Three coupled channels analysis with OBE potential on $Z_c(3900)$

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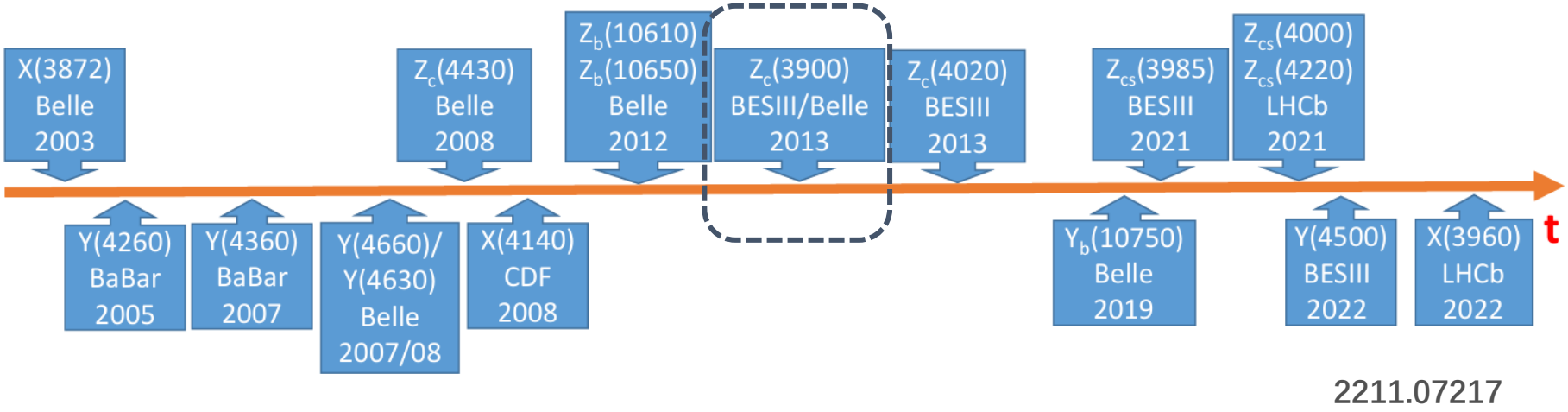
in preparation

The workshop on Super Tau Charm Facility @LanZhou, 2024

Observation of Zc(3900)

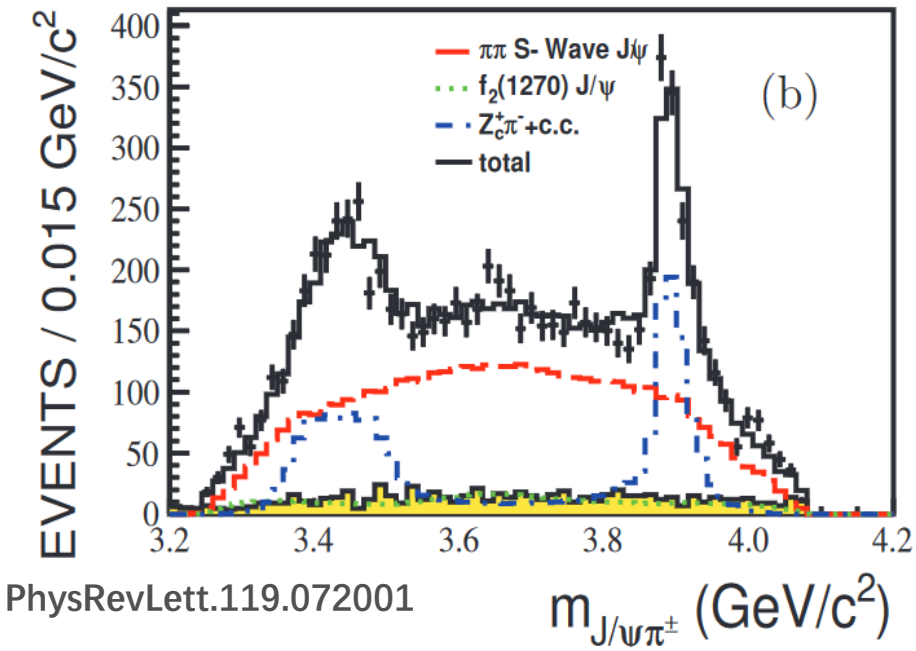


2211.07217

Observation of Zc(3900)/Zc(3885) in :

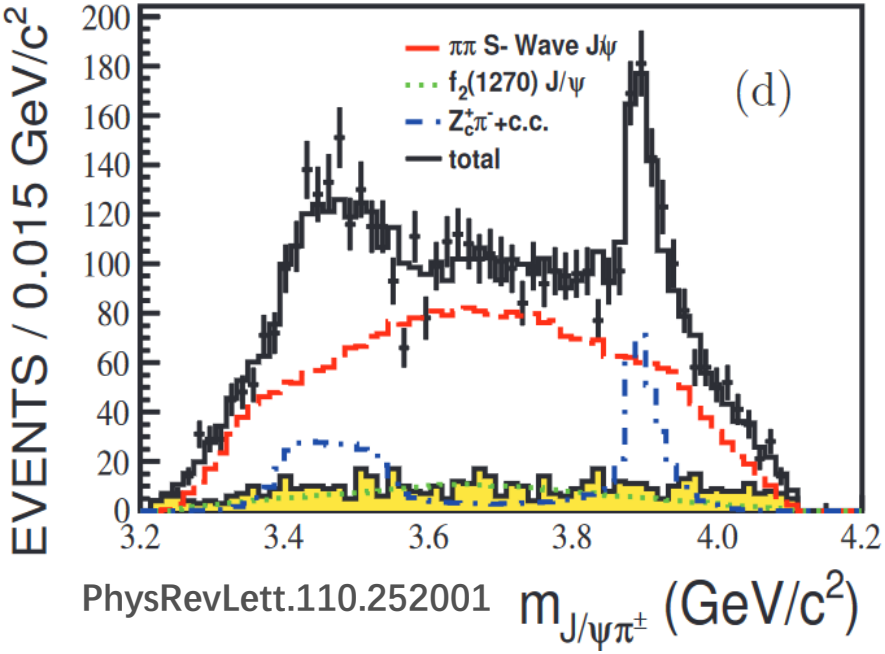
- $e^+e^- \to Y \to \pi^+\pi^- J/\psi \ @\sqrt{s} = 4.23, 4.26\ GeV$
- $e^+e^- \to Y \to \pi^\pm (D\bar{D}^*)^\mp \ @\sqrt{s} = 4.23, 4.26\ GeV$
- $e^+e^- \to Y \to \pi^\pm (\rho\eta_c)^\mp \ @\sqrt{s} = 4.23\ GeV$

$e^+e^- \to \pi^+\pi^- J/\psi \ @\sqrt{s} = 4.23\ GeV$



PhysRevLett.119.072001

$e^+e^- \to \pi^+\pi^- J/\psi \ @\sqrt{s} = 4.26\ GeV$

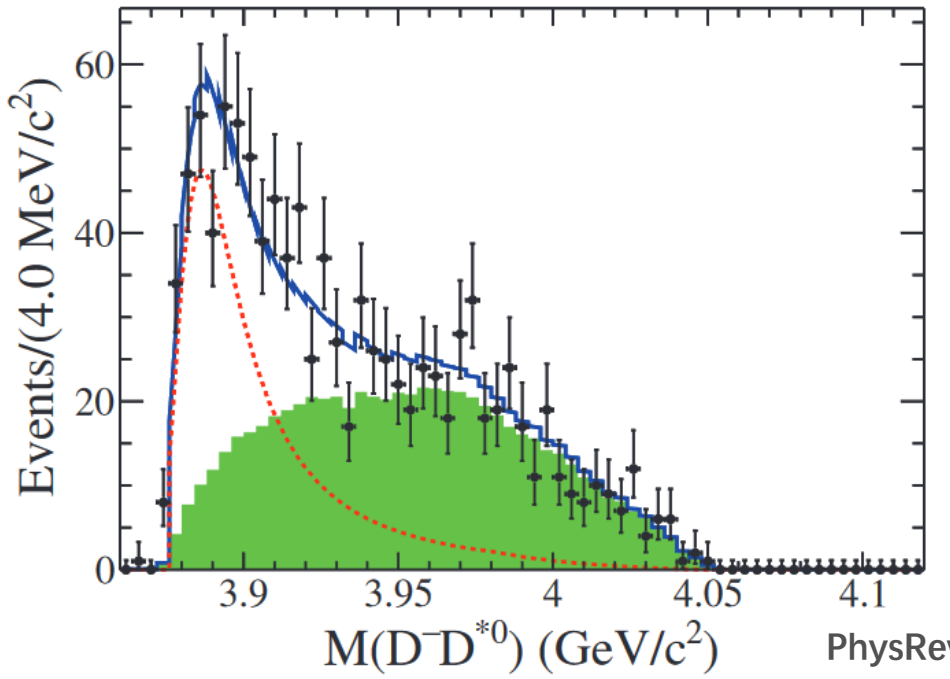


PhysRevLett.110.252001

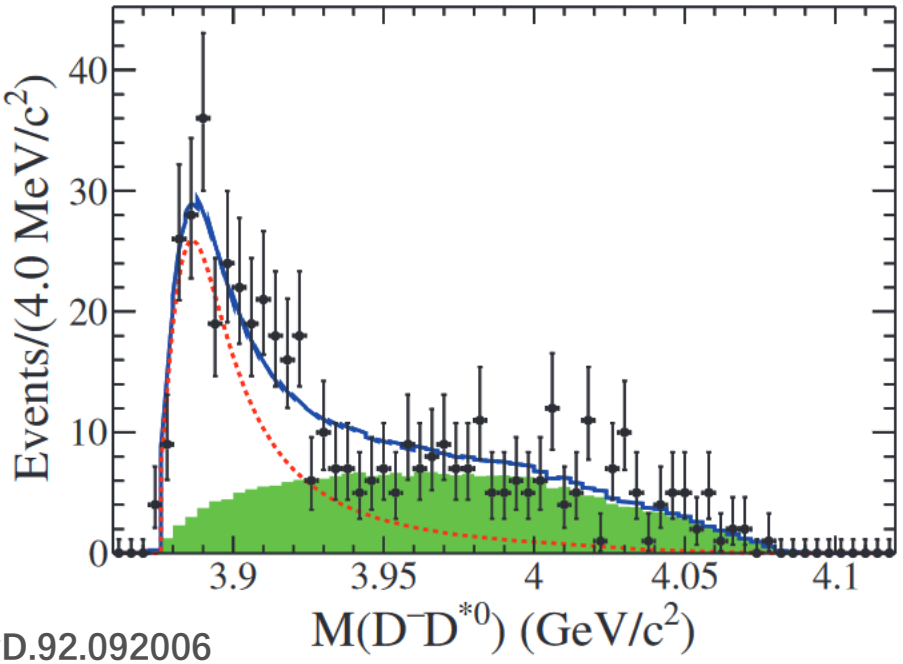
$$m_{BW} = (3894.5 \pm 6.6 \pm 4.5)\ MeV$$

$$\Gamma_{BW} = (63 \pm 24 \pm 26)\ MeV$$

$e^+e^- \to \pi D\bar{D}^*$ @$\sqrt{s} = 4.23\ GeV$



$e^+e^- \to \pi D\bar{D}^*$ @$\sqrt{s} = 4.26\ GeV$

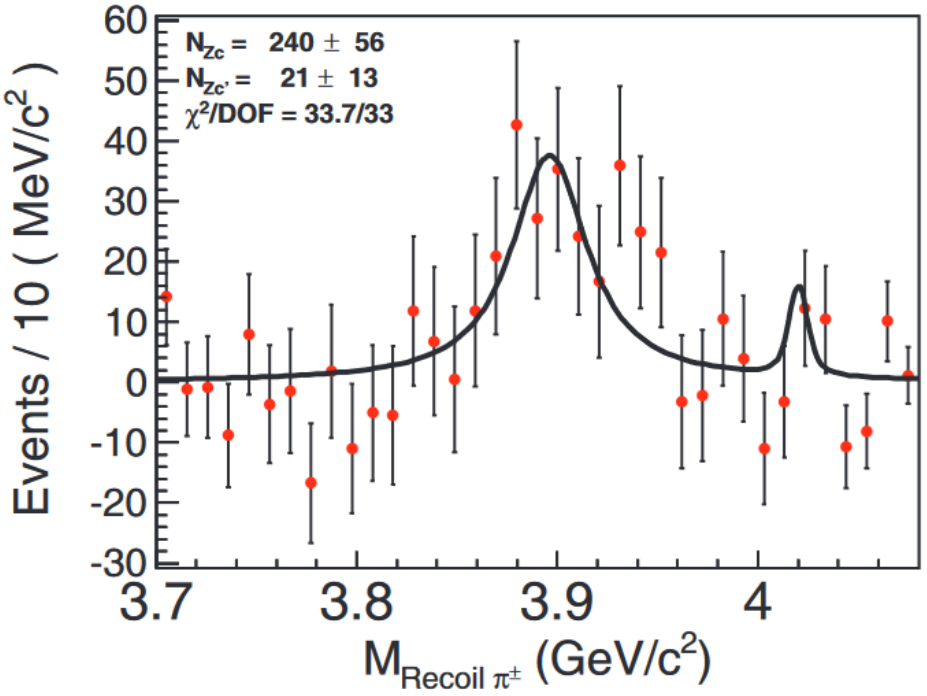


PhysRevD.92.092006

$m_{BW} = (3882.2 \pm 1.1 \pm 1.5)\ MeV$

$\Gamma_{BW} = (26.5 \pm 1.7 \pm 2.1)\ MeV$

$$e^+e^- \to \pi^{\pm}(\rho\eta_c)^{\mp}@\sqrt{s} = 4.23\ GeV$$

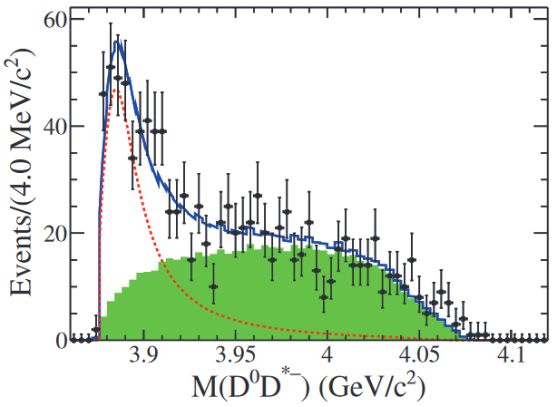


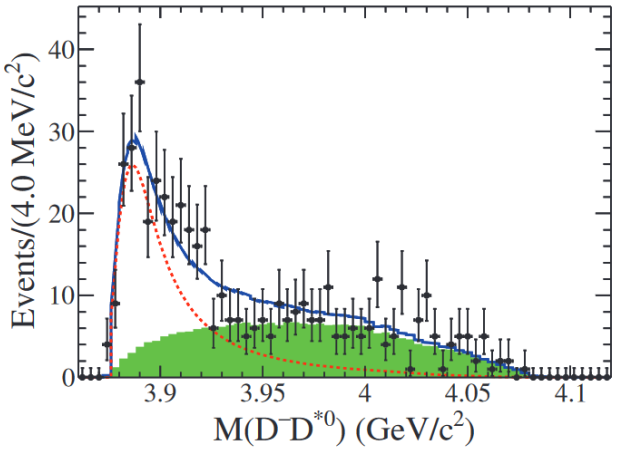
PhysRevD.100.111102

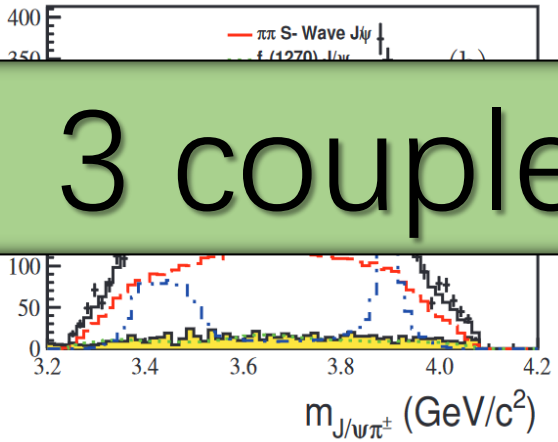
$$\frac{B(Z_c(3900) \to \rho\eta_c)}{B(Z_c(3900) \to \pi J/\psi)} = 2.1 \pm 0.8$$

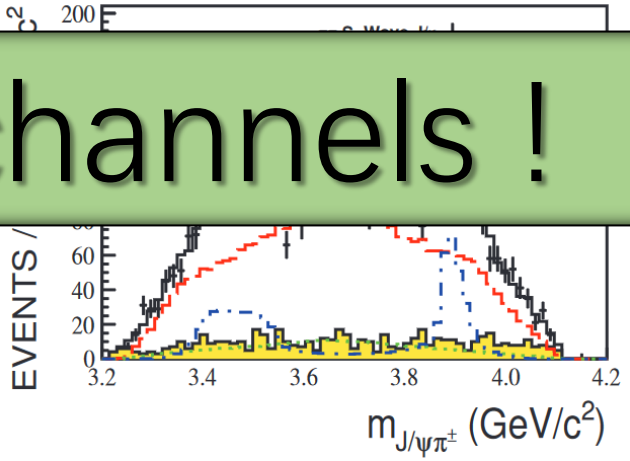
$\sqrt{s} = 4.23GeV$

$\sqrt{s} = 4.26GeV$

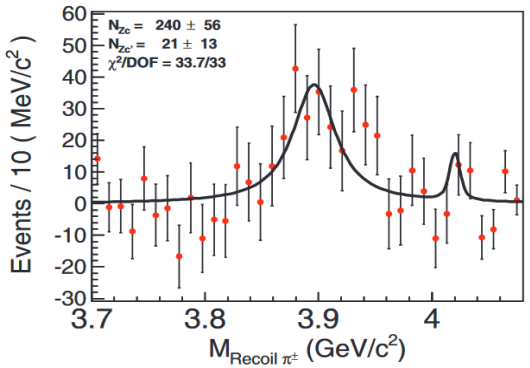








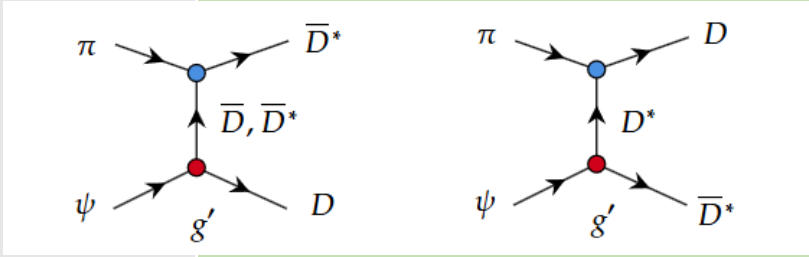
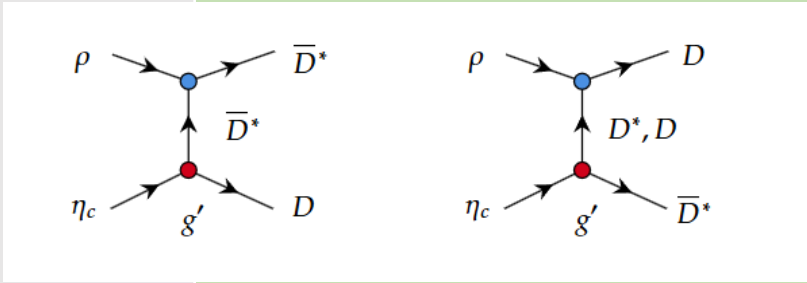
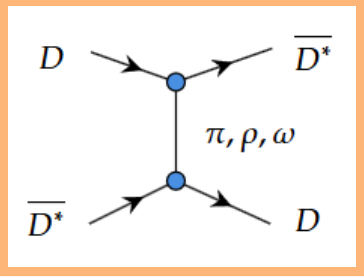
3 couple channels !



recent LQCD study implies that

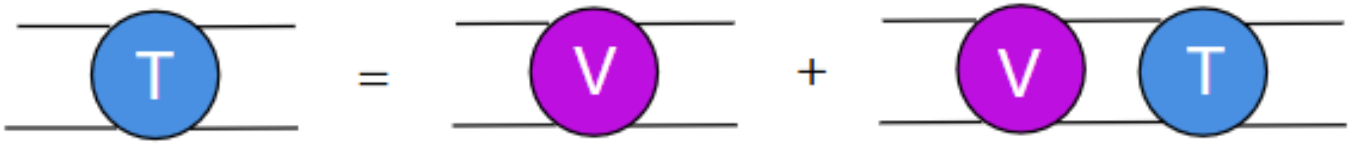
$\rho\eta_c$ may be the same footing as $\pi J/\psi$

PhysRevLett.117.242001

	$\pi J/\psi$	$\rho\eta_c$	$D\bar{D}^*$
$\pi J/\psi$			
$\rho\eta_c$			
$D\bar{D}^*$	$\mathcal{L}_{\psi HH} = g' \mathrm{Tr}\left[\mathcal{J}\bar{H}_2 \overleftrightarrow{\partial}_\mu \gamma^\mu \bar{H}_1\right] + \mathrm{H.c.},$ $(J/\psi, \eta_c)$ superfileds	$(D, D^*), (\bar{D}, \bar{D}^*)$,superfileds	Taken from Tcc work 2306.12406 

OZI-suppression

Lippmann Schwinger Equation



PhysRevLett.119.072001

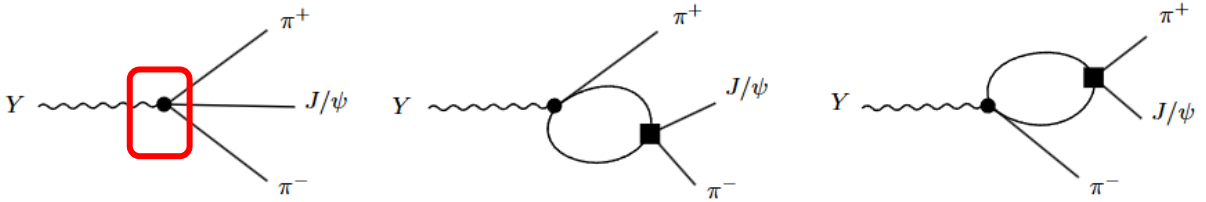
- partial-wave LSE : $J^P = 1^+$ 👉 only L=0 (s wave) is considered

- Dipole Form Factor

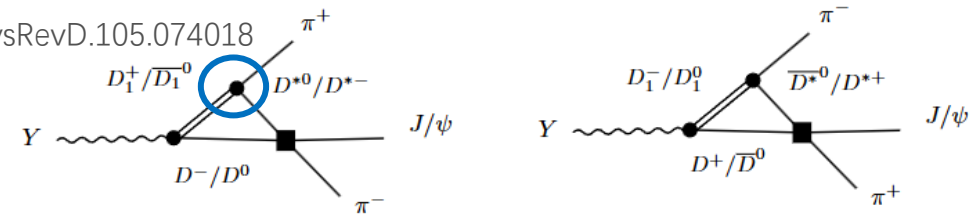
$$V_{\beta\alpha}(\boldsymbol{p},\boldsymbol{k}) \to V_{\beta\alpha}(\boldsymbol{p},\boldsymbol{k}) \left(\frac{\Lambda_\alpha^2}{\Lambda_\alpha^2+\boldsymbol{k}^2}\right)^2 \left(\frac{\Lambda_\beta^2}{\Lambda_\beta^2+\boldsymbol{p}^2}\right)^2$$

$$\Lambda_{D\bar{D}^*} = 1GeV, \qquad \Lambda_{\rho\eta_c} = \Lambda_{\pi J/\psi} = 1.5GeV \qquad \left(\frac{m_{\eta_c}}{m_D} \approx \frac{m_{J/\psi}}{m_D} \approx 1.5\right)$$

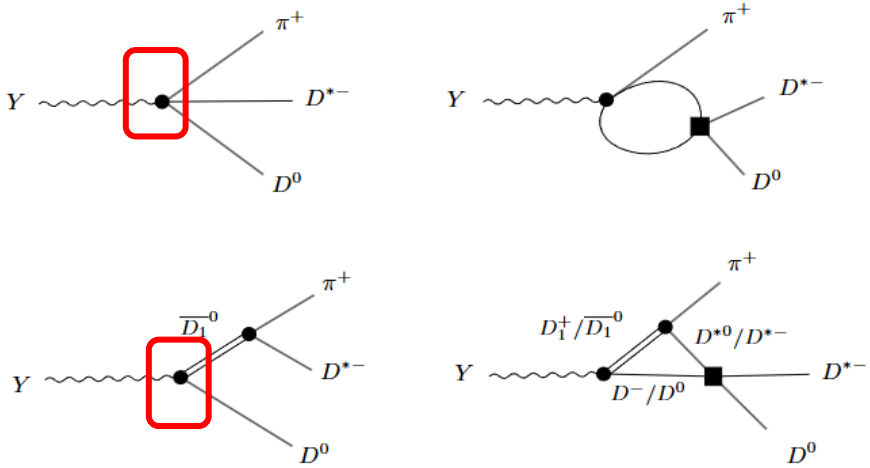
$e^+e^- \to \pi^+\pi^- J/\psi$



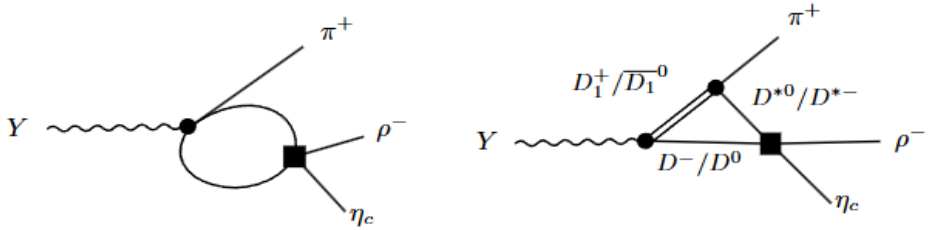
PhysRevD.105.074018



$e^+e^- \to \pi D\bar{D}^*$



$e^+e^- \to \pi\rho\eta_c$



Two incoherent polynomials

$$d\Gamma \sim N \int |M|^2 d\Phi + \boxed{P}$$

scale factor

amplitude squared

phase space

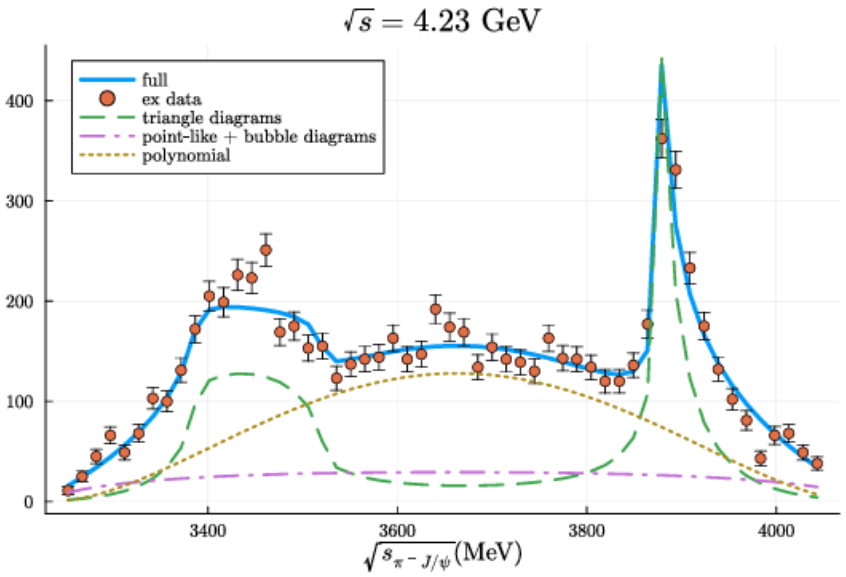
polynomials

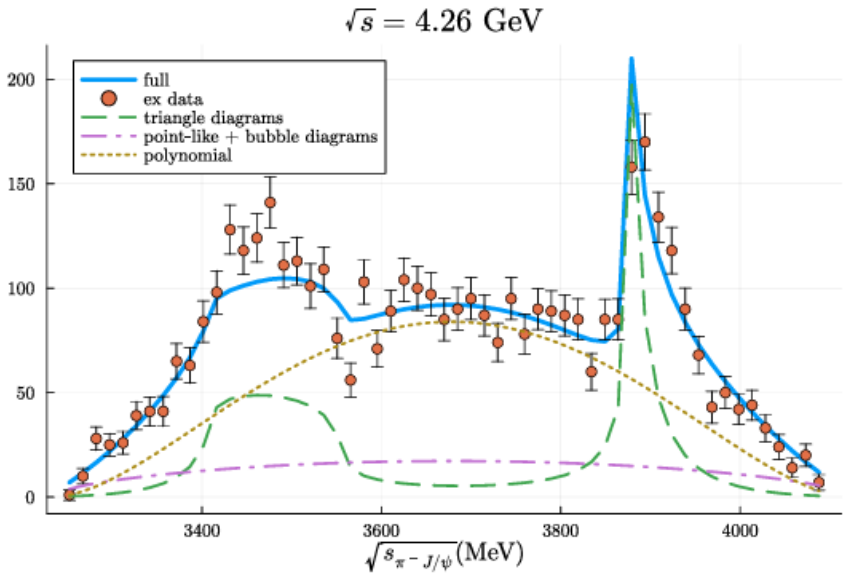
$$b_0 * \left(\sqrt{s_{\pi^- J/\psi}} - m_\pi - m_{J/\psi}\right)^{b_1} * \left(\sqrt{s} - m_\pi - \sqrt{s_{\pi^- J/\psi}}\right)^{b_1}$$ ⇒ mimic $\pi\pi$ FSI

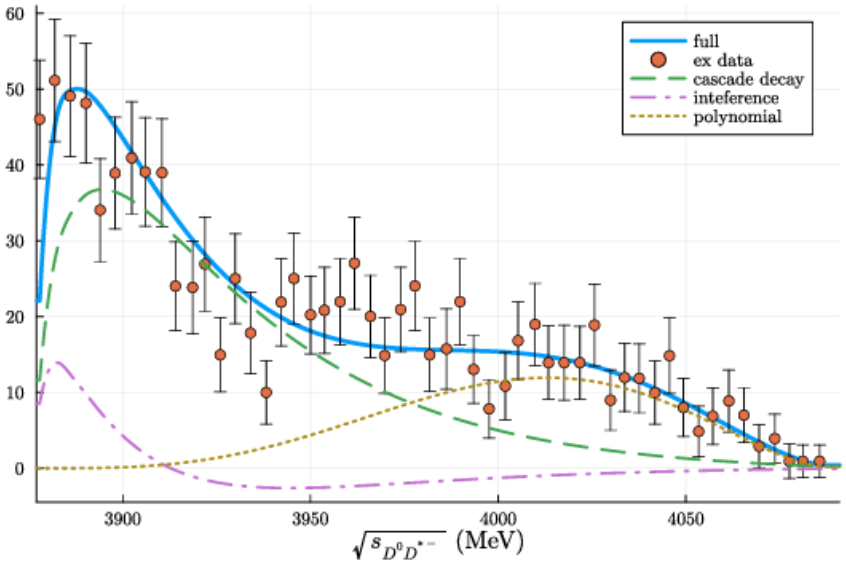
$$c_0 * (\sqrt{s_{D^0 D^{*-}}} - m_D - m_{\bar{D}^*})^{c_1} * (\sqrt{s} - m_\pi - \sqrt{s_{D^0 D^{*-}}})^{c_2}$$

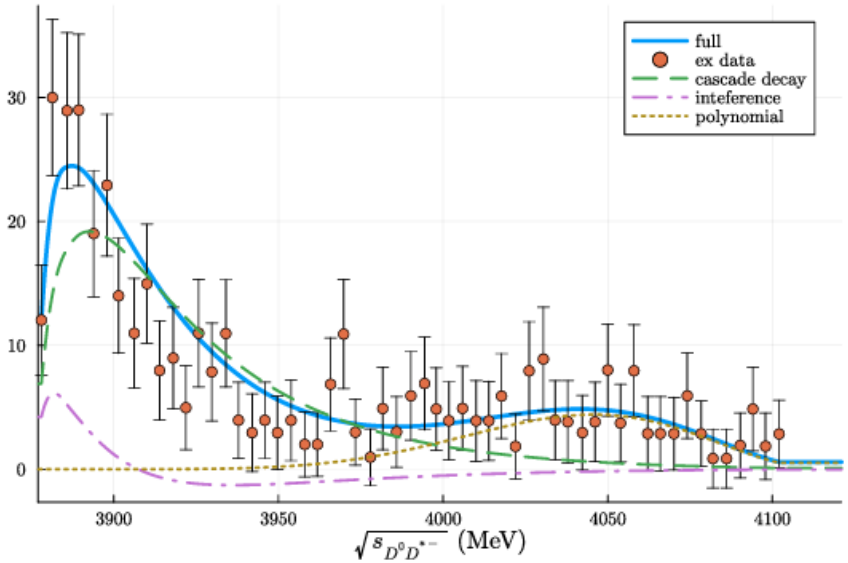
PhysRevLett.112.022001

mimic possible background contribution



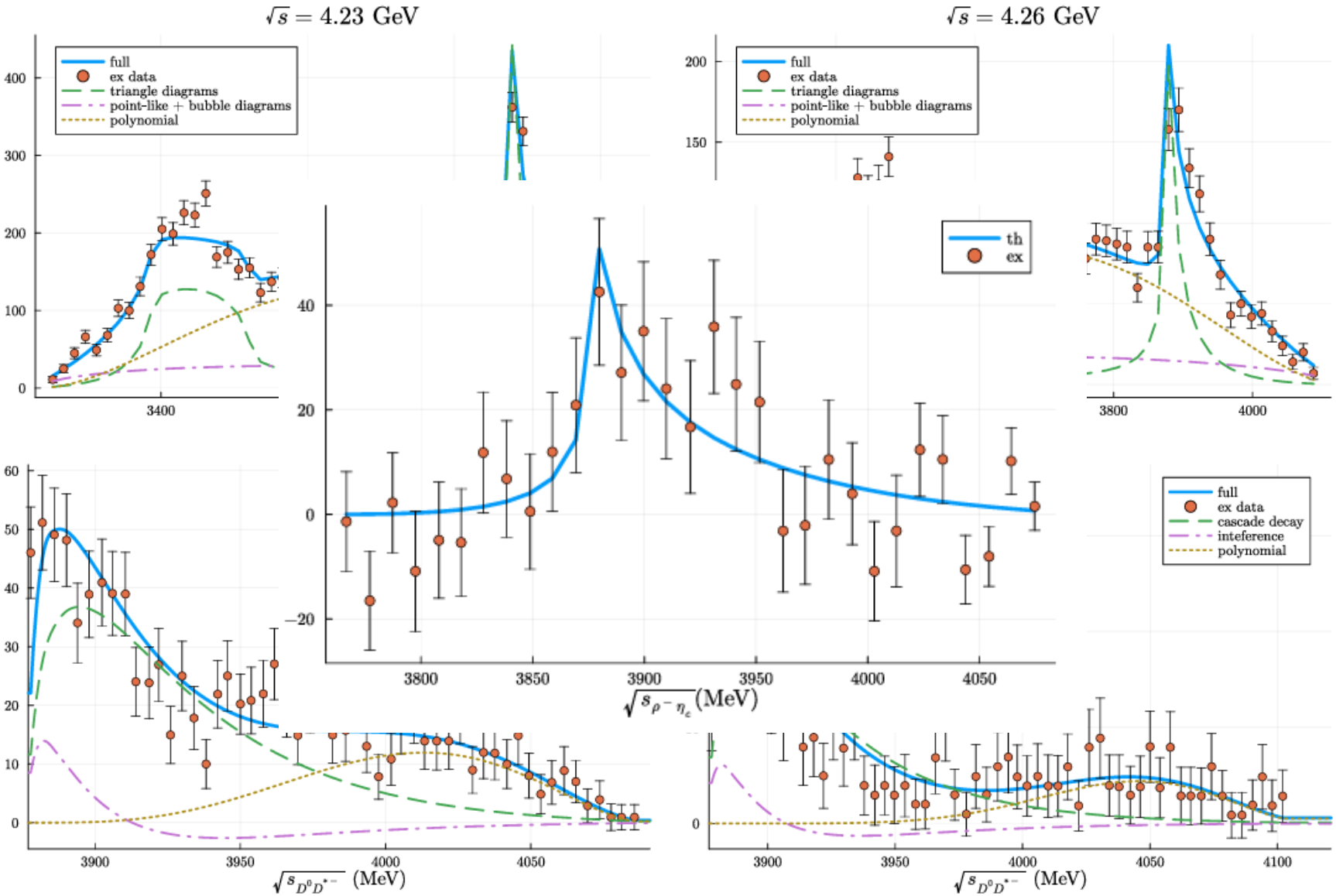






#. par = 23

#. ex dat ~ 250 in total @$\sqrt{s} = 4.23, 4.26\ GeV$

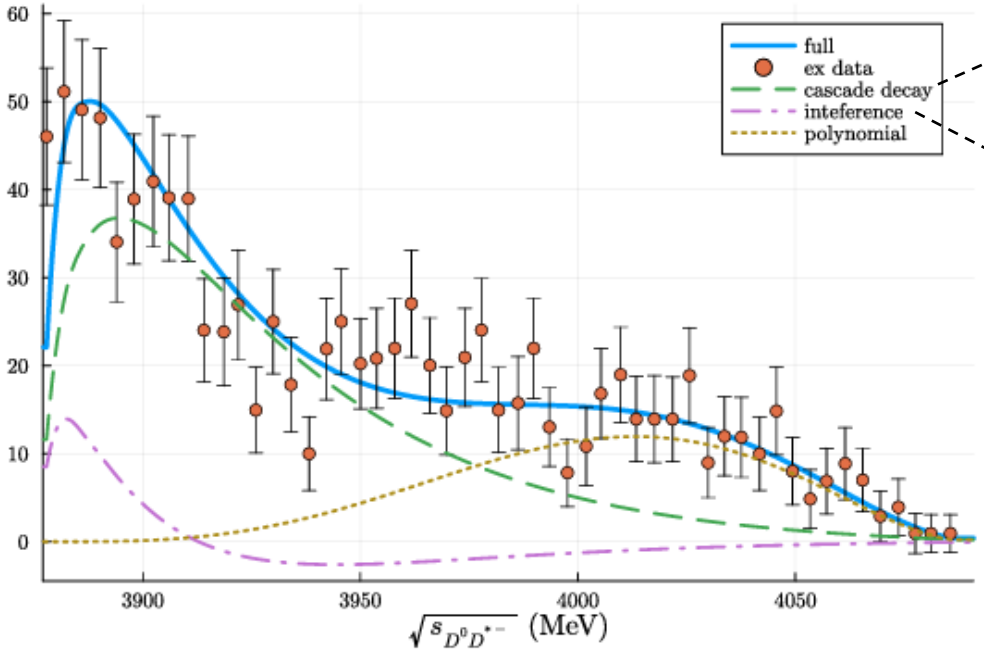


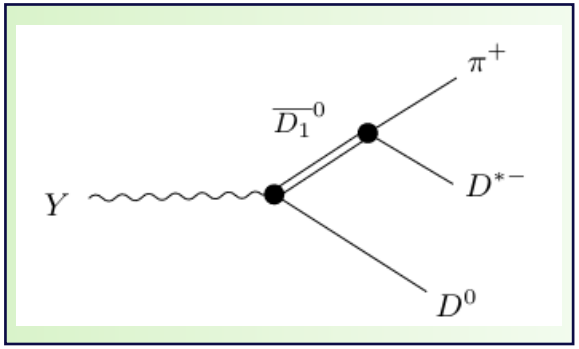
#. par = 23

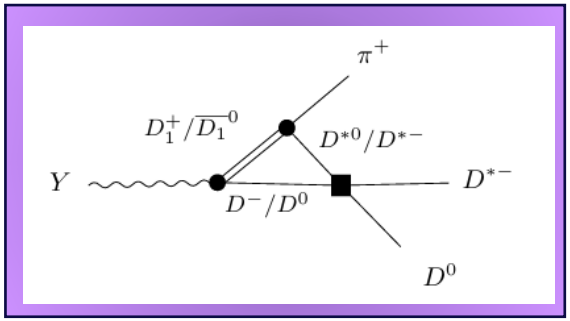
#. ex dat ~ 250 in total @$\sqrt{s} = 4.23, 4.26\ GeV$

$e^+e^- \to \pi D \bar{D}^*$

$@\sqrt{s} = 4.23GeV$



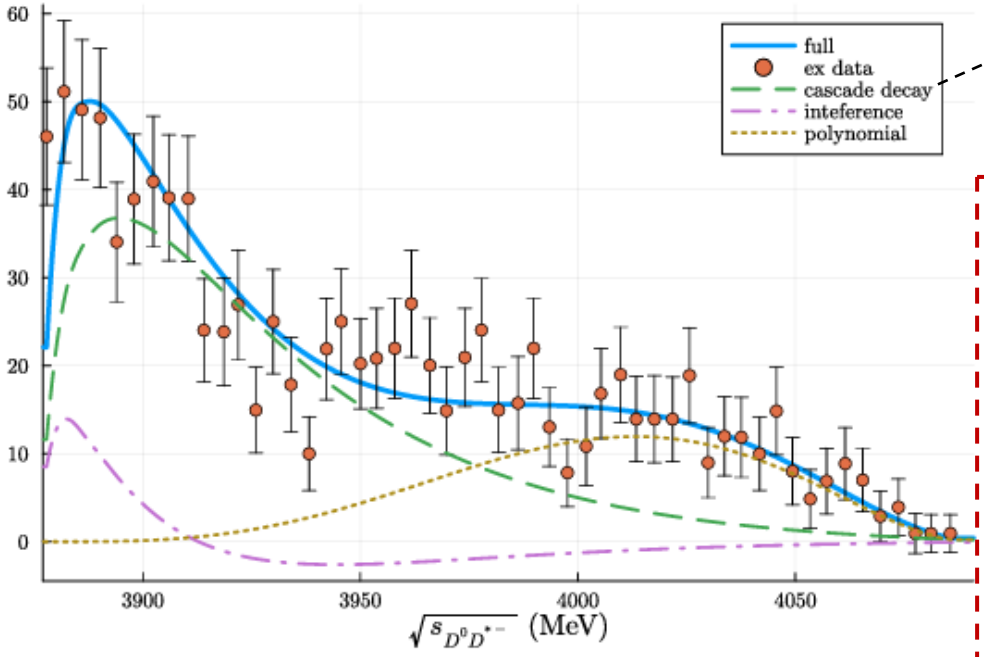


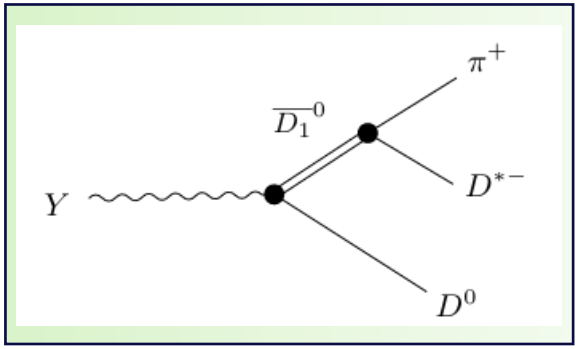


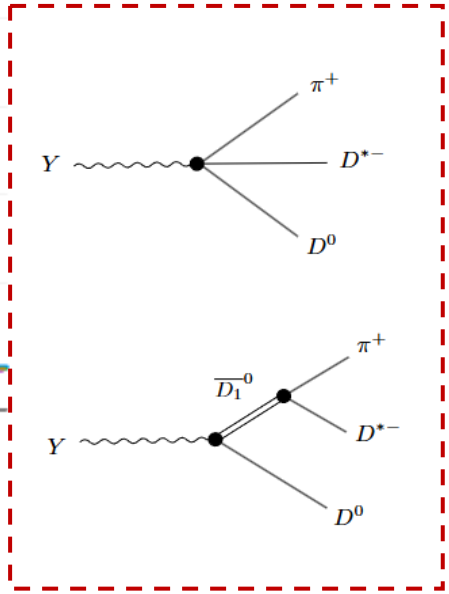
- dominated by cascade decay
- enhanced by interference with triangle diagram
- polynomials is considerable at the tail only (same as BES3)

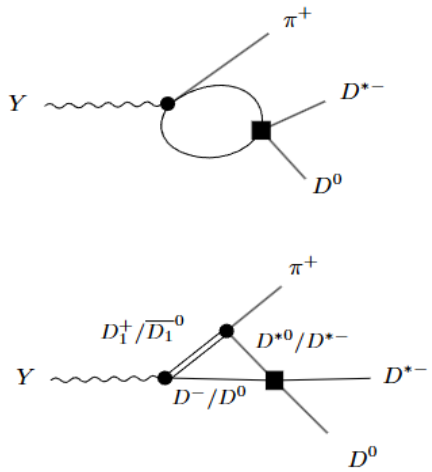
$e^+e^- \to \pi D \bar{D}^*$

$@\sqrt{s} = 4.23GeV$







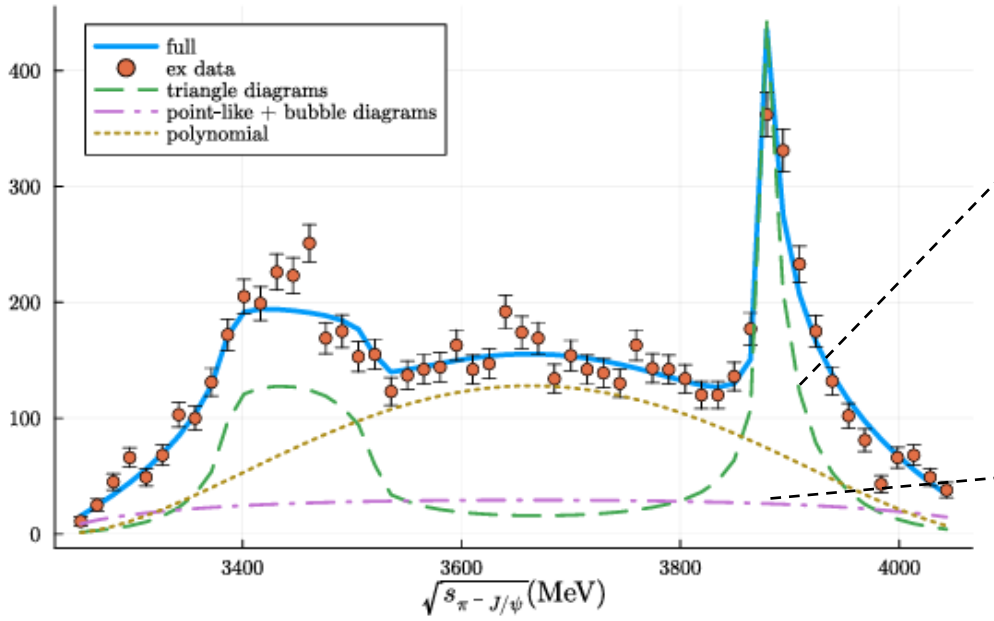


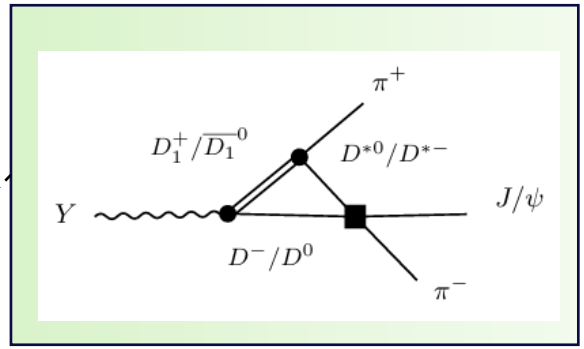
OBE potential is weak, so FSI is perturbative compared to tree diagram.

PhysRevD.91.051504

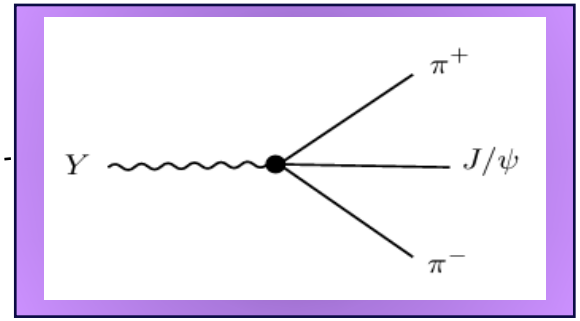
$e^+e^- \to \pi^+\pi^- J/\psi$

$@\sqrt{s} = 4.23GeV$





(also interchange π^+, π^-)

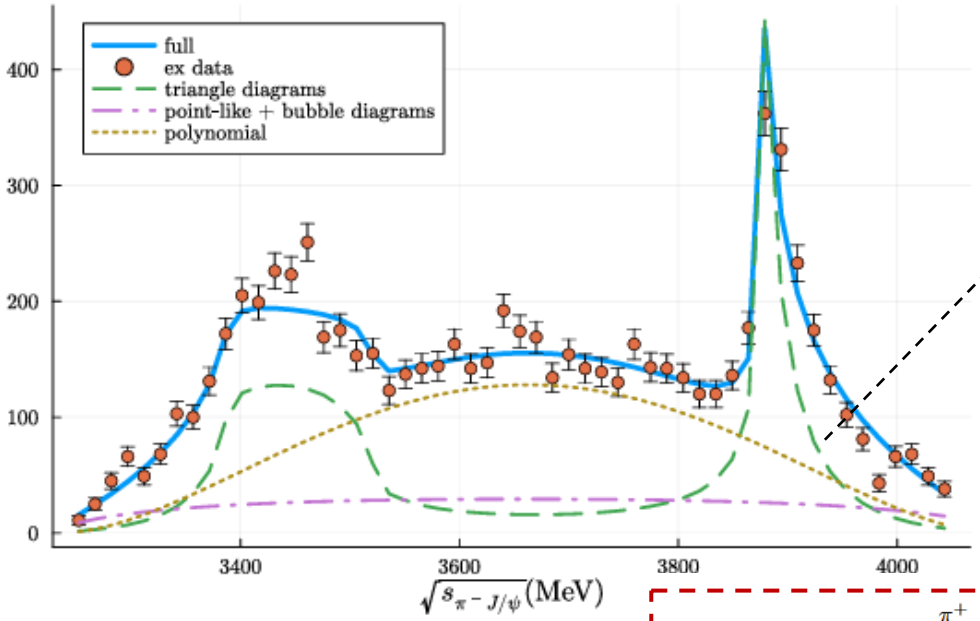


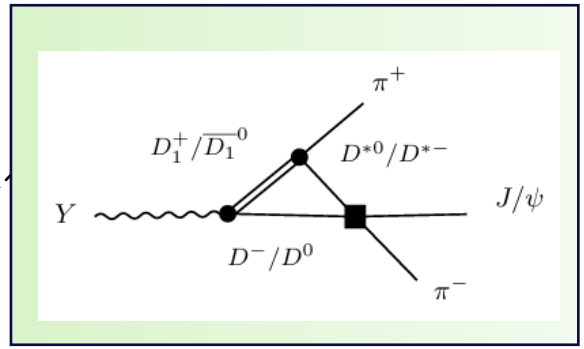
(FSI not shown)

- dominated by triangle diagram
- polynomials gives similar line shape as MC simulation of $\pi\pi$ FSI

$e^+e^- \to \pi^+\pi^- J/\psi$

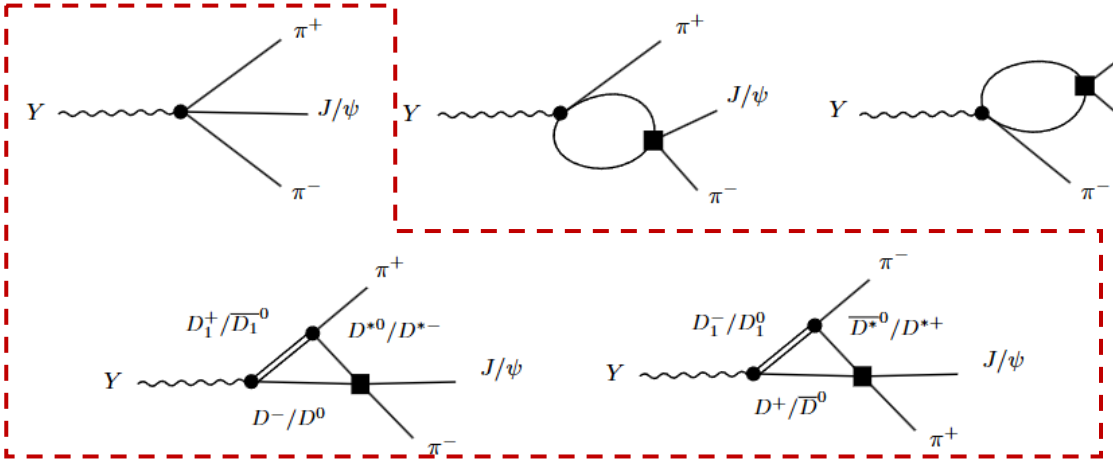
$@\sqrt{s} = 4.23GeV$





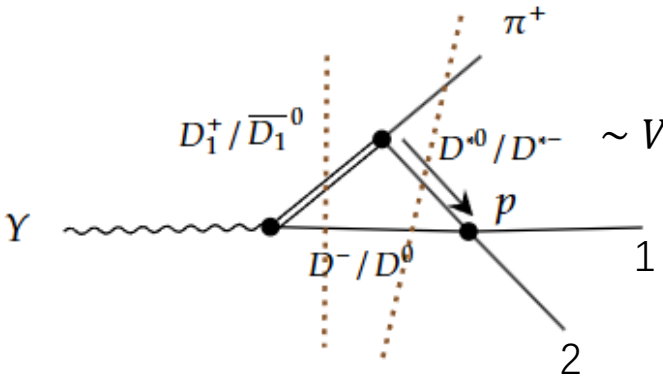
(also interchange π^+, π^-)

recall that FSI is perturbative



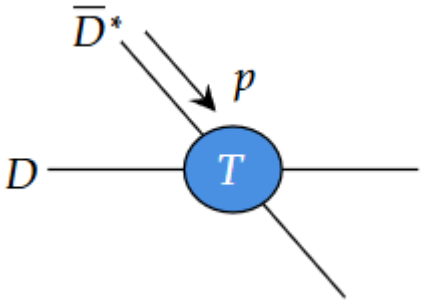
role of triangle diagram

triangle diagram = pure triangle loop + T-matrix (FSI)



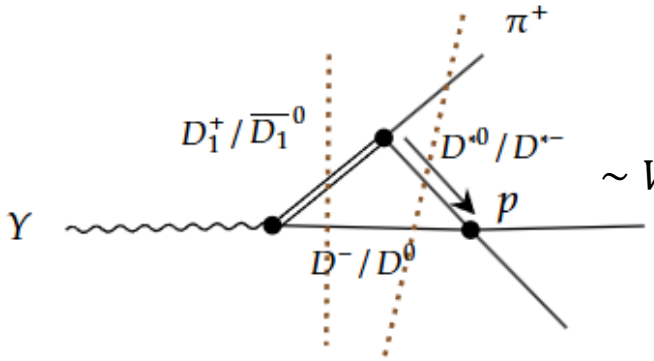
$$\sim V_{D_1 D^* \pi} \int \frac{d^3 p}{\sqrt{s} - \omega_{D_1} - \omega_D + \frac{i\Gamma_{D_1}}{2}} \frac{1}{\sqrt{s} - \omega_\pi(\sqrt{s_{12}}) - \omega_{D^*} - \omega_D + i0^+}$$

width of $D_1(2420)$ is taken into account

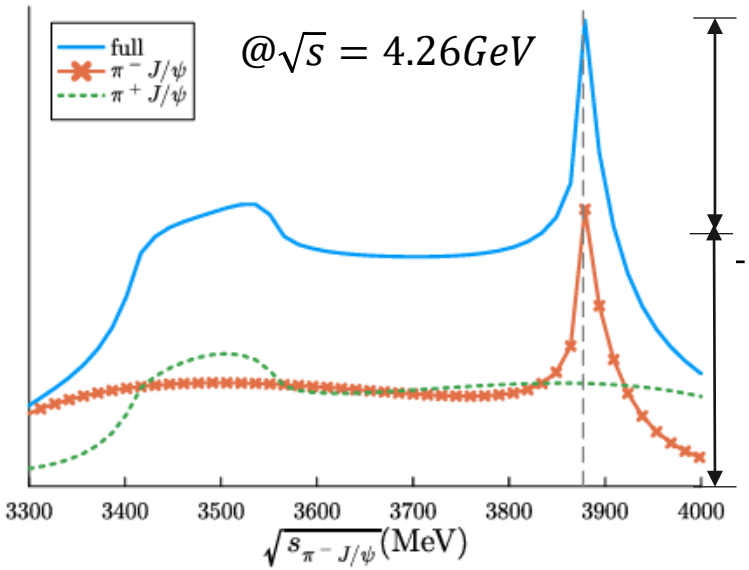


2 body unitary cut : $\pi J/\psi, \rho\eta_C, D\overline{D}^*$ threshold

pure triangle loop in $e^+e^- \to \pi\pi J/\psi$



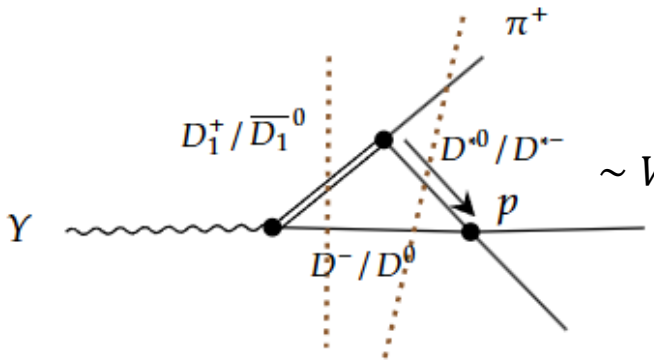
$$\sim V_{D_1 D^* \pi} \int \frac{d^3p}{\sqrt{s} - \omega_{D_1} - \omega_D + \frac{i\Gamma_{D_1}}{2}} \frac{1}{\sqrt{s} - \omega_\pi(\sqrt{s_{\pi J/\psi}}) - \omega_{D^*} - \omega_D + i0^+}$$



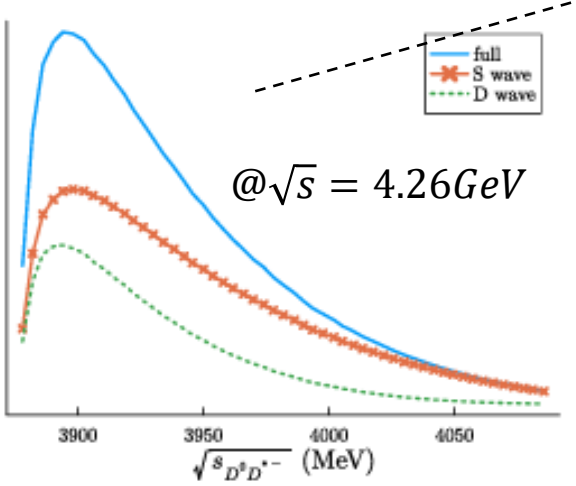
- a cusp EXACTLY at $D\bar{D}^*$ threshold (branch cut)
- ratio of cusp to platform is not large enough

👉 pure triangle loop is important, but not enough.

pure triangle loop in $e^+e^- \to \pi D \bar{D}^*$

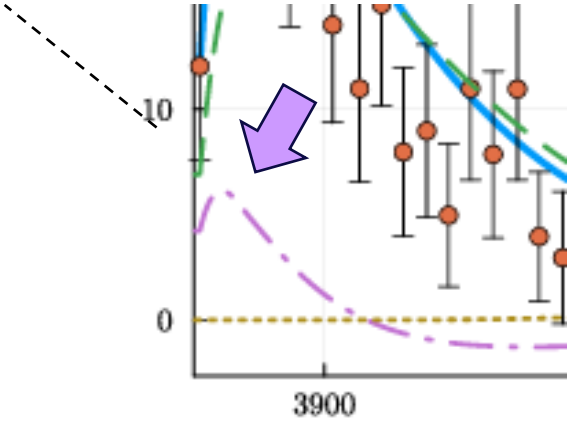


$$\sim V_{D_1 D^* \pi} \int \frac{d^3p}{\sqrt{s} - \omega_{D_1} - \omega_D + \frac{i\Gamma_{D_1}}{2}} \frac{1}{\sqrt{s} - \omega_\pi(\sqrt{s_{D\bar{D}^*}}) - \omega_{D^*} - \omega_D + i0^+}$$

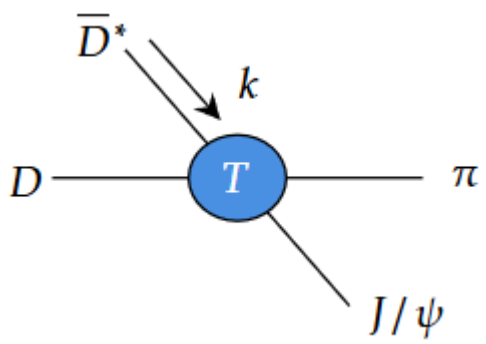


- indeed a peak
- but NOT close enough to threshold

👉 T-matrix still needed



T-matrix

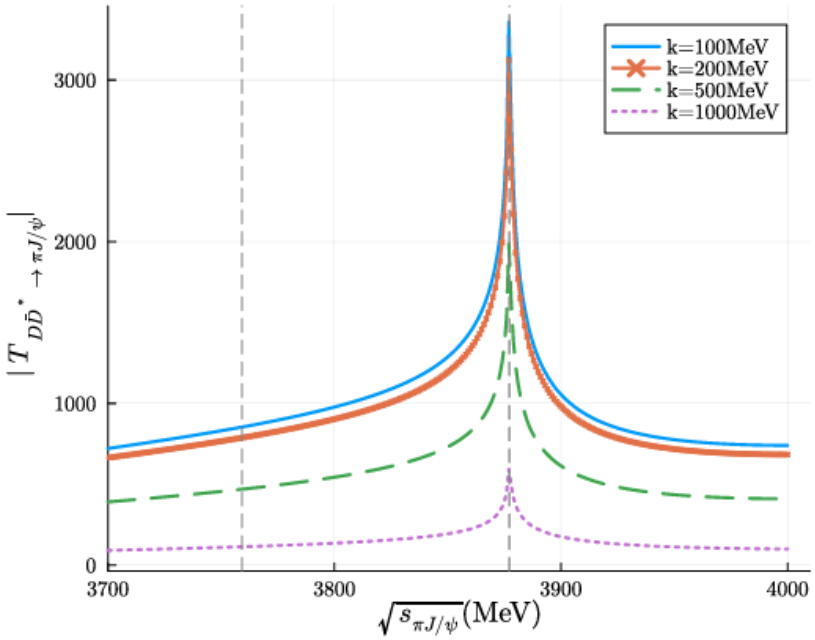


half-on-shell

$$|\, T_{D\bar{D}^* \to \pi J/\psi}(k\,, \bar{p}_{\pi J/\psi}\,; \sqrt{s_{\pi J/\psi}}\,)\,|$$

off energy shell

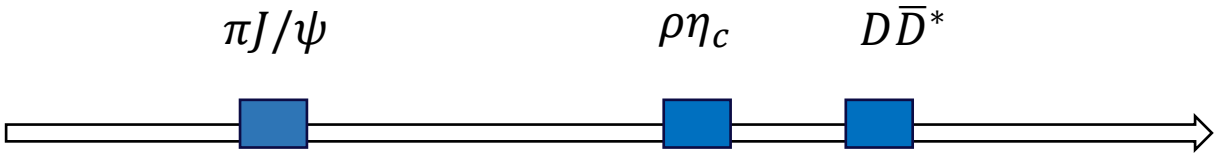
on energy shell



- a sharp cusp EXACTLY at $D\bar{D}^*$ threshold (thanks to one-pion-exchange potential)

👉 **triangle diagram: cusp from T-matrix enhanced by cusp from pure triangle loop**

Pole of T-matrix



3 channels $\rightarrow 2^3 = 8$ Riemann sheets, labelled by $RS^{\pm\pm\pm}$

Adjacent to physical region: $RS^{+++}, RS^{-++}, RS^{--+}, RS^{---}$

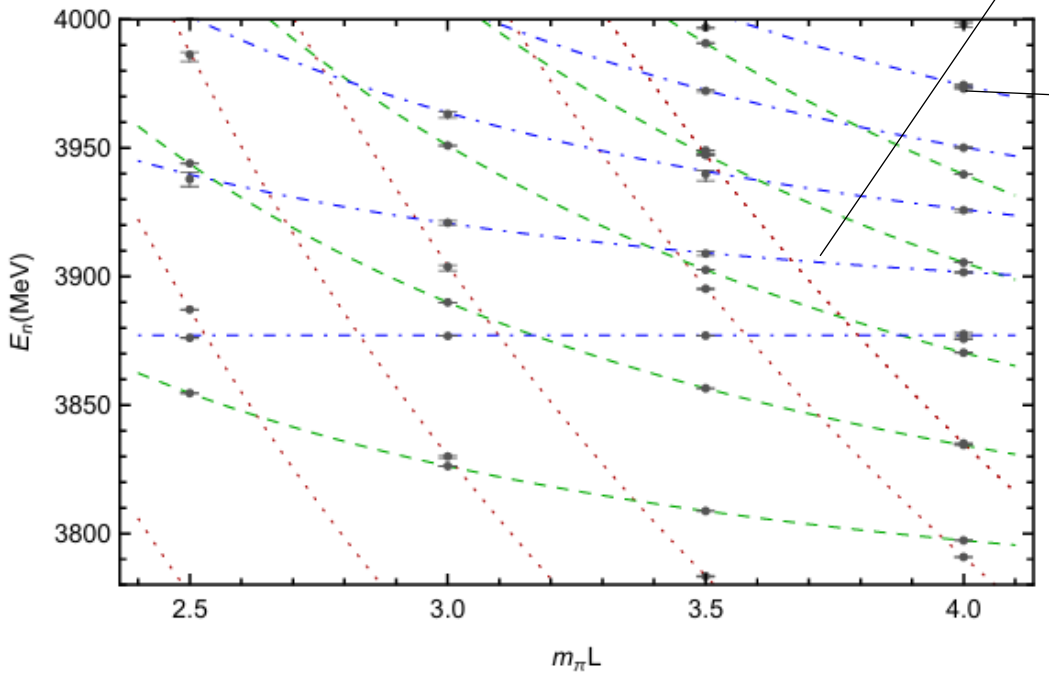
only one pole $\sim$ **3798** MeV is found in RS^{---} ("virtual")

a deeper virtual pole in LQCD at larger m_π

PhysRevLett.117.242001

Ref	Pole Position	Type
Ref. [13]	3798^{+25}_{-31}	Virtual
	$3902(6) - 38(9)i$	Resonance
Ref.([12])	3831^{+27}_{-38}	Virtual
	$3894(6) - 30(13)i$	Resonance
Ref.([14])	3870	Virtual
Ref.([18])	3879	Virtual
Ref.([15])	3872	Virtual
Ref.([19])	3880(3) - 13(1)i	Resonance
Ref.([23])	3884 - 22i	Resonance
Ref.([16])	3840	Virtual

Finite Volume Spectra



free energy

eigenenergy

almost free

qualitatively same as LQCD results

(all current LQCD research show no direct signal of Zc(3900))

possible interpretation of $Z_c(3900)$-structure

For $e^+e^- \to \pi D\bar{D}^*$, Z_c structure:

- mainly from $Y \to D_1(2420)\bar{D} \to \pi D\bar{D}^*$ cascade decay
- enhanced by interference with triangle diagram

For $e^+e^- \to \pi\pi J/\psi$ ($\pi\rho\eta_c$), Z_c structure:

- two cusps both exactly at $D\bar{D}^*$ threshold
- mainly from unitary cut of T-matrix, enhanced by cut of pure triangle loop

possible interpretation of $Z_c(3900)$-structure

For $e^+e^- \to \pi D\bar{D}^*$, Z_c structure:

- mainly from $Y \to D_1(2420)\bar{D} \to \pi D\bar{D}^*$ cascade decay

NOT a genuine particle,
But a threshold cusp ?

- two cusps both exactly at $D\bar{D}^*$ threshold
- mainly from unitary cut of T-matrix, enhanced by cut of pure triangle loop

Summary and Perspective

Summary

- Three-coupled-channels with OBE interaction analysis on $Z_c(3900)$
  (channel-channel interaction almost determined from previous Tcc work)
- Experimental line shapes are successfully reproduced.
- Our model is qualitatively same as all current LQCD results
- $Z_c(3900)$ may not a genuine particle but a threshold cusp

Perspective

- $\pi\pi$ FSI is not incorporated in a self-contained way
- $D^*\bar{D}^*$ channel and $Z_c(4020)$
- improvement of the quality of data on STCF in the future is suggested

Summary and Perspective

Summary

- Three-coupled-channels with OBE interaction analysis on Zc
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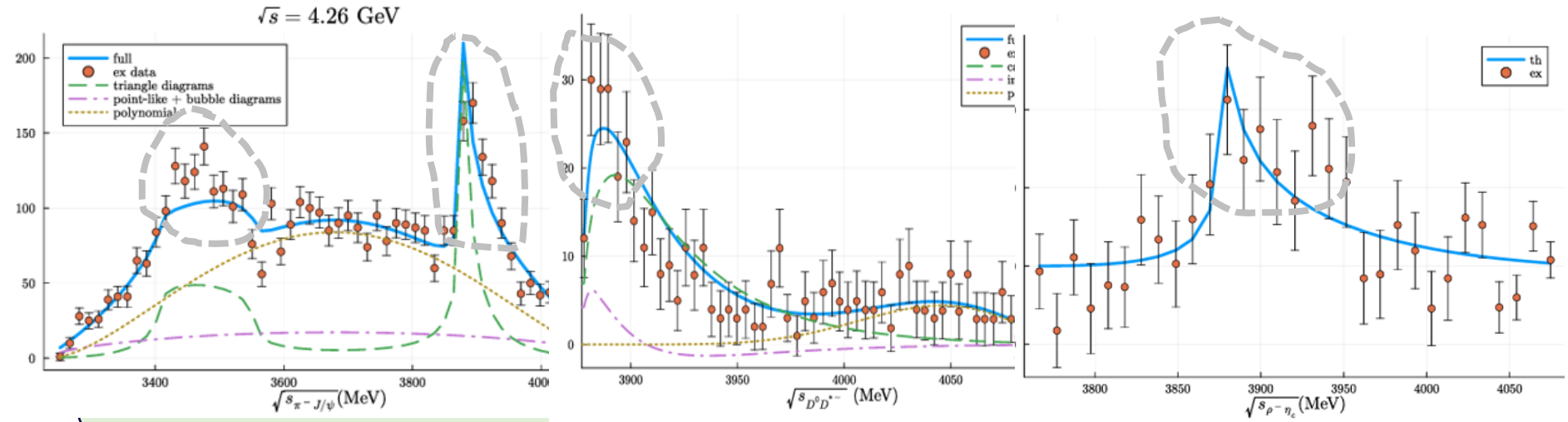
- Zc may not a genuine particle but a threshold cusp



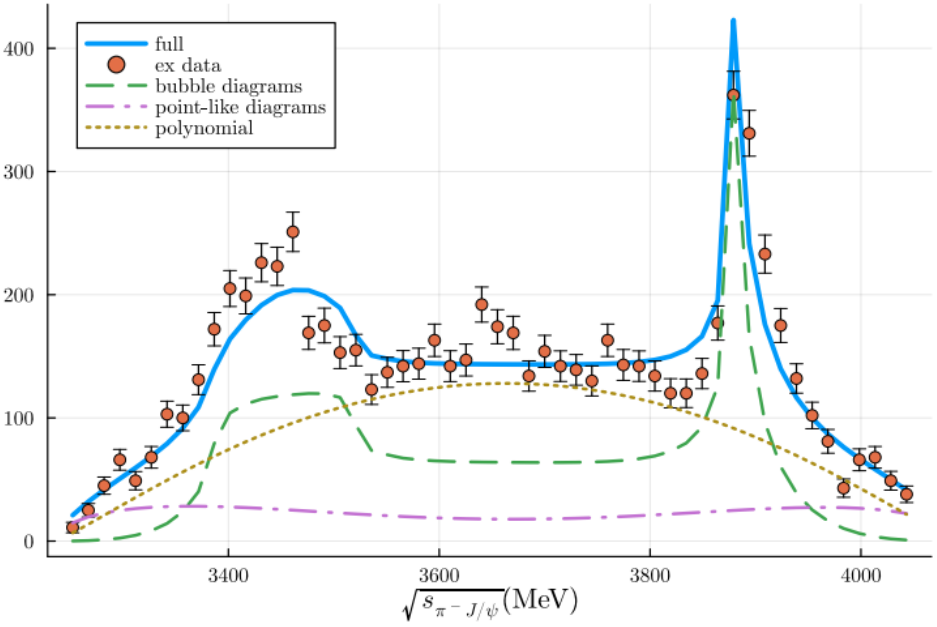
Backup

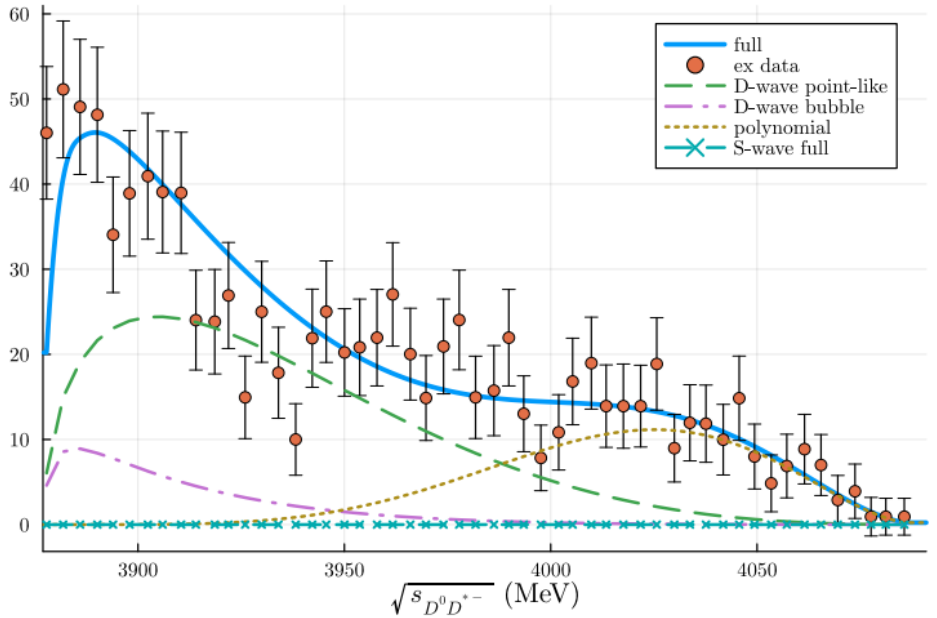
	Scheme	$\Lambda_{\pi J/\psi(\rho\eta_c)}$ (fixed)	g' (10^{-5} MeV$^{-\frac{3}{2}}$)
This work	1	1.3 GeV	7.506 ± 0.120
		1.5 GeV	6.143 ± 0.070
		1.7 GeV	5.129 ± 0.070
	2	1.5 GeV	5.584 ± 0.470
		1.7 GeV	4.69 ± 0.280
Other refs	-	-	3.68 [14]
	-	-	3.85 [39]

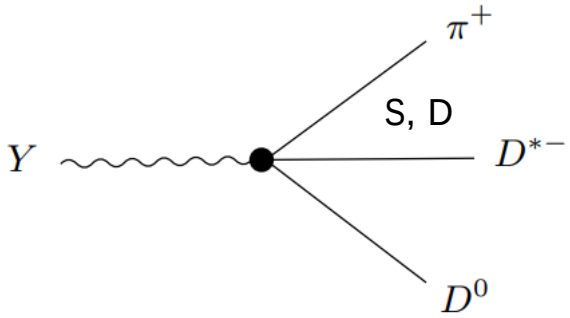
$$\hat{\chi}^2 \sim 1.6$$

Backup

w/o triangle diagram



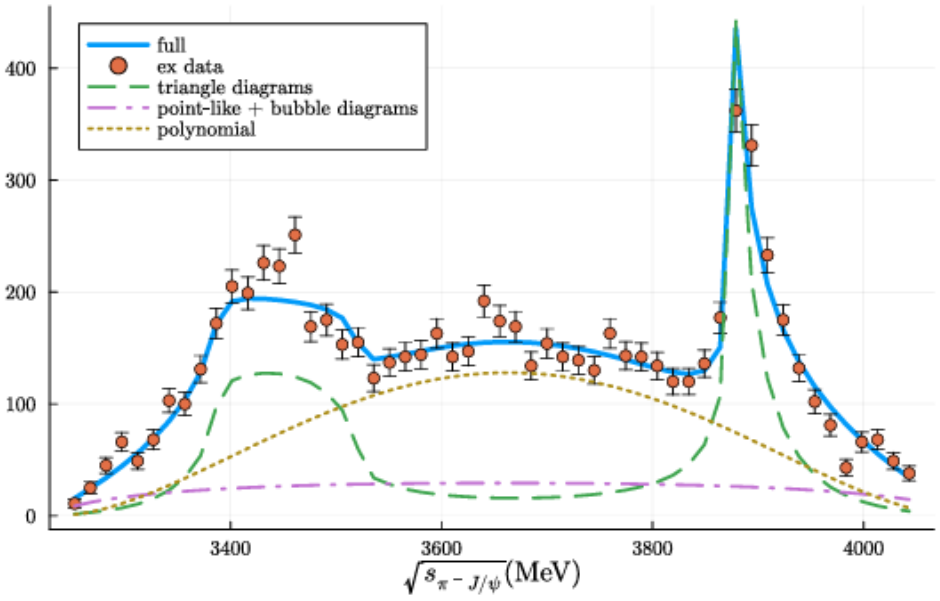


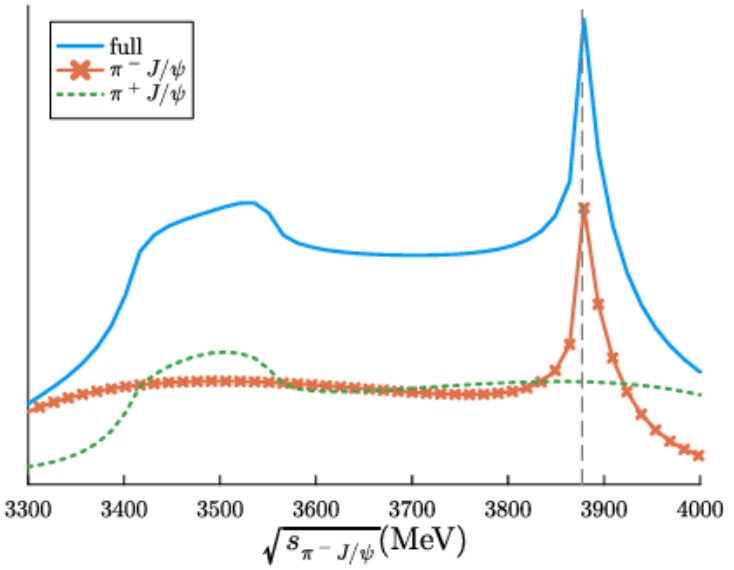


Backup

	Pole Position	Type	Scheme($\Lambda_{\pi J/\psi}$)
This work	3798.72 - 1.10i	Virtual	1(1.3GeV)
	3798.46 - 1.71i		1(1.5GeV)
	3798.12 - 2.26i		1(1.7GeV)
	3798.27 - 2.02i		2(1.5GeV)
	3797.80 - 2.64i		2(1.7GeV)

Backup


full
ex data
triangle diagrams
point-like + bubble diagrams
polynomial
400
300
200
100
0
3400
3600
3800
4000
$\sqrt{s_{\pi^- J/\psi}}$(MeV)


full
$\pi^- J/\psi$
$\pi^+ J/\psi$
3300
3400
3500
3600
3700
3800
3900
4000
$\sqrt{s_{\pi^- J/\psi}}$(MeV)