QCD sum rule study on the light hybrid states

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Contents

- **Present status of hadron spectroscopy**
- QCD sum rule studies on hybrid states
- Decay analyses on hybrid states
- Studies on the $J^{PC} = 1^{-+}$ hybrid states

Quark model

Phys.Lett. 8 (1964) 214-215

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

...

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowest baryon configuration (qqq) gives just the representations **1**, **8**, and **10** that have been observed, while

8419/TH.412
21 February 1964

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

II *)

G. Zweig
CERN---Geneva

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

...

6) In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $AAAA\bar{A}$, $AAAAAA\bar{A}\bar{A}$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $A\bar{A}$, $AA\bar{A}\bar{A}$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $A\bar{A}$ and AAA, that is, "deuces and treys".

Quark model

LIGHT UNFLAVORED ($S = C = B = 0$)	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$
• π^0	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• $f_0(600)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$
• $\rho(770)$	$1^+(1^{--})$	$a_2(1700)$	$1^-(2^{++})$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $\phi(1020)$	$0^-(1^{--})$	$X(1835)$	$?^?(?^{-+})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	• $f_2(1950)$	$0^+(2^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$
• $a_2(1320)$	$1^-(2^{++})$	• $f_2(2010)$	$0^+(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$	$f_0(2020)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$	• $a_4(2040)$	$1^-(4^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$	• $f_4(2050)$	$0^+(4^{++})$
• $\eta(1405)$	$0^+(0^{-+})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $f_1(1420)$	$0^+(1^{++})$	$f_0(2100)$	$0^+(0^{++})$
• $\omega(1420)$	$0^-(1^{--})$	$f_2(2150)$	$0^+(2^{++})$
$f_2(1430)$	$0^+(2^{++})$	$\rho(2150)$	$1^+(1^{--})$
• $a_0(1450)$	$1^-(0^{++})$	$\phi(2170)$	$0^-(1^{--})$
• $\rho(1450)$	$1^+(1^{--})$	$f_0(2200)$	$0^+(0^{++})$
• $\eta(1475)$	$0^+(0^{-+})$	$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
• $f_0(1500)$	$0^+(0^{++})$	$\eta(2225)$	$0^+(0^{-+})$
$f_1(1510)$	$0^+(1^{++})$	$\rho_3(2250)$	$1^+(3^{--})$

STRANGE ($S = \pm1$, $C = B = 0$)	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
$K^*_0(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K^*_0(1430)$	$1/2(0^+)$
• $K^*_2(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K^*_3(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K^*_0(1950)$	$1/2(0^+)$
$K^*_2(1980)$	$1/2(2^+)$
• $K^*_4(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K^*_5(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{?})$

CHARMED

CHARMED, STRANGE ($C = S = \pm1$)	$I(J^P)$
• $D^\pm_s$	$0(0^-)$
• $D^{*\pm}_s$	$0(?^?)$
• $D^*_{s0}(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)^\pm$	$0(?^?)$
$D_{s1}(2700)^\pm$	$0(1^-)$

BOTTOM ($B = \pm1$)	
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B^0_s/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
$B^*_J(5732)$	$?(?^?)$
• $B_1(5721)^0$	$1/2(1^+)$
• $B^*_2(5747)^0$	$1/2(2^+)$

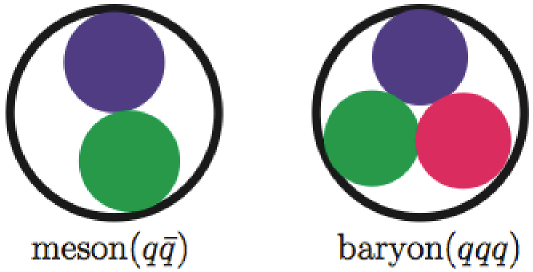
BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)	
• B^0_s	$0(0^-)$
• B^*_s	$0(1^-)$
• $B_{s1}(5830)^0$	$1/2(1^+)$
• $B^*_{s2}(5840)^0$	$1/2(2^+)$
$B^*_{sJ}(5850)$	$?(?^?)$

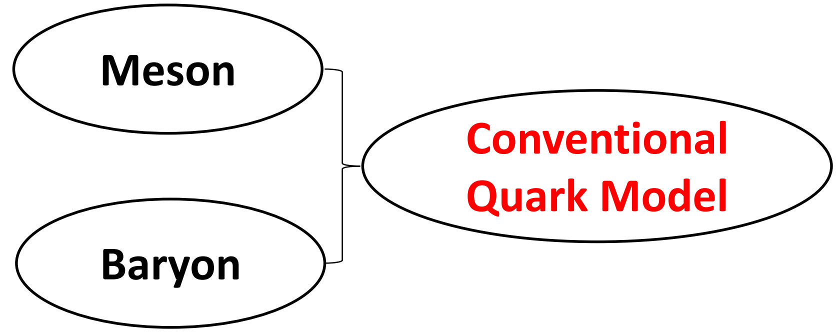
BOTTOM, CHARMED

$c\bar{c}$	$I^G(J^{PC})$
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$?^?(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
• $X(3872)$	$0^?(?^{?+})$
$\chi_{c2}(2P)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
$X(3945)$	$?^?(?^{??})$
• $\psi(4040)$	$0^-(1^{--})$
• $\psi(4160)$	$0^-(1^{--})$
• $X(4260)$	$?^?(1^{--})$
$X(4360)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$

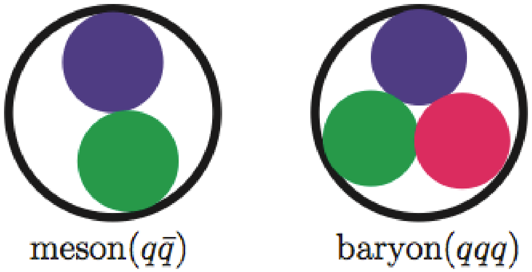
$b\bar{b}$	
$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
• $\Upsilon(2S)$	$0^-(1^{--})$
$\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$

Categorizations





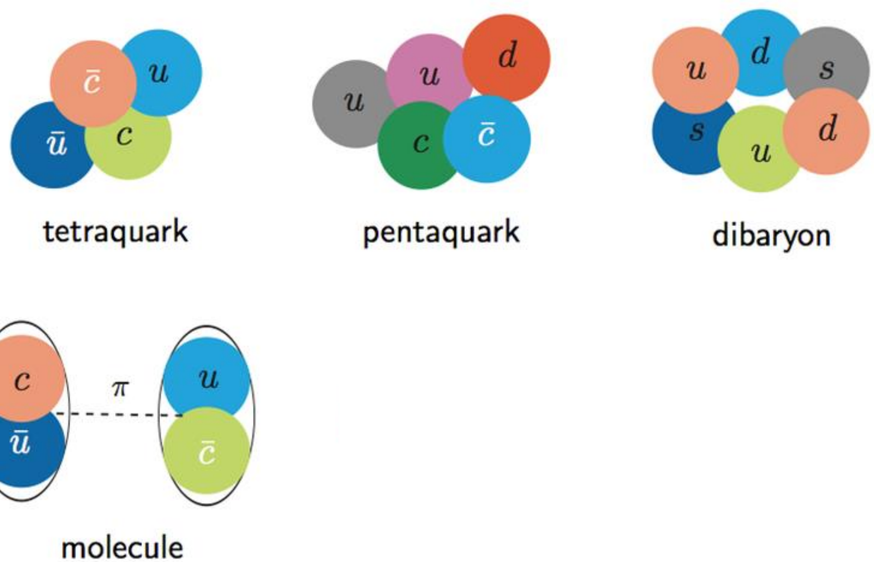
Categorizations



Meson

Baryon

Conventional Quark Model

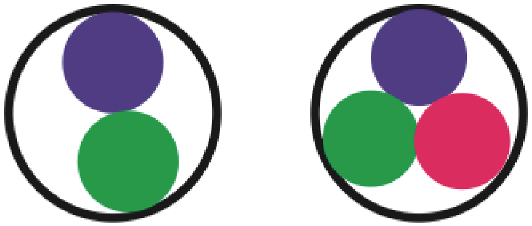


Multiquark

Molecular

Exotic hadron

Categorizations

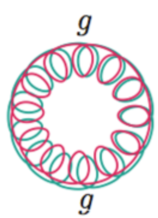


meson($q\bar{q}$) baryon(qqq)

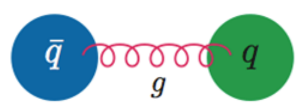
Meson

Baryon

Conventional Quark Model



glueball



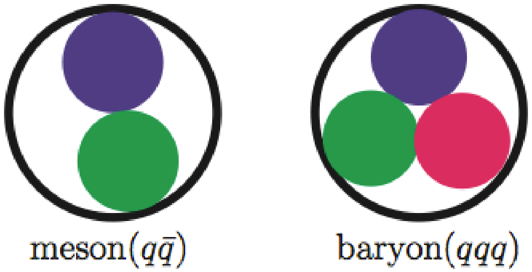
hybrid

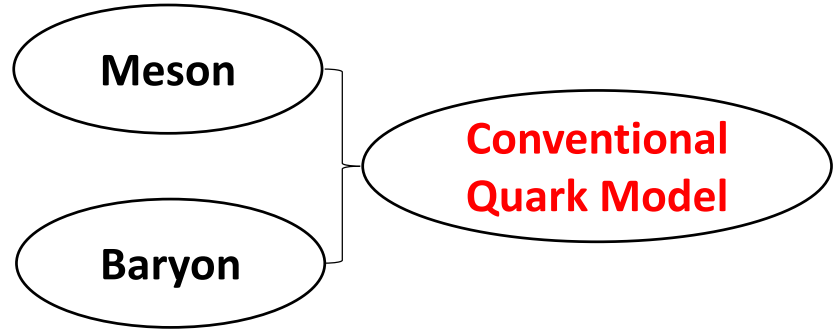
Glueball

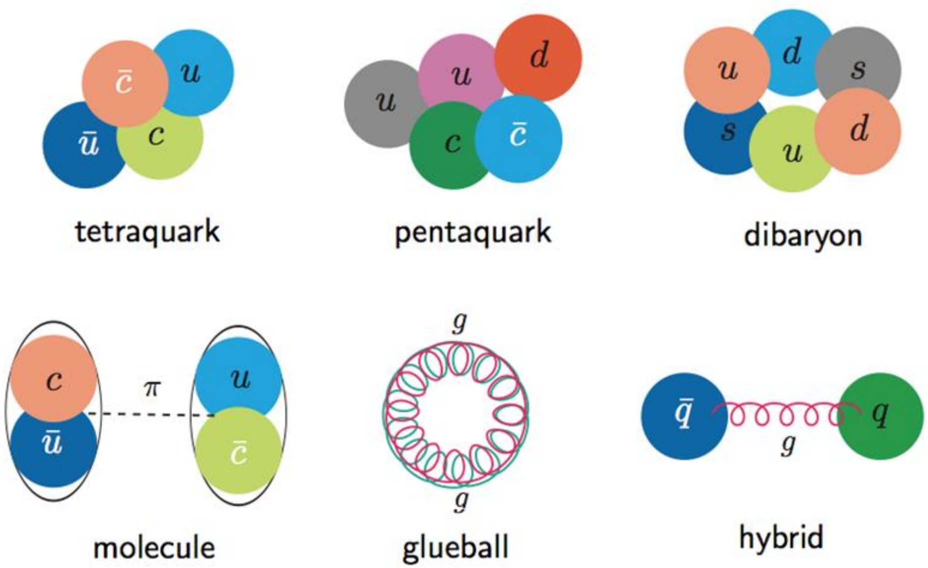
Hybrid

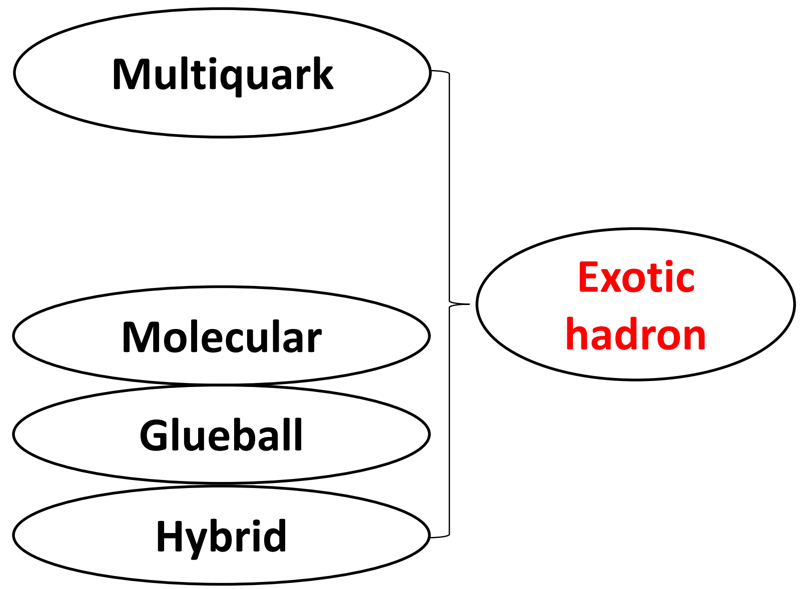
Exotic hadron

Categorizations









Previous Studies

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R. L. Jaffe and K. Johnson, Phys. Lett. 60B, 201 (1976).

Flux tube model: N. Isgur and J. E. Paton, Phys. Rev. D31, 2910 (1985).

Coulomb Gauge: A. Szczepaniak et al., Phys. Rev. Lett. 76, 2011 (1996);
F. J. Llanes-Estrada, P. Bicudo and S. R. Cotanch, Phys. Rev. Lett. 96, 081601 (2006).

Glueball trajectories: I. Szanyi et al., Nucl. Phys. A998, 121728 (2020).

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S. Narison, Z. Phys. C26, 209 (1984);
S. Narison, Nucl. Phys. B509, 312 (1998);
J. I. Latorre, S. Narison and S. Paban, Phys. Lett. B191, 437 (1987);
E. Bagan and T. G. Steele, Phys. Lett. B243, 413 (1990);
G. Hao, C. F. Qiao and A. L. Zhang, Phys. Lett. B642, 53 (2006);
C. F. Qiao and L. Tang, Phys. Rev. Lett. 113, 221601 (2014);
A. Pimikov, H. J. Lee, N. Kochelev and P. Zhang, Phys. Rev. D95, 071501(R) (2017);
A. Pimikov, Phys. Rev. D106, 056011 (2022).

Recent D0 and TOTEM experiments

- **There is currently no definite evidence for the glueball's existence.**

- Recently, D0 and TOTEM studied pp and $p\bar{p}$ cross sections, and found them differ with a significance of 3.4σ (which can be increased to be $5.2-5.7\sigma$).

  D0 Collaboration, Phys. Rev. D 86, 012009 (2012);

  D0 and TOTEM Collaborations, Phys. Rev. Lett. 127 , 062003 (2021);

  TOTEM Collaboration, Eur. Phys. J. C 79, 785 (2019).

- **The above difference leads to the evidence of a *t*-channel exchanged odderon, i.e., predominantly a three-gluon glueball of C = −.**

  COMPETE Collaboration, Phys. Rev. Lett. 89, 201801 (2002);

  V. A. Khoze, A. D. Martin and M. G. Ryskin, Phys. Rev. D 97, 034019 (2018);

  E. Martynov and B. Nicolescu, Eur. Phys. J. C 79, 461 (2019).

Interests in glueballs are reviving recently!

Recent D0 and TOTEM experiments

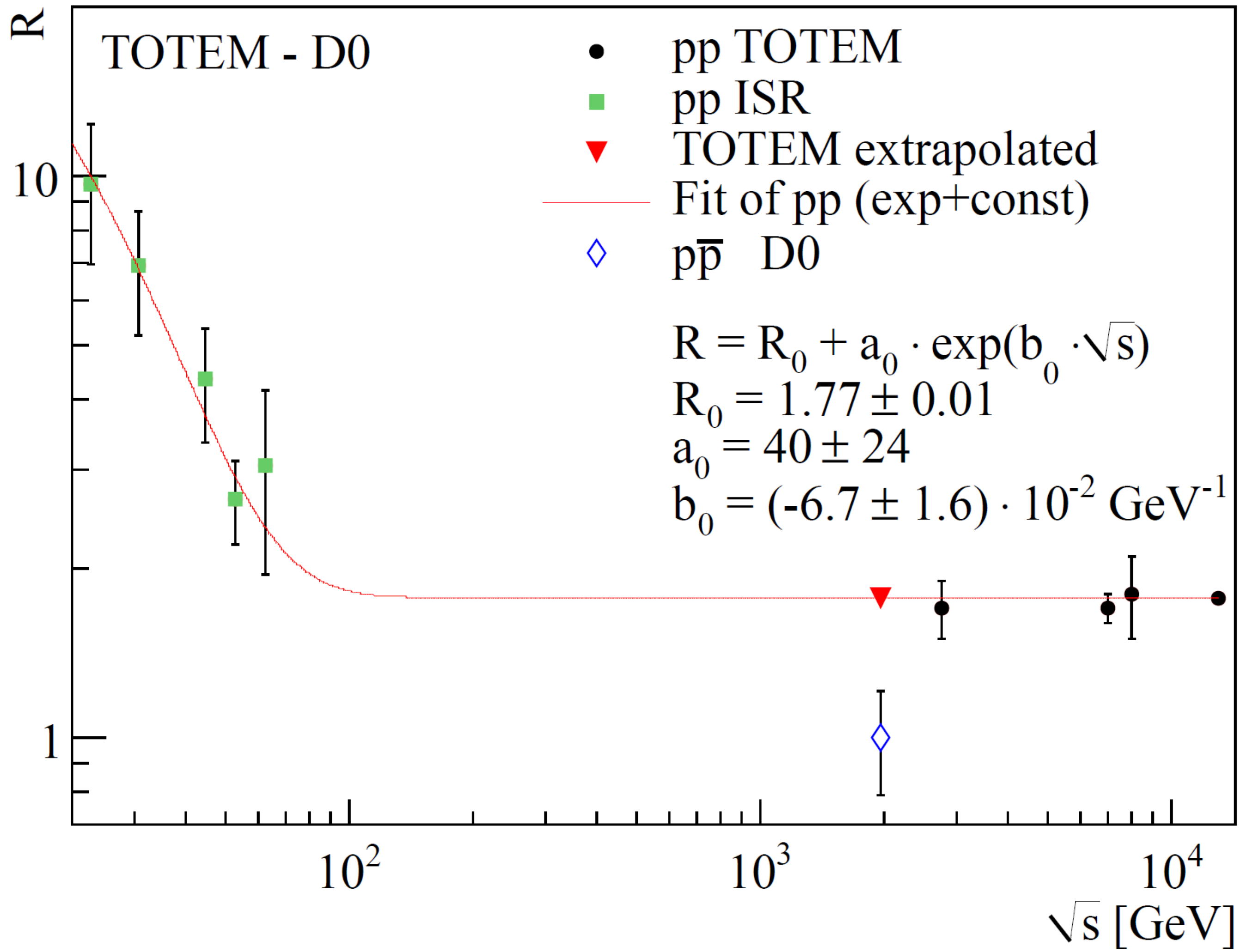
- **There is** ... **nce.**

- Recently ... found them di ... to be 5.2 – 5.

  D0 Collab

  D0 and T

  TOTEM C



- **The abo** ... **anged odderor**

  COMPETI

  V. A. Kho

  E. Martyr

Interests in glueballs are reviving recently!

Recent BESIII experiment

- **There is currently no definite evidence for the hybrid's existence.**

- Up to now there are three candidates observed in experiments with the exotic quantum number $I^G J^{PC} = 1^- 1^{-+}$:

$$\pi_1(1400),\ \pi_1(1600),\ \text{and}\ \pi_1(2015).$$

D. Alde, et al., Phys. Lett. B 205 (1988) 397.

G. S. Adams, et al., Phys. Rev. Lett. 81 (1998) 5760-5763.

J. Kuhn, et al., Phys. Lett. B 595 (2004) 109-117.

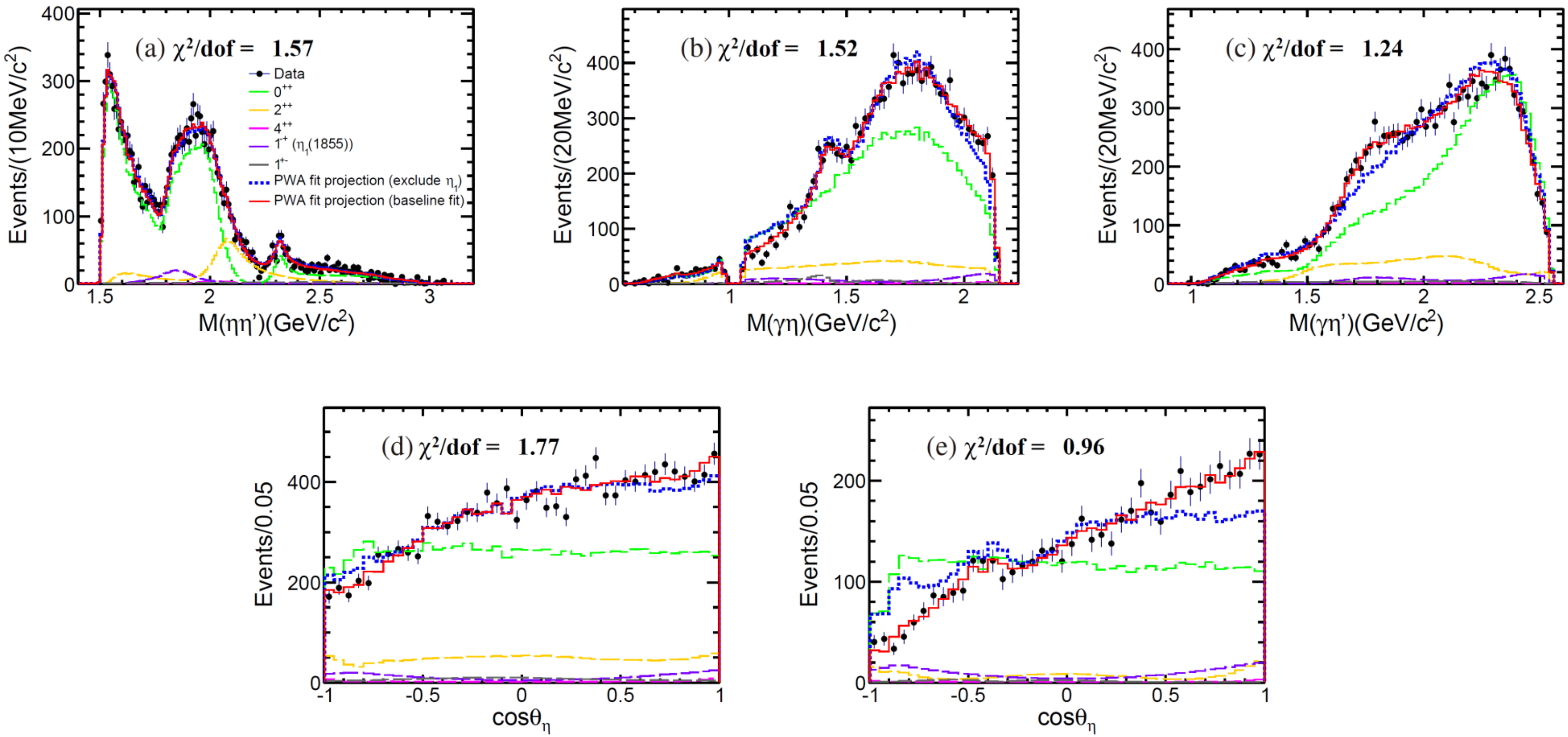
- **Recently, BESIII studied the $J/\psi \to \gamma\eta\eta'$ decay, and observed the $\eta_1(1855)$ with the exotic quantum number $I^G J^{PC} = 0^+ 1^{-+}$, which is a good candidate for the hybrid state.**

BESIII Collaboration, Phys. Rev. Lett. 129, 192002 (2022);

BESIII Collaboration, Phys. Rev. D 106, 072012 (2022).

Interests in hybrid states are reviving recently!

Recent BESIII experiment



The statistical significance is larger than 19σ.

Interests in hybrid states are reviving recently!

Recent theoretial studies

- **Hybrid interpretation:**

  L. Qiu and Q. Zhao, Chin. Phys. C46, 051001 (2022);
  V. Shastry, C. S. Fischer and F. Giacosa, Phys. Lett. B834, 137478 (2022);
  F. Chen, et al., Phys. Rev. D107 , 054511 (2023);
  E. S. Swanson, Phys. Rev. D107, 074028 (2023);
  C. Shi, et al., Phys. Rev. D109, 094513 (2024);
  B. Chen, S. Q. Luo and X. Liu, Phys. Rev. D108, 054034 (2023);
  C. Farina and E. S. Swanson, Phys. Rev. D109, 094015 (2024).

- **Molecular interpretation:**

  X. K. Dong, et al., Sci. China Phys. Mech. Astron. 65, 261011 (2022);
  X. Zhang and J.-J. Xie, Chin. Phys. C44, 054104 (2020);
  F. Yang, H. Q. Zhu and Y. Huang, Nucl. Phys. A1030, 122571 (2023);
  X. Y. Wang, F. C. Zeng and X. Liu, Phys. Rev. D106, 036005 (2022);
  M. J. Yan, et al. Universe 9, 109 (2023);
  Q. H. Shen and J. J. Xie, Phys. Rev. D107, 034019 (2023);
  Y. Yu, et al., Phys. Lett. B842, 137965 (2023);
  X. K. Dong, Phys. Lett. B853, 138646 (2024).

- **Tetraquark interpretation:**

  H. X. Chen, A. Hosaka, and S. L. Zhu, Phys. Rev. D78, 117502 (2008);
  B. D. Wan, S. Q. Zhang and C. F. Qiao, Phys. Rev. D106, 074003 (2022).

Recent theoretial studies

➢Hybrid interpretation:

L. Qiu and Q. Zhao, Chin. Phys. C46, 051001 (2022);

V. Shastry, C. S. Fischer and F. Giacosa, Phys. Lett. B834, 137478 (2022);

F. Chen, et al., Phys. Rev. D107 , 054511 (2023);

E. S. Swanson, Phys. Rev. D107, 074028 (2023);

C. Shi, et al., Phys. Rev. D109, 094513 (2024);

B. Chen, S. Q. Luo and X. Liu, Phys. Rev. D108, 054034 (2023);

C.

➢M

X.

X.

F.

Hua-Xing Chen, Wei Chen, Xiang Liu, Yan-Rui Liu and Shi-Lin Zhu,
An updated review of the new hadron states,
Rept. Prog. Phys. 86, 026201 (2023), [arXiv:2204.02649 [hep-ph]].

X. Y. Wang, F. C. Zeng and X. Liu, Phys. Rev. D106, 036005 (2022);

M. J. Yan, et al. Universe 9, 109 (2023);

Q. H. Shen and J. J. Xie, Phys. Rev. D107, 034019 (2023);

Y. Yu, et al., Phys. Lett. B842, 137965 (2023);

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- **QCD sum rule studies on hybrid states**
- Decay analyses on hybrid states
- Studies on the $J^{PC} = 1^{-+}$ hybrid states

QCD sum rule approach

H. X. Chen, Z. X. Cai, P. Z. Huang and S. L. Zhu, Phys. Rev. D83, 014006 (2011);

P. Z. Huang, H. X. Chen and S. L. Zhu, Phys. Rev. D83, 014021 (2011).

Predicted the $\eta\eta'$ decay mode of the $\eta_1(1855)$

H. X. Chen, W. Chen and S. L. Zhu, Phys. Rev. D103, L091503 (2021);

H. X. Chen, W. Chen and S. L. Zhu, Phys. Rev. D104, 094050 (2021);

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H. X. Chen, N. Su and S. L. Zhu, Chin. Phys. Lett. 39, 051201 (2022);

N. Su, H. X. Chen, W. Chen and S. L. Zhu, Phys. Rev. D107, 034010 (2023);

N. Su, W. H. Tan, H. X. Chen, W. Chen and S. L. Zhu, Phys. Rev. D107, 114005 (2023);

N. Su, H. X. Chen, W. Chen and S. L. Zhu, Phys. Rev. D109, L011502 (2024);

W. H. Tan, N. Su, H. X. Chen, arxiv: 2405.06958 [hep-ph].

Separated operators

R. L. Jaffe, K. Johnson and Z. Ryzak
Annals Phys. 168, 344 (1986)

$$\vec{E}_a = G^a_{i0} \quad \text{and} \quad \vec{B}_a = -\frac{1}{2}\epsilon_{ijk} G^a_{jk}$$

C=+
three-gluon
operators

J^{PC}	Operator
0^{++}	$f_{abc}(\vec{E}_a \times \vec{E}_b) \cdot \vec{B}_c$
0^{-+}	$f_{abc}(\vec{E}_a \times \vec{E}_b) \cdot \vec{E}_c$
1^{-+}	$f_{abc}(\vec{B}_a \cdot \vec{E}_b)\vec{B}_c$
1^{++}	$f_{abc}(\vec{E}_a \cdot \vec{B}_b)\vec{E}_c$
2^{++}	$f_{abc}\{(\vec{B}_a \times \vec{B}_b)^i B^j_c - (\vec{E}_a \times \vec{E}_b)^i B^j_c + 2(\vec{B}_a \times \vec{E}_b)^i E^j_c$ $+(i \leftrightarrow j) - \frac{2}{3}\delta^{ij}[(\vec{B}_a \times \vec{B}_b)\cdot \vec{B}_c + (\vec{E}_a \times \vec{E}_b)\cdot \vec{B}_c]\}$
2^{-+}	$f_{abc}\{(\vec{E}_a \times \vec{E}_b)^i E^j_c - (\vec{B}_a \times \vec{B}_b)^i E^j_c + 2(\vec{E}_a \times \vec{B}_b)^i B^j_c$ $+(i \leftrightarrow j) - \frac{2}{3}\delta^{ij}[(\vec{E}_a \times \vec{E}_b)\cdot \vec{E}_c + (\vec{B}_a \times \vec{B}_b)\cdot \vec{E}_c]\}$

Combined currents

$\boldsymbol{G^a_{\mu\nu}}$ and $\boldsymbol{\tilde{G}^a_{\mu\nu}}$

$$
\begin{aligned}
\eta_0 &= f^{abc} g_s^3 G_a^{\mu\nu} G_{b,\nu\rho} G^{\rho}_{c,\mu} \,, \\
\tilde{\eta}_0 &= f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} \tilde{G}_{b,\nu\rho} \tilde{G}^{\rho}_{c,\mu} \,, \\
\eta_1^{\alpha\beta} &= f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} G_{b,\mu\nu} \tilde{G}_c^{\alpha\beta} \,, \\
\tilde{\eta}_1^{\alpha\beta} &= f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} G_{b,\mu\nu} G_c^{\alpha\beta} \,, \\
\eta_2^{\alpha_1\alpha_2,\beta_1\beta_2} &= f^{abc} \mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} G_b^{\alpha_2\mu} G^{\beta_2}_{c,\mu} - \{\alpha_2 \leftrightarrow \beta_2\}] \,, \\
\tilde{\eta}_2^{\alpha_1\alpha_2,\beta_1\beta_2} &= f^{abc} \mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\mu} \tilde{G}^{\beta_2}_{c,\mu} - \{\alpha_2 \leftrightarrow \beta_2\}] \,.
\end{aligned}
$$

Separated operators	Combined currents

$\vec{E}_a = G^a_{i0}$ and $\vec{B}_a = -\frac{1}{2}\epsilon_{ijk}G^a_{jk}$

$G^a_{\mu\nu}$ and $\tilde{G}^a_{\mu\nu}$

$$
\begin{array}{ll}
0^{++} & f^{abc}(\vec{E}_a \times \vec{E}_b)\cdot\vec{B}_c\,, \\
0^{-+} & f^{abc}(\vec{E}_a \times \vec{E}_b)\cdot\vec{E}_c\,, \\
1^{++} & f^{abc}(\vec{B}_a \cdot \vec{E}_b)\;\vec{E}_c\,, \\
1^{-+} & f^{abc}(\vec{B}_a \cdot \vec{E}_b)\;\vec{B}_c\,, \\
2^{++} & f^{abc}\mathcal{S}'[(\vec{B}_a \times \vec{B}_b)^i B^j_c] + \cdots\,, \\
2^{-+} & f^{abc}\mathcal{S}'[(\vec{E}_a \times \vec{E}_b)^i E^j_c] + \cdots\,.
\end{array}
\longleftrightarrow
$$

$$
\begin{aligned}
\eta_0 &= f^{abc} g_s^3 G_a^{\mu\nu} G_{b,\nu\rho} G^{\rho}_{c,\mu}\,, \\
\tilde{\eta}_0 &= f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} \tilde{G}_{b,\nu\rho} \tilde{G}^{\rho}_{c,\mu}\,, \\
\eta_1^{\alpha\beta} &= f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} G_{b,\mu\nu} \tilde{G}^{\alpha\beta}_{c}\,, \\
\tilde{\eta}_1^{\alpha\beta} &= f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} G_{b,\mu\nu} G^{\alpha\beta}_{c}\,, \\
\eta_2^{\alpha_1\alpha_2,\beta_1\beta_2} &= f^{abc}\mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} G_b^{\alpha_2\mu} G^{\beta_2}_{c,\mu} - \{\alpha_2 \leftrightarrow \beta_2\}]\,, \\
\tilde{\eta}_2^{\alpha_1\alpha_2,\beta_1\beta_2} &= f^{abc}\mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\mu} \tilde{G}^{\beta_2}_{c,\mu} - \{\alpha_2 \leftrightarrow \beta_2\}]\,.
\end{aligned}
$$

Separated operators

$\vec{E}_a = G^a_{i0}$ and $\vec{B}_a = -\frac{1}{2}\epsilon_{ijk}G^a_{jk}$

$$
\begin{array}{ll}
0^{++} & f^{abc}(\vec{E}_a \times \vec{E}_b) \cdot \vec{B}_c \,, \\
0^{-+} & f^{abc}(\vec{E}_a \times \vec{E}_b) \cdot \vec{E}_c \,, \\
1^{++} & f^{abc}(\vec{B}_a \cdot \vec{E}_b)\ \vec{E}_c \,, \quad \neq 0 \\
1^{-+} & f^{abc}(\vec{B}_a \cdot \vec{E}_b)\ \vec{B}_c \,, \quad \neq 0 \\
2^{++} & f^{abc}\mathcal{S}'[(\vec{B}_a \times \vec{B}_b)^i B^j_c] + \cdots \,, \\
2^{-+} & f^{abc}\mathcal{S}'[(\vec{E}_a \times \vec{E}_b)^i E^j_c] + \cdots \,.
\end{array}
$$

Combined currents

$G^a_{\mu\nu}$ and $\tilde{G}^a_{\mu\nu}$

$$
\begin{aligned}
\eta_0 &= f^{abc} g_s^3 G_a^{\mu\nu} G_{b,\nu\rho} G^{\rho}_{c,\mu} \,, \\
\tilde{\eta}_0 &= f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} \tilde{G}_{b,\nu\rho} \tilde{G}^{\rho}_{c,\mu} \,, \\
\eta_1^{\alpha\beta} &= f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} G_{b,\mu\nu} \tilde{G}^{\alpha\beta}_c \,, \quad = 0 \\
\tilde{\eta}_1^{\alpha\beta} &= f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} G_{b,\mu\nu} G^{\alpha\beta}_c \,, \quad = 0 \\
\eta_2^{\alpha_1\alpha_2,\beta_1\beta_2} &= f^{abc}\mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} G_b^{\alpha_2\mu} G^{\beta_2}_{c,\mu} - \{\alpha_2 \leftrightarrow \beta_2\}] \,, \\
\tilde{\eta}_2^{\alpha_1\alpha_2,\beta_1\beta_2} &= f^{abc}\mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\mu} \tilde{G}^{\beta_2}_{c,\mu} - \{\alpha_2 \leftrightarrow \beta_2\}] \,.
\end{aligned}
$$

Color-Singlet Meson Currents

$\bar{q}_a q_a \qquad J^P = 0^+$

$\bar{q}_a \gamma_5 q_a \qquad J^P = 0^-$

$\bar{q}_a \gamma_\mu q_a \qquad J^P = 1^-$

$\bar{q}_a \gamma_\mu \gamma_5 q_a \qquad J^P = 1^+$

$\bar{q}_a \sigma_{\mu\nu} q_a \qquad J^P = 1^\pm$

Lorentz indices

color indices

flavor indices needed

Color-Octet Meson Currents

$\bar{q}_a \lambda_n^{ab} q_b \qquad J^P = 0^+$

$\bar{q}_a \gamma_5 \lambda_n^{ab} q_b \qquad J^P = 0^-$

$\bar{q}_a \gamma_\mu \lambda_n^{ab} q_b \qquad J^P = 1^-$

$\bar{q}_a \gamma_\mu \gamma_5 \lambda_n^{ab} q_b \qquad J^P = 1^+$

$\bar{q}_a \sigma_{\mu\nu} \lambda_n^{ab} q_b \qquad J^P = 1^\pm$

Lorentz indices

color indices

flavor indices needed

Single-gluon hybrid curents

$\bar{q}q$

$\bar{q}_a \lambda_n^{ab} q_b$ $\bar{q}_a \gamma_5 \lambda_n^{ab} q_b$ $\bar{q}_a \sigma_{\mu\nu} \lambda_n^{ab} q_b$

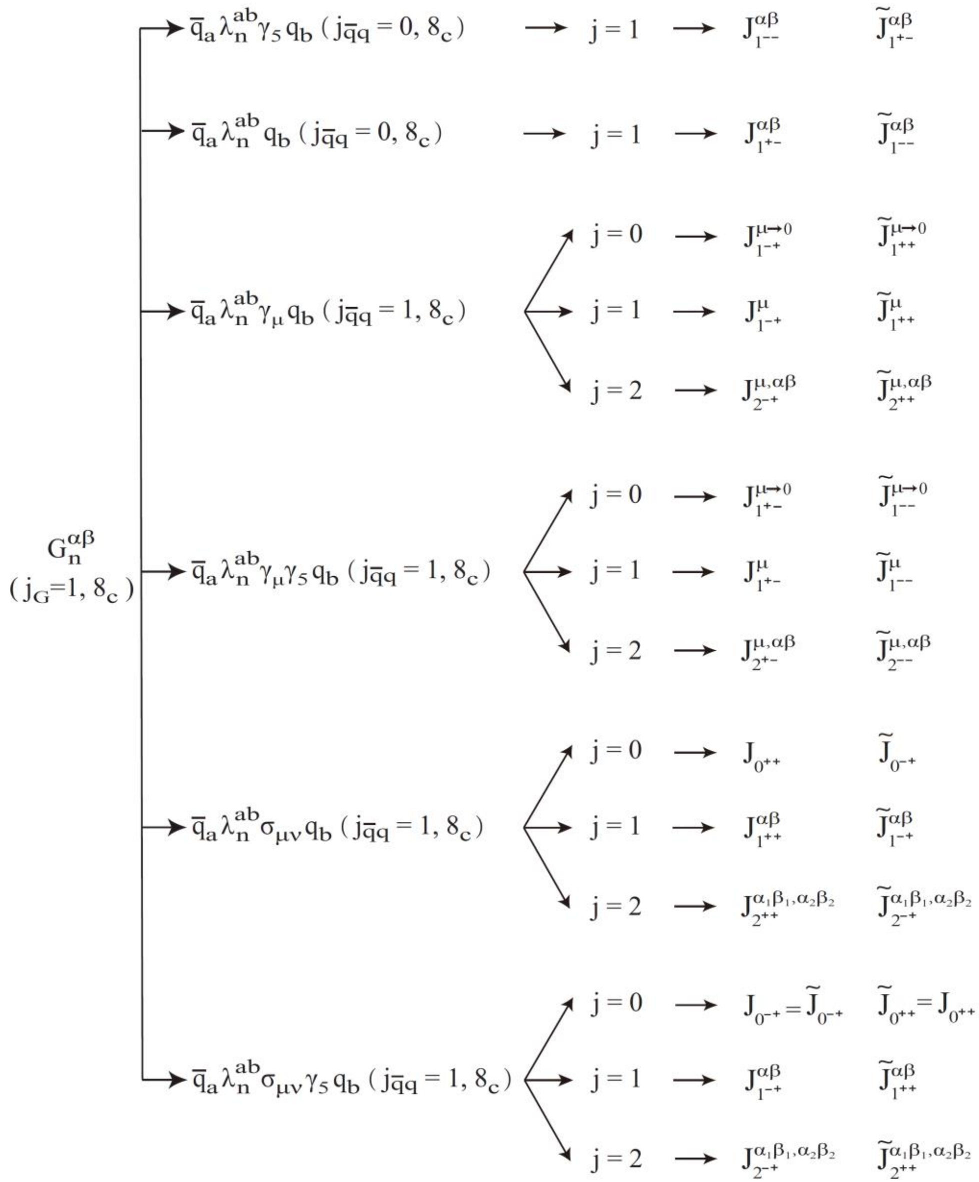
$\bar{q}_a \gamma_\mu \lambda_n^{ab} q_b$ $\bar{q}_a \gamma_\mu \gamma_5 \lambda_n^{ab} q_b$

G

G_n and $\tilde{G}_n$

Single-gluon hybrid curents

$$J_{1--}^{\alpha\beta} = \bar{q}_a \lambda_n^{ab} \gamma_5 q_b \; g_s G_n^{\alpha\beta} \,, \tag{7}$$
$$\widetilde{J}_{1+-}^{\alpha\beta} = \bar{q}_a \lambda_n^{ab} \gamma_5 q_b \; g_s \widetilde{G}_n^{\alpha\beta} \,, \tag{8}$$
$$J_{1+-}^{\alpha\beta} = \bar{q}_a \lambda_n^{ab} q_b \; g_s G_n^{\alpha\beta} \,, \tag{9}$$
$$\widetilde{J}_{1--}^{\alpha\beta} = \bar{q}_a \lambda_n^{ab} q_b \; g_s \widetilde{G}_n^{\alpha\beta} \,, \tag{10}$$
$$J_{1-+}^{\mu} = \bar{q}_a \lambda_n^{ab} \gamma_\beta q_b \; g_s G_n^{\mu\beta} \,, \tag{11}$$
$$\widetilde{J}_{1++}^{\mu} = \bar{q}_a \lambda_n^{ab} \gamma_\beta q_b \; g_s \widetilde{G}_n^{\mu\beta} \,, \tag{12}$$
$$J_{1+-}^{\mu} = \bar{q}_a \lambda_n^{ab} \gamma_\beta \gamma_5 q_b \; g_s G_n^{\mu\beta} \,, \tag{13}$$
$$\widetilde{J}_{1--}^{\mu} = \bar{q}_a \lambda_n^{ab} \gamma_\beta \gamma_5 q_b \; g_s \widetilde{G}_n^{\mu\beta} \,, \tag{14}$$
$$J_{2-+}^{\mu,\alpha\beta} = \bar{q}_a \lambda_n^{ab} \gamma^\mu q_b \; g_s G_n^{\alpha\beta} \,, \tag{15}$$
$$\widetilde{J}_{2++}^{\mu,\alpha\beta} = \bar{q}_a \lambda_n^{ab} \gamma^\mu q_b \; g_s \widetilde{G}_n^{\alpha\beta} \,, \tag{16}$$
$$J_{2+-}^{\mu,\alpha\beta} = \bar{q}_a \lambda_n^{ab} \gamma^\mu \gamma_5 q_b \; g_s G_n^{\alpha\beta} \,, \tag{17}$$
$$\widetilde{J}_{2--}^{\mu,\alpha\beta} = \bar{q}_a \lambda_n^{ab} \gamma^\mu \gamma_5 q_b \; g_s \widetilde{G}_n^{\alpha\beta} \,, \tag{18}$$
$$J_{0++} = \bar{q}_a \lambda_n^{ab} \sigma_{\mu\nu} q_b \; g_s G_n^{\mu\nu} \,, \tag{19}$$
$$\widetilde{J}_{0-+} = \bar{q}_a \lambda_n^{ab} \sigma_{\mu\nu} q_b \; g_s \widetilde{G}_n^{\mu\nu} \,, \tag{20}$$
$$J_{0-+} = \bar{q}_a \lambda_n^{ab} \sigma_{\mu\nu} \gamma_5 q_b \; g_s G_n^{\mu\nu} \,, \tag{21}$$
$$\widetilde{J}_{0++} = \bar{q}_a \lambda_n^{ab} \sigma_{\mu\nu} \gamma_5 q_b \; g_s \widetilde{G}_n^{\mu\nu} \,, \tag{22}$$
$$J_{1++}^{\alpha\beta} = \mathcal{A}[\bar{q}_a \lambda_n^{ab} \sigma^{\alpha\mu} q_b \; g_s G_{n,\mu}^{\beta}] \,, \tag{23}$$
$$\widetilde{J}_{1-+}^{\alpha\beta} = \mathcal{A}[\bar{q}_a \lambda_n^{ab} \sigma^{\alpha\mu} q_b \; g_s \widetilde{G}_{n,\mu}^{\beta}] \,, \tag{24}$$
$$J_{1-+}^{\alpha\beta} = \mathcal{A}[\bar{q}_a \lambda_n^{ab} \sigma^{\alpha\mu} \gamma_5 q_b \; g_s G_{n,\mu}^{\beta}] \,, \tag{25}$$
$$\widetilde{J}_{1++}^{\alpha\beta} = \mathcal{A}[\bar{q}_a \lambda_n^{ab} \sigma^{\alpha\mu} \gamma_5 q_b \; g_s \widetilde{G}_{n,\mu}^{\beta}] \,, \tag{26}$$
$$J_{2++}^{\alpha_1\beta_1,\alpha_2\beta_2} = \mathcal{S}[\bar{q}_a \lambda_n^{ab} \sigma^{\alpha_1\beta_1} q_b \; g_s G_n^{\alpha_2\beta_2}] \,, \tag{27}$$
$$\widetilde{J}_{2-+}^{\alpha_1\beta_1,\alpha_2\beta_2} = \mathcal{S}[\bar{q}_a \lambda_n^{ab} \sigma^{\alpha_1\beta_1} q_b \; g_s \widetilde{G}_n^{\alpha_2\beta_2}] \,, \tag{28}$$
$$J_{2-+}^{\alpha_1\beta_1,\alpha_2\beta_2} = \mathcal{S}[\bar{q}_a \lambda_n^{ab} \sigma^{\alpha_1\beta_1} \gamma_5 q_b \; g_s G_n^{\alpha_2\beta_2}] \,, \tag{29}$$
$$\widetilde{J}_{2++}^{\alpha_1\beta_1,\alpha_2\beta_2} = \mathcal{S}[\bar{q}_a \lambda_n^{ab} \sigma^{\alpha_1\beta_1} \gamma_5 q_b \; g_s \widetilde{G}_n^{\alpha_2\beta_2}] \,. \tag{30}$$



Double-gluon hybrid curents

$\bar{q}q$

$\bar{q}_a \lambda_n^{ab} q_b$ $\boxed{\bar{q}_a \gamma_5 \lambda_n^{ab} q_b}$ $\boxed{\bar{q}_a \sigma_{\mu\nu} \lambda_n^{ab} q_b}$

$\bar{q}_a \gamma_\mu \lambda_n^{ab} q_b$ $\bar{q}_a \gamma_\mu \gamma_5 \lambda_n^{ab} q_b$

GG

$d^{npq} G_p G_q$ $d^{npq} G_p \tilde{G}_q$ $d^{npq} \tilde{G}_p \tilde{G}_q$

$f^{npq} G_p G_q$ $f^{npq} G_p \tilde{G}_q$ $f^{npq} \tilde{G}_p \tilde{G}_q$

Double-gluon hybrid curents

$$J^{\alpha\beta}_{1^{++}(j=0)} = \bar{q}_a\sigma^{\alpha\beta}\lambda_n^{ab}q_b\ f^{npq}\ g_s^2 G_p^{\mu\nu}G_{q,\mu\nu}\,,$$
$$J_{0^{++}(j=1)} = \bar{q}_a\sigma^{\mu\nu}\lambda_n^{ab}q_b\ f^{npq}\ g_s^2 G_{p,\nu\rho}G^{\rho}_{q,\mu}\,,$$
$$J^{\alpha\beta}_{1^{++}(j=1)} = \bar{q}_a\sigma_{\alpha_1\beta_1}\lambda_n^{ab}q_b\ f^{npq}\ g_s^2\left(G_{p,\alpha_2\mu}G^{\mu}_{q,\beta_2} - \{\alpha_2\leftrightarrow\beta_2\}\right)\times g^{\beta_1\beta_2}(g^{\alpha\alpha_1}g^{\beta\alpha_2}$$
$$J^{\alpha_1\beta_1,\alpha_2\beta_2}_{2^{++}(j=1)} = \mathcal{S}[\bar{q}_a\sigma^{\alpha_1\beta_1}\lambda_n^{ab}q_b\ f^{npq}\ g_s^2\left(G_p^{\alpha_2\mu}G^{\beta_2}_{q,\mu} - \{\alpha_2\leftrightarrow\beta_2\}\right)]\,,$$
$$J^{\alpha\beta}_{1^{++}(j=2)} = \bar{q}_a\sigma^{\mu_2\nu_2}\lambda_n^{ab}q_b\ f^{npq}\ \mathcal{S}'[g_s^2 G_{p,\mu_1\nu_1}G_{q,\mu_2\nu_2}]\times g^{\alpha\mu_1}g^{\beta\nu_1}\,,$$
$$J^{\alpha_1\beta_1,\alpha_2\beta_2}_{2^{++}(j=2)} = \mathcal{S}[\bar{q}_a\sigma_{\mu_3\nu_3}\lambda_n^{ab}q_b\ f^{npq}\ \mathcal{S}'[g_s^2 G_{p,\mu_1\nu_1}G_{q,\mu_2\nu_2}]\times g^{\alpha_1\mu_1}g^{\beta_1\nu_1}g^{\nu_2\nu_3}(g^{\alpha_2\mu_2}g$$
$$J^{\alpha_1\beta_1,\alpha_2\beta_2,\alpha_3\beta_3}_{3^{++}(j=2)} = \mathcal{S}[\bar{q}_a\sigma^{\alpha_1\beta_1}\lambda_n^{ab}q_b\ f^{npq}\ g_s^2 G_p^{\alpha_2\beta_2}G_q^{\alpha_3\beta_3}]\,,$$
$$J^{\alpha\beta}_{1^{-+}(j=0)} = \bar{q}_a\sigma^{\alpha\beta}\lambda_n^{ab}q_b\ f^{npq}\ g_s^2 G_p^{\mu\nu}\tilde{G}_{q,\mu\nu}\,,$$
$$J_{0^{-+}(j=1)} = \bar{q}_a\sigma^{\mu\nu}\lambda_n^{ab}q_b\ f^{npq}\ g_s^2 G_{p,\nu\rho}\tilde{G}^{\rho}_{q,\mu}\,,$$
$$J^{\alpha\beta}_{1^{-+}(j=1)} = \bar{q}_a\sigma_{\alpha_1\beta_1}\lambda_n^{ab}q_b\ f^{npq}\ g_s^2\left(G_{p,\alpha_2\mu}\tilde{G}^{\mu}_{q,\beta_2} - \{\alpha_2\leftrightarrow\beta_2\}\right)\times g^{\beta_1\beta_2}(g^{\alpha\alpha_1}g^{\beta\alpha_2}$$
$$J^{\alpha_1\beta_1,\alpha_2\beta_2}_{2^{-+}(j=1)} = \mathcal{S}[\bar{q}_a\sigma^{\alpha_1\beta_1}\lambda_n^{ab}q_b\ f^{npq}\ g_s^2\left(G_p^{\alpha_2\mu}\tilde{G}^{\beta_2}_{q,\mu} - \{\alpha_2\leftrightarrow\beta_2\}\right)]\,,$$
$$J^{\alpha\beta}_{1^{-+}(j=2)} = \bar{q}_a\sigma^{\mu_2\nu_2}\lambda_n^{ab}q_b\ f^{npq}\ \mathcal{S}'[g_s^2 G_{p,\mu_1\nu_1}\tilde{G}_{q,\mu_2\nu_2}]\times g^{\alpha\mu_1}g^{\beta\nu_1}\,,$$
$$J^{\alpha_1\beta_1,\alpha_2\beta_2}_{2^{-+}(j=2)} = \mathcal{S}[\bar{q}_a\sigma_{\mu_3\nu_3}\lambda_n^{ab}q_b\ f^{npq}\ \mathcal{S}'[g_s^2 G_{p,\mu_1\nu_1}\tilde{G}_{q,\mu_2\nu_2}]\times g^{\alpha_1\mu_1}g^{\beta_1\nu_1}g^{\nu_2\nu_3}(g^{\alpha_2\mu_2}g$$
$$J^{\alpha_1\beta_1,\alpha_2\beta_2,\alpha_3\beta_3}_{3^{-+}(j=2)} = \mathcal{S}[\bar{q}_a\sigma^{\alpha_1\beta_1}\lambda_n^{ab}q_b\ f^{npq}\ g_s^2 G_p^{\alpha_2\beta_2}\tilde{G}_q^{\alpha_3\beta_3}]\,,$$
$$J^{\alpha\beta}_{1^{+-}(j=0)} = \bar{q}_a\sigma^{\alpha\beta}\lambda_n^{ab}q_b\ d^{npq}\ g_s^2 G_p^{\mu\nu}G_{q,\mu\nu}\,,$$
$$J_{0^{+-}(j=1)} = \bar{q}_a\sigma^{\mu\nu}\lambda_n^{ab}q_b\ d^{npq}\ g_s^2 G_{p,\nu\rho}G^{\rho}_{q,\mu}\,,$$
$$J^{\alpha\beta}_{1^{+-}(j=1)} = \bar{q}_a\sigma_{\alpha_1\beta_1}\lambda_n^{ab}q_b\ d^{npq}\ g_s^2\left(G_{p,\alpha_2\mu}G^{\mu}_{q,\beta_2} - \{\alpha_2\leftrightarrow\beta_2\}\right)\times g^{\beta_1\beta_2}(g^{\alpha\alpha_1}g^{\beta\alpha_2}$$
$$J^{\alpha_1\beta_1,\alpha_2\beta_2}_{2^{+-}(j=1)} = \mathcal{S}[\bar{q}_a\sigma^{\alpha_1\beta_1}\lambda_n^{ab}q_b\ d^{npq}\ g_s^2\left(G_p^{\alpha_2\mu}G^{\beta_2}_{q,\mu} - \{\alpha_2\leftrightarrow\beta_2\}\right)]\,,$$
$$J^{\alpha\beta}_{1^{+-}(j=2)} = \bar{q}_a\sigma^{\mu_2\nu_2}\lambda_n^{ab}q_b\ d^{npq}\ \mathcal{S}'[g_s^2 G_{p,\mu_1\nu_1}G_{q,\mu_2\nu_2}]\times g^{\alpha\mu_1}g^{\beta\nu_1}\,,$$
$$J^{\alpha_1\beta_1,\alpha_2\beta_2}_{2^{+-}(j=2)} = \mathcal{S}[\bar{q}_a\sigma_{\mu_3\nu_3}\lambda_n^{ab}q_b\ d^{npq}\ \mathcal{S}'[g_s^2 G_{p,\mu_1\nu_1}G_{q,\mu_2\nu_2}]\times g^{\alpha_1\mu_1}g^{\beta_1\nu_1}g^{\nu_2\nu_3}(g^{\alpha_2\mu_2}g$$
$$J^{\alpha_1\beta_1,\alpha_2\beta_2,\alpha_3\beta_3}_{3^{+-}(j=2)} = \mathcal{S}[\bar{q}_a\sigma^{\alpha_1\beta_1}\lambda_n^{ab}q_b\ d^{npq}\ g_s^2 G_p^{\alpha_2\beta_2}G_q^{\alpha_3\beta_3}]\,,$$
$$J^{\alpha\beta}_{1^{--}(j=0)} = \bar{q}_a\sigma^{\alpha\beta}\lambda_n^{ab}q_b\ d^{npq}\ g_s^2 G_p^{\mu\nu}\tilde{G}_{q,\mu\nu}\,,$$
$$J_{0^{--}(j=1)} = \bar{q}_a\sigma^{\mu\nu}\lambda_n^{ab}q_b\ d^{npq}\ g_s^2 G_{p,\nu\rho}\tilde{G}^{\rho}_{q,\mu}\,,$$

$\bar{q}_a\sigma_{\mu\nu}\lambda_n^{ab}q_b$ $(j_{\bar{q}q}=1,\ 8_c)$

$\rightarrow f^{npq}G_pG_q\ (j=0/1/2,\ 8_c^A)$
- $j=0\ (S) \rightarrow J=1$ $|X;1^{++}(j=0)\rangle=0$
- $j=1\ (A)$: $J=0$ $|X;0^{++}(j=1)\rangle$; $J=1$ $|X;1^{++}(j=1)\rangle$; $J=2$ $|X;2^{++}(j=1)\rangle$
- $j=2\ (S)$: $J=1$ $|X;1^{++}(j=2)\rangle=0$; $J=2$ $|X;2^{++}(j=2)\rangle=0$; $J=3$ $|X;3^{++}(j=2)\rangle=0$

$\rightarrow f^{npq}G_p\tilde{G}_q\ (j=0/1/2,\ 8_c)$
- $j=0 \rightarrow J=1$ $|X;1^{-+}(j=0)\rangle=0$
- $j=1$: $J=0$ $|X;0^{-+}(j=1)\rangle$; $J=1$ $|X;1^{-+}(j=1)\rangle$; $J=2$ $|X;2^{-+}(j=1)\rangle$
- $j=2$: $J=1$ $|X;1^{-+}(j=2)\rangle$; $J=2$ $|X;2^{-+}(j=2)\rangle$; $J=3$ $|X;3^{-+}(j=2)\rangle$

$\rightarrow d^{npq}G_pG_q\ (j=0/1/2,\ 8_c^S)$
- $j=0\ (S) \rightarrow J=1$ $|X;1^{+-}(j=0)\rangle$
- $j=1\ (A)$: $J=0$ $|X;0^{+-}(j=1)\rangle=0$; $J=1$ $|X;1^{+-}(j=1)\rangle=0$; $J=2$ $|X;2^{+-}(j=1)\rangle=0$
- $j=2\ (S)$: $J=1$ $|X;1^{+-}(j=2)\rangle$; $J=2$ $|X;2^{+-}(j=2)\rangle$; $J=3$ $|X;3^{+-}(j=2)\rangle$

$\rightarrow d^{npq}G_p\tilde{G}_q\ (j=0/1/2,\ 8_c)$
- $j=0 \rightarrow J=1$ $|X;1^{--}(j=0)\rangle$
- $j=1$: $J=0$ $|X;0^{--}(j=1)\rangle=0$; $J=1$ $|X;1^{--}(j=1)\rangle=0$; $J=2$ $|X;2^{--}(j=1)\rangle=0$
- $j=2$: $J=1$ $|X;1^{--}(j=2)\rangle$; $J=2$ $|X;2^{--}(j=2)\rangle$; $J=3$ $|X;3^{--}(j=2)\rangle$

QCD sum rule method

- In sum rule analyses, we consider two-point correlation functions:

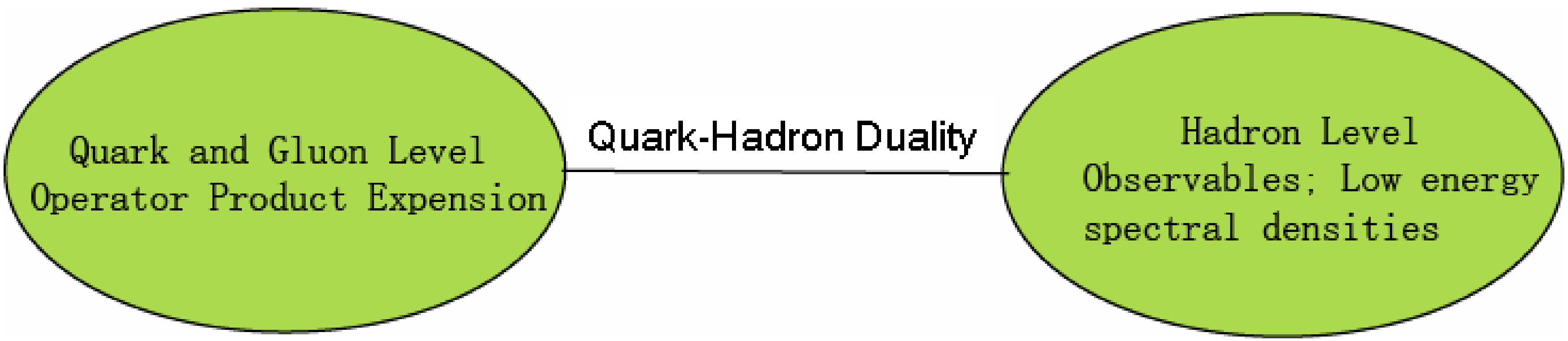
$$\Pi\left(q^2\right) \stackrel{\text{def}}{=} i \int d^4 x e^{iqx} \langle 0|T\eta(x)\eta^+(0)\;|0\rangle$$
$$\approx \sum_n \langle 0|\eta|n\rangle\langle n|\eta^+|0\rangle$$

  where η is the current which can couple to hadronic states.

- By using the dispersion relation, we can obtain the spectral density

$$\Pi\left(q^2\right) = \int_{s_<}^{\infty} \frac{\rho(s)}{s - q^2 - i\varepsilon} ds$$

- In QCD sum rule, we can calculate these matrix elements from QCD (OPE) and relate them to observables by using dispersion relation.



SVZ sum rule

Quark and Gluon Level

(Convergence of OPE)

$$\Pi_{OPE}(q^2) \xrightarrow[s = -q^2]{\text{dispersion relation}} \rho_{OPE}(s) = a_n\, s^n + a_{n-1}\, s^{n-1}$$

Quark-Hadron Duality

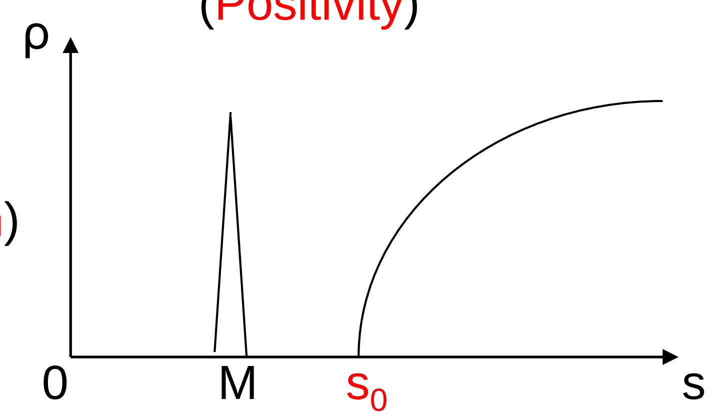
Hadron Level

$$\Pi_{phys}(q^2) = f_x^2 \frac{1}{q^2 - M^2} \longleftrightarrow \rho_{phys}(s) = \lambda_x^2 \delta(s - M_x^2) + \cdots$$

(for boson case)

(Positivity)

(Sufficient amount of Pole contribution)



Nielsen et al, Phys. Rept. 497, 41 (2010); Albuquerque et al, JPG46, 093002 (2019).

SVZ sum rule

Quark and Gluon Level

(Convergence of OPE)

$$\Pi_{OPE}(q^2) \xrightarrow[s\,=\,-q^2]{\text{dispersion relation}} \rho_{OPE}(s) = a_n\, s^n + a_{n-1}\, s^{n-1}$$

Quark-Hadron Duality

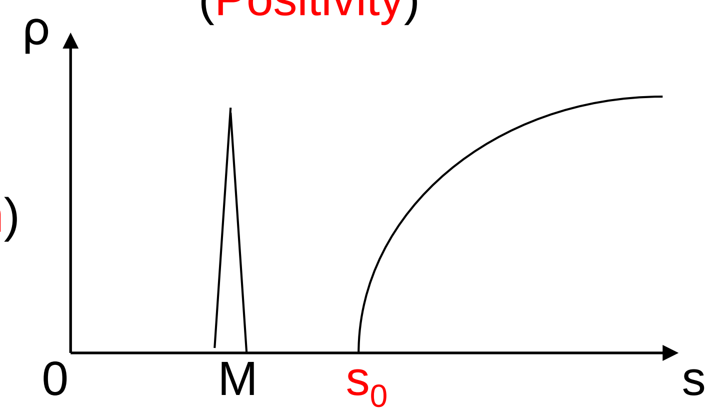
Hadron Level

$$\Pi_{phys}(q^2) = f_x^2 \frac{1}{q^2 - M^2} \longleftrightarrow \rho_{phys}(s) = \lambda_x^2 \delta(s - M_x^2) + \cdots$$

(for boson case)

(Positivity)

(Sufficient amount of Pole contribution)



QCD sum rule method

- Borel transformation to suppress the higher order terms:

$$\Pi(M_B^2) \equiv f^2 e^{-M^2/M_B^2} = \int_{s_<}^{s_0} e^{-s/M_B^2} \rho(s) ds$$

- Two free parameters

$$M_B, \quad s_0$$

We need to choose certain region of (M_B, s_0).

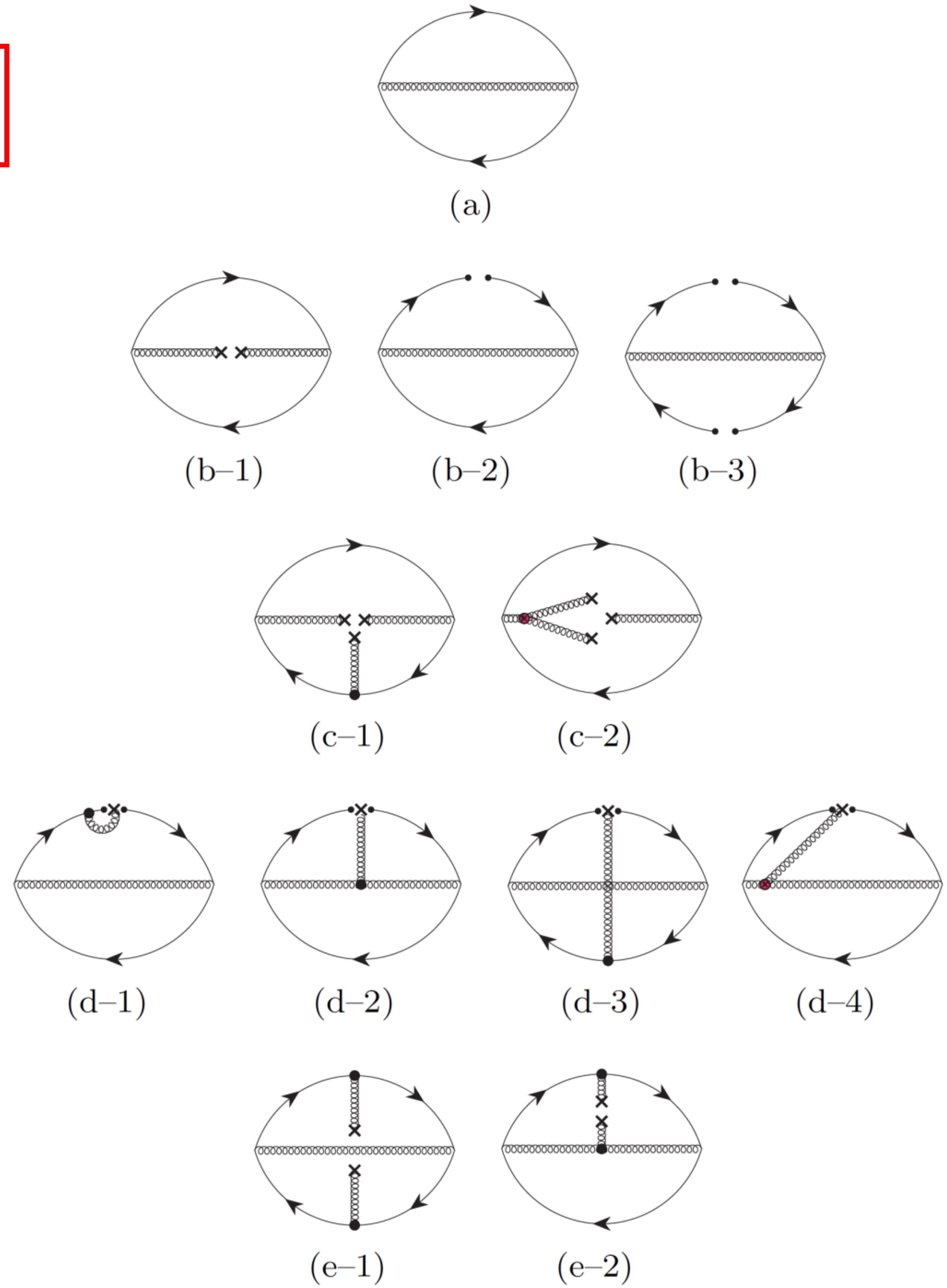
- **Criteria**
  1. Stability
  2. Convergence of OPE
  3. Positivity of spectral density
  4. Sufficient amount of pole contribution

QCD sum rule results

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$

QCD sum rule results

$$G^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$



QCD sum rule results

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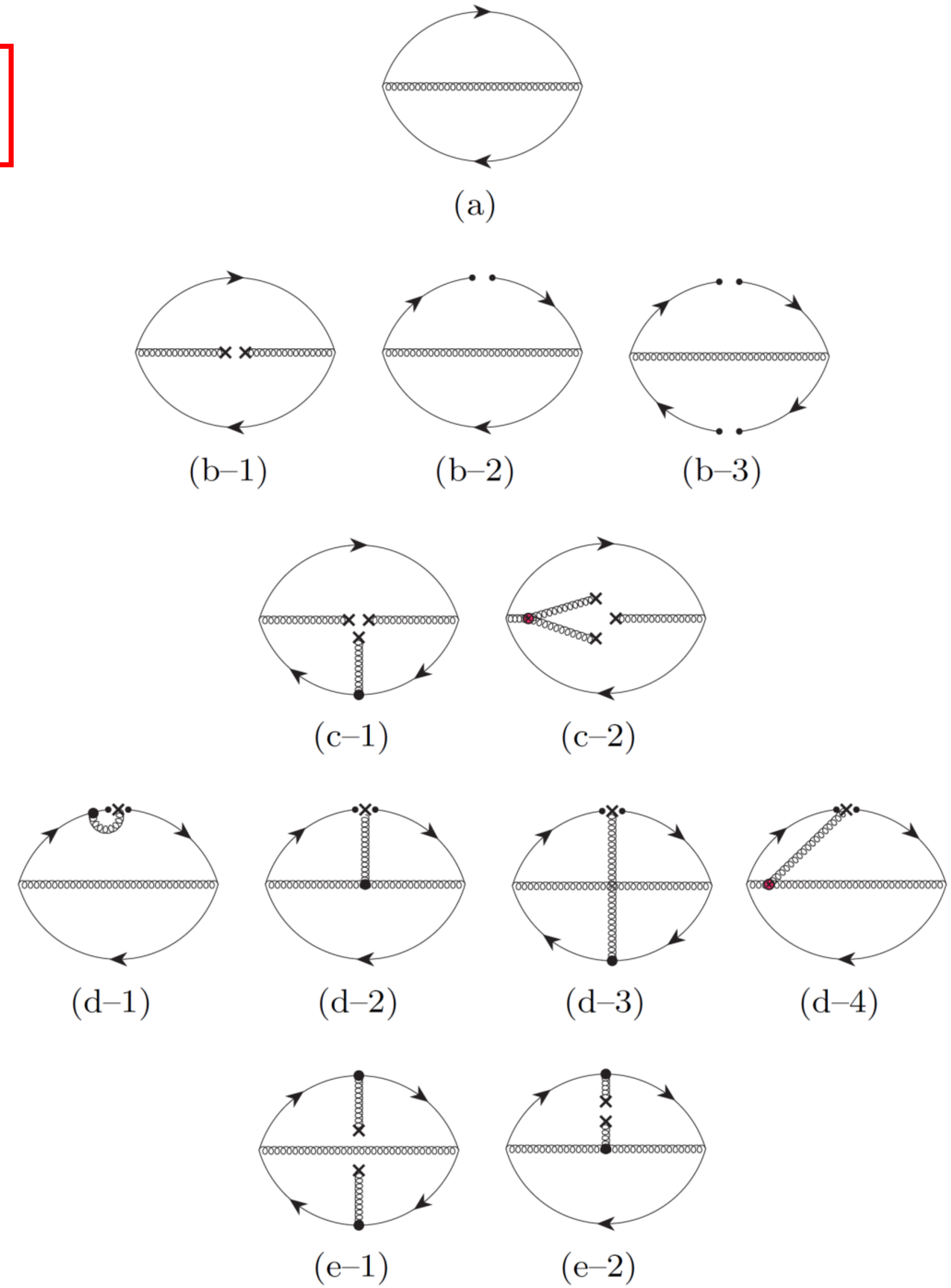


g_s^2

g_s^2

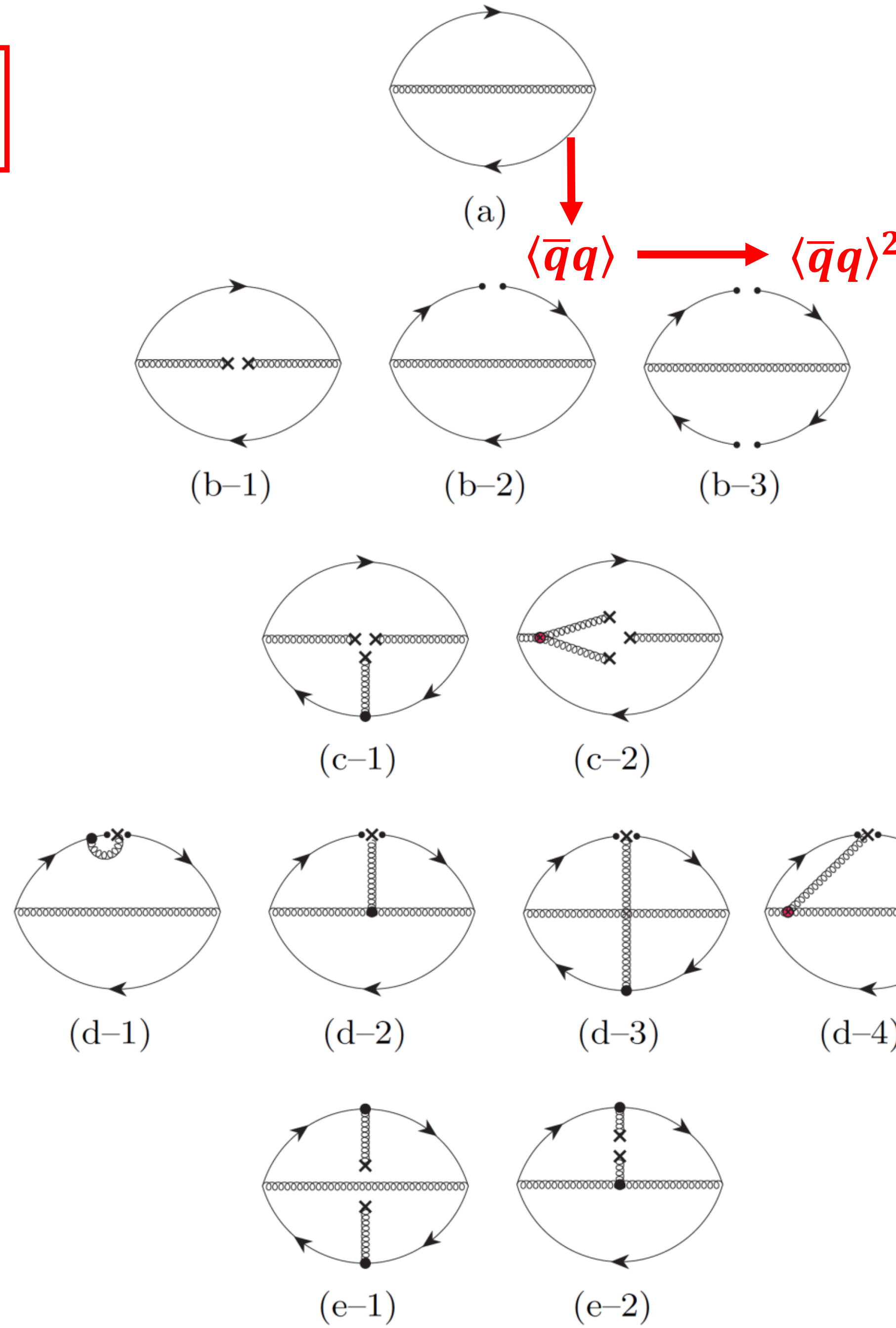
g_s^3

g_s^3

g_s^4

QCD sum rule results

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$



QCD sum rule results

$$J^{\mu}_{1^{-+}} = \bar{s}_a \lambda_n^{ab} \gamma_\nu s_b \; g_s G_n^{\mu\nu}$$

QCD sum rule results

$$J^{\mu}_{1^{-+}} = \bar{s}_a \lambda_n^{ab} \gamma_\nu s_b \, g_s G_n^{\mu\nu}$$

$$\begin{aligned}
&\Pi^{\mu}_{1^{-+}}\left(M_B^2, s_0\right) \\
= &\int_{4m_s^2}^{s_0} \Big(\frac{s^3\alpha_s}{60\pi^3} - \frac{m_s^2 s^2 \alpha_s}{3\pi^3} + s\big(\frac{\langle\alpha_s GG\rangle}{36\pi^2} \\
&+\frac{13\langle\alpha_s GG\rangle\alpha_s}{432\pi^3} + \frac{8m_s\langle\bar{s}s\rangle\alpha_s}{9\pi}\big) + \frac{\langle g_s^3 G^3\rangle}{32\pi^2} \\
&-\frac{3\langle\alpha_s GG\rangle m_s^2\alpha_s}{64\pi^3} - \frac{3m_s\langle g_s\bar{s}\sigma G s\rangle\alpha_s}{4\pi}\Big) \times e^{-s/M_B^2} ds \\
+ &\Big(\frac{\langle\alpha_s GG\rangle^2}{3456\pi^2} - \frac{\langle g_s^3 G^3\rangle m_s^2}{16\pi^2} - \frac{2}{9}\langle\alpha_s GG\rangle m_s\langle\bar{s}s\rangle \\
&+\frac{11\pi\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma G s\rangle\alpha_s}{9}\Big).
\end{aligned}$$

QCD sum rule results

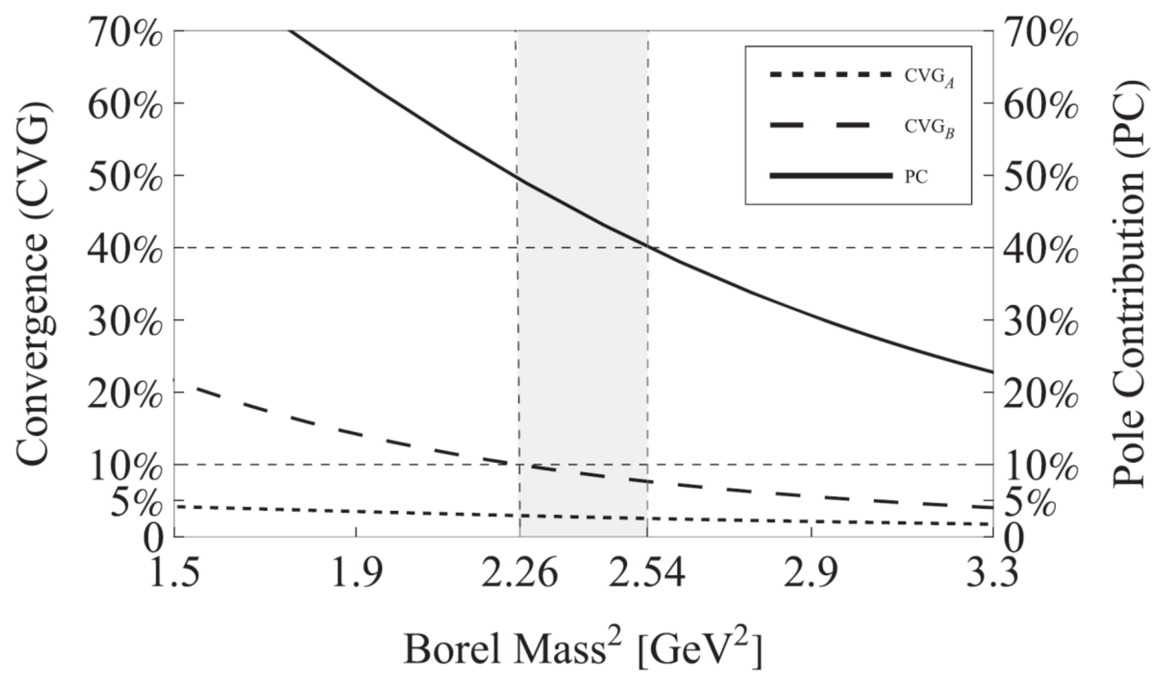
$$J^{\mu}_{1^{-+}} = \bar{s}_a \lambda_n^{ab} \gamma_\nu s_b \, g_s G_n^{\mu\nu}$$

$$\Pi^{\mu}_{1^{-+}}(M_B^2, s_0)$$
$$= \int_{4m_s^2}^{s_0} \Big(\frac{s^3 \alpha_s}{60\pi^3} - \frac{m_s^2 s^2 \alpha_s}{3\pi^3} + s\Big(\frac{\langle \alpha_s G}{36}$$
$$+ \frac{13\langle \alpha_s GG\rangle \alpha_s}{432\pi^3} + \frac{8 m_s \langle \bar{s}s\rangle \alpha_s}{9\pi}\Big)$$
$$- \frac{3\langle \alpha_s GG\rangle m_s^2 \alpha_s}{64\pi^3} - \frac{3 m_s \langle g_s \bar{s}\sigma G}{4\pi}$$
$$+ \Big(\frac{\langle \alpha_s GG\rangle^2}{3456\pi^2} - \frac{\langle g_s^3 G^3\rangle m_s^2}{16\pi^2} - \frac{2}{9}\langle \alpha$$
$$+ \frac{11\pi \langle \bar{s}s\rangle \langle g_s \bar{s}\sigma G s\rangle \alpha_s}{9}\Big).$$

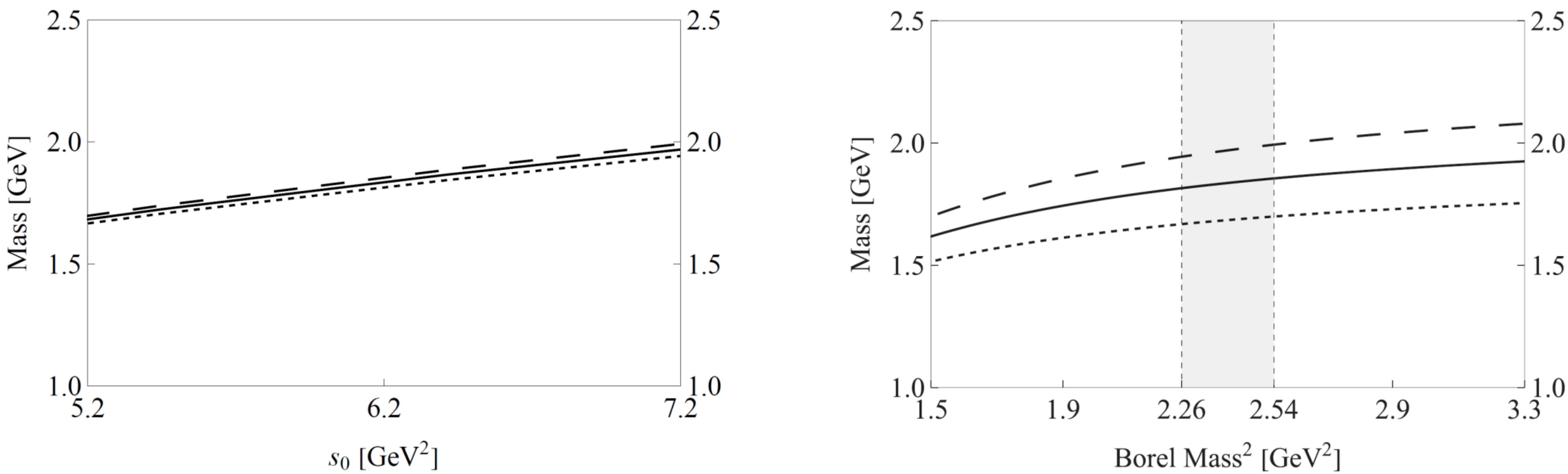
$$\mathrm{CVG}_A \equiv \left| \frac{\Pi^{g_s^{n=4}}(\infty, M_B^2)}{\Pi(\infty, M_B^2)} \right| \le 5\%$$

$$\mathrm{CVG}_B \equiv \left| \frac{\Pi^{\mathrm{D}=6+8}(\infty, M_B^2)}{\Pi(\infty, M_B^2)} \right| \le 10\%$$

$$\mathrm{PC} \equiv \left| \frac{\Pi(s_0, M_B^2)}{\Pi(\infty, M_B^2)} \right| \ge 40\%$$



QCD sum rule results



$$M_{\left|\bar{s}sg;1^{-+}\right\rangle} = 1.84^{+0.14}_{-0.15}\ \mathrm{GeV}$$

$$f_{\left|\bar{s}sg;1^{-+}\right\rangle} = 0.30^{+0.06}_{-0.06}\ \mathrm{GeV}^4$$

QCD sum rule results: Single-gluon hybrid states

State $[J^{PC}]$	Current	s_0^{min} [GeV^2]	Working Regions M_B^2 [GeV^2]	Working Regions s_0 [GeV^2]	Pole [%]	Mass [GeV]	Decay Constant
$\vert\bar{q}gq;1^{--}\rangle$	$J^{\alpha\beta}_{1--}$	4.2	2.03–2.48	5.5	40–54	$1.80^{+0.13}_{-0.16}$	$0.051^{+0.004}_{-0.004}$ GeV^3
$\vert\bar{q}gq;1^{+-}\rangle$	$\tilde{J}^{\alpha\beta}_{1+-}$	16.2	3.61–4.58	18.0	40–53	$4.05^{+0.24}_{-0.12}$	$0.063^{+0.020}_{-0.020}$ GeV^3
$\vert\bar{q}gq;1^{+-}\rangle$	$J^{\alpha\beta}_{1+-}$	5.0	2.29–2.45	5.5	40–45	$1.84^{+0.12}_{-0.14}$	$0.049^{+0.004}_{-0.004}$ GeV^3
$\vert\bar{q}gq;1^{--}\rangle$	$\tilde{J}^{\alpha\beta}_{1--}$	16.3	3.52–4.56	18.0	40–53	$4.09^{+0.29}_{-0.14}$	$0.064^{+0.021}_{-0.020}$ GeV^3
$\vert\bar{q}gq;0^{++}\rangle$	$J^{\mu\to 0}_{1-+}$	20.6	5.11–6.59	24.0	40–56	$4.45^{+0.22}_{-0.17}$	$0.124^{+0.032}_{-0.036}$ GeV^3
$\vert\bar{q}gq;0^{-+}\rangle$	$\tilde{J}^{\mu\to 0}_{1++}$	7.7	3.58–3.81	8.5	40–45	$2.14^{+0.17}_{-0.19}$	$0.105^{+0.005}_{-0.004}$ GeV^3
$\vert\bar{q}gq;0^{--}\rangle$	$J^{\mu\to 0}_{1+-}$	21.6	5.48–6.52	24.0	40–50	$4.49^{+0.21}_{-0.14}$	$0.123^{+0.032}_{-0.037}$ GeV^3
$\vert\bar{q}gq;0^{+-}\rangle$	$\tilde{J}^{\mu\to 0}_{1--}$	7.1	3.32–3.73	8.5	40–49	$2.16^{+0.16}_{-0.19}$	$0.100^{+0.005}_{-0.005}$ GeV^3
$\vert\bar{q}gq;1^{-+}\rangle$	J^{μ}_{1-+}	4.8	2.19–2.28	5.2	40–43	$1.67^{+0.15}_{-0.17}$	$0.243^{+0.057}_{-0.052}$ GeV^4
$\vert\bar{q}gq;1^{++}\rangle$	$\tilde{J}^{\mu}_{1++}$	13.8	3.59–4.10	15.0	40–48	$3.54^{+0.16}_{-0.12}$	$1.370^{+0.494}_{-0.450}$ GeV^4
$\vert\bar{q}gq;1^{+-}\rangle$	J^{μ}_{1+-}	4.6	2.10–2.27	5.2	40–46	$1.68^{+0.14}_{-0.16}$	$0.242^{+0.055}_{-0.051}$ GeV^4
$\vert\bar{q}gq;1^{--}\rangle$	$\tilde{J}^{\mu}_{1--}$	13.7	3.57–4.10	15.0	40–49	$3.53^{+0.16}_{-0.12}$	$1.366^{+0.493}_{-0.450}$ GeV^4
$\vert\bar{q}gq;0^{++}\rangle$	J_{0++}	11.1	3.48–3.91	12.5	40–49	$2.94^{+0.20}_{-0.25}$	$2.893^{+1.029}_{-0.948}$ GeV^4
$\vert\bar{q}gq;0^{-+}\rangle$	J_{0-+}	11.1	3.47–3.92	12.5	40–49	$2.93^{+0.20}_{-0.25}$	$2.882^{+1.026}_{-0.945}$ GeV^4
$\vert\bar{q}gq;1^{++}\rangle$	$J^{\alpha\beta}_{1++}$	5.8	1.84–2.06	6.5	40–48	$2.11^{+0.17}_{-0.21}$	$0.056^{+0.012}_{-0.013}$ GeV^3
$\vert\bar{q}gq;1^{-+}\rangle$	$\tilde{J}^{\alpha\beta}_{1-+}$	5.5	1.81–2.00	6.2	40–48	$2.00^{+0.13}_{-0.16}$	$0.055^{+0.007}_{-0.008}$ GeV^3
$\vert\bar{q}gq;1^{-+}\rangle$	$J^{\alpha\beta}_{1-+}$	5.5	1.81–2.00	6.2	40–48	$2.00^{+0.13}_{-0.16}$	$0.055^{+0.007}_{-0.008}$ GeV^3
$\vert\bar{q}gq;1^{++}\rangle$	$\tilde{J}^{\alpha\beta}_{1++}$	5.8	1.84–2.06	6.5	40–48	$2.11^{+0.17}_{-0.21}$	$0.056^{+0.012}_{-0.013}$ GeV^3

QCD sum rule results: Double-gluon hybrid states

State $[J^{PC}]$	Current	s_0^{min} [GeV^2]	Working Regions M_B^2 [GeV^2]	Working Regions s_0 [GeV^2]	Pole [%]	Mass [GeV]
$\lvert\bar{q}qgg;0^{++}\rangle$	J_{0++}	34.9	6.12–6.92	38 ± 8.0	40–50	$5.61^{+0.29}_{-0.27}$
$\lvert\bar{q}qgg;0^{-+}\rangle$	J_{0-+}	24.4	5.34–5.78	27 ± 5.0	40–48	$4.25^{+0.32}_{-0.39}$
$\lvert\bar{q}qgg;1^{+-}\rangle$	$J_{1+-}^{\alpha\beta}$	32.1	5.51–6.31	35 ± 7.0	40–50	$5.46^{+0.25}_{-0.18}$
$\lvert\bar{q}qgg;1^{--}\rangle$	$J_{1--}^{\alpha\beta}$	20.0	4.60–4.91	22 ± 4.0	40–47	$3.74^{+0.30}_{-0.35}$
$\lvert\bar{q}qgg;2^{++}\rangle$	$J_{2++}^{\alpha_1\beta_1,\alpha_2\beta_2}$	20.0	5.39–5.76	22 ± 4.0	40–46	$3.74^{+0.27}_{-0.32}$
$\lvert\bar{q}qgg;2^{+-}\rangle$	$J_{2+-}^{\alpha_1\beta_1,\alpha_2\beta_2}$	6.4	1.61–1.78	7 ± 2.0	40–48	$2.26^{+0.20}_{-0.25}$
$\lvert\bar{q}qgg;2^{-+}\rangle$	$J_{2-+}^{\alpha_1\beta_1,\alpha_2\beta_2}$	16.8	4.39–4.81	19 ± 4.0	40–49	$3.51^{+0.29}_{-0.35}$
$\lvert\bar{s}sgg;0^{++}\rangle$	J_{0++}	35.3	6.22–7.61	41 ± 8.0	40–57	$5.72^{+0.29}_{-0.32}$
$\lvert\bar{s}sgg;0^{-+}\rangle$	J_{0-+}	24.5	5.36–5.95	28 ± 6.0	40–50	$4.34^{+0.36}_{-0.46}$
$\lvert\bar{s}sgg;1^{+-}\rangle$	$J_{1+-}^{\alpha\beta}$	32.5	5.60–6.79	37 ± 8.0	40–55	$5.52^{+0.29}_{-0.27}$
$\lvert\bar{s}sgg;1^{--}\rangle$	$J_{1--}^{\alpha\beta}$	20.2	4.62–5.07	23 ± 5.0	40–50	$3.84^{+0.35}_{-0.44}$
$\lvert\bar{s}sgg;2^{++}\rangle$	$J_{2++}^{\alpha_1\beta_1,\alpha_2\beta_2}$	20.4	5.45–6.11	24 ± 5.0	40–51	$3.91^{+0.32}_{-0.39}$
$\lvert\bar{s}sgg;2^{+-}\rangle$	$J_{2+-}^{\alpha_1\beta_1,\alpha_2\beta_2}$	7.1	1.79–2.01	8 ± 2.0	40–50	$2.38^{+0.19}_{-0.25}$
$\lvert\bar{s}sgg;2^{-+}\rangle$	$J_{2-+}^{\alpha_1\beta_1,\alpha_2\beta_2}$	17.1	4.44–5.00	20 ± 4.0	40–51	$3.61^{+0.28}_{-0.34}$

QCD sum rule results: Two- and three-gluon glueballs

Glueball	Current	s_0^{min} [GeV^2]	Working Regions		Pole [%]	Mass [GeV]
			s_0 [GeV^2]	M_B^2 [GeV^2]		
$\lvert GG; 0^{++}\rangle$	J_0	7.8	9.0 ± 1.0	3.70–4.19	40–48	$1.78^{+0.14}_{-0.17}$
$\lvert GG; 2^{++}\rangle$	$J_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	8.5	10.0 ± 1.0	3.99–4.60	40–50	$1.86^{+0.14}_{-0.17}$
$\lvert GG; 0^{-+}\rangle$	$\tilde{J}_0$	8.2	9.0 ± 1.0	3.28–3.70	40–47	$2.17^{+0.11}_{-0.11}$
$\lvert GG; 2^{-+}\rangle$	$\tilde{J}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	8.1	10.0 ± 1.0	3.27–4.20	40–55	$2.24^{+0.11}_{-0.11}$
$\lvert GGG; 0^{++}\rangle$	η_0	31.6	33.0 ± 3.0	7.25–7.61	40–44	$4.46^{+0.17}_{-0.19}$
$\lvert GGG; 2^{++}\rangle$	$\eta_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	16.0	35.0 ± 3.0	4.77–9.04	40–90	$4.18^{+0.19}_{-0.42}$
$\lvert GGG; 0^{-+}\rangle$	$\tilde{\eta}_0$	17.0	33.0 ± 3.0	4.48–8.13	40–88	$4.13^{+0.18}_{-0.36}$
$\lvert GGG; 2^{-+}\rangle$	$\tilde{\eta}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	33.1	35.0 ± 3.0	8.10–8.53	40–44	$4.29^{+0.20}_{-0.22}$
$\lvert GGG; 1^{+-}\rangle$	$\xi_1^{\alpha\beta}$	9.0	34.0 ± 4.0	3.16–9.09	40–99	$4.01^{+0.26}_{-0.95}$
$\lvert GGG; 2^{+-}\rangle$	$\xi_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	32.7	35.0 ± 4.0	7.53–8.09	40–46	$4.42^{+0.24}_{-0.29}$
$\lvert GGG; 3^{+-}\rangle$	$\xi_3^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}$	30.2	33.0 ± 4.0	7.69–8.40	40–47	$4.30^{+0.23}_{-0.26}$
$\lvert GGG; 1^{--}\rangle$	$\tilde{\xi}_1^{\alpha\beta}$	31.2	34.0 ± 4.0	5.81–6.77	40–51	$4.91^{+0.20}_{-0.18}$
$\lvert GGG; 2^{--}\rangle$	$\tilde{\xi}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	19.7	36.0 ± 4.0	5.80–9.47	40–81	$4.25^{+0.22}_{-0.33}$
$\lvert GGG; 3^{--}\rangle$	$\tilde{\xi}_3^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}$	35.8	38.0 ± 4.0	6.15–7.22	40–49	$5.59^{+0.33}_{-0.22}$

QCD sum rule results

Lattice QCD results

Glueball	QCD sum rules	Ref. [11] (quenched)	Ref. [12] (quenched)	Ref. [13] (quenched)	Ref. [14] (unquenched)
$\|GG;0^{++}\rangle$	$1.78^{+0.14}_{-0.17}$	$1.71 \pm 0.05 \pm 0.08$	$1.73 \pm 0.05 \pm 0.08$	$1.48 \pm 0.03 \pm 0.07$	1.80 ± 0.06
$\|GG;2^{++}\rangle$	$1.86^{+0.14}_{-0.17}$	$2.39 \pm 0.03 \pm 0.12$	$2.40 \pm 0.03 \pm 0.12$	$2.15 \pm 0.03 \pm 0.10$	2.62 ± 0.05
$\|GG;0^{-+}\rangle$	$2.17^{+0.11}_{-0.11}$	$2.56 \pm 0.04 \pm 0.12$	$2.59 \pm 0.04 \pm 0.13$	$2.25 \pm 0.06 \pm 0.10$	–
$\|GG;2^{-+}\rangle$	$2.24^{+0.11}_{-0.11}$	$3.04 \pm 0.04 \pm 0.15$	$3.10 \pm 0.03 \pm 0.15$	$2.78 \pm 0.05 \pm 0.13$	3.46 ± 0.32
$\|GGG;0^{++}\rangle$	$4.46^{+0.17}_{-0.19}$	–	$2.67 \pm 0.18 \pm 0.13$	$2.76 \pm 0.03 \pm 0.12$	3.76 ± 0.24
$\|GGG;2^{++}\rangle$	$4.18^{+0.19}_{-0.42}$	–	–	$2.88 \pm 0.10 \pm 0.1$[illegible]	–
$\|GGG;0^{-+}\rangle$	$4.13^{+0.18}_{-0.36}$	–	$3.64 \pm 0.06 \pm 0.18$	$3.37 \pm 0.15 \pm 0.15$	4.49 ± 0.59
$\|GGG;2^{-+}\rangle$	$4.29^{+0.20}_{-0.22}$	–	–	$3.48 \pm 0.14 \pm 0.16$	–
$\|GGG;1^{+-}\rangle$	$4.01^{+0.26}_{-0.95}$	$2.98 \pm 0.03 \pm 0.14$	$2.94 \pm 0.03 \pm 0.14$	$2.67 \pm 0.07 \pm 0.12$	3.27 ± 0.34
$\|GGG;2^{+-}\rangle$	$4.42^{+0.24}_{-0.29}$	$4.23 \pm 0.05 \pm 0.20$	$4.14 \pm 0.05 \pm 0.20$	–	–
$\|GGG;3^{+-}\rangle$	$4.30^{+0.23}_{-0.26}$	$3.60 \pm 0.04 \pm 0.17$	$3.55 \pm 0.04 \pm 0.17$	$3.27 \pm 0.09 \pm 0.15$	3.85 ± 0.35
$\|GGG;1^{--}\rangle$	$4.91^{+0.20}_{-0.18}$	$3.83 \pm 0.04 \pm 0.19$	$3.85 \pm 0.05 \pm 0.19$	$3.24 \pm 0.33 \pm 0.15$	–
$\|GGG;2^{--}\rangle$	$4.25^{+0.22}_{-0.33}$	$4.01 \pm 0.05 \pm 0.20$	$3.93 \pm 0.04 \pm 0.19$	$3.66 \pm 0.13 \pm 0.17$	4.59 ± 0.74
$\|GGG;3^{--}\rangle$	$5.59^{+0.33}_{-0.22}$	$4.20 \pm 0.05 \pm 0.20$	$4.13 \pm 0.09 \pm 0.20$	$4.33 \pm 0.26 \pm 0.20$	–

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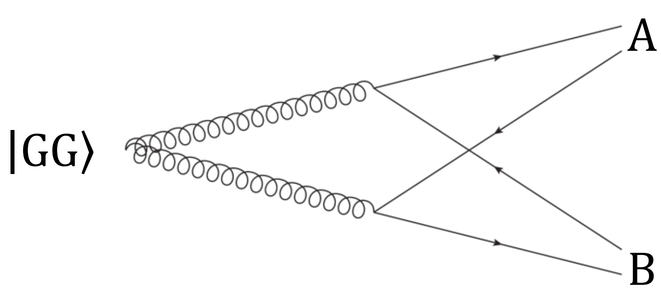
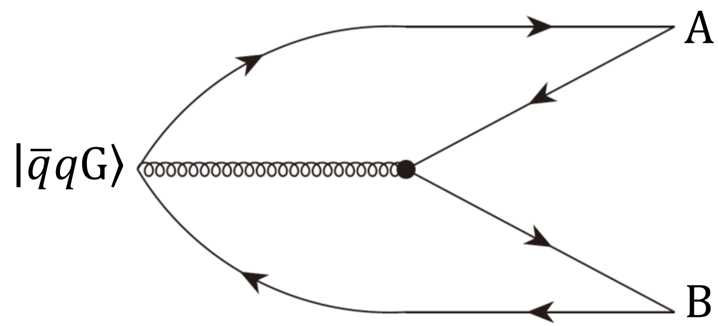
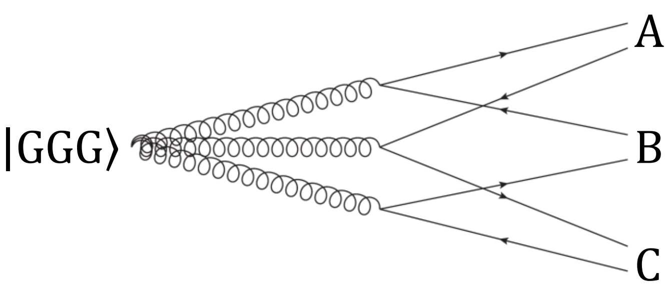
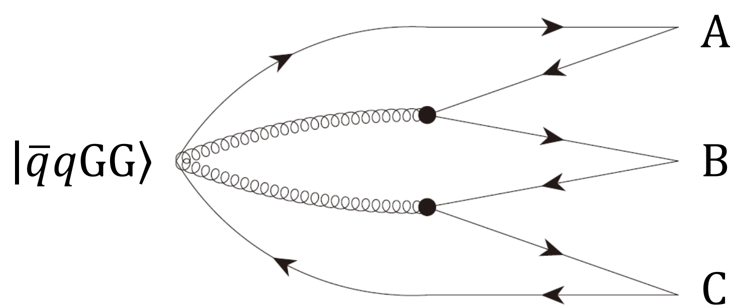
- Present status of hadron spectroscopy
- QCD sum rule studies on hybrid states
- **Decay analyses on hybrid states**
- Studies on the $J^{PC} = 1^{-+}$ hybrid states

Decay analyses

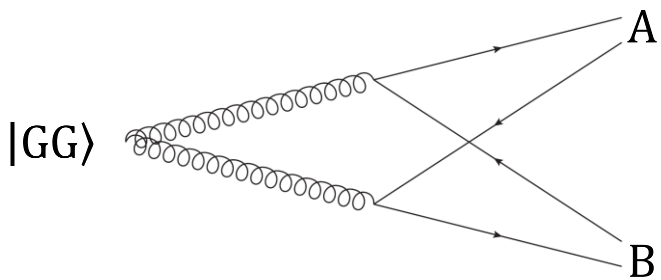


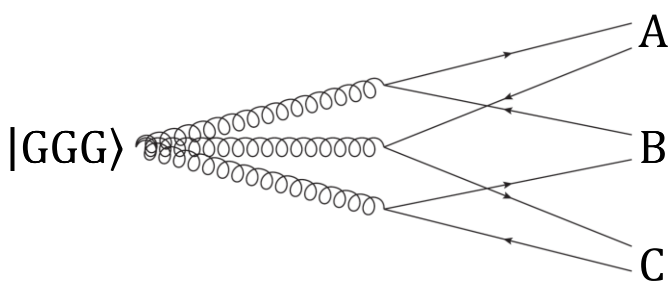






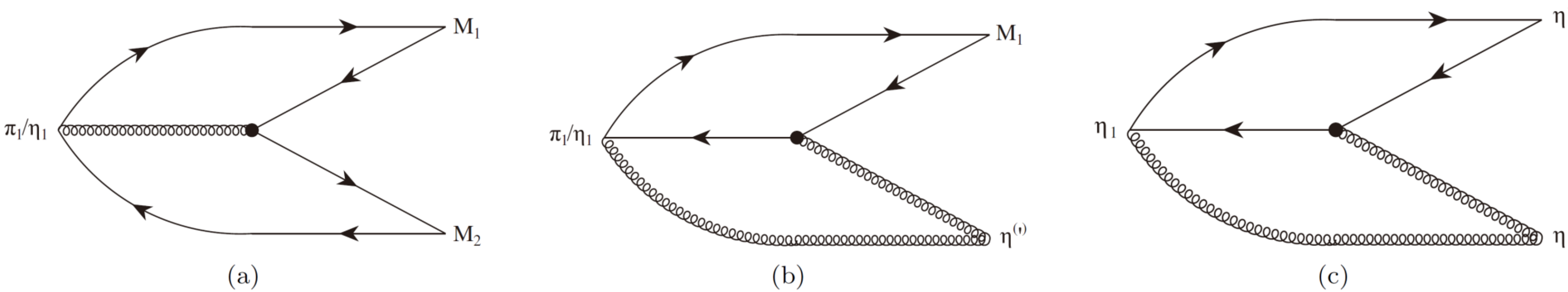
Decay analyses – Glueball





0^{-+}	$\rightarrow$	VVP, VVV	(S-wave)
0^{++}	$\rightarrow$	VPP, VVP, VVV	(P-wave)
1^{--}	$\rightarrow$	VPP, VVP, VVV	(S-wave)
1^{+-}	$\rightarrow$	PPP, VPP, VVP, VVV	(P-wave)
$2^{-\pm}$	$\rightarrow$	VVP, VVV	(S-wave)
$2^{+\pm}$	$\rightarrow$	VPP, VVP, VVV	(P-wave)
3^{--}	$\rightarrow$	VVV	(S-wave)
3^{+-}	$\rightarrow$	VVP, VVV	(P-wave)

Decay analyses – Hybrid



Normal process

Abnormal processes due to the QCD axial anomaly

The QCD axial anomaly ensures the $\eta\eta'$ decay mode to be a characteristic signal of the hybrid nature of the $\eta_1(1855)$.

Decay results – $J^{PC} = 1^{-+}$ Hybrid

Channel	$\lvert \bar{q}qg; 1^- 1^{-+} \rangle$ $M = 1.67^{+0.15}_{-0.17}$ GeV	$\lvert \bar{q}qg; 0^+ 1^{-+} \rangle$ $M = 1.67^{+0.15}_{-0.17}$ GeV	$\lvert \bar{s}sg; 0^+ 1^{-+} \rangle$ $M = 1.84^{+0.14}_{-0.15}$ GeV
$\pi_1/\eta_1 \to \rho\pi$	242^{+310}_{-179}	–	–
$\pi_1/\eta_1 \to b_1(1235)\pi$	$14.5^{+25.9}_{-13.9}$	–	–
$\pi_1/\eta_1 \to f_1(1285)\pi$	$35.9^{+53.9}_{-36.4}$	–	–
$\pi_1/\eta_1 \to \eta\pi$	$2.3^{+2.5}_{-1.2}$	–	–
$\pi_1/\eta_1 \xrightarrow{b} \eta\pi$	$57.8^{+65.0}_{-31.4}$	–	–
$\pi_1/\eta_1 \to \eta'\pi$	$0.43^{+0.50}_{-0.28}$	–	–
$\pi_1/\eta_1 \xrightarrow{c} \eta'\pi$	149^{+162}_{-78}	–	–
$\pi_1/\eta_1 \to a_1(1260)\pi$	–	$79.5^{+112.4}_{-74.9}$	–
$\pi_1/\eta_1 \xrightarrow{a} \eta\eta'$	–	$0.07^{+0.12}_{-0.07}$	$0.93^{+1.04}_{-0.69}$
$\pi_1/\eta_1 \xrightarrow{b} \eta\eta'$	–	$1.62^{+2.13}_{-1.61}$	$1.64^{+1.51}_{-1.01}$
$\pi_1/\eta_1 \xrightarrow{c} \eta\eta'$	–	$11.5^{+11.7}_{-11.5}$	$5.0^{+4.6}_{-3.1}$
$\pi_1/\eta_1 \to K^*(892)\bar{K} + c.c.$	$25.3^{+34.7}_{-24.7}$	$25.3^{+34.7}_{-24.7}$	$73.9^{+85.7}_{-58.0}$
$\pi_1/\eta_1 \to K_1(1270)\bar{K} + c.c.$	–	–	$14.6^{+19.8}_{-14.6}$
$\pi_1/\eta_1 \to K^*(892)\bar{K}^*(892)$	–	–	$0.08^{+0.39}_{-0.08}$
Sum	530^{+540}_{-330}	120^{+160}_{-110}	100^{+110}_{-80}

Contents

- Present status of hadron spectroscopy
- QCD sum rule studies on hybrid states
- Decay analyses on hybrid states
- **Studies on the $J^{PC} = 1^{-+}$ hybrid states**

$J^{PC} = 1^{-+}$ Hybrid

Experiments

$I^G J^{PC} = 1^- 1^{-+}$: $\pi_1(1400)$ $\pi_1(1600)$ $\pi_1(2015)$

$I^G J^{PC} = 0^+ 1^{-+}$: $\eta_1(1855)$

$J^{PC} = 1^{-+}$ Hybrid

Experiments

$I^G J^{PC} = 1^- 1^{-+}$: $\pi_1(1400)$ $\pi_1(1600)$ $\pi_1(2015)$

?

$\pi_1(1600)$

COMPASS: $M = 1564 \pm 24 \pm 86$ MeV
$\Gamma = 492 \pm 54 \pm 102$ MeV

$I^G J^{PC} = 0^+ 1^{-+}$: $\eta_1(1855)$

BESIII: $M = 1855 \pm 9^{+6}_{-1}$ MeV
$\Gamma = 188 \pm 18^{+3}_{-8}$ MeV

$J^{PC} = 1^{-+}$ Hybrid

Experiments

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$\pi_1(1600)$

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$\Gamma = 188 \pm 18^{+3}_{-8}$ MeV

Theory ($q = up/down, s = strange$)

Hybrid picture:

- **One isovector: $\bar{q}qg$**
- **Two isosinglets: $\bar{q}qg$, $\bar{s}sg$**

Compact tetraquark picture:

- Two isovectors: $\bar{q}\bar{q}qq$, $\bar{q}\bar{s}qs$
- Three isosinglets: $\bar{q}\bar{q}qq$, $\bar{q}\bar{s}qs$, $\bar{s}\bar{s}ss$

Hadronic molecular picture:

- Maybe not as many states as the tetraquark picture
- Near thresholds
- Widths are possibly limited

$J^{PC} = 1^{-+}$ Hybrid

Experiments

$I^G J^{PC} = 1^- 1^{-+}$: $\pi_1(1400)$ $\pi_1(1600)$ $\pi_1(2015)$

?

$\pi_1(1600)$

COMPASS: $M = 1564 \pm 24 \pm 86$ MeV
$\Gamma = 492 \pm 54 \pm 102$ MeV

$I^G J^{PC} = 0^+ 1^{-+}$: $\eta_1(1855)$

BESIII: $M = 1855 \pm 9^{+6}_{-1}$ MeV
$\Gamma = 188 \pm 18^{+3}_{-8}$ MeV

Indistinguishable!

Theory ($q = up/down$, $s = strange$)

$|\bar{q}qg; 1^- 1^{-+}\rangle$: $\mathrm{M} = 1670^{+150}_{-170}$ MeV
$\Gamma = 530^{+540}_{-330}$ MeV

$|\bar{q}qg; 0^+ 1^{-+}\rangle$: $\mathrm{M} = 1670^{+150}_{-170}$ MeV
$\Gamma = 120^{+160}_{-110}$ MeV

$|\bar{s}sg; 0^+ 1^{-+}\rangle$: $\mathrm{M} = 1840^{+140}_{-150}$ MeV
$\Gamma = 100^{+110}_{-80}$ MeV

Hybrid picture:

- One isovector: $\bar{q}qg$
- Two isosinglets: $\bar{q}qg$, $\bar{s}sg$

Decay results – $J^{PC} = 1^{-+}$ Hybrid

Channel	$\vert\bar{q}qg; 1^- 1^{-+}\rangle$ $M = 1.67^{+0.15}_{-0.17}$ GeV	$\vert\bar{q}qg; 0^+ 1^{-+}\rangle$ $M = 1.67^{+0.15}_{-0.17}$ GeV	$\vert\bar{s}sg; 0^+ 1^{-+}\rangle$ $M = 1.84^{+0.14}_{-0.15}$ GeV
$\pi_1/\eta_1 \to \rho\pi$	242^{+310}_{-179}	–	–
$\pi_1/\eta_1 \to b_1(1235)\pi$	$14.5^{+25.9}_{-13.9}$	–	–
$\pi_1/\eta_1 \to f_1(1285)\pi$	$35.9^{+53.9}_{-36.4}$	–	–
$\pi_1/\eta_1 \to \eta\pi$	$2.3^{+2.5}_{-1.2}$	–	–
$\pi_1/\eta_1 \xrightarrow{b} \eta\pi$	$57.8^{+65.0}_{-31.4}$	–	–
$\pi_1/\eta_1 \to \eta'\pi$	$0.43^{+0.50}_{-0.28}$	–	–
$\pi_1/\eta_1 \xrightarrow{c} \eta'\pi$	149^{+162}_{-78}	–	–
$\pi_1/\eta_1 \to a_1(1260)\pi$	–	$79.5^{+112.4}_{-74.9}$	–
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Summary

- **We systematically study two- and three-gluon glueballs as well as single-gluon and double-gluon hybrid states through QCD sum rule method.**
- **The $J^{PC} = 1^{-+}$ hybrid states are still of particular interest. We propose to investigate the $a_1(1260)\pi$ channel in future experiments to further understand $\eta_1(1855)$.**

Thank you very much!