



Charm mixing and indirect CPV studies at STCF

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超级陶粲装置研讨会，兰州，09/07/2024



Outline

- Charm mixing and indirect CPV
- Time-integrated measurement
- Sensitivity study with STCF
- Summary and future plan



Neutral meson mixing

First observation of neutral meson mixing in K^0 mesons.

Evidence for a Long-Lived Neutral Unstable Particle*

W. F. FRY, J. SCHNEPS, AND M. S. SWAMI

Department of Physics, University of Wisconsin, Madison, Wisconsin

(Received July 19, 1956)

SEARCH FOR $B^0 - \bar{B}^0$ OSCILLATIONS AT THE CERN PROTON-ANTIPROTON COLLIDER

UA1 Collaboration, CERN, Geneva, Switzerland

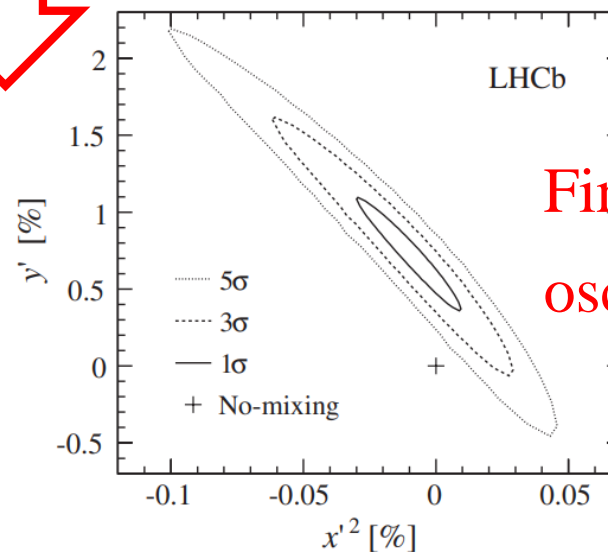
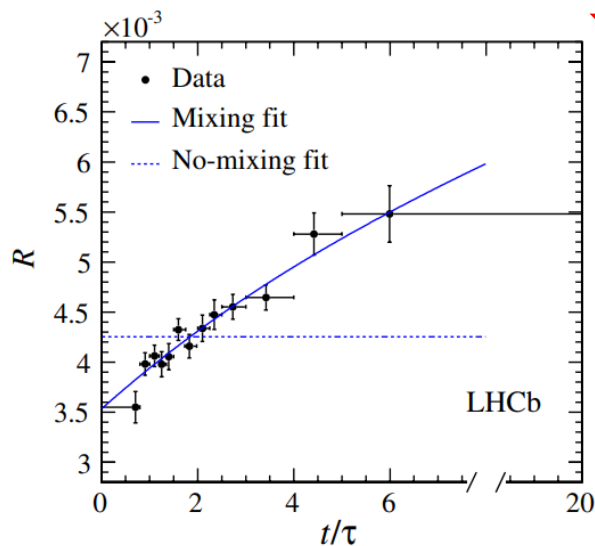
PRL 97, 242003 (2006)

PHYSICAL REVIEW LETTERS

week ending
15 DECEMBER 2006

Observation of $B_s^0 - \bar{B}_s^0$ Oscillations

Search for D^0 meson mixing (Belle, BaBar, CDF and LHCb)



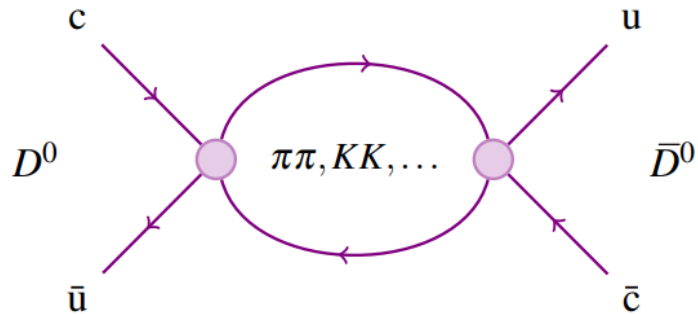
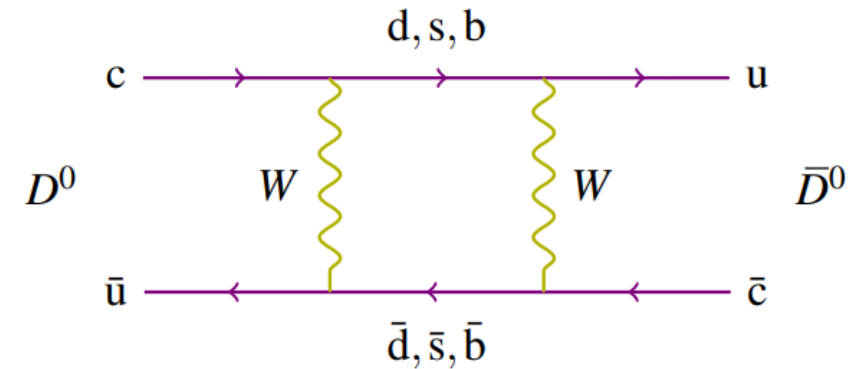
First observation of $D^0 \bar{D}^0$ oscillations.

Charm mixing

- Mixing amplitudes governed by two contributions:

- Short distance:

- ✓ Via box diagrams, $(x, y) \sim 10^{-7}$
- ✓ Suppressed by GIM cancellation



- Long distance:

- ✓ re-scattering diagram, $(x, y) \sim 10^{-3}$
- ✓ Theoretical prediction of x and y very challenging

Plots from arXiv:1503.00032

Charm mixing

- Mass eigenstates of D^0 and \bar{D}^0 mesons can be written as superpositions of flavor eigenstates:

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle \quad \boxed{|p|^2 + |q|^2 = 1}$$

- Time evolution of a initially flavor eigenstates of D^0 and \bar{D}^0 mesons:

$$\begin{aligned} |D_{phys}^0(t)\rangle &= g_+(t)|D^0\rangle + \frac{q}{p}g_-(t)|\bar{D}^0\rangle & g_+(t) &= \exp\left(-\left(im + \frac{\Gamma}{2}\right)t\right) \cosh\left(\left(i\Delta m - \frac{\Delta\Gamma}{2}\right)\frac{t}{2}\right) & m &\equiv \frac{m_1 + m_2}{2}, \Delta m \equiv m_2 - m_1 \\ |\bar{D}_{phys}^0(t)\rangle &= g_+(t)|\bar{D}^0\rangle + \frac{q}{p}g_-(t)|D^0\rangle & g_-(t) &= \exp\left(-\left(im + \frac{\Gamma}{2}\right)t\right) \sinh\left(\left(i\Delta m - \frac{\Delta\Gamma}{2}\right)\frac{t}{2}\right) & \Gamma &\equiv \frac{\Gamma_1 + \Gamma_2}{2}, \Delta\Gamma \equiv \Gamma_1 - \Gamma_2 \end{aligned}$$

- Practically, it is more popular to use the following two dimensionless parameters for describing D^0 - \bar{D}^0 mixing:

$$\boxed{x \equiv \frac{\Delta m}{\Gamma}, y \equiv \frac{\Delta\Gamma}{2\Gamma}}$$

Indirect charm CPV

- CPV in mixing:

Probability of $|D_{phy}^0(t)\rangle \rightarrow |\bar{D}^0\rangle \neq |\bar{D}_{phy}^0(t)\rangle \rightarrow |D^0\rangle$: $|q/p| \neq 1$.

- CPV in mixing/decay interference:

Non-vanishing ϕ for $\frac{q}{p} = \left|\frac{q}{p}\right| e^{i\phi}$.

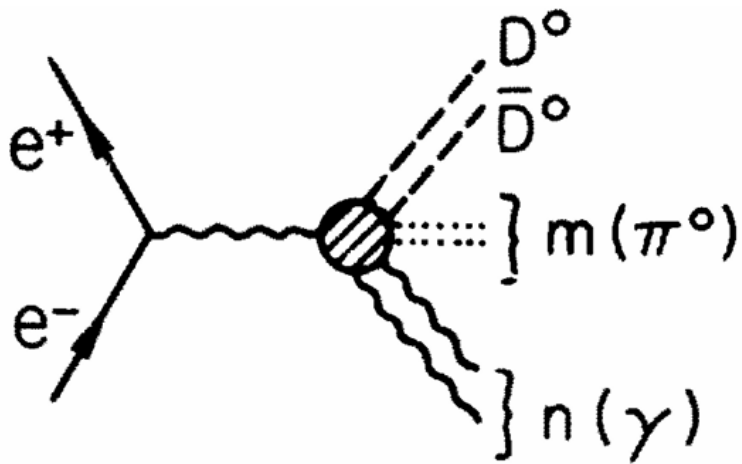
Decay rates: $R(D_{phys}^0(t) \rightarrow f) \propto |A_f|^2 \exp(-\Gamma t) [1 + \frac{1}{4}(x_D^2 + y_D^2)|\lambda_f|^2 \Gamma^2 t^2 - \frac{1}{4}(x_D^2 - y_D^2)\Gamma^2 t^2 - (y_D \text{Re}\lambda_f + x_D \text{Im}\lambda_f)\Gamma t]$

$$\lambda_f = r_f \left|\frac{q}{p}\right| e^{-i(\delta_f + \phi)}$$

Phys. Rev. D 55 (1997) 196

- Currently, the mixing and CPV parameters are usually measured with the time-dependent analysis in B factory.

Quantum correlated $D^0\bar{D}^0$



Phys. Rev. D 15 (1977) 1254

- $D^0\bar{D}^0$ pairs produced by e^+e^- annihilations near threshold are in quantum correlated state with $C = (-1)^{n+1}$.
- @3770 MeV
 - $C = -1, e^+e^- \rightarrow D^0\bar{D}^0$
- @>4009 MeV
 - $C = +1$ for $e^+e^- \rightarrow D^{*0}\bar{D}^0 + c.c, D^{*0} \rightarrow \gamma D^0$
 - $C = -1$ for $e^+e^- \rightarrow D^{*0}\bar{D}^0 + c.c, D^{*0} \rightarrow \pi^0 D^0$

Exploring the QC in $D^0\bar{D}^0$ allows to extract the mixing and CPV parameters with time-integrated decay rates.



Quantum correlated decay rates

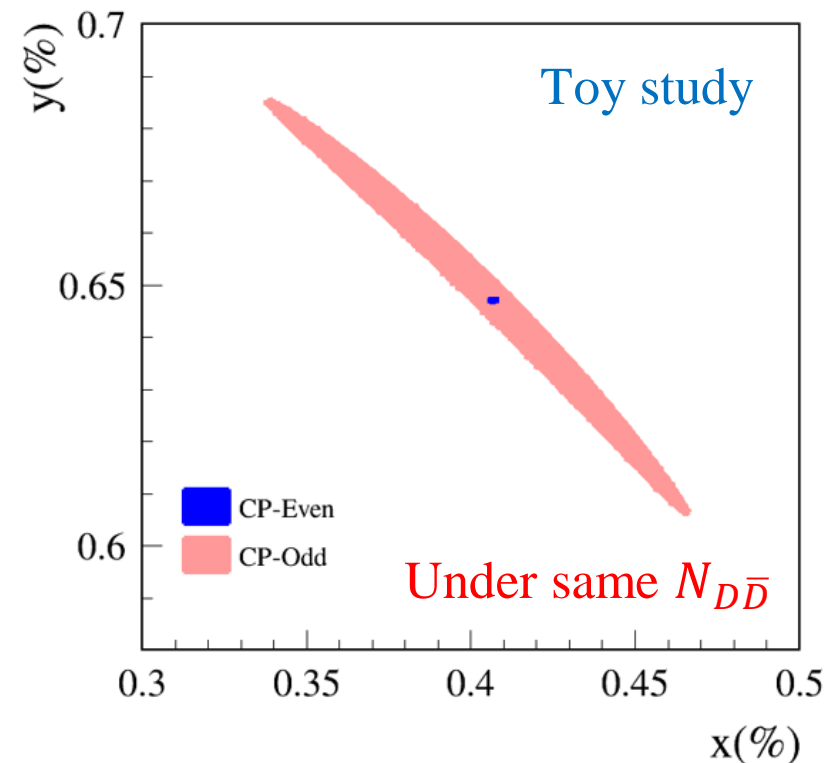
$c = +1$

$$\begin{aligned}
W(f_1, f_2) = & 3(x^2 + y^2)(|\lambda_{D^0}|^2 + |\lambda_{\bar{D}^0}|^2 + 2R_{D^0}R_{\bar{D}^0}\lambda_{D^0}\lambda_{\bar{D}^0}) \\
& + (2 - 3(x^2 - y^2))(1 + 2R_{D^0}R_{\bar{D}^0}\text{Re}(\lambda_{D^0}\lambda_{\bar{D}^0}) + |\lambda_{D^0}\lambda_{\bar{D}^0}|^2) \\
& - 4y[R_{\bar{D}^0}(1 + |\lambda_{D^0}|^2)\text{Re}(\lambda_{\bar{D}^0}) + R_{D^0}(1 + |\lambda_{\bar{D}^0}|^2)\text{Re}(\lambda_{D^0})] \\
& - 4x[R_{\bar{D}^0}(1 - |\lambda_{D^0}|^2)\text{Im}(\lambda_{\bar{D}^0}) + R_{D^0}(1 - |\lambda_{\bar{D}^0}|^2)\text{Im}(\lambda_{D^0})]
\end{aligned}$$

$c = -1$

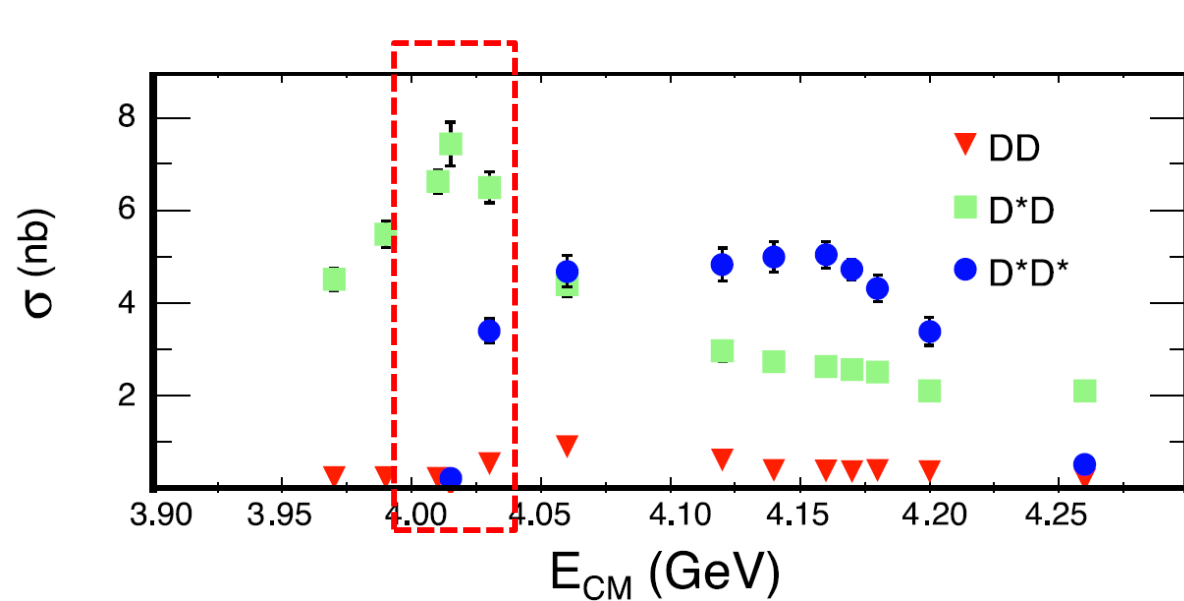
$$\begin{aligned}
W(f_1, f_2) = & (x^2 + y^2)(|\lambda_{D^0}|^2 + |\lambda_{\bar{D}^0}|^2 - 2R_{D^0}R_{\bar{D}^0}\text{Re}(\lambda_{D^0}\lambda_{\bar{D}^0})) \\
& + (2 - (x^2 - y^2))(1 - 2R_{D^0}R_{\bar{D}^0}\text{Re}(\lambda_{D^0}\lambda_{\bar{D}^0}) + |\lambda_{D^0}\lambda_{\bar{D}^0}|^2)
\end{aligned}$$

- C -even quantum correlation samples are better at measuring mixing and CPV parameters.



Fake data study at STCF

- $E_{cms} = 4009\text{MeV}, 4015\text{ MeV}$ and 4030 MeV , $\int L = 1\text{ab}^{-1}$ for each energy point.
- Coherent samples: $e^+e^- \rightarrow D^{*0}\bar{D}^0 + c.c.$ and $e^+e^- \rightarrow D^{*0}\bar{D}^{*0}$.
- Incoherent samples: $e^+e^- \rightarrow D^{*+}D^- + c.c.$ and $e^+e^- \rightarrow D^{*+}D^{*-}$.
- Sim and reconstruction framework is based on FastSim software.



	Component	Final State	$\mathcal{C}(D^0\bar{D}^0)$
Coherent	$D^0\bar{D}^0$	$D^0\bar{D}^0$	Odd
	$D^{*0}\bar{D}^0$	$\gamma D^0\bar{D}^0$	Even
		$\pi^0 D^0\bar{D}^0$	Odd
		$D^{*0}\bar{D}^{*0}$	$\gamma\pi^0 D^0\bar{D}^0$
		$\gamma\gamma(\pi^0\pi^0)D^0\bar{D}^0$	Odd
Incoherent	$D^{*+}D^{*-}$	$\pi^+\pi^-D^0\bar{D}^0$	-
	$D^{*+}D^-$	$\pi^+D^0D^-$	-

QC correction for MC samples

- QCMC package is developed for both C – odd/even $D^0\bar{D}^0$.
- For each event, QC weight is determined by the ratio of decay rate with or without QC effect.
- Input of amplitude models for multibody decays, mixing and CPV parameters from HFLAV.

C-even

- $$W(f_1, f_2) = 3(x^2 + y^2)(|\lambda_{D^0}|^2 + |\lambda_{\bar{D}^0}|^2 + 2R_{D^0}R_{\bar{D}^0}\lambda_{D^0}\lambda_{\bar{D}^0})$$
$$+ (2 - 3(x^2 - y^2))(1 + 2R_{D^0}R_{\bar{D}^0}\text{Re}(\lambda_{D^0}\lambda_{\bar{D}^0}) + |\lambda_{D^0}\lambda_{\bar{D}^0}|^2)$$
$$- 4y[R_{\bar{D}^0}(1 + |\lambda_{D^0}|^2)\text{Re}(\lambda_{\bar{D}^0}) + R_{D^0}(1 + |\lambda_{\bar{D}^0}|^2)\text{Re}(\lambda_{D^0})]$$
$$- 4x[R_{\bar{D}^0}(1 - |\lambda_{D^0}|^2)\text{Im}(\lambda_{\bar{D}^0}) + R_{D^0}(1 - |\lambda_{\bar{D}^0}|^2)\text{Im}(\lambda_{D^0})]$$

C-odd

- $$W(f_1, f_2) = (x^2 + y^2)(|\lambda_{D^0}|^2 + |\lambda_{\bar{D}^0}|^2 - 2R_{D^0}R_{\bar{D}^0}\text{Re}(\lambda_{D^0}\lambda_{\bar{D}^0}))$$
$$+ (2 - (x^2 - y^2))(1 - 2R_{D^0}R_{\bar{D}^0}\text{Re}(\lambda_{D^0}\lambda_{\bar{D}^0}) + |\lambda_{D^0}\lambda_{\bar{D}^0}|^2)$$

Analysis strategy: **signal modes**

- Decay modes: $D^0 \rightarrow K\pi\pi\pi$, $K\pi\pi^0$ and $K_S\pi\pi$.
- For the coherent samples, the **double tag method** is performed.
 - Flavor tag: $K\pi\pi\pi$, $K\pi\pi^0$, $K\pi$ (like-sign and opposite sign for $K\eta\pi$).
 - CP -even tag: KK , $\pi\pi$, $\pi\pi\pi^0$, $K_S\pi^0\pi^0$.
 - CP -odd tag: $K_S\pi^0$, $K_S\eta$, $K_S\omega$, $K\eta'$.
 - Self-conjugate tag: $K_S\pi\pi$.
- For the incoherent samples, the **flavor tag method** is adopted.

Analysis strategy: fitting method of coherent sample

- The charm mixing parameters are extracted in a ratio between C -even and C -odd $D^0\bar{D}$ production processes.
- In quantum correlations:

$$N_{sig}^{C-even} = N_{D^{*0}\bar{D}^0} \cdot R^{C-even} \cdot \mathcal{B}(D^{*0} \rightarrow D^0\gamma) \cdot \varepsilon_{sig}$$

decay rate

$$N_{sig}^{C-odd} = N_{D^{*0}\bar{D}^0} \cdot R^{C-odd} \cdot \mathcal{B}(D^{*0} \rightarrow D^0\pi^0) \cdot \varepsilon_{sig}$$

$$\frac{N_{sig}^{C-even}}{N_{sig}^{C-odd}} = \frac{R^{C-even} \cdot \mathcal{B}(D^{*0} \rightarrow D^0\gamma)}{R^{C-odd} \cdot \mathcal{B}(D^{*0} \rightarrow D^0\pi^0)}$$

- This will cancel the reconstruction efficiencies and the number of $D^0\bar{D}^0$ pairs.
- Cancel out some systematic uncertainty.
- Similar treatment in $D^{*0}\bar{D}^{*0}$ sample.

$$R^{C-even}(K^-\pi^+\pi^-\pi^+; K^+\pi^-) \propto A_{K3\pi}^2 A_{K\pi}^2 \{3(x^2 + y^2)[K_i(r_{CP}^{-1})^2(r_D^{K\pi})^2 + \bar{K}_i(r_{CP})^2(r_D^{K3\pi})^2 + 2\sqrt{K_i\bar{K}_i}R_{K3\pi}^i r_D^{K3\pi} r_D^{K\pi} \cos(\delta_D^{i,K3\pi} - \delta_D^{K\pi} - 2\phi)] + [2 - 3(x^2 - y^2)][K_i + \bar{K}_i(r_D^{K\pi})^2(r_D^{K3\pi})^2 + 2\sqrt{K_i\bar{K}_i}r_D^{K\pi} R_{K3\pi}^i r_D^{K3\pi} \cos(\delta_D^{i,K3\pi} + \delta_D^{K\pi})] - 4y[r_D^{K\pi}(K_i r_{CP}^{-1} + \bar{K}_i r_{CP}(r_D^{K3\pi})^2) \cos(\delta_D^{K\pi} + \phi) + \sqrt{K_i\bar{K}_i}R_{K3\pi}^i r_D^{K3\pi}(r_{CP} + r_{CP}^{-1}(r_D^{K\pi})^2) \cos(\delta_D^{i,K3\pi} - \phi)] - 4x[r_D^{K\pi}(K_i r_{CP}^{-1} - \bar{K}_i r_{CP}(r_D^{K3\pi})^2) \sin(\delta_D^{K\pi} + \phi) + \sqrt{K_i\bar{K}_i}R_{K3\pi}^i r_D^{K3\pi}(r_{CP} - r_{CP}^{-1}(r_D^{K\pi})^2) \sin(\delta_D^{i,K3\pi} - \phi)]\}$$

$$R^{C-odd}(K^-\pi^+\pi^-\pi^+; K^+\pi^-) \propto A_{K3\pi}^2 A_{K\pi}^2 \{(x^2 + y^2)[K_i(r_{CP}^{-1})^2(r_D^{K\pi})^2 + \bar{K}_i(r_{CP})^2(r_D^{K3\pi})^2 - 2\sqrt{K_i\bar{K}_i}R_{K3\pi}^i r_D^{K3\pi} r_D^{K\pi} \cos(\delta_D^{i,K3\pi} - \delta_D^{K\pi} - 2\phi)] + [2 - (x^2 - y^2)][K_i + \bar{K}_i(r_D^{K\pi})^2(r_D^{K3\pi})^2 - 2\sqrt{K_i\bar{K}_i}r_D^{K\pi} R_{K3\pi}^i r_D^{K3\pi} \cos(\delta_D^{i,K3\pi} + \delta_D^{K\pi})] - 2\sqrt{K_i\bar{K}_i}r_D^{K\pi} R_{K3\pi}^i r_D^{K3\pi} \cos(\delta_D^{i,K3\pi} + \delta_D^{K\pi})\}$$

Strong phase parameters are fixed.

Analysis strategy: fitting method of incoherent sample

- The time-independent decay rates turn out to be:

$$R(D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-) \propto A_{K3\pi}^2 [K_i + \frac{1}{2}(x^2 + y^2)(r_{CP})^2 (r_D^{K3\pi})^2 \bar{K}_i - \frac{1}{2}(x^2 - y^2)K_i - \sqrt{K_i \bar{K}_i} r_{CP} r_D^{K3\pi} R_{K3\pi}^i (y \cos(\delta_D^{i,K3\pi} - \phi) + x \sin(\delta_D^{i,K3\pi} - \phi))]$$

$$R(\bar{D}^0 \rightarrow K^- \pi^+ \pi^+ \pi^-) \propto A_{K3\pi}^2 [\bar{K}_i (r_D^{K3\pi})^2 + \frac{1}{2}(x^2 + y^2)(r_{CP}^{-1})^2 K_i - \frac{1}{2}(x^2 - y^2)\bar{K}_i (r_D^{K3\pi}) - \sqrt{K_i \bar{K}_i} r_{CP}^{-1} r_D^{K3\pi} R_{K3\pi}^i (y \cos(\delta_D^{i,K3\pi} - \phi) - x \sin(\delta_D^{i,K3\pi} - \phi))]$$

$$R(D^0 \rightarrow K^+ \pi^- \pi^- \pi^+) \propto A_{K3\pi}^2 [\bar{K}_i (r_D^{K3\pi})^2 + \frac{1}{2}(x^2 + y^2)(r_{CP})^2 K_i - \frac{1}{2}(x^2 - y^2)\bar{K}_i (r_D^{K3\pi})^2 - \sqrt{K_i \bar{K}_i} r_{CP} r_D^{K3\pi} R_{K3\pi}^i (y \cos(\delta_D^{i,K3\pi} + \phi) + x \sin(\delta_D^{i,K3\pi} + \phi))]$$

$$R(\bar{D}^0 \rightarrow K^+ \pi^- \pi^- \pi^+) \propto A_{K3\pi}^2 [K_i + \frac{1}{2}(x^2 + y^2)(r_{CP}^{-1})^2 (r_D^{K3\pi})^2 \bar{K}_i - \frac{1}{2}(x^2 - y^2)K_i - \sqrt{K_i \bar{K}_i} r_{CP}^{-1} r_D^{K3\pi} R_{K3\pi}^i (y \cos(\delta_D^{i,K3\pi} + \phi) - x \sin(\delta_D^{i,K3\pi} + \phi))]$$

- The charm mixing parameters are extracted by:

$$N_{sig} = \mathcal{L} \cdot \sigma_{D^{*+}D^-}^{obs} \cdot R \cdot \mathcal{B}(D^{*+} \rightarrow D^0 \pi^+) \cdot \varepsilon_{sig}$$

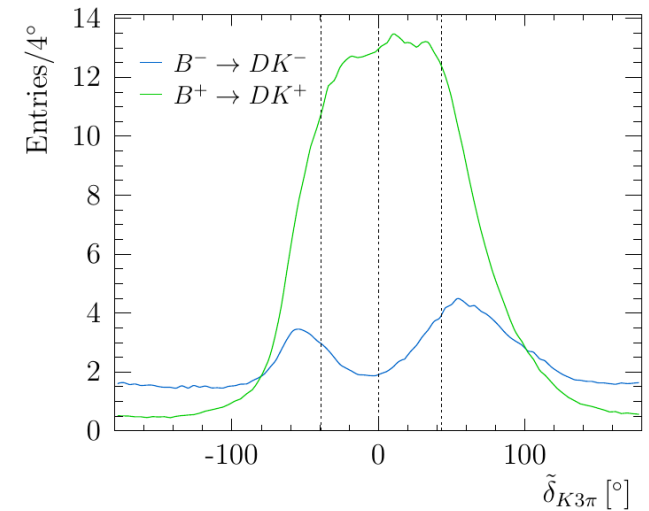
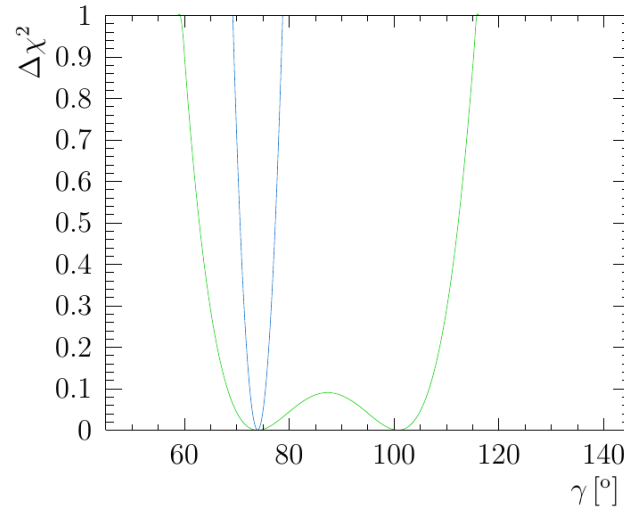
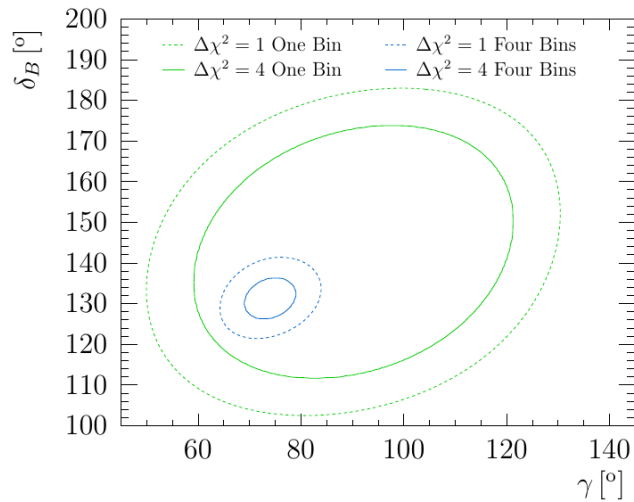
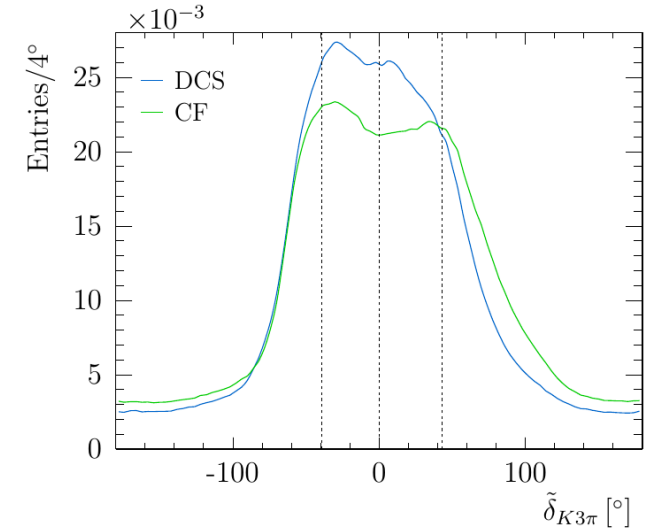
- Similar treatment in $D^{*+}D^{*-}$ sample.

Binning scheme of $K^- \pi^+ \pi^+ \pi^-$

- Local strong phase has been divided into 4 bins with approximately equal population of CF decay events.

$$\tilde{\delta}_D^{K3\pi} = \arg(A_{\bar{D}^0 \rightarrow K^+ 3\pi}(\mathbf{x}) A_{D^0 \rightarrow K^+ 3\pi}^*(\mathbf{x})) - \arg\left(\int A_{\bar{D}^0 \rightarrow K^+ 3\pi}(\mathbf{x}') A_{D^0 \rightarrow K^+ 3\pi}^*(\mathbf{x}') d\mathbf{x}'\right)$$

- Binned parameters $R_{K3\pi,i}$ and $\delta_D^{K3\pi,i}$ in each bin has been calculated by the amplitude model.



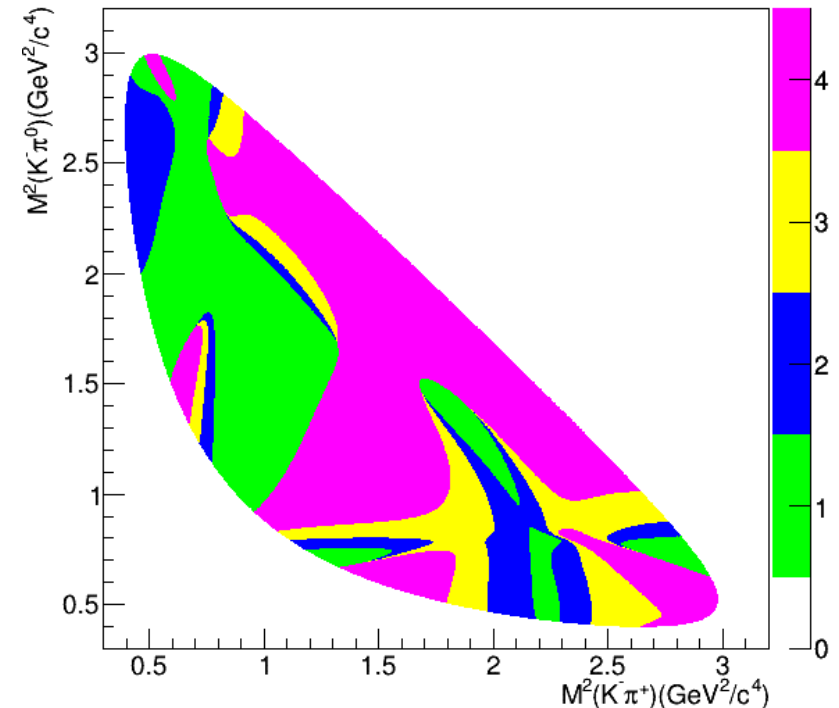
Binning scheme of $K^- \pi^+ \pi^0$

- The phase space of the D meson decay is divided into disjoint regions using the model predictions for the strong-phase difference between the CF(BESIII) and DCS(BABAR) amplitudes.

$$\Delta\delta_{K\pi\pi^0} = \arg(A_{\bar{D}^0 \rightarrow K^+ \pi^- \pi^0} A_{D^0 \rightarrow K^+ \pi^- \pi^0}^*)$$

- Similar to the binning scheme for the decay $D^0 \rightarrow K\pi\pi\pi$, we initially categorize the decay $D^0 \rightarrow K\pi\pi^0$ into four bins.

Bin	Range
1	$-180^\circ < \Delta\delta_{K\pi\pi^0} < -18^\circ$
2	$-18^\circ < \Delta\delta_{K\pi\pi^0} < 0^\circ$
3	$0^\circ < \Delta\delta_{K\pi\pi^0} < 22^\circ$
4	$22^\circ < \Delta\delta_{K\pi\pi^0} < 180^\circ$



Sensitivity study results

- The results of the charm mixing and CPV parameters are summarized in the following table, where the uncertainty is only statistical.

$K^- \pi^+ \pi^0$	4009 MeV	4015 MeV	4030 MeV
$x(\%)$	0.044	0.041	0.040
$y(\%)$	0.017	0.016	0.016
r_{CP}	0.034	0.032	0.030
$\phi(^{\circ})$	2.51	2.34	2.25

The most accurate results will be obtained at $E_{cms} = 4030$ MeV, as more samples are available at this energy point.

$K^- \pi^+ \pi^+ \pi^-$	4009 MeV	4015 MeV	4030 MeV	LHCb(50fb ⁻¹)
$x(\%)$	0.047	0.043	0.043	-
$y(\%)$	0.025	0.021	0.021	-
r_{CP}	0.042	0.038	0.038	0.005
$\phi(^{\circ})$	3.10	2.85	2.83	0.30

$K_S \pi \pi$	4009 MeV	4015 MeV	4030 MeV	LHCb(50fb ⁻¹)	Belle II(50ab ⁻¹)
$x(\%)$	0.069	0.064	0.063	0.012	0.030
$y(\%)$	0.050	0.046	0.046	0.013	0.020
r_{CP}	0.077	0.071	0.070	0.011	0.022
$\phi(^{\circ})$	4.57	4.24	4.19	0.48	1.50

Summary and to do list

Summary:

- Precision measurement of charm mixing is an important goal in heavy flavor physics for the next decade.
- Time-integrated measurements with C -even $D\bar{D}$ and incoherent $D\bar{D}$ at STCF are essential contributions.
- The most accurate results will be obtained at $E_{cms} = 4030$ MeV, as more samples are available at this energy point.

Plan:

- Study of more double tag channels, such as $K_S\pi^+\pi^-\pi^0$ and $K^-\pi^+\pi^0\pi^0$.
- Sensitivity of direct CPV at STCF with K^+K^- , $\pi^+\pi^-$ and some CP eigenstates.

Thanks!

Backup



量子关联 $D^0 \bar{D}^0$

$$\psi(3770) \rightarrow D^0 \bar{D}^0$$

$$\psi(3770): I^G(J^{PC}) = 0^-(1^{--})$$

角动量守恒

$$D^0: I(J^P) = \frac{1}{2}(0^-)$$



$D^0 \bar{D}^0$ 轨道角动量 $L = 1$, $D^0 \bar{D}^0$ 可看作全同玻色子。

两玻色子体系的总波函数:
$$\psi = \psi(r_1, r_2) \cdot \chi_{s, s_z} \cdot \chi_{I, I_3} \cdot \psi(D^0, \bar{D}^0)$$

空间波函数, 自旋波函数, 同位旋波函数, 态函数

$$L = 1$$

两末态粒子总自旋为0

两末态粒子总同位旋为0

母粒子为玻色子, 满足玻色对称关系

空间波函数满足反对称关系

自旋波函数是对称的

同位旋波函数是对称的

态函数必须满足反对称关系

- 由于对称性关系的要求, $D^0 \bar{D}^0$ 不可能出于相同的量子态, 例如 $D^0 \rightarrow K^- \pi^+$ vs $\bar{D}^0 \rightarrow K^- \pi^+$ 在奇C宇称关联的 $D^0 \bar{D}^0$ 样本中是完全禁戒的。



量子关联 $D^0 \bar{D}^0$

$$e^+ e^- \rightarrow \gamma^* \rightarrow \gamma D^0 \bar{D}^0$$

$$\gamma^*: J^P = 1^-$$

$$\gamma: J = 1$$

$$D^0: I(J^P) = \frac{1}{2}(0^-)$$

角动量守恒



$D^0 \bar{D}^0$ 轨道角动量 $L = 0$ 。

两玻色子体系的总波函数:

$$\gamma^*: J^P = 1^-$$

空间波函数, 自旋波函数, 同位旋波函数, 态函数

$$L = 0$$

两末态粒子总自旋为0

两末态粒子总同位旋为0

母粒子为玻色子, 满足玻色对称关系

空间波函数满足对称关系

自旋波函数是对称的

同位旋波函数是对称的

态函数必须满足对称关系

$e^+ e^- \rightarrow n \gamma m \pi^0 D^0 \bar{D}^0$, 当n为奇数时, 态函数满足交换对称关系, n为偶数时, 态函数满足交换反对称关系。