Hyperon-Nucleon Interaction from Lattice QCD:

some preliminary analyses

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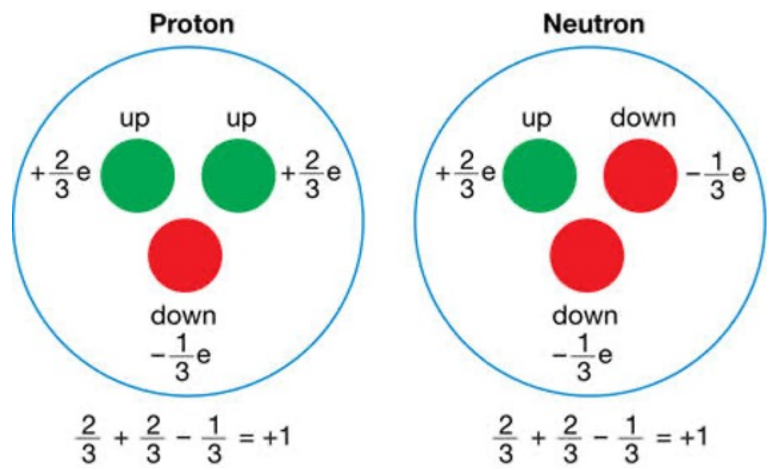
2024超级陶粲装置研讨会

Outline

- Motivation: Hyperon-nucleon interactions
- $p - \Lambda$ scattering from the HALQCD approach
- $p - \Lambda$ scattering from the Lüscher′s finite volume method
- Prospect

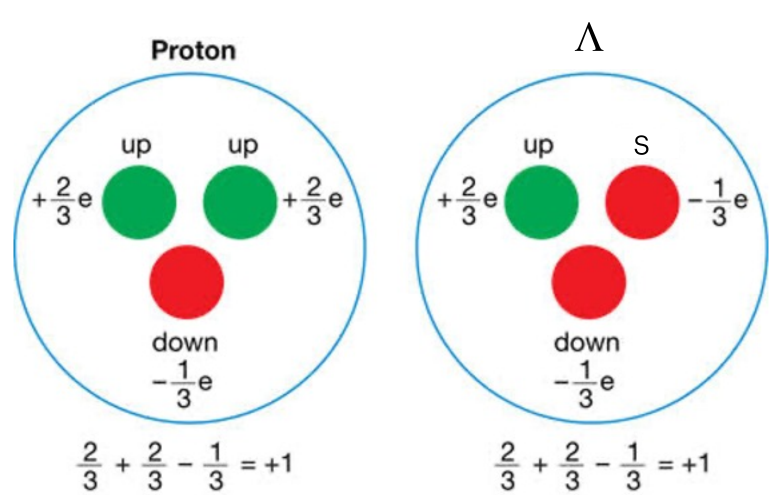
Motivation

- Nucleon-nucleon interactions



✓ hadronic molecule?
✓ multiquark state?
✓ binding energy?

- Hyperon-Nucleon interactions

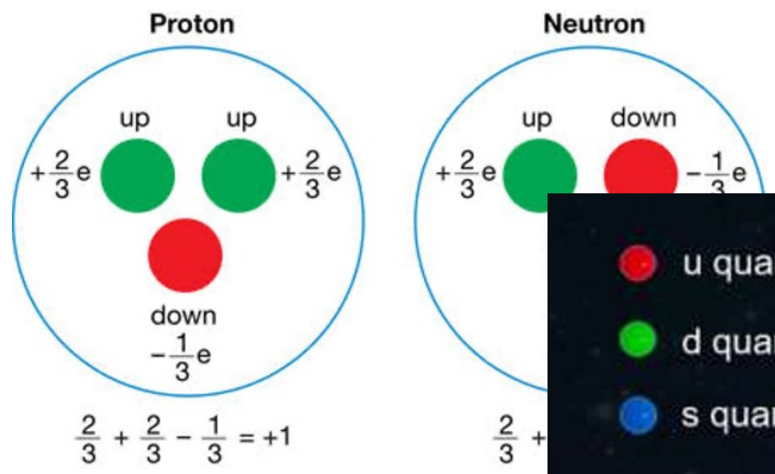


Also concerned

✓ hadronic molecule?
✓ multiquark state?
✓ binding energy?
✓ "hyperon puzzle" in neutron stars

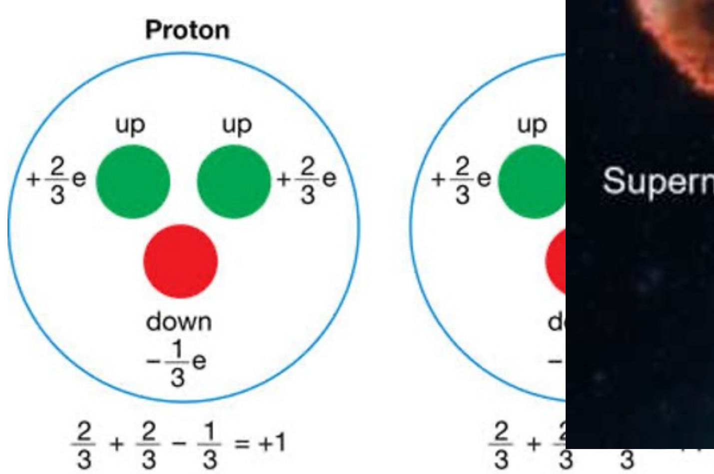
Motivation

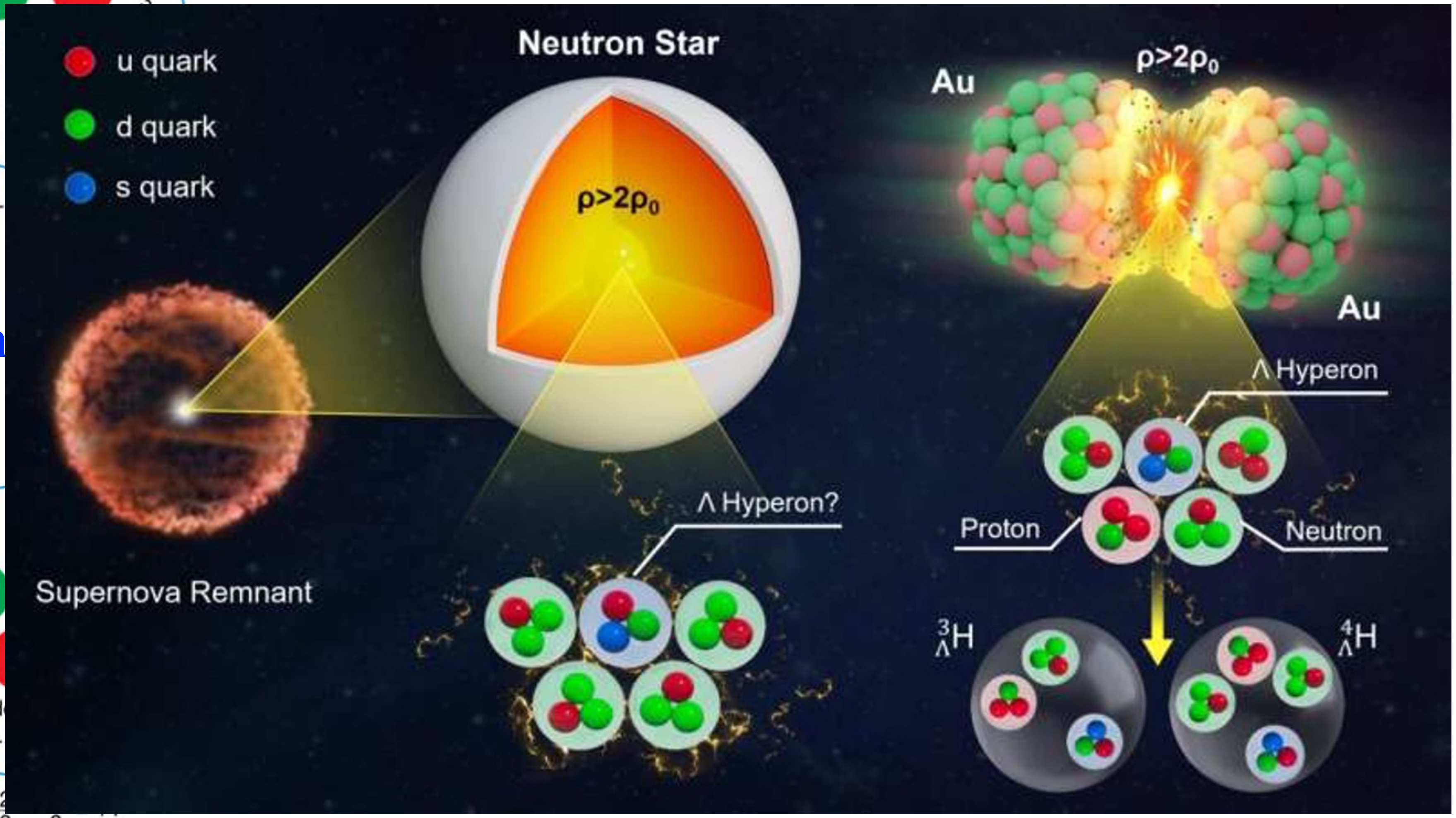
- Nucleon-nucleon interactions



- ✓ hadronic molecule?
- ✓ multiquark state?

- Hyperon-Nucleon in

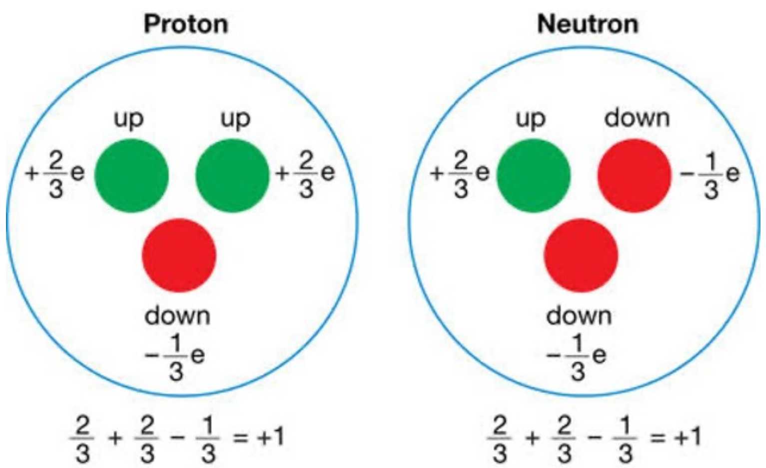




From Karen McNulty Walsh, Brookhaven National Laboratory

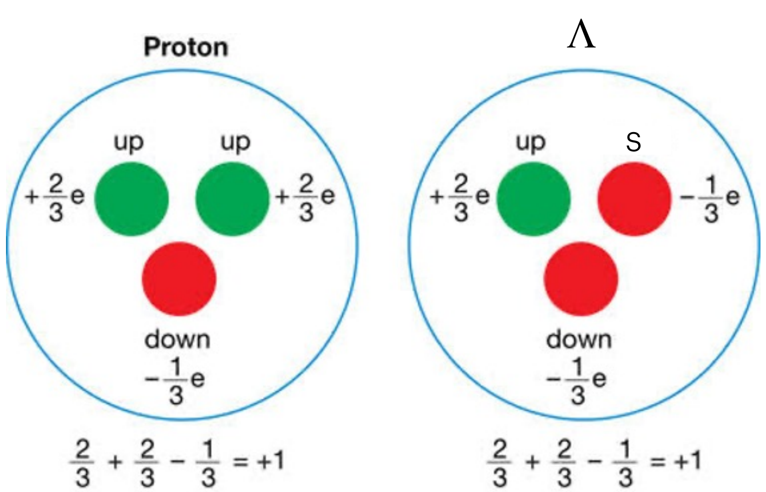
Motivation

- Nucleon-Nucleon interactions



✓ 强子分子态?
✓ 多夸克态?
✓ 结合能?

- Hyperon-Nucleon interactions



Hep-ex:

✓ YN correlation functions in heavy-ion collisions:
- J. Adams et al. [STAR Collaboration], Phys. Rev. C 74, 064906 (2006)
- J. Adam et al. [STAR Collaboration], Phys. Lett. B 790, 490 (2019)
- S. Acharya et al. [ALICE Collaboration], Phys. Rev. Lett. 123, 112002 (2019)
- S. Acharya et al. [ALICE Collaboration], Nature 588, 232 (2020)

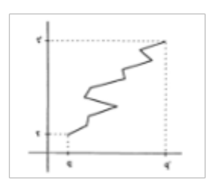
✓ hypernuclei:
- [J-PARC E07 Collaboration], Phys. Rev. Lett. 126, 062501 (2021)

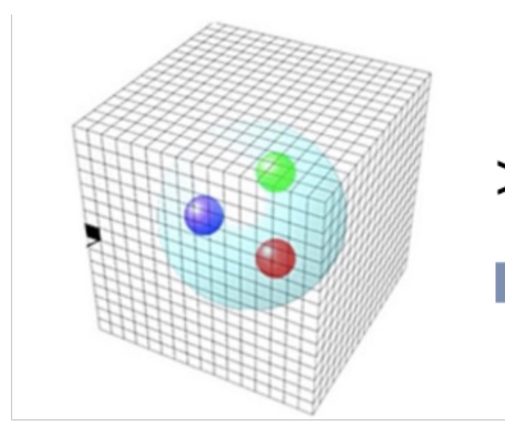
✓ YN scattering:
- G. Alexander, et al. Phys. Rev. 173, 1452 (1968)
- B. Sechi-Zorn, et al. Phys. Rev. 175, 1735 (1968)
- J. A. Kadyk, et al. Nucl. Phys. B 27, 13 (1971)
- BESIII Collaboration,PhysRevLett.132.231902(2024)

Lattice QCD

- 描述强相互作用的理论被称为量子色动力学QCD。
- 当能标降低时，相互作用增强，QCD进入到非微扰区域。
- 格点量子色动力学(Wilson, 1974)：从第一性原理出发的非微扰方法

K. G. Wilson





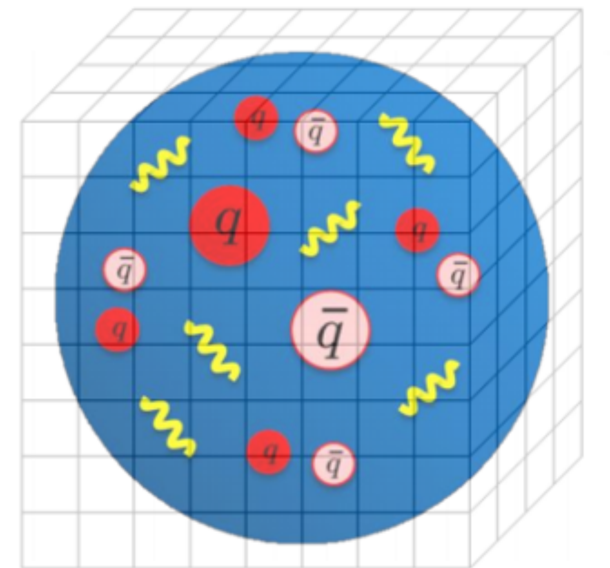
格点QCD

$> 2^{1000000}$

超级计算机

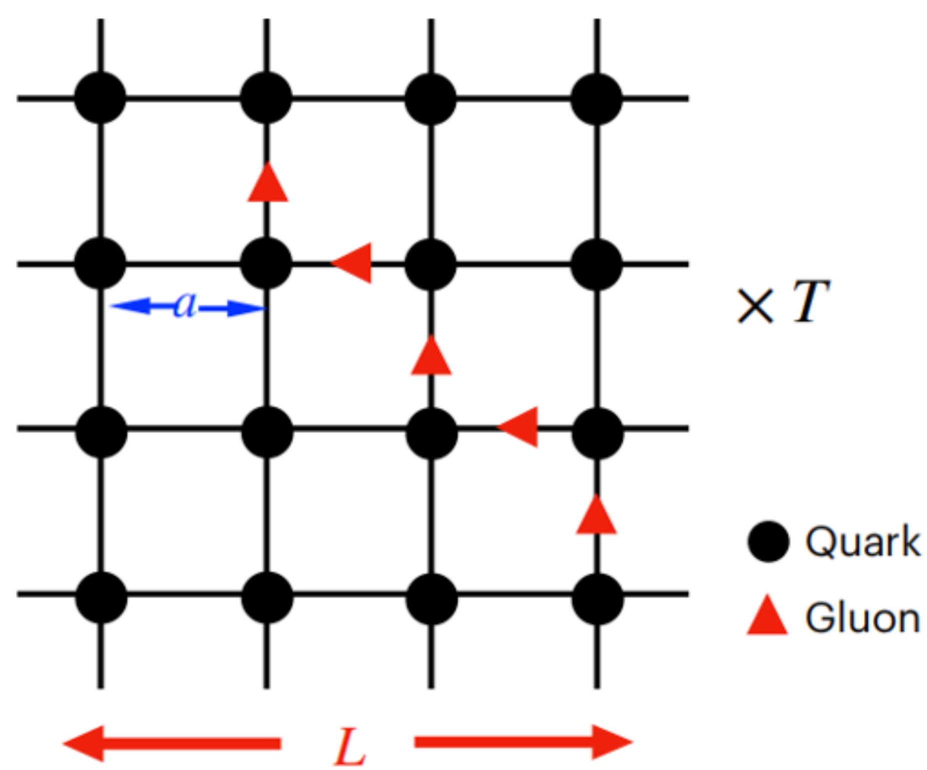
Lattice QCD

$$S_E^{latt} = \underbrace{\sum \frac{6}{g^2} ReTr(1 - U_P)}_{\text{wilson gauge action}} + \underbrace{\sum_q \bar{q}(D_\mu^{lat}\gamma_\mu + am_q)q}_{\text{lattice fermion action}}$$



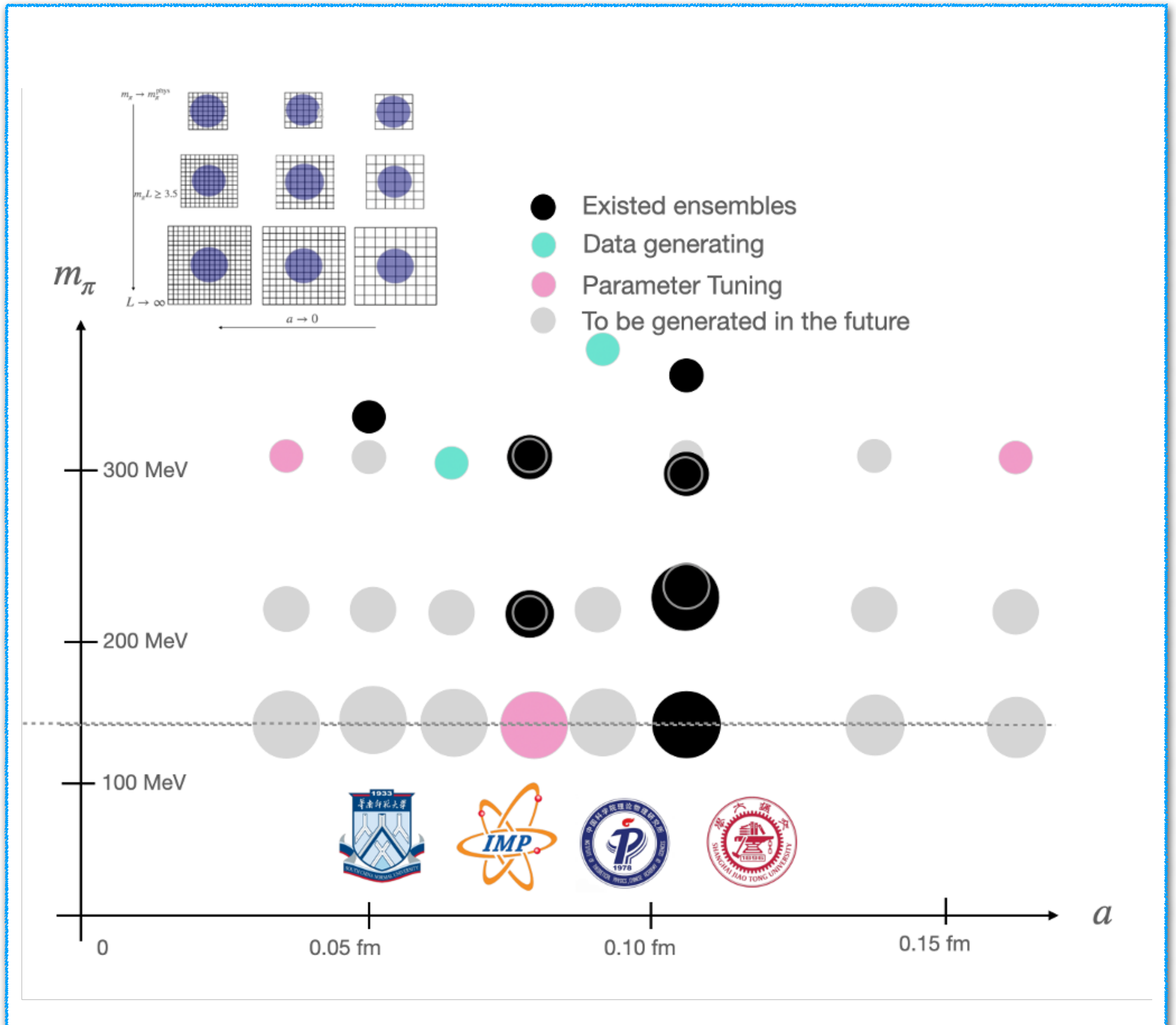
- Quark on discrete lattice:
  consider both IR and UV effects:

$$m_\pi L \gtrsim 4, \quad \text{and} \quad a^{-1} \gg \text{mass scale}$$

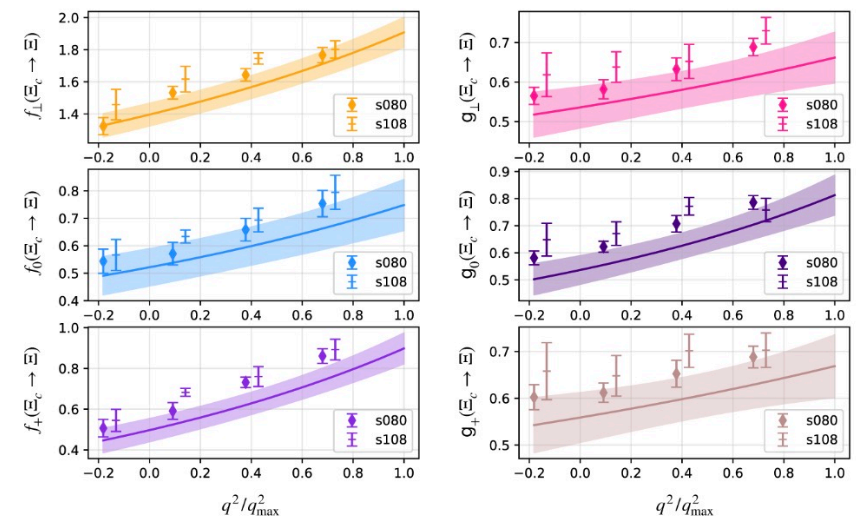


New Lattice QCD configurations

CLQCD



Hu, et.al., PRD 109, 054507 (2024)



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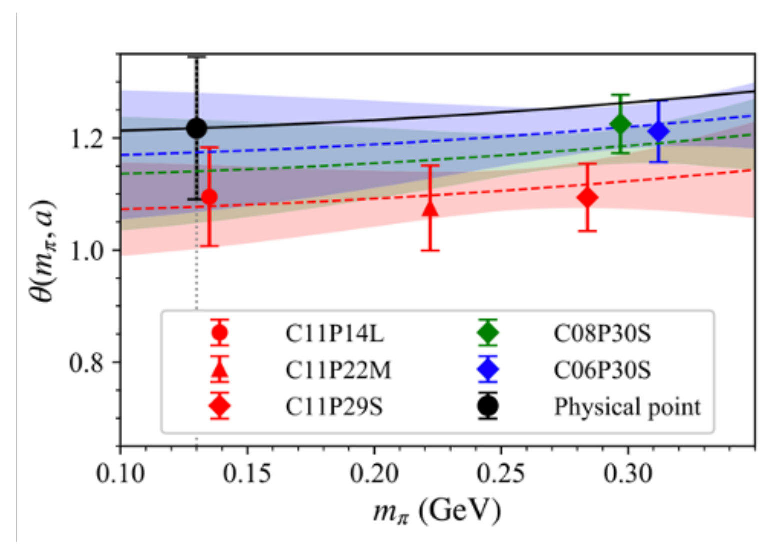

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The HALQCD method

The key quantity in HALQCD method is Nambu-Bethe-Salpeter wave function:

$$\Psi^W(\vec{r}) = \sum_{\vec{x}} \langle 0 | T\{p(\vec{x},0)\Lambda(\vec{x}+\vec{r},0)\} | p\Lambda, W\rangle$$

In lattice simulations, NBS wave function is obtained from three-point correlator:

$$C_{p\Lambda}(\vec{r},t) = \sum_{\vec{x}} \langle 0 | p(\vec{x},t)\Lambda(\vec{x}+\vec{r},t)\bar{J}_{p\Lambda}(0) | 0\rangle$$

By inserting the complete set of energy eigenstates:

$$\sum_{\vec{x}} \langle 0 | p(\vec{x},t)\Lambda(\vec{x}+\vec{r},t)\bar{J}_{p\Lambda}(0) | 0\rangle = \sum_{n} A_n \Psi^{W_n}(\vec{r}) e^{-W_n t}$$

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The HALQCD method

Defining a nonlocal potential $U(\vec{r}, \vec{r'})$ so as to satisfy

$$(E_k - H_0)\Psi^W(\vec{r}) = \int d^3\vec{r'}\, U(\vec{r}, \vec{r'})\Psi^W(\vec{r'})$$

assume the nonlocal potential is energy independent

$$\left[-H_0 - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right] R(\vec{r}, t) = \int d^3\vec{r'}\, U(\vec{r}, \vec{r'}) R(\vec{r'}, t)$$

$$R_{p\Lambda}(\vec{r}, t) = \frac{C_{p\Lambda}(\vec{r}, t)}{C_p(t)C_\Lambda(t)} = \sum_n A'_n \Psi^{W_n}(\vec{r}) e^{-\Delta W_n t}$$

Then the leading order analysis neglecting higher orders leads to

$$U(\vec{r}, \vec{r'}) = V_0^{LO}(\vec{r})\delta(\vec{r} - \vec{r'})$$

$$V_0^{LO}(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2 R(\vec{r}, t)}{R(\vec{r}, t)} - \frac{(\partial/\partial t)R(\vec{r}, t)}{R(\vec{r}, t)} + \frac{1}{8\mu}\frac{(\partial^2/\partial t^2)R(\vec{r}, t)}{R(\vec{r}, t)}$$

Lattice setup for $p - \Lambda$

- C24P29, $n_s^3 \times n_t = 24^3 \times 72$
- $a = 0.10530(18)\text{fm}$;
- $m_\pi \simeq 293\text{MeV}, m_K \simeq 509\text{MeV}$
- Coulomb gauge fixed-wall source for HALQCD method

Two-point Correlation Function

We define the map from color-spin index to weight index:

$$p_\sigma = \epsilon^{abc}\frac{1}{\sqrt{2}}[u^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi}d^b_\xi(x) - d^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi}u^b_\xi(x)] \times [P_+(1-(-1)^\sigma i\gamma_1\gamma_2)]_{\sigma\rho}u^c_\rho(x)$$

$$\Lambda_\sigma = \epsilon^{abc}\frac{1}{\sqrt{2}}[d^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi}u^b_\xi(x) - u^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi}d^b_\xi(x)] \times [P_+(1-(-1)^\sigma i\gamma_1\gamma_2)]_{\sigma\rho}s^c_\rho(x)$$

$$p_\sigma(x) = \sum_\alpha w_\alpha^{[N]\sigma} u^{i(\alpha)}(x) d^{j(\alpha)}(x) u^{k(\alpha)}(x)$$

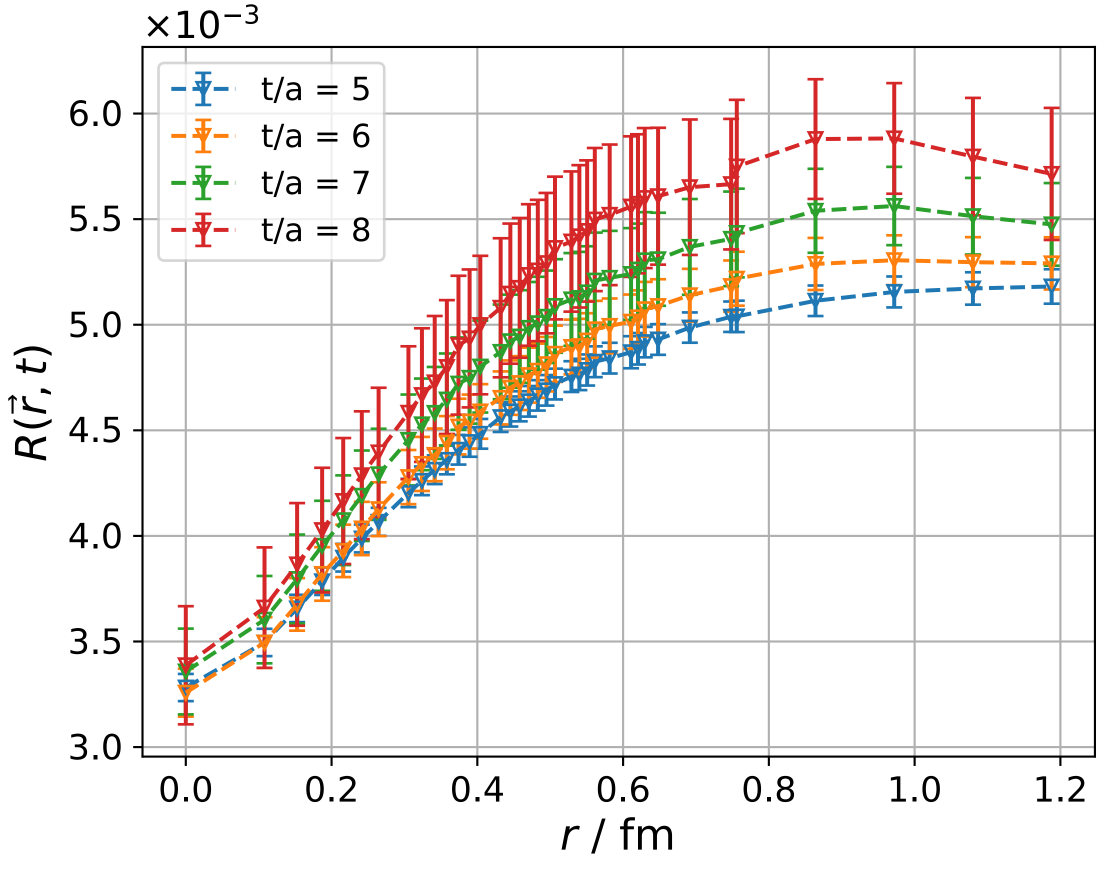
$$\Lambda_\sigma(x) = \sum_\alpha w_\alpha^{[N]\sigma} d^{i(\alpha)}(x) u^{j(\alpha)}(x) s^{k(\alpha)}(x)$$

After test different types of operators, we choose construct the correlation functions with operator D(sink)H(source)

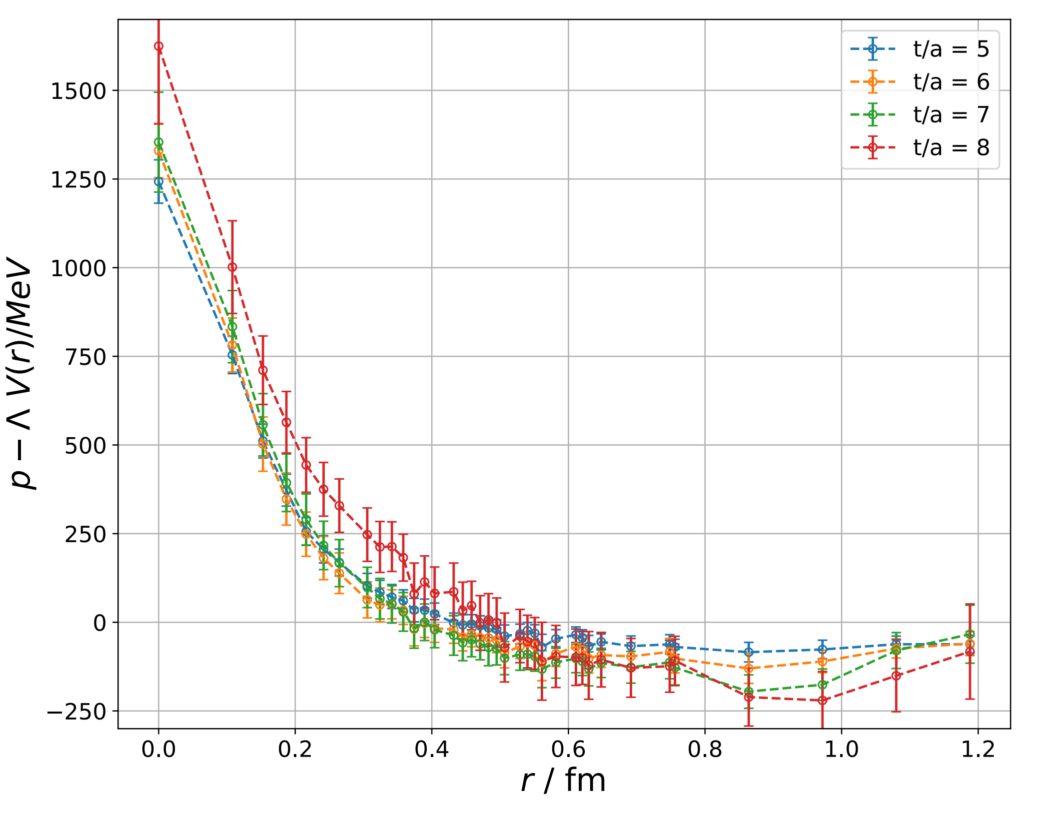
$$C_{p\Lambda}(\vec{r},t) = \sum_{\vec{x}} \langle 0 | (p(\vec{x},t)\Lambda(\vec{x}+\vec{r},t))^D \bar{J}^H_{p\Lambda}(0) | 0 \rangle$$

The HALQCD method

We extract the R wave function and the effective potential for the $^3S_1$ channel on the timeslice $t/a = 5,6,7,8$.



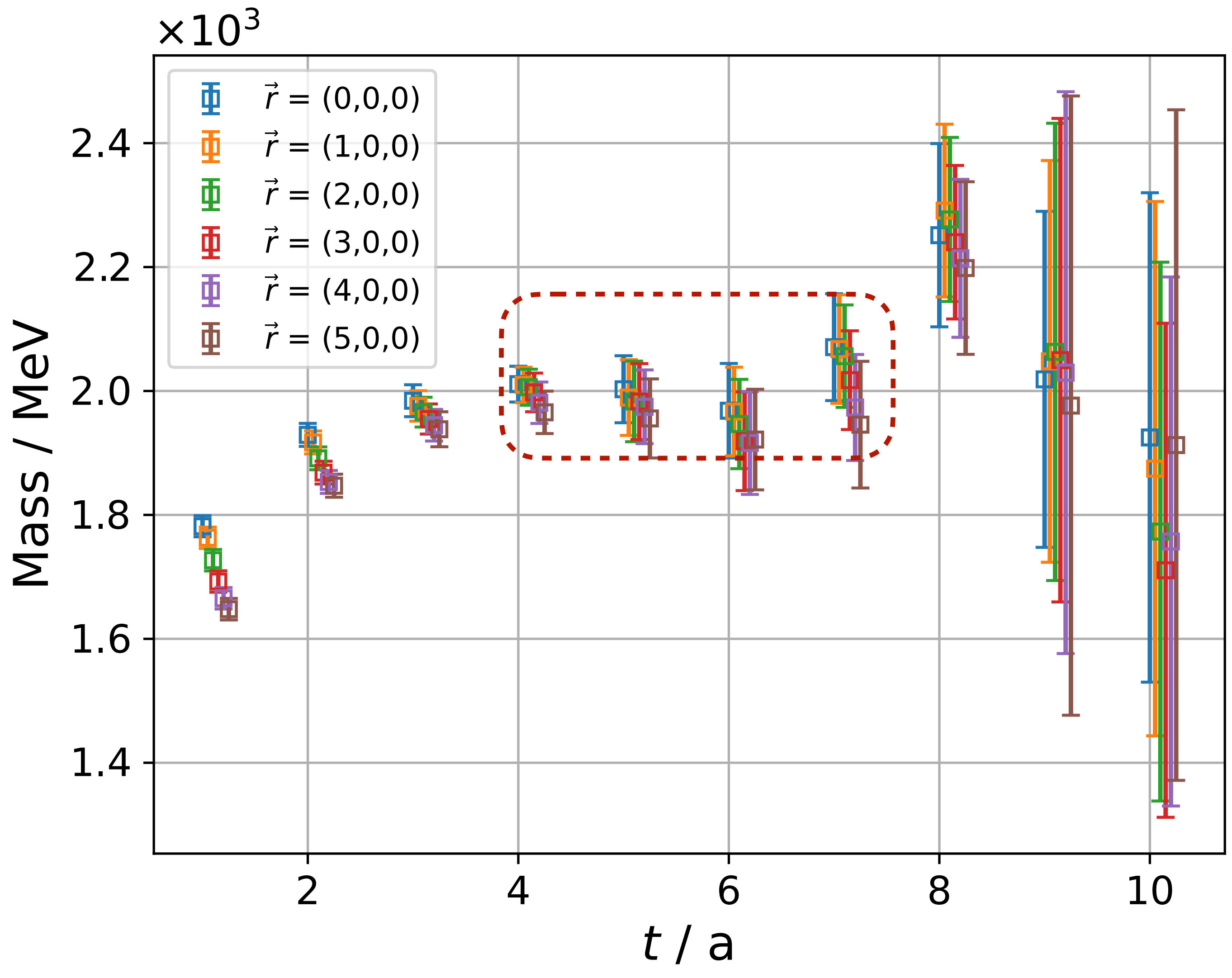
R wave function



effective potential

Effective mass

We try to find the appropriate time slice for the ground state saturation of the system from the effective mass of the dibaryon system.



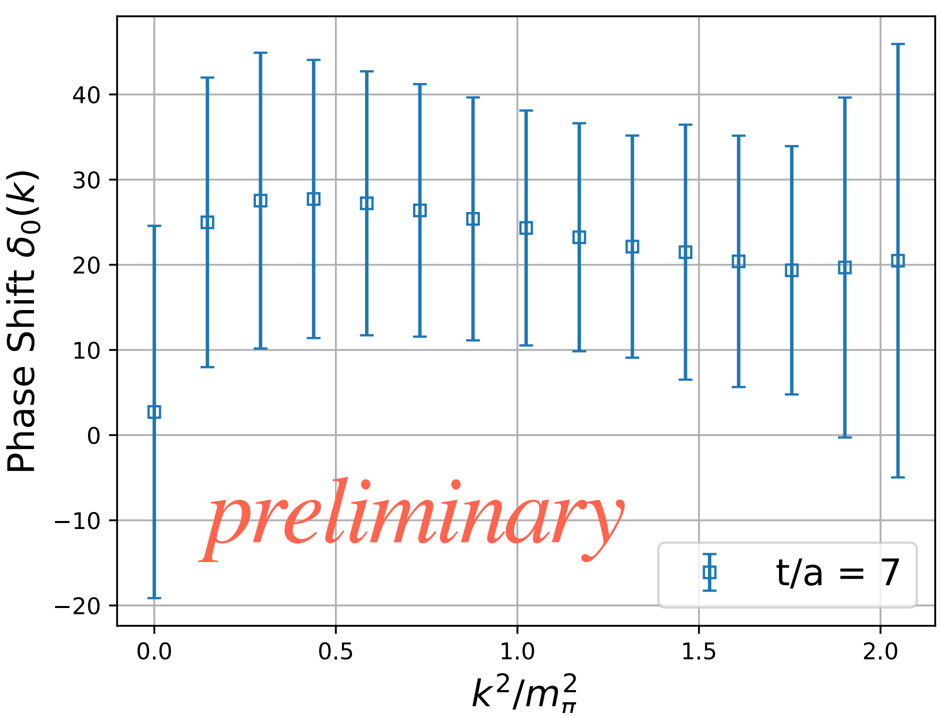
Phase shift from the HALQCD method

We parameterize the effective potential in this form

$$V(r) = a_1 e^{-a_2 r^2} + a_3 e^{-a_4 r^2} + A\frac{e^{-Br}}{r}$$

$a_1 = -0.293(10)$ $a_2 = 0.0444(16)$ $a_3 = 0.499(16)$ $a_4 = 0.0969(34)$ $A = 1.348(53)$ $B = 1.142(45)$

scattering phase shift can be obtained by solving the Schrodinger equation



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Lüscher′s **finite volume formula**

The direct method for scattering on the lattice: Lüscher′s **finite volume method**

Finite volume energies ⟷ (Lüscher formula) ⟷ **Infinite volume scattering**

$$E_{1\cdot 2}(k) = \sqrt{m_1^2 + \bar{k}^2} + \sqrt{m_2^2 + \bar{k}^2}$$

$$q^2 = \bar{k}^2 L^2 / (2\pi)^2$$

⟷

$$q \cot \delta_0(q) = \frac{1}{\pi^{3/2}} Z_{00}(1; q^2)$$

↓ ERE(effective range expansion)

$$k^{2l+1} cot \delta_l(k) = a_l^{-1} + \frac{1}{2} r_l k^2 + \ldots$$

scattering length　　effective range

Lüscher′s **finite volume formula**

We construct the two-particle operators on the ${}^3S_1(T_1^+)$ and ${}^1S_0(A_1^+)$ channels

$$
\begin{aligned}
&A_1^+ : \\
\mathcal{O}_{A_1^+} &= p_{\frac{1}{2}}(0)\Lambda_{-\frac{1}{2}}(0) - p_{-\frac{1}{2}}(0)\Lambda_{\frac{1}{2}}(0) \\
\mathcal{O}'_{A_1^+} &= p_{\frac{1}{2}}(e_x)\Lambda_{-\frac{1}{2}}(-e_x) - p_{-\frac{1}{2}}(e_x)\Lambda_{\frac{1}{2}}(-e_x) + p_{\frac{1}{2}}(-e_x)\Lambda_{-\frac{1}{2}}(e_x) - p_{-\frac{1}{2}}(-e_x)\Lambda_{\frac{1}{2}}(e_x) \\
&+ p_{\frac{1}{2}}(e_y)\Lambda_{-\frac{1}{2}}(-e_y) - p_{-\frac{1}{2}}(e_y)\Lambda_{\frac{1}{2}}(-e_y) + p_{\frac{1}{2}}(-e_y)\Lambda_{-\frac{1}{2}}(e_y) - p_{-\frac{1}{2}}(-e_y)\Lambda_{\frac{1}{2}}(e_y) \\
&+ p_{\frac{1}{2}}(e_z)\Lambda_{-\frac{1}{2}}(-e_z) - p_{-\frac{1}{2}}(e_z)\Lambda_{\frac{1}{2}}(-e_z) + p_{\frac{1}{2}}(-e_z)\Lambda_{-\frac{1}{2}}(e_z) - p_{-\frac{1}{2}}(-e_z)\Lambda_{\frac{1}{2}}(e_z)
\end{aligned}
$$

We evaluate a correlation matrix of the form

$$C_i^{\alpha\beta}(t) = \langle 0 | \mathcal{O}_i^{\alpha}(t)\mathcal{O}_i^{\beta\dagger}(0) | 0 \rangle$$

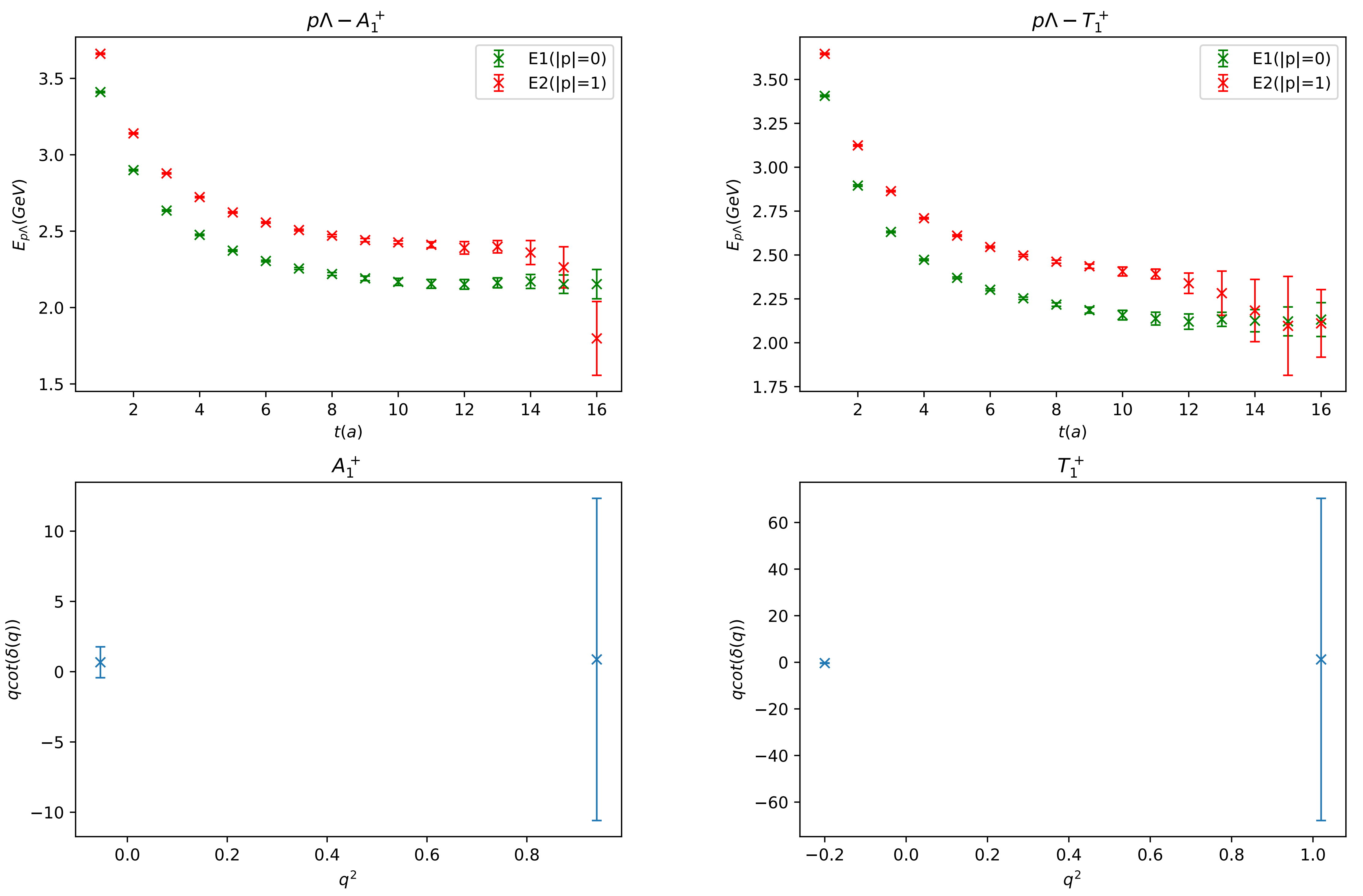
Solving the so-called Generalized Eigenvalue Problem

$$C(t)v_n(t) = \lambda_n(t)C(t_r)v_n(t)$$

$$\lambda_n(t) = c_0 e^{-m_n(t-t_r)}\left(1 + c_1 e^{-\Delta E(t-t_r)}\right)$$

Effective mass of the YN system

preliminary results



Prospect

✓ HALQCD method: preliminary results for p-Λ NBS wave function, the interaction potential, and phase shift;

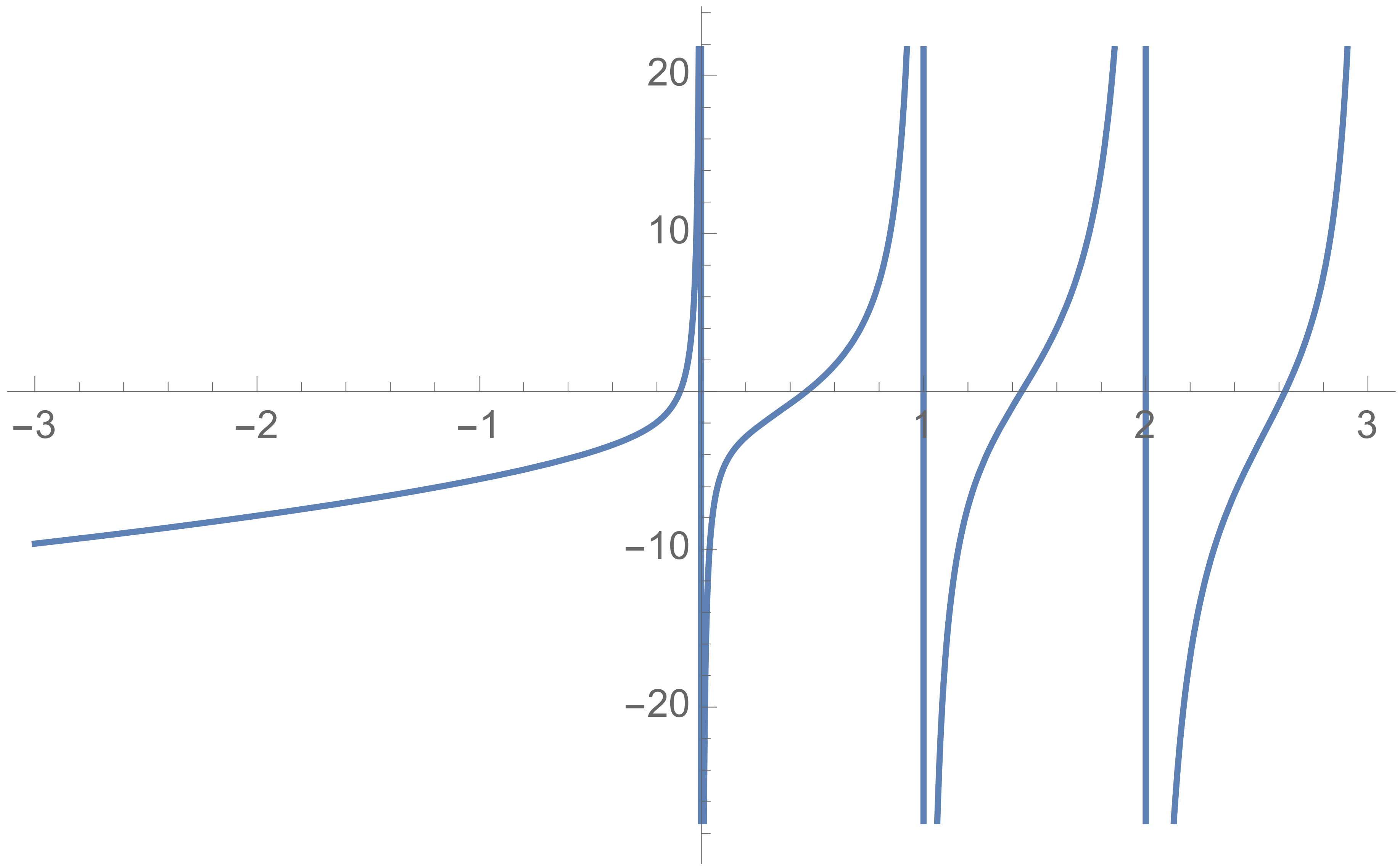
✓ Lüscher′s finite volume method: preliminary results for finite volume energies and phase shift on one ensemble;

➡ a coarse lattice is adopted: more ensembles, discretization error, pion mass, finite volume effect

➡ Effective range expansion: scattering length and effective range

➡ p-Λ: p-Σ coupled channel

Thank you!

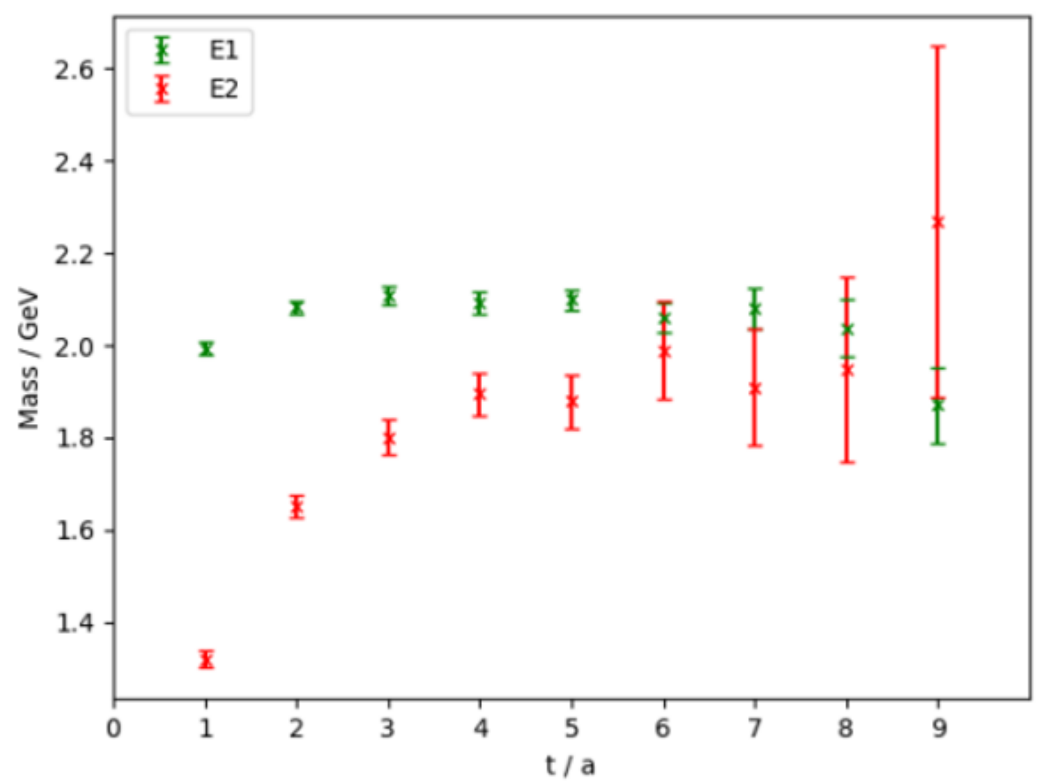


20
10
−10
−20
−3
−2
−1
1
2
3

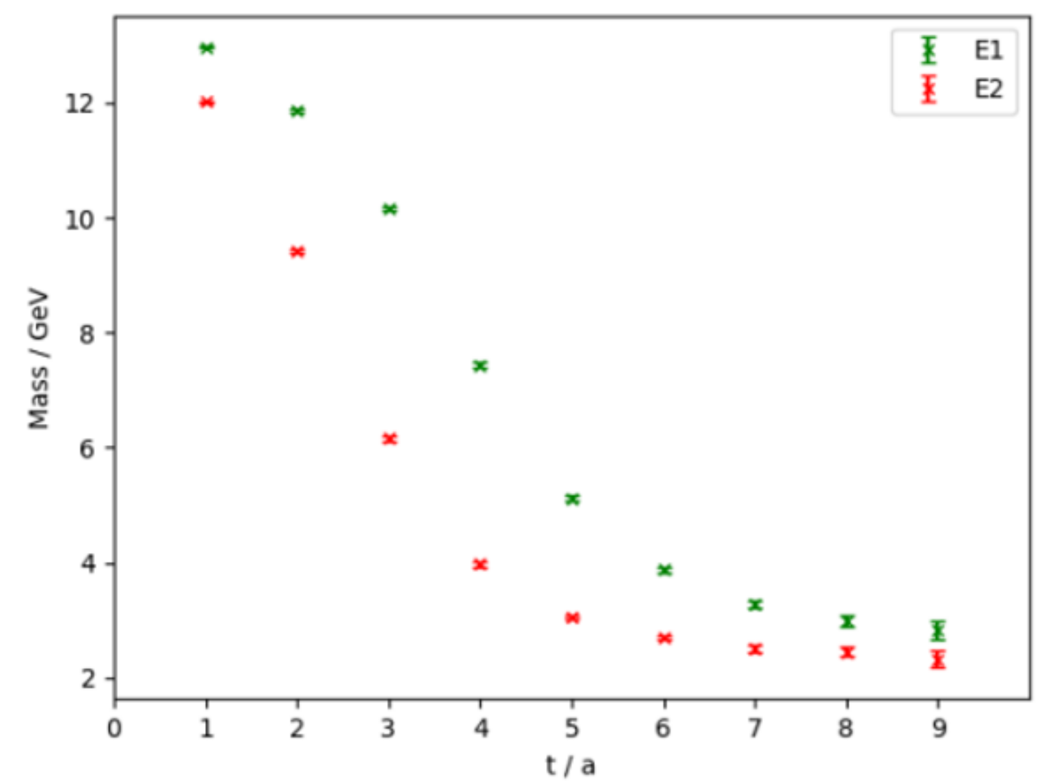
Variational Analysis

We constructed the correlation matrix and calculate the eigenvalues

$$C = \begin{pmatrix} C_{HH} & C_{DH} \\ C_{DH} & C_{DD} \end{pmatrix}$$

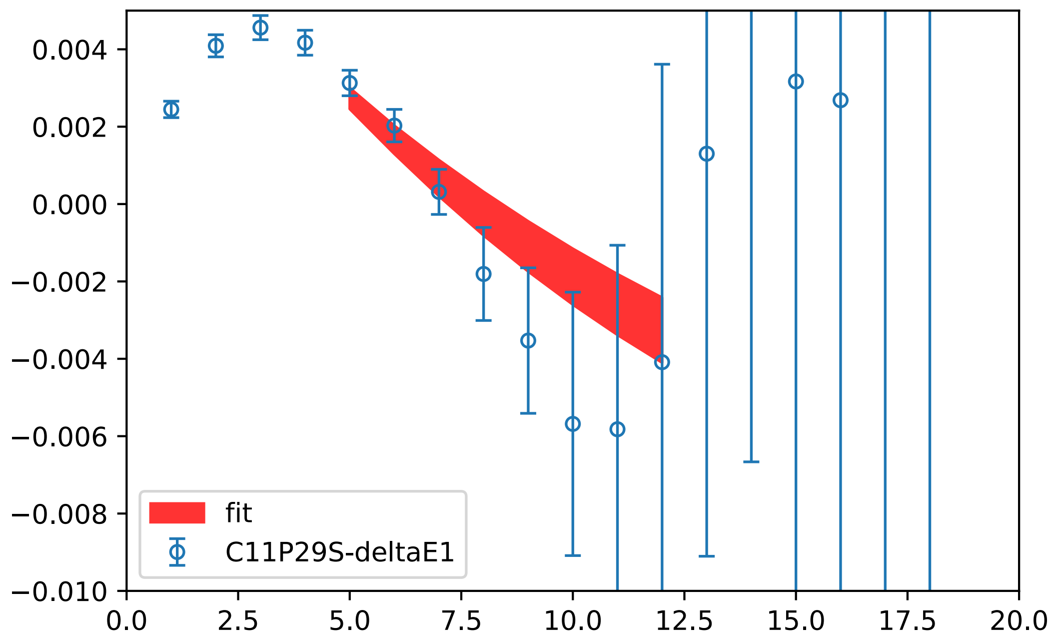


Wallsource

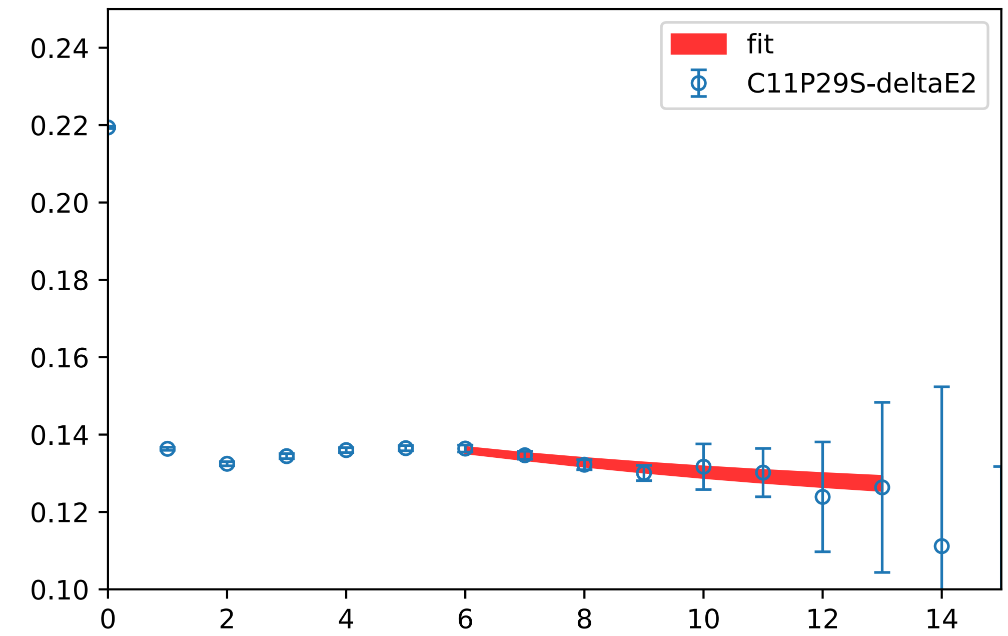


Pointsource

A1+:

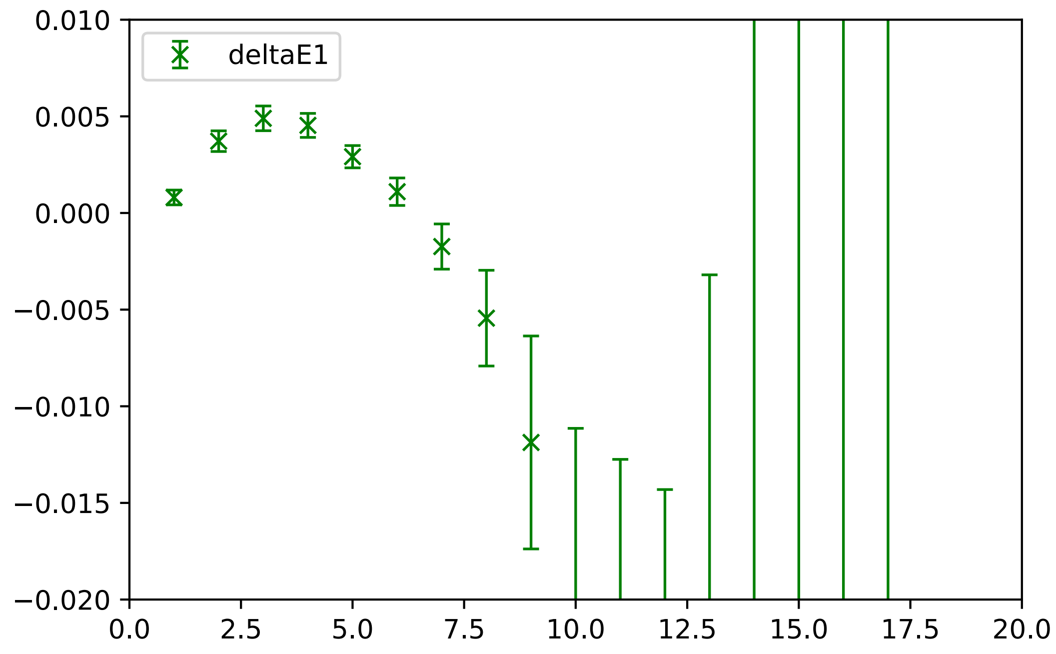


0.004
0.002
0.000
−0.002
−0.004
−0.006
−0.008
−0.010
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
fit
C11P29S-deltaE1

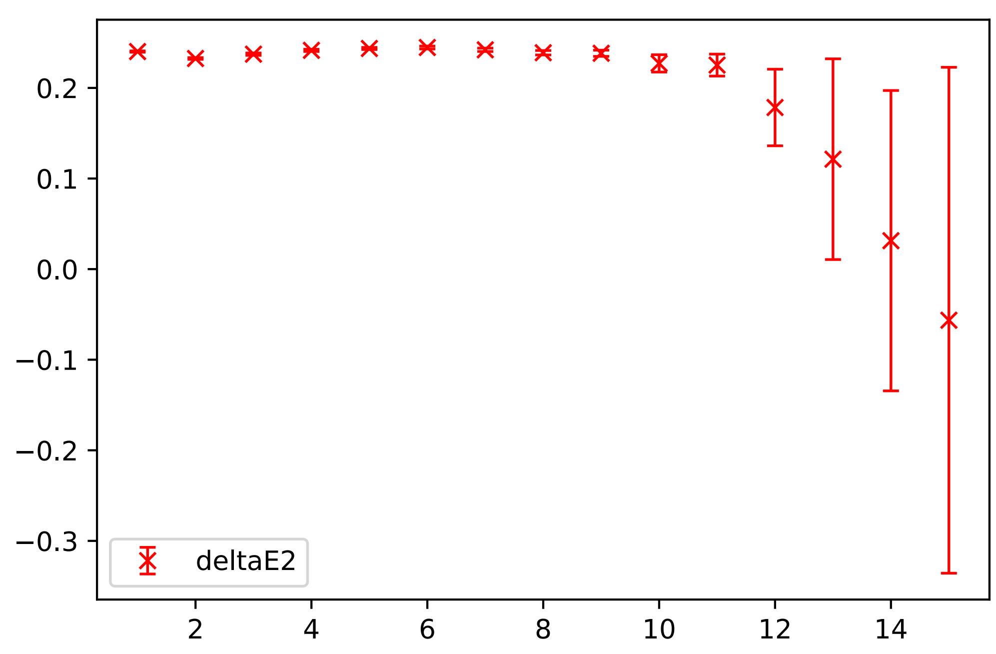


0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0
2
4
6
8
10
12
14
fit
C11P29S-deltaE2

T1+:



0.010
0.005
0.000
−0.005
−0.010
−0.015
−0.020
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
deltaE1



0.2
0.1
0.0
−0.1
−0.2
−0.3
2
4
6
8
10
12
14
deltaE2