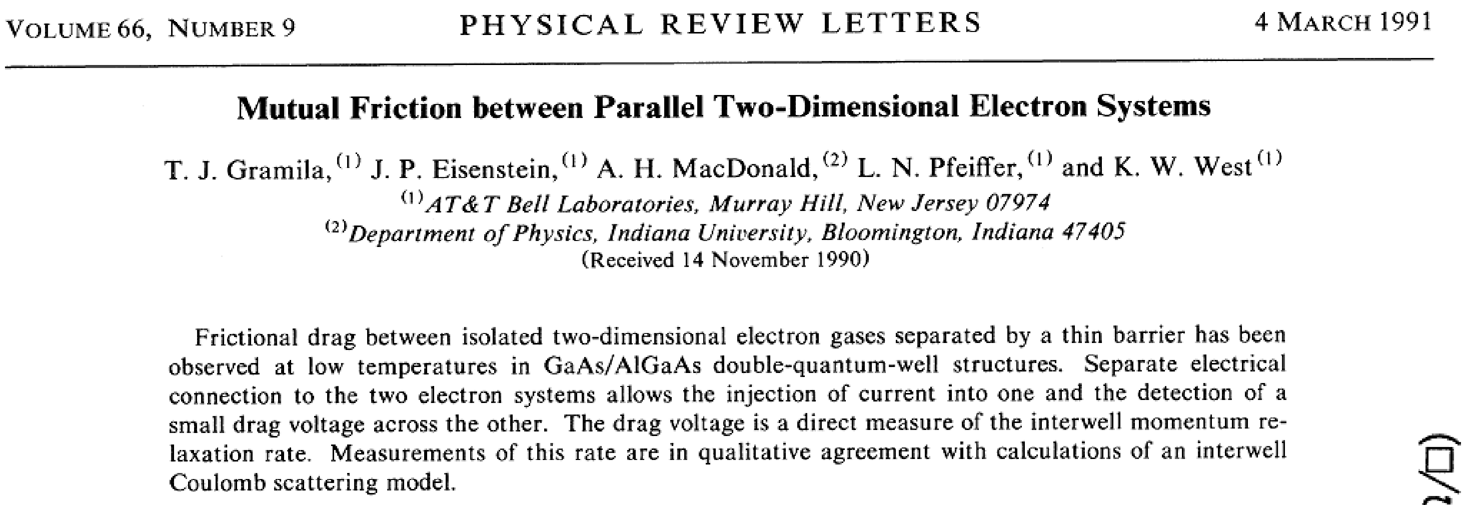
Drag measurements: an unique toolbox in condensed matter physics





- Probe to detect the properties of the constituent layers
- Interlayer quasiparticle interactions
- Novel drag effect & inter-layer correlated states

First observation of drag effect



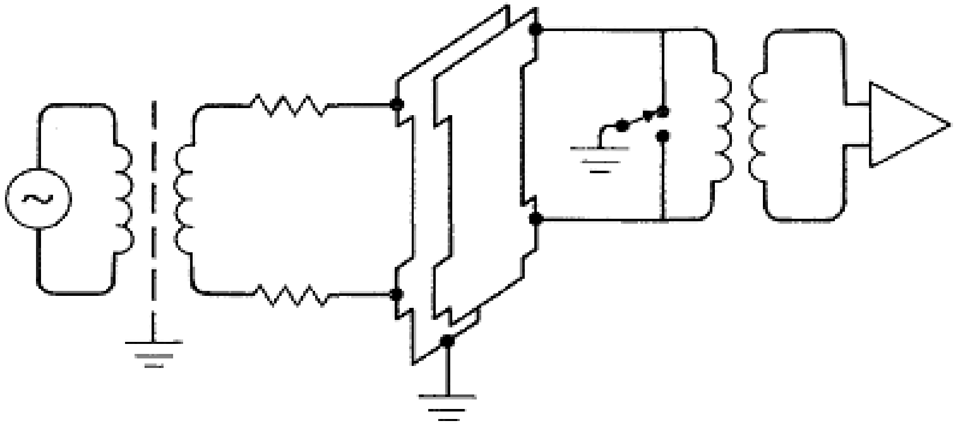
VOLUME 66, NUMBER 9 PHYSICAL REVIEW LETTERS 4 MARCH 1991

Mutual Friction between Parallel Two-Dimensional Electron Systems

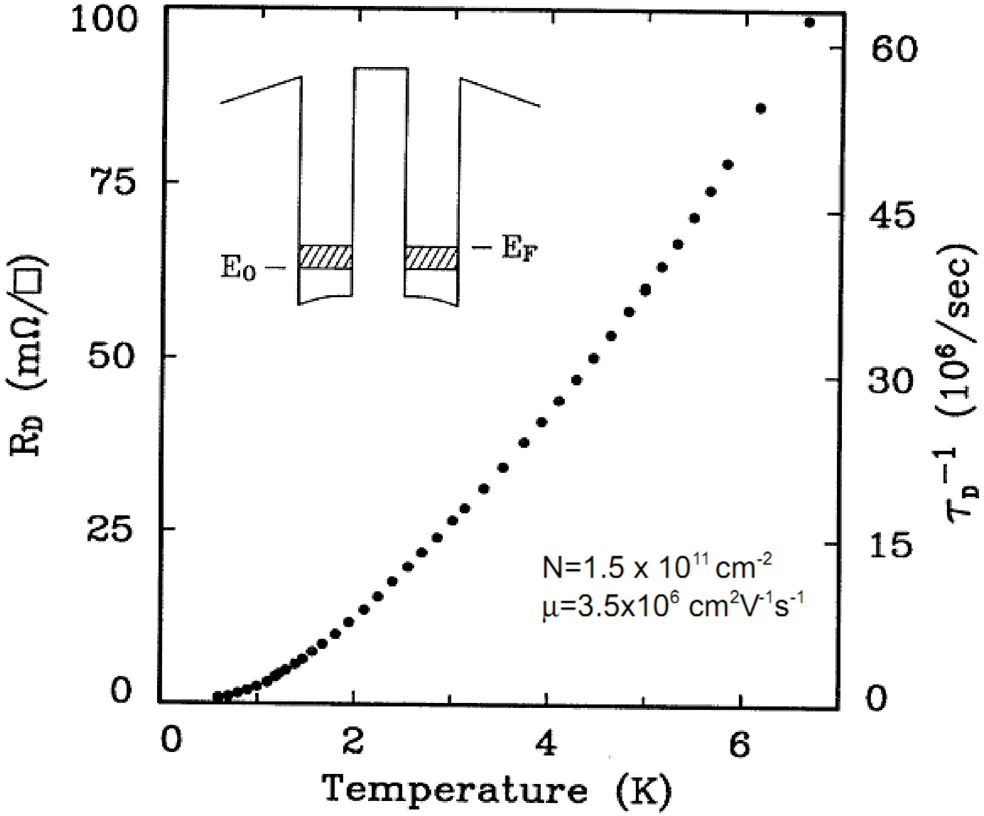
T. J. Gramila,(1) J. P. Eisenstein,(1) A. H. MacDonald,(2) L. N. Pfeiffer,(1) and K. W. West(1)
(1)*AT&T Bell Laboratories, Murray Hill, New Jersey 07974*
(2)*Department of Physics, Indiana University, Bloomington, Indiana 47405*

(Received 14 November 1990)

Frictional drag between isolated two-dimensional electron gases separated by a thin barrier has been observed at low temperatures in GaAs/AlGaAs double-quantum-well structures. Separate electrical connection to the two electron systems allows the injection of current into one and the detection of a small drag voltage across the other. The drag voltage is a direct measure of the interwell momentum relaxation rate. Measurements of this rate are in qualitative agreement with calculations of an interwell Coulomb scattering model.



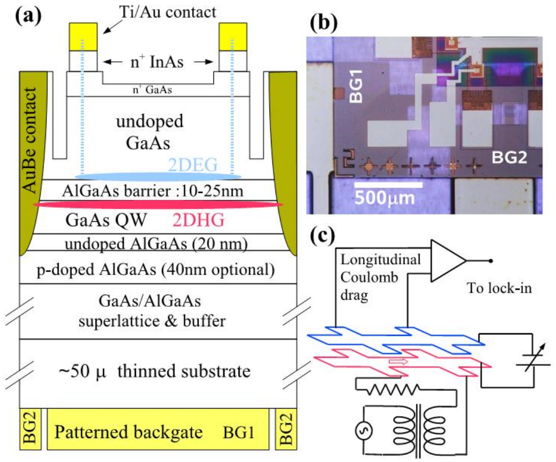
Interlayer spacing: 17.5 nm

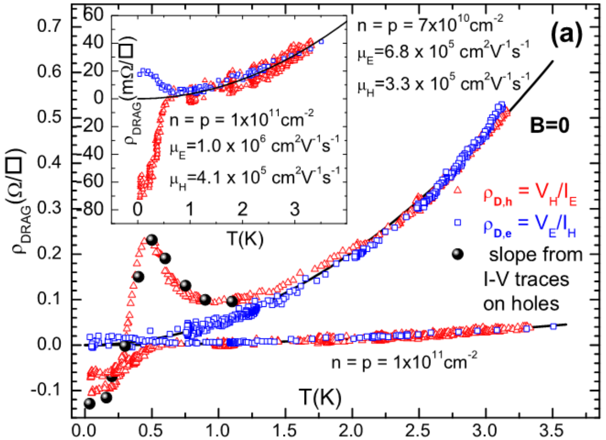


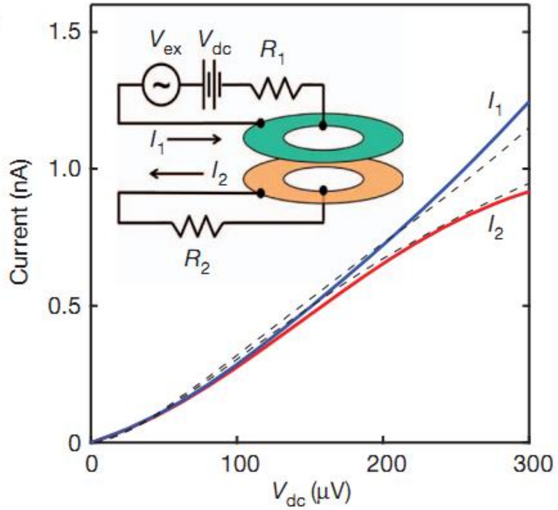
Consistent with the Coulomb scattering model

Typical systems

☐ GaAs/AlGaAs double-quantum well



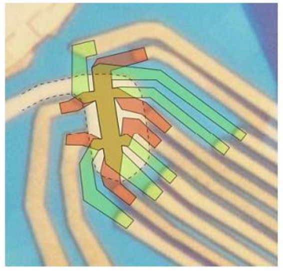


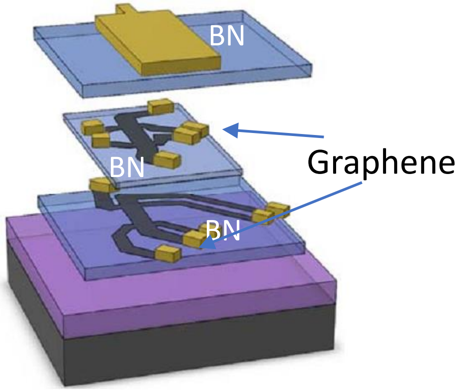


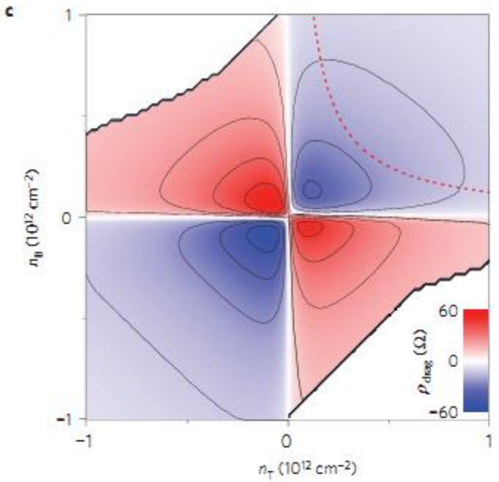
A. F. Croxall *et al.*, Phys. Rev. Lett. 101, 246801 (2008)

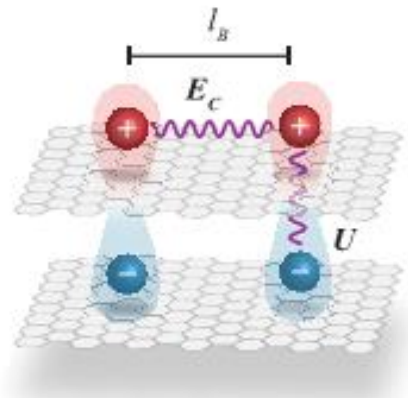
D. Namdi. *et al.*, Nature 488, 481(2012)

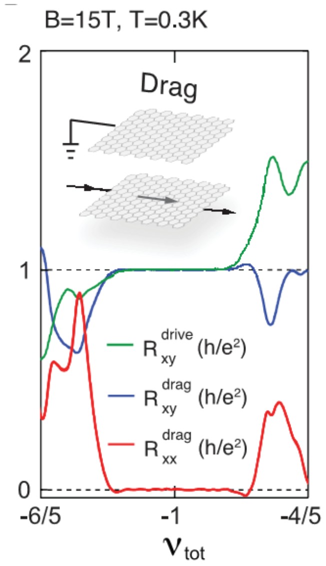
☐ Graphene based double-layer system









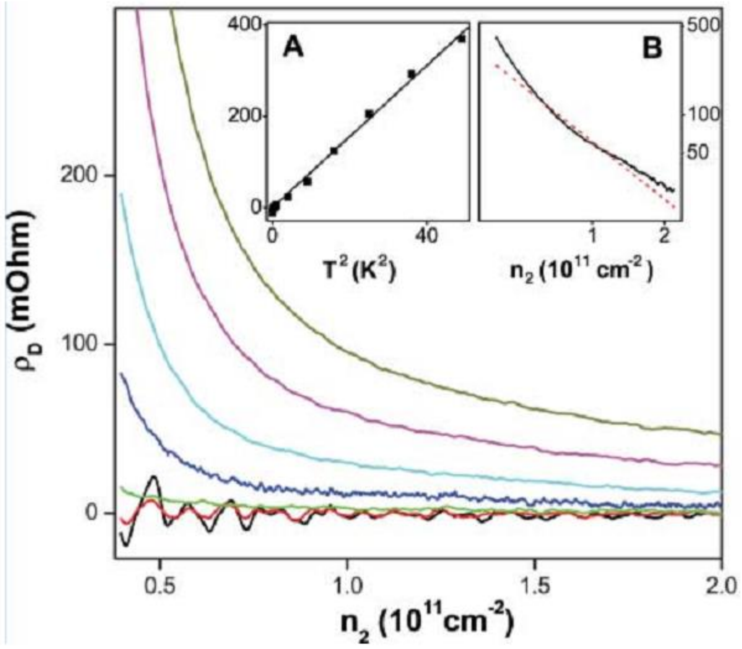


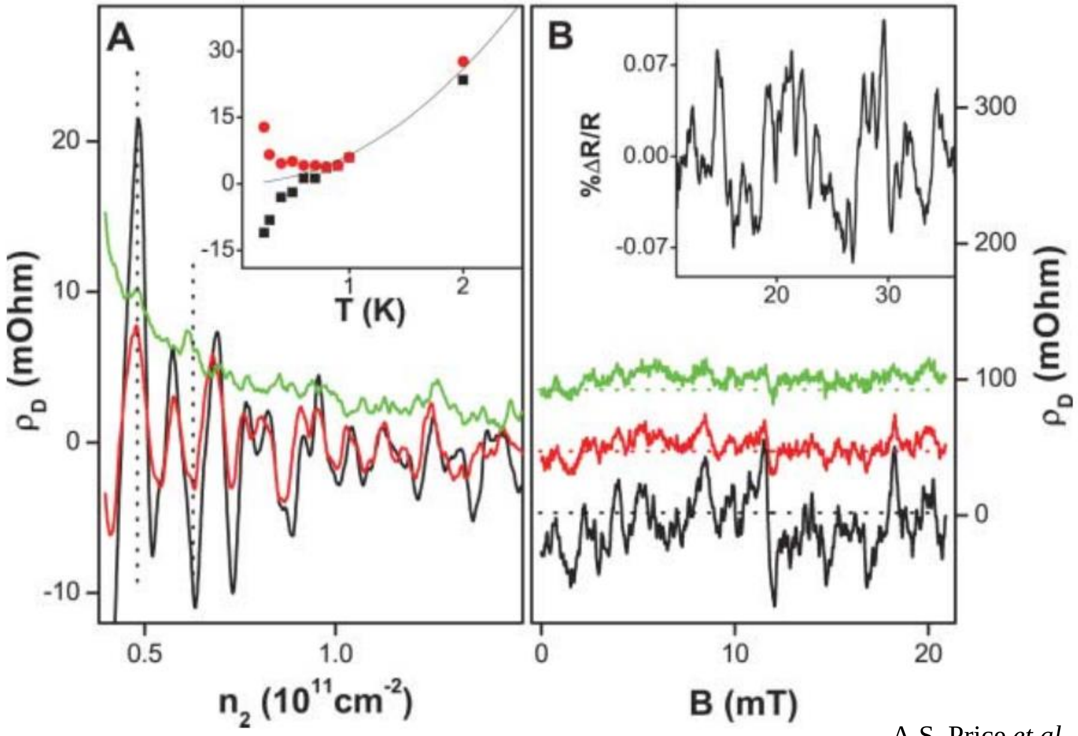
Stronger coupling & Higher tunability

R. V. Gorbachev *et al.*, Nat. Phys. 8, 896 (2012)

X. Liu *et al.*, Science 375, 205 (2022)

Interaction-enhanced quantum effects





A.S. Price *et al.*, Science 316, 99 (2007)

- **Giant drag fluctuations at low T**
- **Intra-layer interference effect + Inter-layer electron-electron interactions**

(short interaction distance → large momentum transfer → enhanced mesoscopic fluctuations)

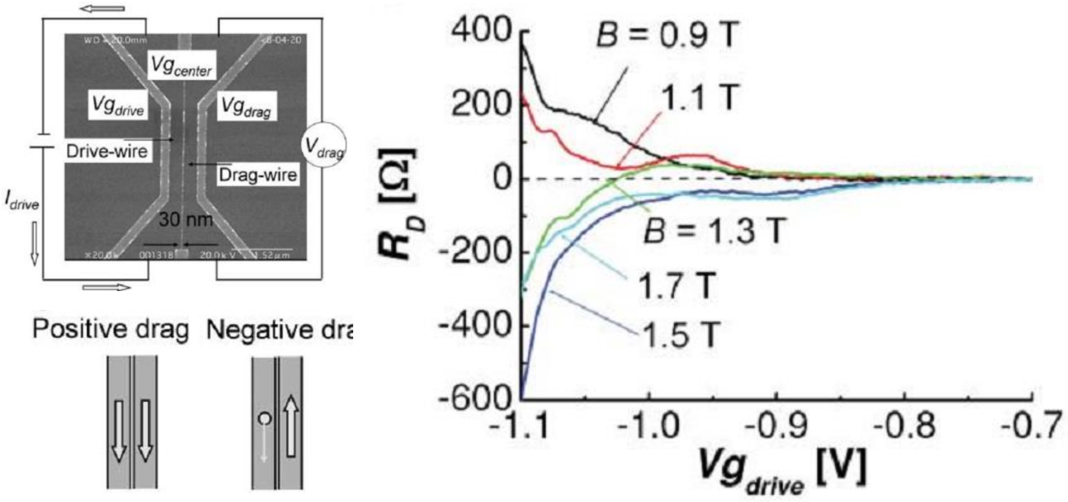
Probing new quantum states

Negative Coulomb Drag in a One-Dimensional Wire

Science 313, 204 (2006)

M. Yamamoto,[1,2] M. Stopa,[3] Y. Tokura,[4,5] Y. Hirayama,[2,4] S. Tarucha[1,5]

We observed negative Coulomb drag for parallel coupled quantum wires, in which electrons flow in the opposite directions between the wires. This only occurred under the conditions of strong correlation in the wires, that is, low density, high magnetic field, and low temperature, and cannot be addressed by a standard theory of momentum transfer. We propose a Coulomb drag model in which formation of a Wigner crystal state in the drag wire and a particle-like state in the drive wire is taken into account.



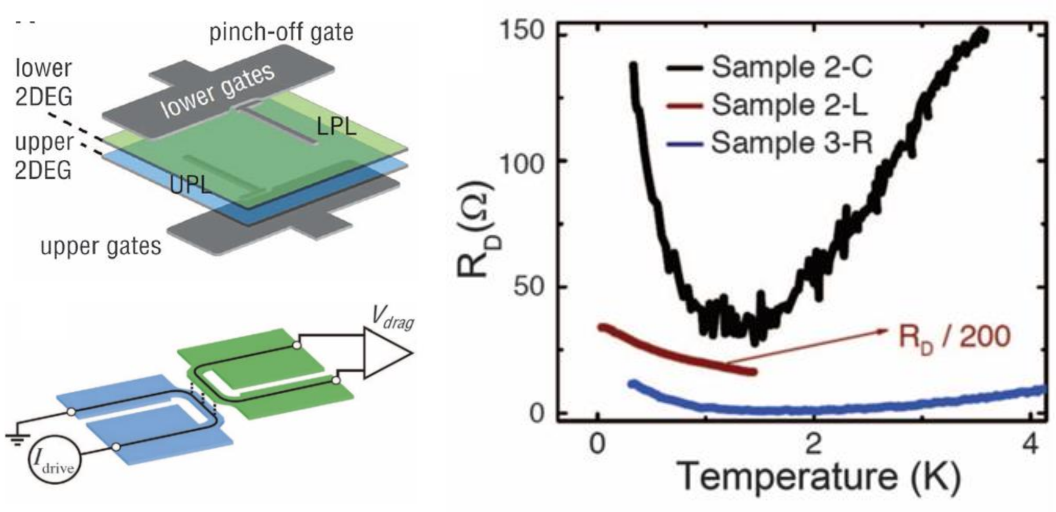
- **Negative drag due to strong correlations**
- **Wigner crystal state**

1D-1D Coulomb Drag Signature of a Luttinger Liquid

Science 343, 631 (2014)

D. Laroche,[1,2] G. Gervais,[1*] M. P. Lilly,[2] J. L. Reno[2]

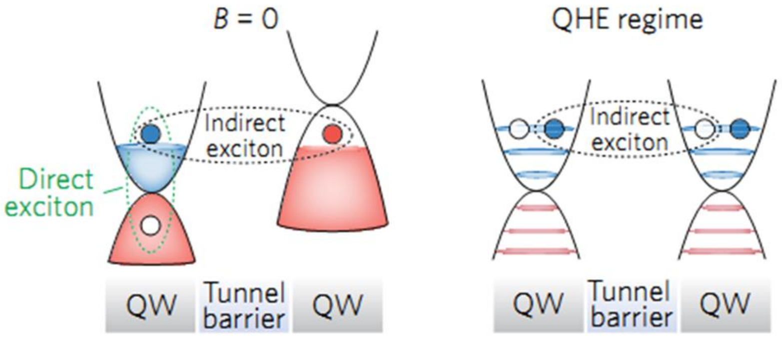
One-dimensional (1D) interacting electronic systems exhibit distinct properties when compared to their counterparts in higher dimensions. We report Coulomb drag measurements between vertically integrated quantum wires separated by a barrier only 15 nanometers wide. The temperature dependence of the drag resistance is measured in the true 1D regime where both wires have less than one 1D subband occupied. As a function of temperature, an upturn in the drag resistance is observed below a temperature *T** ~ 1.6 kelvin. This crossover in Coulomb drag behavior is consistent with Tomonaga-Luttinger liquid models for the 1D-1D drag between quantum wires.

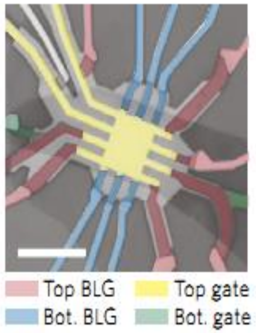


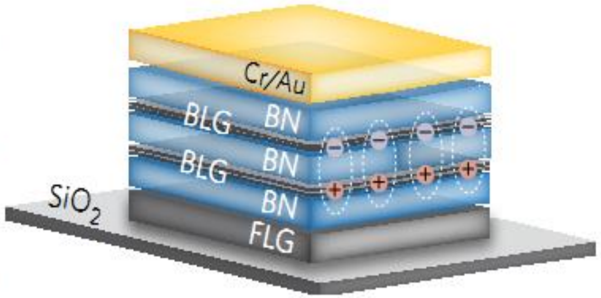
- **An upturn in the drag resistance at low T**
- **1D Tomonaga-Luttinger liquid**

New inter-layer correlated states

Superfluidity phase
in bilayer quantum Hall system

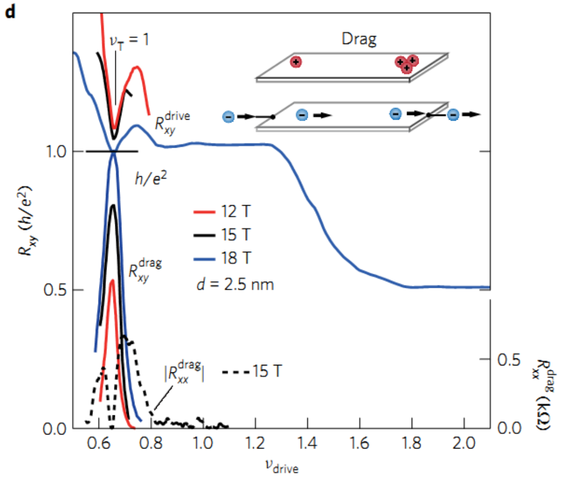






- Quantized Hall drag
- Zero resistance via counterflow

d



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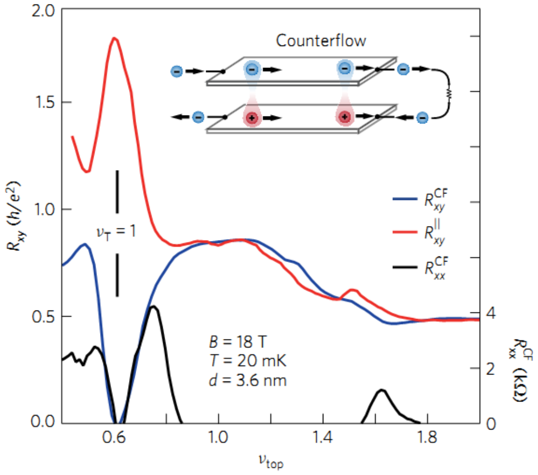


Li *et al.*, Nat. Phys. 13, 751 (2017)

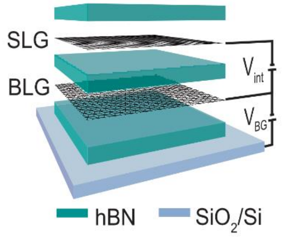
Outline

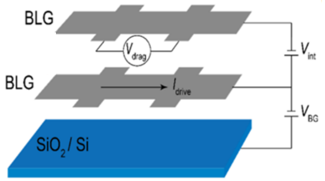
- Interlayer quasiparticle interactions
  - Interactions between massless and massive fermions

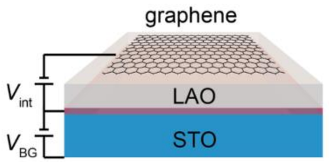
    Monolayer graphene + Bilayer graphene
- New interlayer coupling effects
  - Inter-layer quantum interference effect

    Bilayer graphene + Bilayer graphene
  - Giant supercurrent drag effect (Josephson-Coulomb drag)

    Graphene + Superconducting $LaAlO_3/SrTiO_3$



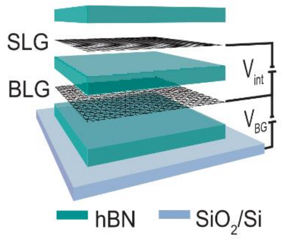




Outline

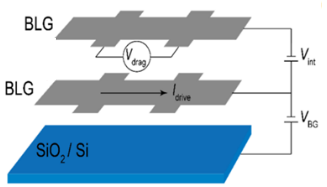
- Interlayer quasiparticle interactions
  - Interactions between massless and massive fermions

    Monolayer graphene + Bilayer graphene



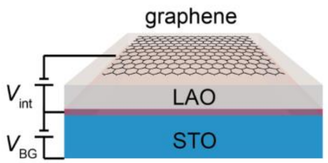
- New interlayer coupling effects
  - Inter-layer quantum interference effect

    Bilayer graphene + Bilayer graphene



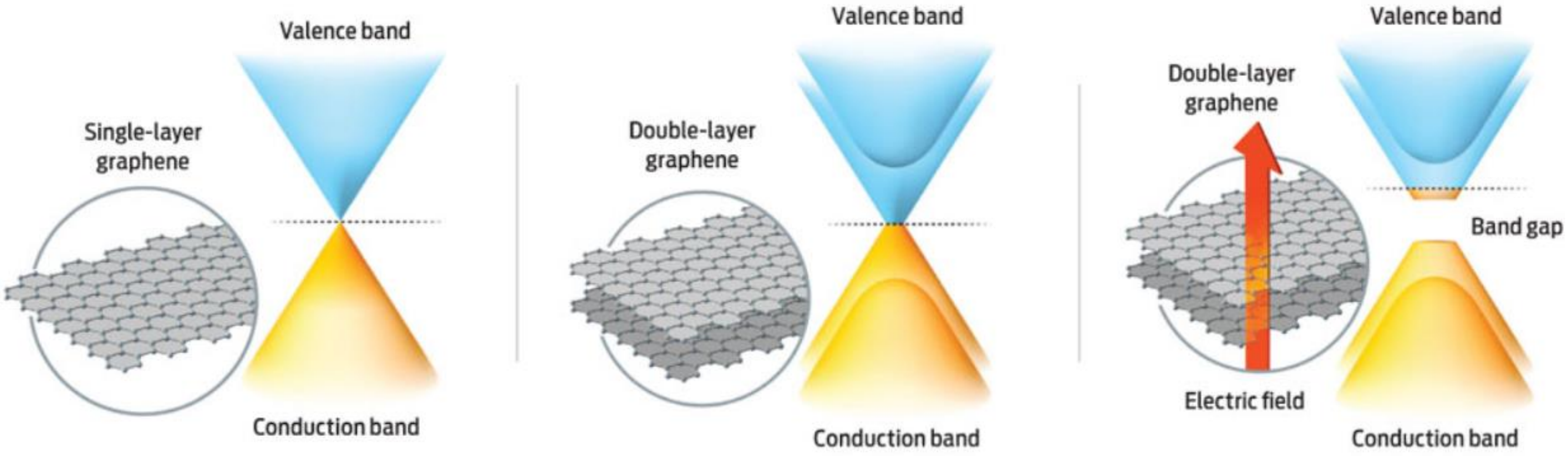
  - Giant supercurrent drag effect (Josephson-Coulomb drag)

    Graphene + Superconducting $LaAlO_3/SrTiO_3$



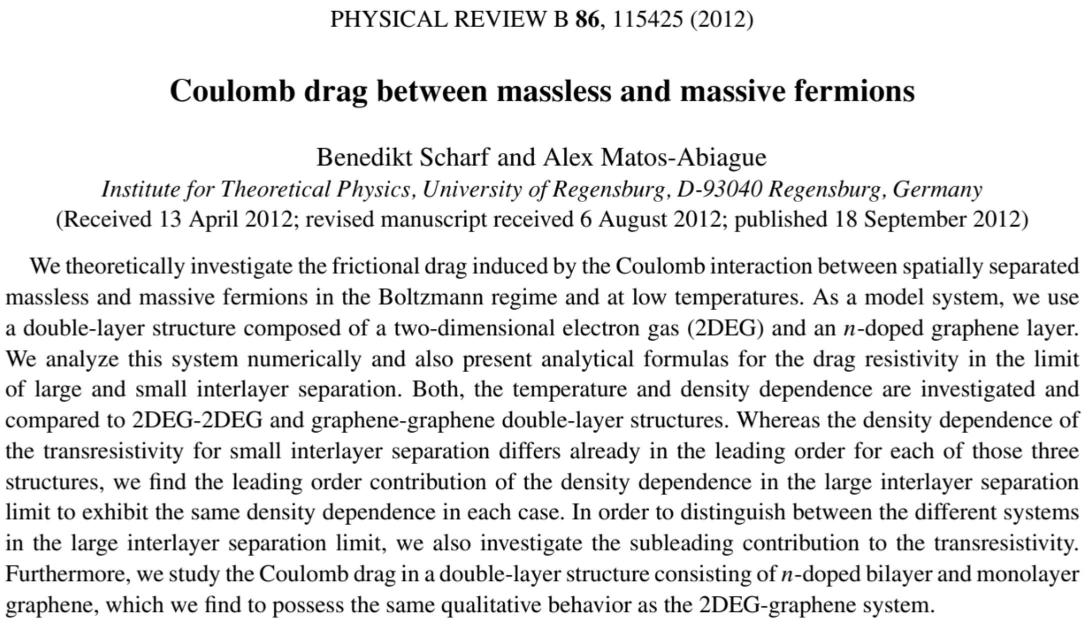
Thickness dependent electronic structure in graphene

- Massless fermions in SLG
- Massive fermions in BLG



Interactions between massless and massive fermions ?

Coulomb drag between massless and massive fermions



PHYSICAL REVIEW B **86**, 115425 (2012)

Coulomb drag between massless and massive fermions

Benedikt Scharf and Alex Matos-Abiague
Institute for Theoretical Physics, University of Regensburg, D-93040 Regensburg, Germany

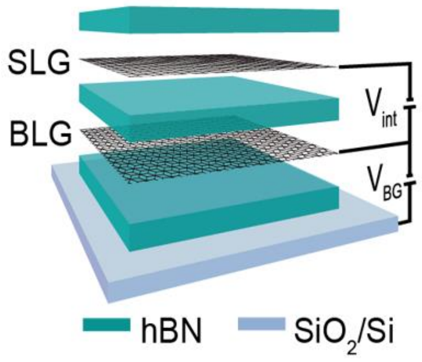
(Received 13 April 2012; revised manuscript received 6 August 2012; published 18 September 2012)

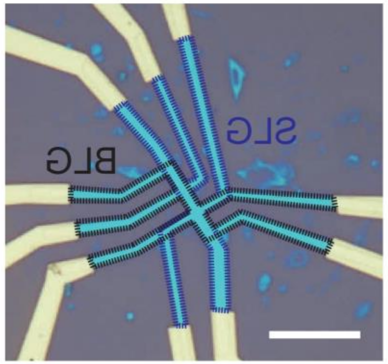
We theoretically investigate the frictional drag induced by the Coulomb interaction between spatially separated massless and massive fermions in the Boltzmann regime and at low temperatures. As a model system, we use a double-layer structure composed of a two-dimensional electron gas (2DEG) and an n-doped graphene layer. We analyze this system numerically and also present analytical formulas for the drag resistivity in the limit of large and small interlayer separation. Both, the temperature and density dependence are investigated and compared to 2DEG-2DEG and graphene-graphene double-layer structures. Whereas the density dependence of the transresistivity for small interlayer separation differs already in the leading order for each of those three structures, we find the leading order contribution of the density dependence in the large interlayer separation limit to exhibit the same density dependence in each case. In order to distinguish between the different systems in the large interlayer separation limit, we also investigate the subleading contribution to the transresistivity. Furthermore, we study the Coulomb drag in a double-layer structure consisting of n-doped bilayer and monolayer graphene, which we find to possess the same qualitative behavior as the 2DEG-graphene system.

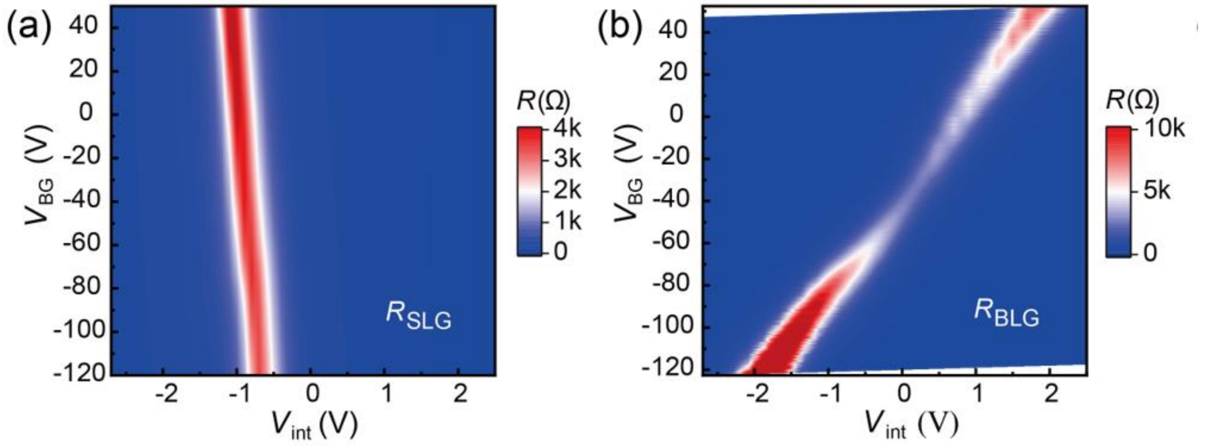
System (active-passive)	ρ_D Strong coupling regime ($k_{\mathrm{F}}d \ll 1$)	ρ_D Weak coupling regime ($k_{\mathrm{F}}d \gg 1$)
Massive-massive	$\propto 1/n^3$	$\propto 1/n^3$
Massless-massless	$\propto 1/n$	$\propto 1/n^3$
Massless-massive	$\propto 1/n^2$	$\propto 1/n^3$

Density matched cases

Carrier density dependence

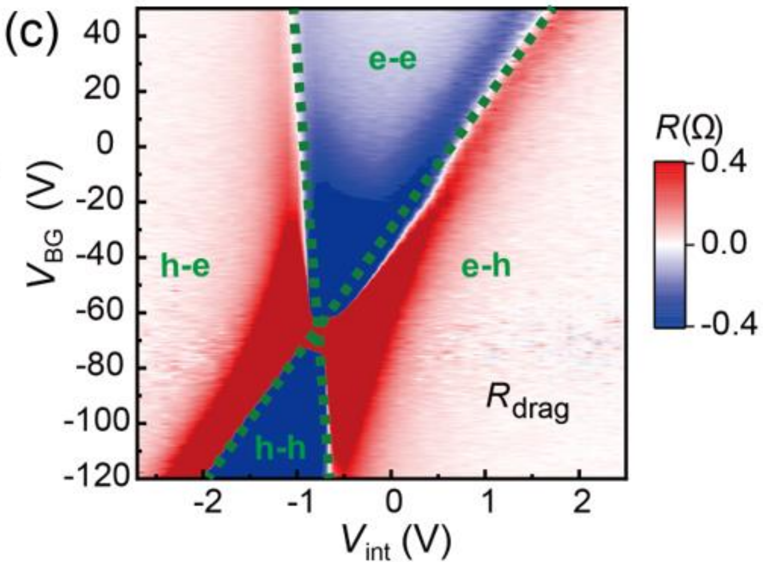




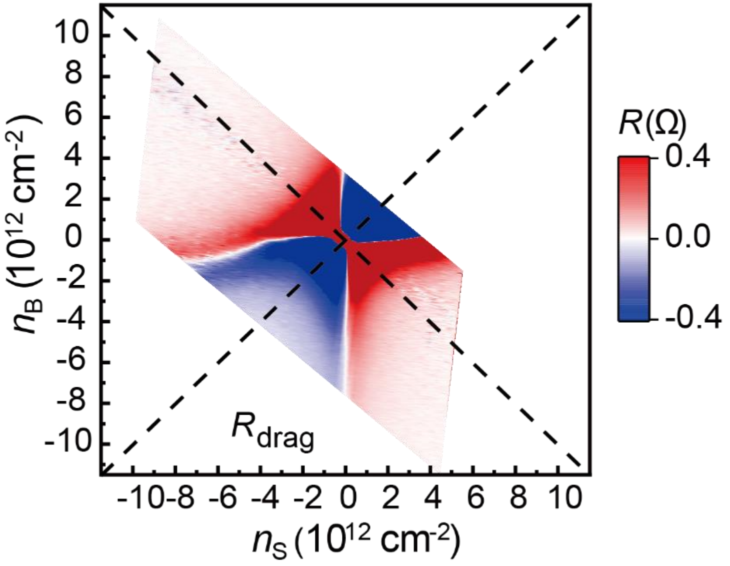


T = 140 K

R_{drag} vs (V_{int}, V_{BG})



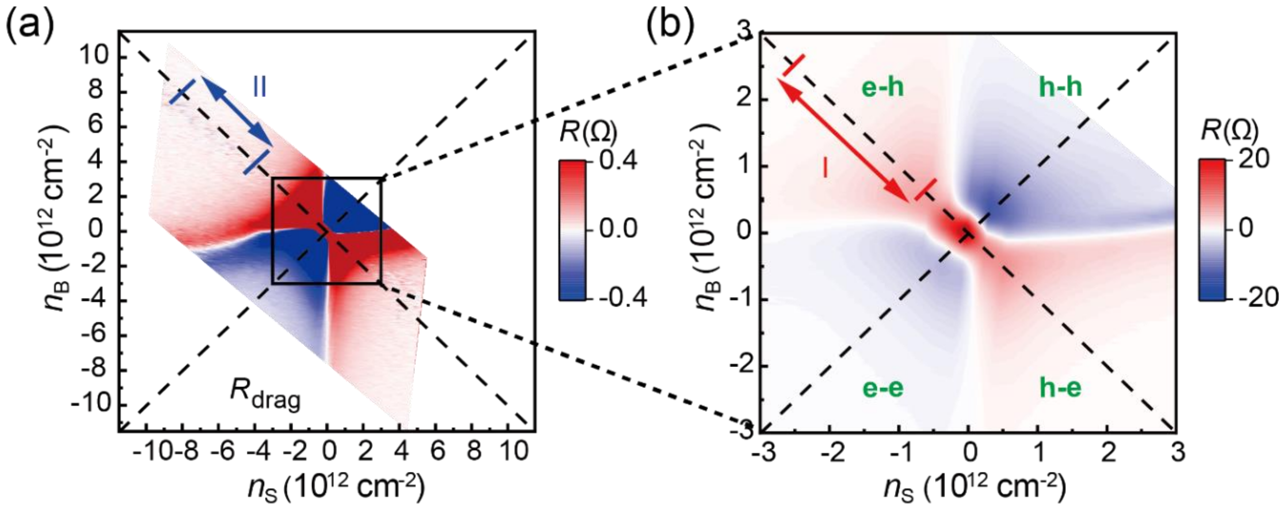
R_{drag} vs (n_S, n_B)

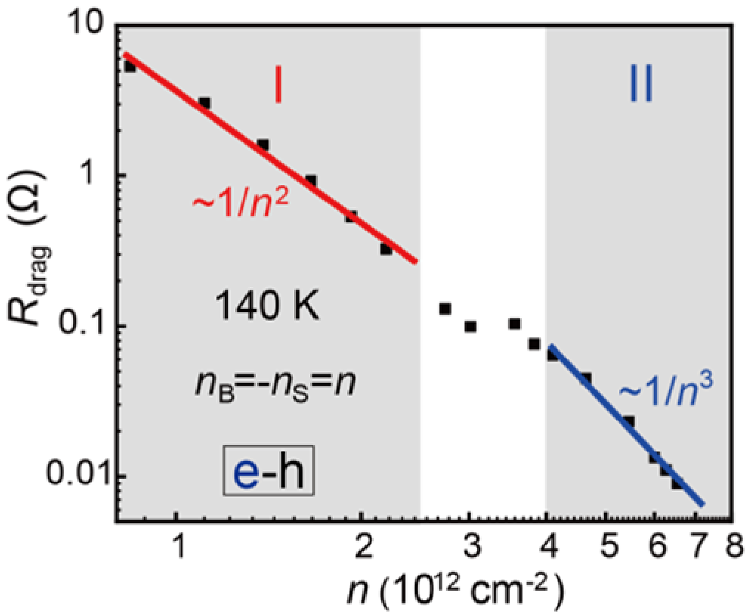


R_{drag} vs carrier density

Density matched cases

$|n_S| = |n_B|$





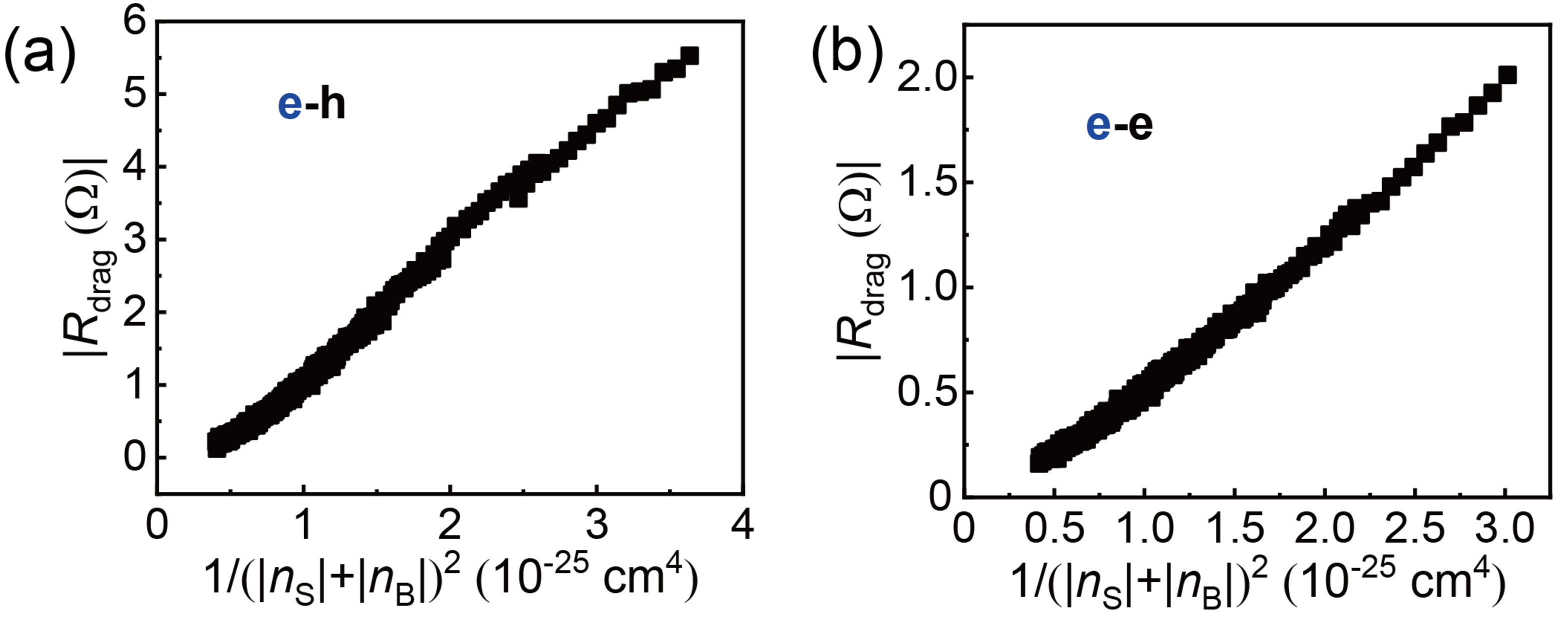
- **Regime I ($k_F d$ <1)**
  $R_{\mathrm{drag}} \propto 1/n^2$
- **Regime II ($k_F d$ >1)**
  $R_{\mathrm{drag}} \propto 1/n^3$

Consistent with theoretical prediction

R_{drag} vs carrier density

Density mismatched cases

$|n_S| \neq |n_B|$



Strong coupling regime

$R_{\text{drag}} \propto 1/(|n_S|+|n_B|)^2$

Theoretically unclear

Summary

- Inter-layer drag interactions between massless and massive fermions
- Weak coupling → strong coupling regime

  R_{drag}: $1/n^3 \rightarrow 1/n^2$ Fingerprint feature for massless-massive systems
- A generalized carrier dependent expression $1/(|n_S|+|n_B|)^2$ for the strong coupling regime

Lijun Zhu *et al.*, Nano Lett. 20, 1396 (2020)

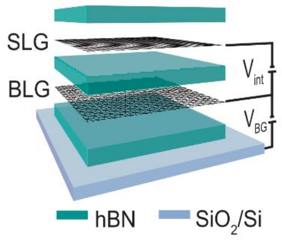
Interlayer quasiparticle interaction ⇒ **New drag effect**

Outline

➢ Interlayer quasiparticle interactions

- Interactions between massless and massive fermions

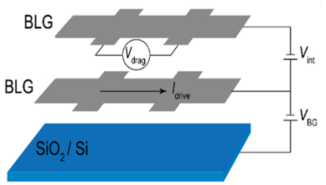
  Monolayer graphene + Bilayer graphene



➢ New interlayer coupling effects

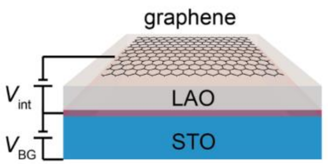
- Inter-layer quantum interference effect

  Bilayer graphene + Bilayer graphene



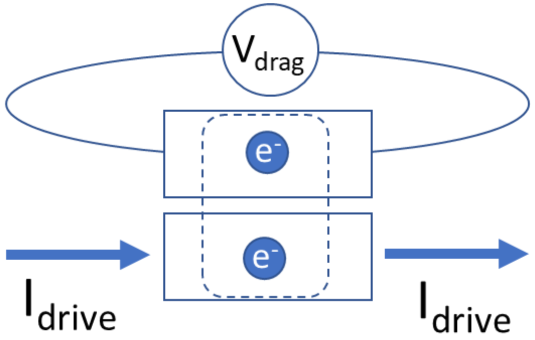
- Giant supercurrent drag effect (Josephson-Coulomb drag)

  Graphene + Superconducting $LaAlO_3/SrTiO_3$



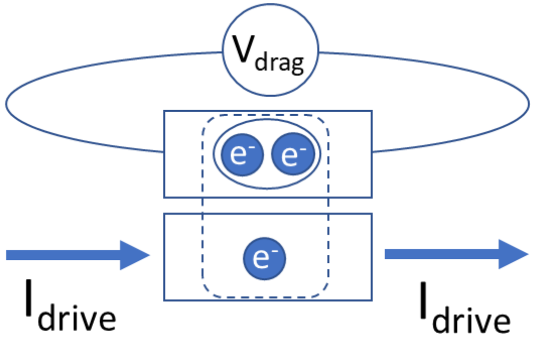
Superconducting drag?

- Metal ⇔ Metal

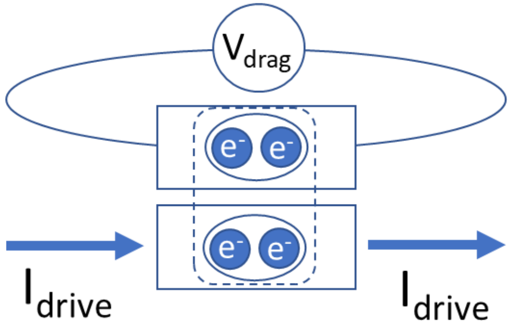


Drag current << drive current

- Superconductor ⇔ Metal

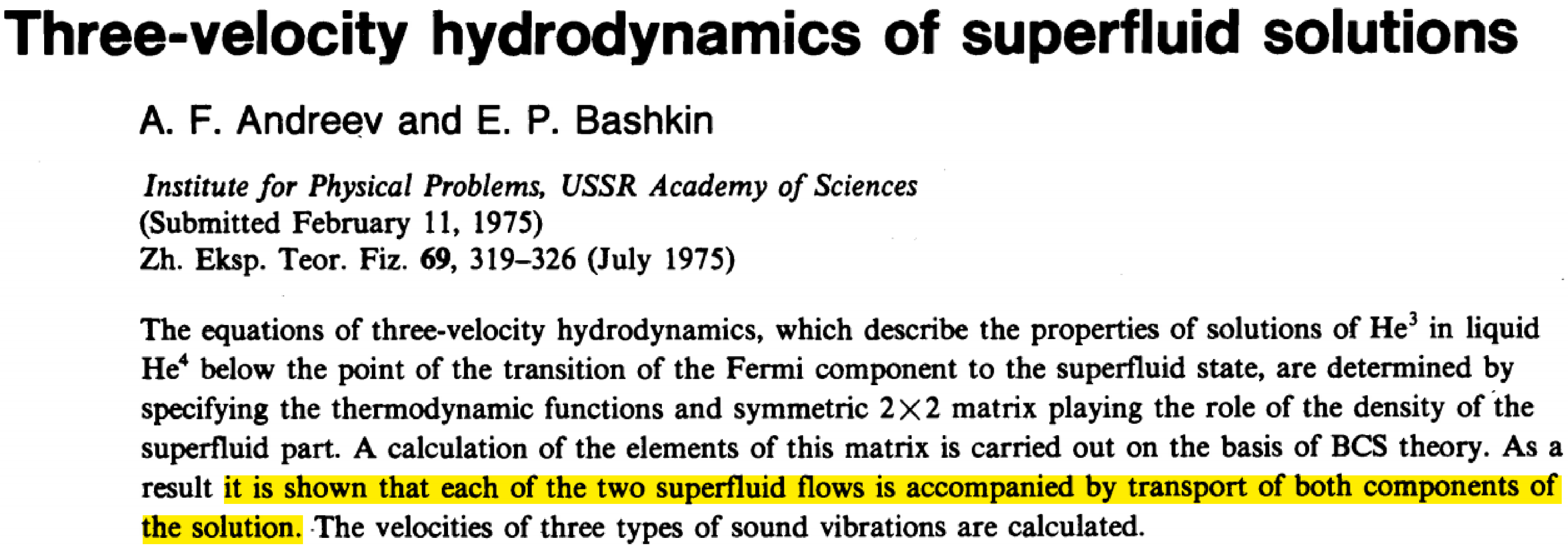


- Superconductor ⇔ Superconductor



Novel and giant drag effect?

Prediction of Superconducting drag in ^{3}He-^{4}He mixtures



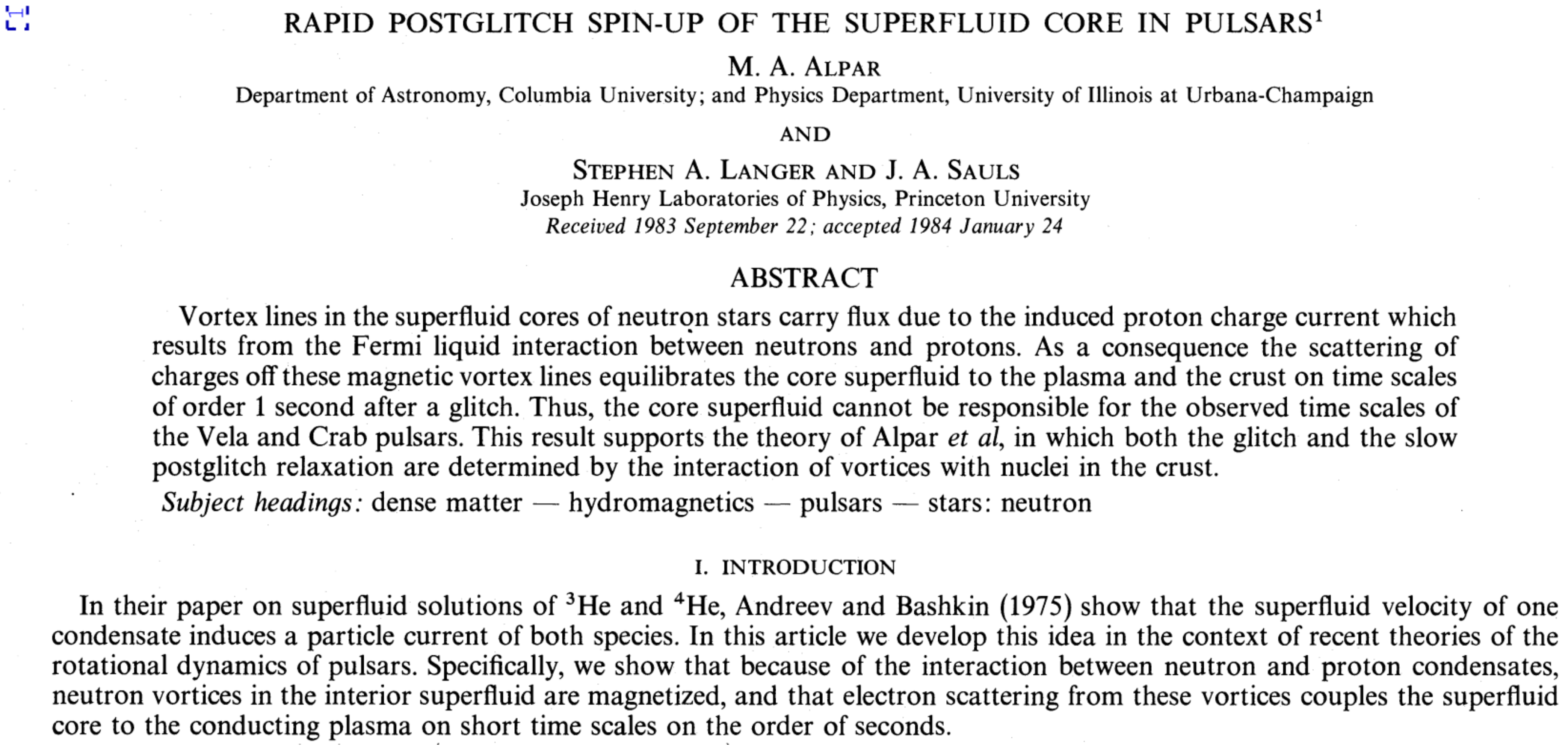
Three-velocity hydrodynamics of superfluid solutions

A. F. Andreev and E. P. Bashkin

Institute for Physical Problems, USSR Academy of Sciences
(Submitted February 11, 1975)
Zh. Eksp. Teor. Fiz. **69**, 319–326 (July 1975)

The equations of three-velocity hydrodynamics, which describe the properties of solutions of He^3 in liquid He^4 below the point of the transition of the Fermi component to the superfluid state, are determined by specifying the thermodynamic functions and symmetric 2×2 matrix playing the role of the density of the superfluid part. A calculation of the elements of this matrix is carried out on the basis of BCS theory. As a result it is shown that each of the two superfluid flows is accompanied by transport of both components of the solution. The velocities of three types of sound vibrations are calculated.

Prediction of Superconducting drag in neutron stars



RAPID POSTGLITCH SPIN-UP OF THE SUPERFLUID CORE IN PULSARS[1]

M. A. Alpar
Department of Astronomy, Columbia University; and Physics Department, University of Illinois at Urbana-Champaign

AND

Stephen A. Langer and J. A. Sauls
Joseph Henry Laboratories of Physics, Princeton University

Received 1983 September 22; accepted 1984 January 24

ABSTRACT

Vortex lines in the superfluid cores of neutron stars carry flux due to the induced proton charge current which results from the Fermi liquid interaction between neutrons and protons. As a consequence the scattering of charges off these magnetic vortex lines equilibrates the core superfluid to the plasma and the crust on time scales of order 1 second after a glitch. Thus, the core superfluid cannot be responsible for the observed time scales of the Vela and Crab pulsars. This result supports the theory of Alpar *et al*, in which both the glitch and the slow postglitch relaxation are determined by the interaction of vortices with nuclei in the crust.

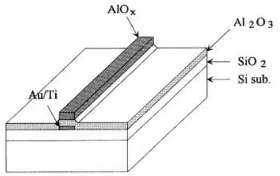
Subject headings: dense matter — hydromagnetics — pulsars — stars: neutron

I. INTRODUCTION

In their paper on superfluid solutions of ^{3}He and ^{4}He, Andreev and Bashkin (1975) show that the superfluid velocity of one condensate induces a particle current of both species. In this article we develop this idea in the context of recent theories of the rotational dynamics of pulsars. Specifically, we show that because of the interaction between neutron and proton condensates, neutron vortices in the interior superfluid are magnetized, and that electron scattering from these vortices couples the superfluid core to the conducting plasma on short time scales on the order of seconds.

Superconducting drag: previous experiments

Au/Ti & SC AlOx



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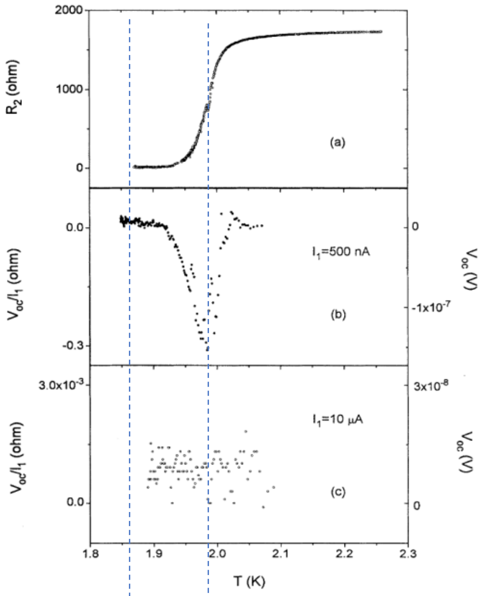
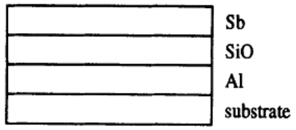


X. Huang *et al.*, Phys. Rev. Lett. 74, 4051 (1995)

Sb & SC AlOx



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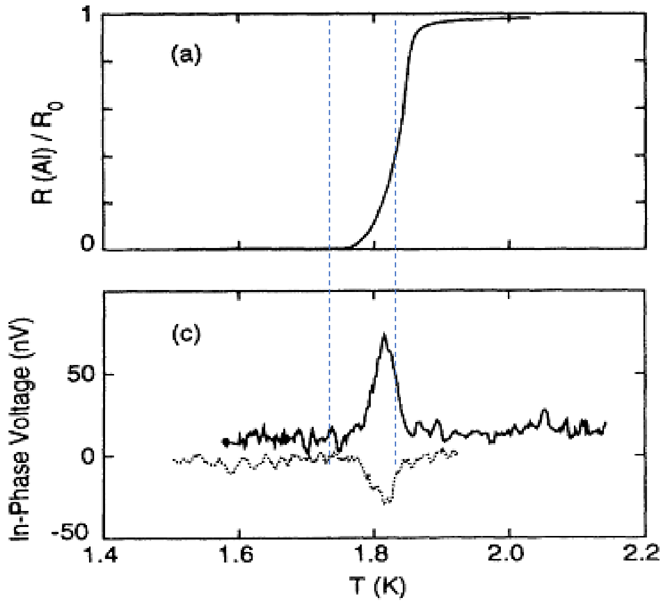


N. Giordano *et al.*, Phys. Rev. B 50, 9363 (1994)

Disadvantages of the conventional systems

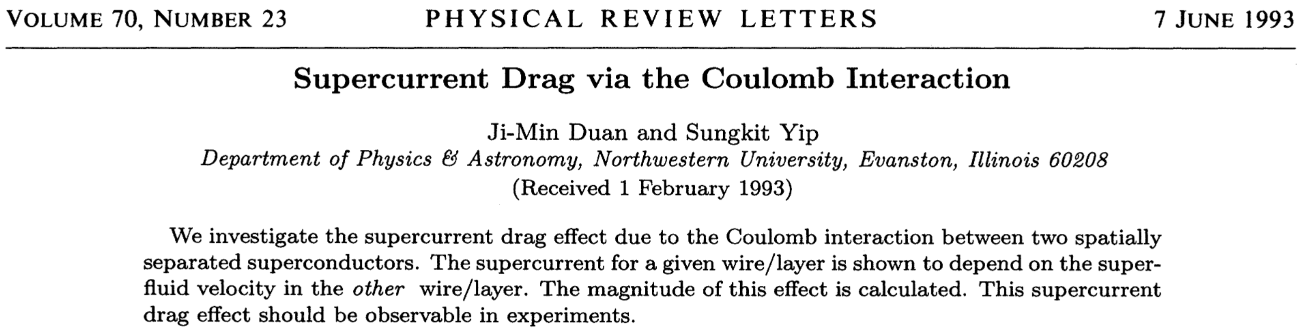
- Lack of tunability
- High carrier density (screening effect)
- Large inter-layer spacing
- Non-uniform inter-layer interaction
- Very weak drag response

Weak and uncontrolled drag responses

Passive-to-active ratio (PAR) r ~ 10^{-3}

Superconducting drag: previous theories

- Coulomb interaction



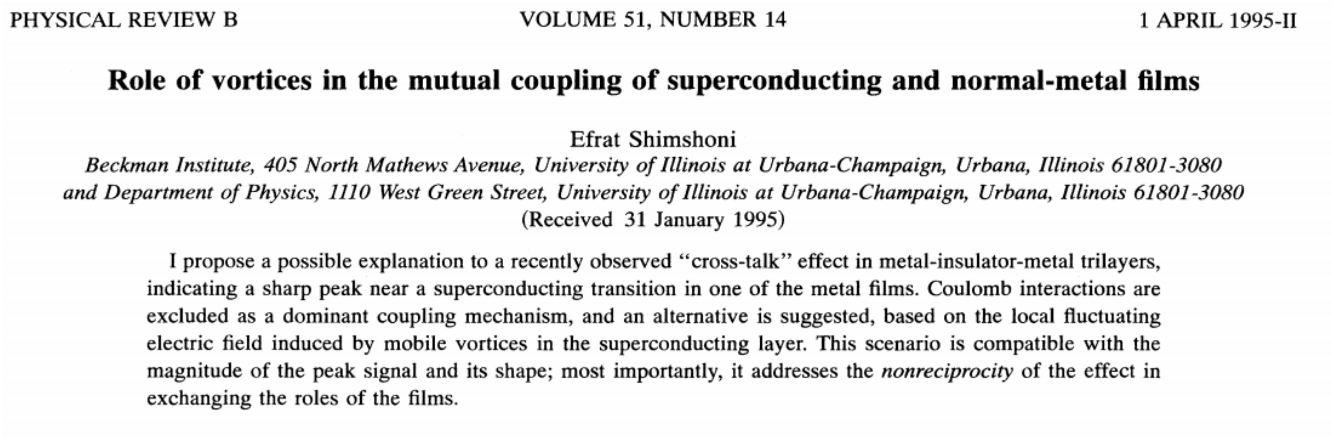
VOLUME 70, NUMBER 23 PHYSICAL REVIEW LETTERS 7 JUNE 1993

Supercurrent Drag via the Coulomb Interaction

Ji-Min Duan and Sungkit Yip
Department of Physics & Astronomy, Northwestern University, Evanston, Illinois 60208
(Received 1 February 1993)

We investigate the supercurrent drag effect due to the Coulomb interaction between two spatially separated superconductors. The supercurrent for a given wire/layer is shown to depend on the superfluid velocity in the *other* wire/layer. The magnitude of this effect is calculated. This supercurrent drag effect should be observable in experiments.

- Magnetic induction



PHYSICAL REVIEW B VOLUME 51, NUMBER 14 1 APRIL 1995-II

Role of vortices in the mutual coupling of superconducting and normal-metal films

Efrat Shimshoni
Beckman Institute, 405 North Mathews Avenue, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801-3080
and Department of Physics, 1110 West Green Street, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801-3080
(Received 31 January 1995)

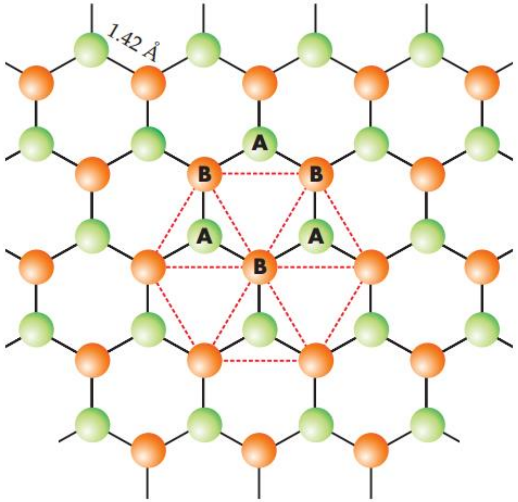
I propose a possible explanation to a recently observed "cross-talk" effect in metal-insulator-metal trilayers, indicating a sharp peak near a superconducting transition in one of the metal films. Coulomb interactions are excluded as a dominant coupling mechanism, and an alternative is suggested, based on the local fluctuating electric field induced by mobile vortices in the superconducting layer. This scenario is compatible with the magnitude of the peak signal and its shape; most importantly, it addresses the *nonreciprocity* of the effect in exchanging the roles of the films.

Mechanism remains unclear

2D materials to construct drag devices

2D conductor

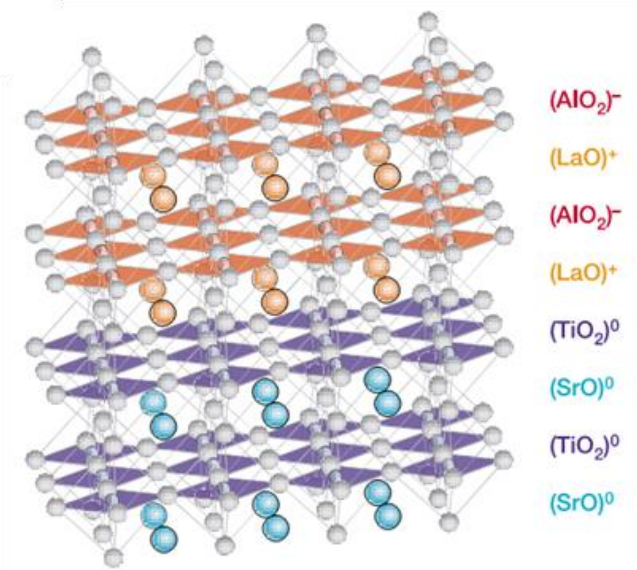
Graphene



+

2D superconductor

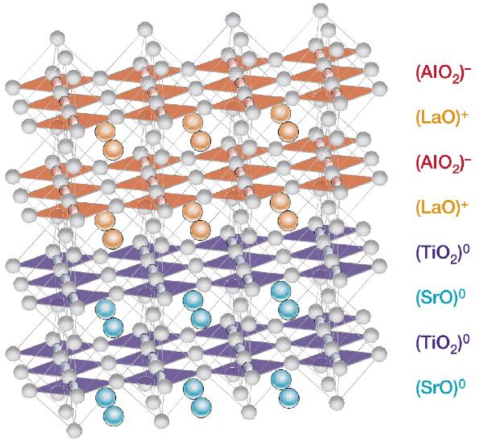
$LaAlO_3/SrTiO_3$ interface

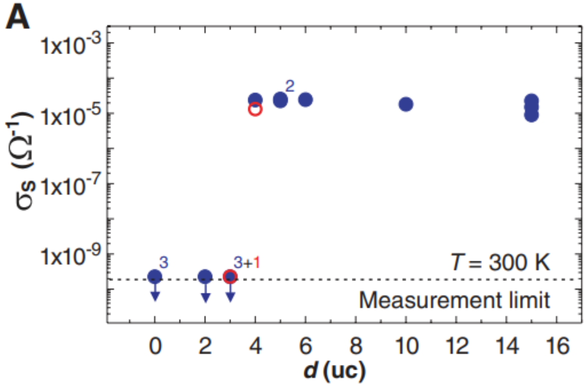


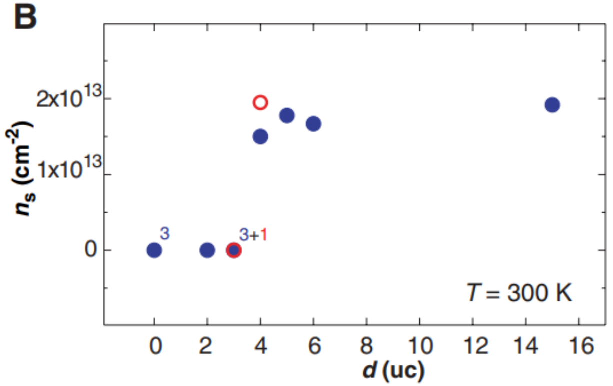
LAO/STO interface

☐ Thickness dependent interfacial conductivity

LAO thickness ≥ 4 uc: conducting



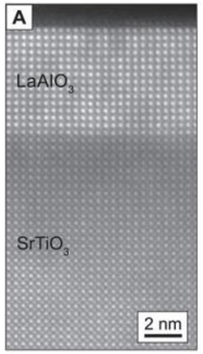


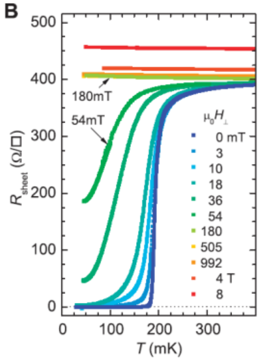


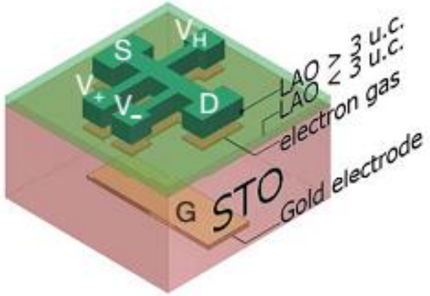
A. Ohtomo *et al*., Nature 427, 423 (2004)

S. Thiel *et al*., Science 313, 1942 (2006)

☐ Interfacial 2D superconductivity and its high tunability

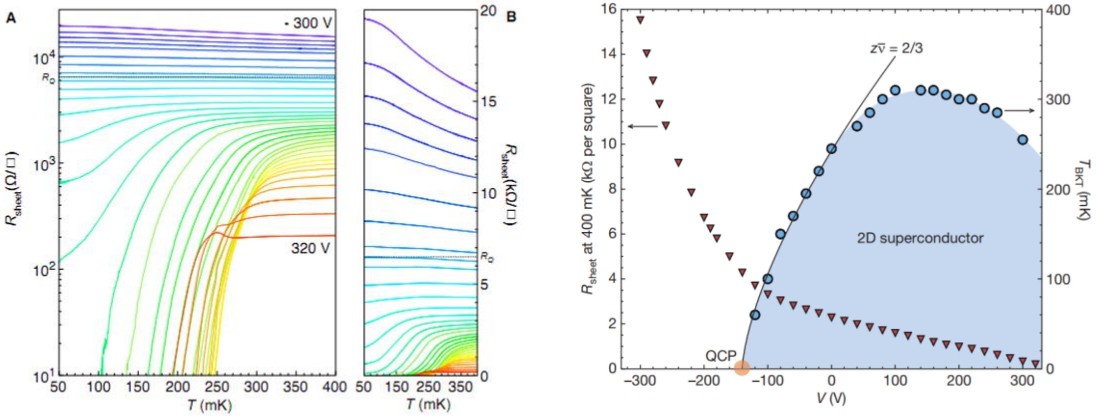






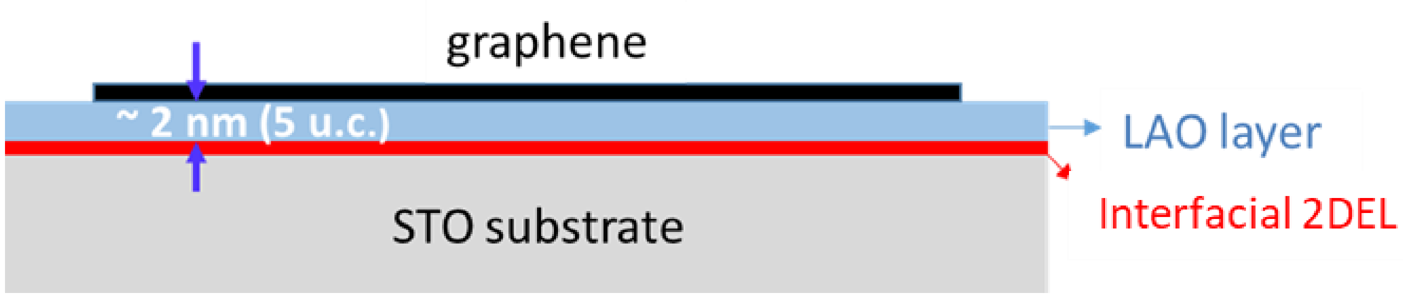
N. Reyren *et al*., Science 317, 1196 (2007)

Tc ~300 mK



A. D. Caviglia *et al*., Nature 456, 624 (2008)

Advantages of Graphene/LAO/STO



- **High tunability for both layers**

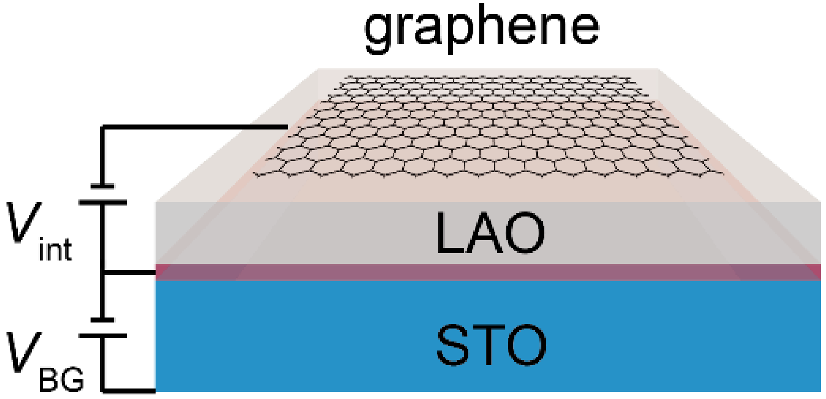
  Graphene: tunable carrier type and density

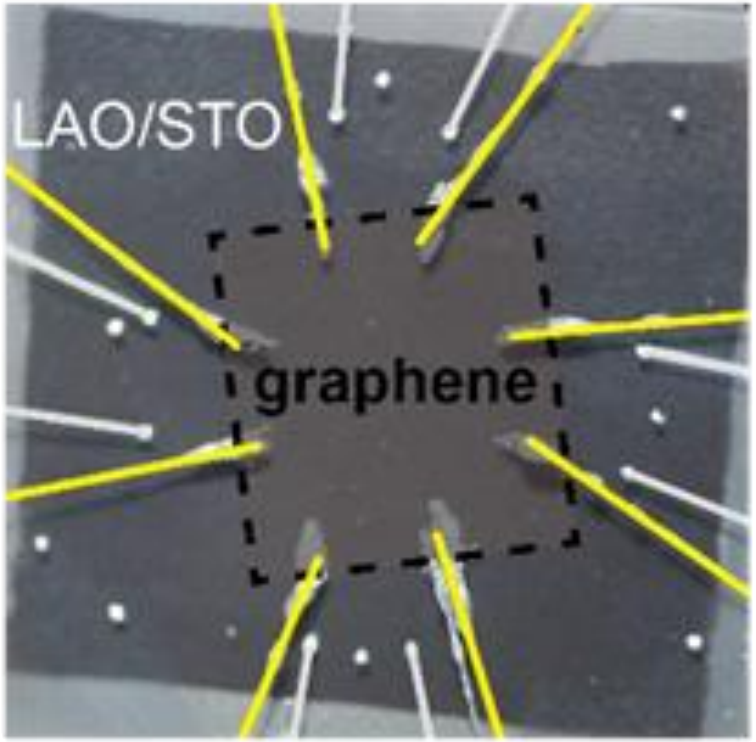
  LAO/STO interface: tunable superconductivity, band filling, and SOC

- **Strong coupling due to ultra-small inter-layer distance**

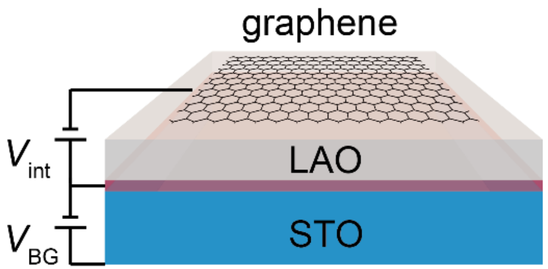
  LAO layer: a natural and ideal spacer (< 2 nm for 5 uc LAO)

Hybrid graphene/$LaAlO_3$/$SrTiO_3$ device

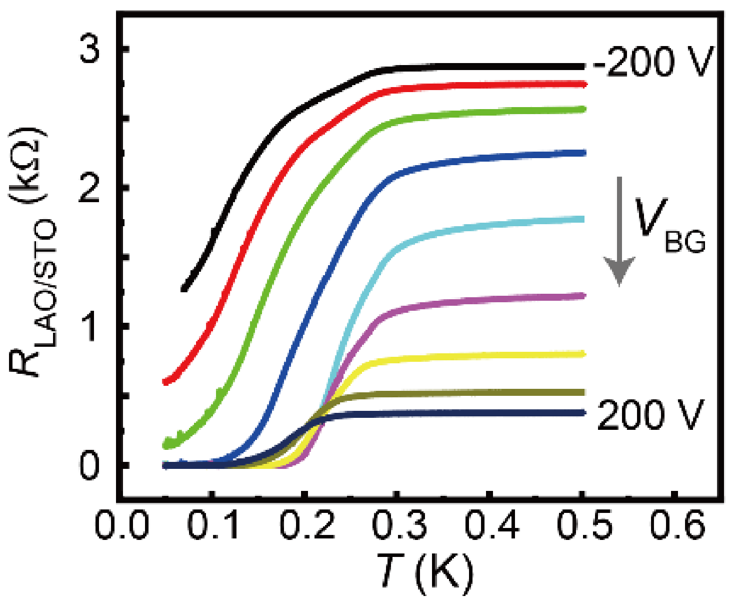




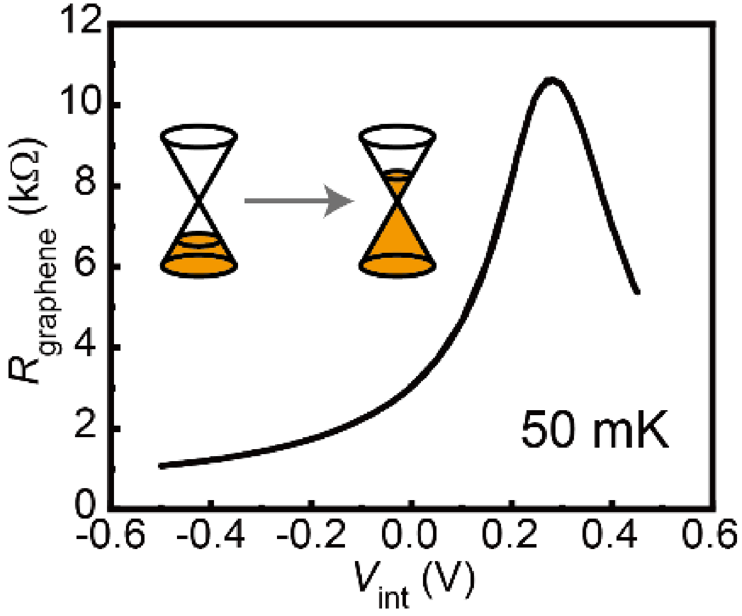
Basic transport characterizations



☐ Tunning LAO/STO using V_{BG}

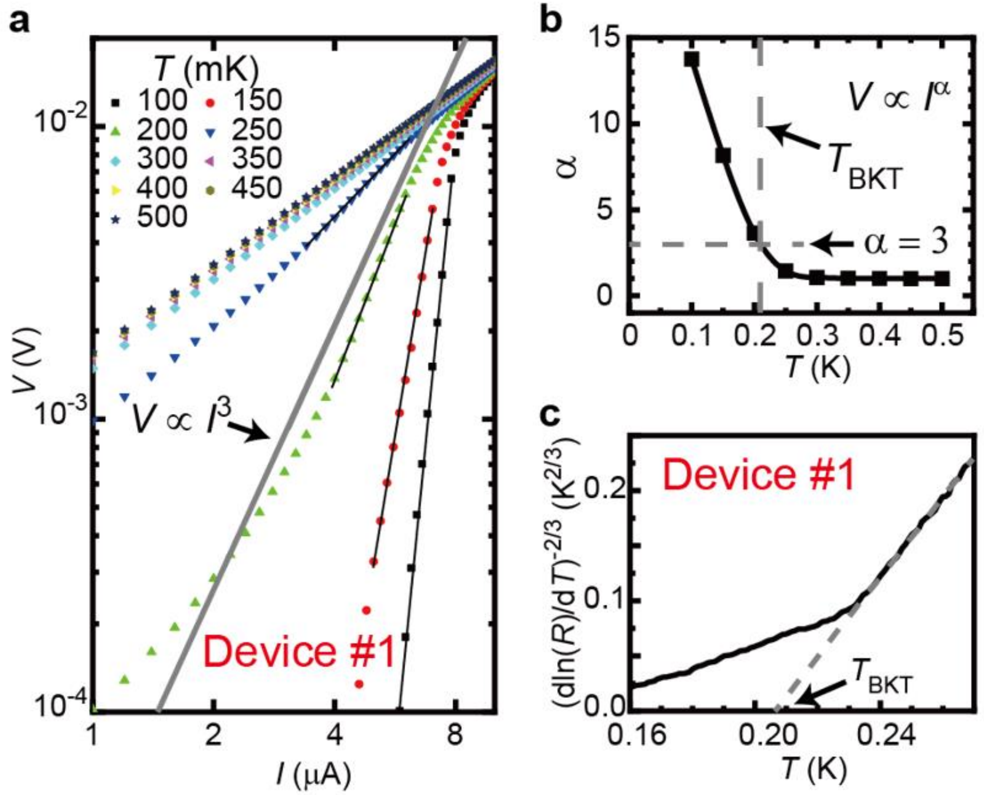


☐ Tunning graphene layer using V_{int}



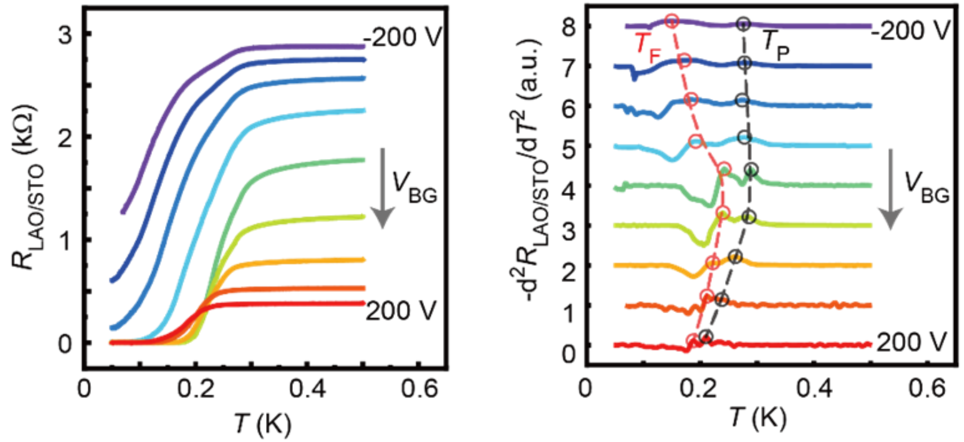
Superconductivity of LAO/STO

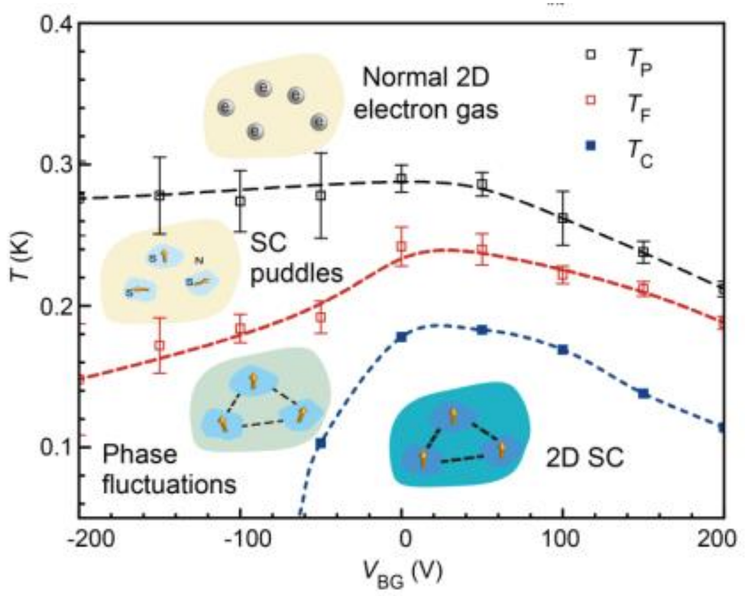
☐ 2D superconductivity



- Berezinskii-Kosterlitz-Thouless (BKT) transition
  → $V \propto I^3$ T_{BKT} ~210 mK
- $R \propto \exp[-b/(T/T_{BKT}-1)^{1/2}]$ → $[d\ln R/dT]^{-2/3} \propto (T-T_{BKT})$
  → T_{BKT} ~206 mK

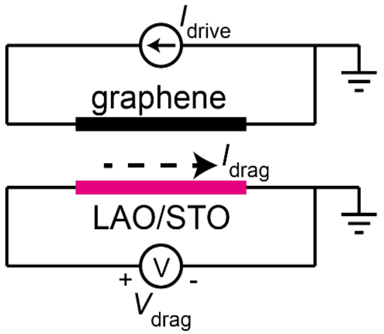
☐ **Electronic phase separation in LAO/STO**

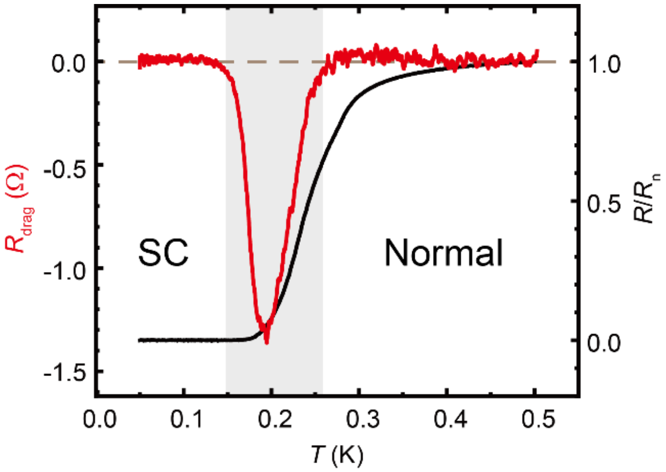




Inter-layer drag effect

☐ Temperature dependence





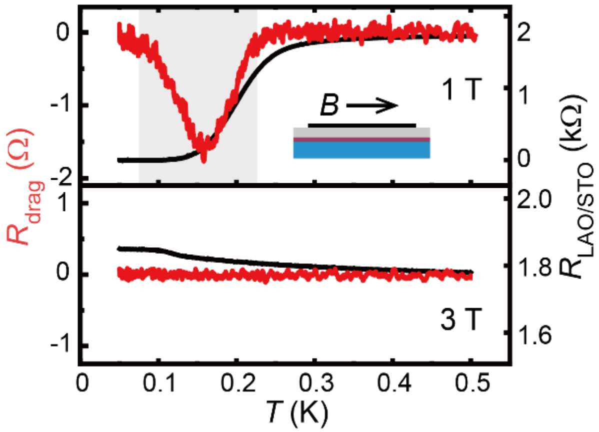
- Negative drag signal at the SC transition
- Peaked at ~195 mK

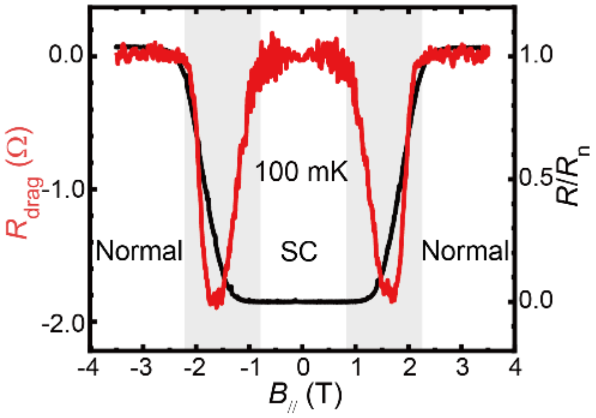
Intimate correlation between the drag response and the SC transition of the LAO/STO interface

⇩

Supercurrent drag effect

☐ Magnetic field dependence

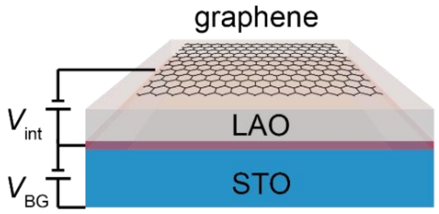




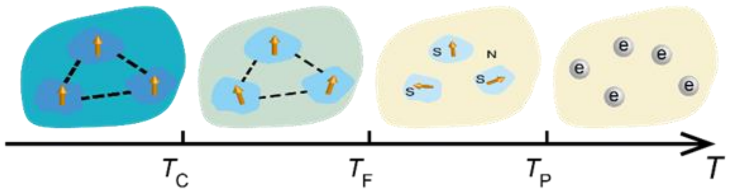
- Suppression of SC by magnetic field

→ weakening (disappearance) of drag response

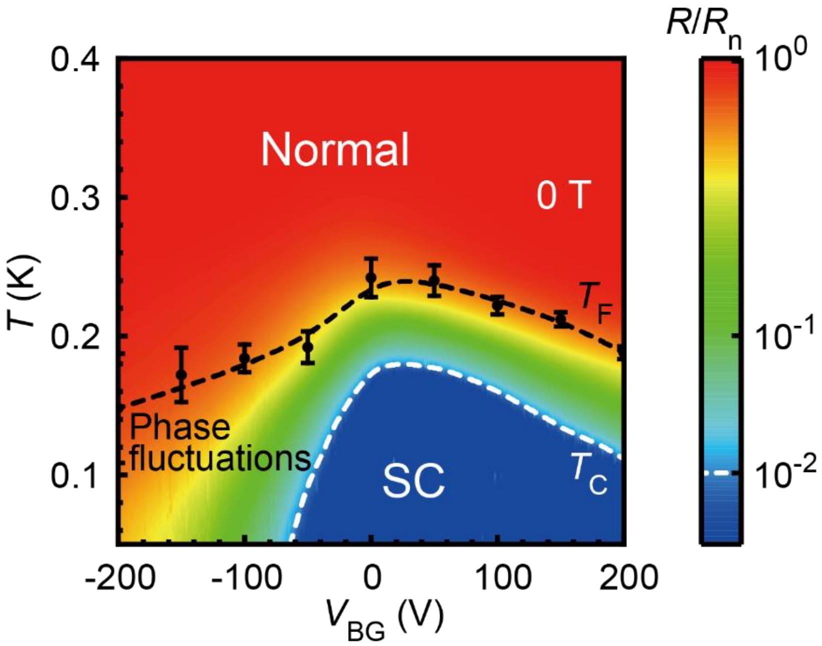
Phase diagrams of drag effect



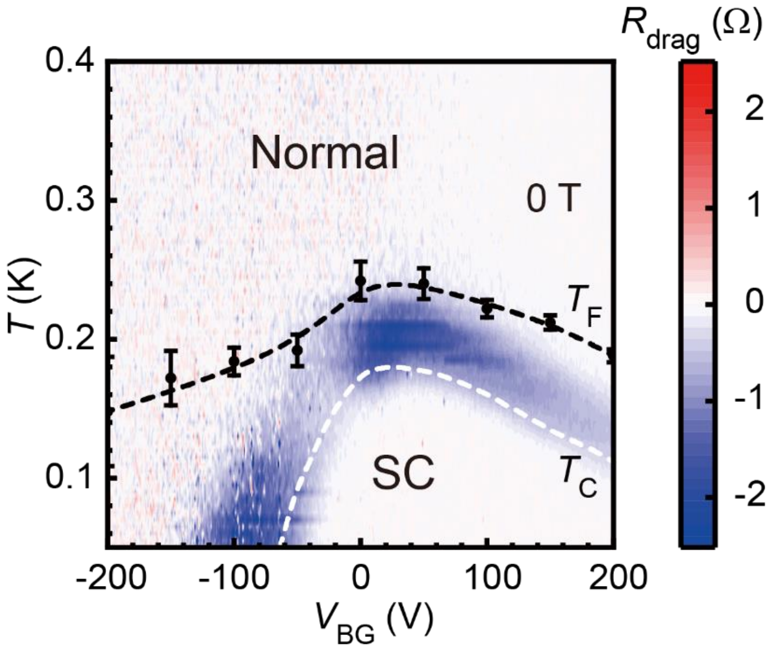
V_{BG}, T



☐ Resistance of LAO/STO



☐ Drag resistance R_{drag}

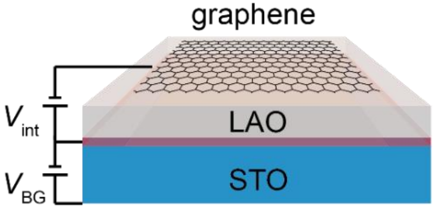


Why no drag signal below T_C

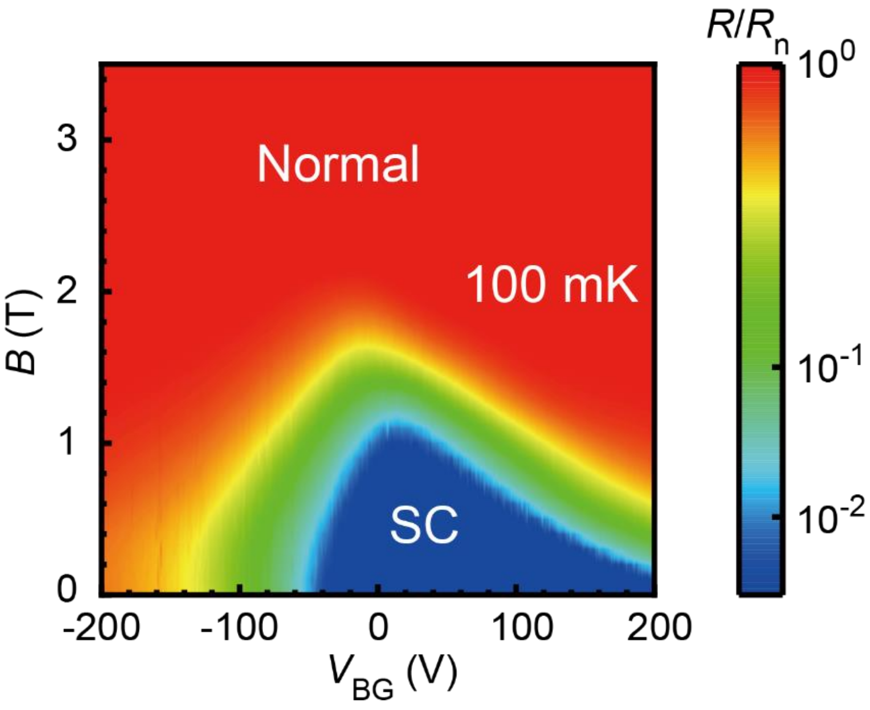
applying an active current (I_{drive}) to the graphene layer, and measuring the passive voltage drop (V_{drag}) at the LAO/STO interface in open circuits

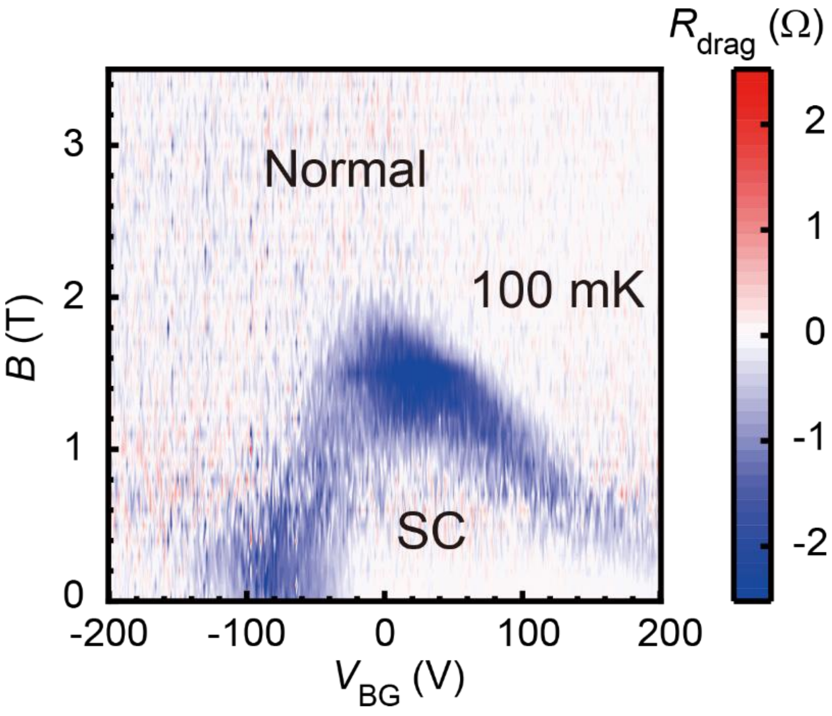
$R_{LAO/STO} = 0$ below T_C ⇨ $V_{drag} = 0$

Phase diagrams of drag effect

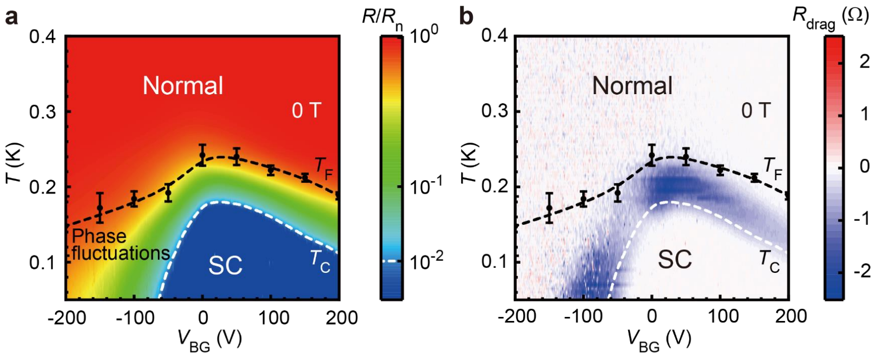


V_{BG}, B





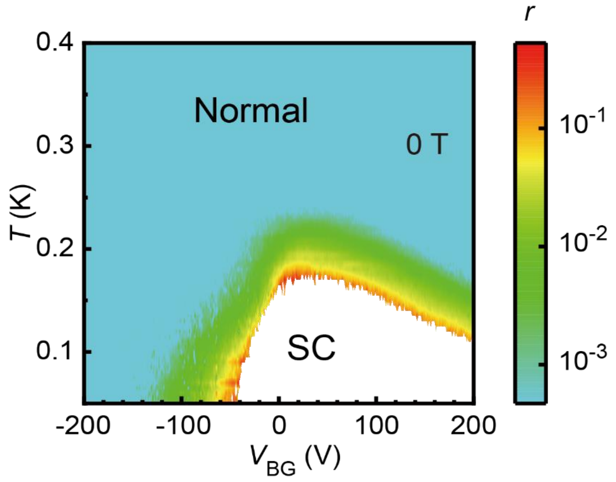
Strong inter-layer coupling

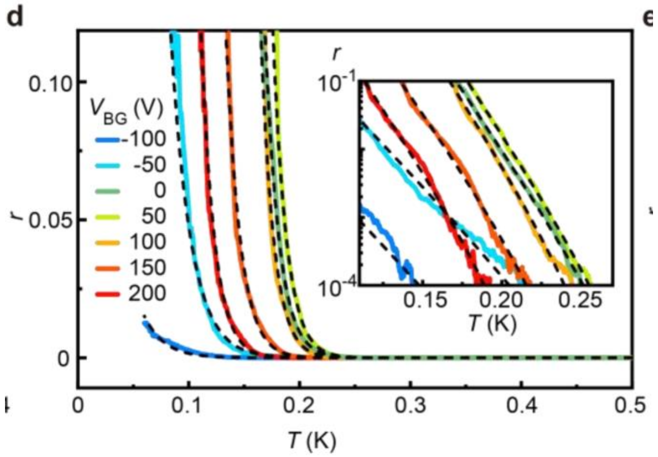


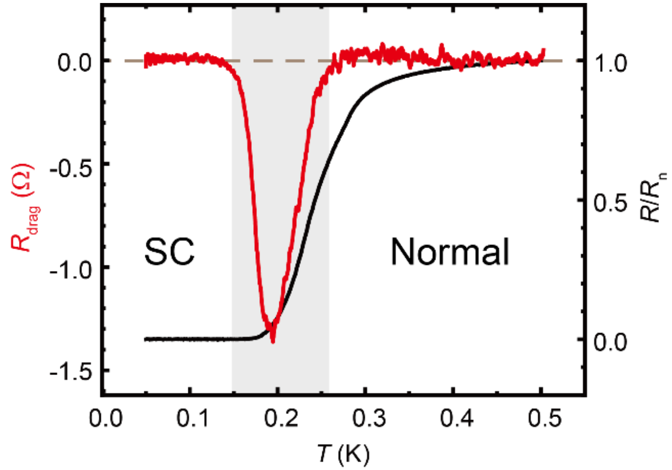
☐ Passive-to-active ratio (PAR) *r*

$$r = \frac{I_{drag}}{I_{drive}} = -\frac{V_{drag}}{R_{LAO/STO} I_{drive}} = -\frac{R_{drag}}{R_{LAO/STO}}$$

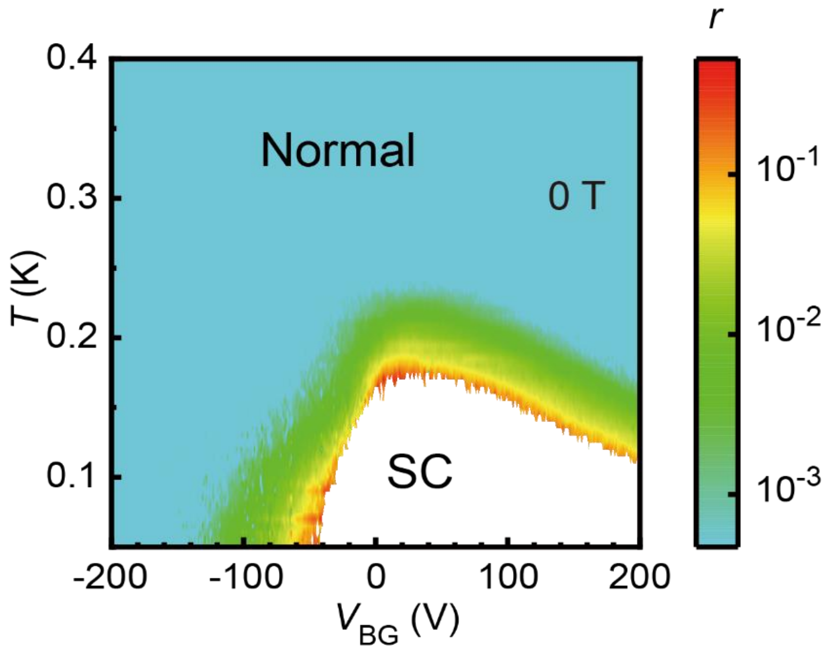
PAR *r* eliminates the passive-layer resistance and manifests the intrinsic correlations between the two layers

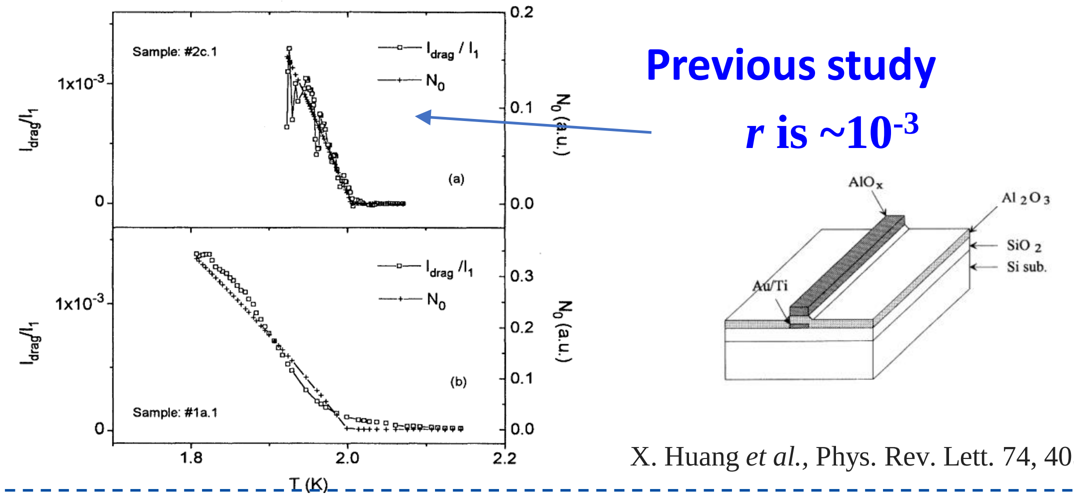






Strong inter-layer coupling





Previous study

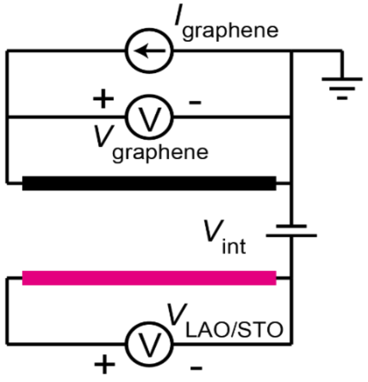
***r* is ~10^{-3}**

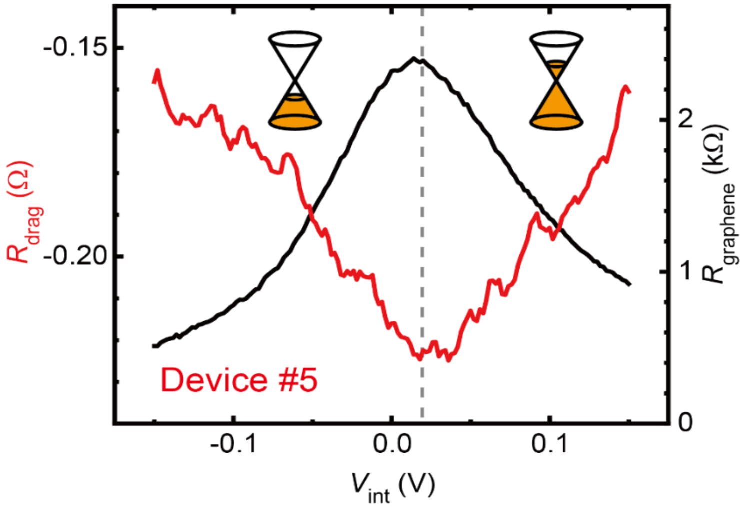
X. Huang *et al.*, Phys. Rev. Lett. 74, 4051 (1995).

***r* is ~0.3 at V_{BG} ~ 20 V and T ~ 170 mK**

New mechanism

☐ Unique behavior distinct from the conventional Coulomb drag phenomena

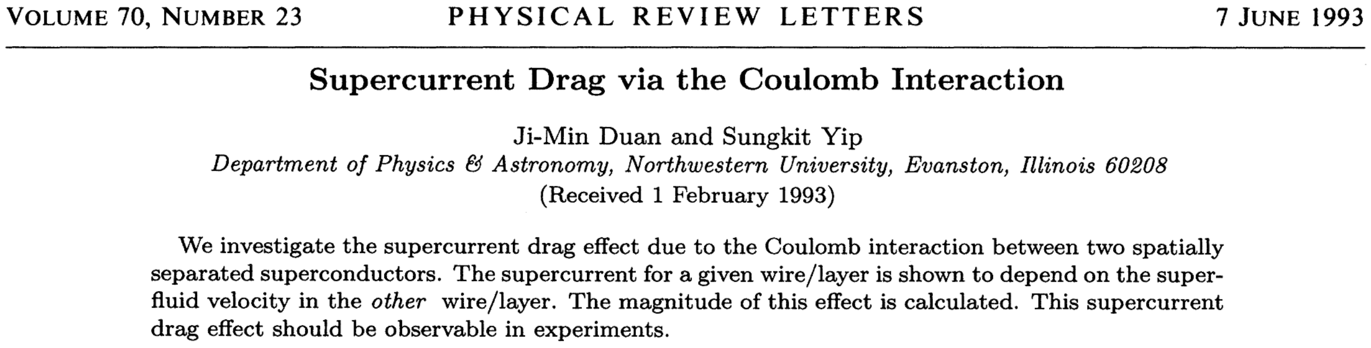




- Polarity: always negative, **carrier-polarity independent**
- Magnitude: anti-correlations between the drag signal and the carrier density of the drive layer

- **Momentum transfer mechanism**

Carrier polarity dependence


VOLUME 70, NUMBER 23 PHYSICAL REVIEW LETTERS 7 JUNE 1993

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Ji-Min Duan and Sungkit Yip
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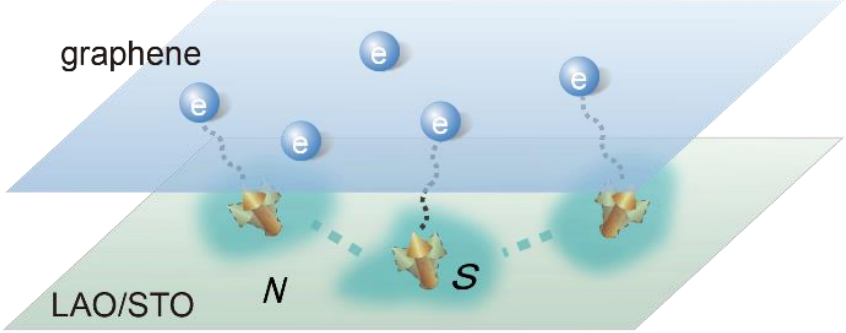
New mechanism: Josephson-Coulomb (JC) drag

Model: **2D electrons coupling to a 2D Josephson Junction (JJ) array via Coulomb interaction**



JC drag action

$$S_c = -\sum_j \int u_j(\boldsymbol{r})\rho(\boldsymbol{r},t)V_j(t)\mathrm{d}^2\boldsymbol{r}\mathrm{d}t$$

electrostatic interaction

$$u_j(\boldsymbol{r}) = (a^2 d/2\pi)[(\boldsymbol{r}-\boldsymbol{R}_j)^2+d^2]^{-3/2}$$

Charge density of graphene

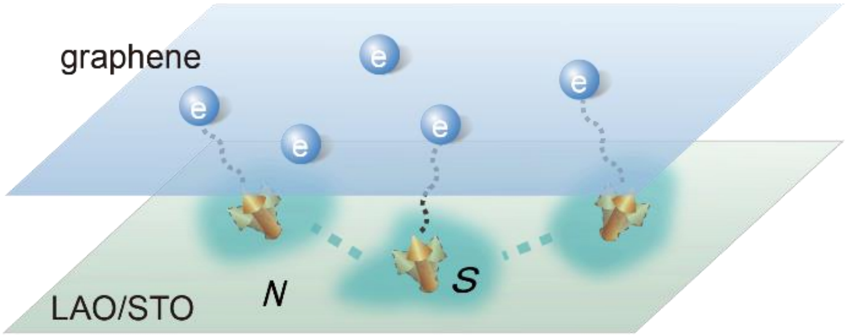
Electric potential at the Jth SC puddle

$$V_j(t) = -(\hbar/2q)\partial_t\varphi_j(t)$$

$\varphi_j(t)$: phase of the jth SC puddle

S_c: interactions between the SC phases and the graphene electrons

New mechanism: Josephson-Coulomb (JC) drag



Evolutions of the SC phases

Time-dependent electric potentials in graphene and thus charge density fluctuations

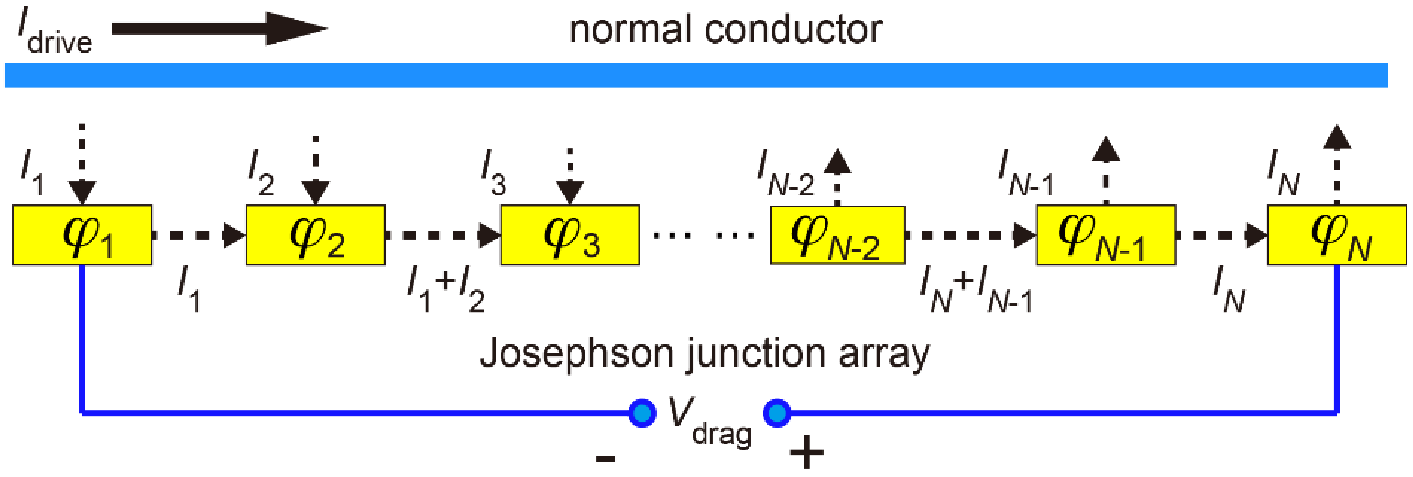
Cooper-pair tunnelings in the JJ array and thus SC phases variations

New category in drag physics

- Inherently nonequilibrium
- Quantum fluctuations dominating the interlayer processes

Josephson-Coulomb (JC) drag effect

JC drag processes between 1D JJ array and a graphene strip (a simplified model)



$$I_j = I_G\,[u_j(L)-u_j(0)]$$

Josephson energy between puddles:

PAR: $r \sim aNE_J(T)/d\max(|E_F|,T)$

$$E_J(T) = E_J(0)[1-(T/T_P)^2]\exp(-T/T_0)$$

superfluid density

Cooper-pair dephasing effect

Josephson-Coulomb (JC) drag effect

Prediction from the JC drag model

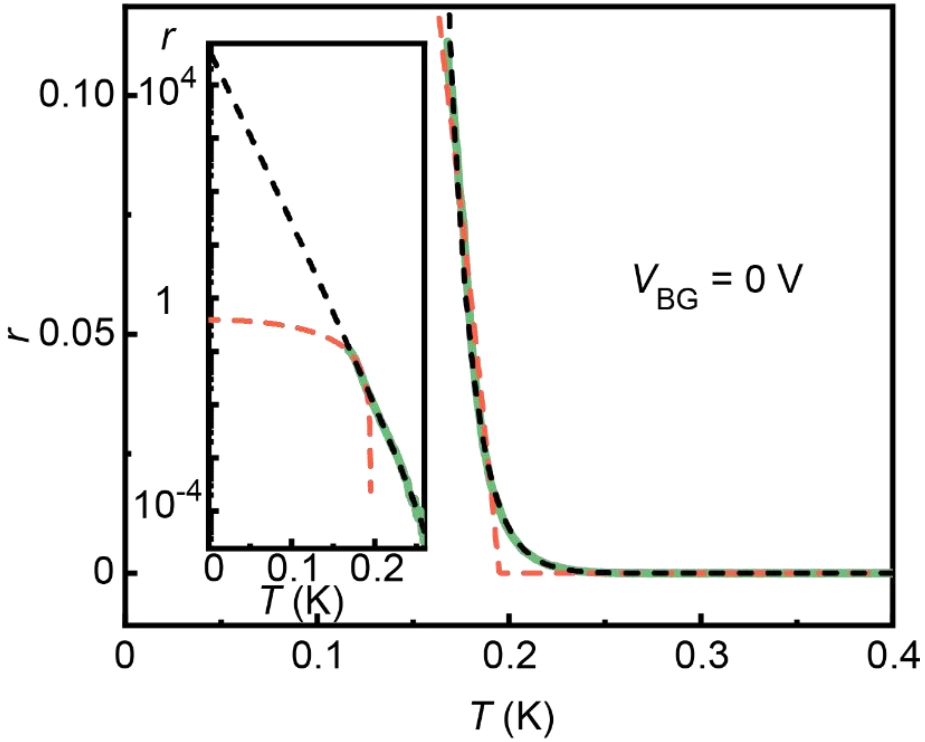
Polarity: independent of carrier type √
determined by the effective coupling $u_j(r)$

Magnitude: maximized as graphene approaching the Dirac point √
Coulomb interaction is less screened

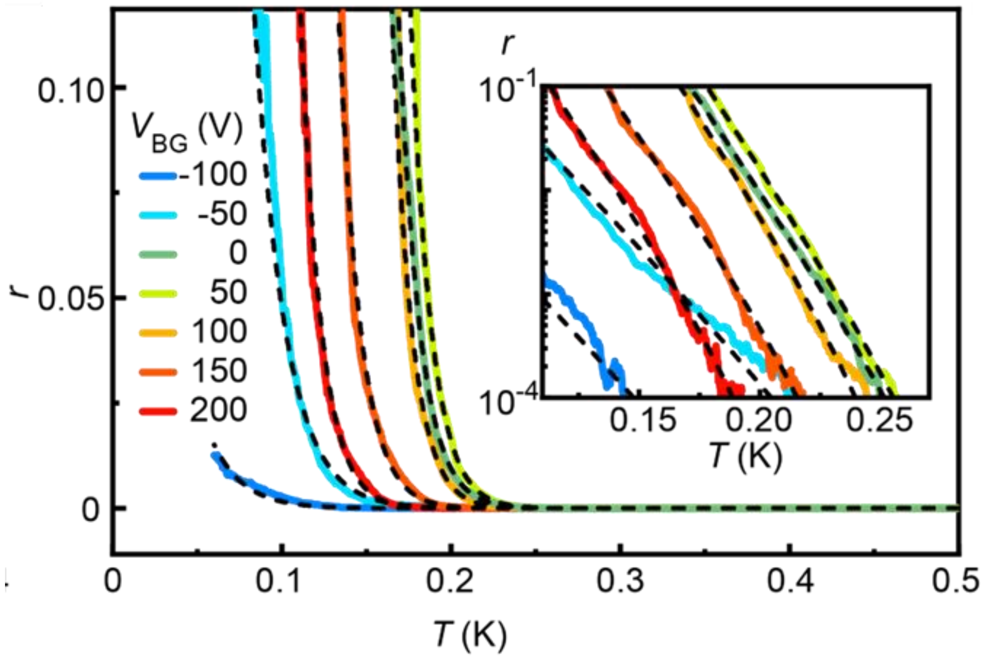
Temperature dependence: $\boldsymbol{r \sim r_0[1-(T/T^*)^2]\exp(-T/a)}$?

Quantitative analysis

☐ **Fitting of the *r* vs *T* curve**



☐ **Fittings at different V_{BG}**



Equation from JJ drag mechanism: $r = r_0[1-(T/T^*)^2]\exp(-T/a)$

Excellent consistency

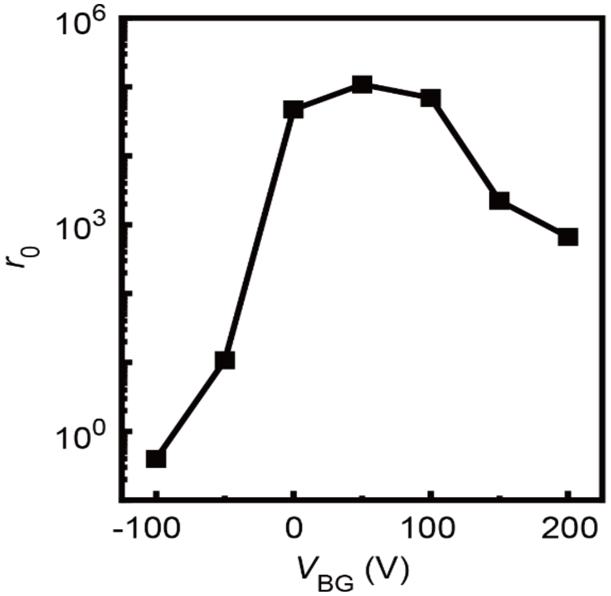
$$r_0 \sim aNE_J(0)/dE_F$$

Equation: $r = r_0[1-(T/T^*)^2]$ ❌

Giant passive-to-active ratio (PAR)

☐ Giant PAR

PAR which is not attainable below T_C now can be extrapolated down to zero temperature



$$\boldsymbol{r = r_0[1-(T/T^*)^2]\exp(-T/a)}$$

$$\boldsymbol{r_0 \sim aNE_J(0)/dE_F}$$

r_0 : PAR at the zero-temperature limit $\sim 10^5$

applying an active current in graphene can induce anstonishing passive current 10^5 times larger in the superconductor layer

- Giant amplification
- $E_J(0) >> E_F$
- Innumerable SC puddles ($N \gg 1$)
- Relatively large puddles compared to the interlayer distance ($a/d \gg 1$)

Summary

- Giant and highly gate-tunable drag responses
- Josephson-Coulomb (JC) drag mechanism

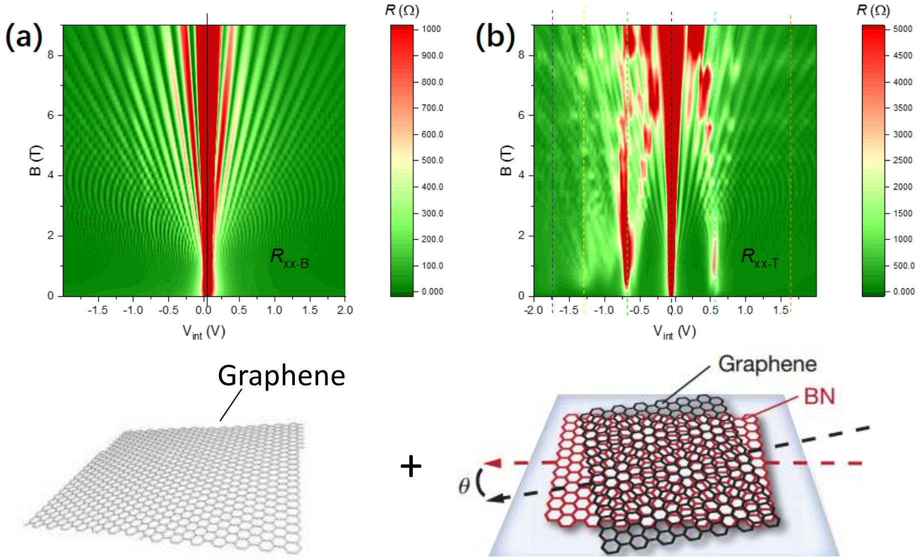
Effective Coulomb coupling between the quantum fluctuations of the SC phases in a superconductor and the charge densities in a normal conductor

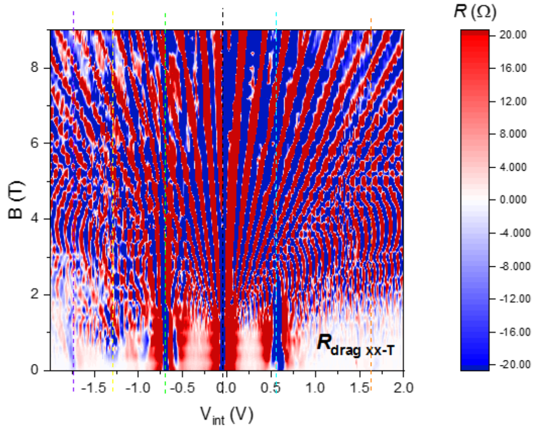
Ran Tao *et al.*, Nat. Phys. accepted

Unique role of quantum fluctuations in Superconducting drag

Perspective

- **Drag experiment:** Quasiparticle interactions, new drag effects, new correlated electronic states
- **Graphene based double layer electronic systems:** ideal and versatile platform
- **Newly-emerging 2D electronic systems beyond graphene :** more fascinating physics





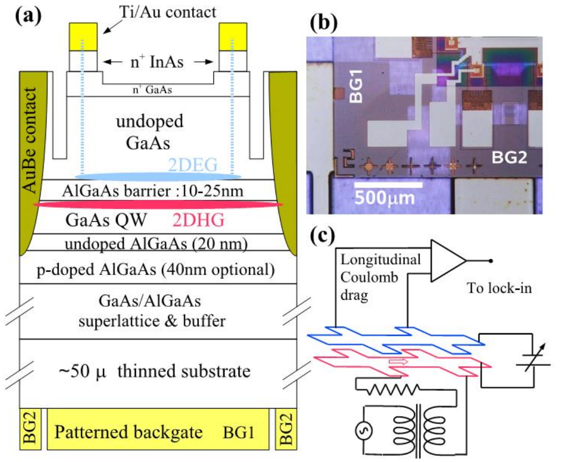
創寰宇學府 育天下英才

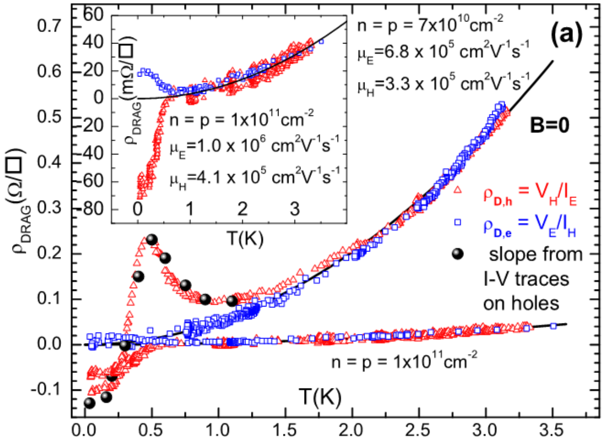
UNIVERSITY OF SCIENCE AND
TECHNOLOGY OF CHINA

Thank you

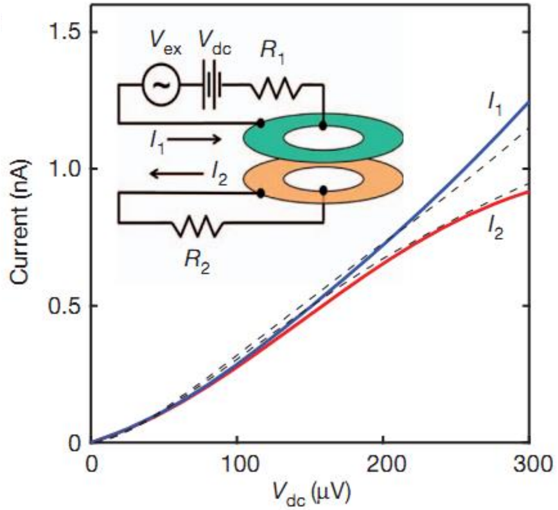
Typical systems

☐ GaAs/AlGaAs double-quantum well



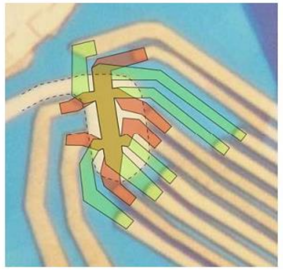


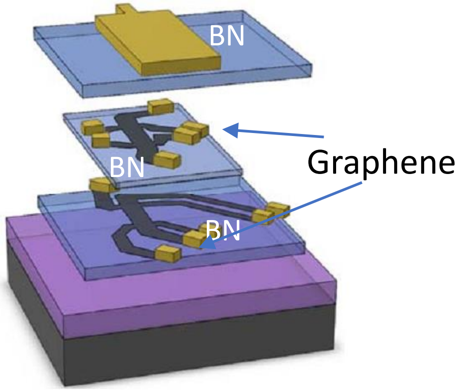
A. F. Croxall *et al.*, Phys. Rev. Lett. 101, 246801 (2008)



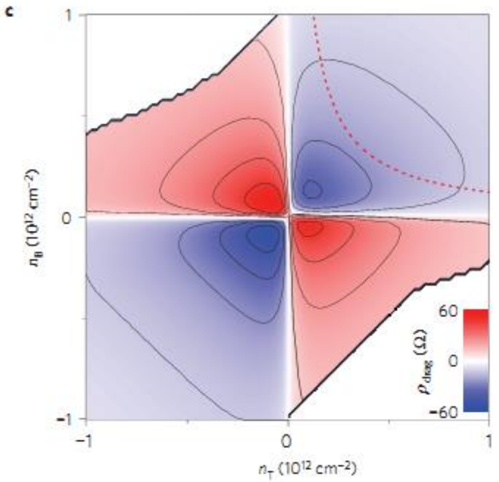
D. Namdi. *et al.*, Nature 488, 481(2012)

☐ Graphene based double-layer system

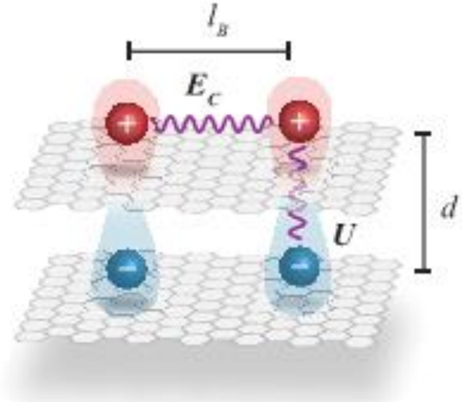


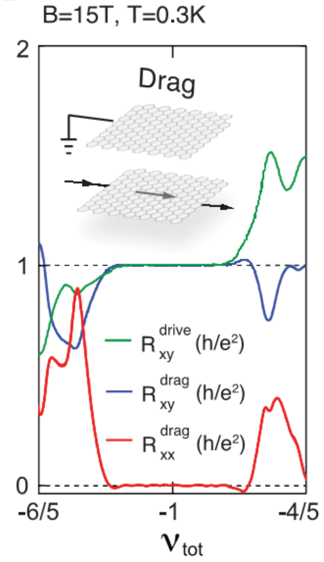


Stronger coupling & Higher tunability



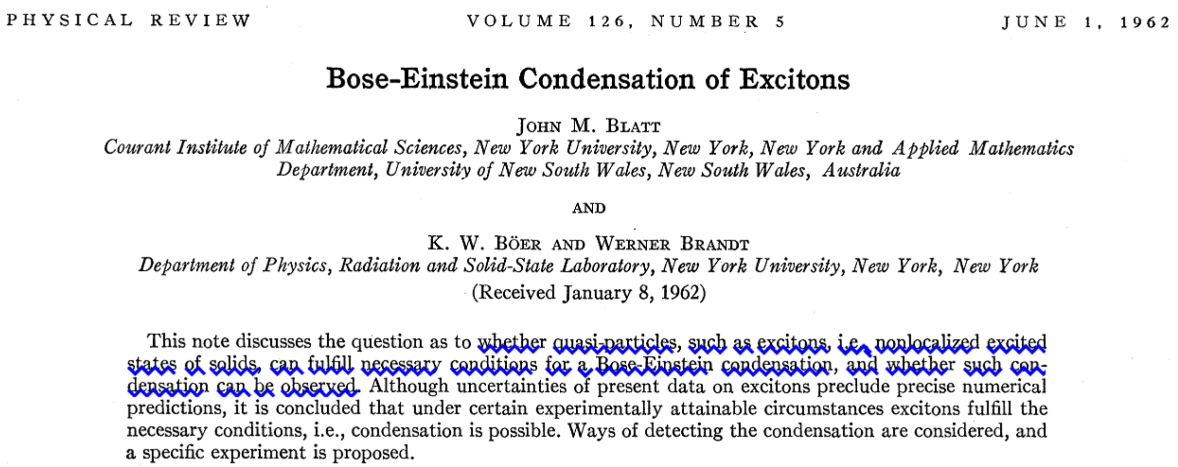
R. V. Gorbachev *et al.*, Nat. Phys. 8, 896 (2012)





X. Liu *et al.*, Science 375, 205 (2022)

Novel inter-layer correlated states



PHYSICAL REVIEW VOLUME 126, NUMBER 5 JUNE 1, 1962

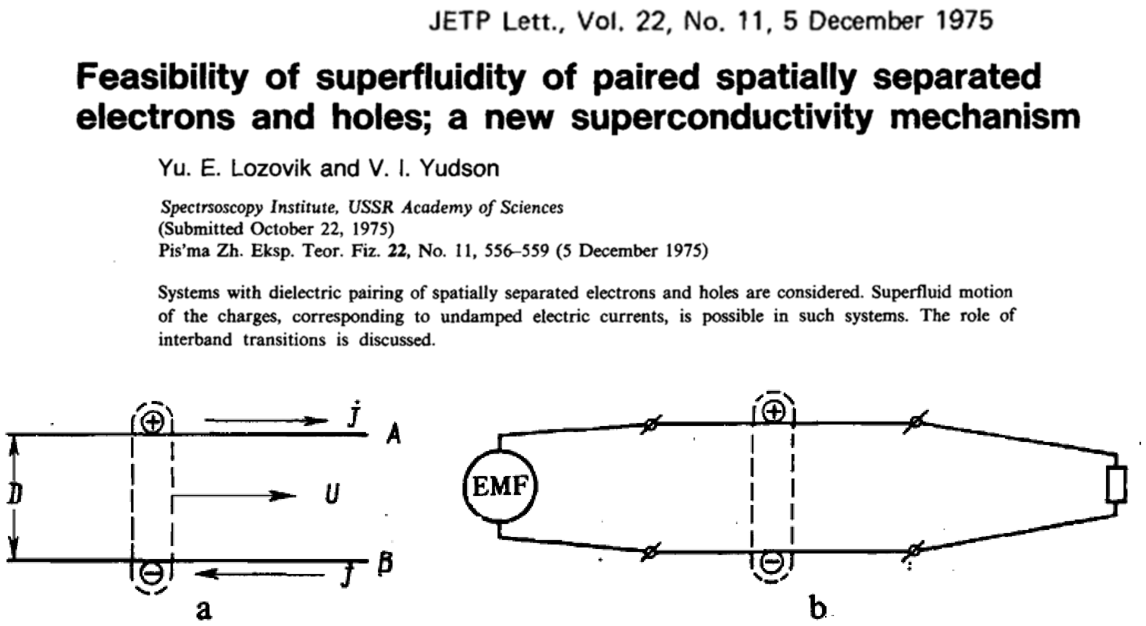
Bose-Einstein Condensation of Excitons

John M. Blatt
Courant Institute of Mathematical Sciences, New York University, New York, New York and Applied Mathematics Department, University of New South Wales, New South Wales, Australia

and

K. W. Böer and Werner Brandt
Department of Physics, Radiation and Solid-State Laboratory, New York University, New York, New York
(Received January 8, 1962)

This note discusses the question as to whether quasi-particles, such as excitons, i.e., nonlocalized excited states of solids, can fulfill necessary conditions for a Bose-Einstein condensation, and whether such condensation can be observed. Although uncertainties of present data on excitons preclude precise numerical predictions, it is concluded that under certain experimentally attainable circumstances excitons fulfill the necessary conditions, i.e., condensation is possible. Ways of detecting the condensation are considered, and a specific experiment is proposed.



JETP Lett., Vol. 22, No. 11, 5 December 1975

Feasibility of superfluidity of paired spatially separated electrons and holes; a new superconductivity mechanism

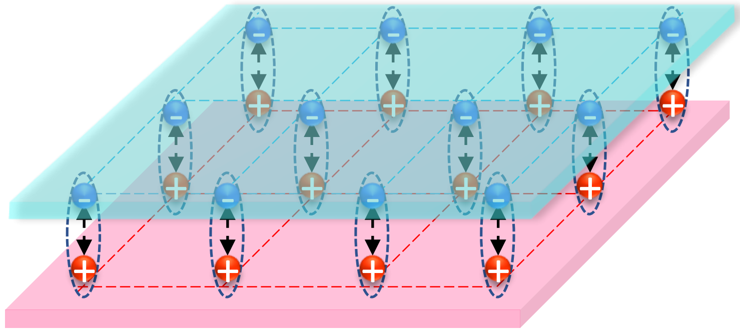
Yu. E. Lozovik and V. I. Yudson

Spectrsoscopy Institute, USSR Academy of Sciences
(Submitted October 22, 1975)
Pis'ma Zh. Eksp. Teor. Fiz. **22**, No. 11, 556–559 (5 December 1975)

Systems with dielectric pairing of spatially separated electrons and holes are considered. Superfluid motion of the charges, corresponding to undamped electric currents, is possible in such systems. The role of interband transitions is discussed.

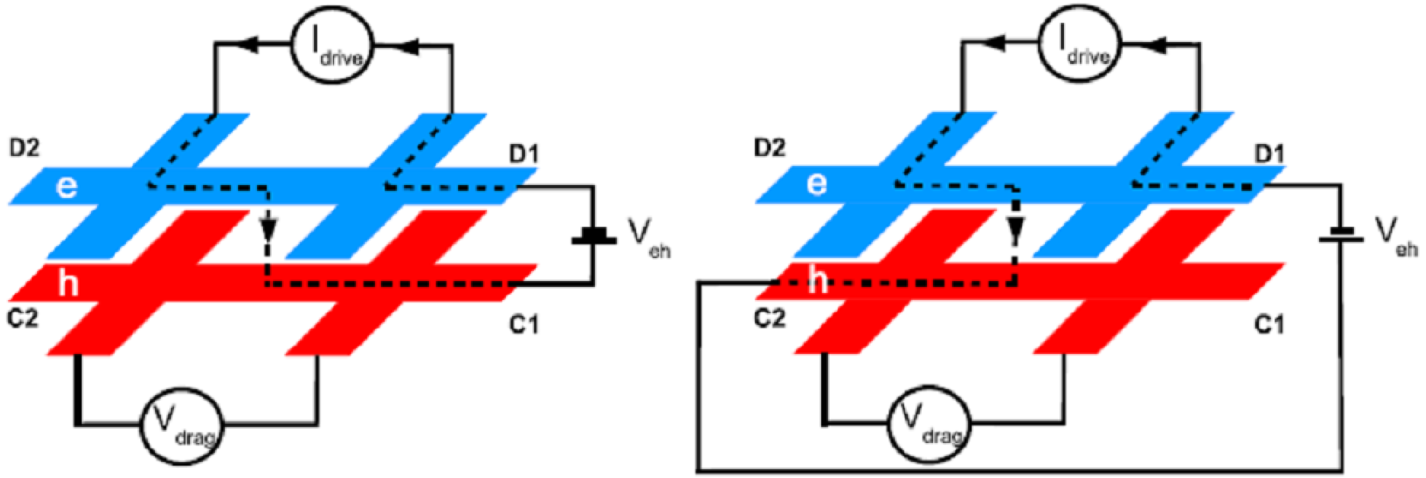
Separating the electrons and holes spatially

- ➢ Preventing recombination, long lifetime
- ➢ No need to pump, a true ground state
- ➢ **Possible superfluid state**



Challenges in drag measurements

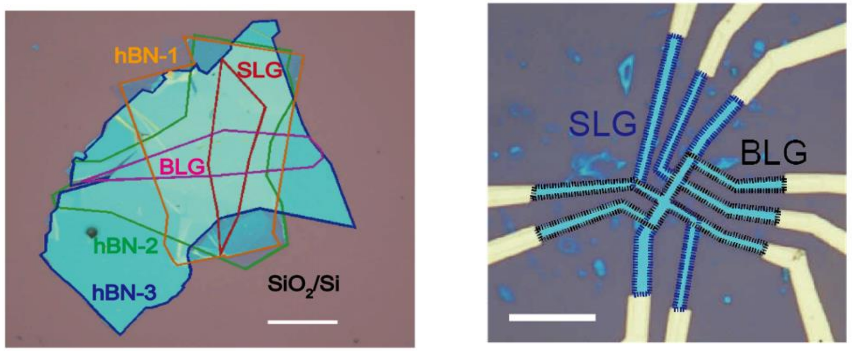
- **Independent contacts for the two layers**
- **Small inter-layer spacing, while very low interlayer leakage**
- Low carrier densities for both layers
- Gate-tunable for both layers



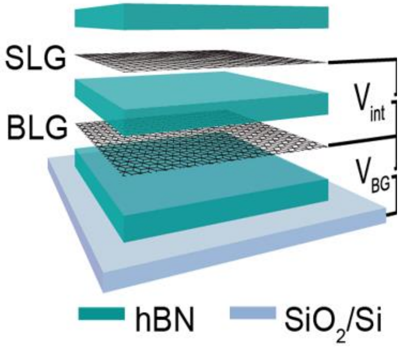
Leakage check: shift the bias points and check if the signal changes

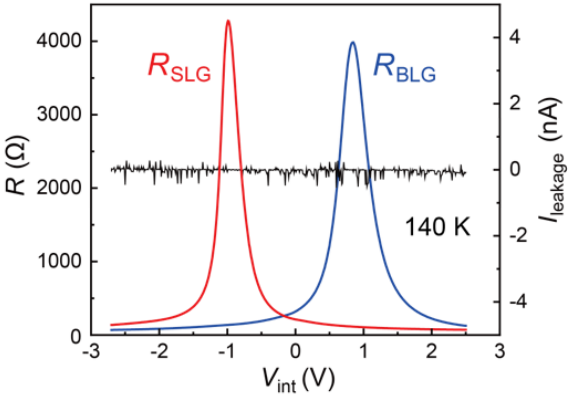
Device and basic drag characterizations

☐ SLG/BLG device

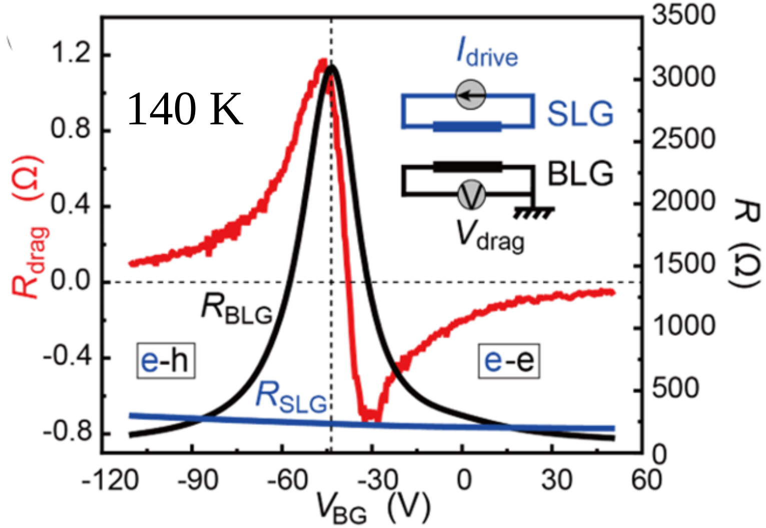


Inter-layer thickness ~3.5 nm



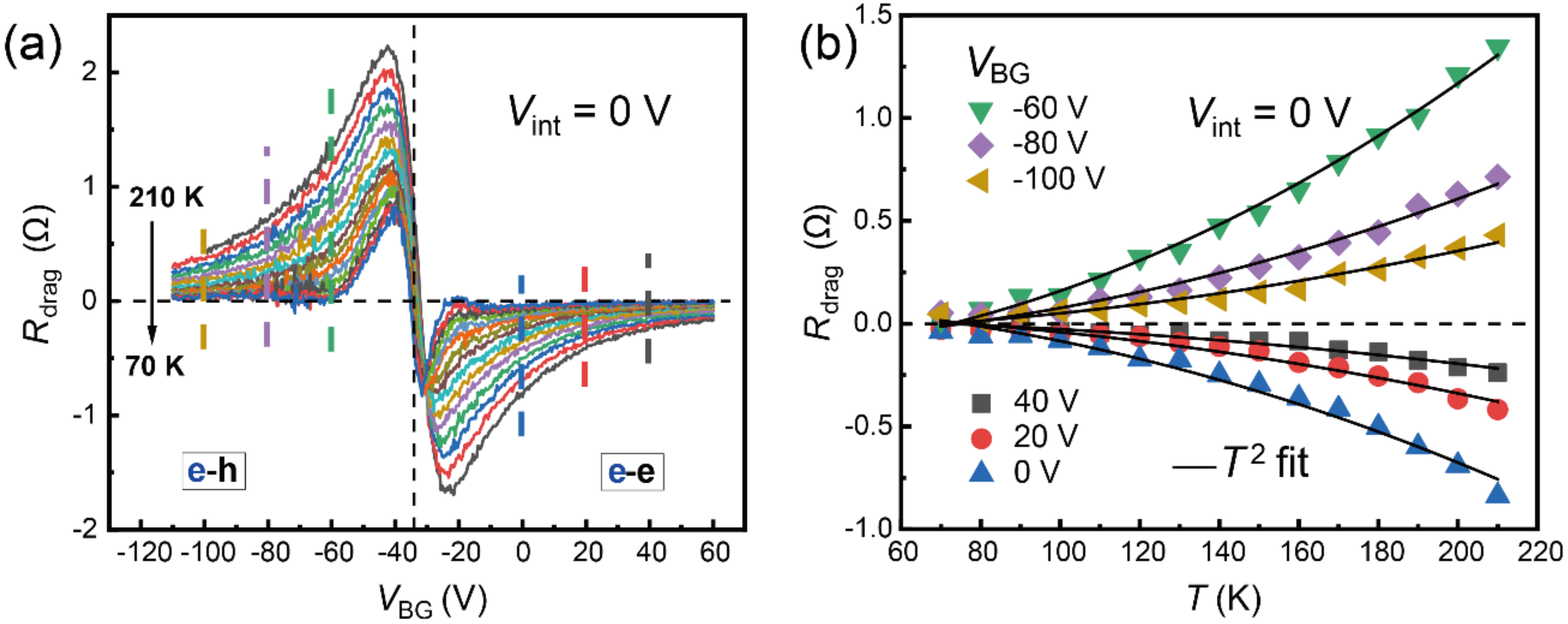


- Effective field effect
- Negligible inter-layer leakage current



- R_{Drag}: nonmonotonic carrier density dependent
- Momentum drag theory considering disorder-scattering

Temperature-dependent characters



Away from the CNP (charge neutrality point)

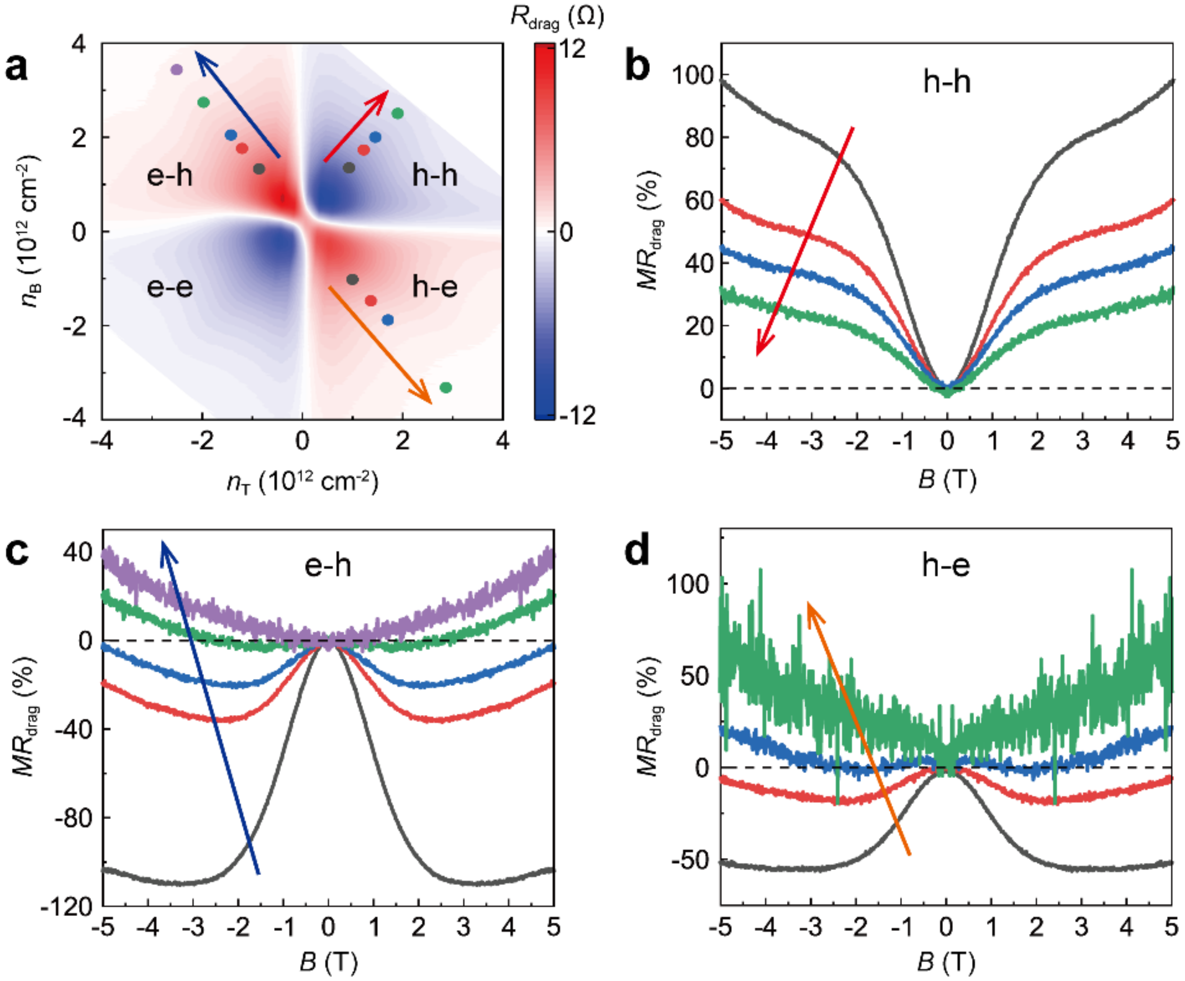
- e-e: negative e-h: positive
- $R_{\mathrm{drag}} \propto T^2$

Coulomb scattering mechanism in Fermi liquid regime

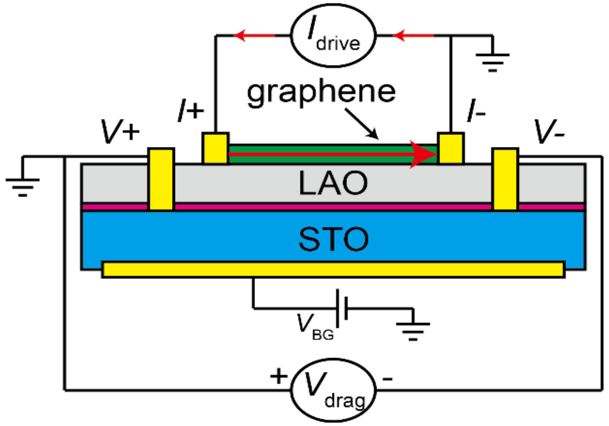
$E_F >> k_B T,\ k_F d >> 1 \qquad k_F = \sqrt{\pi n}$

Carrier density dependence

- ☐ **Similar evolution behavior in other three regions**

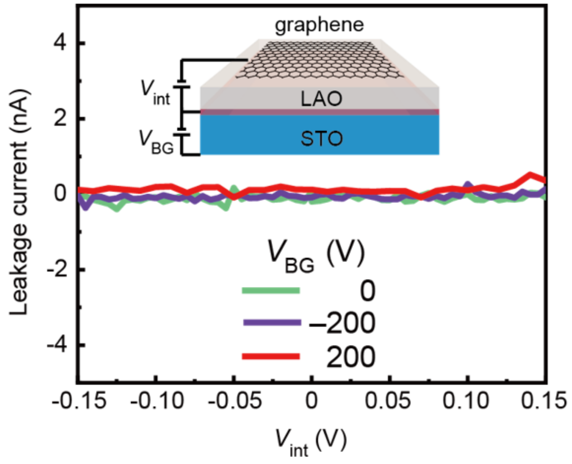


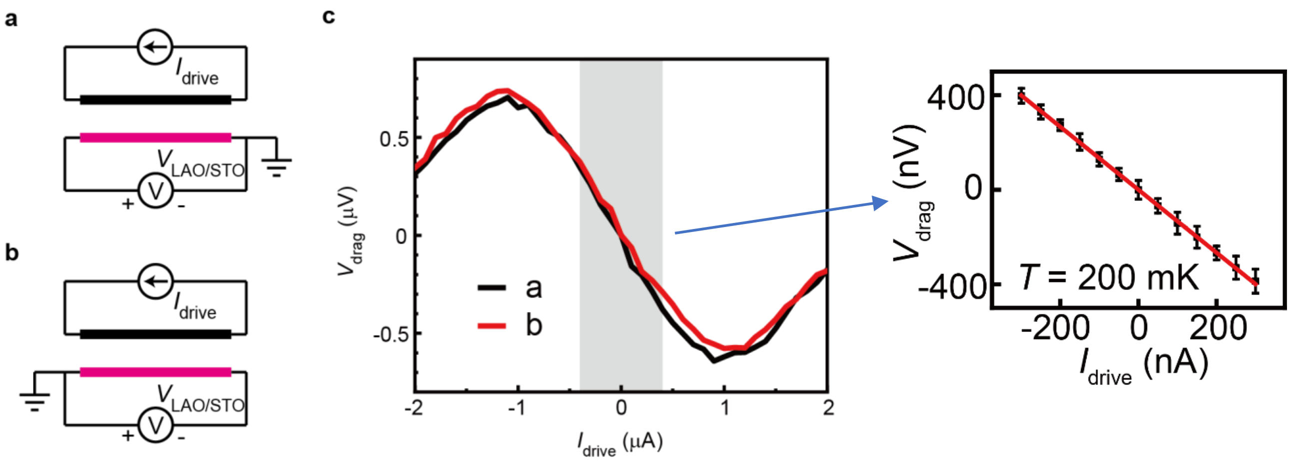
Drag experiment



☐ Negligible inter-layer leakage

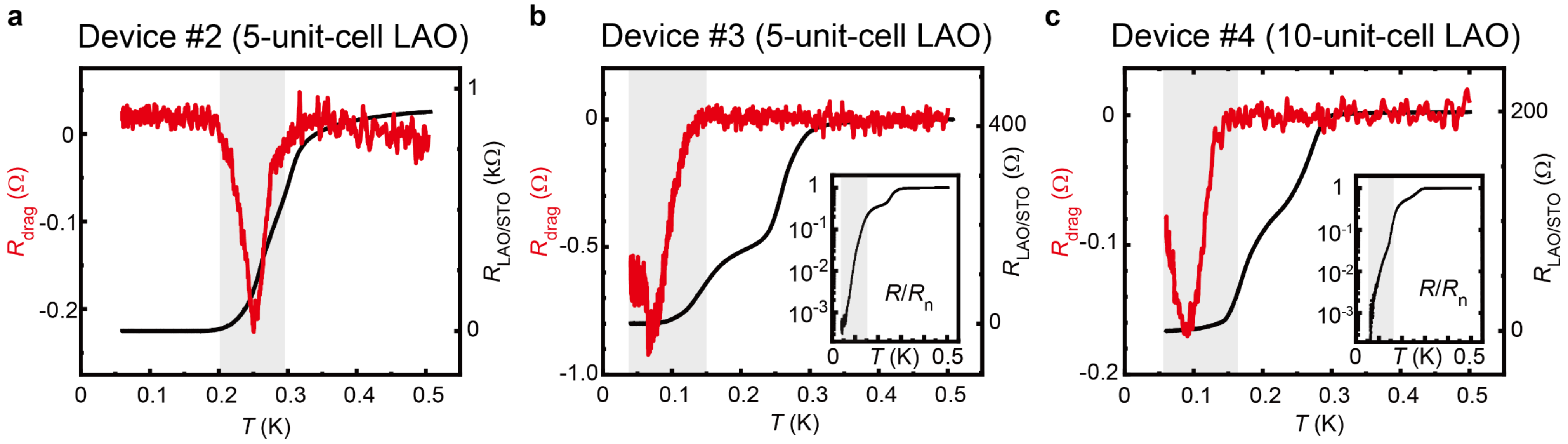
☐ Good linear dependence of V_{drag}-I_{drive}





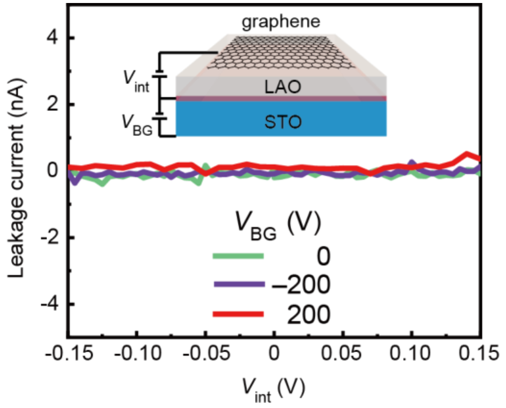
Repeatability

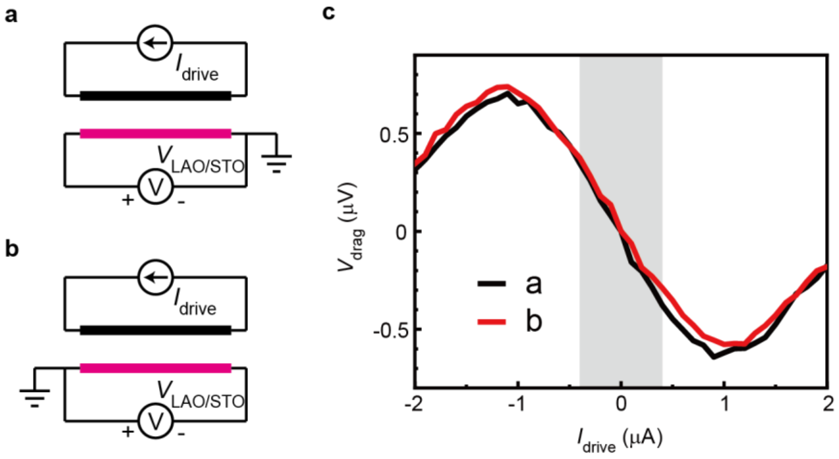
Typical characteristics are well reproduced in other devices.



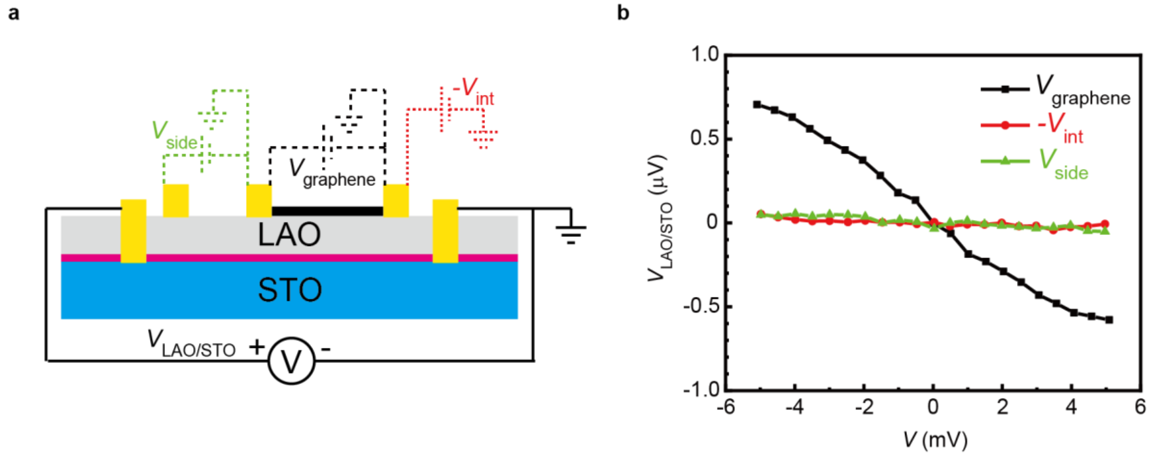
Validity check

- Exclusion of the impact of inter-layer leakage/tunneling

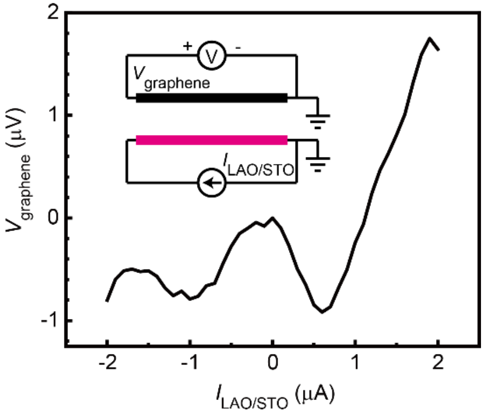




- Exclusion of the impact of electrostatic field

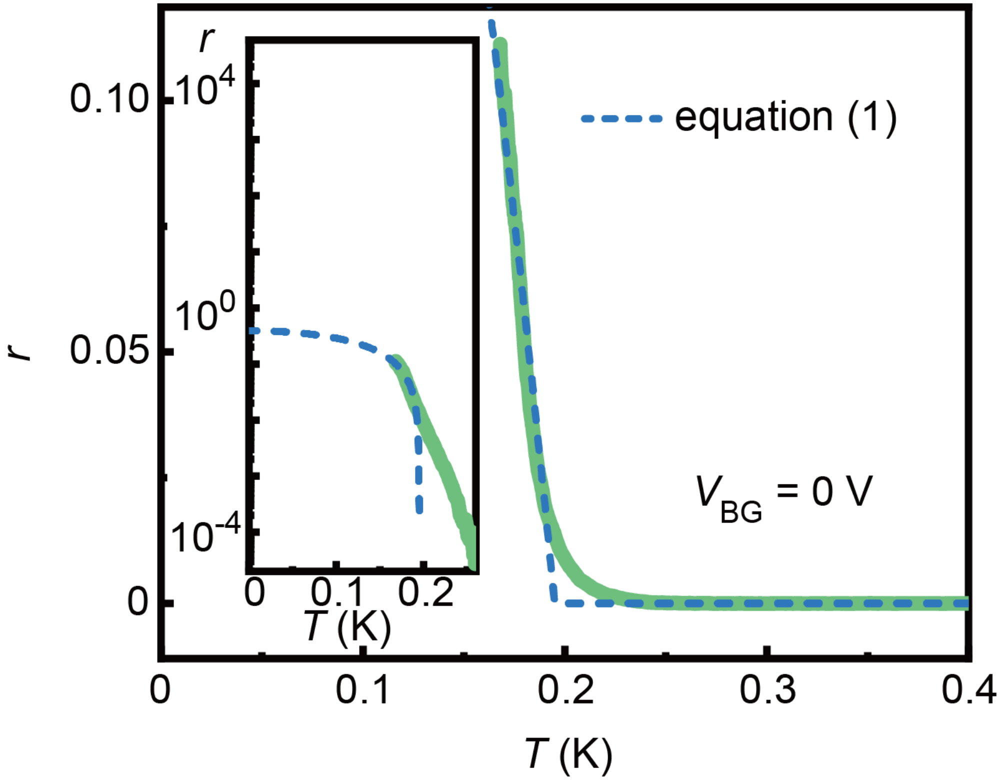


- Drag response when graphene serves as the drag layer



Quantitative analysis

☐ **Fitting of the *r* vs *T* curve？**



Equation (1): $r = r_0[1-(T/T^*)^2]$

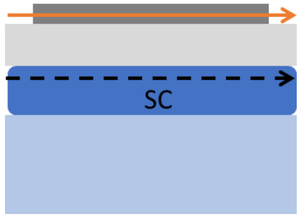
—a clear deviation occurs…

Other issues? e.g. the inhomogeneity of superconductivity

- r increases with decreasing temperature

  - consistent with the enhanced superconductivity

- The superfluid density of **a homogeneous superconductor**:

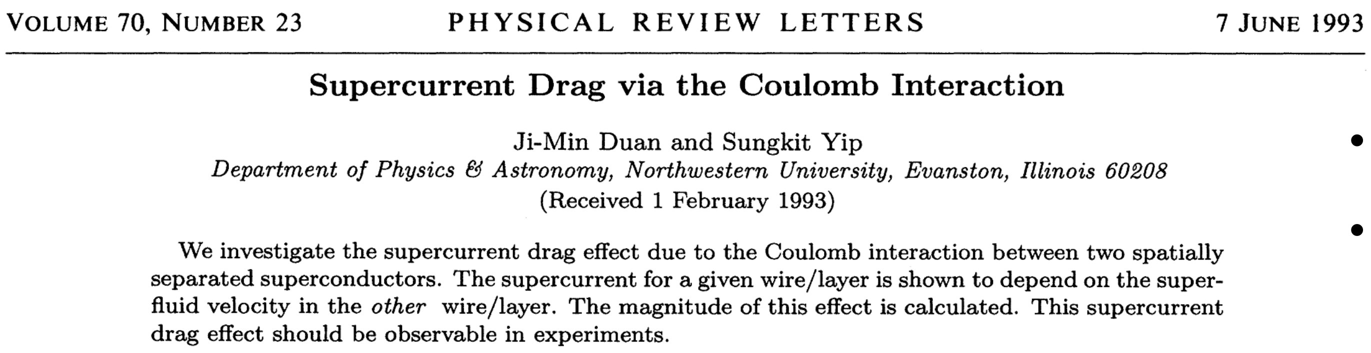


$$n_s = n_0[1-(T/T_{SC})^b]$$

n_0: superfluid density at absolute zero.
T_{SC}: temperature when Cooper pairs start to emerge.
b=2 for a s-wave superconductor.

Possible explanations?

☐ Coulomb Interaction



VOLUME 70, NUMBER 23 PHYSICAL REVIEW LETTERS 7 JUNE 1993

Supercurrent Drag via the Coulomb Interaction

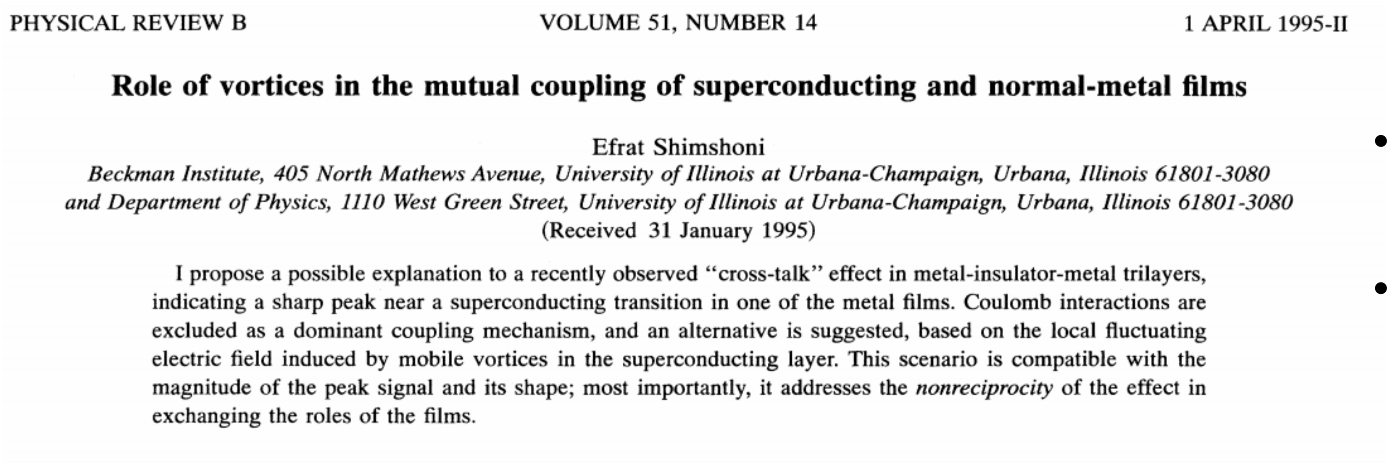
Ji-Min Duan and Sungkit Yip
Department of Physics & Astronomy, Northwestern University, Evanston, Illinois 60208

(Received 1 February 1993)

We investigate the supercurrent drag effect due to the Coulomb interaction between two spatially separated superconductors. The supercurrent for a given wire/layer is shown to depend on the superfluid velocity in the *other* wire/layer. The magnitude of this effect is calculated. This supercurrent drag effect should be observable in experiments.

- Based on the inter-layer momentum transfer
- V_{drag} should be **negative/positive** when the carrier polarity in the two layers is the **same/opposite**

☐ Magnetic Induction



PHYSICAL REVIEW B VOLUME 51, NUMBER 14 1 APRIL 1995-II

Role of vortices in the mutual coupling of superconducting and normal-metal films

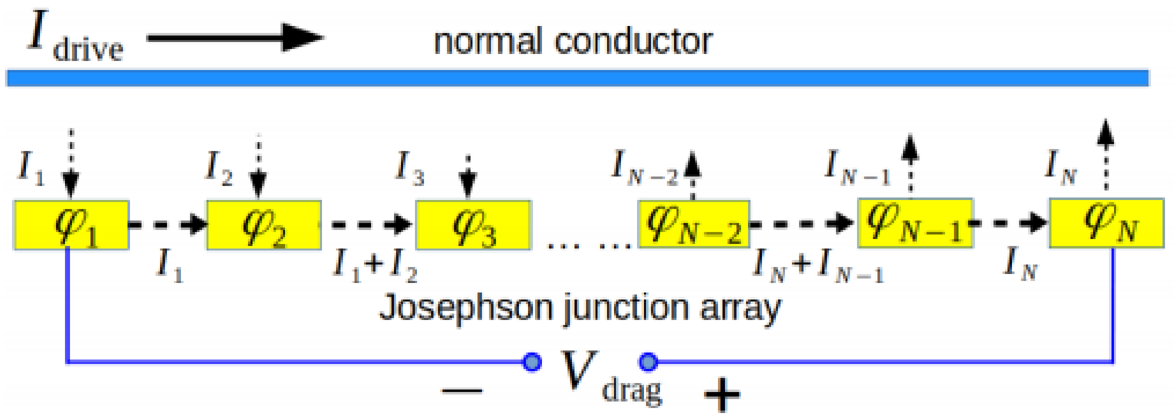
Efrat Shimshoni
Beckman Institute, 405 North Mathews Avenue, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801-3080
and Department of Physics, 1110 West Green Street, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801-3080

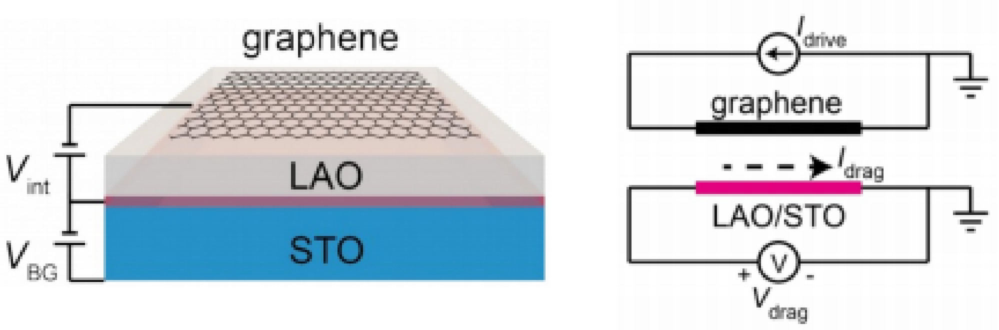
(Received 31 January 1995)

I propose a possible explanation to a recently observed "cross-talk" effect in metal-insulator-metal trilayers, indicating a sharp peak near a superconducting transition in one of the metal films. Coulomb interactions are excluded as a dominant coupling mechanism, and an alternative is suggested, based on the local fluctuating electric field induced by mobile vortices in the superconducting layer. This scenario is compatible with the magnitude of the peak signal and its shape; most importantly, it addresses the *nonreciprocity* of the effect in exchanging the roles of the films.

- Key factor: local fluctuating electric field induced by mobile vortices in the superconductor layer;
- V_{drag} should be **always positive**.

Josephson-Coulomb drag effects





$$S_n[\bar{\Psi}_n,\Psi_n]/\hbar = \int_\tau\int_{\mathbf{r}} \bar{\Psi}_n(\mathbf{r},\tau)\,(\hbar\partial_\tau + H_n)\,\Psi_n(\mathbf{r},\tau) + \frac{1}{2}\int_\tau\int_{\mathbf{r},\mathbf{r}'} \rho_n(\mathbf{r},\tau)U(\mathbf{r}-\mathbf{r}')\rho_n(\mathbf{r}',\tau)$$

$$S_s/\hbar = \sum_j \int_t \frac{\hbar^2}{2E_C}(\partial_t\varphi_j)^2 - \sum_{\langle j,k\rangle}\int_t E_J(\varphi_j-\varphi_k) + S_D[\{\varphi_j\}],$$

$$S_c/\hbar = -\frac{\hbar}{2e_s}\int_t\sum_j\int_{\mathbf{r}} u_j(\mathbf{r})\varphi_j(t)\dot{\rho}_n(\mathbf{r},t), \quad u_j(\mathbf{r}) = \frac{a_j^2 d}{2\pi}\frac{1}{[(\mathbf{r}-\mathbf{R}_j)^2+d^2]^{3/2}}$$

$$I_j(t) = \int d^2\mathbf{r}\, u_j(\mathbf{r})\,\dot{\rho}_g(\mathbf{r},t) \qquad V(\mathbf{r},t) = -\sum\nolimits_j u_j(\mathbf{r})\frac{\hbar}{2e}\dot{\varphi}_j(t)$$

- **New category in drag physics**:

  Inherently nonequilibrium
- **Quantum-fluctuation-dominant**:

  Distinct from the Casimir effect
- An important piece to **modern SC electronics:**

  Current (voltage) transformer when conductor (SC) is active; Synchronize *terahertz radiators* based on JJ arrays