格点QCD中多夸克态的研究

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CLQCD

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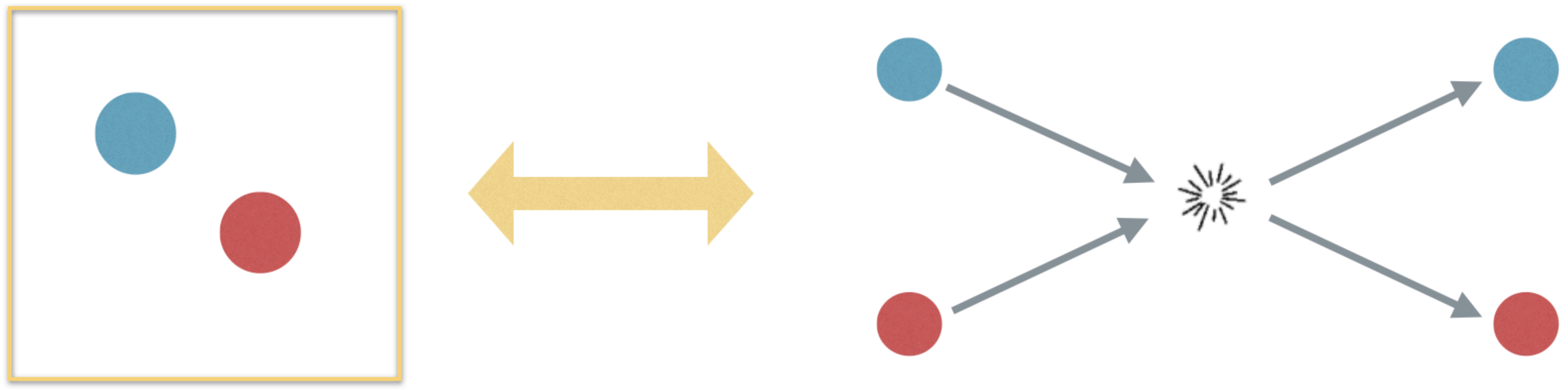
Outline

- Spectroscopy and scattering on lattice
- Lattice setup
- Preliminary results
  - $D_s\bar{D}^*$ scattering and Z_{cs}
  - $\Lambda_c\Lambda_c$ scattering
  - $D\pi$ scattering and $D_0^*(2300)$

Scattering on lattice

Lüscher's finite volume method: M. Lüscher, Nucl. Phys. B354, 531(1991)



Finite volume spectrum

Infinite volume scattering parameters

Scattering on lattice

- General Lüscher's formula for two-body scattering:

$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (1 + i\mathbf{M})] = 0$$

Diagonal matrix of phase-space factors $\rho_{ij} = \delta_{ij}\frac{2k_i}{E_{cm}}$

Infinite-volume scattering matrix

Finite volume information $M(E_{cm}, L)$

- Resonances/bound states are formally defined as poles in scattering amplitudes.

Scattering on lattice

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Finite volume spectrum:

✦ build large basis of operators $\{\mathcal{O}_1, \mathcal{O}_2, \cdots\}$ with desired quantum numbers, construct the matrix of correlation function:

$$C_{ij} = < 0 | \mathcal{O}_i \mathcal{O}_j^\dagger | 0 > = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

✦ Solve the generalized eigenvalue problem(GEVP): $C_{ij} v_j^n(t) = \lambda_n(t) C_{ij}^0 v_j^n(t)$

✦ Eigenvalues: $\lambda_n(t) \sim e^{-E_n t}(1 + e^{-\Delta E\, t})$

✦ Optimal linear combinations of the operators to overlap on the n'th state:

$$\Omega_n = \sum_i v_i^n \mathcal{O}_i$$

✦ Computational technique: distillation quark smearing.

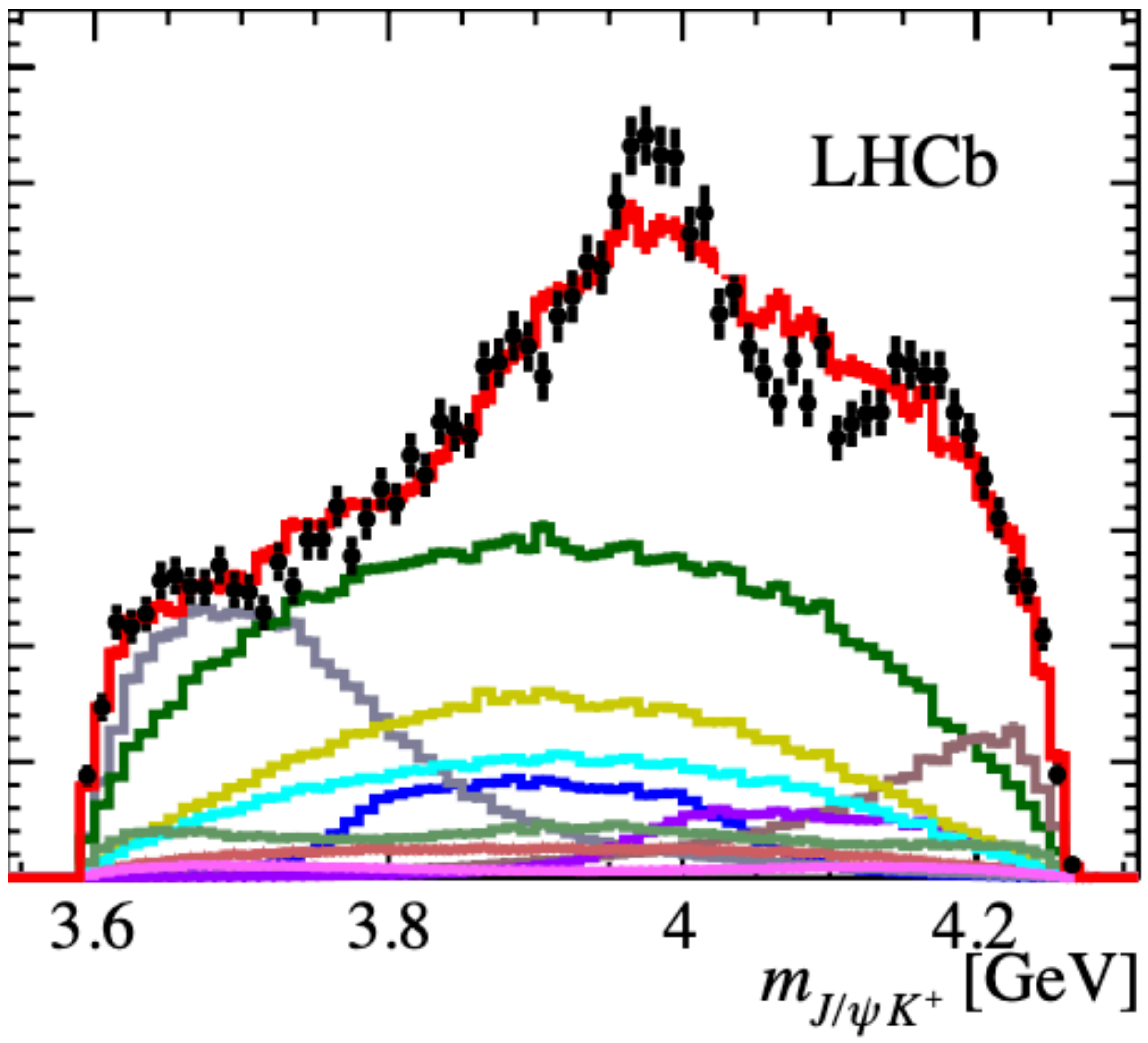
- Improve precision
- Disconneted diagrams
- Efficient for large numbers of ops

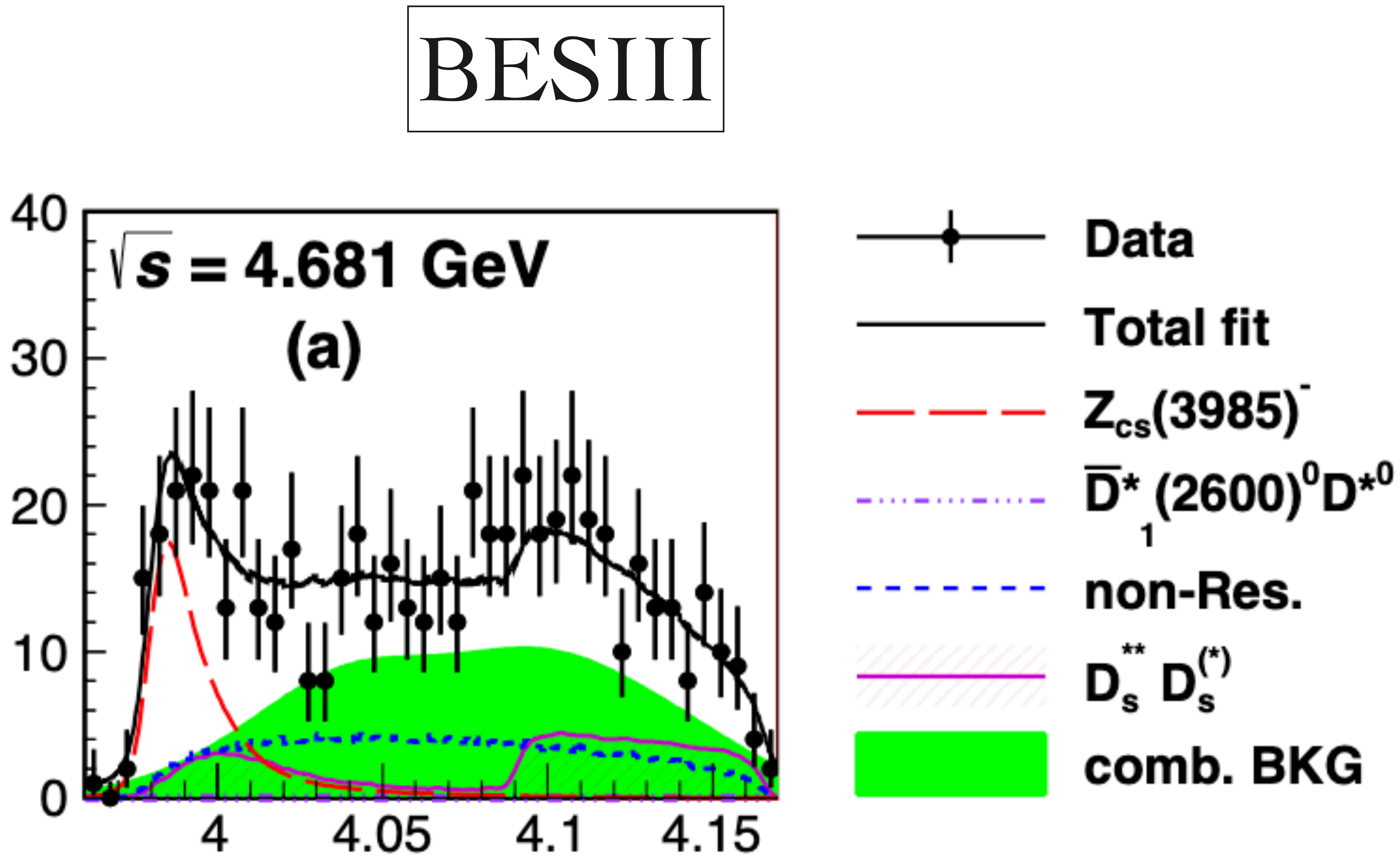
Lattice QCD configurations

- 2+1 flavor Wilson-clover configurations generated by CLQCD.

Lattice spacing	Volume($L^3 \times T$)	M_π (MeV)	# of confs
~0.105fm	$24^3 \times 72$	290	1000
	$32^3 \times 64$	290	1000
	$32^3 \times 64$	230	450
	$48^3 \times 96$	230	200
	$48^3 \times 96$	135	200
~0.077fm	$32^3 \times 96$	300	800
	$48^3 \times 96$	300	200
	$32^3 \times 64$	210	460
	$48^3 \times 96$	210	200
~0.052fm	$48^3 \times 144$	320	200

$D_s\bar{D}^*$ scattering and Z_{cs}





Phys. Rev. Lett. 127, 082001 (2021)

Phys. Rev. Lett. 126, 102001 (2021)

$D_s\bar{D}^*$ scattering and Z_{cs}

$D_s^+\bar{D}^{0*}$ and $J/\Psi K$ scattering:

Interpolating operators:

$$\mathcal{O}(D_s^+\bar{D}^{0*}) = D_s^+(\mathbf{p})\bar{D}^{0*}(-\mathbf{p}), \quad |\mathbf{p}|^2 = 0,1,2,3$$

$$\mathcal{O}(J/\Psi K^+) = J/\Psi(\mathbf{p})K^+(-\mathbf{p}), \quad |\mathbf{p}|^2 = 0,1,2,3$$

Lattice spacing	Volume($L^3 \times T$)	M_π (MeV)	# of confs
~0.077fm	$32^3 \times 96$	300	566
	$48^3 \times 96$	300	200

$D_s\bar{D}^*$ scattering and Z_{cs}

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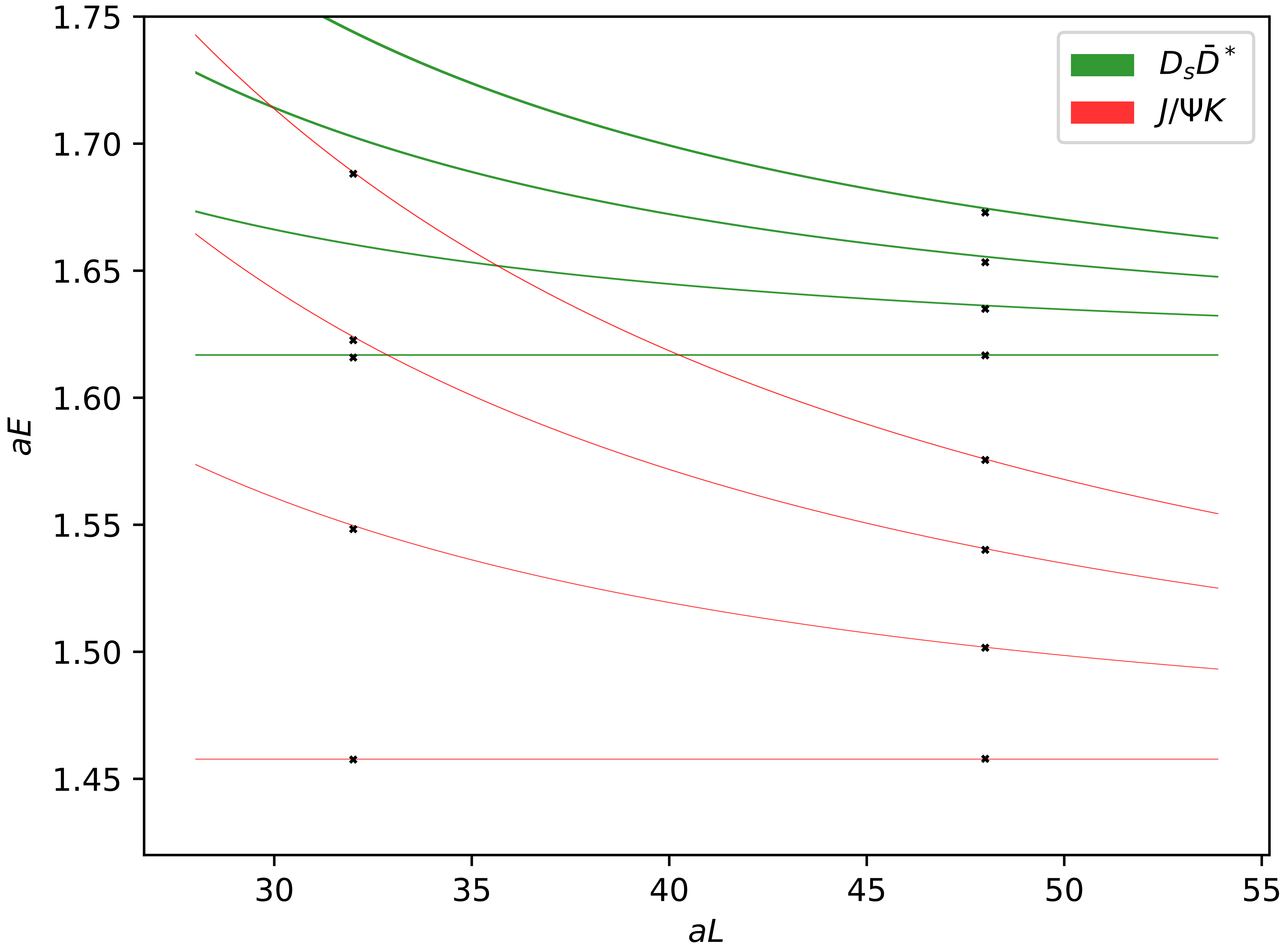
Finite volume sepctrum



$D_s\bar{D}^*$ scattering and Z_{cs}

- Effective range expansion:

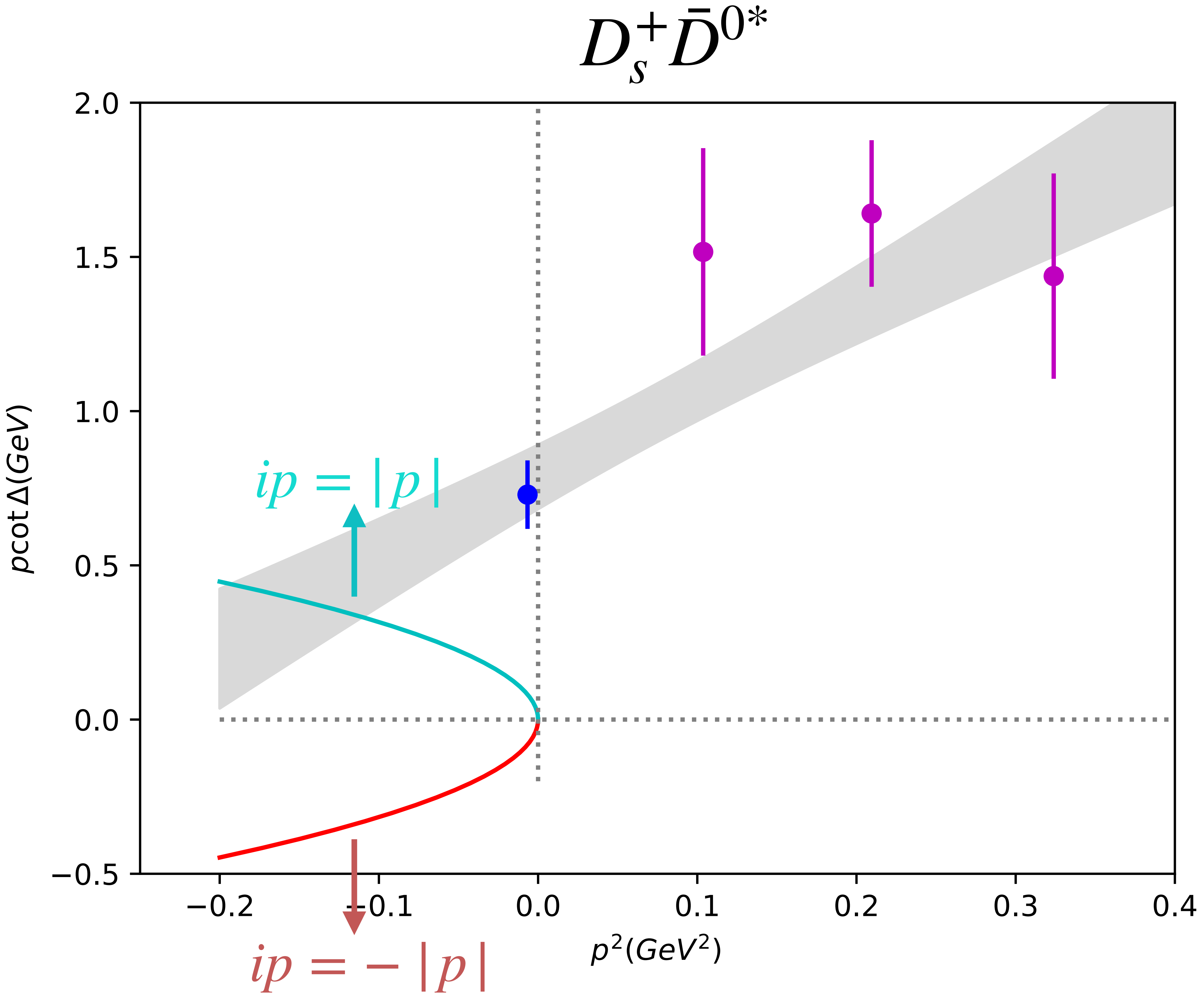
$$k\cot\delta_l = \frac{1}{a_0} + \frac{1}{2}r_0 k^2$$

- Scattering amplitude:

$$T \sim \frac{1}{pcot\delta - ip}$$

- Luscher's formula:

$$pcot\delta(p) = \frac{2Z_{00}(1;(\frac{pL}{2\pi})^2)}{L\sqrt{\pi}}$$



$$a_0 = 0.26(0.03)\text{fm}, \quad r_0 = 1.1(0.2)\text{fm}$$

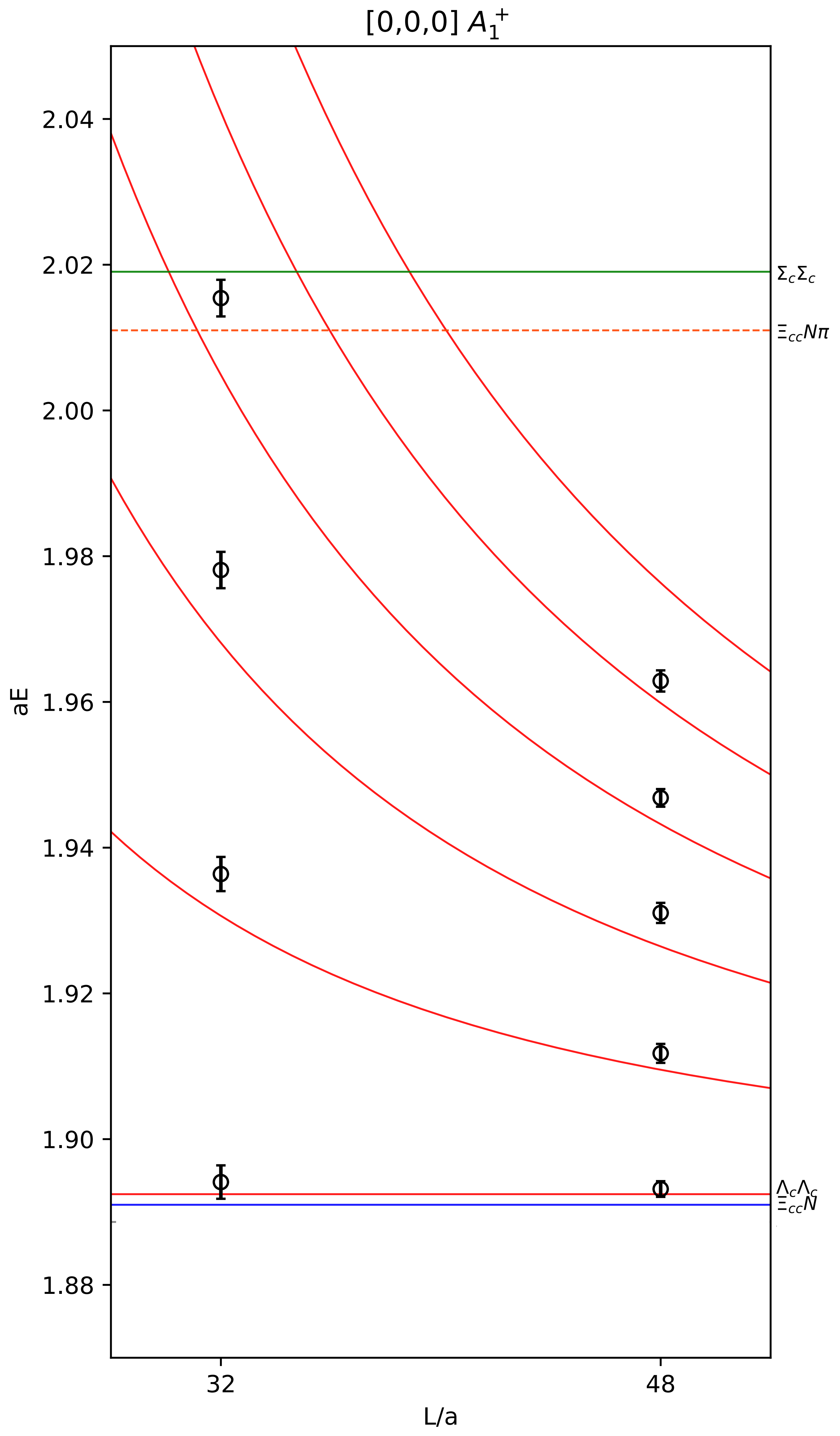
$\Lambda_c \Lambda_c$ scattering

- In lattice calculations, baryon-baryon scattering is more challenge than meson-meson scattering due to poor signal.

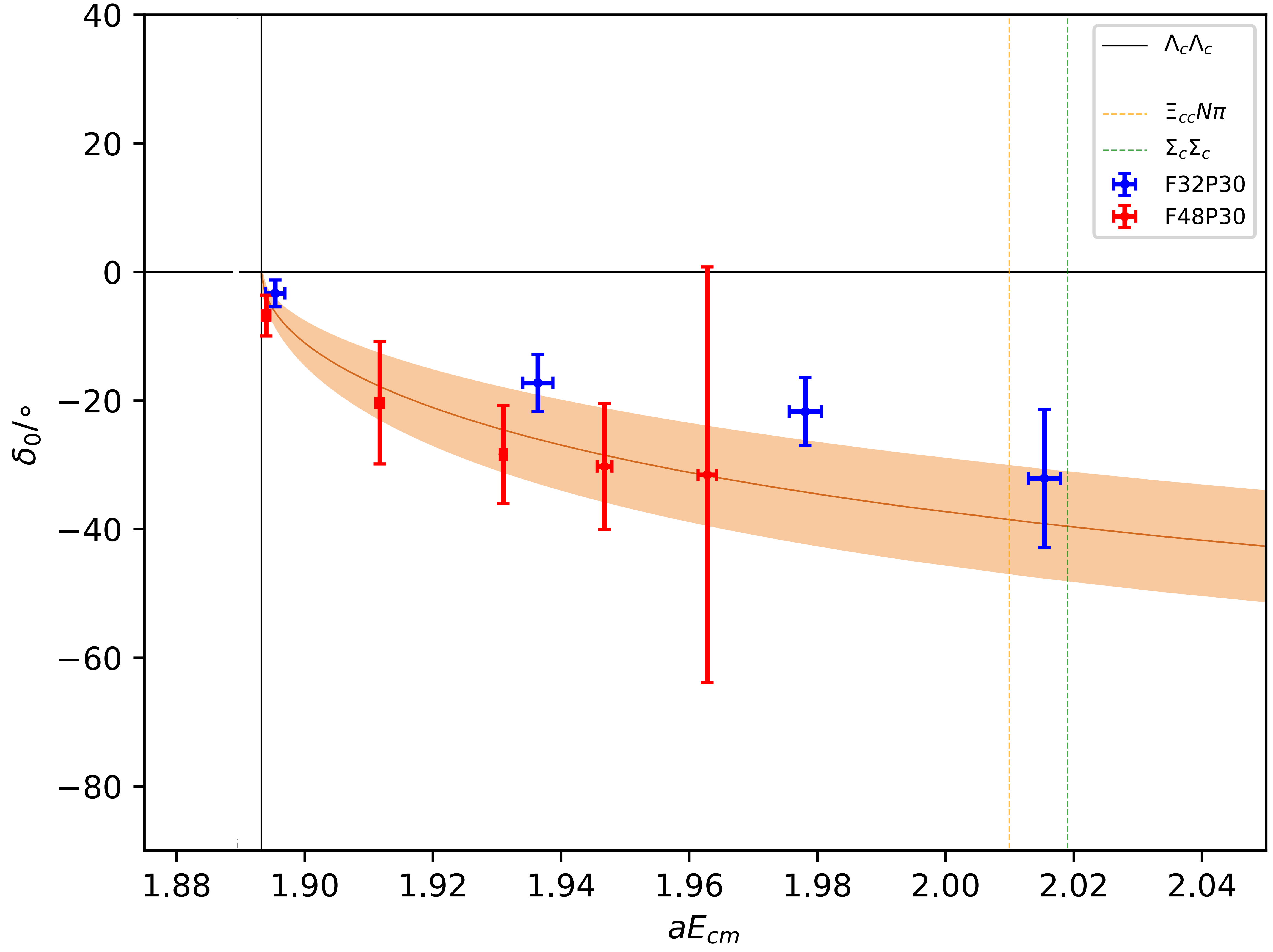
- The H dibaryon($\Lambda\Lambda$) , a scalar six quark state with flavor content *uuddss* is proposed long time ago.

- How would the binding nature depend on quark mass? — $\Lambda_c \Lambda_c$

$\Lambda_c\Lambda_c$ scattering



Phase shift



$\Lambda_c\Lambda_c$ scattering

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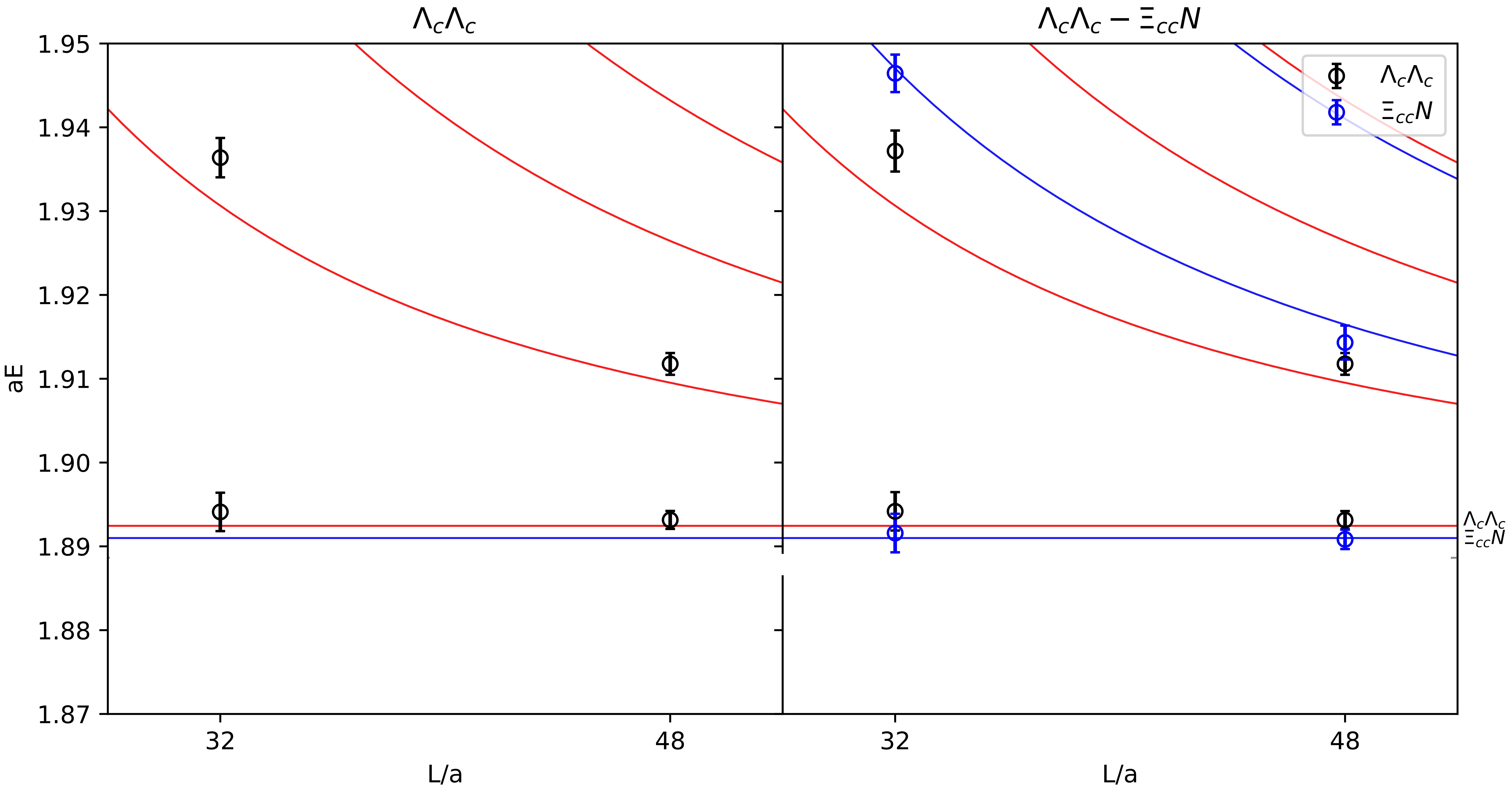
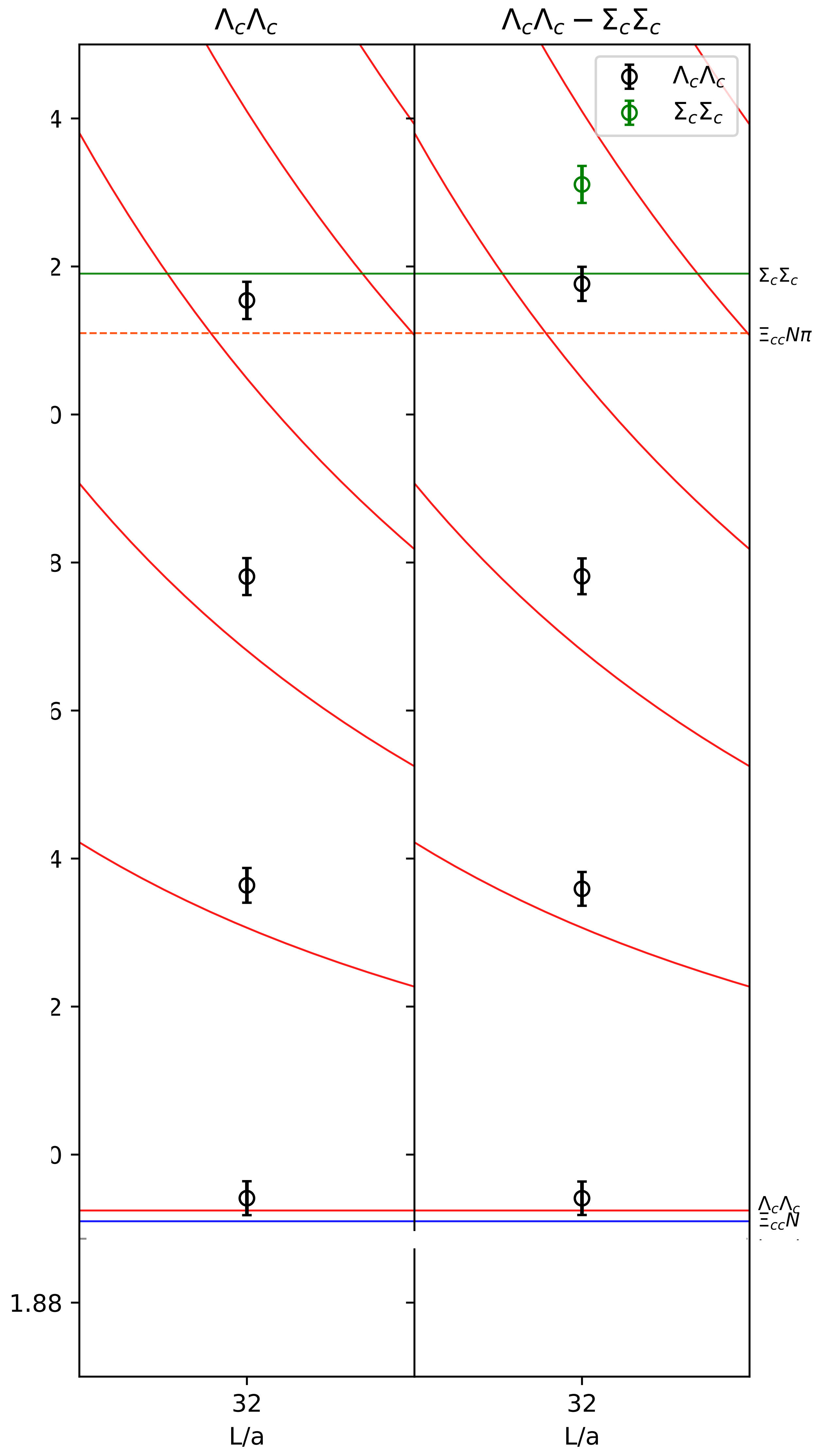
Coupled channels $\Xi_{cc}N$, $\Sigma_c\Sigma_c$





$D\pi$ scattering and $D_0*(2300)$

- The positive parity charmed mesons D_0^*、D_1 and their strange partners D_{s0}^*、D_{s1} are particularly interesting due to some puzzles.

$$M_{D_0^*} = 2343 \pm 10 \text{ MeV}, \quad M_{D_{s0}^*} = 2317.8 \pm 0.5 \text{ MeV}$$

- Previous lattice studies on D_0^* at unphysical pion masses do not agree with experiments.
- We calculated the $D\pi$ scattering at serveral pion masses including the physical point using the CLQCD configurations. The pion mass dependence of the pole positions is investigated.

Haobo Yan, Chuan Liu, Liuming Liu, Yu Meng and Hanyang Xing, arXiv:2404.13479
"Pion mass dependence in Dπ scattering and the $D_0^*(2300)$resonance from lattice QCD"

$D\pi$ scattering and $D_0^*(2300)$

- Six ensembles with four different pion masses and three lattice spacings.
- Large number of interpolating operators are constructed to reliably extract the full spectrum.

Lattice spacing	Volume($L^3 \times T$)	M_π (MeV)	# of confs
~0.105fm	$48^3 \times 96$	135	131
~0.077fm	$32^3 \times 96$	300	566
	$48^3 \times 96$	300	200
	$32^3 \times 64$	210	460
	$48^3 \times 96$	210	250
~0.052fm	$48^3 \times 144$	320	270

$$O_{D\pi}^{I=\frac{1}{2}}(P) = \sqrt{2}D^0(p_1)\pi^+(p_2) - D^+(p_1)\pi^0(p_2), (P = p_1 + p_2, \quad P^2 = 0,1,2,3,4)$$

$$\mathcal{O}_1(P) = \bar{d}\Gamma c(P)$$

$D\pi$ scattering and $D_0^*(2300)$

CLQCD

Finite volume sepctrum

$D\pi$ scattering and $D_0^*(2300)$

CLQCD

Parametrization of the scattering amplitude:

- Effective range expansion:

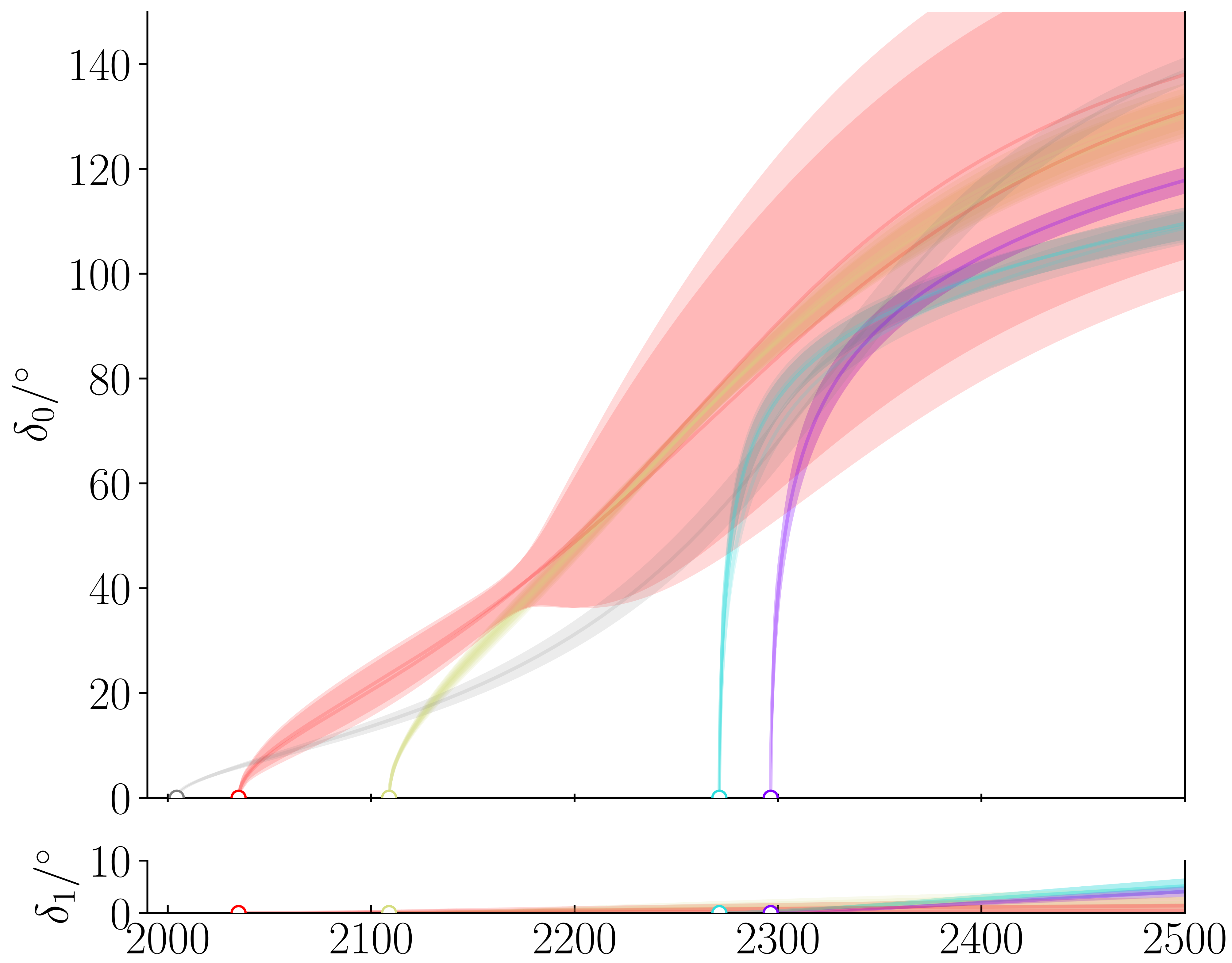
$$k^{2l+1}\cot\delta_l = \frac{1}{a_l} + \frac{1}{2}r_l k^2 + P_2 k^4 + \cdots$$

- K-matrix:

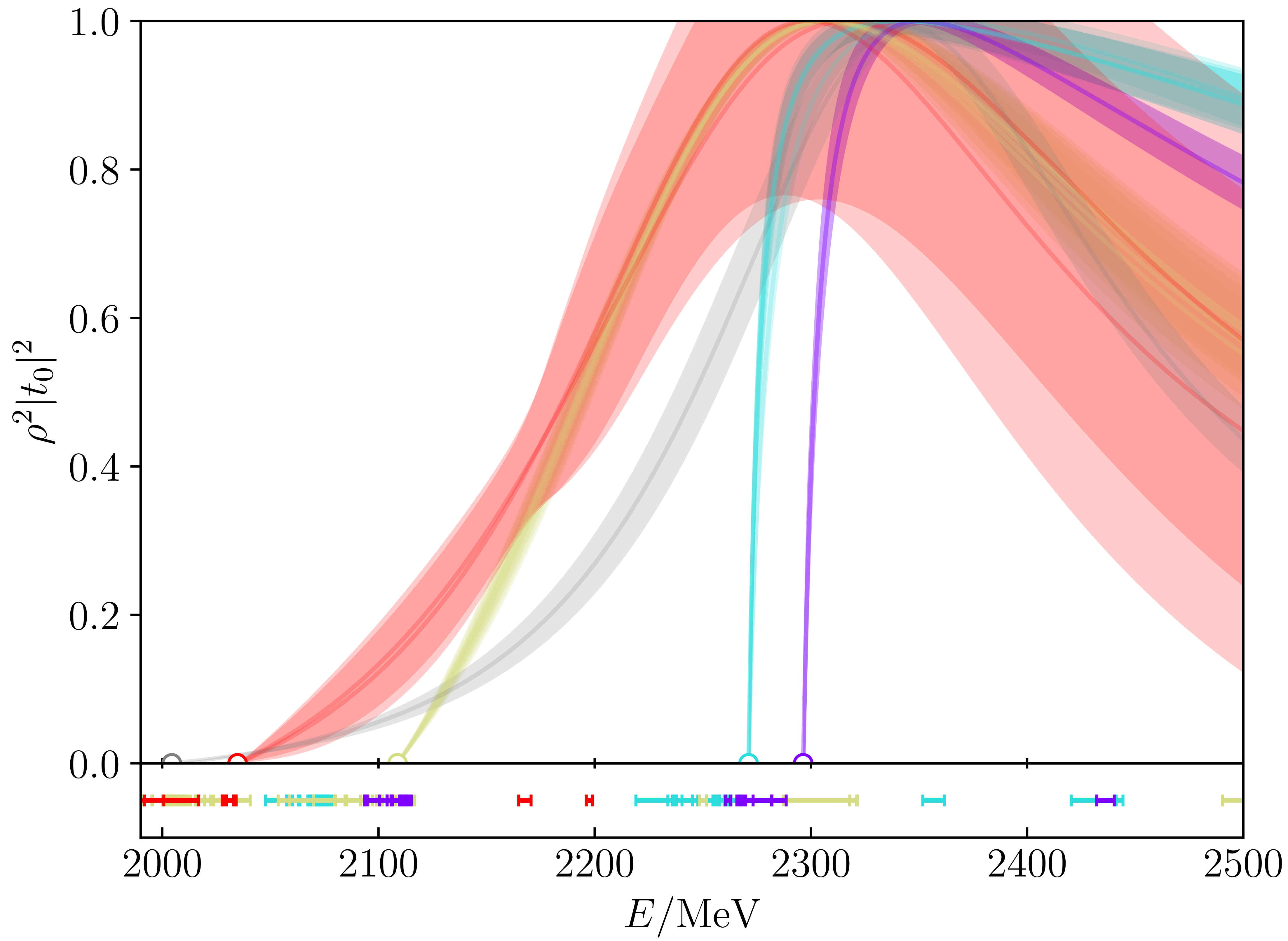
$$(t^{(l)})^{-1}(s) = \frac{1}{(2k)^l}K^{-1}(s)\frac{1}{(2k)^l} + I(s)$$

$$K_l = \frac{g_l^2}{m_l^2 - s}$$

$D\pi$ scattering and $D_0^*(2300)$



S- and P-wave phase shift



Cross section

$D\pi$ scattering and $D_0^*(2300)$

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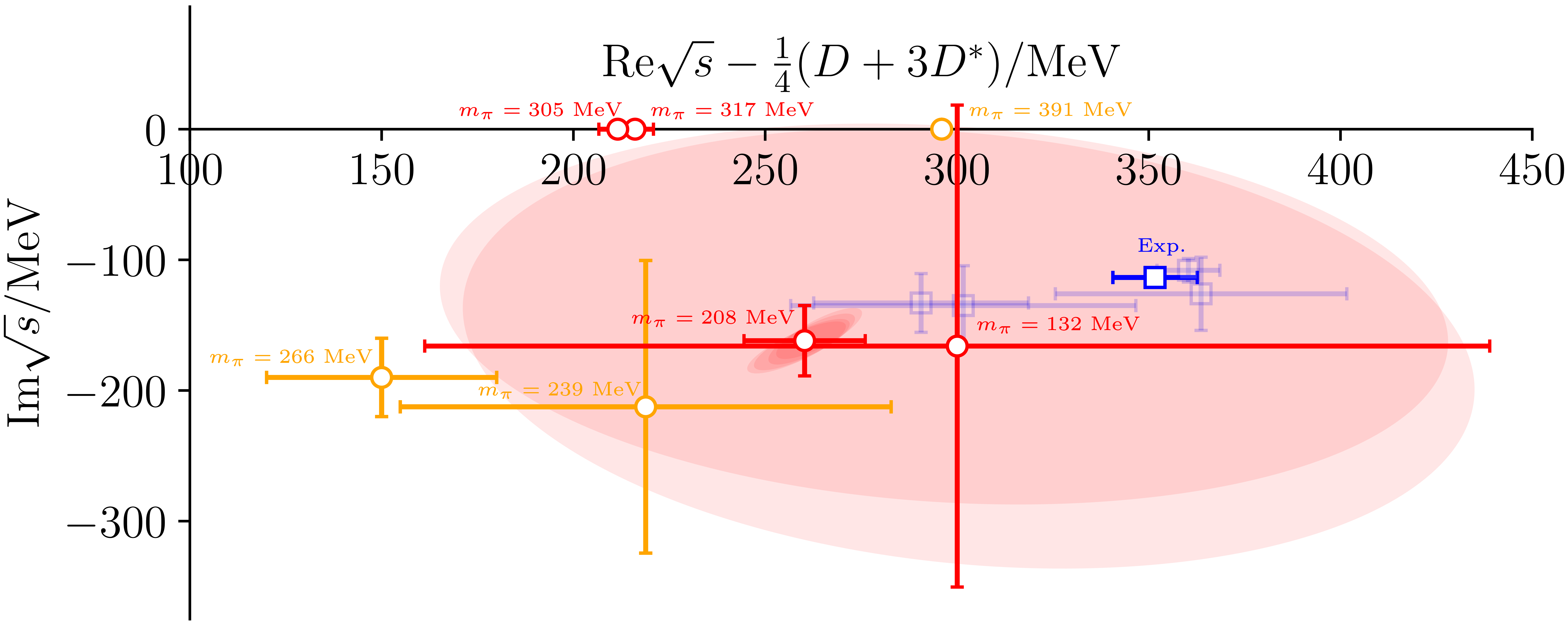
Pole position



- At $m_\pi \sim 300$MeV, there is a virtual state pole. When pion mass decreases, it becomes a resonance and the pole position gets closer to the experimental value.

Summary

- Z_{cs}: At pion mass $m_\pi \sim 300$MeV, a virtual state pole is found in $D_s\bar{D}^*$ scattering. The interaction of $J/\Psi K$ is negeligibly weak.
- $\Lambda_c\Lambda_c$: found repulsive interaction, coupled channels $\Xi_{cc}N$, $\Sigma_c\Sigma_c$ need furthur investigation.
- $D_0^*(2300)$: Pion mass dependence of the pole position are studied. At the physical pion mass, the result agrees with experiments.
- Coupled channels:
  - Need robust determination of the spectrum with a complete set of interpolating operators and perform coupled channel scattering analysis.