Collider Physics

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Deep Inelastic Scattering and Structure Functions

1. Introduction to Deep Inelastic Scattering, Kinematics & Variables
2. Probing the Structure of Nucleons
3. Elastic & Inelastic Scattering
4. The Bjorken Scaling Variable
5. Form Factors
6. Structure Functions in the Parton Model
7. $F_2(x)$
8. Colour and Hadrons
9. Running Coupling Constant α_s
10. Asymptotic Freedom and Confinement
11. Scaling Violations
12. DGCLAP Equations

Scattering of electrons on nucleus/proton

Calculation	electron	Electron with spin	Point-like target, infinite Mass	Point-like target with mass M	Point-like proton	Point-like proton with spin	Finite size proton with spin	Expression
	electron		Target, charge Ze (Z=1 proton)					
Rutherford	✅		✅					$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{Z^2e^4}{4E_0^2\,(\sin\theta/2)^4}$
Mott		✅		✅				$\left(\frac{d\sigma}{d\Omega}\right)_M = \left(\frac{d\sigma}{d\Omega}\right)_R \cdot \left(\cos\frac{\theta}{2}\right)^2$
σ_{NS}		✅			✅			$\left(\frac{d\sigma}{d\Omega}\right)_{NS} = \left(\frac{d\sigma}{d\Omega}\right)_M \cdot 1/\left(1 - \frac{2E_0}{M}\sin\theta/2^2\right)$
σ		✅				✅		$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_M \cdot \left(1 + \frac{q^2}{2M^2}\tan\theta/2^2\right)$
Rosenbluth		✅					✅	$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_M \cdot \left[\frac{G_E^2(Q^2)+\tau\cdot G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan\theta/2^2\right]$

e nucleus ⟺ e proton

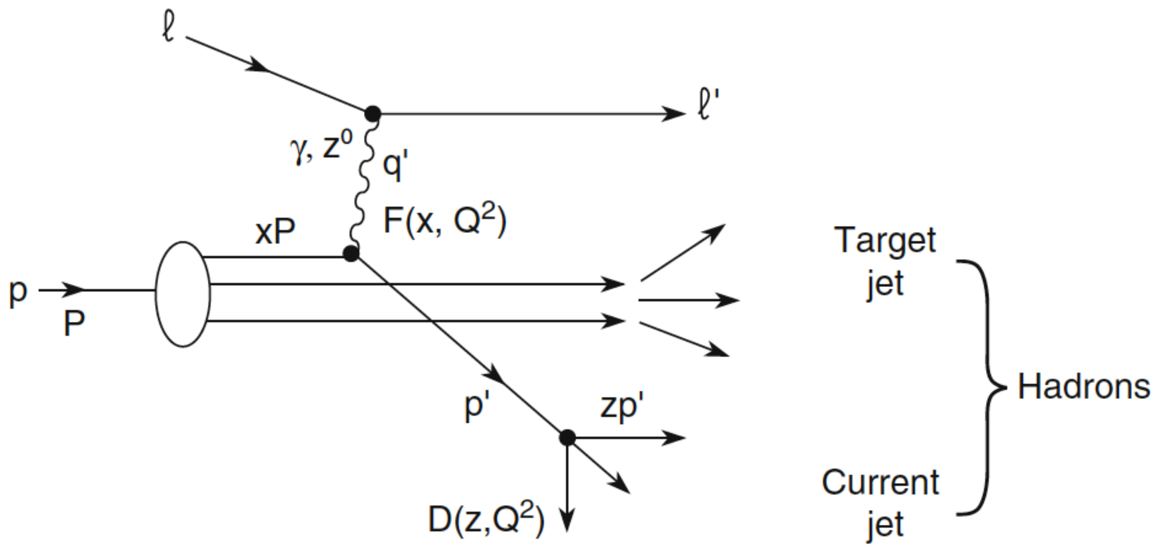
Why Deep Inelastic Scattering?

This course is titled "Collider Physics"

- Colliders are today the most powerful instrument to study the innermost structure of matter
- Proton-proton colliders are the accelerators that can reach the highest energies, for reasons that will be clear when discussing about accelerators
- Proton are very complex objects, with a complex internal structure
- The interpretation of scattering experiments need to be based on the understanding of the proton structure
- The scattering lepton-nucleon allows us to study the structure of the proton

Deep Inelastic Scattering
→ *DIS*

Basis of QCD, the theory of hadronic interactions



Many generation of scattering experiments.

- Initially they used leptons (mostly electrons) produced in accelerators and sent on a target
- The last generation was the HERA collider at Desy, Germany

30 GeV electrons against 900 GeV protons

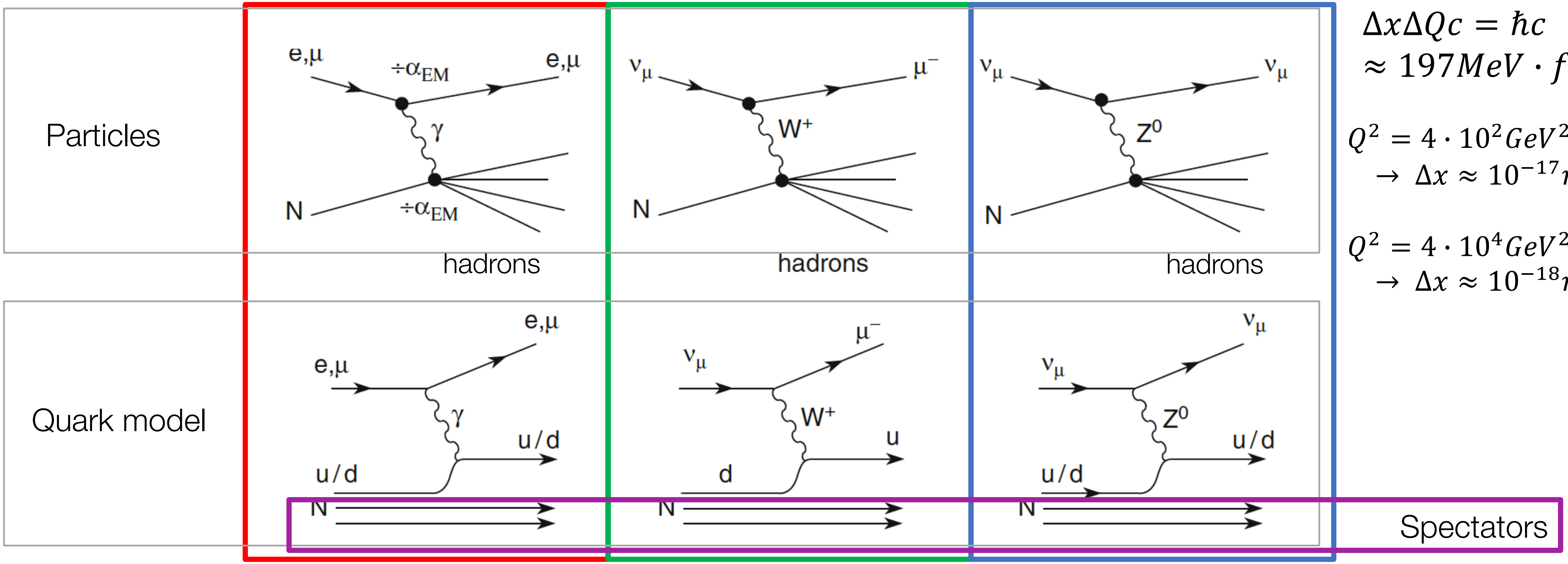
Inelastic lepton-nucleus scattering

$$ep: \quad e^{\pm} + p \rightarrow e^{\pm} + X^{+}$$

$$\mu p: \quad \mu^{\pm} + p \rightarrow \mu^{\pm} + X^{+}$$

$$\nu_\mu p(CC): \nu_\mu + p \rightarrow \mu^{-} + X^{++}, \; \overline{\nu}_\mu + p \rightarrow \mu^{+} + X^{0}$$

$$\nu_\mu p(NC): \quad \nu_\mu + p \rightarrow \nu_\mu + X^{+}, \; \overline{\nu}_\mu + p \rightarrow \overline{\nu}_\mu + X^{+}.$$



$$\Delta x \Delta Q c = \hbar c \approx 197 MeV \cdot fm$$

$$Q^2 = 4 \cdot 10^2 GeV^2/c^2 \rightarrow \Delta x \approx 10^{-17} m$$

$$Q^2 = 4 \cdot 10^4 GeV^2/c^2 \rightarrow \Delta x \approx 10^{-18} m$$

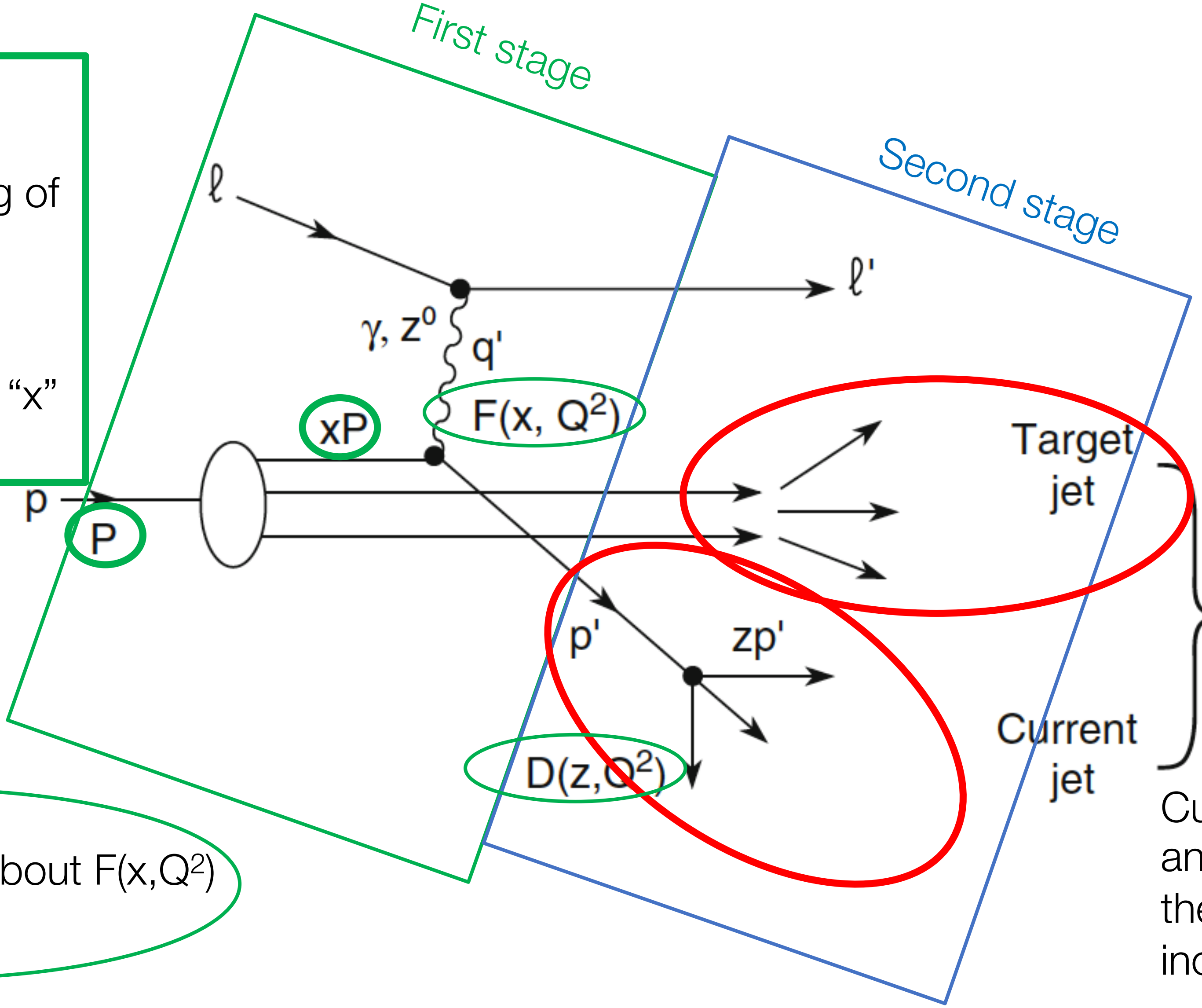
N = p / n

Electromagnetic | Weak Charged Current | Weak Neutral Current

The Story of an Inelastic Lepton-Nucleon Scattering

First stage:

~ elastic scattering of the virtual particle (boson) with one quark of the N carrying a fraction "x" of the N



Second stage:

Fragmentation → quarks cannot exist 'alone' → 'dress' into two jets

Target jet ~ in the same direction of the incoming lepton

Hadrons

Current jet ~ at large angle with respect to the direction of the incoming lepton

Later we will talk about $F(x,Q^2)$ and $D(z,Q^2)$

Deep Inelastic Scattering, Kinematics & Variables

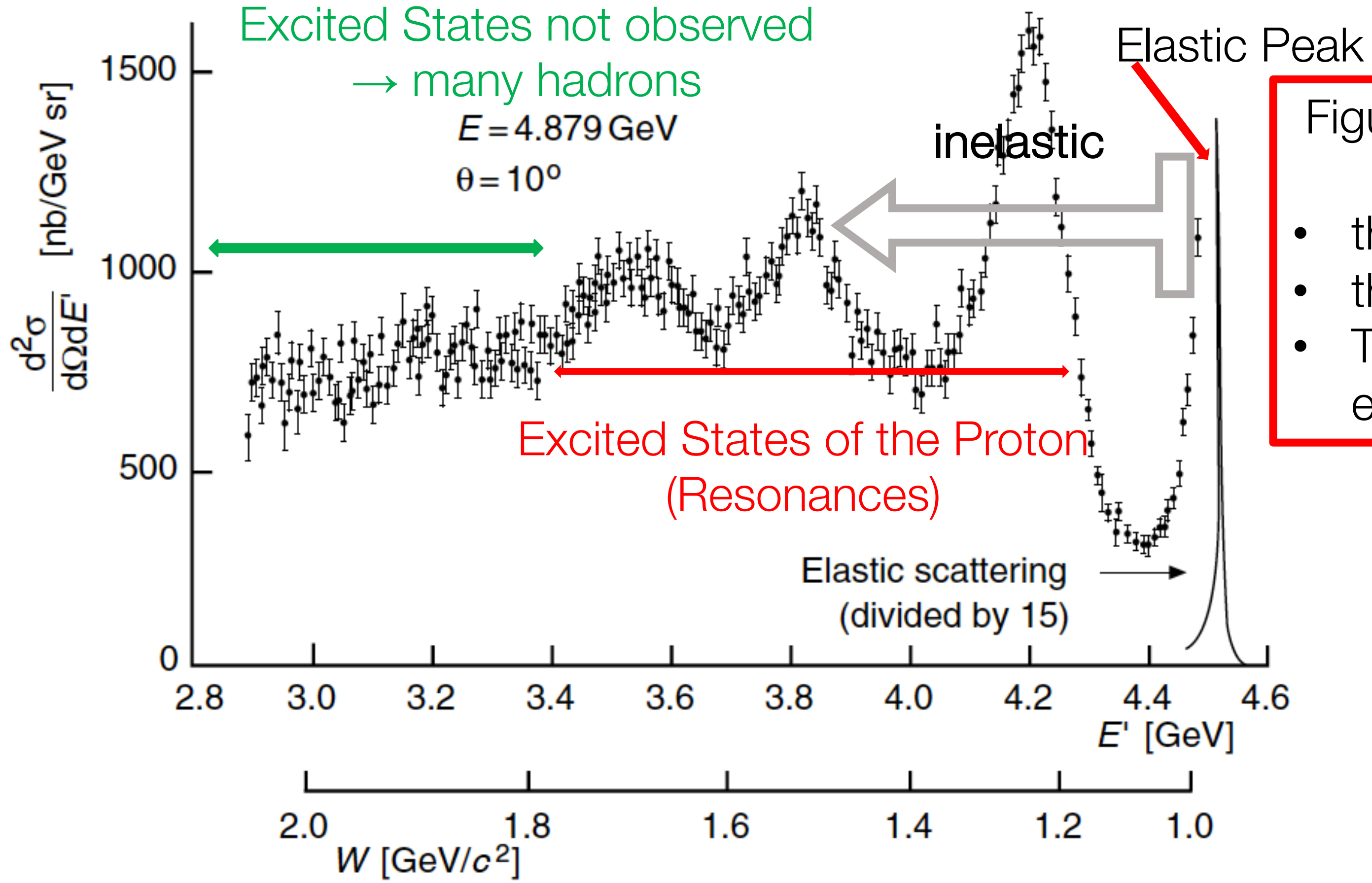


Figure ← electron-proton scattering.

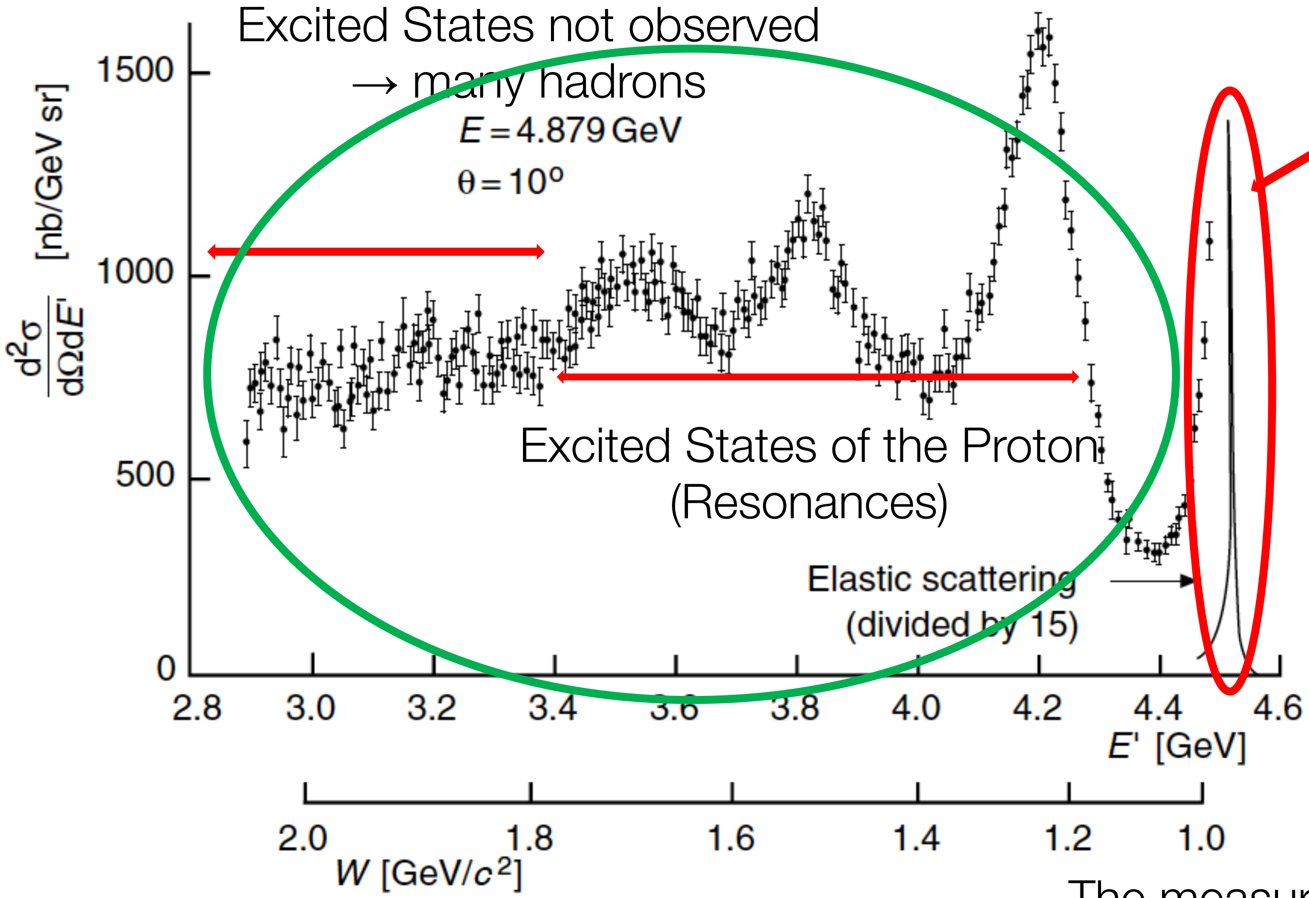
- the incoming electron energy was E = 4. 9 GeV
- the scattering electron angle was fixed to θ = 10°
- The electron scattering energy is shown (part of the energy to the proton!)

We see

- sharp elastic scattering peak (scaled down by 15)
- peaks at lower electron energies associated with inelastic excitations (excited states of the nucleon which we call nucleon resonances.
- Further down in energy, states with many hadrons

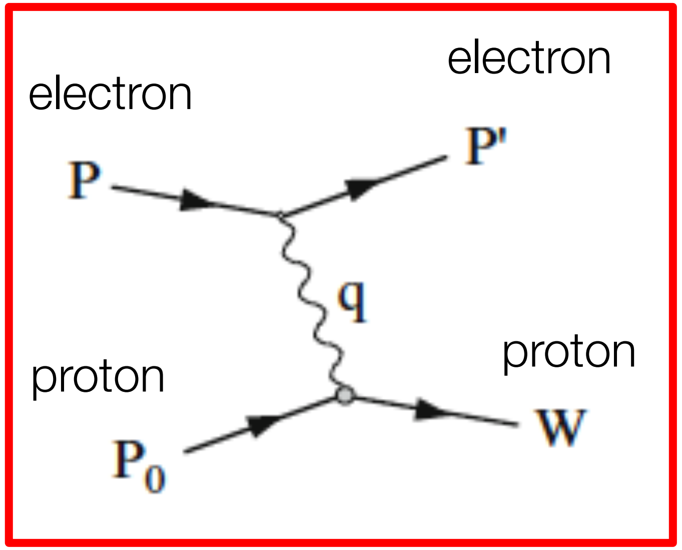
These excited states of the proton are an indication that **the proton is a composite system.** → quark model.

Deep Inelastic Scattering, Kinematics & Variables



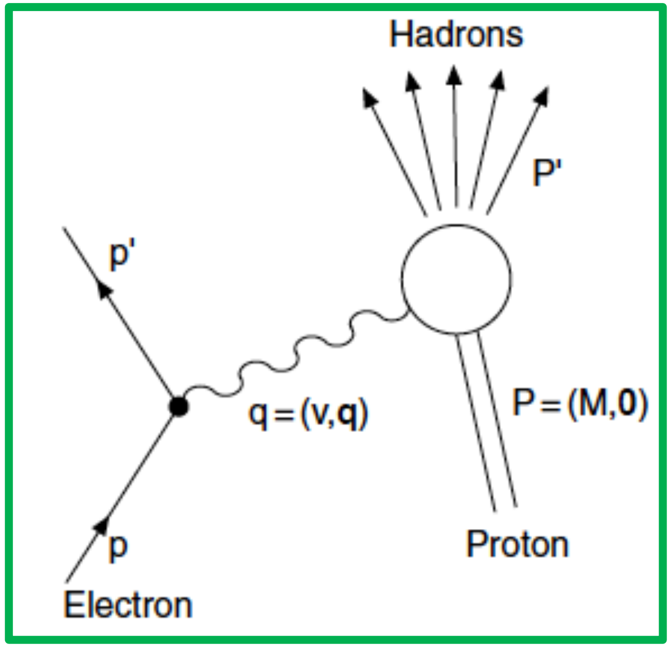
Elastic Peak → 2 particles in the final state./

The measurement of the electron scattering angle determines the kinematics of the event



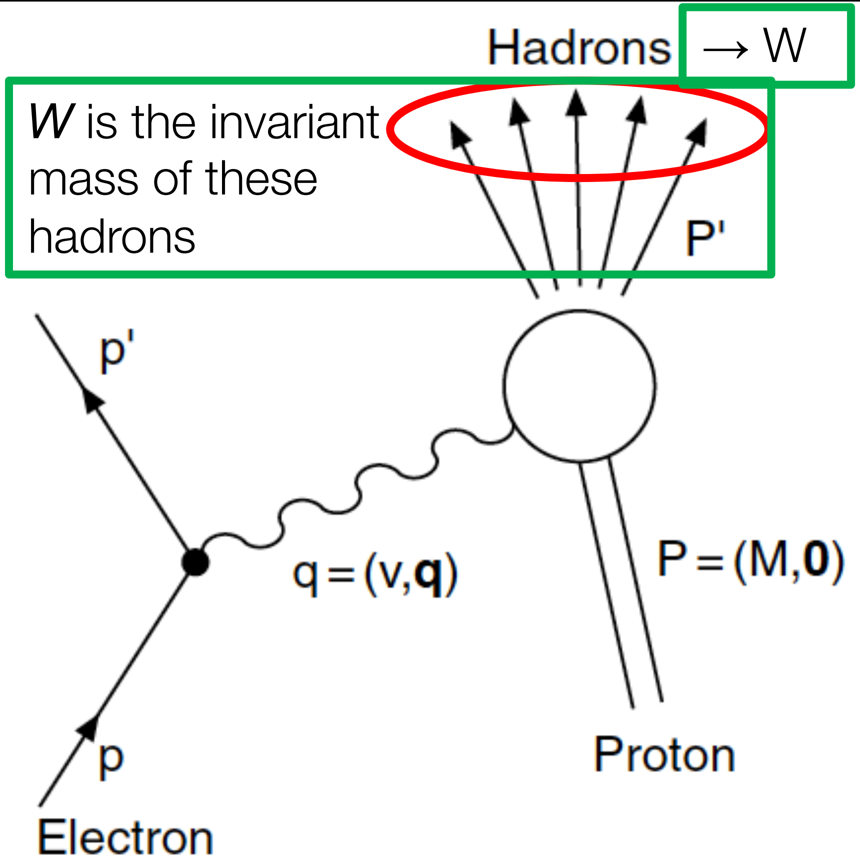
Inelastic scattering → many particles in the final state)

The measurement of the electron scattering angle does not determine the kinematics of the event → 2 variables needed (E',Θ) or (Q^2,υ)



Elastic Scattering	Inelastic Scattering
Form Factors	Structure Functions
$G_E(Q^2)$, $G_M(Q^2)$	$W_2(Q^2,\nu)$, $W_1(Q^2,\nu)$

Vocabulary and Kinematics of DIS



Electron-proton inelastic scattering: more than the two incoming particles in the final state.

The scattering occurs between a proton at rest and an exchanged photon. In this representation the kinematics is defined as follows (*use quadri-momenta*):

W is defined as the invariant mass of all hadrons of the final state (*W>M*)

$$W^2 = P'^2 = (P+q)^2 = M^2 + 2Pq + q^2 =$$
$$= M^2 + 2M\nu - Q^2 \qquad (Q^2 = -q^2)$$

And where

$$\nu = \frac{Pq}{M}$$

Quadri-momenta of particles are as follows: the target proton is at rest P=(Mc,**0**), the exchanged photon is

$$q=((E-E')/c,\mathbf{q}) \rightarrow \frac{Pq}{M} = \nu = \frac{Mc \cdot \frac{E-E'}{c} - q \cdot 0}{M}$$

Therefore the energy transferred by the virtual photon from the electron to the proton in the laboratory frame is: ν = E – E'

Elastic and Inelastic Scattering

- Elastic scattering: $G_E(Q^2)$ and $G_M(Q^2)$ form factors.

$$\left(\frac{d\sigma}{d\Omega}\right)=\left(\frac{d\sigma}{d\Omega}\right)_{Mott}\cdot\left[\frac{G_E^2(Q^2)+\tau\cdot G_M^2(Q^2)}{1+\tau}+2\tau G_M^2(Q^2)\tan\frac{\theta^2}{2}\right] \text{where } \tau=\frac{Q^2}{M^2c^2}$$

- The Q^2 dependence of the form factors gives us information about the radial distributions of the charge and the magnetic moments.

In elastic scattering, one parameter only fixes the kinematics of the event.
Example: the scattering angle θ is fixed, → squared four-momentum transfer Q^2, the energy transfer ν, the energy of the scattered electron E are also fixed. Since

$$W = M,$$

(and remembering that $W^2 = P'^2 = (P+q)^2 = M^2 + 2Pq + q^2 = M^2 + 2M\nu - Q^2$
We get

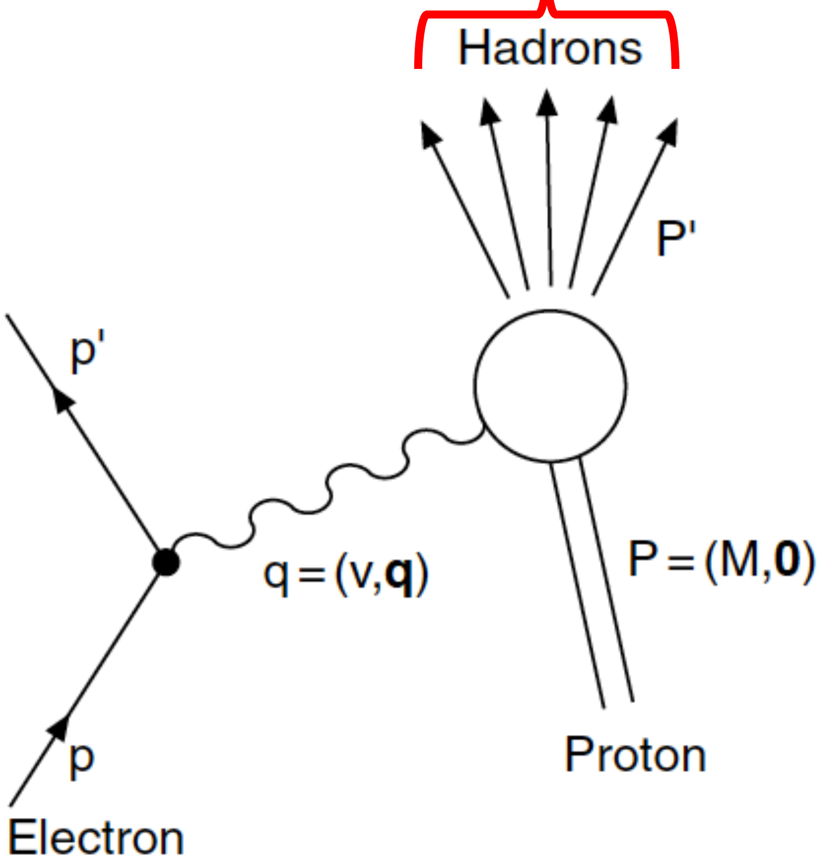
Elastic Scattering

$$M^2 = M^2 + 2M\nu - Q^2 \quad \rightarrow \quad 2M\nu - Q^2 = 0$$

Inelastic scattering: W_1 and W_2 structure functions.

The Bjorken Scaling Variable

$W^2 = M^2 + 2M\nu - Q^2$



The structure of the proton is best studied introducing a new Lorentz-invariant variable $\boldsymbol{x}$ defined as

$$\boldsymbol{x = \frac{Q^2}{2Pq} = \frac{Q^2}{2M\nu}}$$

This variable is generally known as “Bjorken scaling variable” and gives an indication of the inelasticity of the process.

$$Q^2 = 4E_e E_{e'} sin^2(\theta_e/2)$$
$$x = Q^2/2m_p(E_p - E_{p'})$$

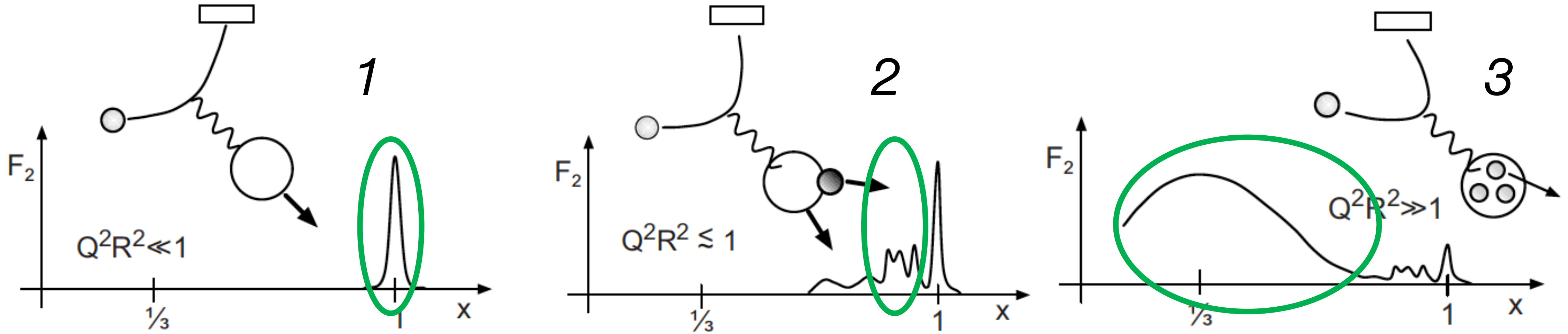
In the elastic scattering W=M and the relation $2M\nu - Q^2 = 0$ gives x=1.

In the case of inelastic processes however W>M and $0 \leq x \leq 1$.

To deduce the momentum transfer Q^2 and the energy loss v , *the energy and the scattering angle of the electron have to be determined in the experiment*

$W = M$	$W > M$
$x = 1$	$0 \leq x \leq 1$

Understanding ‘x’



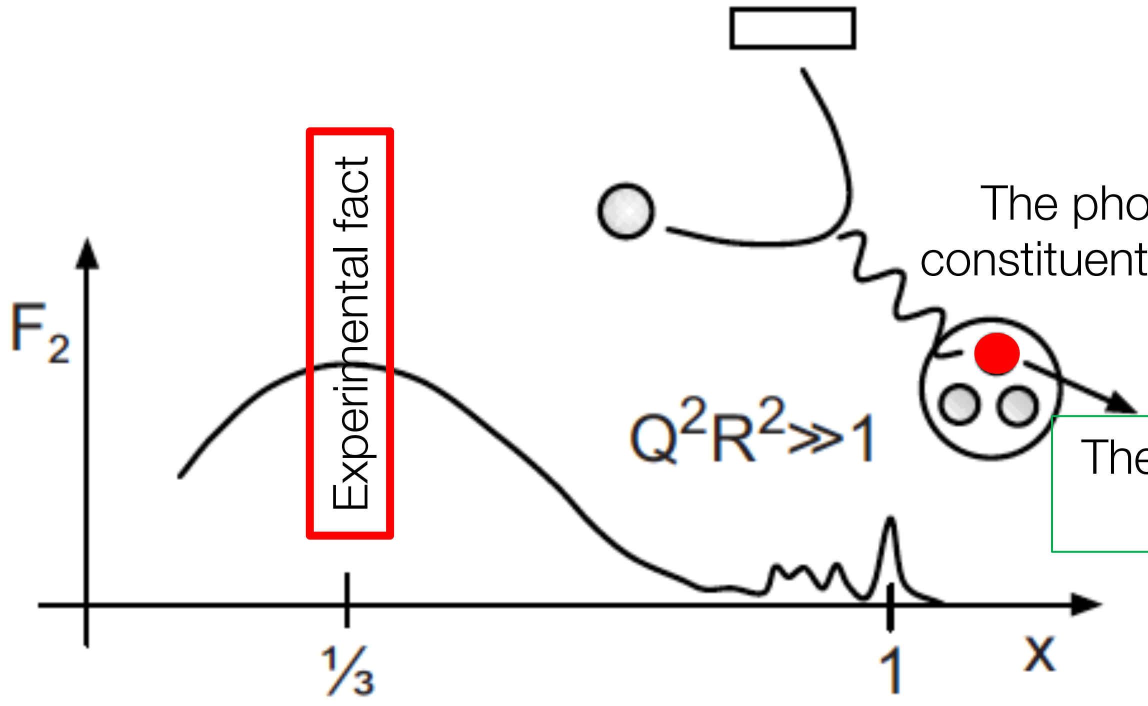
What do we see with increasing Q^2?

See above 3 different cases

Q2 ↑ wave length of the probe particle ↓

1. The Q^2 of the reaction is ~low, the **nucleon** is seen by the exchanged photon as **a unique object**. We have elastic scattering
2. The Q^2 of the reaction is not as ~low as in 1, not enough to probe the inner structure but enough to **excite the nucleon**
3. The Q^2 of the reaction is ~large enough to see the internal structure of the proton and the photon scatters elastically on one of the **internal constituents of the nucleon**

More Understanding of 'x'



The photon hits one constituent of the nucleon

The constituent of the nucleon is scattered elastically

The constituent of the quark cannot be observed → it 'transforms' into several hadrons ("it hadronizes") (will see this later)

The peak at ~1/3 can be understood as the "most probable" x value corresponding to the *elastic scattering of the photon and one of the nucleon constituents.*

If we assume that the 'x' budget is equally shared by 'n' nucleon constituents then

$$x = \frac{1}{n}\frac{Q^2}{2Pq} = \frac{1}{n}\frac{Q^2}{2M\nu}$$

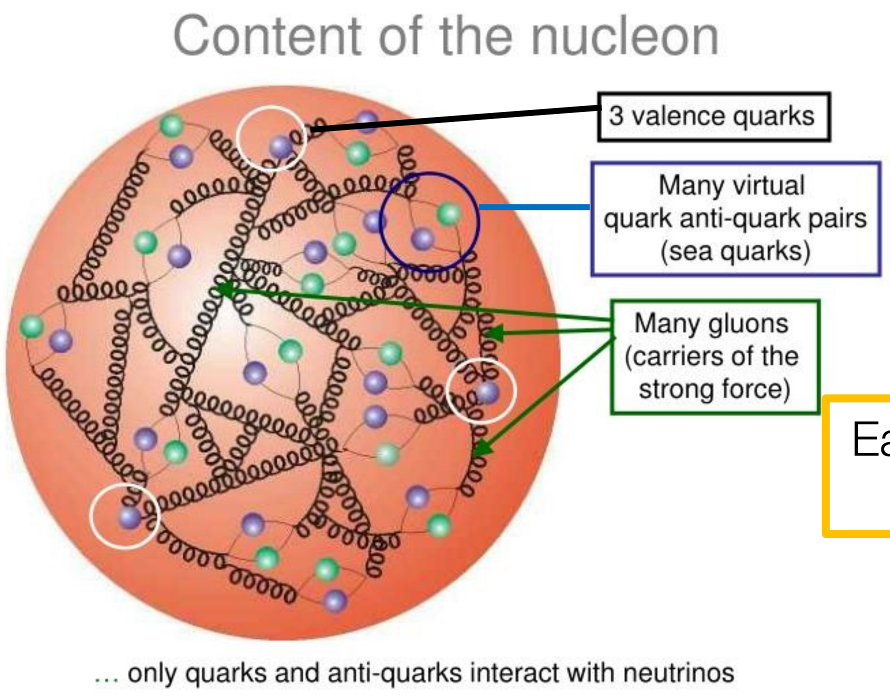
This term is equal to 1 in case of elastic scattering

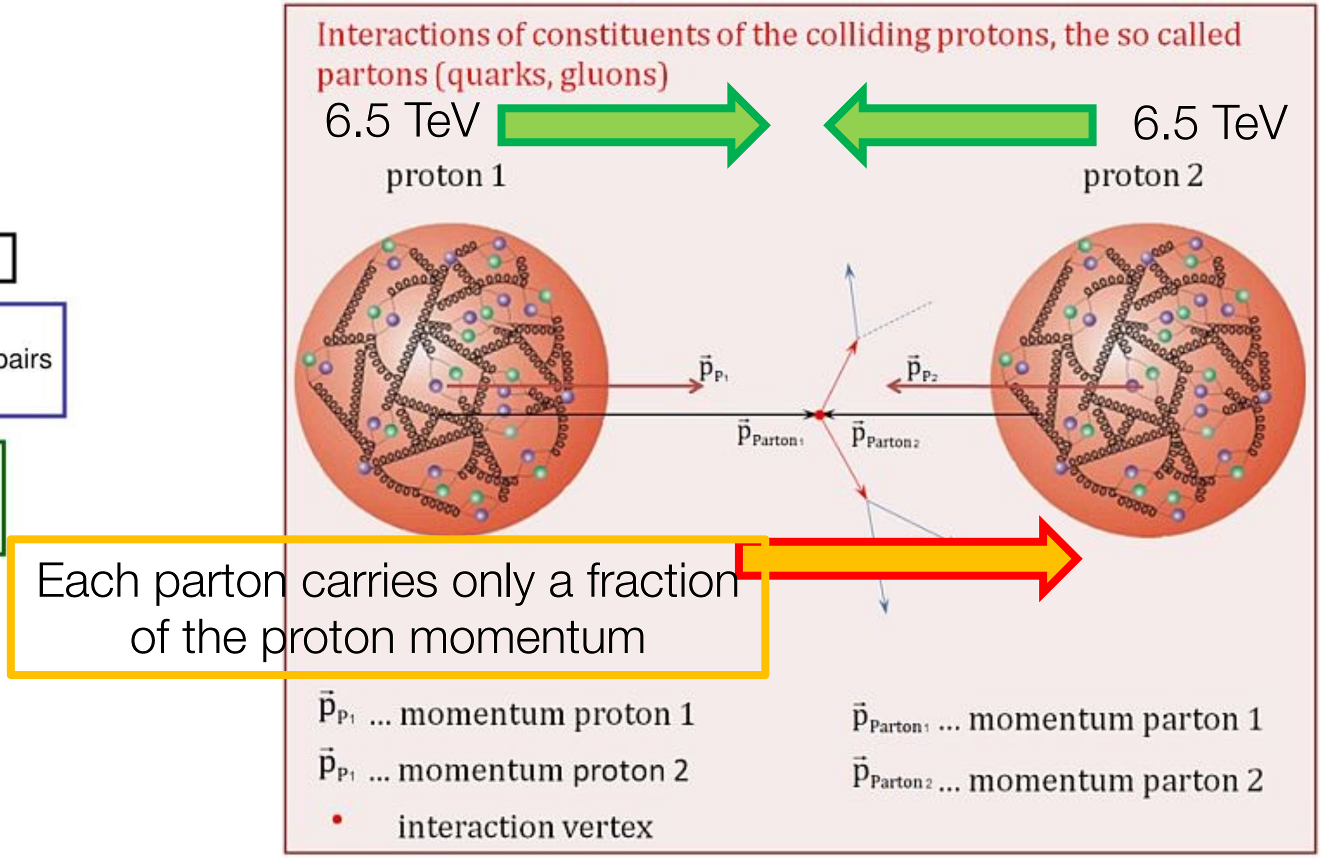
$\frac{1}{3} = \frac{1}{n}$ → there are 3 components in the nucleon

Why do we Need to Study e Scattering on p/Nuclei ?

LHC is nowadays the largest accelerator in the world (more in next lectures). It accelerates and collides protons against protons at an energy of 6.5 TeV + 6.5 TeV.
Collisions recorded at LHC are due to collisions between two very complex objects (will discuss more about it!)

→ To interpret these collisions you need to know the structure of the proton





Electron – Proton scattering: History

Studying the nucleon's constituents the wave length of the probe particle λ has to be small compared to the nucleon's radius, R

$$\lambda \ll R \;\rightarrow\; Q^2 \gg \hbar^2/R^2$$

Large Q^2 values are needed → high energies are required.

- The first generation ~1960 @ **SLAC** 25 GeV electrons on a target
- The second generation ~ 1980 @ **CERN** using beams of *muons* of up to 300 GeV (*).

Beams on target

- The last generation ~1990 → 2007 @ **DESY** Collider **HERA:** 30 GeV electrons against 900 GeV protons (see next slides).

Collider

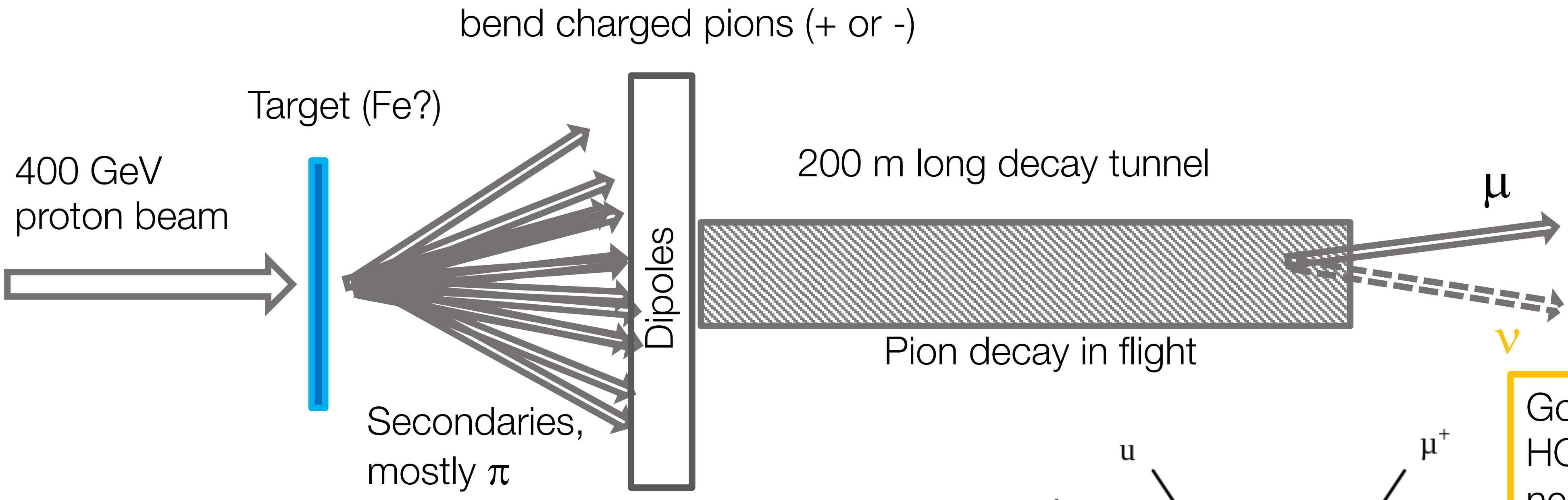
- In the SLAC experiments, the basic properties of the quark and gluon structure of the hadrons were established.
- The second and the third generations of experiments are at the basis of the

Quantum Chromodynamics,

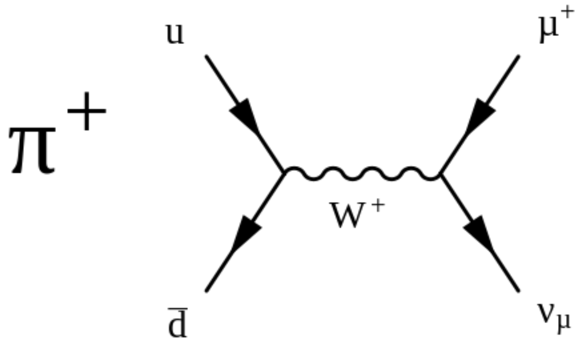
the theory of the strong interaction.

(*) Protons of 400 GeV on a target produced pions which were kept confined in a 200 meters tunnel. During the flight part of the pions decayed into muons which were collected into a beam with energies up to 300 GeV.

Producing Muon Beams



Goes ~undetected
HOWEVER may give a
neutrino beam



$\pi^0 \rightarrow \gamma + \gamma$

p + Fe → X (~1/3 π^+ + 1/3 π^- + 1/3 π^0) + ~ p (1/2 energy)

$\pi^- \rightarrow \mu^- + \bar{\nu}$

$\pi^+ \rightarrow \mu^+ + \nu$

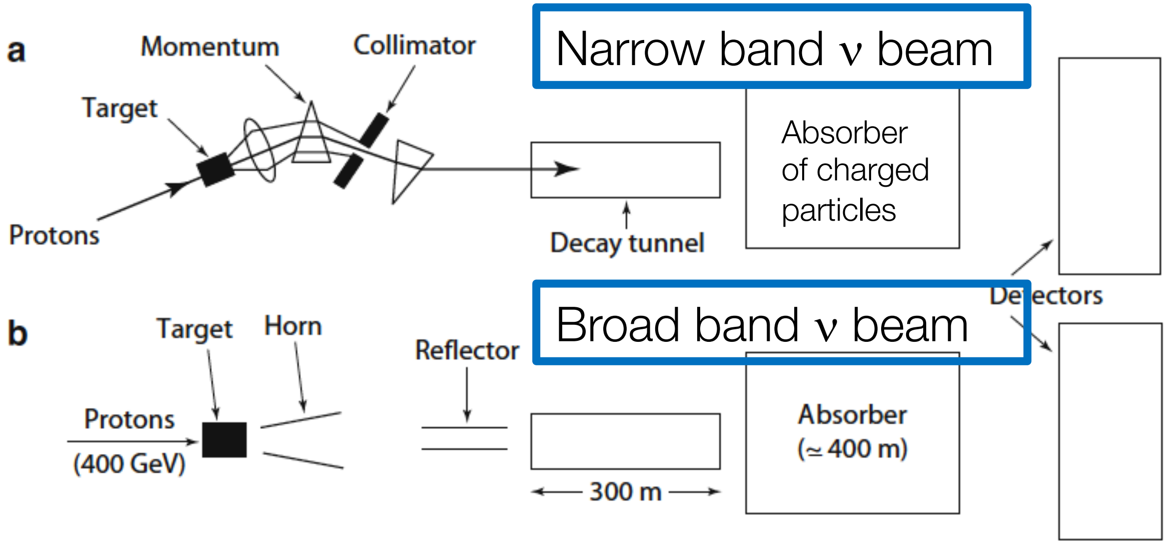
π^+ *gives* ν *and* μ^+

π^- *gives* $\bar{\nu}$ *and* μ^-

Producing Neutrino Beams

$\pi^- \rightarrow \mu^- + \bar{\nu}$

$\pi^+ \rightarrow \mu^+ + \nu$

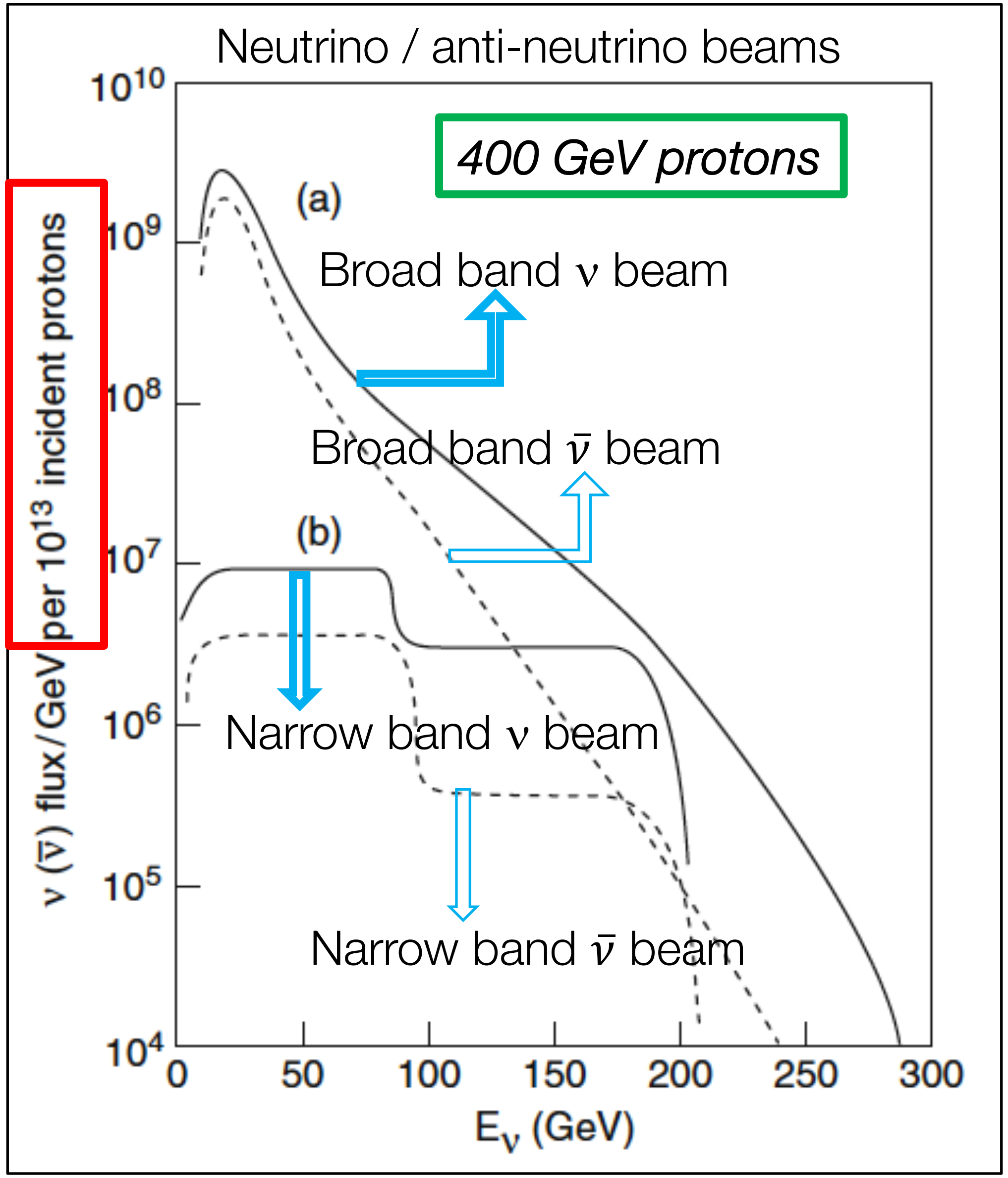


Narrow band ν beam: ~ selected in momentum ~ low intensity
Broad band ν beam: ~ not selected in momentum ~ high intensity

Experiments:

The mean free path in iron of 10GeV neutrinos is $\lambda \approx 2.6 \cdot 10^9 Km$ (~ 20 cm for hadrons!). This means that only a very small fraction $3 \cdot 10^{-13}$ of 10 GeV neutrinos interact in a meter of iron. With a flux of 10^{12} neutrinos (for 10^{13} accelerated protons incident on the target), there are only 0.3 interactions in one meter of iron.

→ very long and massive detectors



Hera, Hadron-Electron Ring in Desy-DE

Circular $e + p$ accelerator @ Desy, Hamburg-DE.

- 15 to 30 m underground and circumference of 6.3 km. Leptons and protons → two independent rings

- At HERA, 27.5 GeV electrons (or positrons) collided with 920 GeV protons, cms energy of 318 GeV (*).
- electrons or positrons: 450 MeV, 7.5 GeV, 14 GeV, 27.5 GeV.
- Protons: 50 MeV, 7 GeV, 40 GeV, 920 GeV.

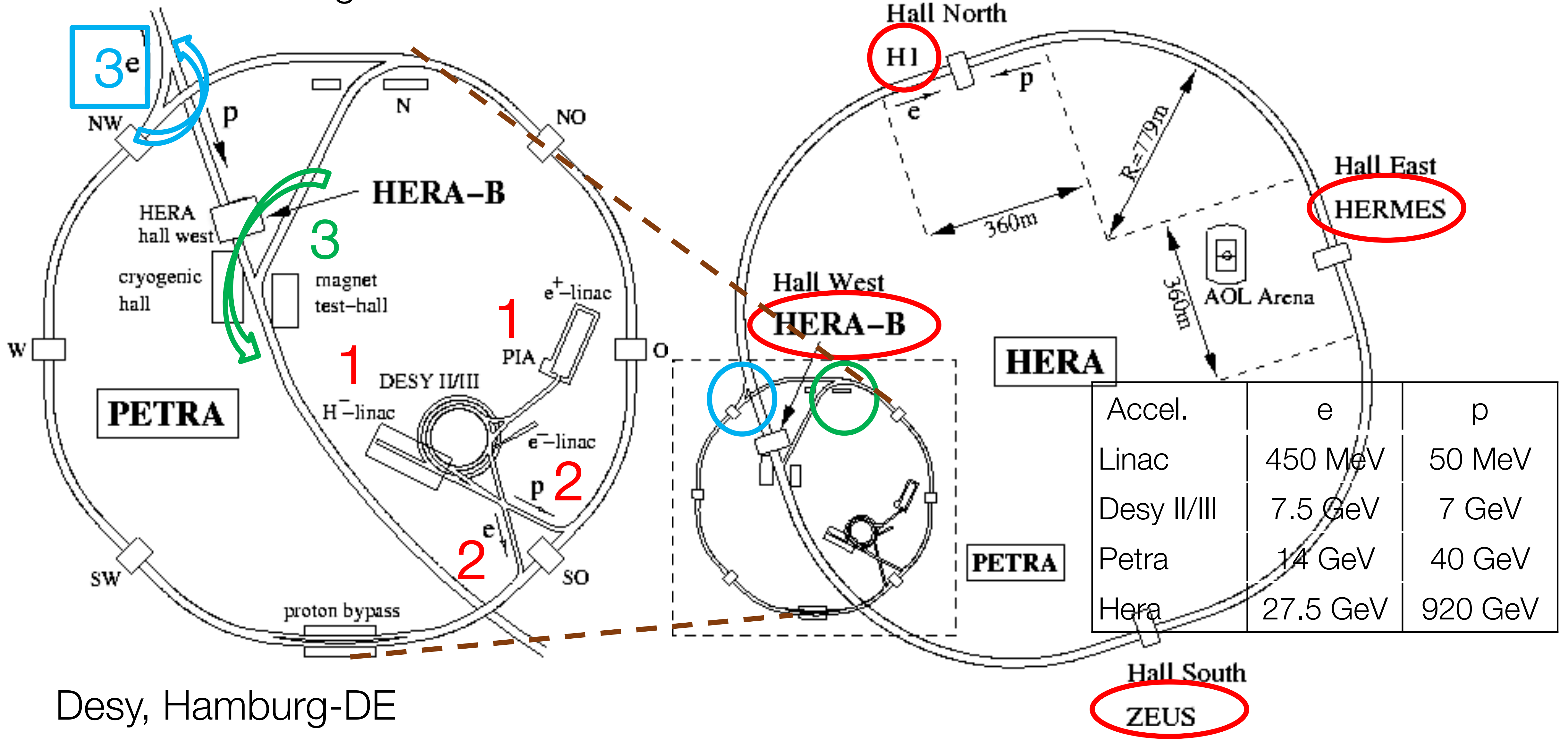
- 4 interaction regions, 4 experiments H1, ZEUS, HERMES and Hera-B.
- About 40 minutes to fill the machine

- Operated between 1992 and 2007.

$$(*)\ E_{cm}(or\ cms) = \sqrt{m_p^2 + m_e^2 + 2E_pE_e(1 - \beta_1\beta_2\cos(\theta))} \approx \sqrt{2E_pE_e \cdot 2}$$

HERA Accelerator Complex

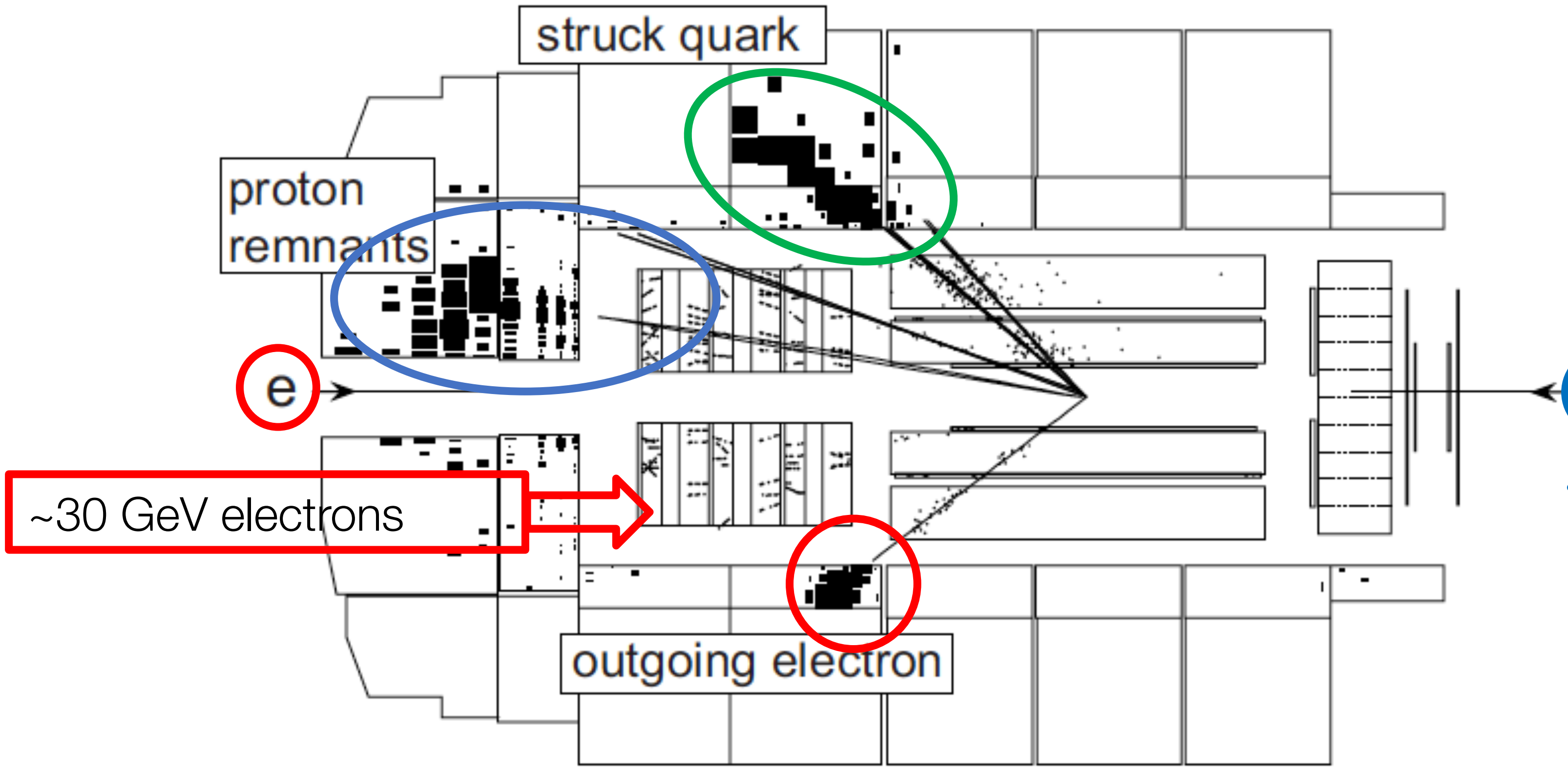
Three stage acceleration: Linac and *Petra and then Hera*



Accel.	e	p
Linac	450 MeV	50 MeV
Desy II/III	7.5 GeV	7 GeV
Petra	14 GeV	40 GeV
Hera	27.5 GeV	920 GeV

Desy, Hamburg-DE

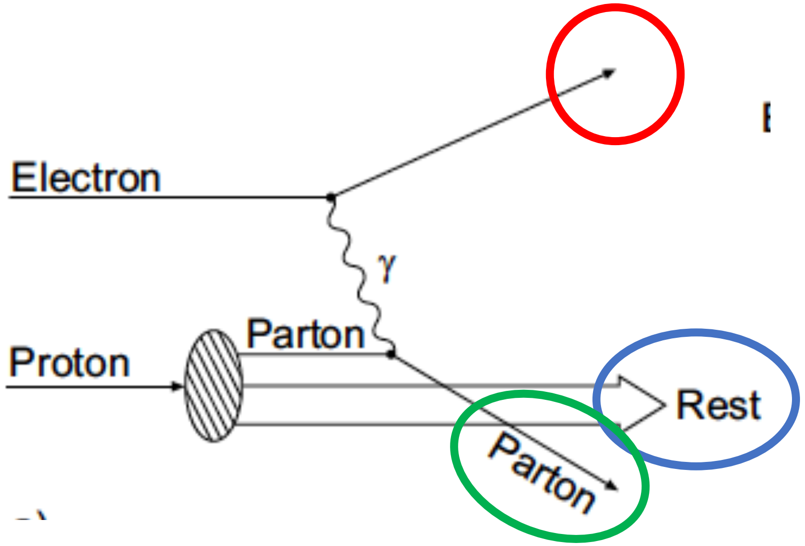
Display of one DIS event in Hera



To deduce the momentum transfer Q^2 and the **energy loss** ν=E'-E, the **energy** and the **scattering angle** of the electron have to be determined in the experiment.

Very asymmetric event topology!

The direction of all charged particles is measured in the inner tracking detector. The energy of the scattered electron is measured in the electromagnetic calorimeter, that of the hadrons in the hadron calorimeter.



From W_2 and W_1 to F_2 and F_1

For elastic scattering, two form factors G_E^2, G_M^2 are necessary to describe the electric and magnetic distributions. The cross-section for the scattering of an electron off a nucleon is described by the Rosenbluth formula,.

$$\tau = \frac{Q^2}{M^2c^2} \qquad \left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[\frac{G_E^2(Q^2) + \tau \cdot G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2) \tan\frac{\theta^2}{2}\right].$$

← Elastic Scattering, Q^2

In the e-p inelastic scattering, it transforms into

$$\left(\frac{d\sigma}{d\Omega dE'}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[W_2(Q^2,\nu) + 2 \cdot W_1(Q^2,\nu) \tan\frac{\theta^2}{2}\right]$$

← Inelastic Scattering, Q^2,ν

where the first term describes electrical interactions and the second term represents the magnetic interaction.

One variable, Q^2, in the elastic case → two variables, Q^2 and ν, in the inelastic case (x, Q^2)

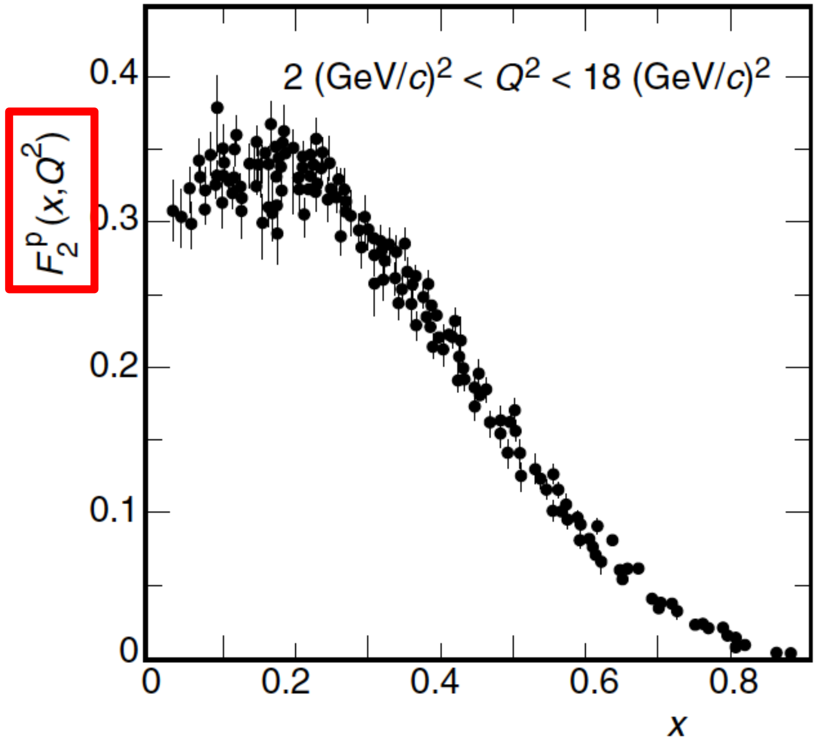
The two structure functions $W_1(Q^2,\nu)$ and $W_2(Q^2,\nu)$ above can be replaced by two dimensionless functions (much more in the following)

$$\nu = \frac{Pq}{M}$$

$$F_1(x, Q^2) = Mc^2 W_1(Q^2,\nu)$$
$$F_2(x, Q^2) = \nu W_2(Q^2,\nu)$$

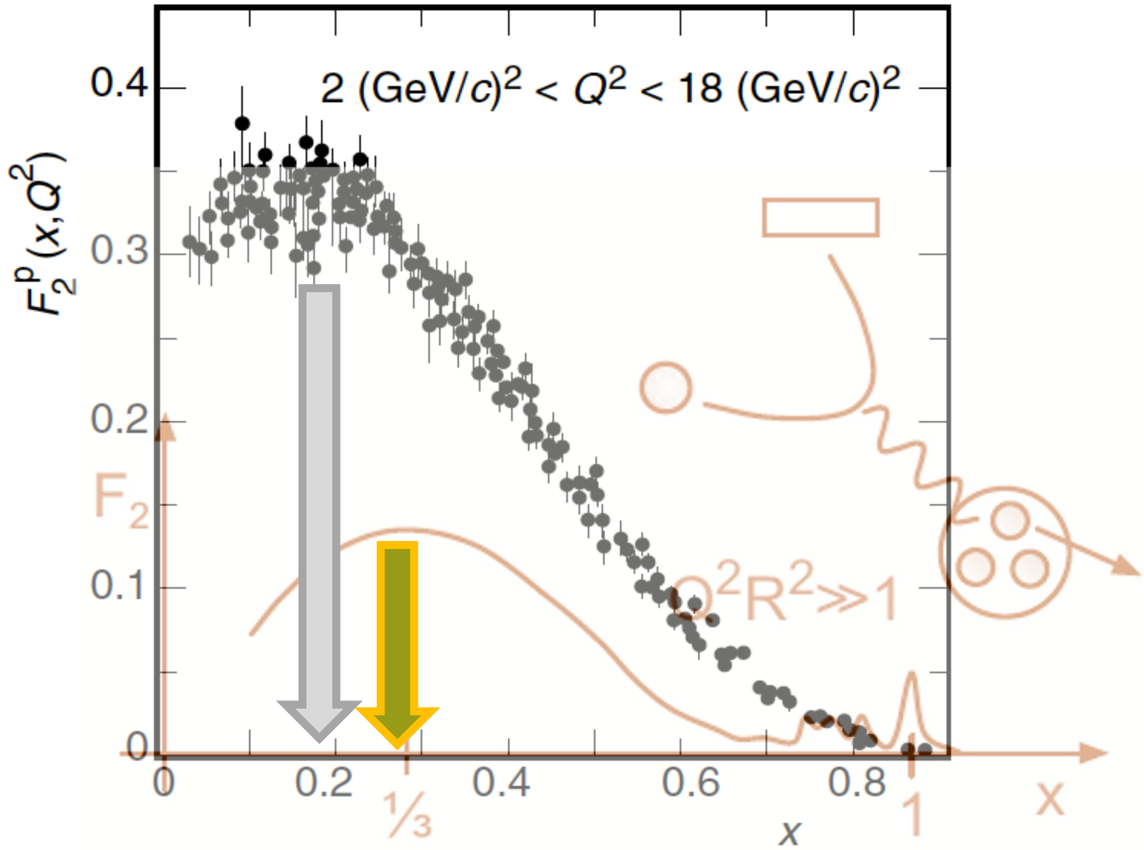
Magnetic interaction term: F_1 vanishes for scattering off spin 0 particles

From W_2 and W_1 to F_2 and F_1



The measured structure function $F_2(x,Q^2)$ is shown in the figure ← for a Q^2 interval between 2 and 18 $(GeV/c)^2$.

$F_2^p(x,Q^2)$ → measured in protons, you cannot choose a single quark!



- Experimental points taken at different Q^2 are seen to be superimposed.
  - **It can be shown that this implies the scattering off point like objects.**
- The peak of the distribution at ~ 0.2, less than 1/3 . The shift is due to understood effects that will be discussed later

How to measure the F_2 Structure Function?

- Scattering ep/μp (at accelerators)

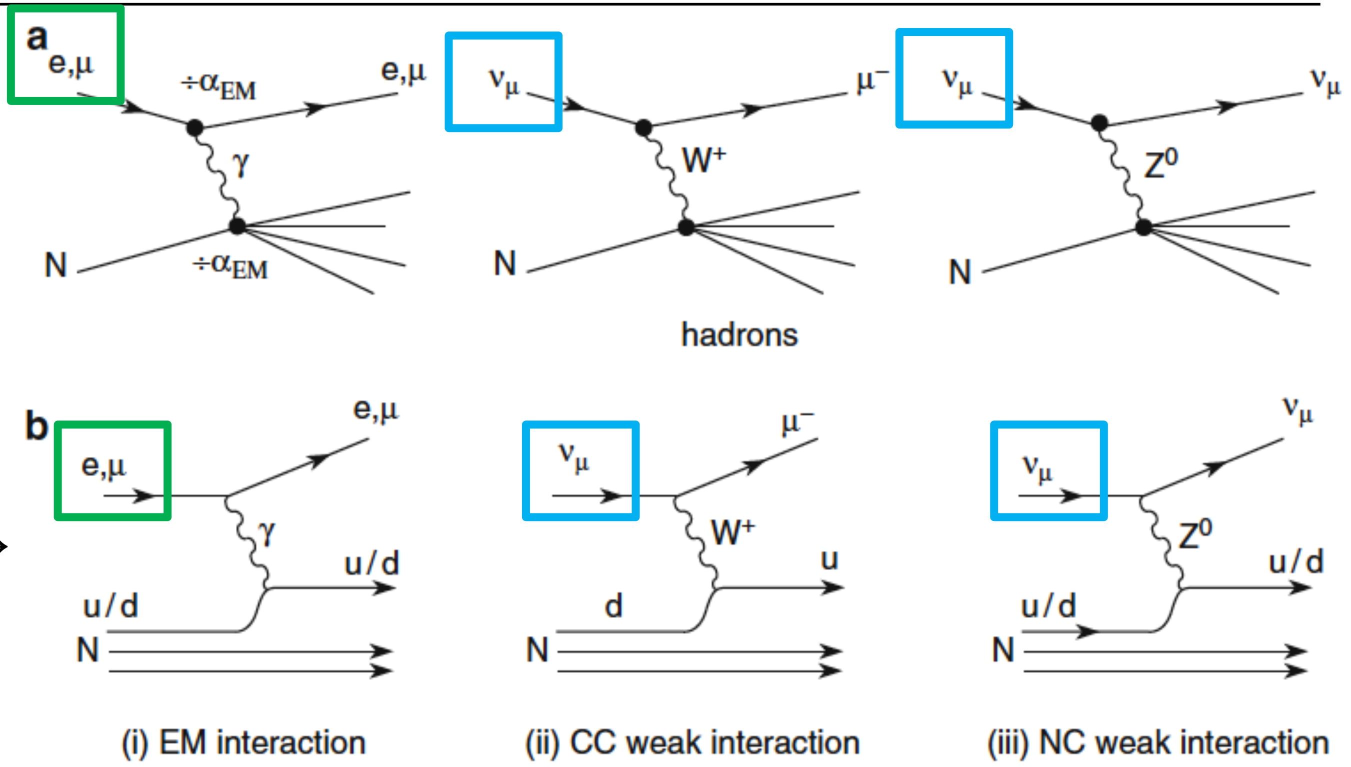
$$ep: \quad e^{\pm} + p \rightarrow e^{\pm} + X^{+}$$

$$\mu p: \quad \mu^{\pm} + p \rightarrow \mu^{\pm} + X^{+}$$

- Scattering of leptons (electrons and neutrinos) on a hydrogen (1p), deuterium (1p+1n) and heavier nuclei target (targets with #protons=#neutrons).
- F_2^d : Scattering on nuclei the structure function is always given per nucleon (protons and neutrons) → How to distinguish F_2^p $from$ F_2^n?
- The structure function of the deuteron F^d_2 is equal to the average structure function of the nucleons

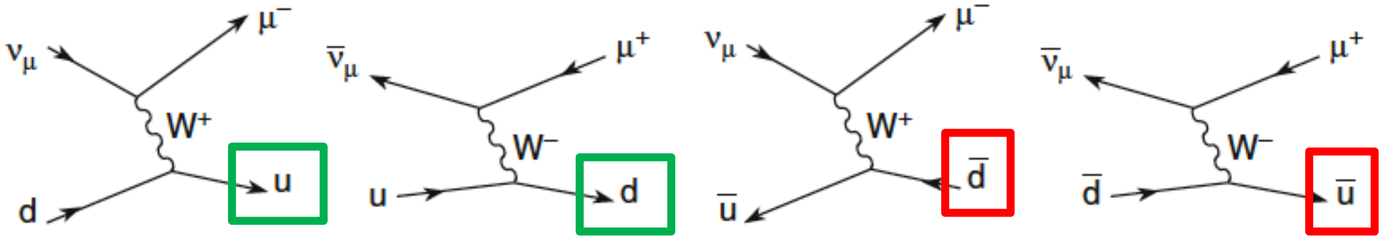
$$F_2^d \approx \frac{F_2^p + F_2^n}{2} = F_2^N$$

- Neutrinos on a target → it is possible to distinguish between ‘valence’ and ‘sea’ quarks



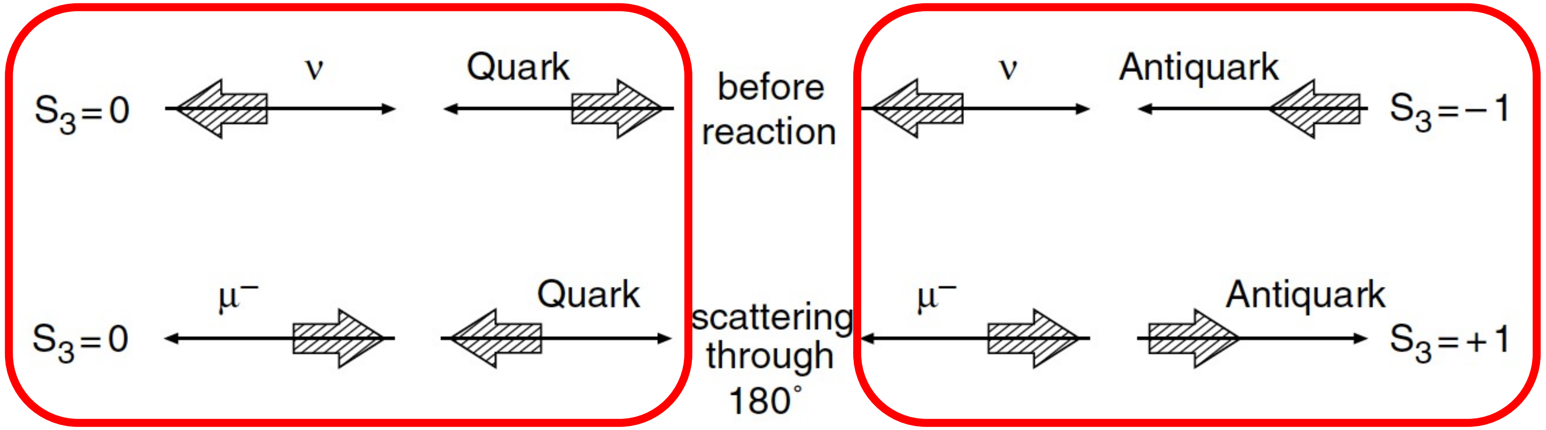
$$\nu_\mu p(CC): \nu_\mu + p \rightarrow \mu^- + X^{++}, \ \overline{\nu}_\mu + p \rightarrow \mu^+ + X^0$$

$$\nu_\mu p(NC): \quad \nu_\mu + p \rightarrow \nu_\mu + X^{+}, \ \overline{\nu}_\mu + p \rightarrow \overline{\nu}_\mu + X^{+}$$



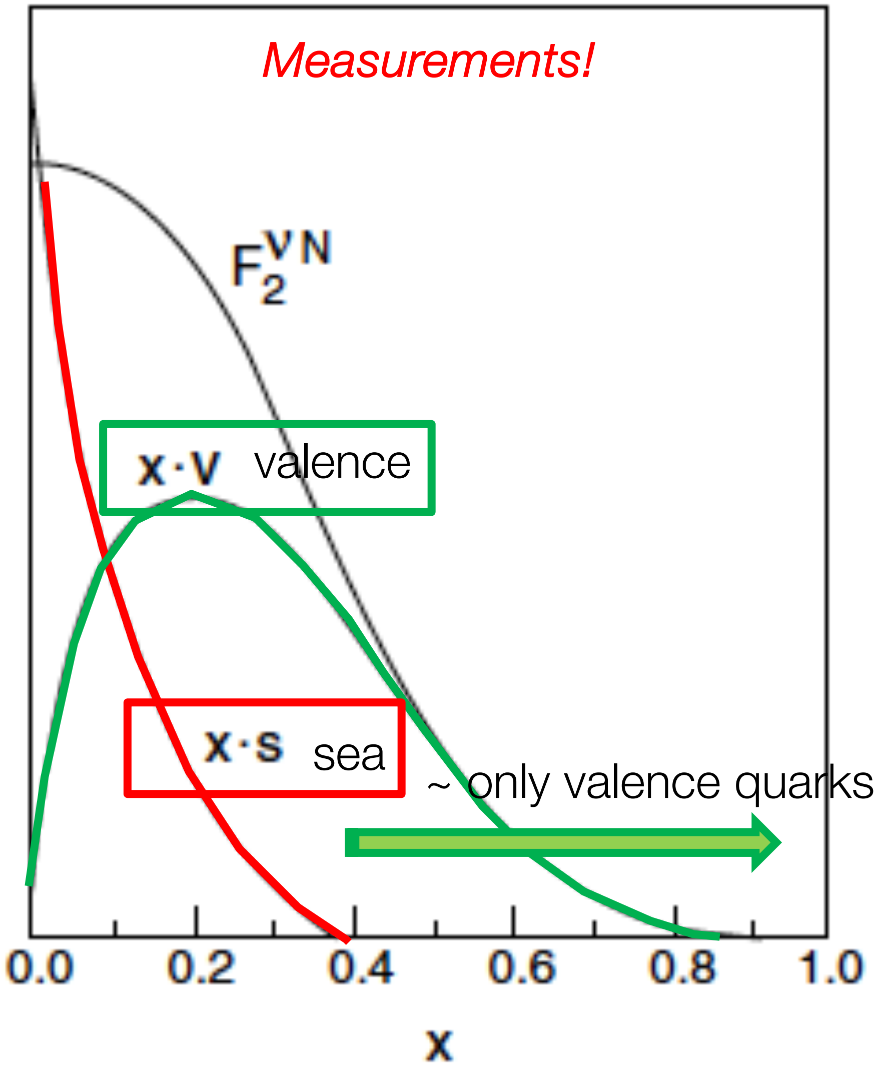
Helicity in neutrino quark scattering (Bogdan & Povh)

The scattering off the quarks and antiquarks is characterised by different angle and energy distributions for the outgoing leptons. This becomes plausible if one (analogously to our considerations in the case of Mott scattering in Sect. 5.3) considers the extreme case of *scattering through $\theta_{c.m.} = 180^\circ$ in the centre of mass frame for the neutrino and the quark.* We choose the quantisation axis ***z*** to be the direction of the incoming neutrino's momentum. Since the W boson only couples to left handed fermions, both the neutrino and the quark have in the high energy limit negative helicities and the projection of the total spin on the ***z*** axis is, both before and after scattering through 180° $S_3 = 0$.



This also holds for all other scattering angles, i.e., the scattering is isotropic. On the other hand if a left handed neutrino interacts with a right handed antiquark, the spin projection before the scattering is $S_3 = -1$ but after being scattered through 180° it is $S_3 = +1$. Hence scattering through 180° is forbidden by conservation of angular momentum. An angular dependence, proportional to $(1 + \cos\theta_{c.m.})^2$, is found in the cross-section.

$F_2^{\nu N}$ from Neutrino-Nucleon Scattering



- Neutrino scattering yields complementary information about the quark distribution.
- Neutrinos couple to the weak charge of the quarks via the weak interaction.
- In neutrino scattering, it is possible to distinguish between the different types of quarks, and also between quarks and antiquarks. Details in previous slide.

- Neutrino and antineutrino scattering give the momentum distribution of the sea quarks and of the valence quarks separately.
- Fig. ← shows that sea quarks contribute to the structure function only at small values of x. Their momentum distribution drops off rapidly with x and is negligible above $x \approx 0.35$.
- The distribution of the valence quarks has a maximum at about $x \approx 0.2$ and approaches zero for $x \rightarrow 1$ and $x \rightarrow 0$.
- For large x, F_2 becomes extremely small. Thus it is very unlikely that one quark alone carries the major part of the momentum of the nucleon.

Structure Functions in the Parton Model

Structure Functions describe the internal structure of a nucleon. Let's say that

- A nucleon is made of quarks of type *f*;
- Each quark carries a charge $z_f \cdot e$;
- The **electro-magnetic** cross section for a scattering on a quark is $|z_f \cdot e|^2$
- $q_f(x)$ is the probability of finding a quark of type f inside the nucleon carrying a fraction of the nucleon momentum in the interval $(x, x + dx)$ (similarly $\overline{q_f}(x)$ for anti-quarks)
- There are two types of quarks:
  - *valence* quarks: they determine the quantum numbers of the nucleon
  - *sea quarks*, they exist in pairs, quark + anti-quark. They are produced and annihilated as virtual particles in the field of the strong interaction. Something similar happens in the production of virtual electron–positron pairs in the Coulomb field
- The nucleon also contains neutral components, *gluons*, with momentum distribution *g(x)*

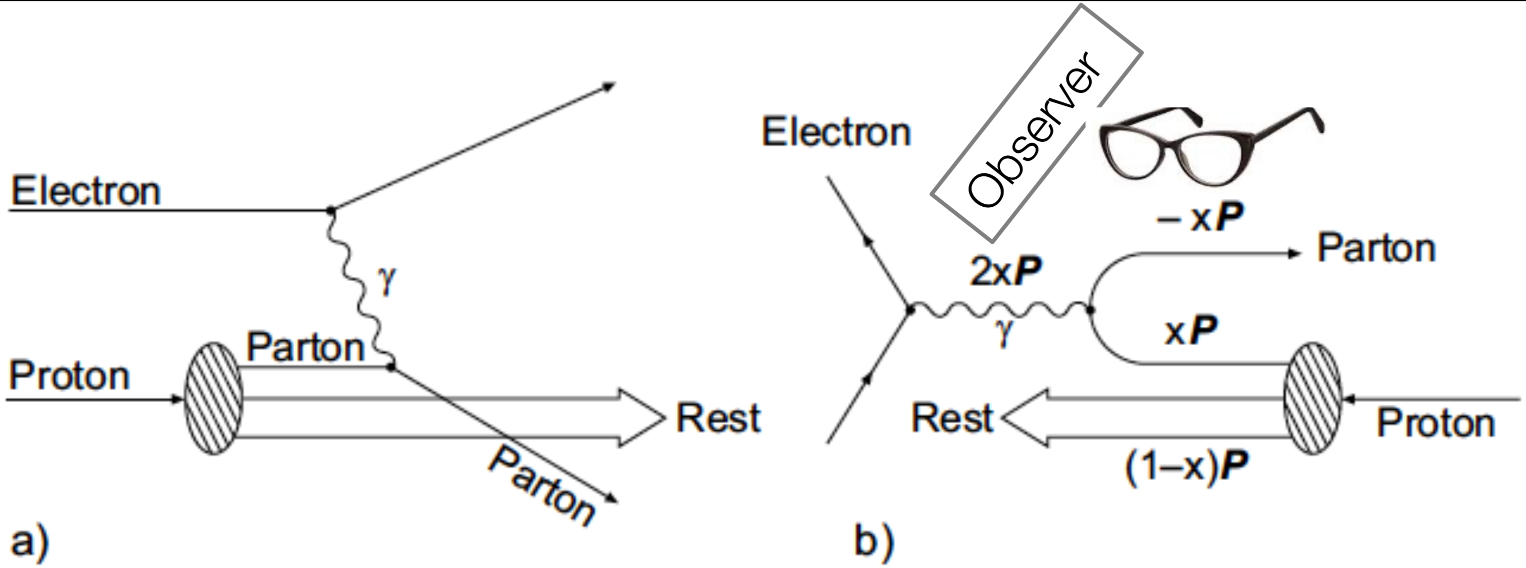
The Structure Function $F_2(x)$ is the superposition of the momentum distributions carried by the quarks and weighted by x and ${z_f}^2$

$$F_{2(x)} = x \cdot \sum z_f^2 \cdot (q_f(x) + \overline{q_f}(x))$$

$F_2(x)$ is not sensitive to gluons (gq interaction)

Choosing the Reference Frame of the DIS: Parton Model

- Physics is independent of reference frame
- Proton observed in a reference system where it appears to be very fast → only longitudinal components, neglect p_T
- Masses can be neglected



Partons =
quarks (charged) and neutral (gluons)

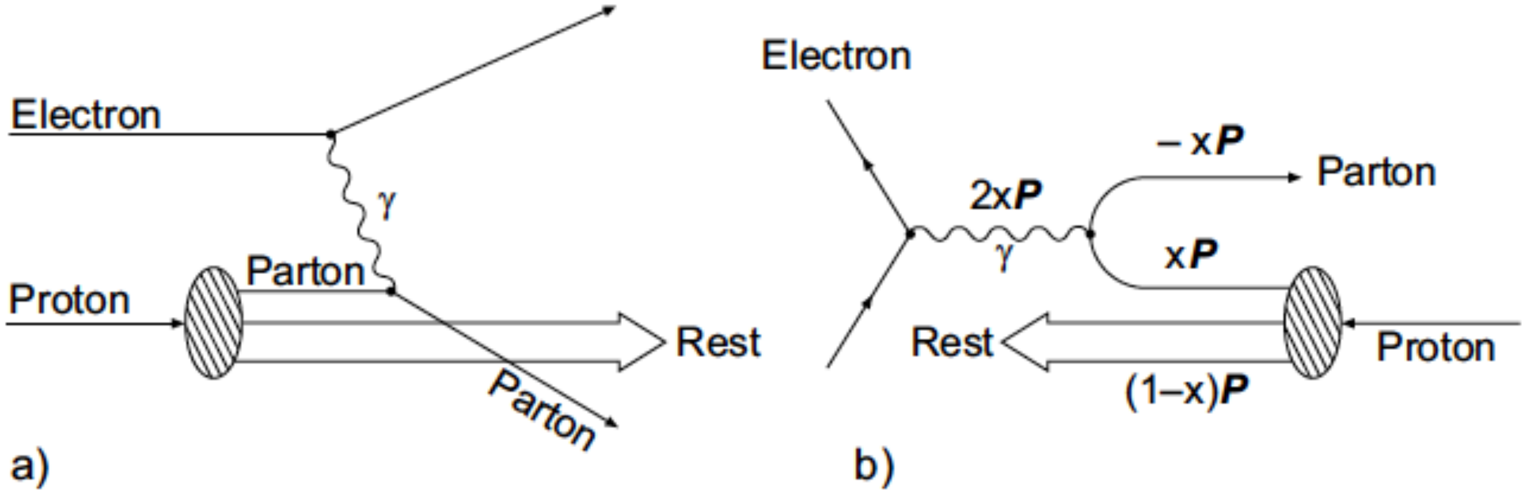
parton point of view of deep inelastic e-p scattering:

- (a) in the laboratory system
- (b) in a fast moving system (the Breit frame) in which the momentum transferred by the virtual photon is zero. Hence the momentum of the parton hit by the electron is turned around but its magnitude is unchanged.

Decomposing the proton into a sum of independent components allows us to see the

Interaction electron proton = sum of elastic interactions between the electron (via photon exchange) and partons

The Impulse Approximation



It is assumed that

- the duration of the interaction photon – parton is so short that partons do not have time to interact between themselves →

  Impulse Approximation

- Masses can be neglected →
  $Q^2 \gg M^2c^2$

remember: ν = energy transferred by the virtual γ from the e to the p

In the laboratory system the photon which has four-momentum q=(ν/c, **q**) interacts with a parton carrying the four-momentum xP

The reduced wave-length λ– of the virtual photon is given by

$$\lambda = \frac{\hbar}{|\boldsymbol{q}|} = \frac{\hbar}{\sqrt{Q^2}} .$$

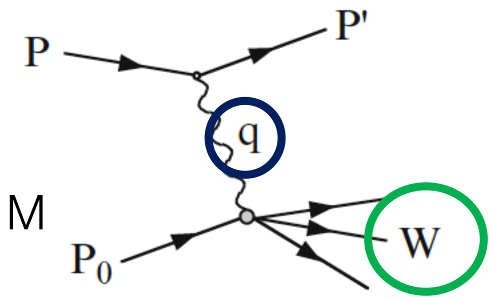
This gives the size of the structures of the proton we can study using a photon with momentum transfer Q^2

One step back: Elaborating more on W_2 & W_1

$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{point}\\ \text{spin } 1/2}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left[1 + 2\tau \tan^2\frac{\theta}{2}\right] \qquad \tau = \frac{Q^2}{4M^2c^2}$$

Elastic scattering, one variable
Spin ½ e on spin ½ proton

If the reaction $\ell + N \rightarrow \ell' + X$ is inelastic → the lepton scattering angle and energy are independent



$$W^2 = (P_0 + q)^2 = M^2 + q^2 + 2M\nu = M^2 - Q^2 + 2M\nu > M^2$$

(while in elastic scattering $W^2 = M^2 \rightarrow Q^2 = 2M\nu \rightarrow \nu - \frac{Q^2}{2M} = 0$)

$P = (E, \mathbf{p});\ P' = (E', \mathbf{p}')$ for the incident and scattered electron

$P_0 = (M, 0)\ ;\ W = (E_0', \mathbf{p}_0')$ for the proton before and after impact

p at rest, M =proton mass;
$P_0^2 = M^2; P^2 = m_e^2;$

$$q = P - P' = (E - E', \boldsymbol{p} - \boldsymbol{p}') = (\nu, \boldsymbol{q});$$
$$q^2 = 2m_e^2 - 2E'E + 2pp'\cos(\theta) \rightarrow m_e \sim 0; p \sim E \rightarrow$$
$$q^2 = -2E'E\,(1 - \cos\theta) = -\,4EE'\sin^2(\frac{\theta}{2})$$

One step back: Elaborating more on W_2 & W_1

$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{point}\\ \text{spin } 1/2}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left[1 + 2\tau \tan^2\frac{\theta}{2}\right] \qquad \tau = \frac{Q^2}{4M^2c^2}$$

Elastic scattering, one variable
Spin ½ e on spin ½ proton

Condition for DIS: $Q^2 \gg M^2; \quad \nu = E - E' \gg M.$

$$\frac{d^2\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)^*_{\text{Mott}} \left[W_2(Q^2,\nu) + 2W_1(Q^2,\nu)\tan^2\frac{\theta}{2}\right]$$

Inelastic scattering, two variables

Mott

$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\cos^2\frac{\theta}{2}\left(W_2(Q^2,\nu) + W_1(Q^2,\nu)2\tan^2\frac{\theta}{2}\right)$$

DIS of point-like particles with nucleons (p or n) → sum of elastic scattering on components (with mass *m*) of nucleons

Mott

$$\left(\frac{d^2\sigma}{dQ^2 d\nu}\right)_{ela} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\cos^2\frac{\theta}{2}\left(1 + \frac{Q^2}{4m^2}2\tan^2\frac{\theta}{2}\right)\delta\left(\nu - \frac{Q^2}{2m}\right)$$

δ → condition for elastic scattering, one variable

More on $W_{1,2}$

If we compare the two expressions

$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{point}\\ \text{spin } 1/2}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left[1 + 2\tau \tan^2\frac{\theta}{2}\right] \qquad \tau = \frac{Q^2}{4M^2c^2}$$

Elastic scattering, one variable
Spin ½ e on spin ½ proton

$$\left(\frac{d^2\sigma}{dQ^2 d\nu}\right)_{ela} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\cos^2\frac{\theta}{2}\left(1 + \frac{Q^2}{4m^2}2\tan^2\frac{\theta}{2}\right)\delta\left(\nu - \frac{Q^2}{2m}\right)$$

Replace *M* with *m* !
Which physical meaning?

Mott

$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\cos^2\frac{\theta}{2}\left(W_2(Q^2,\nu) + W_1(Q^2,\nu)2\tan^2\frac{\theta}{2}\right)$$

From $W_{1,2}$ to $F_{1,2}$ some calculation

We understand more if we use $d^2\sigma/dQ^2dx$

$$\frac{d^2\sigma}{dQ^2dx} = \frac{\nu}{x}\frac{d^2\sigma}{dQ^2d\nu} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\frac{1}{x}\cos^2\frac{\theta}{2}\left(\nu W_2(Q^2,\nu) + \nu W_1(Q^2,\nu) 2\tan^2\frac{\theta}{2}\right)$$

$$= \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\frac{1}{x}\cos^2\frac{\theta}{2}\left(F_2(x) + \frac{\nu F_1(x)}{M} 2\tan^2\frac{\theta}{2}\right)$$

$$x = Q^2/_{2M\nu} \rightarrow dx/_{d\nu} = Q^2/_{\nu^2 2M} = x/\nu$$

$$F_1(x,Q^2) = Mc^2 W_1(Q^2,\nu)$$
$$F_2(x,Q^2) = \nu W_2(Q^2,\nu)\,.$$

multiply and divide F_1 by $2x \rightarrow \frac{2x\,\nu\,F_1}{2x\,M}$ $\quad \nu = \frac{Q^2}{2Mx}$ $\quad \frac{2x\,\nu\,F_1(x)}{2x\,M} = \frac{2x\,Q^2\,F_1}{2x\;2Mx\,M} = \frac{2x\,Q^2\,F_1}{4M^2x^2}$

We also pass from $F_1(x,Q^2)$ to $F_1(x)$ without explaining it → experiments!

$$= \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\frac{1}{x}\cos^2\frac{\theta}{2}\left(F_2(x) + 2xF_1(x)\frac{Q^2}{4M^2x^2} 2\tan^2\frac{\theta}{2}\right).$$

From $W_{1,2}$ to $F_{1,2}$

$$= \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\frac{1}{x}\cos^2\frac{\theta}{2}\left(F_2(x) + 2xF_1(x)\frac{Q^2}{4M^2x^2}2\tan^2\frac{\theta}{2}\right).$$

$$\tau = \frac{Q^2}{4M^2c^2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)=\left(\frac{d\sigma}{d\Omega}\right)_{NS}\left[1+\frac{q^2}{2M^2}\tan^2(\theta/2)\right]$$ Spin 1/2 particle

$M^2x^2 \rightarrow m^2$

$$x = Q^2/_{2M\nu}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_M=\left(\frac{d\sigma}{d\Omega}\right)_R(1-\beta^2\sin^2\theta/2)\simeq\left(\frac{d\sigma}{d\Omega}\right)_R\cos^2(\theta/2).$$ Spin 0 particle

If we compare the expression with F_1 and F_2 with the elastic expression for ***e q*** scattering (2 point-like objects) spin 0 and spin 1/2 (with mass xM) expression we conclude that

If we want to have the same expression for elastic scattering of a point-like object on a point-like spin ½ on spin ½ object

- F_1 = 0 for spin 0 particles and
- $F_2 = 2x F_1$ for spin ½ particles (Callan-Gross relation)

Bjorken scaling : Callan-Gross formula

a) the cross sections of pointlike spin ½ particle of mass m (à la Rosenbluth with $G_E=G_M=1$) :

$$\left[\frac{d^2\sigma}{d\Omega dE'}\right]_{\text{point-like, spin1/2}} = \frac{12\alpha^2 E'^2}{EQ^4}\left[\cos^2\frac{\theta}{2} + 2\tau\sin^2\frac{\theta}{2}\right];$$

$$\left[\frac{d^2\sigma}{d\Omega dE'}\right]_{\text{DIS}} = \frac{4\alpha^2 E'^2}{Q^4}\left[W_2\cos^2\frac{\theta}{2} + 2W_1\sin^2\frac{\theta}{2}\right];$$

$$W_2\cos^2\frac{\theta}{2} + 2W_1\sin^2\frac{\theta}{2} = \frac{3}{E}\left[\cos^2\frac{\theta}{2} + 2\tau\sin^2\frac{\theta}{2}\right];$$

$$W_1 = \frac{3\tau}{E}; \quad W_2 = \frac{3}{E}; \quad \frac{W_1}{W_2} = \frac{F_1(x)}{F_2(x)}\frac{\nu}{M} = \tau = \frac{Q^2}{4m^2};$$

b) from the kinematics of elastic scattering of point-like constituents of mass m :

$$Q^2 = 2m\nu = 2M\nu x \rightarrow m = xM;$$

$$\frac{F_1(x)}{F_2(x)} = \frac{Q^2}{4m^2}\frac{M}{\nu} = \frac{2m\nu}{4m^2}\frac{M}{\nu} = \frac{M}{2m} = \frac{1}{2x}; \quad \rightarrow$$

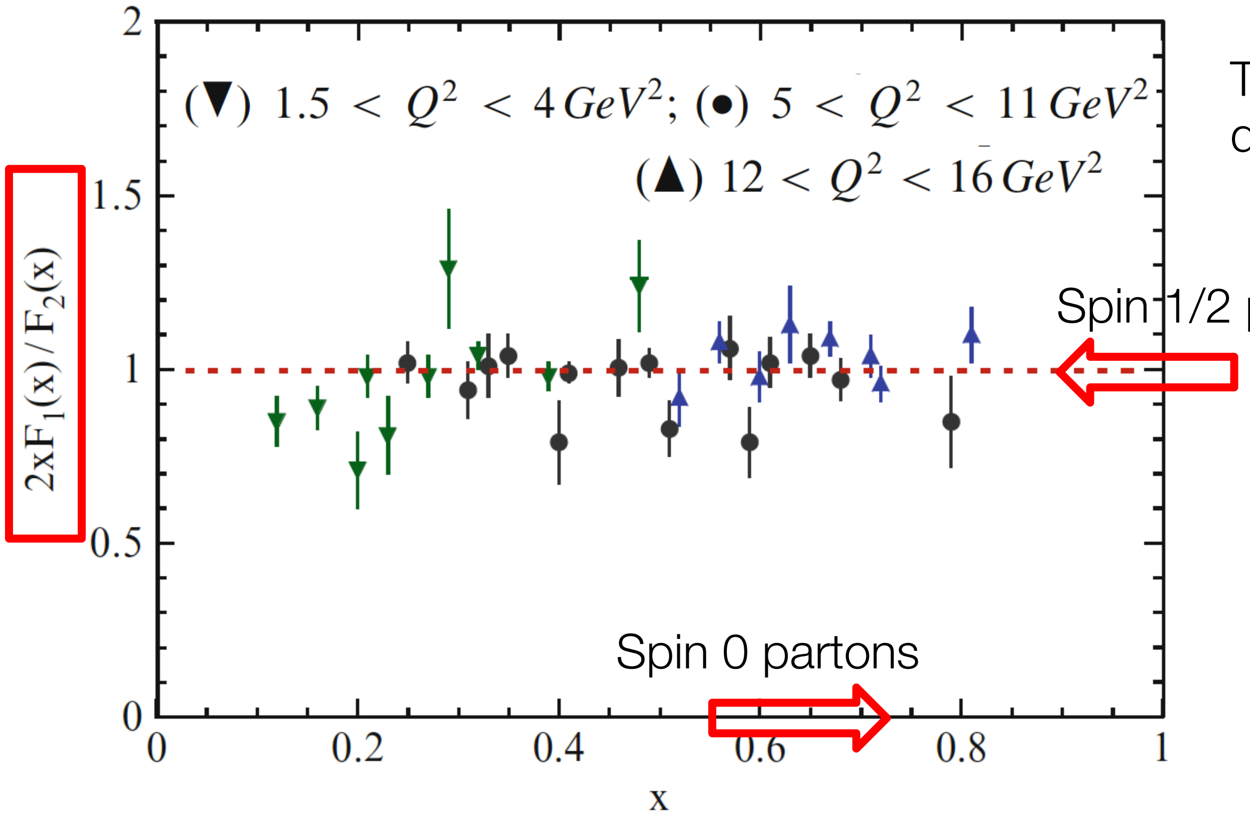
$$2xF_1(x) = F_2(x).$$

Warnings :

- don't confuse M (the nucleon) with m (the constituent);
- don't confuse the inelastic scattering ep with the elastic scattering eq;
- x refers to the inelastic case;
- an hypothetical [*nobody uses it*] variable ξ, analogous to x but for the constituent scattering; in this case, $Q^2=2m\nu\xi$, ξ = **1**;
- we learn that x = m/M [REMEMBER].

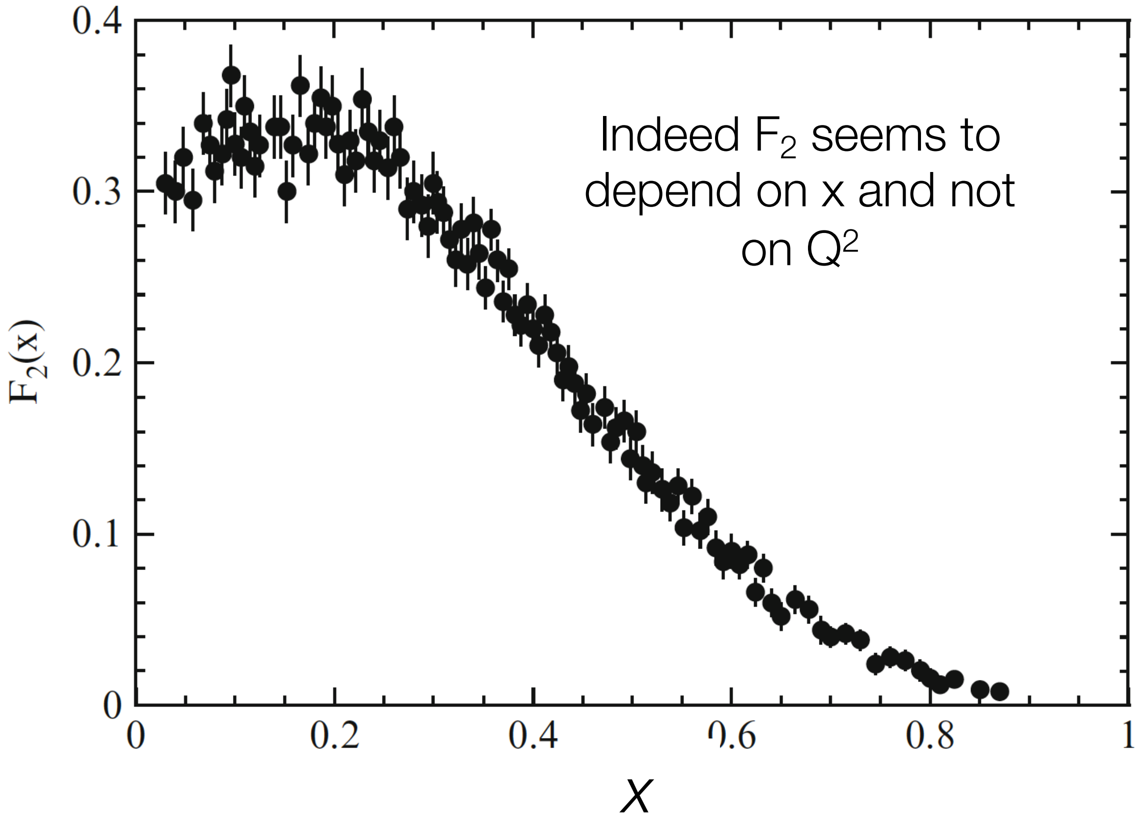
Prof. Paolo Bagnaia
University of Rome “La Sapienza”

The Callan-Gross relation



The Callan-Gross relation is found to be valid for different Q^2 intervals

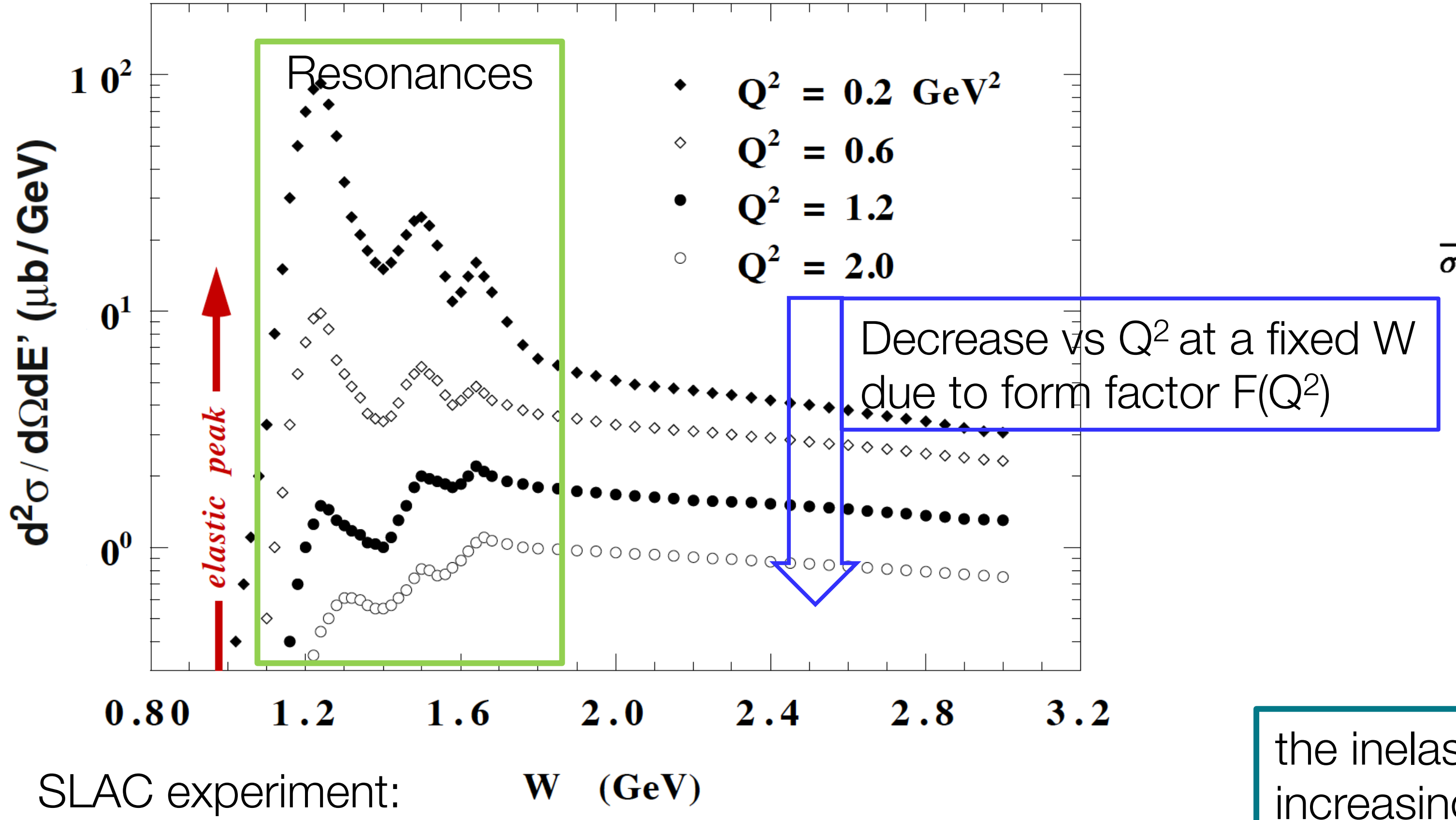
→ partons are spin ½ particles

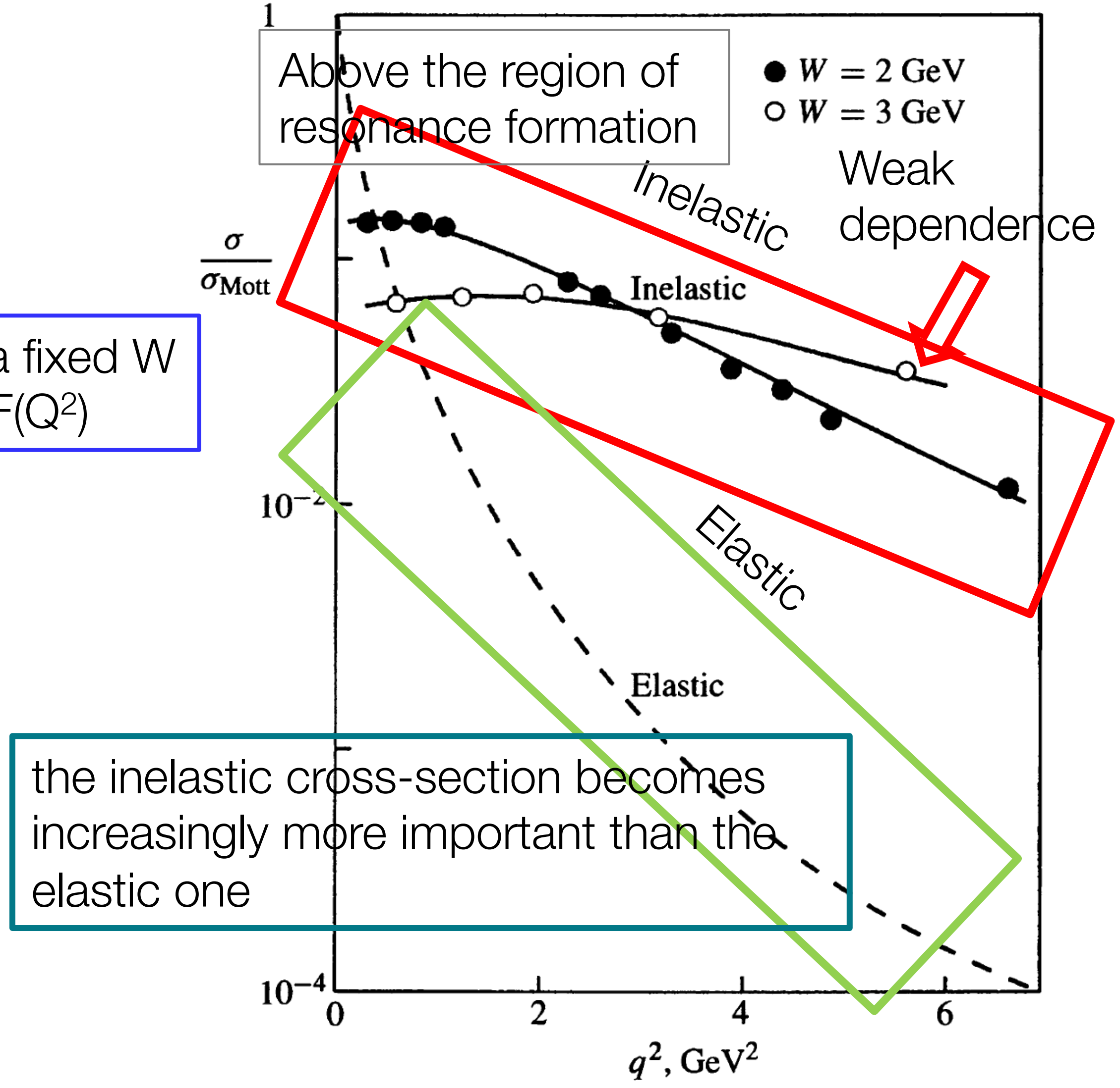


Indeed F_2 seems to depend on x and not on Q^2

- The hadron is made of point-like charged partons
- Partons are spin ½ objects
- Partons share a fraction x of the hadron 4-momentum
- The function $F_2(x)/x$ distribution represents the x-distribution of partons inside the hadron

The "x" scaling

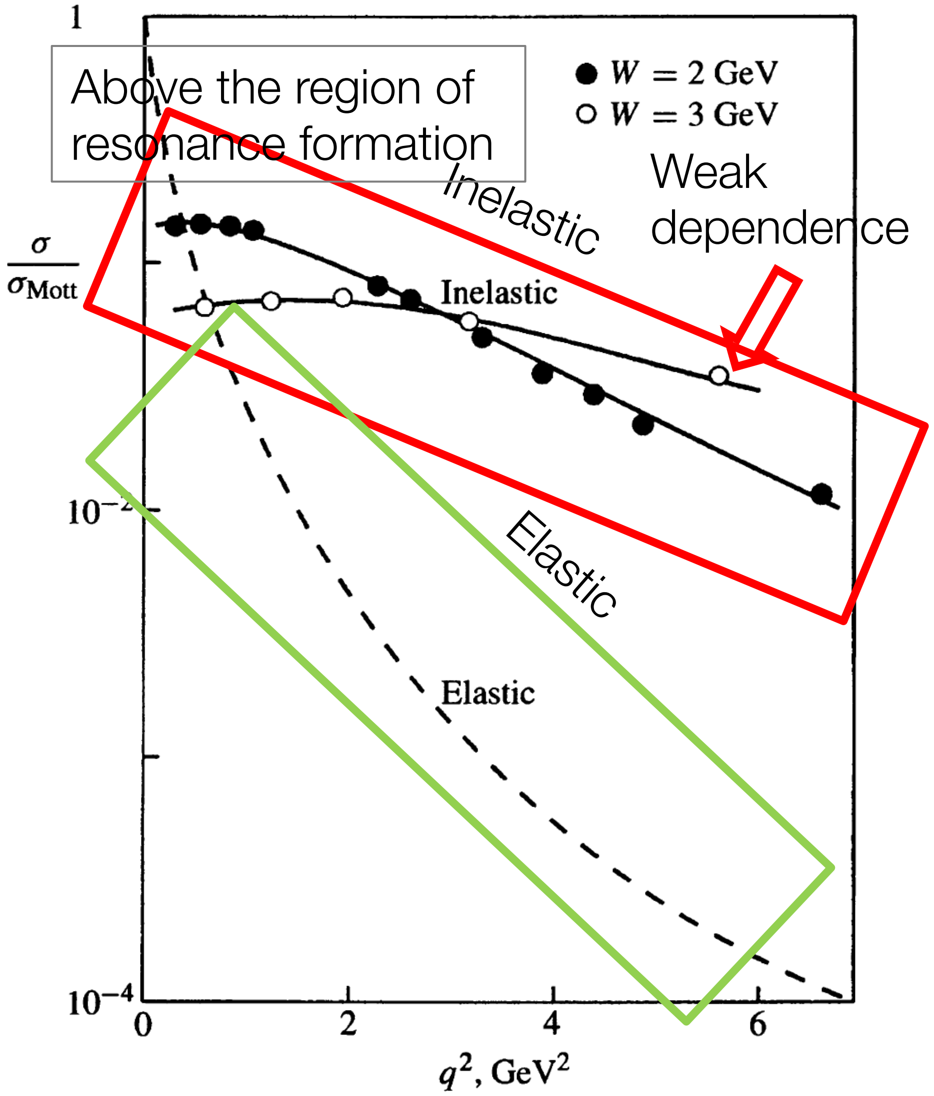




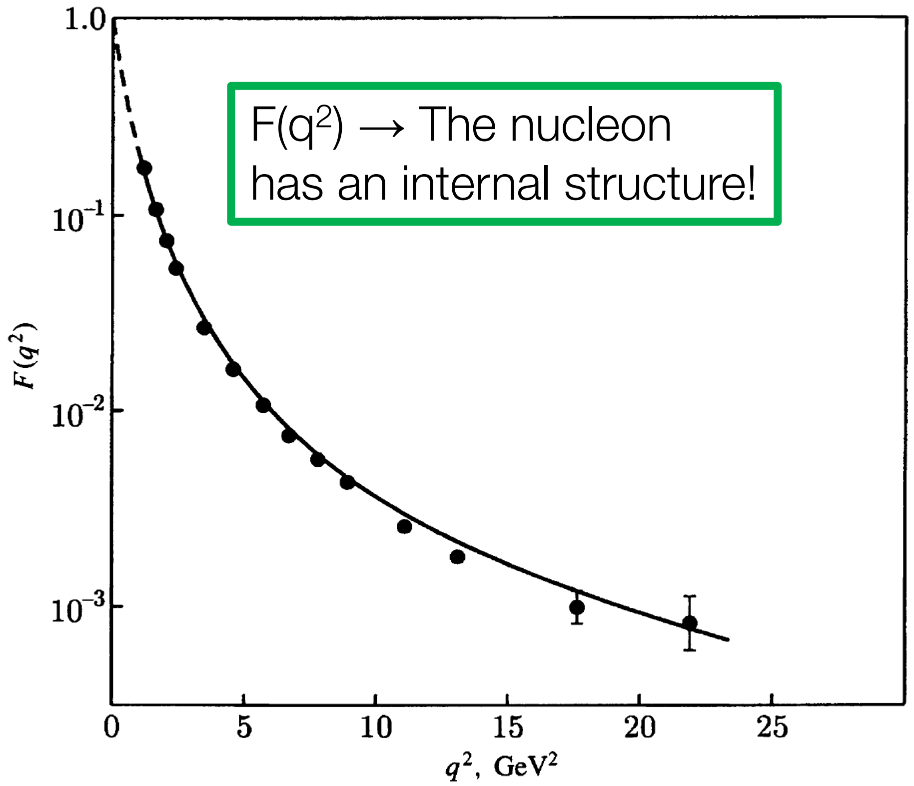
SLAC experiment:

- 20 GeV electron beam on hydrogen and deuterium
- The energy and direction of the scattered electron was measured
- $\rightarrow Q^2, \nu, W$ were measured

The “x” scaling



The weak dependence indicates that the “quark” is a point-like object!



the inelastic cross-section becomes increasingly more important than the elastic one

Toward “x”

In late 60’ Bjorken showed that in the ‘DIS’ region

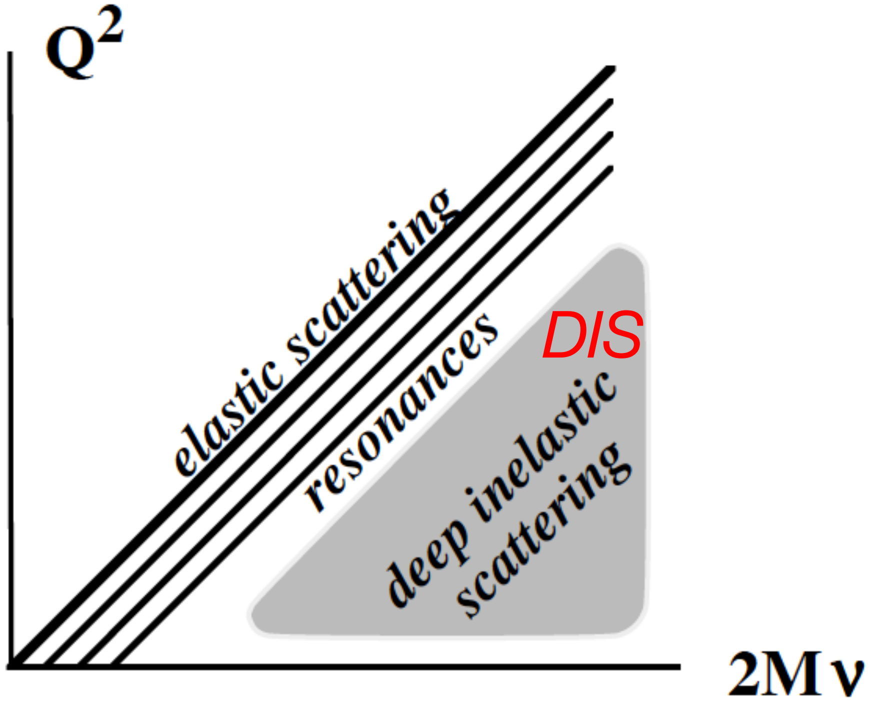
$$Q^2 \gg M^2$$
$$\nu \gg M$$

the ratio

$$x = {Q^2}/{2M\nu}$$

remains finite for

$$Q^2 \rightarrow \infty \text{ and } \nu \rightarrow \infty$$



→ depends on a dimensionless variable and not, separately, on Q^2 *and* ν

$$\lim_{Q^2,\nu\to\infty} \nu W_2(Q^2,\nu) = F_2(x) \qquad \lim_{Q^2,\nu\to\infty} M W_1(Q^2,\nu) = F_1(x)$$

Bjorken scaling

This hypothesis was

- derived in the assumption DIS consists of elastic lepton scattering on proton constituents → $Q^2 = 2m\nu$ →

  $x = m/M$ can be seen as the fraction of nucleon mass carried by the parton
- experimentally tested in the years after using a 20 GeV electron beam on hydrogen and deuterium

The Structure of Hadrons

- deep inelastic scattering may be used as a tool to study the structure and composition of the nucleons.
- the spectroscopy of these particles also gives information about the structure of the hadrons and the forces acting between them.
- By the mid-sixties a large number of apparently different hadrons were known. The quark model was invented to accommodate the ‘zoo’ of hadrons which had been discovered

		u	d	p (uud)	n (udd)
Charge	z	$+2/3$	$-1/3$	1	0
Isospin	I	$1/2$		$1/2$	
	I_3	$+1/2$	$-1/2$	$+1/2$	$-1/2$
Spin	s	$1/2$	$1/2$	$1/2$	$1/2$

Quantum numbers of u, d quarks and of protons and neutrons

Use information from both

- deep inelastic scattering and
- spectroscopy to extract the properties of the quarks.

Idea: reconstruct the properties of the nucleons (charge, mass, magnetic moment, isospin, etc.) by combining the quantum numbers of these constituents.

Combining Quantum Numbers

		u	d	p (uud)	n (udd)
Charge	z	$+2/3$	$-1/3$	1	0
Isospin	I	1/2		1/2	
	I_3	$+1/2$	$-1/2$	$+1/2$	$-1/2$
Spin	s	1/2	1/2	1/2	1/2

- The quarks have spin ½
- in the quark model, their spins must combine to give the total spin 1/2 of the nucleon → nucleons are built up out of at least 3 quarks. The proton has two u-quarks and one d-quark, while the neutron has two d-quarks and one u-quark.

- **u** and **d** quarks form an isospin doublet, it is natural to assume that also the proton and the neutron form an isospin doublet (I = 1/ 2) u-quark and d-quark can be exchanged (isospin symmetry) → proton → neutron.
- The fact that the charges of the quarks *are multiples of 1/3 is derived by* the fact that the maximum positive charge found in hadrons is two (e. g., Δ^{++}), and the maximum negative charge is one (e. g., Δ^{-}). Hence the charges of these hadrons are attributed to 3 u quarks (charge:$3 \cdot \left(\frac{2e}{3}\right) = 2e$)) and 3 d-quarks (charge: $3 \cdot \left(\frac{-1e}{3}\right) = -1e$)) respectively.

Combining the Quarks (Recap!)

- Nucleons: three *valence quarks* determine the *quantum numbers*
- Virtual quark-antiquark pairs, *sea quarks:*
  - *quantum numbers average out to zero*
  - carry a very small fractions x of the nucleon's momentum.
- There are not only "u" and "d" quarks but also s (strange), c (charm), b (bottom) and t (top). These heavy quarks contribute very little to the 'sea'.
- *Electrically charged*, *sea quarks* → "*visible*" in deep inelastic scattering.

The cross-section for electro-magnetic interactions is proportional charge[2], e_k^2

$$\Longrightarrow \quad F_2(x) = \sum_k e_k^2 \cdot x \cdot f_k(x).$$

- The six quark types can be arranged in doublets (called families or generations), according to their increasing mass :

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

Charge 2/3 e
Charge -1/3 e

Very heavy quarks, contribute very little to Deep Inelastic Scattering at ~low or moderate Q^2 . They can be neglected

Exploded View of the Proton & Neutron F_2

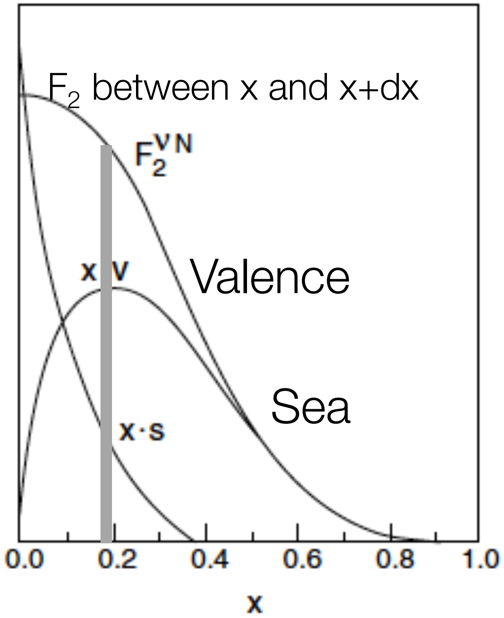
$F_2^{e,p}$ and $F_2^{e,n}$: structure functions of of protons and neutrons;
$d_{v,s}, \overline{d_{v,s}}$: x-distribution of d-valence and of sea quarks (similarly for other quarks)

$$F_2^{\mathrm{e,p}}(x) = x \cdot \left[\frac{1}{9}\left(d_\mathrm{v}^\mathrm{p} + d_\mathrm{s} + \bar{d}_\mathrm{s}\right) + \frac{4}{9}\left(u_\mathrm{v}^\mathrm{p} + u_\mathrm{s} + \bar{u}_\mathrm{s}\right) + \frac{1}{9}\left(s_\mathrm{s} + \bar{s}_\mathrm{s}\right)\right]$$

$$F_2^{\mathrm{e,n}}(x) = x \cdot \left[\frac{1}{9}\left(d_\mathrm{v}^\mathrm{n} + d_\mathrm{s} + \bar{d}_\mathrm{s}\right) + \frac{4}{9}\left(u_\mathrm{v}^\mathrm{n} + u_\mathrm{s} + \bar{u}_\mathrm{s}\right) + \frac{1}{9}\left(s_\mathrm{s} + \bar{s}_\mathrm{s}\right)\right]$$

Valence quarks

Sea quarks



The proton and the neutron can be interchanged by exchanging d and u quarks (isospin symmetry)

$$u_\mathrm{v}^\mathrm{p}(x) = d_\mathrm{v}^\mathrm{n}(x)\,,$$
$$d_\mathrm{v}^\mathrm{p}(x) = u_\mathrm{v}^\mathrm{n}(x)\,,$$
$$u_\mathrm{s}^\mathrm{p}(x) = d_\mathrm{s}^\mathrm{p}(x) = d_\mathrm{s}^\mathrm{n}(x) = u_\mathrm{s}^\mathrm{n}(x)$$

The proton has two u-quarks and one d-quark, the neutron has two d-quarks and one u-quark.

And the 'average' Nucleon structure function can be written as

$$F_2^{\mathrm{e,N}}(x) = \frac{F_2^{\mathrm{e,p}}(x) + F_2^{\mathrm{e,n}}(x)}{2} = \frac{5}{18}\, x \cdot \sum_{q=d,u}\left(q(x) + \bar{q}(x)\right) + \frac{1}{9}\, x \cdot \left[s_\mathrm{s}(x) + \bar{s}_\mathrm{s}(x)\right]$$

Term with sea quarks only → negligible

5/18 is ~ the mean square charge of u + d quarks

Comparing $F_2^{\upsilon,N}$ and $F_2^{e,N}$

- $\nu,\bar{\nu}$ DIS: same *weak charge for all quarks* no charge factors z_f^2
- Because of charge conservation and helicity, neutrinos and antineutrinos couple differently to the different types of quarks and antiquarks. These differences, however, cancel out when the structure function of an average nucleon is considered. One then obtains:

e,N

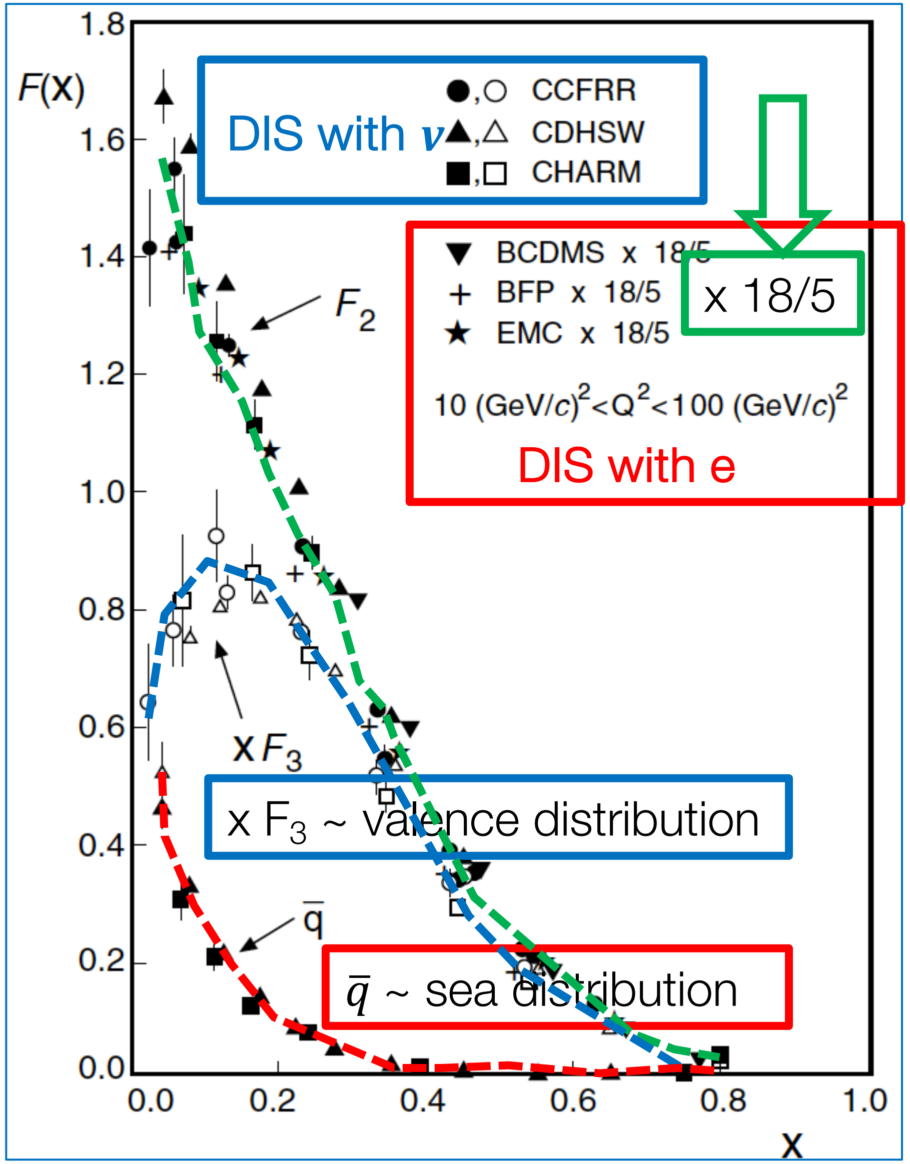
$$F_2^{e,N}(x) = \frac{5}{18} x \cdot \sum_{q=d,u} (q(x) + \bar{q}(x))$$

νN

$$F_2^{\nu,N}(x) = x \cdot \sum_f (q_f(x) + \bar{q}_f(x))$$

Experiments show that $F_2^{\upsilon,N}$ and $F_2^{e,N}$ are identical ((but for the factor 5/18 due to charge) → This means that the charge numbers +2/3 and −1/3 have been correctly attributed to the u- and d-quarks.

- Valence quarks peak at $x \approx 0.17$ and an average value of $\langle x_v \rangle \approx 0.12$
- Sea quark →low x values with an average value of $\langle x_s \rangle \approx 0.04$



Comparing $F_2^{\upsilon,N}$ and $F_2^{e,N}$

WARNING!

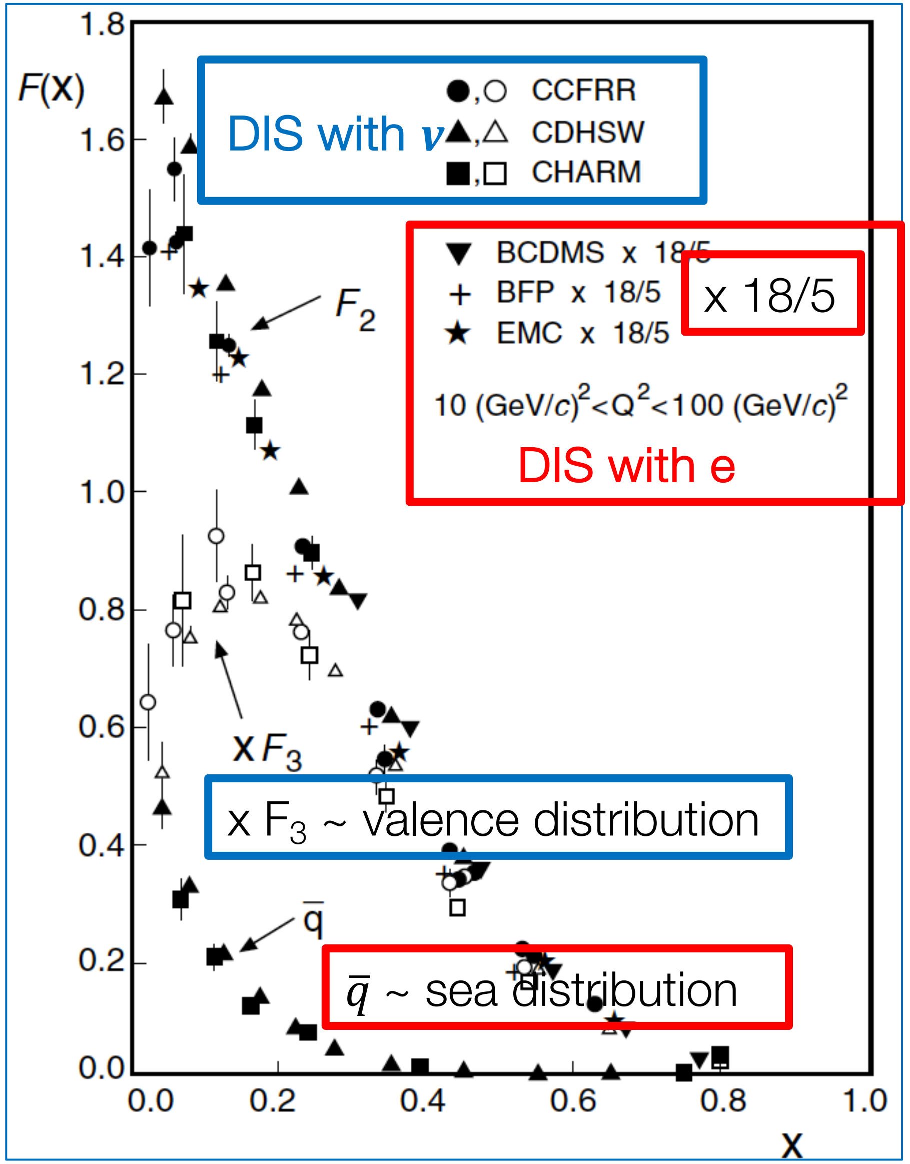
The integral of $F_2^{\upsilon,N}$ and $F_2^{e,N}$ gives about 0.5 → IMPORTANT INFORMATION: half of the momentum of a nucleon is carried by components that are NOT quarks

$$\int_0^1 F_2^{\nu,\mathrm{N}}(x)\,\mathrm{d}x \approx \frac{18}{5}\int_0^1 F_2^{\mathrm{e,N}}(x)\,\mathrm{d}x \approx 0.5$$

This component is not detected in $F_2^{\upsilon,N}$ or $F_2^{e,N}$. This means it is sensible

- neither to electromagnetic interactions
- nor to weak interactions

→ *gluons*



Looking at F_2^n / F_2^p

$$F_2^{e,N}(x) = \frac{F_2^{e,p}(x) + F_2^{e,n}(x)}{2} = \frac{5}{18} x \cdot \sum_{q=d,u} (q(x) + \bar{q}(x)) + \frac{1}{9} x \cdot [s_s(x) + \bar{s}_s(x)]$$

- F_2^n / F_2^p is about 1 at very low x, the contribution of valence quarks is very small and the ratio is sensitive to sea quarks expected to be equally present in protons and neutrons

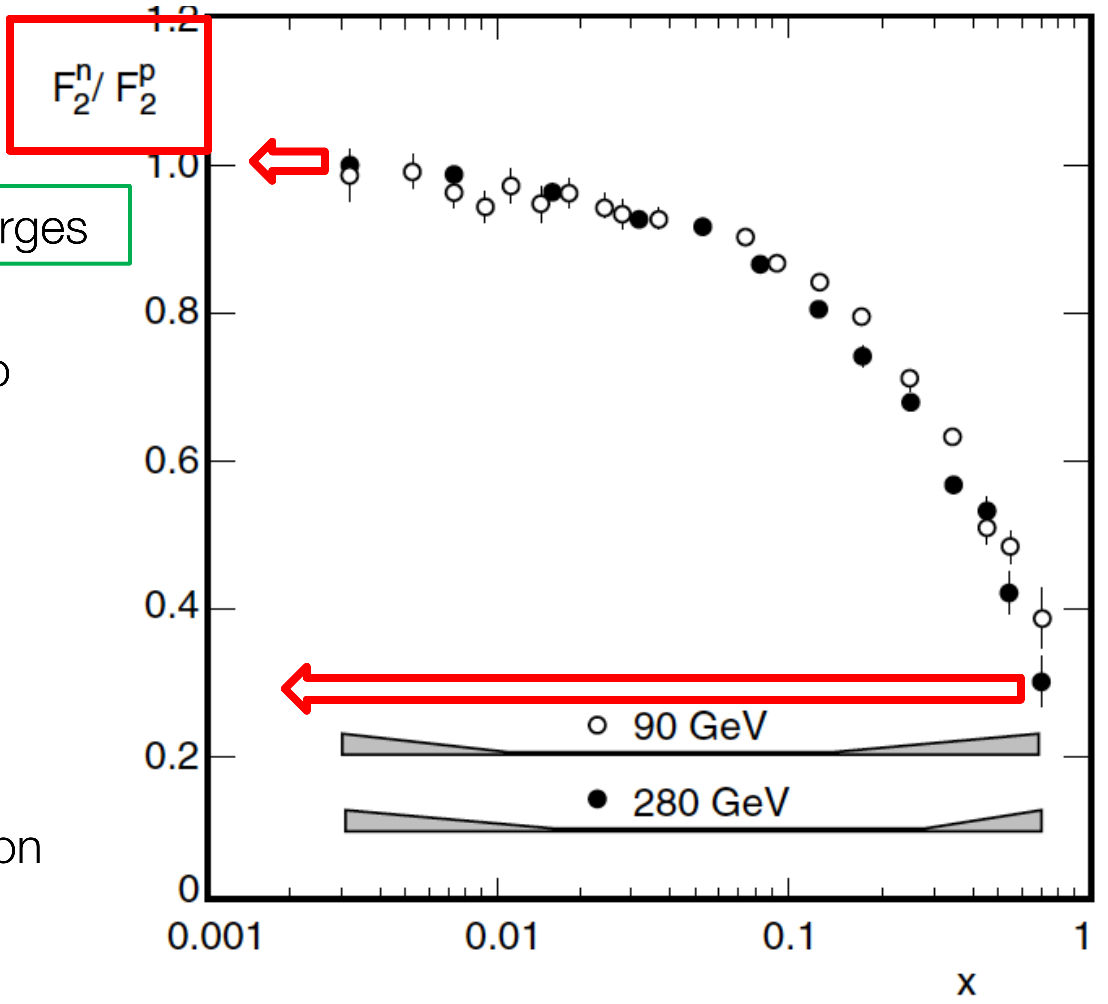
- F_2^n / F_2^p at x ~ 1 (mostly valence quarks) is expected to be about $(2z_d^2 + z_u^2)/(2z_u^2 + z_d^2) \approx \frac{2}{3}$ (neutron / proton) the ratio of the mean square charges of the valence quarks of the neutron and proton

z_d, z_u charges

- It is found to be ~ ¼ ~ $(2/3)^2/(1/3)^2$

  - $(2z_d^2 + z_u^2)/(2z_u^2 + z_d^2)$

- This means that the ratio is about $(z_u^2)/(z_d^2)$, neutron / proton implying that large momentum fractions in the proton are carried by u-quarks, and, in the neutron, by d-quarks.



Constituent Quarks and their Masses

- About 1/2 of the momentum of a nucleon is carried by valence and sea quarks.
- Nucleons can be constructed using only the valence quarks.
- *Quark masses cannot really be measured because quarks are never free (will discuss this!).*
- Bare u and d quarks are are (expected to be) small: m_u =1.5 – 5 MeV/c^2 , m_d =3 – 9 MeV/c^2 . These masses are commonly called ***current quark masses***.

- One can assume that there are only three valence quarks, with enlarged masses (~"incorporating sea & gluons") but unchanged quantum numbers, call them "***constituent quarks***".
- The ***constituent quark masses*** are much larger (300 MeV/c^2). The ***constituent masses*** must be mainly due
  - the electromagnetic interaction → mass differences of a few MeV;
  - Additional effects must be due to differences between quark–quark interaction.

- It is often assumed that m_u ~ m_d ~ few MeV and m_s ~ m_u + 150MeV.
- The masses of heavier quarks are m_c ~1.550 MeV and m_b ~ 4.300 MeV.
- Hadrons and mesons made of the t quarks cannot be formed because the quark t is free for a very short time.

Quark	Colour	Electr. Charge	Mass $[\mathrm{MeV}/c^2]$ Bare Quark	Const. Quark
down	b, g, r	$-1/3$	3 – 9	≈ 300
up	b, g, r	$+2/3$	1.5 – 5	≈ 300
strange	b, g, r	$-1/3$	60 – 170	≈ 450
charm	b, g, r	$+2/3$	1 100 – 1 400	
bottom	b, g, r	$-1/3$	4 100 – 4 400	
top	b, g, r	$+2/3$	$168 \cdot 10^3 - 179 \cdot 10^3$	

Quarks in Hadrons: Baryons and Mesons

Hadrons can be classified in two groups:

1. the baryons , fermions with half-integral spin
2. the mesons, bosons with integral spin.

Baryons.

.

- baryons are composed of three quarks.
- quarks have spin 1/2, baryons have half-integral spin.
- baryons are produced in pairs. Baryon number B = 1 for baryons and B = – 1 to antibaryons → B = +1/ 3 for quarks, and B = – 1/ 3 for antiquarks.
- Experiments indicate that baryon number is conserved in all particle reactions and decays.
- The B(quark) – B(antiquark) number is conserved.
- This would be violated by, e. g., the hypothetical **decay of the proton: $p \rightarrow \pi^0 + e^+$** . Without baryon number conservation this decay mode would be energetically favoured. Yet, it has not been observed.

Mesons.

- Pions are the lightest hadrons ~ 140 MeV/c^2.
- They are found in three different charge states: π^-, π^0 and π^+ .
- Pions have spin 0. It is, therefore, natural to assume that they are composed of a quark and an antiquark: this is the only way to build the three charge states out of quarks.

$$|\pi^+\rangle = |u\bar{d}\rangle \quad |\pi^-\rangle = |d\bar{u}\rangle \quad |\pi^0\rangle = \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle$$

- The pions are the lightest systems of quarks. Hence, they can only decay into the even lighter leptons or into photons.

- The pion mass is considerably smaller than the constituent quark mass → the interquark interaction energy has a substantial effect on hadron masses.
- The total angular momentum = vector sum of the quark, antiquark spins, integer orbital angular momentum contribution.

- Mesons eventually decay into electrons, neutrinos and/or photons; there is no “meson number conservation (the number of quarks minus the number of antiquarks is zero) → any number of mesons may be produced or annihilated.

Introducing Coloured Quarks and Coloured Gluons

Quarks have another important property called colour. This is needed to ensure that quarks in hadrons obey the Pauli principle.

Δ^{++} resonance (***baryon***!)

- It is made of three u-quarks, has spin J = 3/2 and positive parity; it is the lightest baryon with $J^P = 3/2^+$ → we therefore can assume that its orbital angular momentum is = 0;
- it has a symmetric spatial wave function. In order to yield total angular momentum 3/2, the spins of all three quarks have to be parallel:

$$|\Delta^{++}\rangle = |u^{\uparrow}u^{\uparrow}u^{\uparrow}\rangle$$

- Thus, the spin wave function is also symmetric.
- The wave function of this system is furthermore symmetric under the interchange of any two quarks, as only quarks of the same flavour are present.
- The total wave function is symmetric, in violation of the Pauli principle.

To fulfil the Pauli principle the colour, a kind of quark charge, has to be introduced.
HP: The colour quantum number can assume three values, which may be called red, blue and green.
Accordingly, antiquarks carry the anti-colours anti-red, anti-blue , and anti-green.

The strong interaction binds quarks into a hadron → mediated by force carriers → gluons.
... And gluons? Do they carry colour?

Gluons and the QCD

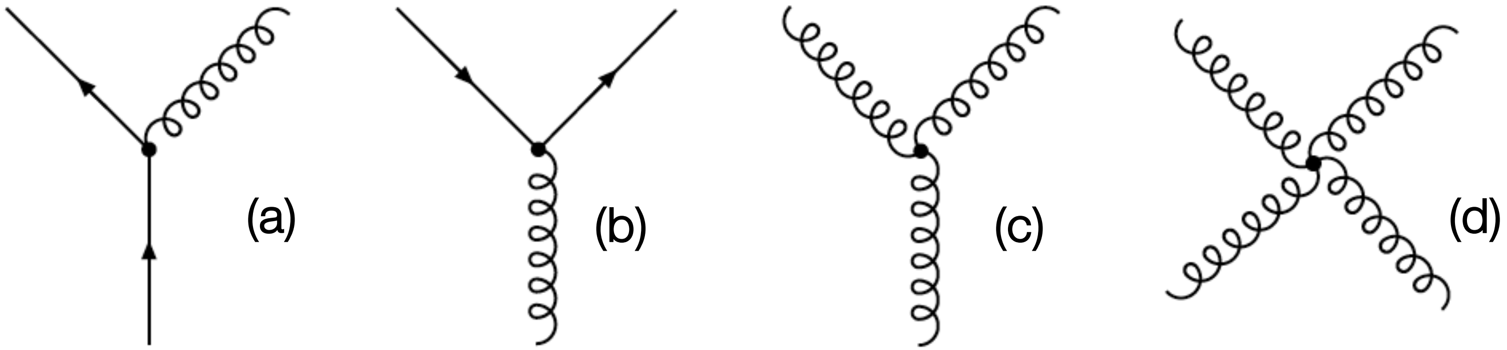
The gluons carry simultaneously colour and anti-colour → 3 colors x 3 anti-colors → 9 combinations.

Colour forms combinations that may be organised in multiplets of states: a singlet and an octet. One possible choice is (others exist):

Octet	$r\bar{g},\quad r\bar{b},\quad g\bar{b},\quad g\bar{r},\quad b\bar{r},\quad b\bar{g},\quad \sqrt{1/2}\,(r\bar{r}-g\bar{g}),\quad \sqrt{1/6}\,(r\bar{r}+g\bar{g}-2b\bar{b})$
Singlet	$\sqrt{1/3}\,(r\bar{r}+g\bar{g}+b\bar{b})$ Net colour of singlet = 0 → do not mediate QCD

Exchange of the eight gluons mediate the interaction between particles carrying colour charge, i.e., not only the quarks but also the gluons themselves.

→ This is an important difference to the electromagnetic interaction, where the photon field quanta have no charge, and therefore cannot couple with each other.



The fundamental interaction diagrams of the strong interaction: emission of a gluon by a quark **(a)**, splitting of a gluon into a quark–antiquark pair **(b)** and "self-coupling" of gluons (**c, d)**.

Colour Carriers

	Quarks	Anti-quarks	Gluon	Photon
Charge	✅	✅		
Colour	✅	✅	✅	

Hadrons and the Colour-Neutrality

In principle each hadron might exist in many different colours (the colours of the constituent quarks involved), would

- have different total (net) colours
- but would be equal in all other respects.

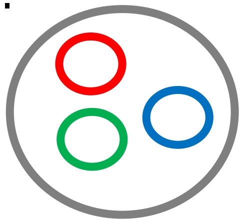
In practice only one type of each hadron is observed (one π^-, p, Δ^0 etc.)

additional condition: only colourless particles can exist as free particles ⟶

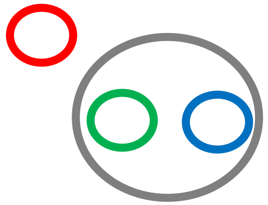
Hadrons as colour-neutral objects.

- colour + anti-colour = “white” (= white objects!)
- Three different colours = “white” as well.

➢ This is why quarks are not observed as free particles. Breaking one hadron into quarks would produce at least two objects carrying colour: the quark, and the rest of the hadron. This would be a violation of the hadron colour-neutrality.



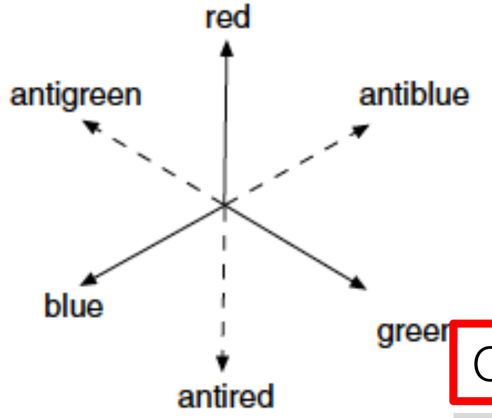
This phenomenon is, therefore, called confinement.

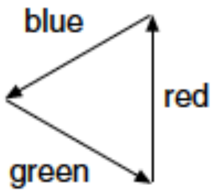


➢ This implies that the potential acting on a quark increases with increasing separation—in sharp contrast to the Coulomb potential. This phenomenon is due to the inter-gluonic interactions.

Colourless –White- Hadrons

Graphically: three vectors in a plane symbolising the three colours, rotated by 120°





$$|\pi^+\rangle = \begin{cases} |u_r \bar{d}_{\bar{r}}\rangle \\ |u_b \bar{d}_{\bar{b}}\rangle \\ |u_g \bar{d}_{\bar{g}}\rangle \end{cases}$$

The pion π^+ Is a superposition of these states

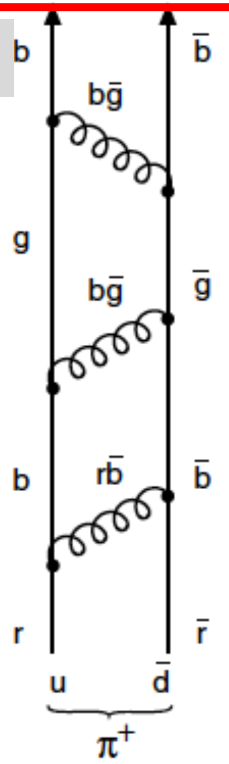
Combination of 2/3 colours giving white: colour+anti-colour or r+b+g

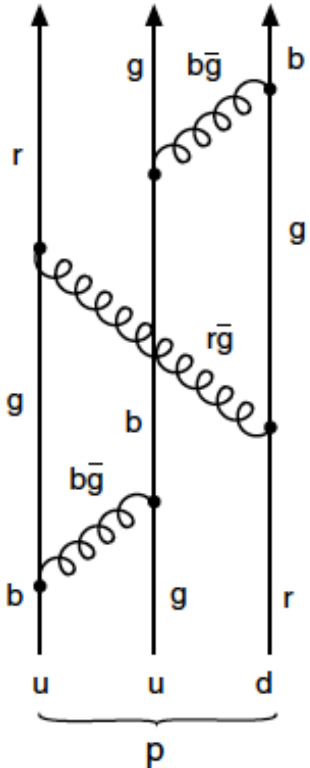
Gluons are not white: they carry colour and anti-colour

- Due to exchange of gluons the colour combination of hafrons continuously changes; **but the net-colour "white" remains.**
- to obtain a colour neutral baryon, each quark must have a different colour. The proton is a mixture of such states:

$$|p\rangle = \begin{cases} |u_b u_r d_g\rangle \\ |u_r u_g d_b\rangle \\ \vdots \end{cases}$$

- From this argument, it also becomes clear why no hadrons exist which are $|qq\rangle$ *or* $|qq\bar{q}\rangle$ combinations, or similar combinations. These states would not be colour neutral



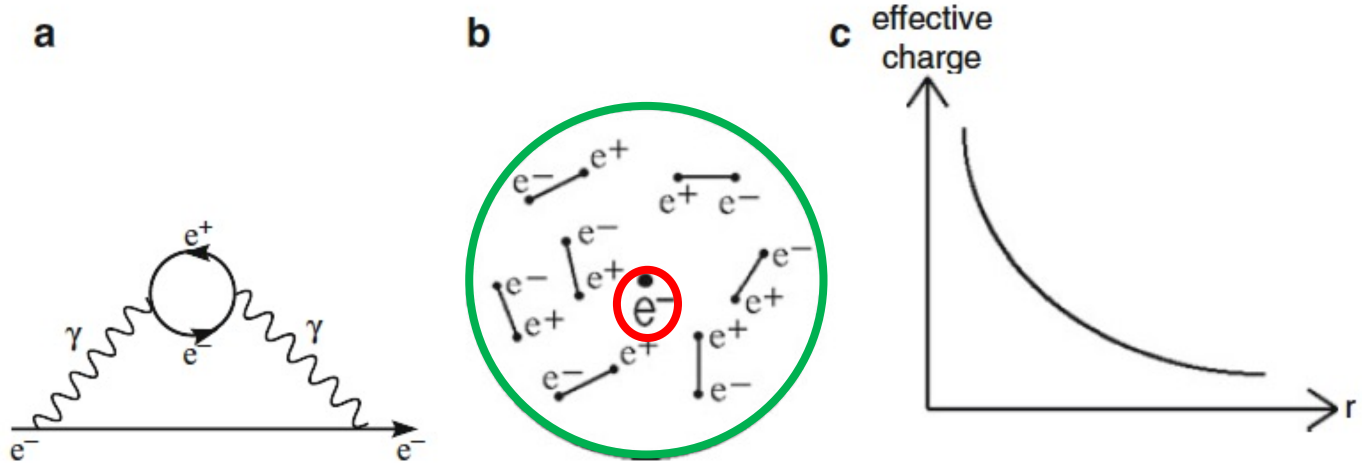


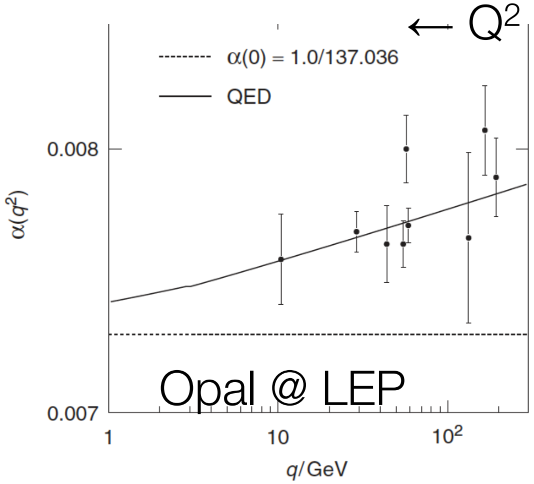
QED: Running $\alpha(Q^2)$: visible charge of electrons

Virtual pairs of e^+e^- in *em* interactions have the effect of screening the real e^- charge.

At low Q^2 is, the the distances between the interacting particles are large →

- the virtual photon sees a cloud of charges
- the effective charge of the interacting particles decreases:
- the coupling constant is small.





At high Q^2 is, the the distances between the interacting particles are small →

- the virtual photon sees the individual charge
- the effective charge of the interacting particles increases:
- the coupling constant is large.

A parametrization describing the variation of α with Q^2 is given here and it is defined at a given scale μ^2.

$\alpha(m_e) = 1/137$ $\alpha(m_Z) = 1/128$

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\ln\left(\frac{Q^2}{\mu^2}\right)}$$

The Running Coupling Constant α_s

- The coupling "constant" α_s describing the strength of the hadronic interaction between two particles depends on Q^2.
- While in the *em* interaction α_{em} depends weakly on Q^2 , in the strong interaction, however, it is very strong.

Why?

The fluctuation of the photon into a electron-positron pair

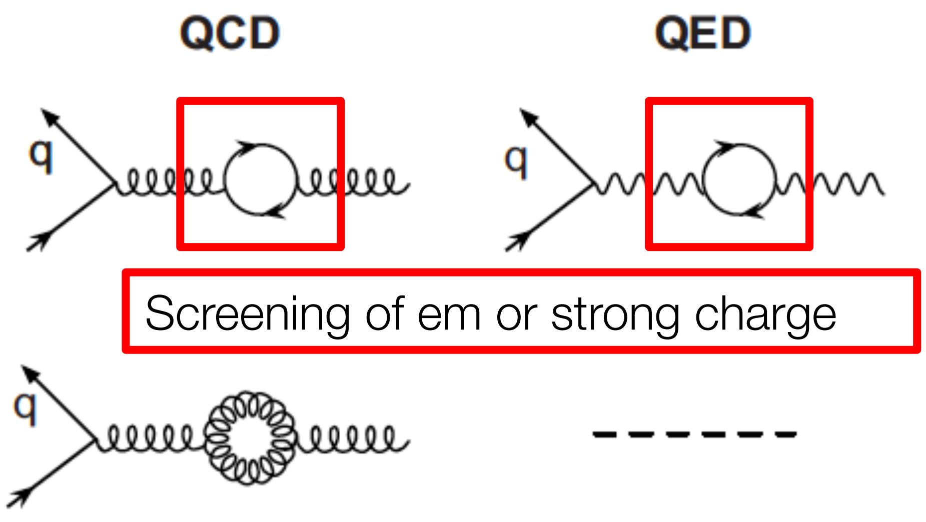
and

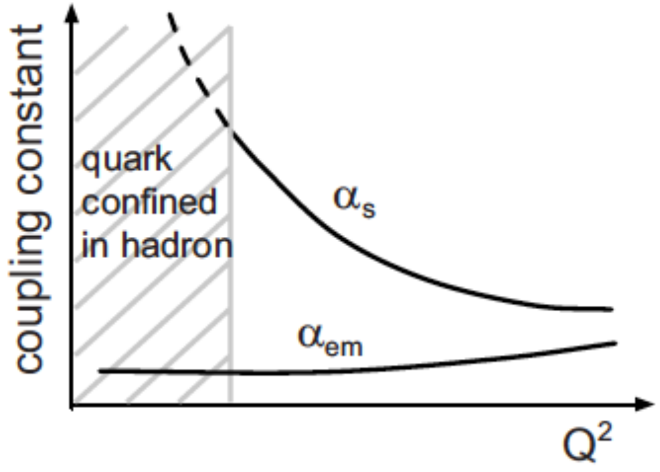
The fluctuation of the gluon into the quark-antiquark pair

generates a

- ➢ repulsive force between two quarks of the same colour (same charge) and
- ➢ the attractive force between quarks with (opposite charge) colour and anticolour

Generates screening of the electric and strong charge.



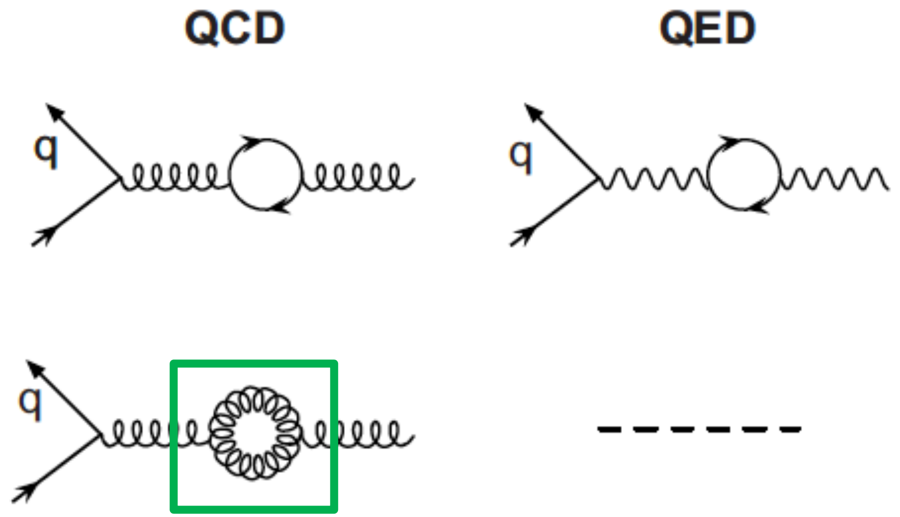


The Running Coupling Constant α_s

Gluons couple with gluons (photons do NOT couple to photons)!

Different colours may give rise to an attractive force if the quantum state is antisymmetric, and a repulsive force if it is symmetric under the interchange of quarks.

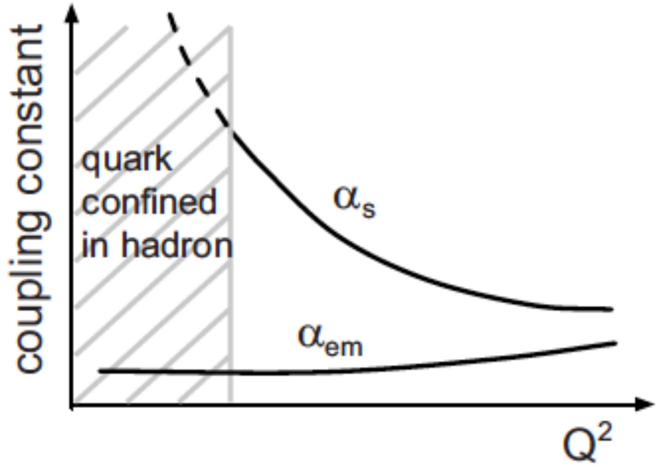
This means that the favourite state of three quarks is the state with three quarks of different colours, q_r q_b q_g, that is, the colourless state of baryons.



Anti-Screening of strong charge

The higher Q^2 is, the smaller are the distances between the interacting particles; effective charge of the interacting particles increases: the coupling constant increases.

Gluons can fluctuate into gluons → this can be shown to give anti-screening. The closer the interacting particles are, the smaller is the charge they see.



a_s decreases with increasing Q^2.

Confinement and Asymptotic Freedom

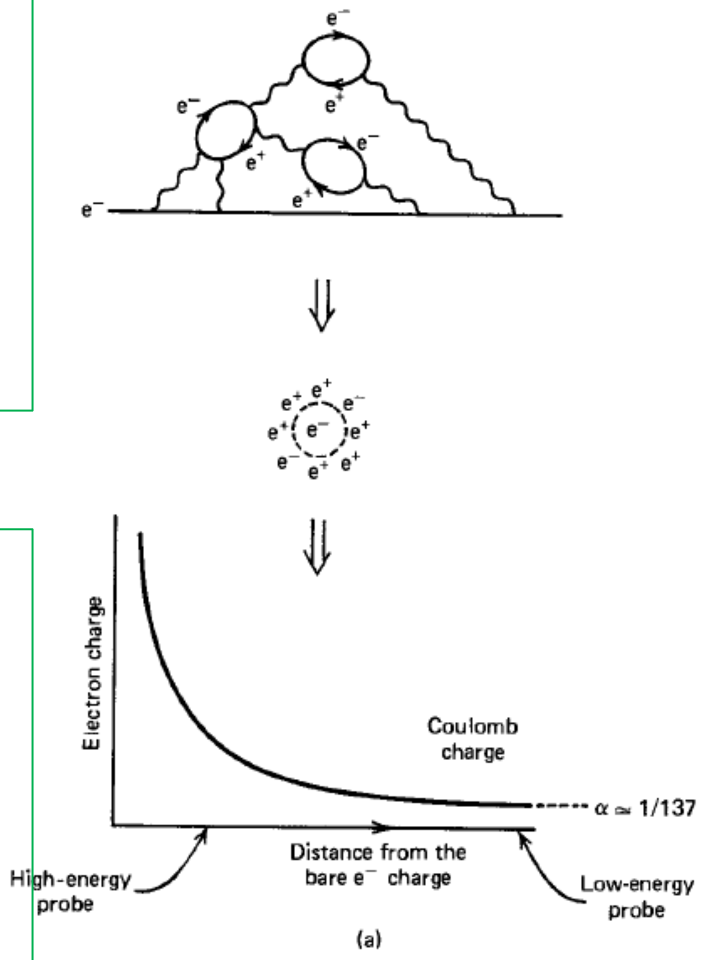
At low Q^2 is, the the distances between the interacting particles are large →

- ➢ the virtual photon sees a cloud of charges
- ➢ the effective charge of the interacting particles decreases:
- ➢ the coupling constant is small.

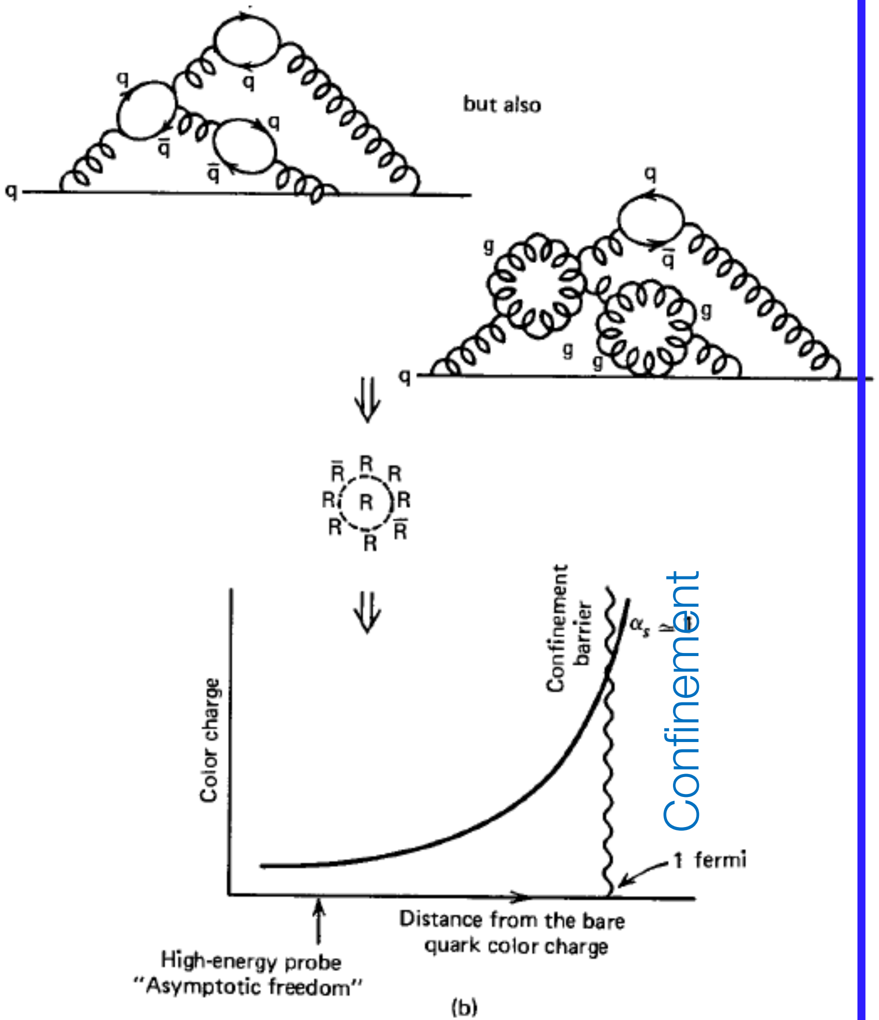
At high Q^2 is, the the distances between the interacting particles are small →

- ➢ the virtual photon sees the individual charge
- ➢ the effective charge of the interacting particles increases:
- ➢ the coupling constant is large.

Quantum electrodynamics



Quantum chromodynamics



Asymptotic Freedom

Asymptotic Freedom and Confinement

In the case of gluons the anti-screening is far stronger than the screening. A first-order perturbation calculation in QCD gives:

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \cdot \ln(Q^2/\Lambda^2)}$$

Colour neutral!

n_f number of flavours that contribute to the interaction

$Q^2 \rightarrow$ separation among different components

Λ parameter of the function determined from data

$33 = 11 \times N_c$

A heavy virtual quark–antiquark pair has a very short lifetime and range, it can be resolved only at very high Q^2. This means that n_f varies with Q^2 between $n_f \approx 3$–6 → when Q^2 increases n_f increases too.

The parameter Λ is the only free parameter of QCD. It was found to be $\Lambda \approx 250$ MeV/c by comparing the prediction with the experimental data. The application of perturbative expansion procedures in QCD is valid only if $\alpha_s << 1$.

This is satisfied for $Q^2 >> \Lambda^2 \approx 0.06\ (\text{GeV/c})^2$.

The formula indicates two regions:

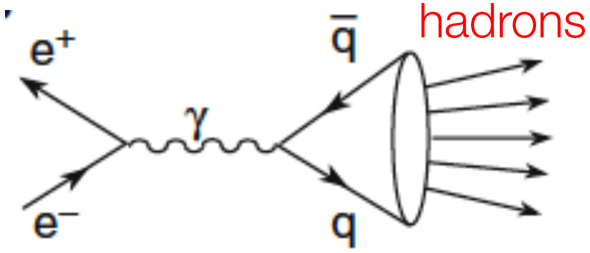
- For very small distances (high values of Q^2) " α_s decreases, vanishing asymptotically. In the limit $Q^2 \rightarrow \infty$, quarks can be considered "free", this is called asymptotic freedom.
- At large distances, (low values of Q^2) α_s increases so strongly that it is impossible to separate individual quarks inside hadrons (confinement).

Measuring the Number of Colours

Study the production of

- $q\bar{q}$ pairs and of
- $\mu^+\mu^-$ pairs

in e^+e^- interactions



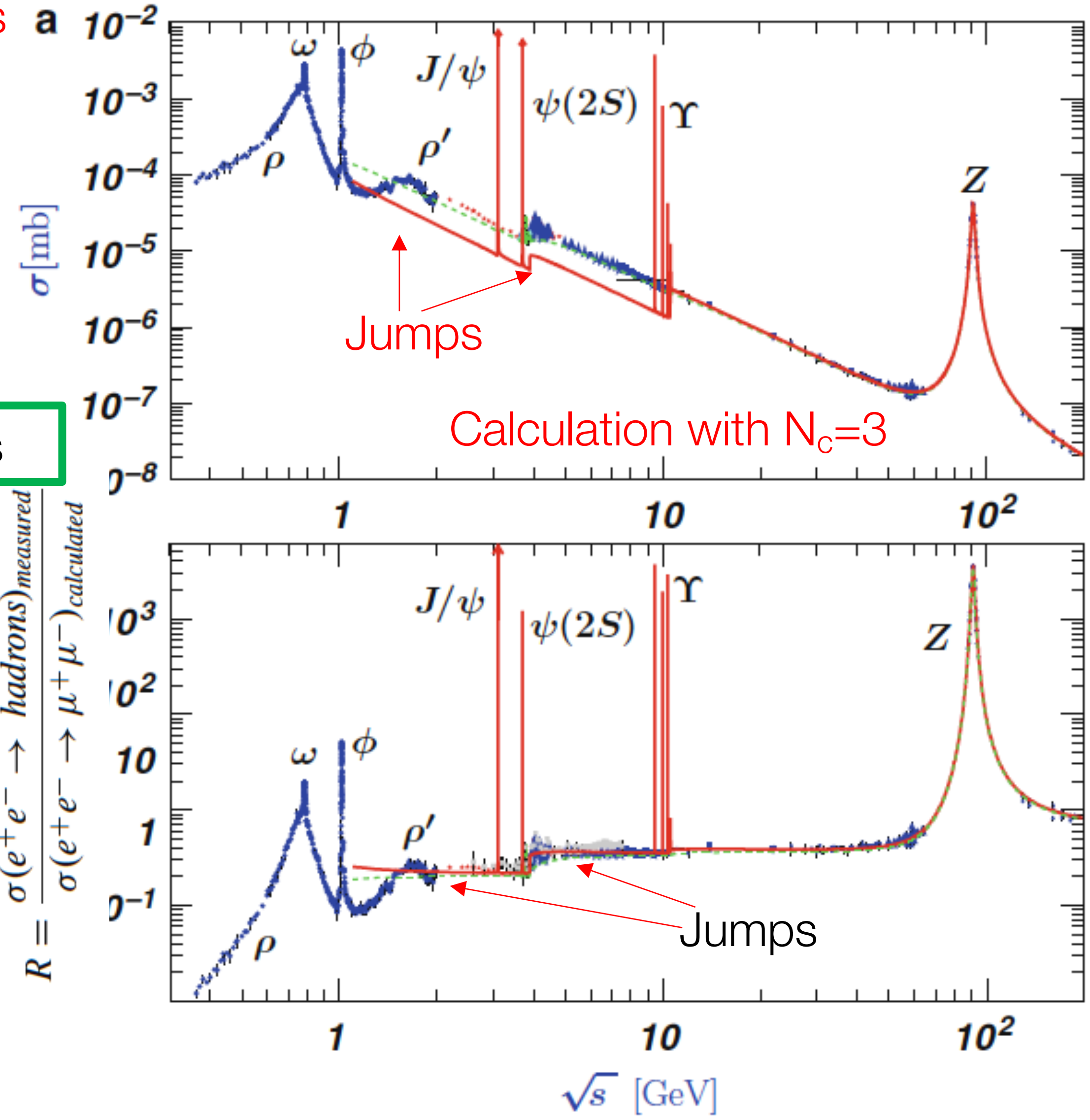
The two cross-sections are due to the exchange of one photon and are described by the two expressions below

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha_{EM}^2(\hbar c)^2}{3s}$$

$$\sigma(e^+e^- \to \gamma \to q\bar{q} \to hadrons) = N_C \frac{4\pi\alpha_{EM}^2}{3}\frac{(\hbar c)^2}{s}\sum_{n=1}^{N_f} Q_n^2$$

N_f # accessible quarks

Q_q^2 *charge of quarks involved*



Jumps are understood with the opening of kinematical windows as soon as $\sqrt{s} > m_q + m_{\bar{q}}$

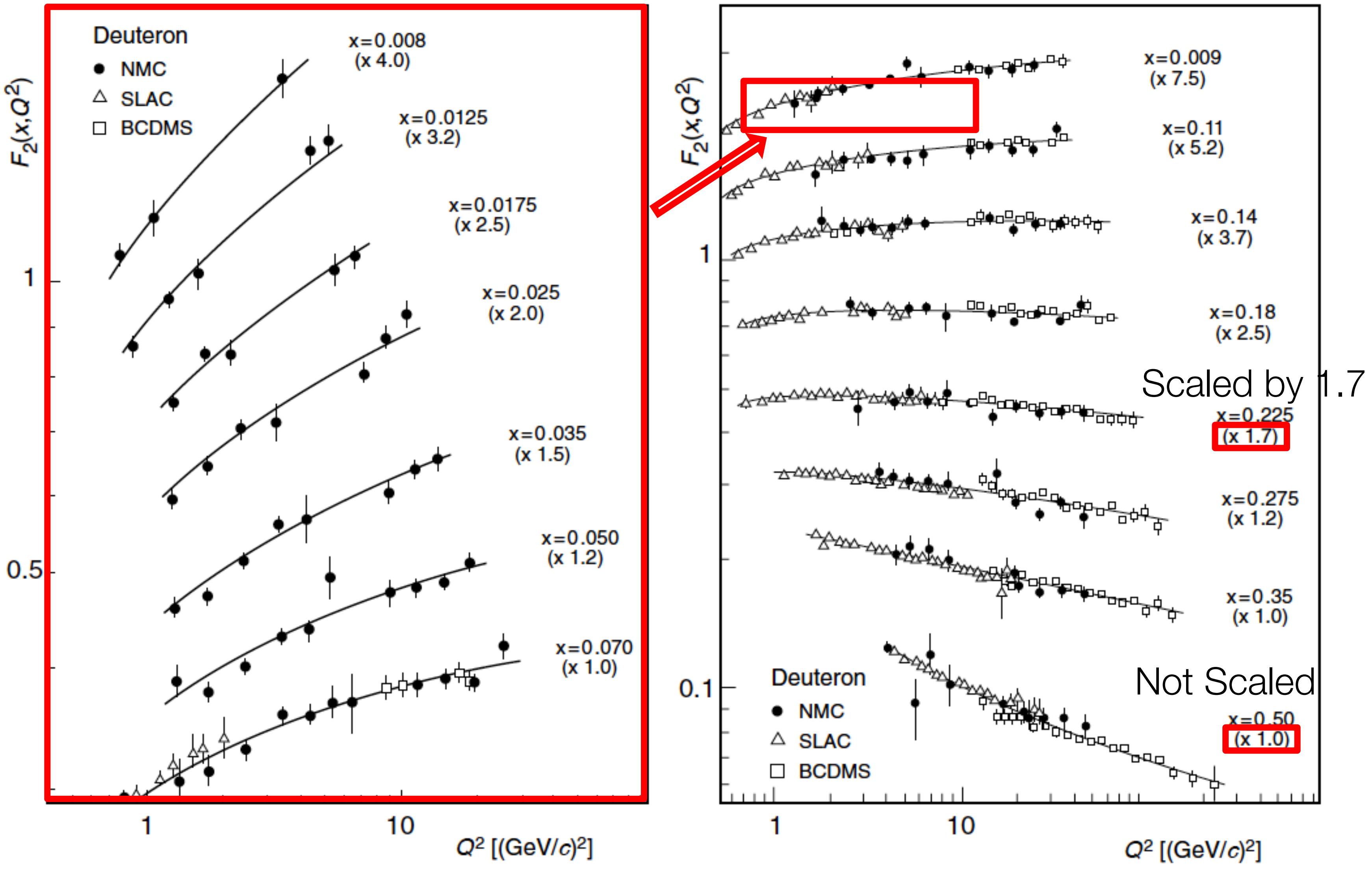
A factor $N_C = 3$ had to be introduced to account for the number of different coloured hadrons

$F_2(x)$ or $F_2(x,Q^2)$? Scaling Violations

We showed that *initial measurements* of the structure function F_2 depend only on the scaling variable *x (Bjorken scaling)* .

High precision measurements show that F_2 does depend also on Q^2 (but weakly).

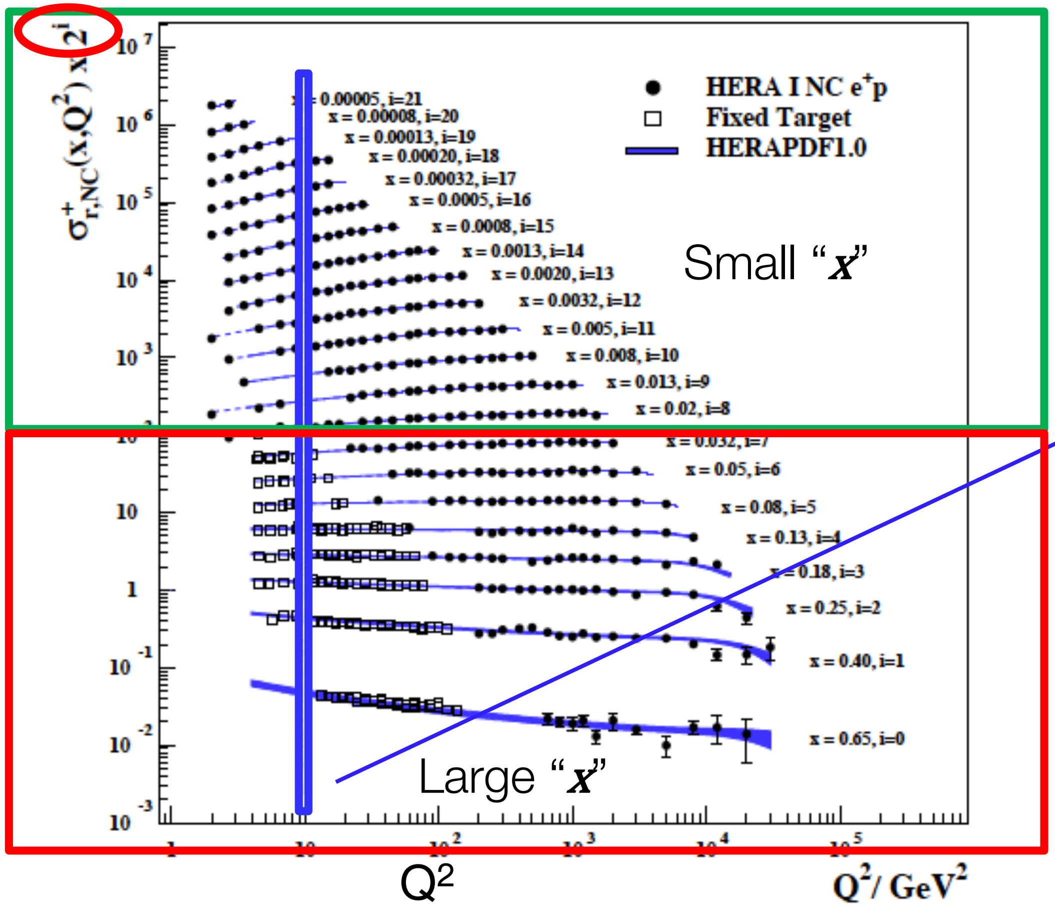
Figure → shows the experimental measurements of F_2 as a function of Q^2 at several fixed values of x .

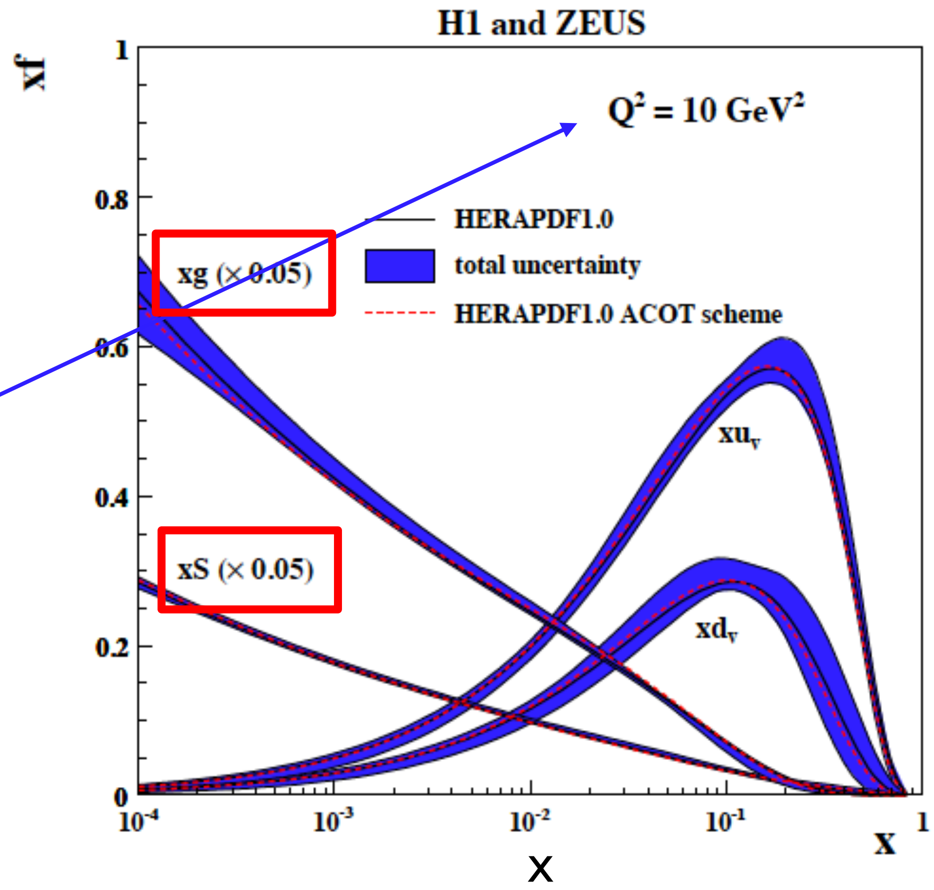


H1 and ZEUS Pdf's

$Q^2 \uparrow \sigma \uparrow$

$Q^2 \uparrow \sigma \downarrow$





On the left the HERA combined NC e+p reduced cross section and fixed-target data as a function of Q^2. The error bars indicate the total experimental uncertainty. An analytic parametrisation is superimposed.

The data shows a large range of x and Q^2.

On the right the bands represent the total uncertainty of the fit.

Scaling Violations in DIS

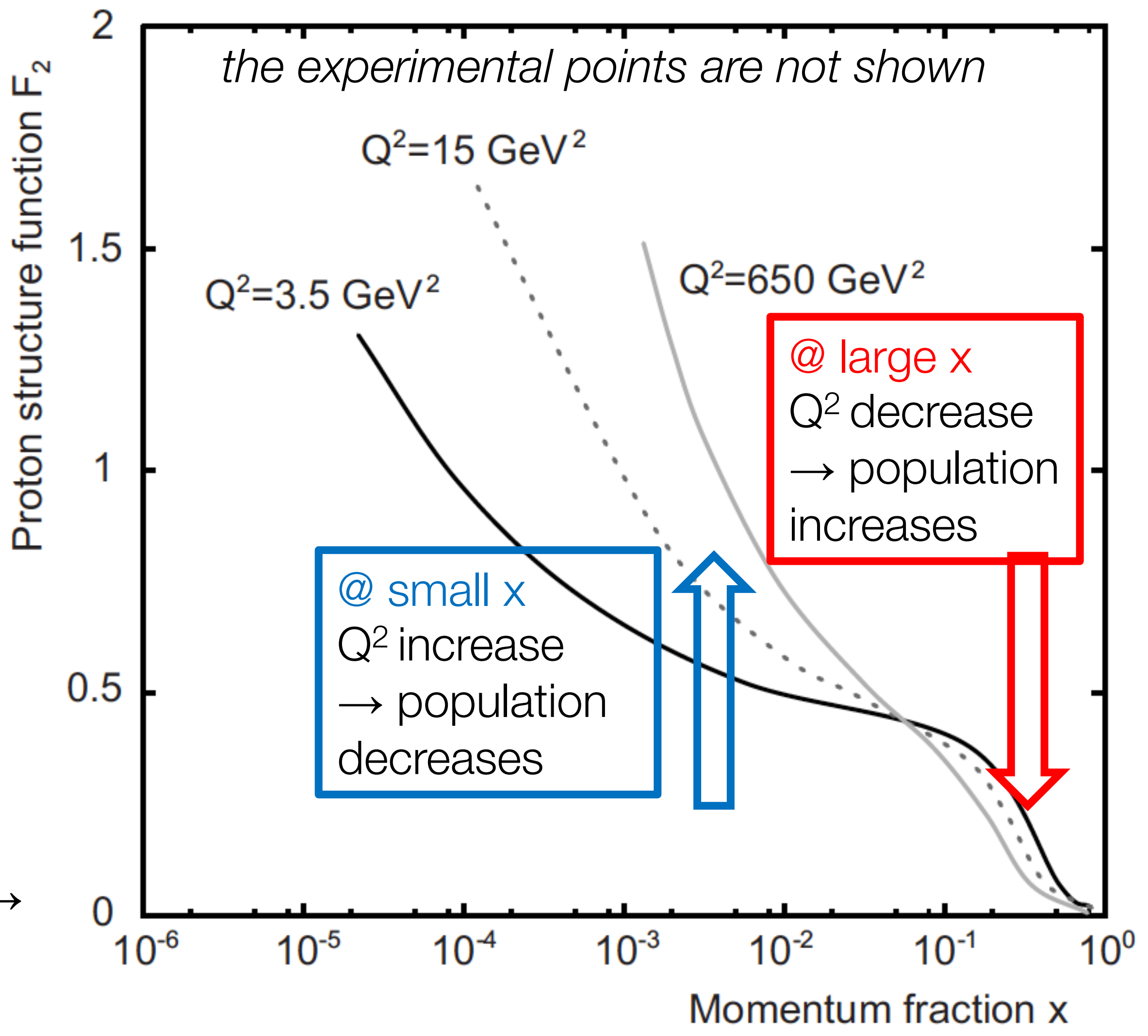
Quarks can emit or absorb gluons, gluons may split into $q\bar{q}$ pairs, or emit gluons themselves. Thus, the momentum distribution between the constituents of the nucleon is changing continuously.

We see that the structure function

- increases with Q^2 at small values of x and
- decreases when Q^2 increases at large values of x .

This behaviour, called **scaling violation**, is sketched in Fig. →.

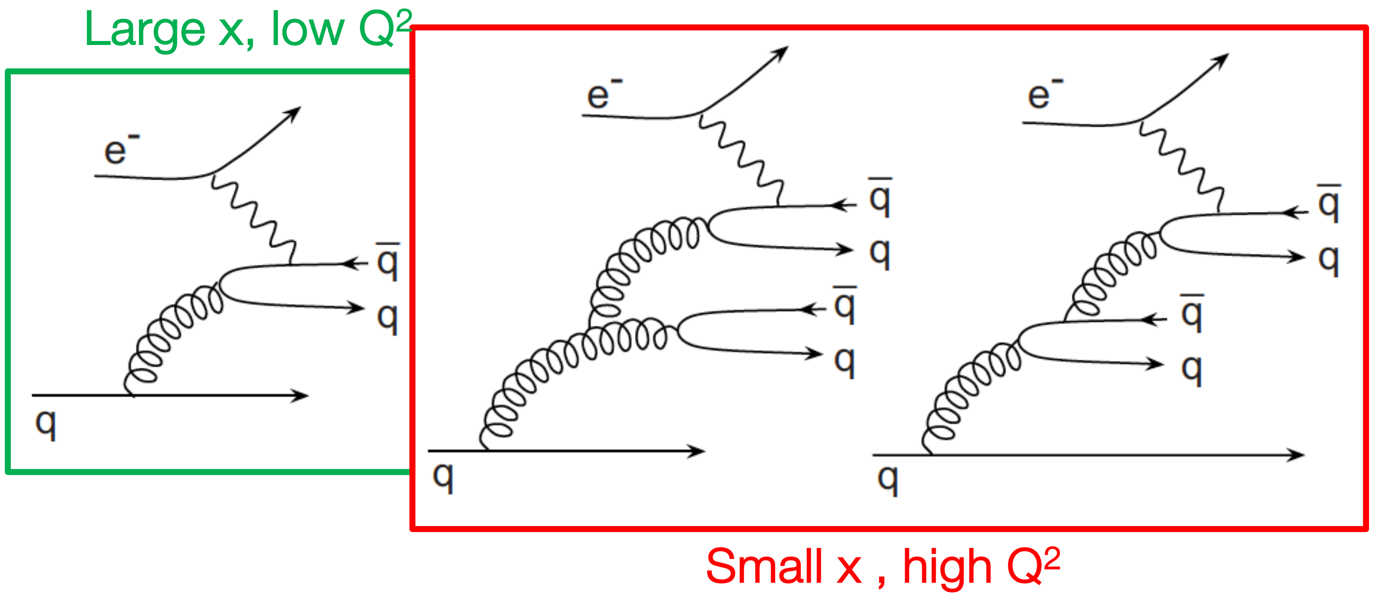
With increasing values of Q^2 many quarks seen → the momentum of the proton is shared among many partons → there are few quarks with large momentum fractions in the nucleon → quarks with small momentum fractions predominate.



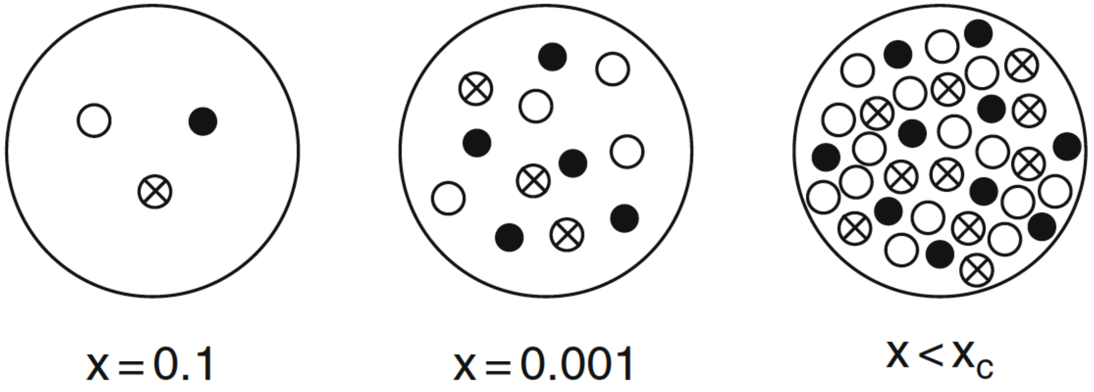
Inside the Nucleon

A virtual photon can resolve dimensions of the order of $\hbar/Q^2$. At small Q^2 quarks and emitted gluons cannot be distinguished and a quark distribution $q(x,Q^2)$ is measured.

At larger Q^2 and higher resolution, emission and splitting processes must be considered → the number of partons that share the momentum of the nucleon increases.



- The quark distribution $q(x,Q^2)$ at small momentum fractions x, therefore, is larger than $q(x,Q^2)$ at high values of x;
- the effect is reversed for large x.



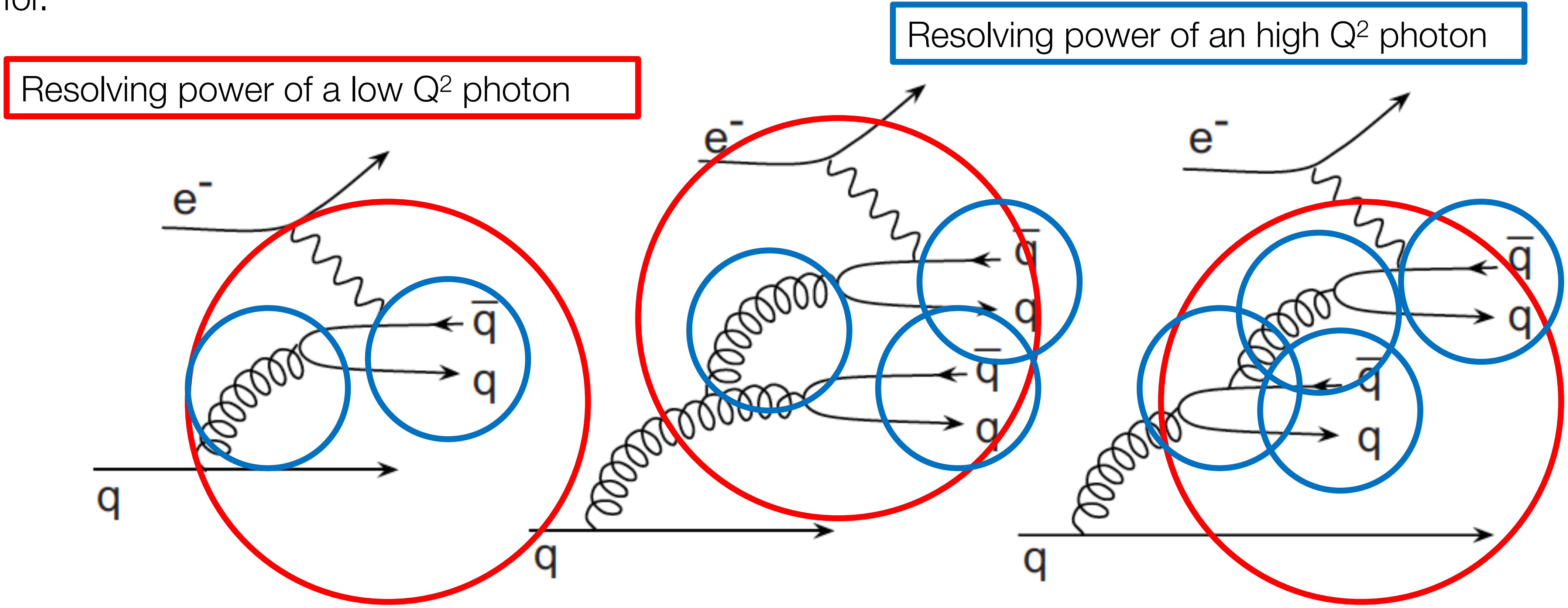
Origin of the increase of the structure function with Q^2 at small values of x and its decrease at large x. The gluon distribution $g(x,Q^2)$ shows a similar Q^2-dependence due to processes of gluon emission by a quark or by another gluon.

Visibility of Quark Components

The photon exchanged in DIS has an equivalent length of $\frac{\hbar}{\sqrt{Q^2}}$ and cannot resolve any structure smaller than this

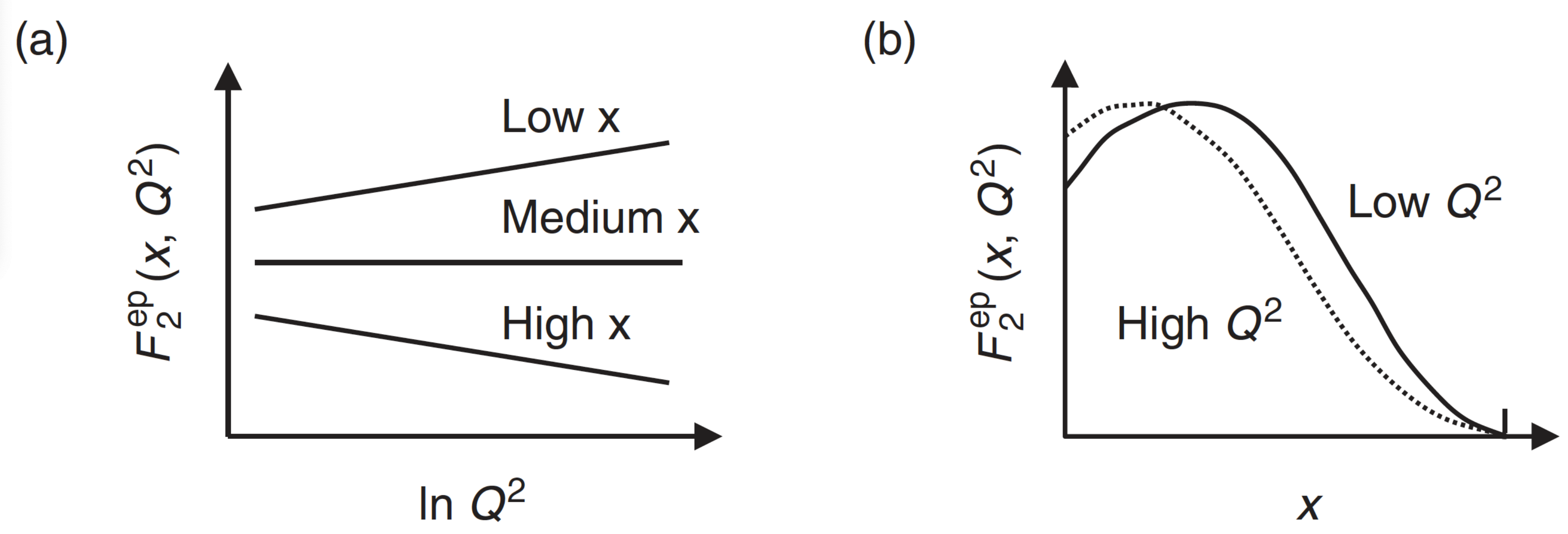
→

- at low Q^2 the photon cannot distinguish gluons and sees the x distribution of quarks
- at high Q^2 the photon starts to resolve inner structure of quarks and splitting processes must be accounted for.



Extrapolating Structure Functions

(a) the Q2 dependence at low and high x and (b) the x dependence at low and high Q2.



- Not possible to calculate the proton PDFs from first principles within QCD,
- the Q2 dependence of the PDFs is calculable: system of Altarelli Parisi equations.
-
- If $\alpha_s(Q^2)$ and the shape of $q(x,Q^2)$ and $g(x,Q^2)$ are known at a given value Q^2
- → $q(x,Q^2)$ and $g(x,Q^2)$ can be predicted from QCD for all other values of Q^2.
- The coupling $\alpha_s(Q^2)$ and the gluon distribution $g(x,Q^2)$, which cannot be directly measured, can be determined from the observed scaling violation of the structure function $F_2(x,Q^2)$.

In QCD, the above process is described in terms of scale-dependent parton distributions $f_a(x, \mu^2)$, where $a = g$ or q and, typically, μ is the scale of the probe Q. For $Q^2 \gg M^2$, the structure functions are of the form

$$F_i = \sum_a C_i^a \otimes f_a, \tag{16.21}$$

where $\otimes$ denotes the convolution integral

$$C \otimes f = \int_x^1 \frac{dy}{y} C(y) f\left(\frac{x}{y}\right), \tag{16.22}$$

and where the coefficient functions C_i^a are given as a power series in α_s. The parton distribution f_a corresponds, at a given x, to the density of parton a in the proton integrated over transverse momentum k_t up to μ. Its evolution in μ is described in QCD by a DGLAP equation (see Refs. 14–17) which has the schematic form

$$\frac{\partial f_a}{\partial \ln \mu^2} \sim \frac{\alpha_s(\mu^2)}{2\pi} \sum_b (P_{ab} \otimes f_b), \tag{16.23}$$

where the P_{ab}, which describe the parton splitting $b \to a$, are also given as a power series in α_s. Although perturbative QCD can predict, via Eq. (16.23), the evolution of the parton distribution functions from a particular scale, μ_0, these DGLAP equations cannot predict them *a priori* at any particular μ_0. Thus they must be measured at a starting point μ_0 before the predictions of QCD can be compared to the data at other scales, μ. In general, all observables involving a hard hadronic interaction (such as structure functions) can be expressed as a convolution of calculable, process-dependent coefficient functions and these universal parton distributions, e.g. Eq. (16.21).

It is often convenient to write the evolution equations in terms of the gluon, non-singlet (q^{NS}) and singlet (q^S) quark distributions, such that

$$q^{NS} = q_i - \bar{q}_i \quad (\text{or } q_i - q_j), \qquad q^S = \sum_i (q_i + \bar{q}_i). \tag{16.24}$$

The non-singlet distributions have non-zero values of flavor quantum numbers, such as isospin and baryon number. The DGLAP evolution equations then take the form

$$\frac{\partial q^{NS}}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} P_{qq} \otimes q^{NS},$$

$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} q^S \\ g \end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi} \begin{pmatrix} P_{qq} & 2n_f P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q^S \\ g \end{pmatrix}, \tag{16.25}$$

where P are splitting functions that describe the probability of a given parton splitting into two others, and n_f is the number of (active) quark flavors. The leading-order

Nomenclature

$f_a(x,q^2)$ parton distributions

P_{ab} parton splitting $b \to a$

n_f number of active quark flavors

Altarelli – Parisi Equations (Review Particles Properties)

into two others, and n_f is the number of (active) quark flavors. The leading-order Altarelli-Parisi [16] splitting functions are

$$P_{qq} = \frac{4}{3}\left[\frac{1+x^2}{(1-x)}\right]_+ = \frac{4}{3}\left[\frac{1+x^2}{(1-x)_+}\right] + 2\delta(1-x) , \quad (16.26)$$

$$P_{qg} = \frac{1}{2}\left[x^2 + (1-x)^2\right] , \quad (16.27)$$

$$P_{gq} = \frac{4}{3}\left[\frac{1+(1-x)^2}{x}\right] , \quad (16.28)$$

$$P_{gg} = 6\left[\frac{1-x}{x} + x(1-x) + \frac{x}{(1-x)_+}\right] + \left[\frac{11}{2} - \frac{n_f}{3}\right]\delta(1-x), \quad (16.29)$$

where the notation $[F(x)]_+$ defines a distribution such that for any sufficiently regular test function, $f(x)$,

$$\int_0^1 dx f(x)[F(x)]_+ = \int_0^1 dx\ (f(x) - f(1))F(x) . \quad (16.30)$$

In general, the splitting functions can be expressed as a power series in α_s. The series contains both terms proportional to $\ln\mu^2$ and to $\ln\ 1/x$. The leading-order DGLAP evolution sums up the $(\alpha_s \ln\mu^2)^n$ contributions, while at next-to-leading order (NLO) the sum over the $\alpha_s(\alpha_s \ln\mu^2)^{n-1}$ terms is included [18,19]. In fact, the NNLO contributions to the splitting functions and the DIS coefficient functions are now also all known [20–22].

In the kinematic region of very small x, it is essential to sum leading terms in $\ln\ 1/x$, independent of the value of $\ln\mu^2$. At leading order, LLx, this is done by the BFKL equation for the unintegrated distributions (see Refs. [23,24]). The leading-order $(\alpha_s \ln(1/x))^n$ terms result in a power-like growth, $x^{-\omega}$ with $\omega = (12\alpha_s \ln 2)/\pi$, at asymptotic values of $\ln\ 1/x$. More recently, the next-to-leading $\ln\ 1/x$ (NLLx) contributions have become available [25,26]. They are so large (and negative) that the result appears to be perturbatively unstable. Methods, based on a combination of collinear and small x resummations, have been developed which reorganize the perturbative series into a more stable hierarchy [27–30]. There are indications that small x resummations become necessary for real precision for $x \lesssim 10^{-3}$ at low scales. On the

Proton contains both quarks and gluons — so DGLAP is a *matrix in flavour space:*

$$\frac{d}{d \ln Q^2} \begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q \leftarrow q} & P_{q \leftarrow g} \\ P_{g \leftarrow q} & P_{g \leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

[In general, matrix spanning all flavors, anti-flavors, $P_{qq'} = 0$ (LO), $P_{\bar{q}g} = P_{qg}$]

Splitting functions are:

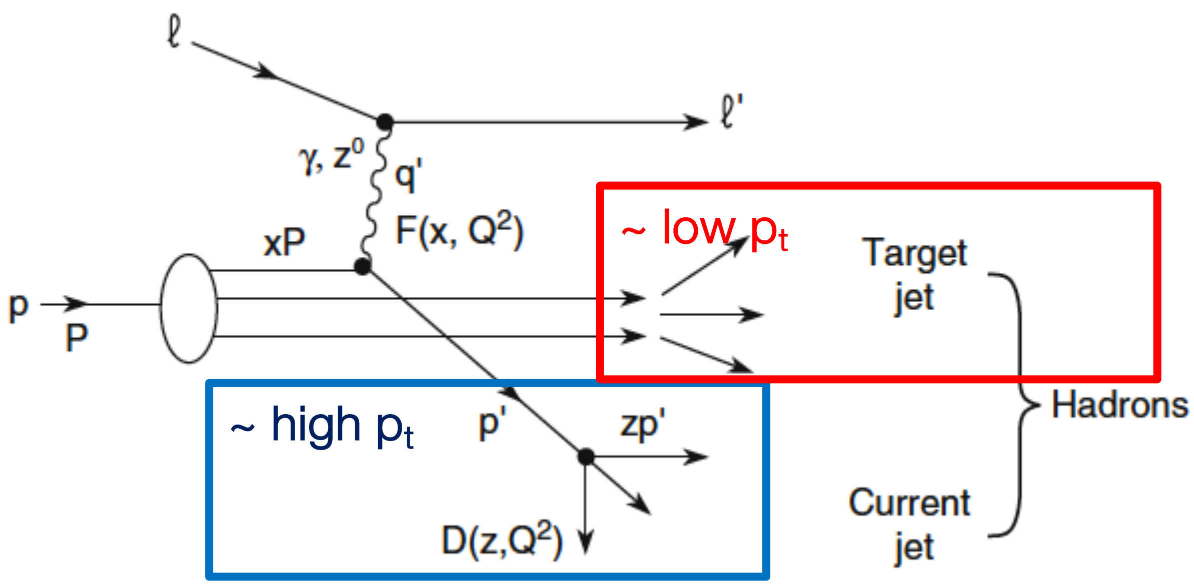
$$P_{qg}(z) = T_R \left[z^2 + (1-z)^2\right], \qquad P_{gq}(z) = C_F \left[\frac{1 + (1-z)^2}{z}\right],$$

$$P_{gg}(z) = 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \delta(1-z)\frac{(11C_A - 4n_f T_R)}{6}.$$

Have various symmetries / significant properties, e.g.

- P_{qg}, P_{gg}: *symmetric $z \leftrightarrow 1-z$* (except virtuals)
- P_{qq}, P_{gg}: *diverge for $z \to 1$* soft gluon emission
- P_{gg}, P_{gq}: *diverge for $z \to 0$* Implies PDFs grow for $x \to 0$

Fragmentation of quarks into hadrons



The second stage of the DIS process is the parton fragmentation into two jets of hadrons (also called hadronization). This is a strong interaction process, which "dresses" naked quarks to form hadrons in the final state.

The fragmentation function, *$D(z;Q^2)$* gives the energy distribution of hadrons from the interacting parton.

$D(z;Q^2)$ gives the probability that a given hadron carries a fraction z of the interacting parton energy. This energy is not experimentally measurable and must be estimated. In this second stage, the gluons play an important role since the strong interaction amongst constituents modifies the structure function, making it dependent on Q^2.

- The virtual γ-parton collision of the first phase occurs within a time $\Delta t_1 \approx \frac{\hbar}{\upsilon}, \upsilon = E - E'$.
- The quark hadronization (or quark dressing) is characterized by a time $\Delta t_2 \approx \frac{\hbar}{m_p^2} \approx 10^{-24} s$ (m_p = proton mass).

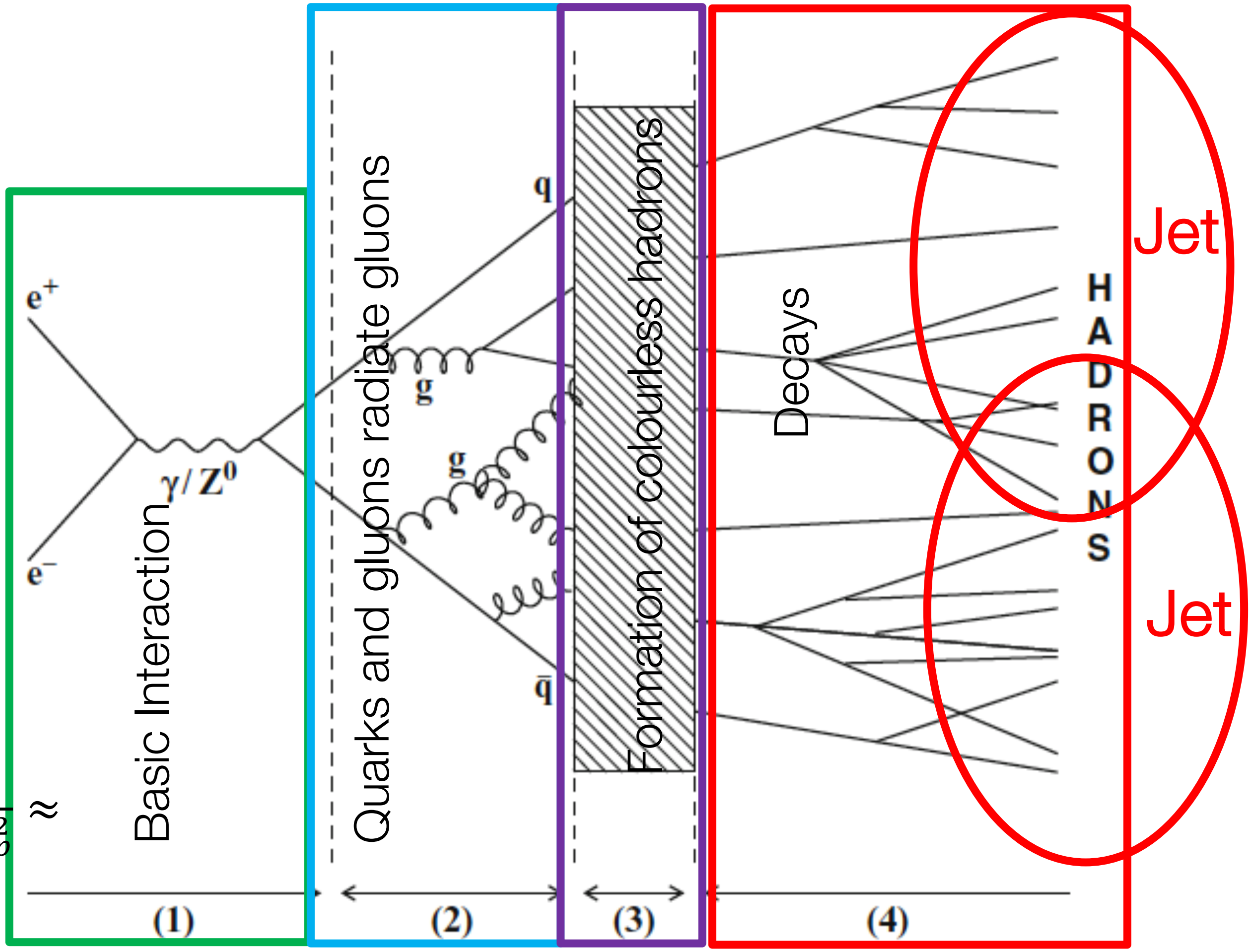
If $\upsilon \gg m_p$, one has $\Delta t_1 \gg \Delta t_2$ and the two subprocesses can be considered as distinct.

The Fragmentation Process

1. Basic (EW) Interaction
2. The quark or the antiquark can radiate a gluon, which can radiate another gluon, or produce a $q\bar{q}$ pair.
3. The coloured partons (quarks and gluons) fragment (hadronise) in colourless hadrons. The process cannot be treated with perturbation methods; → *models*
4. the hadronic resonances decay to hadrons

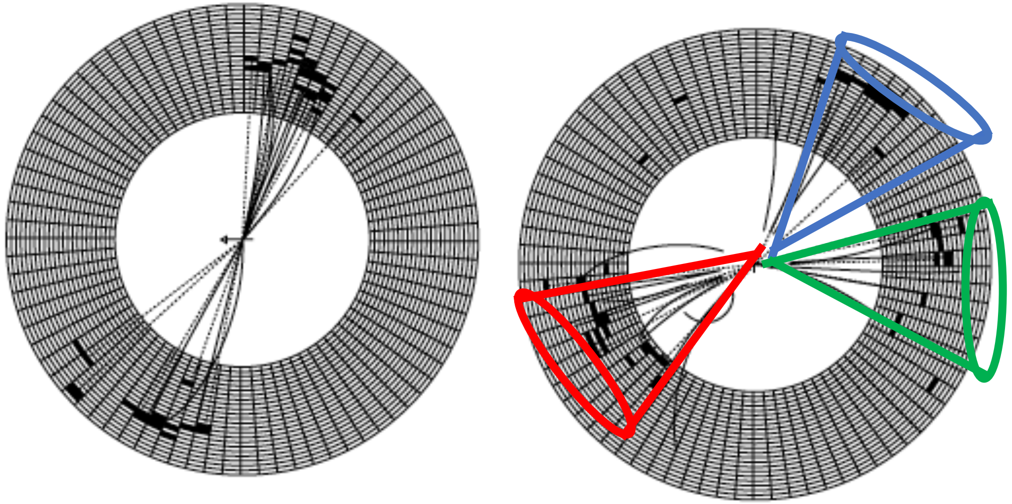
- The virtual γ-parton collision of the first phase occurs within a time $\Delta t_1 \approx \frac{\hbar}{\nu}, \nu = E - E'$.
- The quark hadronization is defined by a time $\Delta t_2 \approx \frac{\hbar}{m_p^2} \approx 10^{-24} s$ (m_p = proton mass).

If $\nu \gg m_p$, one has $\Delta t_1 \gg \Delta t_2$ and the two subprocesses can be considered as distinct.

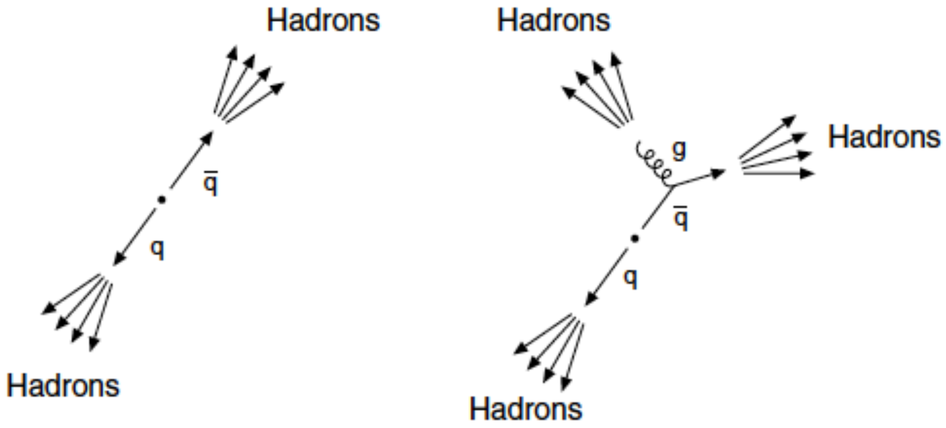


Measuring $\alpha_s(Q^2)$ at different Q^2

Jet production in $pp, p\bar{p}$ *interactions*



- At high energies, hadrons are typically produced in two jets, emitted in opposite directions.
- These jets are produced in the hadronization of the primary quarks and antiquarks.
- In addition to simple qq production, higher-order processes can happen. For example, a high-energy ("hard") gluon can be emitted, which can then manifest itself as a third jet of hadrons.



This is ~ to the emission of a γ in em bremsstrahlung. The em coupling constant α is small → emission of a hard photon is a relatively rare process.
The probability of gluon bremsstrahlung (right part of the Figure) is given by the coupling constant α_s .

A comparison of the 3- and 2-jet event rates → α_s.

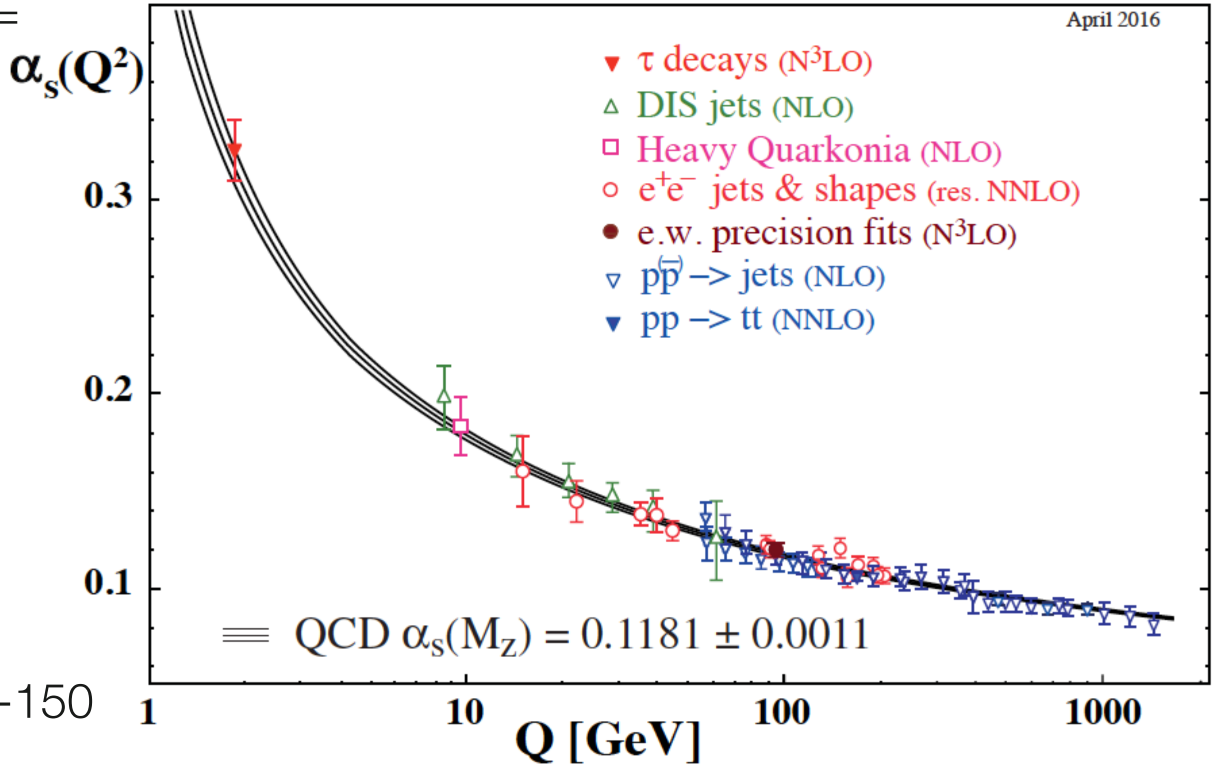
Measurements at different energies show that α_s decreases with increasing Q^2 as predicted by

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \cdot \ln(Q^2/\Lambda^2)}$$

More Ways of Measuring $\alpha_s(Q^2)$

Review of Particles Properties 2018 edition: http://pdg.lbl.gov/2018/reviews/rpp2018-rev-qcd.pdf

- Hadronic decays of the τ lepton: $\tau \rightarrow \upsilon_\tau + hadrons$ (Q=1.77GeV)
- Evolution of the nucleon structure functions measured in inelastic scattering of e,μ,ν on nucleons (Q=2 ÷ 50 GeV)
- Jet production in the inelastic scattering $ep \rightarrow eX$ (Q= 2 ÷ 50 GeV)
- Analyses of the energy levels of bound states $q\bar{q}$ (quarkonium) (Q = 1.5 ÷ 5 GeV)
- Decays of the vector mesons ϒ (Q = 5 GeV)
- Hadronic cross-section of the annihilation $e^+e^- \rightarrow hadrons$ (Q = 10 ÷ 200 GeV)
- Fragmentation function of jets produced in $e^+e^- \rightarrow hadrons$ (Q = 10 ÷ 200 GeV)
- *Hadronic decays of the Z^0 boson (Q = 91 GeV)*
- *Jet production in $pp, p\bar{p}$ interactions (Q = 50 ÷1000 GeV)*
- Photon production in in $pp, p\bar{p}$ interactions (Q = 30 ÷150 GeV)



Material

1. Povh, Rith, Scholz, Zetsche: Particles and Nuclei, An Introduction to the Physical Concepts. Springer
2. Braibant, Giacomelli, Spurio: Particles and Fundamental Interactions, An Introduction to Particle Physics, Springer
3. P.Bagnaia: Sapienza University, Particle Physics, Hadron Structure
4. M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018), Standard Model and Related Topics
5. http://th-www.if.uj.edu.pl/~erichter/dydaktyka/Dydaktyka2017/SpecFizCzast-2017/WyklSpec-4-theory-2017.pdf from (http://th-www.if.uj.edu.pl/~erichter/dydaktyka/Dydaktyka2017/SpecFizCzast-2017/)
6. Collider Physics at Hera, M.Klein and R.Yoshida

Deep Inelastic Scattering

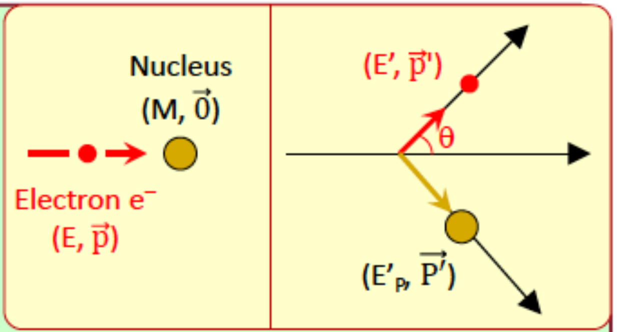
End of Deep Inelastic Scattering

Particle Physics
Toni Baroncelli
Haiping Peng
USTC

E-N Scattering Kinematics

(In)elastic scattering e-N : kinematics

$$\begin{cases} e^-_{init} & (E, \vec{p}; m); \\ N_{init} & (M, \vec{0}; M); \end{cases} \qquad \begin{cases} e^-_{fin} & (E', \vec{p}'; m); \\ N_{fin} & (E'_p, \vec{P}'_p; M); \end{cases}$$



4-momentum conservation
$$\begin{cases} E + M = E' + E'_p \rightarrow E'_p = E + M - E'; \\ \vec{p} + \vec{0} = \vec{p}' + \vec{P}'_p \rightarrow \vec{P}'_p = \vec{p} - \vec{p}'; \end{cases}$$

Square and subtract
$$\begin{cases} (E'_p)^2 = E^2 + M^2 + E'^2 + 2EM - 2EE' - 2ME'; \\ (\vec{P}'_p)^2 = p^2 + p'^2 - 2pp'\cos\theta; \\ (E'_p)^2 - (\vec{P}'_p)^2 = M^2 = E^2 + M^2 + E'^2 + 2EM - 2EE' - 2ME' - p^2 - p'^2 + 2pp'\cos\theta; \end{cases}$$

Ultra-relativistic approx.
$(m_e << E, E') \rightarrow (p \approx E, p' \approx E')$
$$\begin{cases} \cancel{M}^2 = \cancel{E}^2 + \cancel{M}^2 + \cancel{E}'^2 + 2EM - 2EE' - 2ME' - \cancel{E}^2 - \cancel{E}'^2 + 2EE'\cos\theta; \\ 0 = 2EM - 2EE' - 2ME' + 2EE'\cos\theta; \end{cases}$$

$$E' = \frac{EM}{E + M - E\cos\theta} = \frac{EM}{E(1 - \cos\theta) + M}$$ q.e.d.

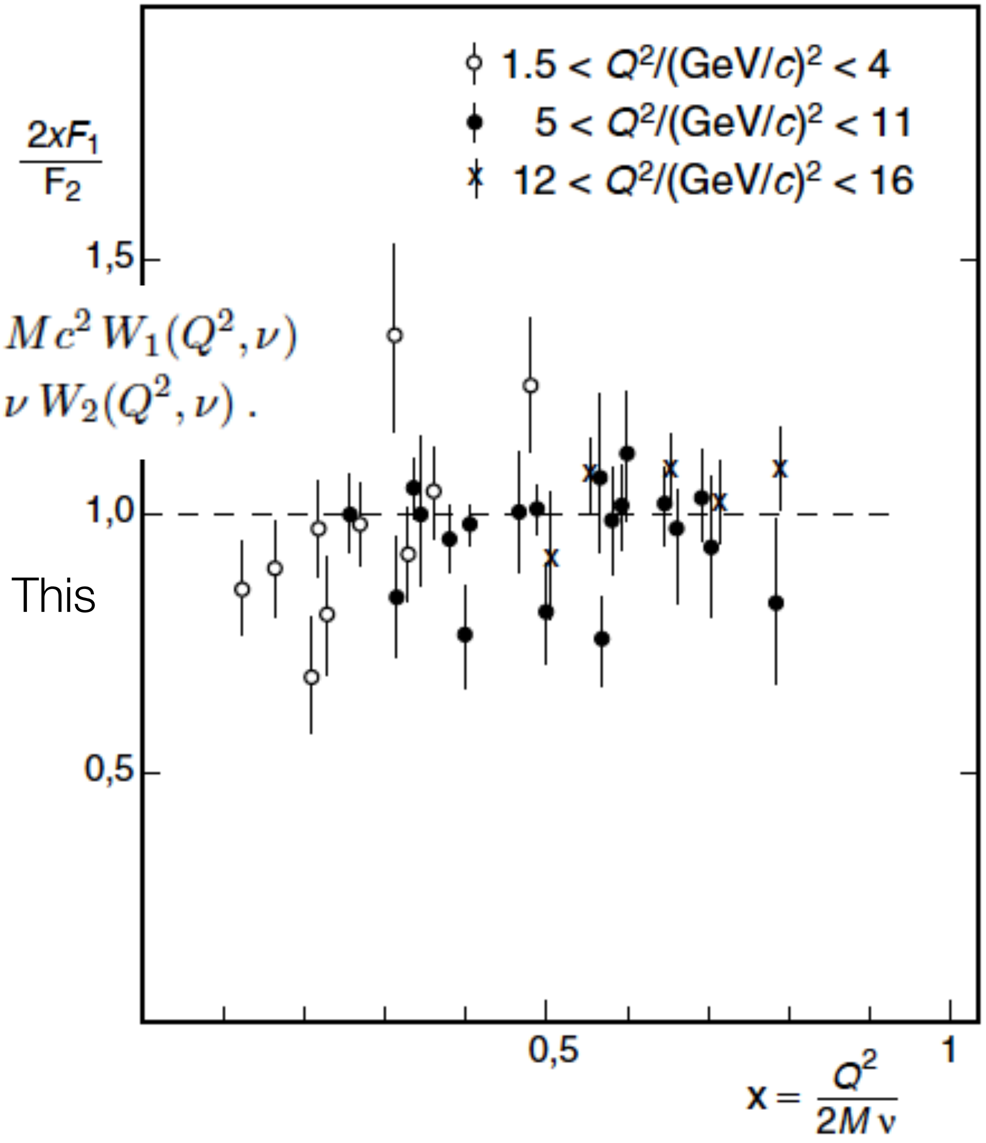
NB – The reaction is planar (why?). The final state is defined by 6 variables. There are 3 (E, p conservations) and 2 ($m^2 = E^2 - p^2$) rules. Therefore : 6-5=1 independent variable.

Callan-Gross Relation (spin ½ target)

The comparison between these two formulas

$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{point}\\ \text{spin } 1/2}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left[1 + 2\tau \tan^2\frac{\theta}{2}\right] \qquad \tau = \frac{Q^2}{4M^2c^2} \,.$$

$$\frac{d^2\sigma}{d\Omega\, dE'} = \left(\frac{d\sigma}{d\Omega}\right)^*_{\text{Mott}} \left[W_2(Q^2,\nu) + 2W_1(Q^2,\nu)\tan^2\frac{\theta}{2}\right] \qquad \begin{aligned} F_1(x,Q^2) &= Mc^2\, W_1(Q^2,\nu) \\ F_2(x,Q^2) &= \nu\, W_2(Q^2,\nu)\,. \end{aligned}$$



Gives the Callan-Gross relation: $2xF_1(x) = F_2(x)\,.$

This relation is expected to be valid ONLY for spin ½ target objects. This is verified experimentally as shown in the figure to the right.

Experimental evidence tells us that

- Nucleons are made of point-like objects
- These point-like objects carry spin 1/2