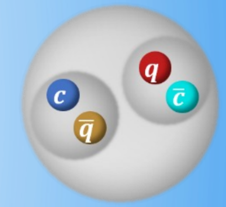
2024年超级陶粲装置研讨会

2024年 7 月 7-11 日, 兰州

Low-energy scattering of heavy mesons: Entanglement suppression and hadronic molecules



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Tao-Ran Hu, Su Chen, FKG, Phys. Rev. D 110, 014002 (2024)
Zhen-Hua Zhang, Teng Ji, Xiang-Kun Dong, FKG, C. Hanhart, U.-G. Meißner, A. Rusetsky, arXiv:2404.11215

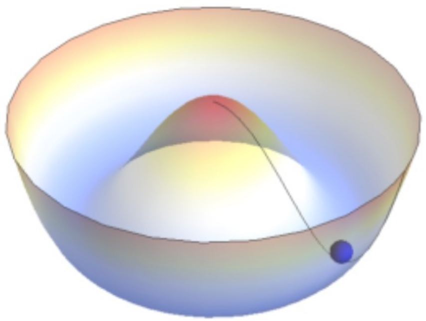
Symmetries

- Symmetries play important role in modern physics
  - Gauge symmetries
  - Global symmetries
- Approximate symmetries in hadron physics

u	d	s		c	b	t
~2 MeV	~5 MeV	~100 MeV	$\ll \Lambda_{\rm QCD} \ll$	~1.3 GeV	~4.2 GeV	~173 GeV

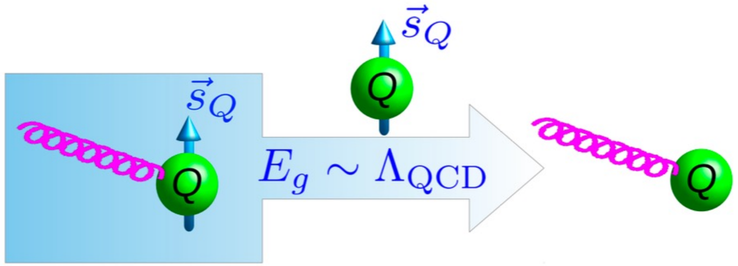
Light quarks

- Spontaneously broken chiral symmetry: $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \xrightarrow{\rm SSB} \mathrm{SU}(N_f)_V$



Heavy quarks

- Heavy quark spin symmetry (HQSS)
- Heavy quark flavor symmetry (HQFS)
- Heavy aiquark-diquark symmetry (HADS)

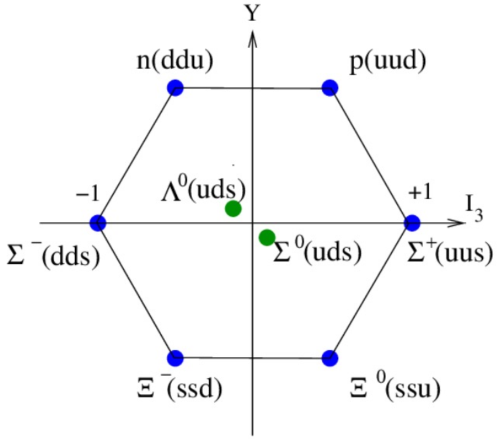


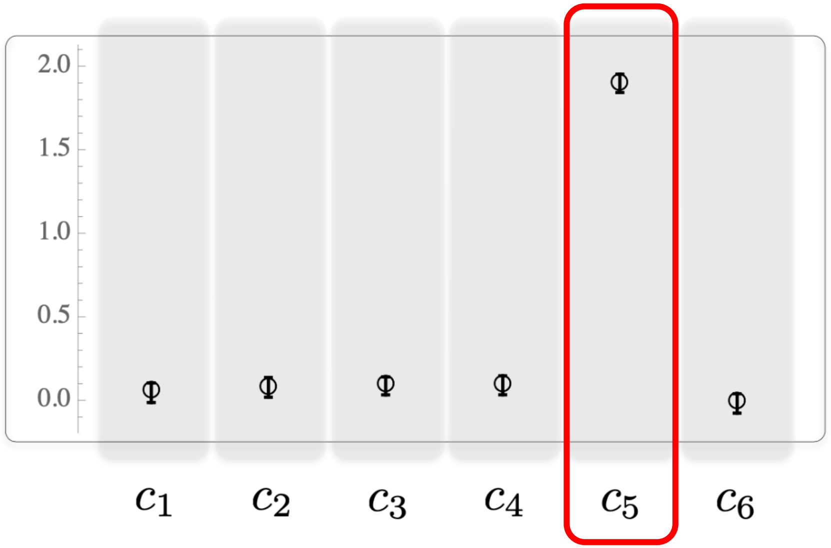
Emergent symmetries

- Emergent symmetries: not derived from Lagrangian
  - SU(4), SU(6) spin-flavor symmetry in nonrelativistic quark model
  - Wigner's SU(4) among neutrons and protons
  - Approximate SU(16) symmetry in low-energy baryon-baryon scattering

$$\begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix}$$

$$B = \begin{pmatrix} \Sigma^0/\sqrt{2}+\Lambda/\sqrt{6} & \Sigma^+ & p \\ \Sigma^- & -\Sigma^0/\sqrt{2}+\Lambda/\sqrt{6} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}$$





- LO contact-term Lagrangian

Savage, Wise (1995)

NPLQCD, PRD 96, 114510 (2017)

$$\mathcal{L}_{\rm LO}^{n_f=3} = -\frac{c_1}{f^2}\langle B_i^\dagger B_i B_j^\dagger B_j\rangle - \frac{c_2}{f^2}\langle B_i^\dagger B_j B_j^\dagger B_i\rangle - \frac{c_3}{f^2}\langle B_i^\dagger B_j^\dagger B_i B_j\rangle - \frac{c_4}{f^2}\langle B_i^\dagger B_j^\dagger B_j B_i\rangle$$
$$- \frac{c_5}{f^2}\langle B_i^\dagger B_i\rangle\langle B_j^\dagger B_j\rangle - \frac{c_6}{f^2}\langle B_i^\dagger B_j\rangle\langle B_j^\dagger B_i\rangle, \qquad i,j=\uparrow,\downarrow$$

SU(16) symmetry

Entanglement suppression

- Conjecture: entanglement suppression is a low-energy property of strong interactions and gives rise to emergent symmetries.
  - explains Wigner's SU(4)
  - explains SU(16) for baryon-baryon interactions

PHYSICAL REVIEW LETTERS **122,** 102001 (2019)

Entanglement Suppression and Emergent Symmetries of Strong Interactions

Silas R. Beane,[1] David B. Kaplan,[2] Natalie Klco,[1,2] and Martin J. Savage[2]
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(Received 20 December 2018; published 14 March 2019)

Entanglement suppression in the strong-interaction S matrix is shown to be correlated with approximate spin-flavor symmetries that are observed in low-energy baryon interactions, the Wigner $SU(4)$ symmetry for two flavors and an $SU(16)$ symmetry for three flavors. We conjecture that dynamical entanglement suppression is a property of the strong interactions in the infrared, giving rise to these emergent symmetries and providing powerful constraints that predict the nature of nuclear and hypernuclear forces in dense matter.

Following studies:

S. Beane, R. Farrell, Annals Phys. 433 (2021) 168581; S. Beane, R. Farrell, M. Varma, IJMPA 36 (2021) 2150205;
I. Low, T. Mehen, PRD 104 (2021) 074014; Q. Liu, I. Low, T. Mehen, PRC 107 (2023) 025204; Q. Liu, I. Low, arXiv:2312.02289

Entanglement measure

- Entanglement measure: measures the degree of entanglement of any given state
  - Many different ways, for bipartite system (density matrix: $\rho = |\psi\rangle\langle\psi|$; partial trace: $\rho_1 = \mathrm{Tr}_2(\rho)$), e.g.,
    - von Neumann entropy:

$$E(\rho) = -\mathrm{Tr}(\rho_1 \ln \rho_1) = -\mathrm{Tr}(\rho_2 \ln \rho_2)$$

    - linear entropy:

$$E(\rho) = -\mathrm{Tr}\big(\rho_1(\rho_1 - 1)\big) = 1 - \mathrm{Tr}(\rho_1^2)$$

  - Common property: vanishes for a direct product state $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$, maximizes for maximally entangled states

For a system with two spin-1/2 particles, let's define the "computational basis:"

$$\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}.$$

Then for a general normalized state,

$$|\psi\rangle = \alpha\,|\uparrow\uparrow\rangle + \beta\,|\uparrow\downarrow\rangle + \gamma\,|\downarrow\uparrow\rangle + \delta\,|\downarrow\downarrow\rangle\,, \qquad |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$$

The reduced density matrix and linear entropy are

$$\rho_1 = \mathrm{Tr}_2\,|\psi\rangle\langle\psi| = \begin{pmatrix} |\alpha|^2 + |\beta|^2 & \alpha\gamma^* + \beta\delta^* \\ \alpha^*\gamma + \beta^*\delta & |\gamma|^2 + |\delta|^2 \end{pmatrix},$$

$$E(|\psi\rangle) = 1 - \mathrm{Tr}_1\rho_1^2 = 2|\alpha\delta - \beta\gamma|^2.$$

Easy to check that

1. It vanishes for a product state.
2. Maximal entanglement is 1/2, which is the case for the Bell states:

$$(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)/\sqrt{2} \qquad (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}$$

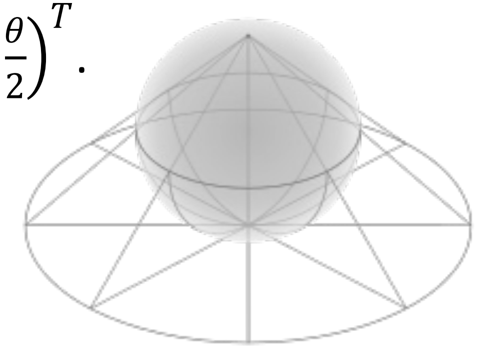
Entanglement power

- Entanglement power: quantifies the ability of an operator U to generate entanglement by averaging over all states obtained by acting it on tensor-product states

$$E(U) = \overline{E\left(U\left|\psi\right\rangle\right)}, \quad \left|\psi\right\rangle = \left|\psi_1\right\rangle \otimes \left|\psi_2\right\rangle.$$

- Entanglement power of S-matrix for 2-body scattering

  - For (iso)spin-1/2 particle, qubit, 2 real parameters for $\mathbb{CP}^1$ manifold, $|\psi\rangle = \left(\cos\frac{\theta}{2}, e^{i\phi}\sin\frac{\theta}{2}\right)^T$.
    For 2-body scattering



$$E(S) = 1 - \int \frac{\mathrm{d}\Omega_1}{4\pi}\frac{\mathrm{d}\Omega_2}{4\pi}\mathrm{Tr}_1[\rho_1^2]$$

  - For two spin-1 particles, we need qutrit, 4 real parameters for $\mathbb{CP}^2$ manifold, $|\psi\rangle = \left(\cos\beta\sin\alpha, e^{i\mu}\sin\beta\sin\alpha, e^{i\nu}\cos\alpha\right)^T$.
    For 2-body scattering

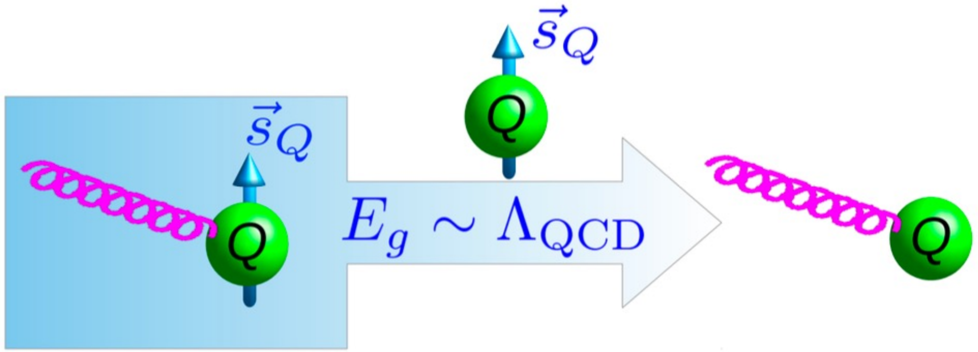
$$E(S) = 1 - \int \mathrm{d}\omega_1\mathrm{d}\omega_2\mathrm{Tr}_1[\rho_1^2],$$

$$\mathrm{d}\omega = (2/\pi^2)\cos\alpha\sin^3\alpha\,\mathrm{d}\alpha\cos\beta\sin\beta\,\mathrm{d}\beta\,\mathrm{d}\mu\,\mathrm{d}\nu$$

I. Bengtsson and K. Życzkowski, Geometry of Quantum States: An Introduction to Quantum Entanglement, 2nd ed. (Cambridge University Press, 2017)

HQSS

- Heavy quark spin symmetry (HQSS)
  - In the heavy quark limit, heavy quark spin decouples
  - Good quantum number s_ℓ: light quark spin + orbital AM



- Consider S-wave interaction between a pair of $s_\ell^P = \frac{1}{2}^-$ (anti-)heavy mesons:

$$0^{++}: \quad D\bar{D}, \quad D^*\bar{D}^*$$

$$1^{+-}: \quad \frac{1}{\sqrt{2}}\left(D\bar{D}^* + D^*\bar{D}\right), \quad D^*\bar{D}^*$$

$$1^{++}: \quad \frac{1}{\sqrt{2}}\left(D\bar{D}^* - D^*\bar{D}\right); \qquad 2^{++}: \quad D^*\bar{D}^*$$

here, phase convention: $D \overset{C}{\to} +\bar{D},\ D^* \overset{C}{\to} -\bar{D}^*$

HQSS for hadronic molecules

- For the HQSS consequences, convenient to use the basis of states: $s_L^{PC} \otimes s_{c\bar{c}}^{PC}$
  - S-wave: $s_L^{PC}, s_{c\bar{c}}^{PC} = 0^{-+}$ or 1^{--}
  - multiplet with $s_L = 0$:

$$0_L^{-+} \otimes 0_{c\bar{c}}^{-+} = 0^{++}, \qquad 0_L^{-+} \otimes 1_{c\bar{c}}^{--} = 1^{+-}$$

  - multiplet with $s_L = 1$:

$$1_L^{--} \otimes 0_{c\bar{c}}^{-+} = 1^{+-}, \qquad 1_L^{--} \otimes 1_{c\bar{c}}^{--} = 0^{++} \oplus \boxed{1^{++}} \oplus 2^{++}$$

Two parameters at LO for each isospin!

- Multiplets in strict heavy quark limit:
  - $X(3872)$ has three partners with 0^{++}, 2^{++} and 1^{+-}

Hidalgo-Duque et al., PLB 727 (2013) 432; Baru et al., PLB 763 (2016) 20

  - Z_b, Z_b' as $B^{(*)}\bar{B}^*$ molecules would imply 6 $I = 1$ hadronic molecules:

$Z_b[1^{+-}], Z_b'[1^{+-}]$ and $W_{b0}[0^{++}], W_{b0}'[0^{++}], W_{b1}[1^{++}]$ and $W_{b2}[2^{++}]$

Bondar et al., PRD 84 (2011) 054010; Voloshin, PRD 84 (2011) 031502; Mehen, Powell, PRD 84 (2011) 114013

Light quark spin symmetry in Z_b resonances? M. Voloshin, PRD 93 (2016) 074011

Heavy-meson scattering

- LO Lagrangians for heavy-meson scattering with HQSS
  - For $D^{(*)}D^{(*)}$scattering S. Fleming, R. Hodges, T. Mehen, PRD 104 (2021) 116010; M.-L. Du et al., PRD 105 (2022) 014024

$$\mathcal{L}_{HH} = -\frac{D_{00}}{8}\mathrm{Tr}\left[H^{a\dagger}H_bH^{b\dagger}H_a\right] - \frac{D_{01}}{8}\mathrm{Tr}\left[H^{a\dagger}H_b\sigma^mH^{b\dagger}H_a\sigma^m\right] - \frac{D_{10}}{8}\mathrm{Tr}\left[H^{a\dagger}H_bH^{c\dagger}H_d\right]\boldsymbol{\tau}_a^d\cdot\boldsymbol{\tau}_c^b - \frac{D_{11}}{8}\mathrm{Tr}\left[H^{a\dagger}H_b\sigma^mH^{c\dagger}H_d\sigma^m\right]\boldsymbol{\tau}_a^d\cdot\boldsymbol{\tau}_c^b$$

  - For $D^{(*)}\bar{D}^{(*)}$scattering

M. T. AlFiky, F. Gabbiani, A. A. Petrov, PLB 640 (2006) 238; J. Nieves, M. P. Valderrama, PRD 86 (2012) 056004; T. Ji et al., PRD 106 (2022) 094002

$$\mathcal{L}_{H\bar{H}} = -\frac{1}{4}\mathrm{Tr}\left[H^{a\dagger}H_b\right]\mathrm{Tr}\left[\bar{H}^c\bar{H}_d^\dagger\right]\left(F_A\delta_a^b\delta_c^d + F_A^\tau\boldsymbol{\tau}_a^b\cdot\boldsymbol{\tau}_c^d\right) + \frac{1}{4}\mathrm{Tr}\left[H^{a\dagger}H_b\sigma^m\right]\mathrm{Tr}\left[\bar{H}^c\bar{H}_d^\dagger\sigma^m\right]\left(F_B\delta_a^b\delta_c^d + F_B^\tau\boldsymbol{\tau}_a^b\cdot\boldsymbol{\tau}_c^d\right)$$

Heavy mesons and anti-heavy mesons: $H_a = P_a + \boldsymbol{P}_a^*\cdot\boldsymbol{\sigma}$, $\bar{H}^a = \bar{P}^a + \bar{\boldsymbol{P}}^{*a}\cdot\boldsymbol{\sigma}$

- Then we compute the entanglement power of the S-matrix, find solutions vanishing it
  - E.g., for the isospin subspace: $E(S_J) = \frac{1}{6}\sin^2[2(\delta_{0J} - \delta_{1J})]$, vanishes for
    - $|\delta_{0J} - \delta_{1J}| = 0$
    - or $|\delta_{0J} - \delta_{1J}| = \frac{\pi}{2}$

Heavy-meson scattering from entanglement suppression

T.-R. Hu, S. Chen, FKG, Phys. Rev. D 110, 014002 (2024)

- Input: $X(3872)$ as isoscalar $D\bar{D}^*$ molecule with $J^{PC} = 1^{++}$: $\bar{\delta}_{01+} = \pi/2$
  - Two possible solutions

TABLE II. Partners of the $X(3872)$ predicted by HQSS or the two solutions of entanglement suppression given in Eqs. (53) and (54). The symbol "$\odot$" denotes the input $X(3872)$, "$\otimes$" represents its predicted partners, "$\oslash$" indicates no near-threshold state is allowed, and "–" signifies that no prediction can be made without further inputs. Moreover, "$\oplus$" means that the corresponding meson pair needs to be mixed with another one to get a spin partner of $X(3872)$, see Eqs. (57) and (58).

Channel	HQSS		Eq. (53) predictions		Eq. (54) predictions	
	$I=0$	$I=1$	$I=0$	$I=1$	$I=0$	$I=1$
$D\bar{D}(0^{++})$	$\oplus$	–	$\otimes$	$\oslash$	$\otimes$	$\otimes$
$D\bar{D}^*(1^{++})$	$\odot$	–	$\odot$	$\oslash$	$\odot$	$\otimes$
$D\bar{D}^*(1^{+-})$	$\oplus$	–	$\otimes$	$\oslash$	$\otimes$	$\otimes$
$D^*\bar{D}^*(0^{++})$	$\oplus$	–	$\otimes$	$\oslash$	$\otimes$	$\otimes$
$D^*\bar{D}^*(1^{+-})$	$\oplus$	–	$\otimes$	$\oslash$	$\otimes$	$\otimes$
$D^*\bar{D}^*(2^{++})$	$\otimes$	–	$\otimes$	$\oslash$	$\otimes$	$\otimes$

Heavy-meson scattering from entanglement suppression

T.-R. Hu, S. Chen, FKG, Phys. Rev. D 110, 014002 (2024)

- Input: $T_{cc}(3875)$ as isoscalar DD^* molecule: $\delta_{01} = \pi/2$
  - Two possible solutions

TABLE I. Partners of the $T_{cc}(3875)^+$ predicted by HQSS or the two solutions of entanglement suppression given in Eqs. (49) and (50). The symbol "⊙" denotes the input $T_{cc}(3875)^+$ state, "⊗" represents its predicted partners, "⊘" indicates that no near-threshold state is allowed (the red ones are forbidden by Bose-Einstein statistics), and "–" signifies that no prediction can be made without further inputs.

Channel	HQSS		Eq. (49) predictions		Eq. (50) predictions	
	$I=0$	$I=1$	$I=0$	$I=1$	$I=0$	$I=1$
$DD(0^+)$	⊘	–	⊘	⊘	⊘	⊗
$D^*D(1^+)$	⊙	–	⊙	⊘	⊙	⊗
$D^*D^*(0^+)$	⊘	–	⊘	⊘	⊘	⊗
$D^*D^*(1^+)$	⊗	⊘	⊗	⊘	⊗	⊘
$D^*D^*(2^+)$	⊘	–	⊘	⊘	⊘	⊗

Heavy-meson scattering from entanglement suppression

T.-R. Hu, S. Chen, FKG, Phys. Rev. D 110, 014002 (2024)

- In both cases, for a given isospin, all possible heavy-meson pairs allowed by Bose-Einstein statistics either at the unitary limit (molecules at threshold) or at the noninteracting limit
  - Meson pairs with different s_ℓ have the same interaction strengths
  - $\Rightarrow$ light quark spin symmetry !
    - SU(2)×SU(2) is enlarged to SU(4)

$$\begin{pmatrix} |0,0\rangle \\ |1,+1\rangle \\ |1,0\rangle \\ |1,-1\rangle \end{pmatrix}$$

- X(3872) has 5 isoscalar partners (v.s. 3 partners predicted with only HQSS)

☞ multiplet with $s_L = 1$:

$$1_L^{--} \otimes 0_{c\bar{c}}^{-+} = 1^{+-}, \qquad 1_L^{--} \otimes 1_{c\bar{c}}^{--} = 0^{++} \oplus \boxed{1^{++}} \oplus 2^{++}$$

- Can we conclude on the existence of isovector hidden-charm molecular state?

One excellent observable for distinguishing different models for X

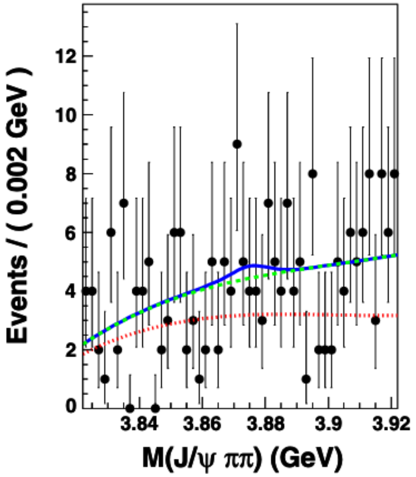
- Isospin-1 partners!
  - No, in charmonium model
  - Quark bound states, in compact tetraquark model L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, PRD 71 (2005) 014028
    - With isospin-independent quark interactions, isoscalar and isovector tetraquarks must coexist

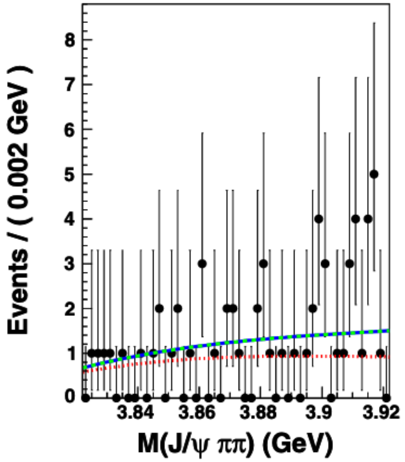
$$I = 1 \text{ multiplet: } [cu][\bar{c}\bar{d}],\ \frac{1}{\sqrt{2}}([cu][\bar{c}\bar{u}] - [cu][\bar{c}\bar{d}]),\ [cd][\bar{c}\bar{u}]$$

  - How about hadronic molecular model?
- $I = 1, J^{PC} = 1^{++}$ states will be called W_{c1}, following the notation by Voloshin M. Voloshin, PRD 84 (2011) 031502

So far negative signal

- No signal in the charged channel so far

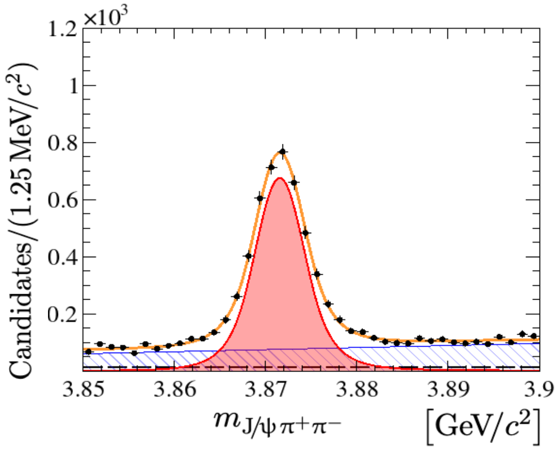




$\bar{B}^0 \to K^- \pi^+ \pi^0 J/\psi$ $B^+ \to K^0 \pi^+ \pi^0 J/\psi$

Belle, PRD 84 (2011) 052004

- No signal around the $D^+ D^{*-}$ threshold

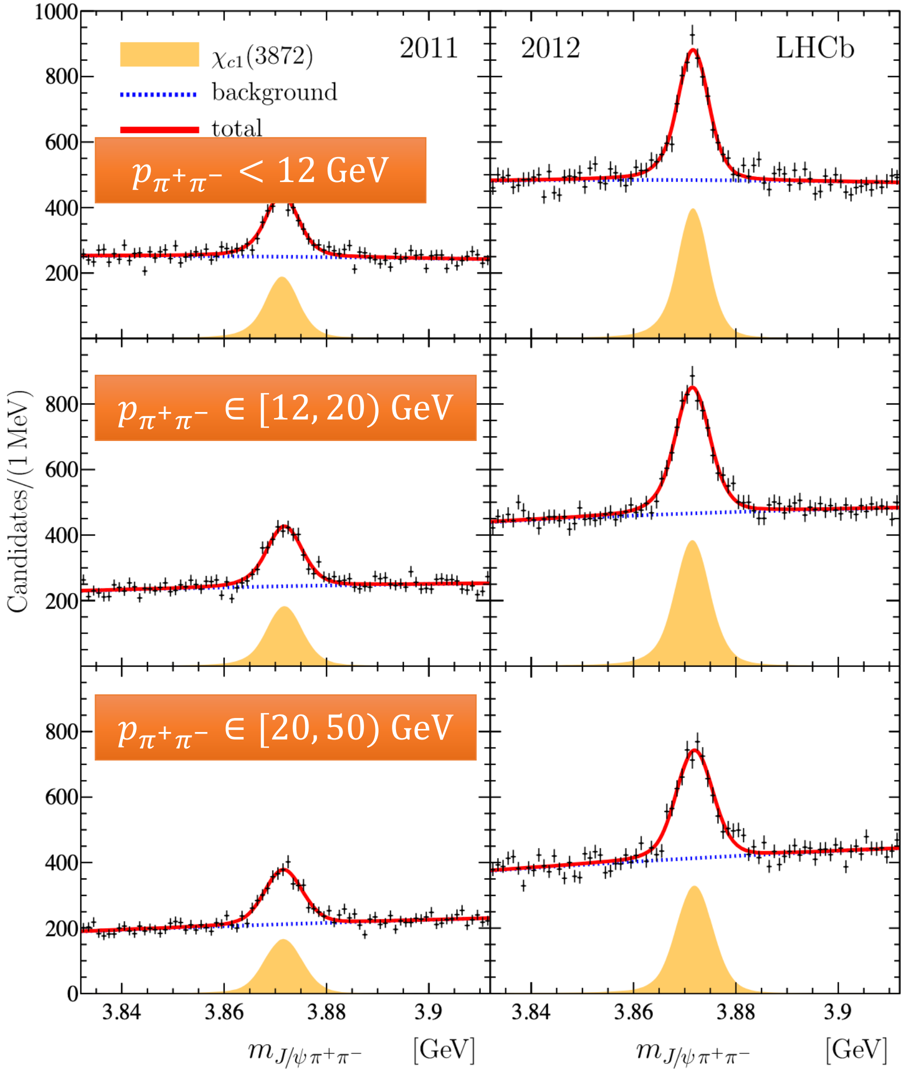


$B^+ \to K^+ \pi^+ \pi^- J/\psi$

LHCb, JHEP 08 (2020) 123

$\pi^+\pi^- J/\psi$ from b-hadrons

LHCb, PRD 102 (2020) 092005



Prediction of an isospin vector partner of $X(3872)$

Z.-H. Zhang, T. Ji, X.-K. Dong, FKG, U.-G. Meißner, A. Rusetsky, arXiv:2404.11215

- How about the $D\bar{D}^*$ hadronic molecular scenario?
- $D^0\bar{D}^{*0}, D^+D^{*-}$ coupled channels: $I = 0, 1$
  - Interactions at leading order: two LECs ($I = 0, 1$) C_{0X}, C_{1X}
  - Two inputs from $X(3872)$ properties

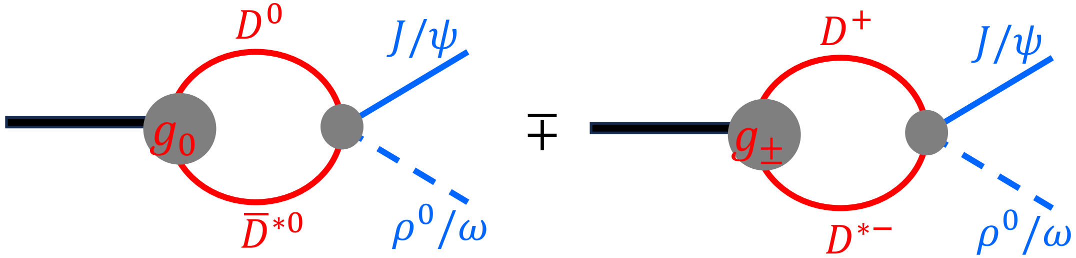
➢ Mass

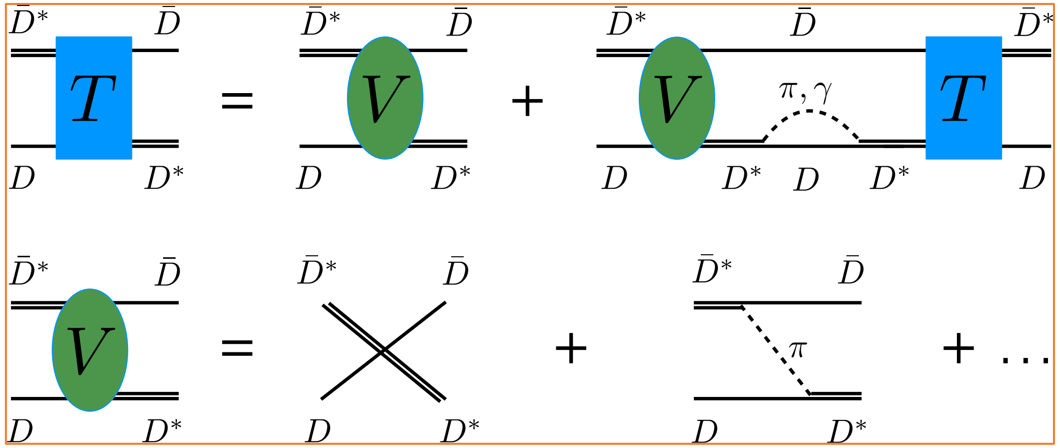
$$M_X = 3871.69^{+0.00+0.05}_{-0.04-0.13}\,\text{MeV}$$ LHCb, PRD 102 (2020) 092005

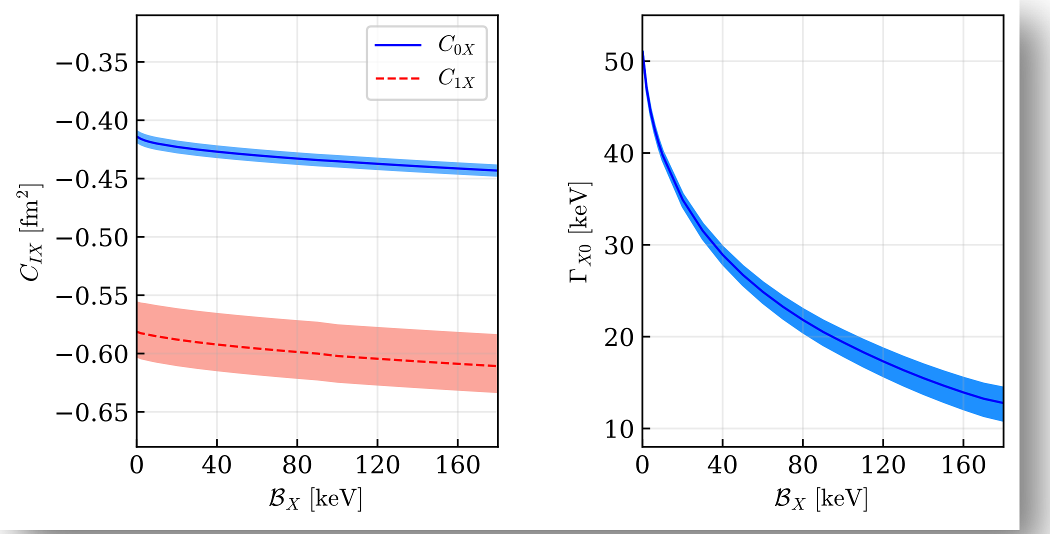
$$M_{D^0} + M_{D^{*0}} = 3871.69(7)\ \text{MeV}$$ PDG 2023

➢ Isospin breaking in decays LHCb, PRD 108 (2023) L011103

$$R_X = \left|\frac{\mathcal{M}_{X(3872)\to J/\psi\rho^0}}{\mathcal{M}_{X(3872)\to J/\psi\omega}}\right| = 0.29 \pm 0.04 = \left|\frac{g_0 - g_\pm}{g_0 + g_\pm}\right|$$







$$\mathcal{B}_X \equiv M_{D^0} + M_{D^{*0}} - M_{X(3872)}$$

Prediction of an isospin vector partner of $X(3872)$

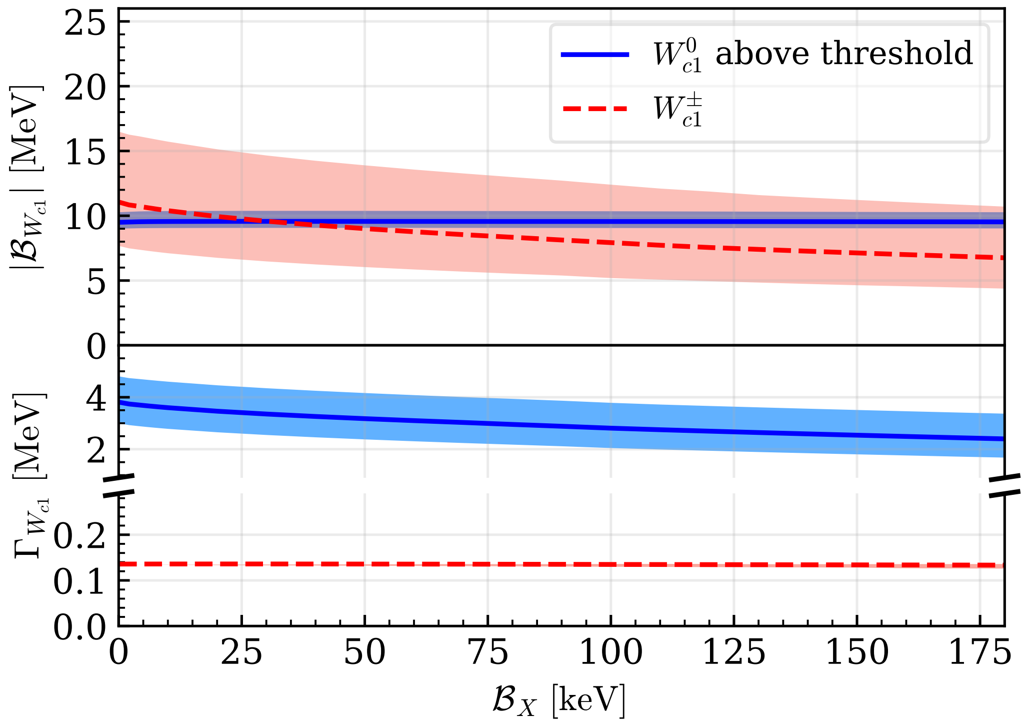
Z.-H. Zhang, T. Ji, X.-K. Dong, FKG, C. Hanhart, U.-G. Meißner, A. Rusetsky, arXiv:2404.11215

- There must be near-threshold isovector W_{c1} states
  - Virtual state pole in the stable D^* limit
    - W_{c1}^{+} in $D^+\bar{D}^{*0}$ single-channel scattering amplitude: pole on the 2nd Riemann sheet (RS), 8^{+8}_{-5} MeV below $D^0 D^{*-}$ threshold

$$W_{c1}^{\pm}: \quad 3866.9^{+4.6}_{-7.7} - i(0.07 \pm 0.01)\ \text{MeV}$$

    - W_{c1}^{0} in $(D\bar{D}^*)_0 - (D\bar{D}^*)_{\pm}$ scattering amplitudes: pole on the 4th RS (RS_{+-}), $1.3^{+0.8}_{-0.0}$ MeV above D^+D^{*-} threshold

$$W_{c1}^{0}: \quad 3881.2^{+0.8}_{-0.0} + i1.6^{+0.7}_{-0.9}\ \text{MeV}$$



Cutoff independence checked: pole positions relative to thresholds changed within 5% for $\Lambda \in [0.5, 1.0]$ GeV

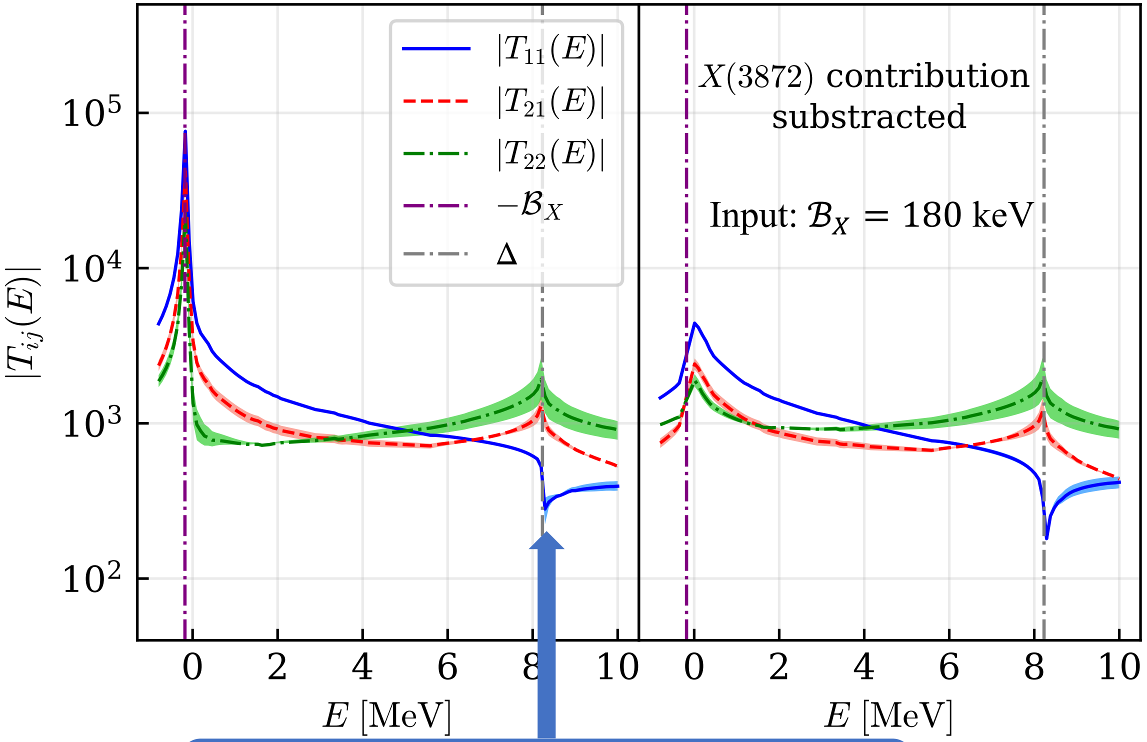
  - Must appear as threshold cusps!!!
  - Compact tetraquarks (Maiani et al. (2005)) cannot be virtual states as they do not feel the thresholds

Why have they not been observed?

Z.-H. Zhang, T. Ji, X.-K. Dong, FKG, C. Hanhart, U.-G. Meißner, A. Rusetsky, arXiv:2404.11215

- The observed $X(3872)$ signals should contain the W_{c1}^0 contribution (but marginal) as well
- W_{c1}^0 lives in the same amplitudes as the $X(3872)$, effects shielded by X

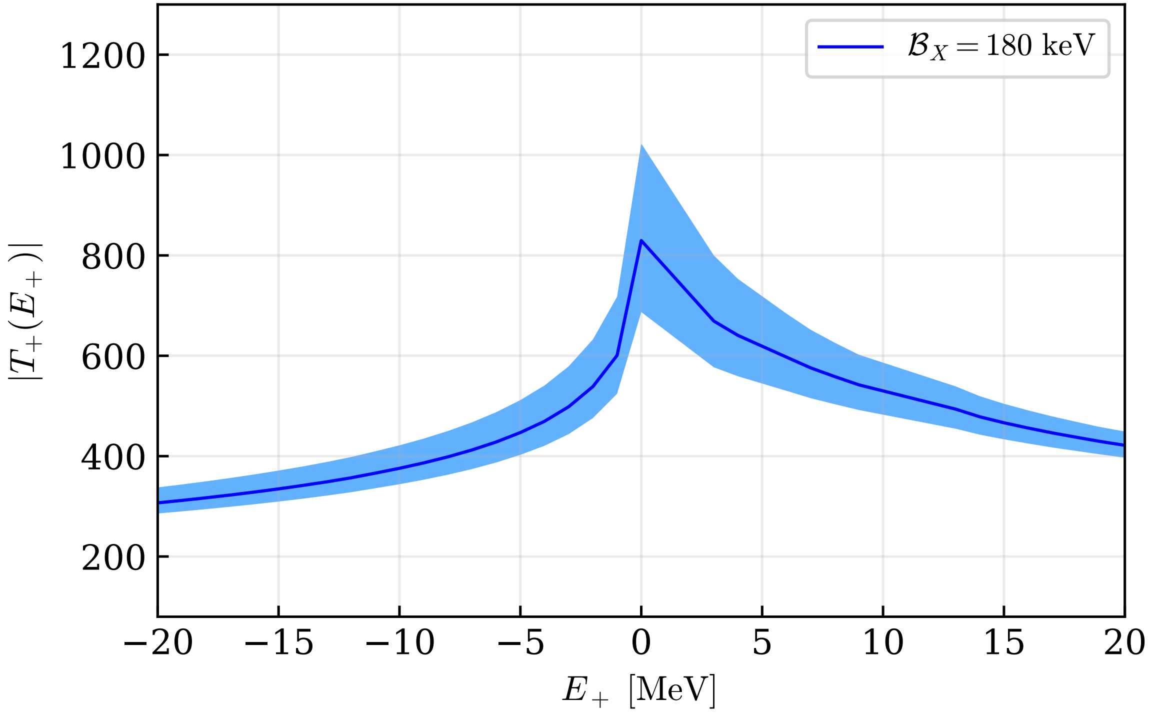
➢ W_{c1}^0 in $D^0\bar{D}^{*0} - D^+D^{*-}$ scattering amplitudes



Threshold cusp!
peak or dip depends on processes

X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

➢ W_{c1}^+ in $D^+\bar{D}^{*0}$ scattering amplitude: height much lower than the X peak

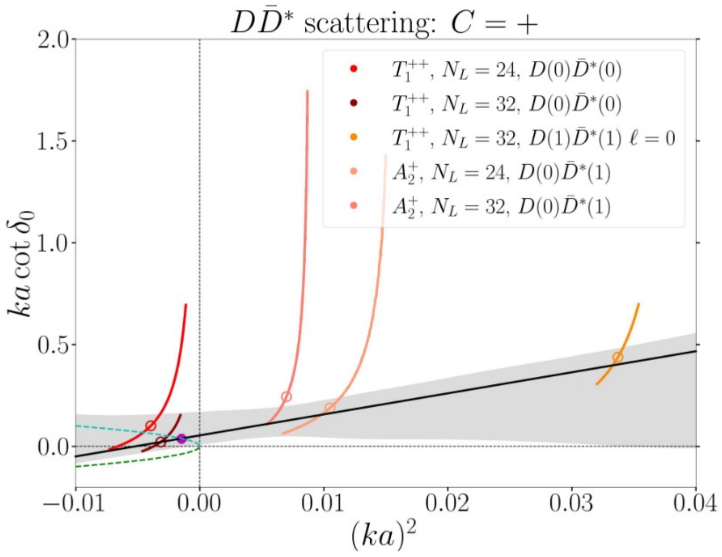


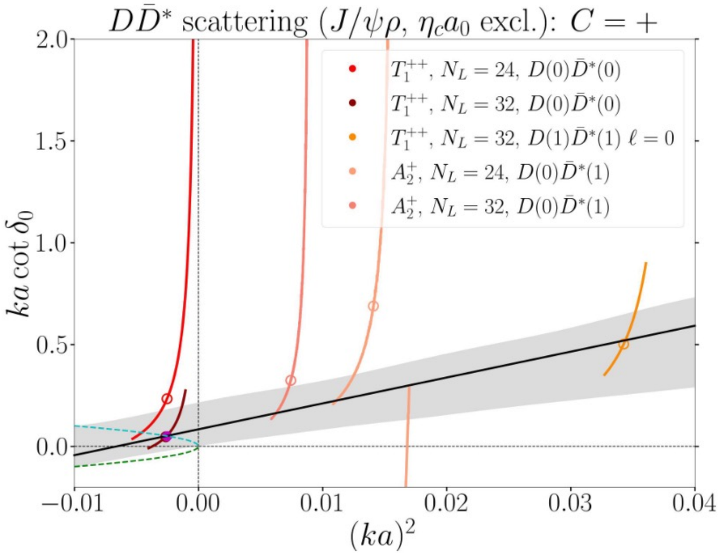
➢ should be searched for in high-statistic $J/\psi\pi^\pm\pi^0$ data

Confirmation from lattice QCD

- The virtual state W_{c1} was confirmed in a very recent lattice QCD calculation with $M_\pi = 280$ MeV

M. Sadl, S. Collins, Z.-H. Guo, M. Padmanath, S. Prelovsek, L.-W. Yan, arXiv:2406.09842 [hep-lat]





J^{PC}	interpolators	$1/a_0$ [fm^{-1}]	r_0 [fm]	$\chi^2/N_{\rm dof}$	$\Delta m_{\rm V}$ [MeV]
1^{+-}	all	$0.46^{+1.16}_{-0.45}$	$0.96^{+0.43}_{-0.73}$	0.13	$-3.0^{+3.0}_{-31.1}$
	$\eta_c\rho$ excl.	$0.54^{+1.07}_{-0.44}$	$2.23^{+0.95}_{-1.08}$	0.24	$-2.8^{+2.6}_{-17.1}$
1^{++}	all	$0.62^{+1.30}_{-0.51}$	$1.78^{+0.25}_{-2.44}$	0.18	$-3.8^{+3.6}_{\rm a}$
	$J/\psi\rho$, $\eta_c a_0$ excl.	$0.96^{+1.42}_{-0.91}$	$2.19^{+0.36}_{-1.00}$	0.15	$-6.7^{+6.7}_{-19.5}$

[a] Uncertainty is so large that it is unbounded from below.

$$\Delta m_{\rm V} = E^{\rm p}_{\rm cm} - m_D - m_{D^*}.$$

versus our prediction: -8^{+5}_{-8} MeV

Summary

- The entanglement suppression conjecture + $X(3872)$ and T_{cc} as molecules leads to an emergent symmetry for low-energy heavy-meson scattering: light quark spin symmetry

- Robust prediction of an isovector partner of $X(3872)$ in the hadronic molecular picture: $W_{c1}^{\pm,0}$
  - Virtual state poles (poles on RSs not directly connected to physical region), thus threshold cusps
  - Search for threshold cusps in high-statistic data
    - at D^0D^{*-} threshold in $J/\psi\pi^{\pm}\pi^0$, such as $e^+e^- \to \rho^{\mp} + J/\psi\pi^{\pm}\pi^0$: distinguishing hadronic molecular and compact tetraquark models for $X(3872)$
    - at D^+D^{*-} threshold in $J/\psi\pi^+\pi^-$, such as $e^+e^- \to \gamma/V + J/\psi\pi^+\pi^-$: could be more difficult due to interference between peak and dip

Thank you for your attention!