束流横向极化和超子CP破坏
Probe CP violation of hyperon by transversely polarized beams

曹　须

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arXiv: 2404.00298

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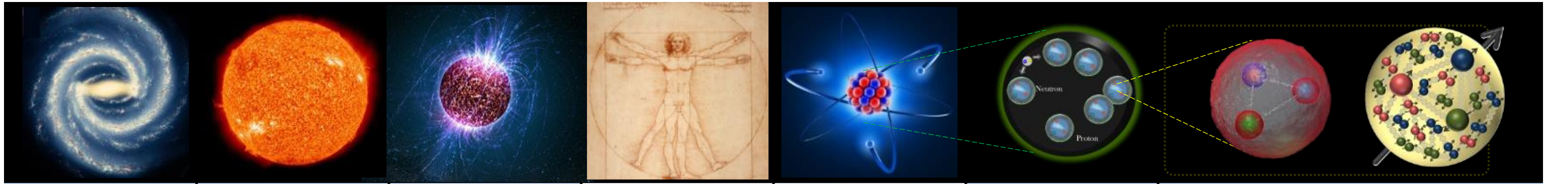
7月7日~11日，兰州大学

Introduction

Gravity (Strength = 10^{-40})	Electro-Magnetic (1)	Strong force (137); Weak force(10^{-8})



Galaxy ~10^{21}m	Sun ~10^{9}m	Neutron Star ~10^{4}m	Human ~10^{0}m	Atom 10^{-10}m	Nuclei $10^{-15\,-\,14}$米	Nucleon（10^{-15}米）: Quarks	Nucleon（10^{-15}米）: Gluons

- *Why does our visiable material world have a hierarchical structure?*
- Before that:

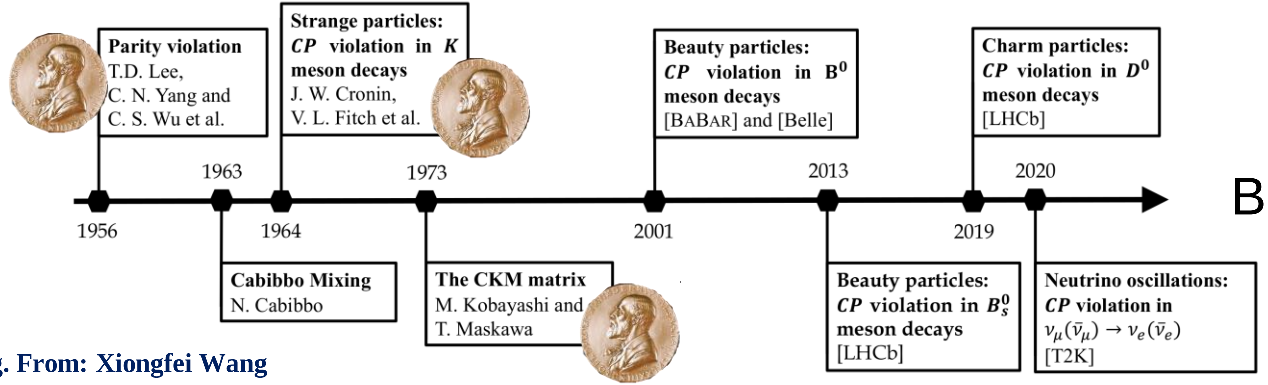


Fig. From: Xiongfei Wang

Baryon

Introduction

- R. Hofstadter, Rev. Mod. Phys. 1956，28：214
- 1961 Nobel Prize in Physics (together with Rudolf Mössbauer)
- "for his pioneering studies of electron scattering in atomic nuclei and for his consequent discoveries concerning the structure of nucleons"

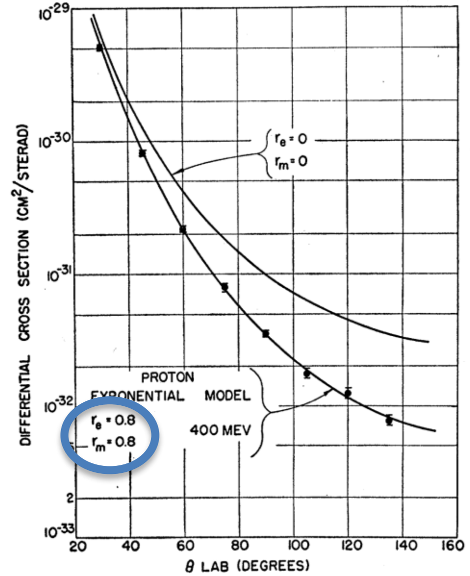
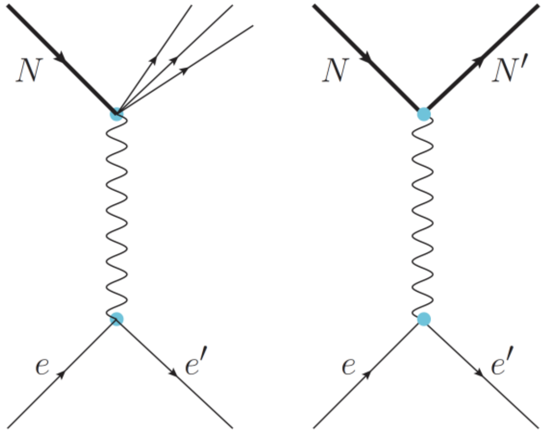


FIG. 26. Typical angular distribution for elastic scattering of 400-Mev electrons against protons. The solid line is a theoretical curve for a proton of finite extent. The model providing the theoretical curve is an exponential with rms radii=0.80×10⁻¹³ cm.

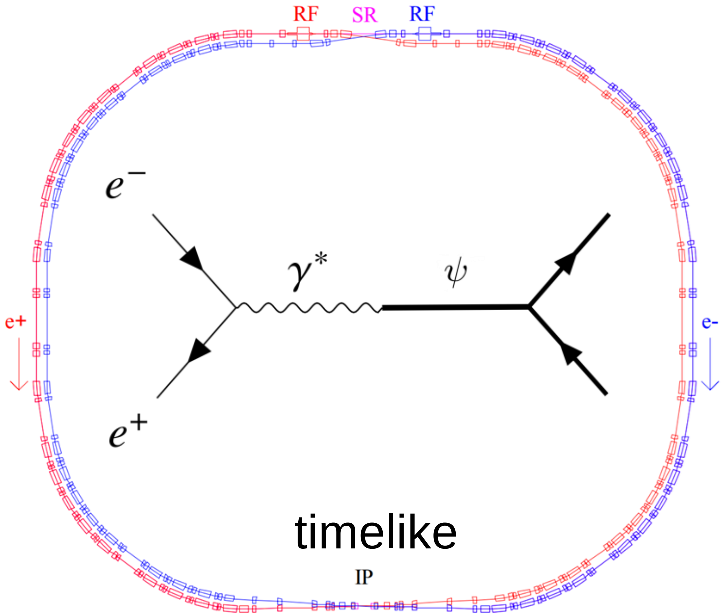


spacelike

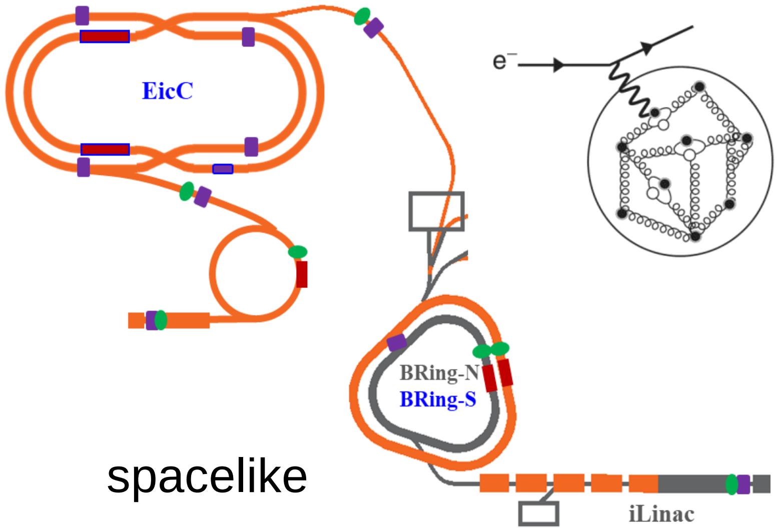
$$r^2_{E/M} = -\frac{6}{G_{E/M}(0)}\frac{dG_{E/M}(Q^2)}{dQ^2}\Big|_{Q^2=0} = (0.84\ \text{fm})^2$$

- **Large Scale Scientific Facility**

hyperon target/beams: difficult if not impossible



The BEPCII Complexity
….and future STCF



曹须等，核技术，43(2): 020001 (2020)
曹须等,中国科学: 物理学力学天文学,50:112005(2020)
D. P. Anderle, V. Bertone, **Xu Cao**, *et al.*, Front. Phys. 16, 64701 (2021)

Introduction

- Surprisingly, study of polarization effects in timelike region appears until 1996
- A. Z. Dubnickova, S. Dubnicka, and M. P. Rekalo, Nuovo Cim. A 109, 241 (1996)
- S. J. Brodsky, C. E. Carlson, J. R. Hiller, and D. S. Hwang, Phys. Rev. D 69, 054022 (2004)
- E. Tomasi-Gustafsson, F. Lacroix, C. Duterte, and G. I. Gakh, Eur. Phys. J. A 24, 419 (2005)
- H. Chen and R.-G. Ping, Phys. Rev. D 76, 036005 (2007)
- G. Fäldt, A. Kupsc, Phys. Lett. B 772, 16 (2017)

finally leading to the most precise test of hyperon CP violation at BESIII: Nature 606, 64 (2022)

- …… polarization observables are totally different between:

	space-like (lab.)	time-like (c.m.)
Unpolarized	$\frac{d\sigma}{d\Omega}_e = \frac{d\sigma_M}{d\Omega_e}\left[2\tau G_M^2 \tan^2(\theta_e/2) + \frac{G_E^2+\tau G_M^2}{1+\tau}\right]$	$\frac{d\sigma}{d\Omega} = \frac{d\sigma_M}{d\Omega}\left[2\tau\lvert G_M\rvert^2\frac{\cot^2\theta}{1+\tau} + \frac{\lvert G_M\rvert^2+\tau\lvert G_E\rvert^2}{1+\tau}\right]$
Long. electron	$\frac{P_t}{P_\ell} = -2\cot(\theta_e/2)\frac{M_p}{\epsilon_1+\epsilon_2}\frac{G_E}{G_M}$	$\mathbf{P}_B = \frac{\gamma_\psi P_e \sin\theta\hat{\mathbf{x}}_1 - \beta_\psi \sin\theta\cos\theta\hat{\mathbf{y}}_1 - (1+\alpha_\psi)P_e\cos\theta\hat{\mathbf{z}}_1}{1+\alpha_\psi\cos^2\theta}$
Long. both beams	$A = -\frac{2\sqrt{\tau(1+\tau)}\tan(\theta_e/2)}{G_E^2+\frac{\tau}{\epsilon}G_M^2}\left[\sin\theta^*\cos\phi^* G_E G_M + \sqrt{\tau[1+(1+\tau)\tan^2(\theta_e/2)]}\cos\theta^* G_M^2\right].$	Nothing New

- How about Transversely Polarized Beams in time-like region?

Introduction

- **Hyperon decay as a polarimeter:**

T. D. Lee and C.-N. Yang, Phys. Rev. 108, 1645 (1957)

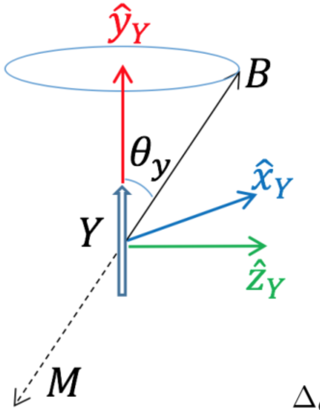
- The amplitude for a spin-1/2 hyperon decaying into a spin-1/2 baryon and a spin-0 meson:

$$M = G_F\, m_\pi^2 \cdot \overline{B}_f\, (A - B\gamma_5)\, B_i$$

- Decay parameters:

$$\alpha = 2\,\mathrm{Re}(s^* p)/(|s|^2 + |p|^2)\ ,$$
$$\beta = 2\,\mathrm{Im}(s^* p)/(|s|^2 + |p|^2)\ ,$$
$$\gamma = (|s|^2 - |p|^2)/(|s|^2 + |p|^2)\ ,$$

where $s = A$ and $p = |\mathbf{p}_f|B/(E_f + m_f)$



- Lee-Yang formula

$$\mathbf{P}_\Lambda = \frac{(\alpha_\Xi + \mathbf{P}_\Xi \cdot \hat{\mathbf{n}})\hat{\mathbf{n}} + \beta_\Xi \mathbf{P}_\Xi \times \hat{\mathbf{n}} + \gamma_\Xi \hat{\mathbf{n}} \times (\mathbf{P}_\Xi \times \hat{\mathbf{n}})}{1 + \alpha_\Xi \mathbf{P}_\Xi \cdot \hat{\mathbf{n}}}$$

- **Strong and weak phase**

J. F. Donoghue and S. Pakvasa, Phys. Rev. Lett. 55, 162 (1985)

J. F. Donoghue, X.-G. He, and S. Pakvasa, Phys. Rev. D 34, 833 (1986)

- The transition amplitudes L = S, P of hyperon can be decomposed as

$$L = \sum_j L_j \exp\{i(\xi_j^L + \delta_j^L)\}$$

while for the the antihyperon c.c. decay

$$\bar{S} = -\sum_j S_j \exp\{i(-\xi_j^S + \delta_{2I}^S)\}$$

$$\bar{P} = \sum_j P_j \exp\{i(-\xi_j^P + \delta_{2I}^P)\},$$

Leading to hyperon CT violation test

$$\Delta_{CP} = \frac{\Gamma_1 + \Gamma_2}{\Gamma_1 - \Gamma_2} \sim L^1 L^3 \sin(\delta_L^1 - \delta_L^3) \sin(\xi_L^1 - \xi_L^3)$$

$$A_{CP} = \frac{\alpha_1 + \alpha_2}{\alpha_1 - \alpha_2} = -\tan(\delta_P - \delta_S)\tan(\xi_P - \xi_S)$$

$$B_{CP} = \frac{\beta_1 + \beta_2}{\alpha_1 - \alpha_2} = -\tan(\xi_P - \xi_S)$$

Introduction

- Known Fact of Unpolarized Beams: measure the hyperon/anti-hyperon decay simultaneously

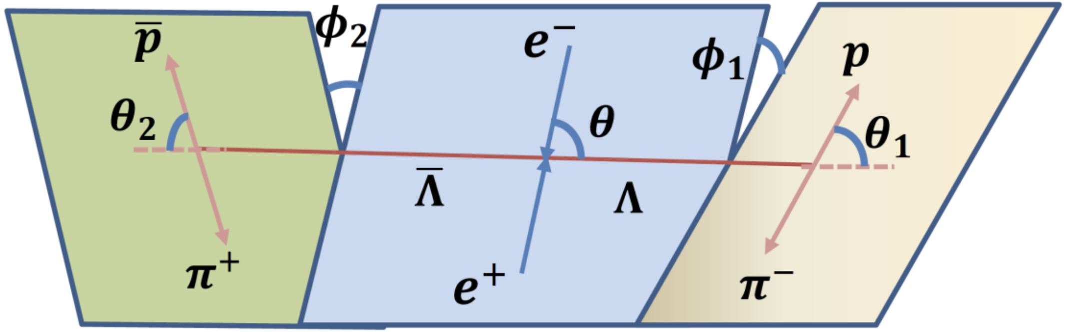
H. Chen, R.-G. Ping, Phys. Rev. D 76, 036005 (2007)

Göran Fäldt, Andrzej Kupsc, Phys.Lett.B 772, 16 (2017)

$$P_y^B = \frac{\sqrt{1-\alpha_\psi^2}\sin\Delta\Phi\sin\theta\cos\theta}{1+\alpha_\psi\cos^2\theta}$$

$$C_{xz}^B = \frac{\sqrt{1-\alpha_\psi^2}\cos\Delta\Phi\sin\theta\cos\theta}{1+\alpha_\psi\cos^2\theta}$$

$$\Delta\Phi = \arg\frac{G_E^B}{G_M^B}$$



$$\mathcal{W}(\xi) = \mathcal{F}_0(\xi) + \sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi)(\alpha_2\cdot\mathcal{F}_3 - \alpha_1\cdot\mathcal{F}_4)$$
$$+\alpha_1\alpha_2(\mathcal{F}_1 + \sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi)\cdot\mathcal{F}_2 + \alpha_\psi\cdot\mathcal{F}_5)$$

$$A_{CP} = \frac{\alpha_1+\alpha_2}{\alpha_1-\alpha_2} = -\tan(\delta_P-\delta_S)\tan(\xi_P-\xi_S)$$

$$\mathcal{F}_0(\xi) = 1+\alpha_\psi\cos^2\theta,$$
$$\mathcal{F}_1(\xi) = \sin^2\theta\sin\theta_1\cos\varphi_1\sin\theta_2\cos\varphi_2 - \cos\theta^2\cos\theta_1\cos\theta_2,$$
$$\mathcal{F}_2(\xi) = \sin\theta\cos\theta(\sin\theta_1\cos\theta_2\cos\varphi_1 - \cos\theta_1\sin\theta_2\cos\varphi_2),$$
$$\mathcal{F}_3(\xi) = \sin\theta\cos\theta\sin\theta_2\sin\varphi_2,$$
$$\mathcal{F}_4(\xi) = \sin\theta\cos\theta\sin\theta_1\sin\varphi_1,$$
$$\mathcal{F}_5(\xi) = \sin^2\theta\sin\theta_1\sin\varphi_1\sin\theta_2\sin\varphi_2 - \cos\theta_1\cos\theta_2,$$

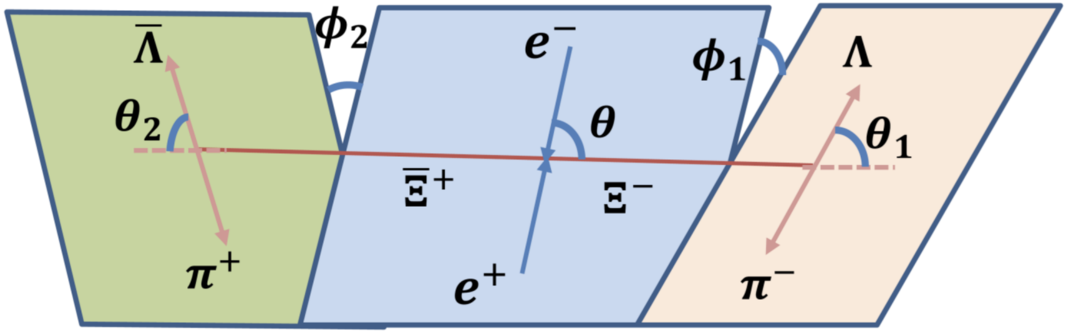
Introduction

- Known Fact of Unpolarized Beams: measure the hyperon/anti-hyperon decay simultaneously

E. Perotti, G. Fäldt, A. Kupsc, *et al.*, Phys. Rev. D 99,056008 (2019)

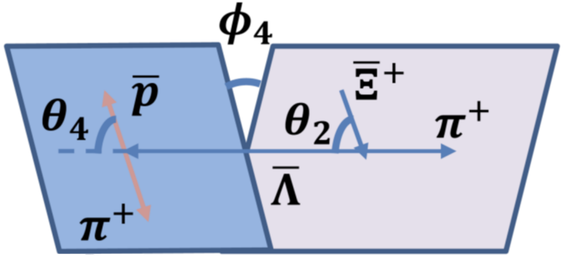
P.-C. Hong, R.-G. Ping, T. Luo, X.-R. Zhou, H. Li,

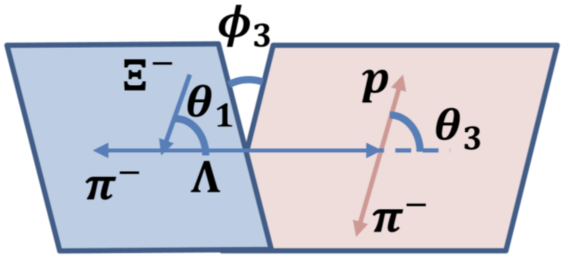
Chin. Phys. C 47, 093103 (2023)



Scattering angle only, no info. in azimuthal angle?

Cascade decay is much more complicated:





$$\mathcal{W}(\boldsymbol{\xi}) = \mathcal{F}_0'(\xi) + \mathcal{F}_{\Xi,\Delta\Phi}(\alpha_2 \cdot \mathcal{F}_3' - \alpha_1 \cdot \mathcal{F}_4') + \alpha_1\alpha_2\mathcal{F}_{\Xi,\Delta\Phi}'$$

$$B_{CP} = \frac{\beta_1 + \beta_2}{\alpha_1 - \alpha_2} = -\tan(\xi_P - \xi_S)$$

The SM prediction for $A_{\rm CP}^{[\Lambda p]}$ is $\sim(1\text{–}5)\times 10^{-5}$, while for $B_{\rm CP}^{[\Xi-]}$, it amounts to $\mathcal{O}(10^{-4})$

	$\sigma(A_{\rm CP}^{[\Lambda p]})$	$\sigma(A_{\rm CP}^{[\Xi-]})$	$\sigma(B_{\rm CP}^{[\Xi-]})$	Comment
BESIII	1.0×10^{-2}	1.3×10^{-2}	3.5×10^{-2}	1.3×10^{9} J/ψ
BESIII	3.6×10^{-3}	4.8×10^{-3}	1.3×10^{-2}	1.0×10^{10} J/ψ (projection)
SCTF	2.0×10^{-4}	2.6×10^{-4}	6.8×10^{-4}	3.4×10^{12} J/ψ (projection)

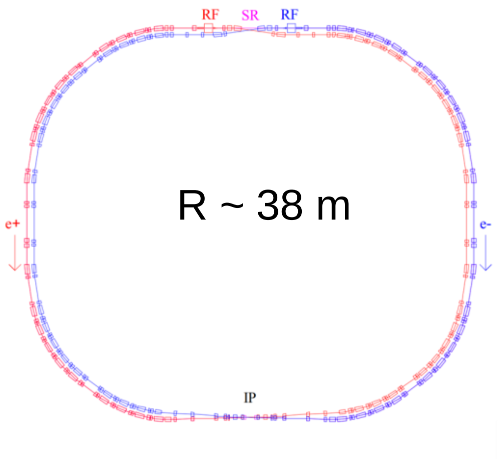
Probe CP violation via Transversely Polarized beams

- Known Fact of Lepton Beams at circular colliders: Sokolov-Ternov effect

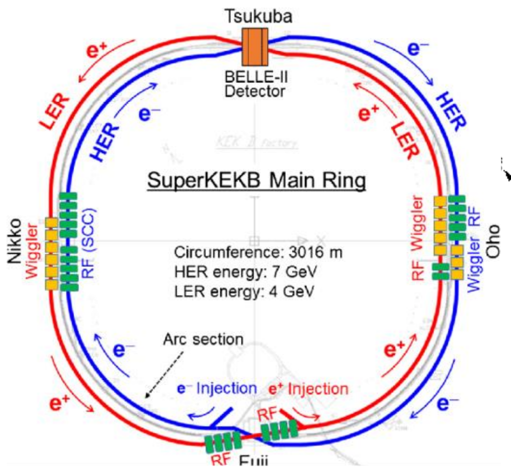
The self-polarization of relativistic electrons or positrons moving in a magnetic field at a storage ring occurs through the emission of spin-flip synchrotron radiation

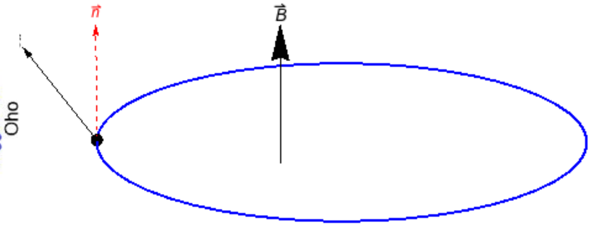
A. A. Sokolov and I. M. Ternov, Dokl. Akad. Nauk SSSR 153, 1052 (1963)

V. N. Baier and V. S. Fadin, Sov. Phys. Dokl. 10, 204 (1965); J. D. Jackson, Rev. Mod. Phys. 48, 417 (1976)



The BEPCII Complexity

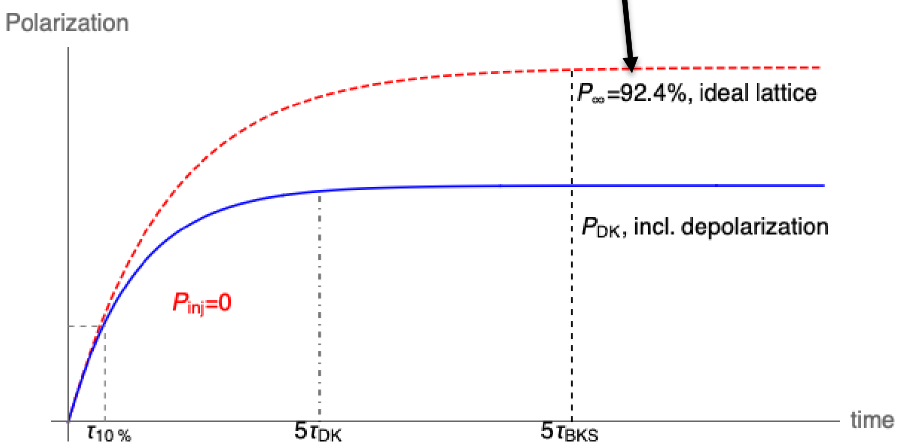




Courtery: 段哲@IHEP

- Zhe Duan@IHEP: thesis
  - Sokolov-Ternov效应引起的束流极化建立时间在1.84 GeV 时约为 4.3 个小时，而在 2.0GeV 时约为 2.8 小时

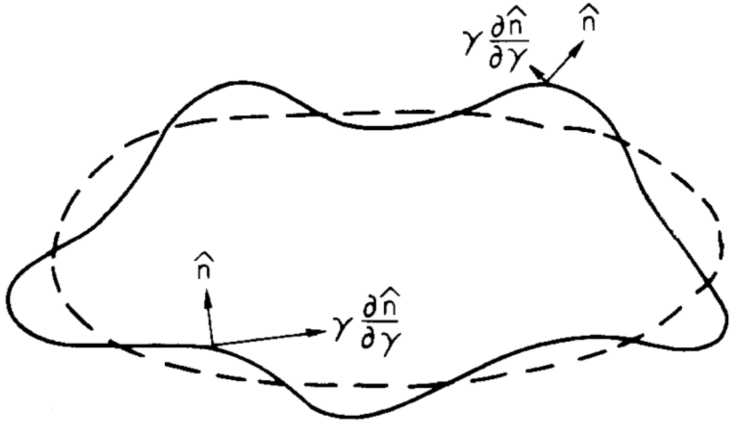
$$P_0\left(1-e^{-t/t_0}\right) \qquad t_0=\left[\frac{5\sqrt{3}}{8}\frac{e^2\hbar\gamma^5}{m^2c^2\rho^3}\right]^{-1}$$



- Alexander W Chao(赵午)：
  - SLAC-PUB-2781(1981)
    POLARIZATION OF A STORED ELECTRON BEAM

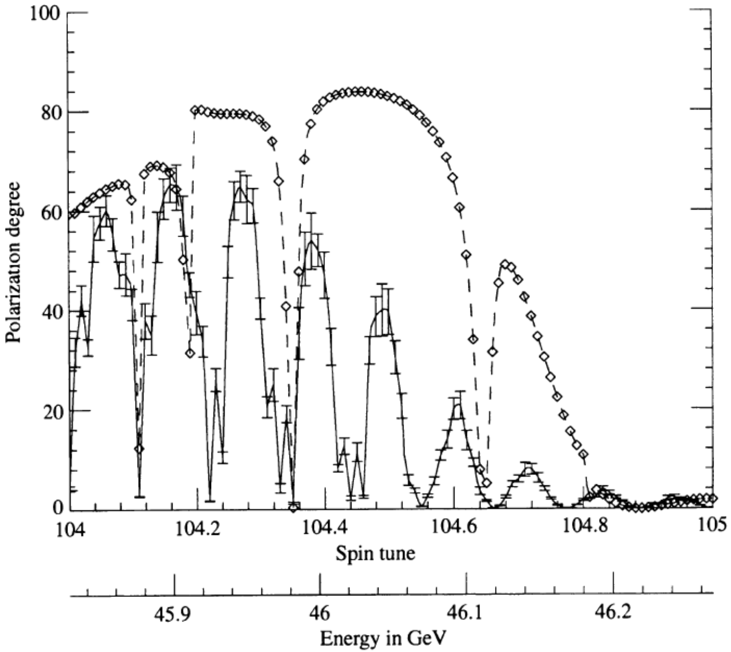
$$E_e=m_e\gamma c^2=\frac{m_ec^2}{G_e}N\simeq 440.5\cdot N \quad \text{MeV},$$

$$G_e\simeq 0.00116 \quad \text{the gyromagnetic anomaly}$$

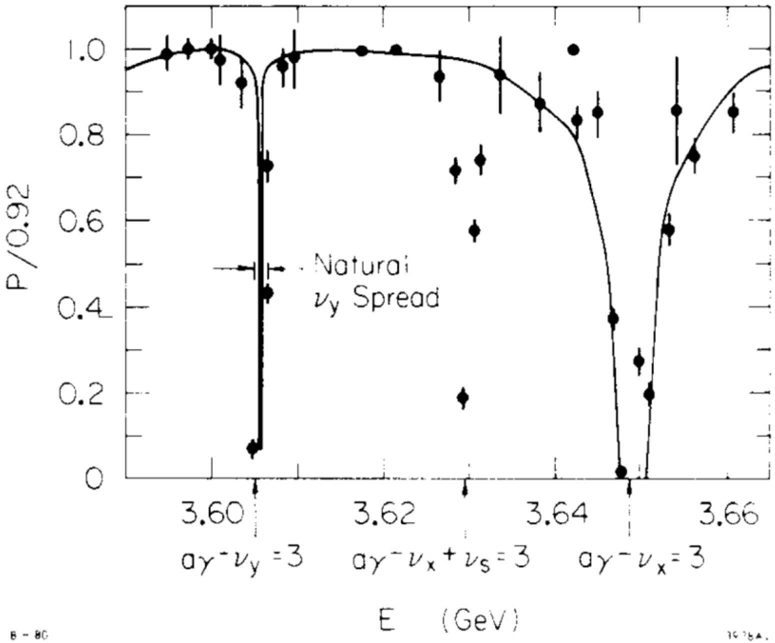


Probe CP violation via Transversely Polarized beams

- Elliot Leader: Spin in particle physics
- LEP:
  The Large Electron-Positron Collider at CERN

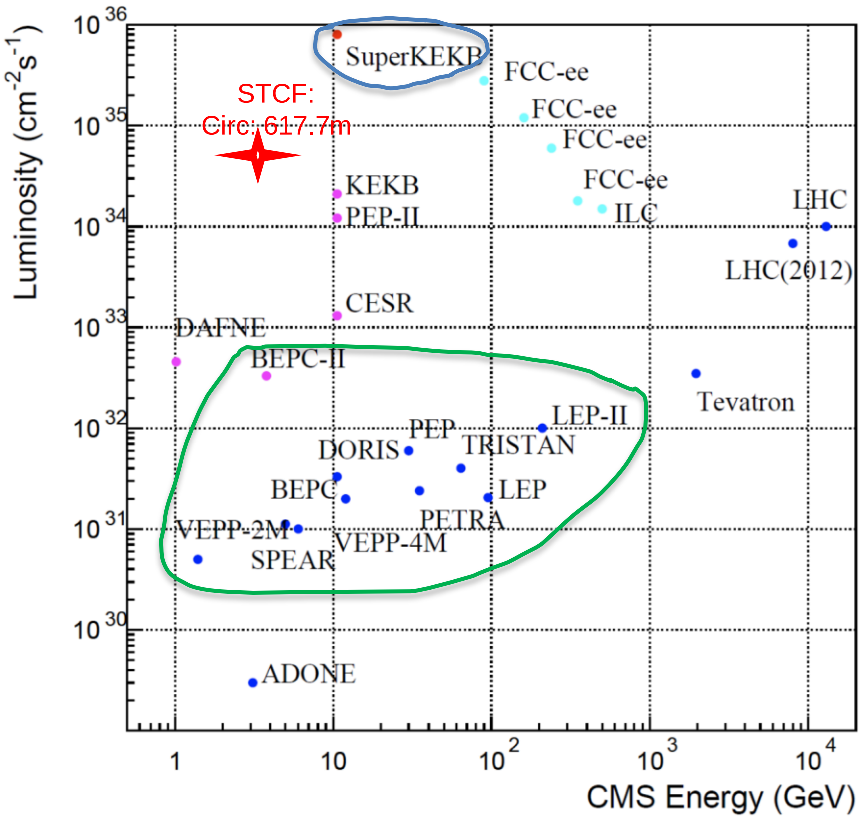


- 赵午：SLAC-PUB-2781(1981)
- SPEAR:
  Stanford Positron Electron Asymmetric Ring



Probe CP violation via Transversely Polarized beams

- Transversely Polarization of Lepton Beams at BEPCII?

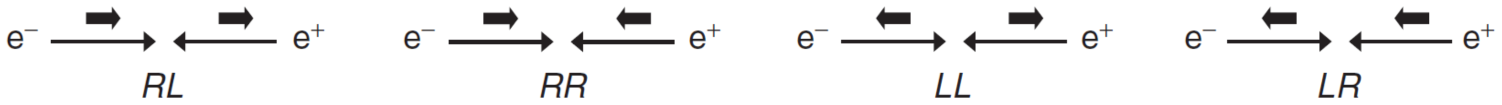


Probe CP violation via Transversely Polarized beams

- Forgotten Facts of Transversely Polarized electron/positron Beams

$$\rho_e^{\pm} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The four possible helicity combinations in the e^+e^- initial state



- $1 = \frac{1}{2} - (-\frac{1}{2})$ $\quad 0 = \frac{1}{2} - \frac{1}{2}$ $\quad 0 = -\frac{1}{2} - (-\frac{1}{2})$ $\quad -1 = -\frac{1}{2} - \frac{1}{2}$

- If.amp 1 1 1 1

$$\rho^{\gamma^*/\psi}_{m,m'} = \sum_{\lambda_1,\lambda_1'\lambda_2,\lambda_2'} D^{1*}_{m,\lambda_1-\lambda_2} D^{1}_{m',\lambda_1'-\lambda_2'} \rho^{+}_{\lambda_1,\lambda_1'} \rho^{-}_{\lambda_2,\lambda_2'} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- **Xu Cao**, Yu-tie Liang, Rong-Gang Ping, 2404.00298 [hep-ph]

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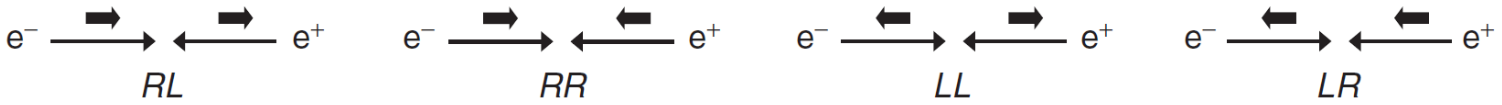


- $1 = ½ - (-½)$ $\quad 0 = ½ - ½$ $\quad 0 = -½ - (-½)$ $\quad -1 = -½ - ½$
- In fact $\quad 1 \quad m_e/\sqrt{s} \quad m_e/\sqrt{s} \quad 1$

$$\rho^{\gamma^*/\psi}_{m,m'} = \sum_{\lambda_1,\lambda_1'\lambda_2,\lambda_2'} D^{1*}_{m,\lambda_1-\lambda_2} D^{1}_{m',\lambda_1'-\lambda_2'} \rho^{+}_{\lambda_1,\lambda_1'} \rho^{-}_{\lambda_2,\lambda_2'} \times \delta_{\lambda_1,-\lambda_2}\delta_{\lambda_1',-\lambda_2'} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

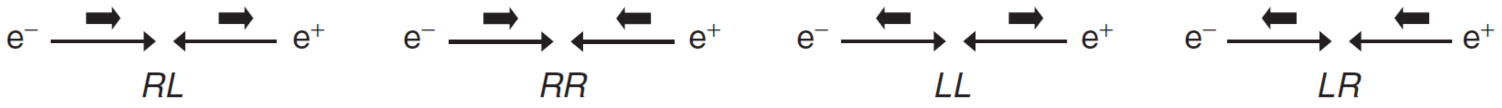
- **Xu Cao**, Yu-tie Liang, Rong-Gang Ping, 2404.00298 [hep-ph]

Probe CP violation via Transversely Polarized beams

- Forgotten Facts of Transversely Polarized electron/positron Beams

$$\rho_e^{\pm} = \frac{1}{2}\begin{pmatrix} 1 & \mathcal{P}_t \\ \mathcal{P}_t^* & 1 \end{pmatrix}$$

The four possible helicity combinations in the e^+e^- initial state



- $1 = ½ - (-½)$ $\quad$ $0 = ½ - ½$ $\quad$ $0 = -½ - (-½)$ $\quad$ $-1 = -½ - ½$

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$$\rho^{\gamma^*/\psi}_{m,m'} = \sum_{\lambda_1,\lambda_1'\lambda_2,\lambda_2'} D^{1*}_{m,\lambda_1-\lambda_2} D^{1}_{m',\lambda_1'-\lambda_2'} \rho^{+}_{\lambda_1,\lambda_1'} \rho^{-}_{\lambda_2,\lambda_2'} = \frac{1}{2}\begin{pmatrix} 1 & 0 & P_T^2 \\ 0 & 0 & 0 \\ P_T^2 & 0 & 1 \end{pmatrix}$$

$$\times\ \delta_{\lambda_1,-\lambda_2}\delta_{\lambda_1',-\lambda_2'}$$

- **Xu Cao**, Yu-tie Liang, Rong-Gang Ping, 2404.00298 [hep-ph]

Probe CP violation via Transversely Polarized beams

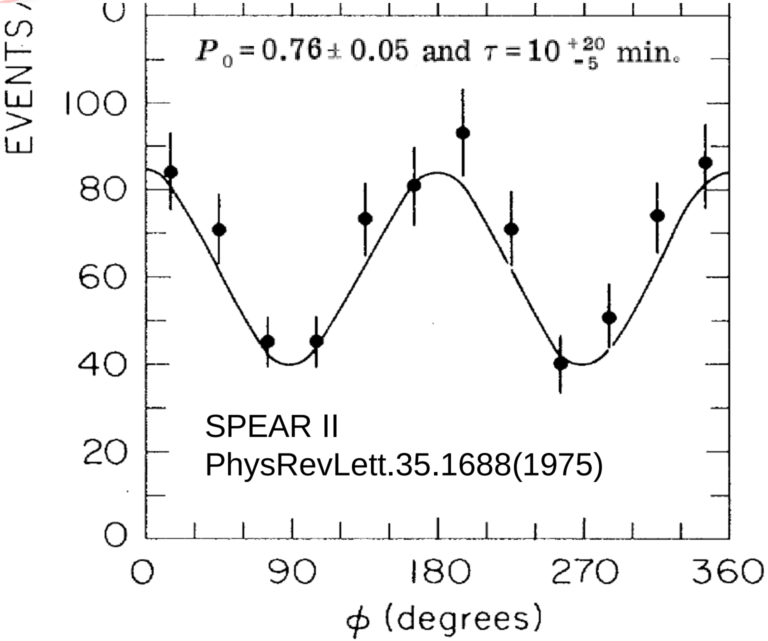
- Forgotten Facts of Transversely Polarized electron/positron Beams

$$\frac{4\pi}{\sigma}\frac{d\sigma}{d\Omega_B} = \frac{3}{3+\alpha_\psi}(1+\alpha_\psi\cos^2\theta + \alpha_\psi P_T^2 \sin^2\theta\cos 2\phi)$$

measured through $e^+e^- \to \mu^+\mu^-$ and e^+e^-

$$P_y^B = \frac{\sqrt{1-\alpha_\psi^2}\,\sin\Delta\Phi\sin\theta\cos\theta(1-P_T^2\cos 2\phi)}{1+\alpha_\psi\cos^2\theta+\alpha_\psi P_T^2\sin^2\theta\cos 2\phi}$$

$$P_x^B = \frac{-P_T^2\sqrt{1-\alpha_\psi^2}\,\sin\Delta\Phi\sin\theta\sin 2\phi}{1+\alpha_\psi\cos^2\theta+\alpha_\psi P_T^2\sin^2\theta\cos 2\phi}$$



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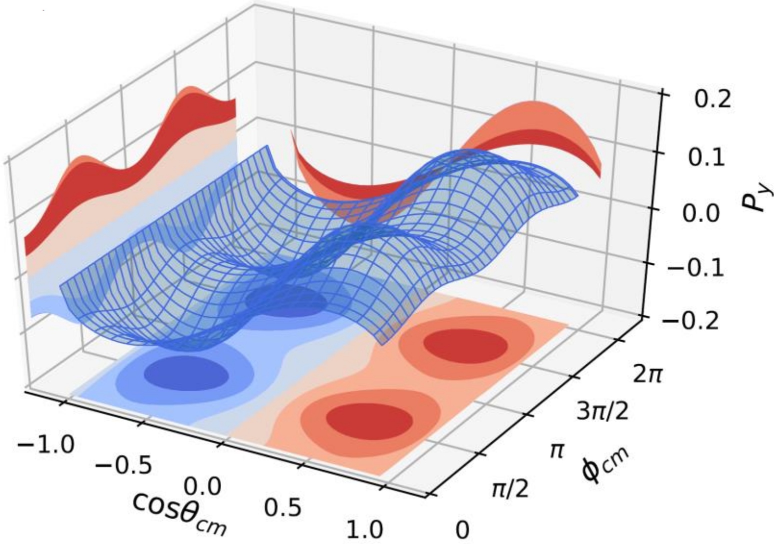

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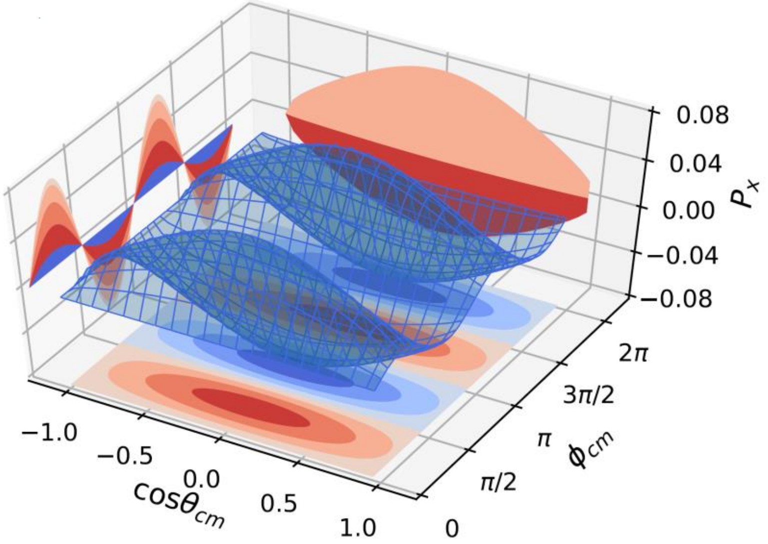
$$P_x^B = \frac{-P_T^2\sqrt{1-\alpha_\psi^2}\,\sin\Delta\Phi\sin\theta\sin 2\phi}{1+\alpha_\psi\cos^2\theta+\alpha_\psi P_T^2\sin^2\theta\cos 2\phi}$$



Integrating out the azimuthal angle is equal to $P_T = 0$

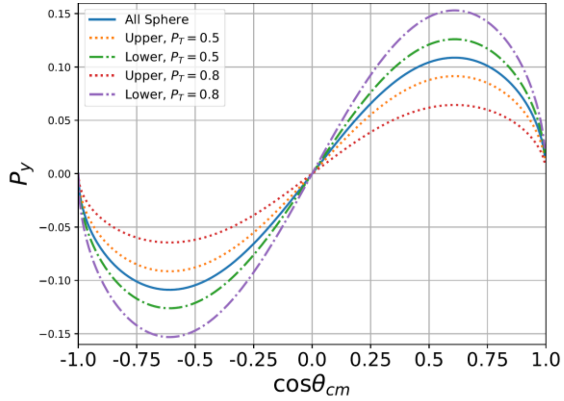
- **Xu Cao**, Yu-tie Liang, Rong-Gang Ping, 2404.00298 [hep-ph]

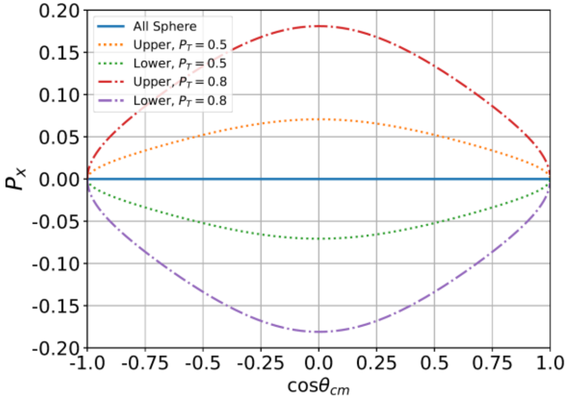
- Forgotten Facts of Transversely Polarized electron/positron Beams

Integrating within different azimuthal angle select P_T terms

$$\mathcal{W}^+_{\cos 2\phi}(\xi) = \int_0^{\pi/4} + \int_{3\pi/4}^{5\pi/4} + \int_{7\pi/4}^{2\pi} \mathcal{W}(\xi)\mathrm{d}\phi$$
$$\mathcal{W}^-_{\cos 2\phi}(\xi) = \int_{\pi/4}^{3\pi/4} + \int_{5\pi/4}^{7\pi/4} \mathcal{W}(\xi)\mathrm{d}\phi,$$

$$\mathcal{W}^+_{\sin 2\phi}(\xi) = \int_0^{\pi/2} + \int_{\pi}^{3\pi/2} \mathcal{W}(\xi)\mathrm{d}\phi$$
$$\mathcal{W}^-_{\sin 2\phi}(\xi) = \int_{\pi}^{\pi/2} + \int_{3\pi/2}^{2\pi} \mathcal{W}(\xi)\mathrm{d}\phi,$$





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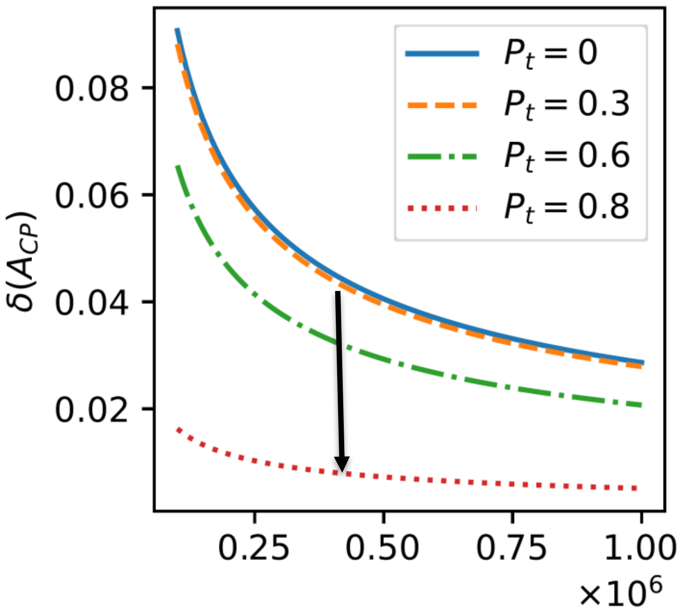
Probe CP violation via Transversely Polarized beams

- Useful for improving the sensitivity of measurements $\sim \frac{1}{\sqrt{N_{events}}}$

$$\mathcal{W}(\boldsymbol{\xi}) = \mathcal{F}_0 + \beta_\psi(\alpha_+\mathcal{F}_3 - \alpha_-\mathcal{F}_4) + \alpha_-\alpha_+(\mathcal{F}_1 + \gamma_\psi\mathcal{F}_2 + \alpha_\psi\mathcal{F}_5)\,,$$

$$\begin{aligned}
\mathcal{F}_0 =& 1 + \alpha_\psi\cos^2\theta + \alpha_\psi P_T^2\sin^2\theta\cos 2\phi\,,\\
\mathcal{F}_1 =& (\sin^2\theta + P_T^2\cos 2\phi\cos^2\theta)\sin\theta_1\cos\phi_1\sin\theta_2\cos\phi_2\\
&- (\cos^2\theta + P_T^2\cos 2\phi\sin^2\theta)\cos\theta_1\cos\theta_2\\
&+ P_T^2\sin\theta_1\sin\theta_2(\sin 2\phi\cos\theta\sin(\phi_1-\phi_2) + \cos 2\phi\sin\phi_1\sin\phi_2)\,,\\
\mathcal{F}_2 =& (1 - P_T^2\cos 2\phi)\sin\theta\cos\theta(\sin\theta_1\cos\theta_2\cos\phi_1 - \cos\theta_1\sin\theta_2\cos\phi_2)\\
&- P_T^2\sin 2\phi\sin\theta(\sin\theta_1\cos\theta_2\sin\phi_1 + \cos\theta_1\sin\theta_2\sin\phi_2)\,,\\
\mathcal{F}_3 =& (1 - P_T^2\cos 2\phi)\sin\theta\cos\theta\sin\theta_2\sin\phi_2 - P_T^2\sin 2\phi\sin\theta\sin\theta_2\cos\phi_2\,,\\
\mathcal{F}_4 =& (1 - P_T^2\cos 2\phi)\sin\theta\cos\theta\sin\theta_1\sin\phi_1 + P_T^2\sin 2\phi\sin\theta\sin\theta_1\cos\phi_1\,,\\
\mathcal{F}_5 =& (\sin^2\theta + P_T^2\cos 2\phi\cos^2\theta)\sin\theta_1\sin\phi_1\sin\theta_2\sin\phi_2 - \cos\theta_1\cos\theta_2\\
&+ P_T^2\sin\theta_1\sin\theta_2[\sin 2\phi\cos\theta\sin(\phi_1-\phi_2) + \cos 2\phi\cos\phi_1\cos\phi_2]\,,
\end{aligned}$$



Observed events of $e^+e^- \to \gamma^*/\psi \to \Lambda(p\pi^-)\bar{\Lambda}(\bar{p}\pi^+)$

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Probe CP violation via Transversely Polarized beams

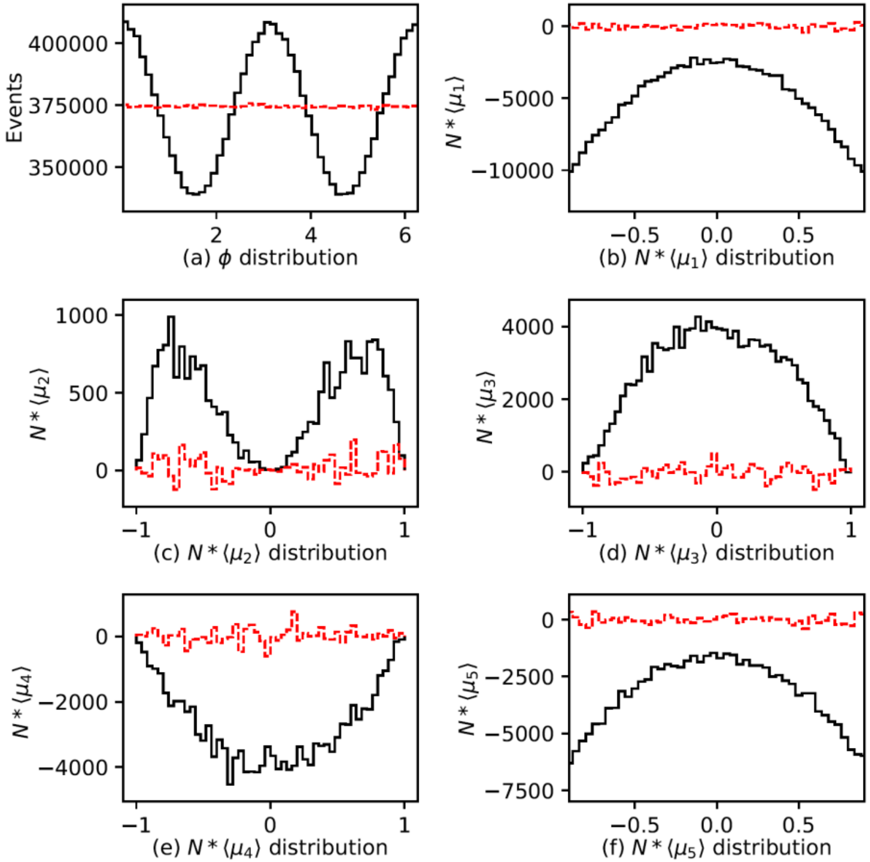
- Or define moments

$$\begin{aligned}
\mu_1 &= \sin\theta_1\sin\theta_2[\sin(2\phi)\cos\theta\sin(\phi_1-\phi_2) \\
&+ \cos(2\phi)\sin\phi_1\sin\phi_2], \\
\mu_2 &= \cos(2\phi)\sin\theta\cos\theta[\sin\theta_1\cos\theta_2\cos\phi_1 \\
&- \cos\theta_1\sin\theta_2\cos\phi_2], \\
\mu_3 &= \sin(2\phi)\sin\theta\cos\phi_2, \\
\mu_4 &= \sin(2\phi)\sin\theta\cos\phi_1, \\
\mu_5 &= \sin\theta_1\sin\theta_2[\sin(2\phi)\cos\theta\sin(\phi_1-\phi_2) \\
&- \cos(2\phi)\cos\phi_1\cos\phi_2].
\end{aligned}$$

$$\frac{d\langle\mu_i\rangle}{d\cos\theta} = \frac{\int \mathcal{W}(\xi)\mu_i d\cos\theta_1 d\cos\theta_2 d\phi_1 d\phi_2}{\int \mathcal{W}(\xi) d\cos\theta_1 d\cos\theta_2 d\phi_1 d\phi_2}, (i=1,2,..,5).$$

Then one has

$$\begin{aligned}
\frac{d\langle\mu_1\rangle}{d\cos\theta} &= \frac{\alpha_-\alpha_+P_T^2\left[(3\alpha_\psi+2)\cos^2\theta+1\right]}{12(\alpha_\psi+3)}, \\
\frac{d\langle\mu_2\rangle}{d\cos\theta} &= -\frac{\alpha_-\alpha_+P_T^2\gamma_\psi\sin^2\theta\cos^2\theta}{6(\alpha_\psi+3)}, \\
\frac{d\langle\mu_3\rangle}{d\cos\theta} &= -\frac{3\alpha_+P_T^2\beta_\psi\sin^2\theta}{8(\alpha_\psi+3)}, \\
\frac{d\langle\mu_4\rangle}{d\cos\theta} &= -\frac{3\alpha_-P_T^2\beta_\psi\sin^2\theta}{8(\alpha_\psi+3)}, \\
\frac{d\langle\mu_5\rangle}{d\cos\theta} &= \frac{\alpha_-\alpha_+P_T^2\left[(2\alpha_\psi+1)\cos^2\theta+\alpha_\psi\right]}{12(\alpha_\psi+3)}.
\end{aligned}$$



Probe CP violation via Transversely Polarized beams

- Comparison of Transversely and Longitudinal Polarized Beams

with the polarization vectors
$$\rho_e^{\pm} = \frac{1}{2}\begin{pmatrix} 1+\mathcal{P}_z & \mathcal{P}_t \\ \mathcal{P}_t^* & 1-\mathcal{P}_z \end{pmatrix} \qquad \rho^{\gamma^*/\psi} = \frac{1}{2}\begin{pmatrix} 1+\mathcal{P}_z & 0 & P_T^2 \\ 0 & 0 & 0 \\ P_T^2 & 0 & 1-\mathcal{P}_z \end{pmatrix}$$

degree of polarization
$$d = \frac{1}{\sqrt{2s}}[(2s+1)\mathrm{Tr}(\rho^{\psi 2})-1]^{1/2} = \sqrt{1+3\mathcal{P}_z^2+3P_T^4}/2$$

$$\begin{aligned}(C_{\mu\nu}) = \frac{3}{3+\alpha_\psi}\cdot &\begin{pmatrix} 1+\alpha_\psi\cos^2\theta & 0 & \beta_\psi\sin\theta\cos\theta & 0 \\ 0 & \sin^2\theta & 0 & \gamma_\psi\sin\theta\cos\theta \\ -\beta_\psi\sin\theta\cos\theta & 0 & \alpha_\psi\sin^2\theta & 0 \\ 0 & -\gamma_\psi\sin\theta\cos\theta & 0 & -\alpha_\psi-\cos^2\theta \end{pmatrix} \\ + \frac{3P_T^2}{3+\alpha_\psi}\cdot &\begin{pmatrix} \alpha_\psi\sin^2\theta\cos2\phi & -\beta_\psi\sin\theta\sin2\phi & -\beta_\psi\sin\theta\cos\theta\cos2\phi & 0 \\ -\beta_\psi\sin\theta\sin2\phi & (\alpha_\psi+\cos^2\theta)\cos2\phi & -(1+\alpha_\psi)\cos\theta\sin2\phi & -\gamma_\psi\sin\theta\cos\theta\cos2\phi \\ \beta_\psi\sin\theta\cos\theta\cos2\phi & (1+\alpha_\psi)\cos\theta\sin2\phi & (1+\alpha_\psi\cos\theta)\cos2\phi & -\gamma_\psi\sin\theta\sin2\phi \\ 0 & \gamma_\psi\sin\theta\cos\theta\cos2\phi & -\gamma_\psi\sin\theta\sin2\phi & -\sin^2\theta\cos2\phi \end{pmatrix} \\ + \frac{3P_L}{3+\alpha_\psi}\cdot &\begin{pmatrix} 0 & \gamma_\psi\sin\theta & 0 & (1+\alpha_\psi)\cos\theta \\ \gamma_\psi\sin\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & -\beta_\psi\sin\theta \\ -(1+\alpha_\psi)\cos\theta & 0 & -\beta_\psi\sin\theta & 0 \end{pmatrix}\end{aligned}$$

- **Xu Cao**, Yu-tie Liang, Rong-Gang Ping, 2404.00298 [hep-ph]
- N. Salone, P. Adlarson, V. Batozskaya, A. Kupsc, S. Leupold, and J. Tandean, Phys. Rev. D 105, 116022 (2022)
- Sheng Zeng, Yue Xu, Xiao Rong Zhou, Jia Jia Qin, Bo Zheng, Chin. Phys. C 47, 113001 (2023)

Probe CP violation via Transversely Polarized beams

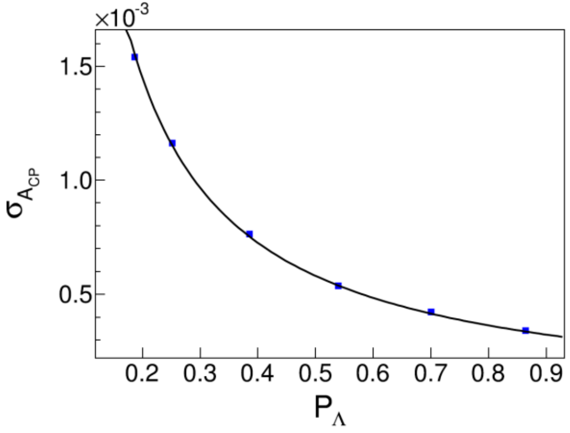
- Comparison of Transversely and Longitudinal Polarized Beams

$$
\begin{aligned}
\mathcal{W}(\boldsymbol{\xi}) = & \mathcal{F}_0 + \beta_\psi(\alpha_+\mathcal{F}_3 - \alpha_-\mathcal{F}_4) \\
& + \alpha_-\alpha_+(\mathcal{F}_1 + \gamma_\psi\mathcal{F}_2 + \alpha_\psi\mathcal{F}_5)\,,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_0 =& 1 + \alpha_\psi\cos^2\theta + \alpha_\psi P_T^2\sin^2\theta\cos 2\phi\,,\\
\mathcal{F}_1 =& (\sin^2\theta + P_T^2\cos 2\phi\cos^2\theta)\sin\theta_1\cos\phi_1\sin\theta_2\cos\phi_2\\
& - (\cos^2\theta + P_T^2\cos 2\phi\sin^2\theta)\cos\theta_1\cos\theta_2\\
& + P_T^2\sin\theta_1\sin\theta_2(\sin 2\phi\cos\theta\sin(\phi_1-\phi_2) + \cos 2\phi\sin\phi_1\sin\phi_2)\,,\\
\mathcal{F}_2 =& (1 - P_T^2\cos 2\phi)\sin\theta\cos\theta(\sin\theta_1\cos\theta_2\cos\phi_1 - \cos\theta_1\sin\theta_2\cos\phi_2)\\
& - P_T^2\sin 2\phi\sin\theta(\sin\theta_1\cos\theta_2\sin\phi_1 + \cos\theta_1\sin\theta_2\sin\phi_2)\,,\\
\mathcal{F}_3 =& (1 - P_T^2\cos 2\phi)\sin\theta\cos\theta\sin\theta_2\sin\phi_2 - P_T^2\sin 2\phi\sin\theta\sin\theta_2\cos\phi_2\,,\\
\mathcal{F}_4 =& (1 - P_T^2\cos 2\phi)\sin\theta\cos\theta\sin\theta_1\sin\phi_1 + P_T^2\sin 2\phi\sin\theta\sin\theta_1\cos\phi_1\,,\\
\mathcal{F}_5 =& (\sin^2\theta + P_T^2\cos 2\phi\cos^2\theta)\sin\theta_1\sin\phi_1\sin\theta_2\sin\phi_2 - \cos\theta_1\cos\theta_2\\
& + P_T^2\sin\theta_1\sin\theta_2[\sin 2\phi\cos\theta\sin(\phi_1-\phi_2) + \cos 2\phi\cos\phi_1\cos\phi_2]\,,
\end{aligned}
$$

$$
+\alpha_-\cdot\mathcal{F}_6 + \alpha_+\cdot\mathcal{F}_7 - \alpha_-\alpha_+\cdot\mathcal{F}_8
$$

$$
\begin{aligned}
\mathcal{F}_6(\xi) &= P_e(\gamma_\psi\sin\theta\sin\theta_1\cos\varphi_1 - (1+\alpha_\psi)\cos\theta\cos\theta_1),\\
\mathcal{F}_7(\xi) &= P_e(\gamma_\psi\sin\theta\sin\theta_2\cos\varphi_2 + (1+\alpha_\psi)\cos\theta\cos\theta_2),\\
\mathcal{F}_8(\xi) &= P_e\beta_\psi\sin\theta(\cos\theta_1\sin\theta_2\sin\varphi_2 + \sin\theta_1\sin\varphi_1\cos\theta_2).
\end{aligned}
$$



- **Xu Cao**, Yu-tie Liang, Rong-Gang Ping, 2404.00298 [hep-ph]
- N. Salone, P. Adlarson, V. Batozskaya, A. Kupsc, S. Leupold, and J. Tandean, Phys. Rev. D 105, 116022 (2022)
- Sheng Zeng, Yue Xu, Xiao Rong Zhou, Jia Jia Qin, Bo Zheng, Chin. Phys. C 47, 113001 (2023)

Summary and Perspective

- Good things (Hyperons) come in pairs

- Transversely Polarization of lepton beams

- … can used to enhance the sensitivity of the CP violation test

- … is required to consider in the data analysis at circular collides: sys. errors

- … technically easier to obtain in comparison of longitudinal polarization of beams

- … spin rotators on either side of the interaction points converted the polarization of the beam from transverse to longitudinal, or vise versa

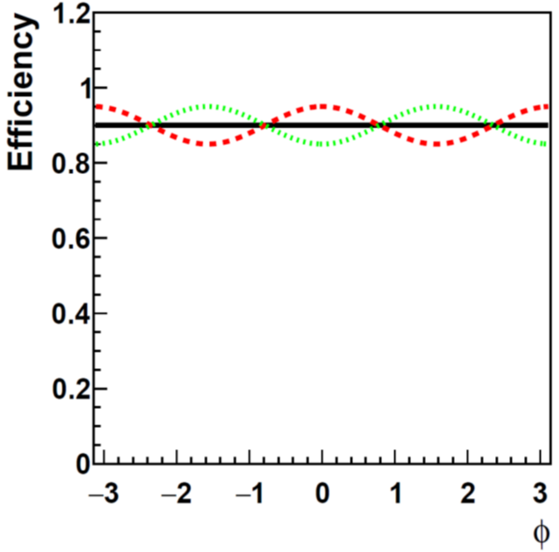
Thank You !

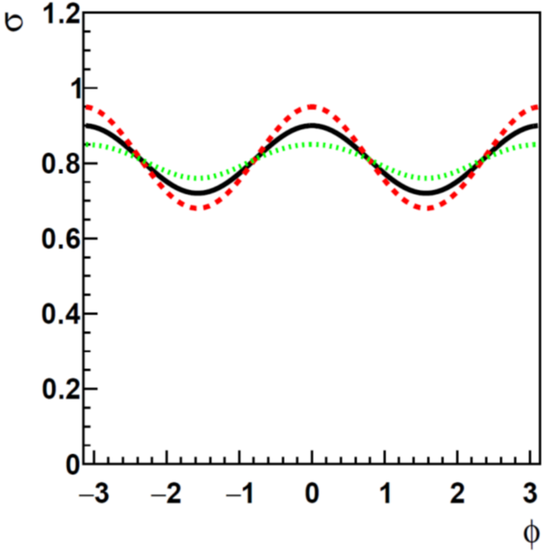
- Requirement of estimation of systematic errors

Toy model:

efficiency curves over angle with 5% oscillation amplitude

a beam polarization of 30%, the measured cross sections will be 0.3% shift from the correct value.





- **Xu Cao**, Yu-tie Liang, Rong-Gang Ping, 2404.00298 [hep-ph]

Spares

- Comparison of Transversely and Longitudinal Polarized Beams

$$
\begin{aligned}
\rho_1^{i,j}(\theta,\phi) &\equiv \sum_{k,k'=\pm 1} \rho_{k,k'}^{\gamma^*/\psi} \mathcal{D}_{k,i}^{1*}(\phi,\theta,0)\, \mathcal{D}_{k',j}^{1}(\phi,\theta,0) \\
&= \sum_{k=\pm 1} \left[\mathcal{D}_{k,i}^{1*}(\phi,\theta,0)\, \mathcal{D}_{k,j}^{1}(\phi,\theta,0) \right. \\
&+ P_T^2 \mathcal{D}_{k,i}^{1*}(\phi,\theta,0)\, \mathcal{D}_{-k,j}^{1}(\phi,\theta,0) \\
&+ \left. P_L k \mathcal{D}_{k,i}^{1*}(\phi,\theta,0)\, \mathcal{D}_{k,j}^{1}(\phi,\theta,0) \right]
\end{aligned}
$$

with the polarization vectors of leptons

$$
\rho_e^{\pm} = \frac{1}{2} \begin{pmatrix} 1+\mathcal{P}_z & \mathcal{P}_t \\ \mathcal{P}_t^* & 1-\mathcal{P}_z \end{pmatrix}
$$

and the spin densiry matrix

$$
\rho^{\gamma^*/\psi} = \frac{1}{2} \begin{pmatrix} 1+\mathcal{P}_z & 0 & P_T^2 \\ 0 & 0 & 0 \\ P_T^2 & 0 & 1-\mathcal{P}_z \end{pmatrix}
$$

$$
\begin{aligned}
\rho_1^{i,j}(\theta,\phi) &\equiv \sum_{k,k'=\pm 1} \rho_{k,k'}^{\gamma^*/\psi} \mathcal{D}_{k,i}^{1*}(\phi,\theta,0)\, \mathcal{D}_{k',j}^{1}(\phi,\theta,0) \\
&= \sum_{k=\pm 1} \left[\mathcal{D}_{k,i}^{1*}(\phi,\theta,0)\, \mathcal{D}_{k,j}^{1}(\phi,\theta,0) \right. \\
&+ P_T^2 \mathcal{D}_{k,i}^{1*}(\phi,\theta,0)\, \mathcal{D}_{-k,j}^{1}(\phi,\theta,0) \\
&+ \left. P_L k \mathcal{D}_{k,i}^{1*}(\phi,\theta,0)\, \mathcal{D}_{k,j}^{1}(\phi,\theta,0) \right]
\end{aligned}
$$

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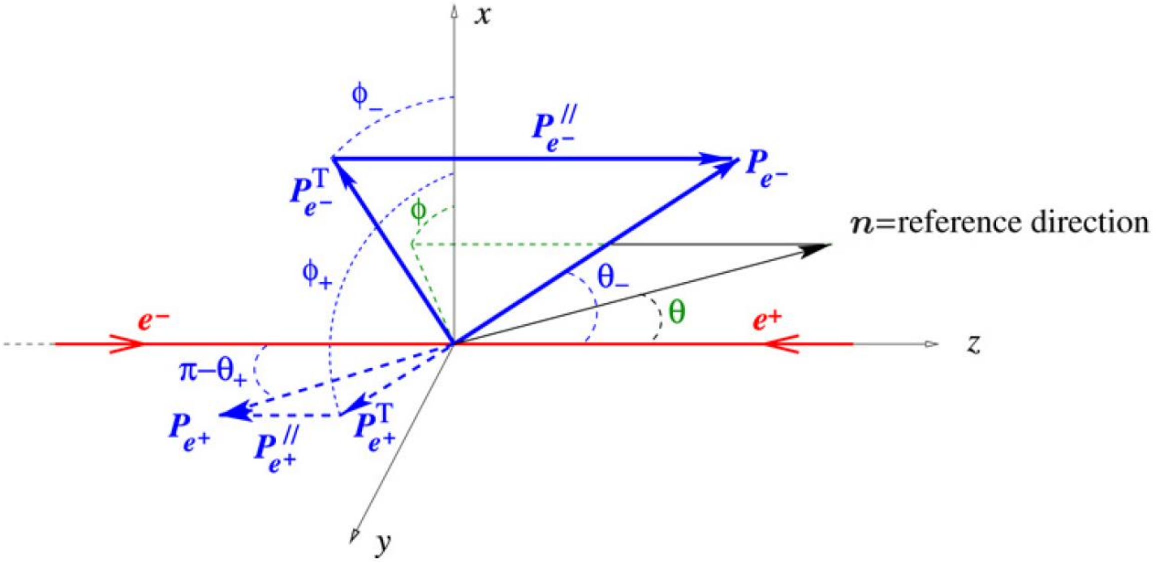
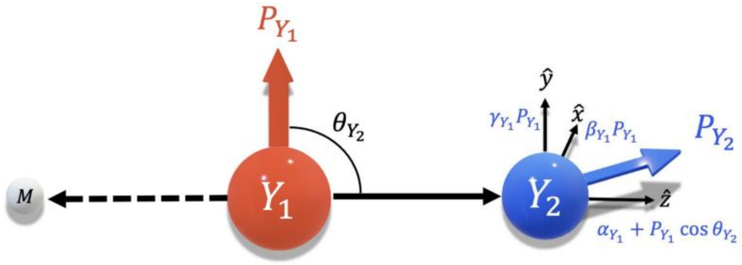
Spares

- Decomposition of the polarization vectors into a longitudinal components in the direction of the electron/positron momentum and transverse components with respect to a fixed coordinate system





- G. Moortgat-Pick et al. Physics Reports 460,2008,131