形状因子和轻介子结构

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Overview

Light-cone distribution amplitudes

Take pion as the example

New physics hunter $D \to \pi l^+ l^-$

DiPion LCDAs and $D_{(s)} \to \pi\pi e^+\nu$ decay

Conclusion

Emergent phenomena of QCD

QCD is believed to confine, that is, its physical states are color singlets with internal quark and gluon degrees of freedom

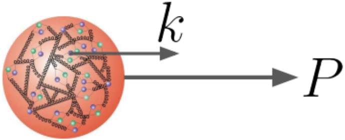
- QCD allow us to study hadron structures in terms of partons
- Factorization theorem to separate the hard partonic physics out of the hadronic physics (soft, nonperturbative objects)
- Define hadron structures by quantum field theories
- Identify theoretical observables in factorizable formulism

$$\frac{d\sigma}{d\Omega} = \int_x^1 \frac{d\zeta}{\zeta} \mathcal{H}(\zeta) f(\frac{x}{\zeta})$$

- The universal nonperturbative objects can be studied by QCD-based analytical (QCDSRs, χPT, instanton) and numerical approaches (LQCD)
- Also can be studied by performing global QCD analysis and fit, an inverse problem !
- CETQ, CT, MMHT, NNPDF, ABM, JAM, et.al.

Pion PDF,TMD,GPD

Definitions of pion distribution



One dimension PDF

△ $f_i(\zeta) = \int \frac{dz^-}{4\pi} e^{-i\zeta P^+ z^-} \langle \pi | \bar{\psi}_i(0, z^-, 0_T)\gamma^+ \psi_i(0) | \pi \rangle$

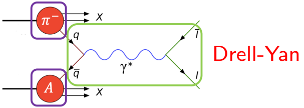
△ $\zeta = \frac{k^+}{P^+}$, the parton momentum fraction

△ $f_i(\zeta) \sim \sum_\alpha \int dk_T^2 \langle \pi | b^\dagger_{k,\alpha} b_{k,\alpha}(\zeta P^+, k_T, \alpha) | \pi \rangle$

number operator

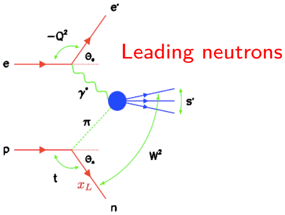
△ transversal momentum distributions (TMD) $f(\zeta, k_T)$

△ Generalized parton distributions (GPD) $f(\zeta, b_T)$



$$\sigma \propto \sum_{i,j} f_i^\pi(x_\pi, \mu) \otimes f_j^A(x_A, \mu) \otimes C_{i,j}(x_\pi, x_A, Q/\mu)$$

Extracted from **fixed target πA data**



Deeply virtuality meson production

△ TDIS at 12GeV JLab, leading proton observable, fixed target instead of collider (HERA);

△ EIC, EIcC, great integrated luminosity to reduce the systematics uncertainties;

△ COMPASS++/AMBER give π-induced DY data.

Pion LCDAs

Colliders: Pion DAs in the light-cone dominated processes

- Define the LCDAs with the Lorentz and gauge invariant ME

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\pi^-(P)\rangle = f_\pi \int_0^1 du\, e^{i\zeta P\cdot x}\left[iP_\mu\left(\phi(u)+\frac{x^2}{4}g_1(u,\mu)\right)\right.$$
$$\left. + \left(x_\mu - \frac{x^2 P_\mu}{2P\cdot x}\right)g_2(u,\mu)\right]$$
$$\langle 0|\bar{u}(x)i\gamma_5 d(-x)|\pi^-(P)\rangle = f_\pi m_0^\pi \int_0^1 du\, e^{i\zeta P\cdot x}\phi^p(u,\mu)$$
$$\langle 0|\bar{u}(x)i\sigma_{\mu\nu}\gamma_5 d(-x)|\pi^-(P)\rangle = -\frac{if_\pi m_0^\pi}{3}\left(P_\mu x_\nu - P_\nu x_\mu\right)\int_0^1 du\, e^{i\zeta P\cdot x}\phi^\sigma(u,\mu)$$

- LCDAs are dimensionless functions of u and renormalization scale μ

△ Expansion in power of large momentum transfer is governed by contributions from small transversal separations x^2 between constituents

△ describe the probability amplitudes to find the π in a state with minimal number of constitutes and have small transversal separation of order $1/\mu$

△ decay constant $\langle 0|\bar{u}(0)\gamma_z\gamma_5 d(0)|\pi^-(P)\rangle = if_\pi p_\mu$ △ normalization $\int_0^1 du\,\Phi(u) = 1$

Pion LCDAs

Conformal spin and collinear twist definition

[Braun, Korchemsky, Müller 2003]

- An application of conformal symmetry in massless QCD
- the underlying idea of *conformal expansion of LCDAs* is similar to *partial-wave expansion of wave function in quantum mechanism*
- *invariance of massless QCD under conformal trans.* *VS* *rotation symmetry*
- *longitudinal* $\otimes$ *transversal* dofs *VS* *angular* $\otimes$ *radial* dofs for spherically symmetry potential
- the transversal-momentum dependence (scale dependence of the relevant operators) is governed by the RGE
- the longitudinal-momentum dependence (orthogonal polynomials) is described in terms of irreducible representations of the corresponding symmetry group
  collinear subgroup of conformal group $SL(2, R) \cong SU(1,1) \cong SO(2,1)$

Pion LCDAs

$$\langle 0|\bar{u}_i(0)\Gamma_{ij}d_j(z)|\pi^-(p)\rangle = \frac{i}{4}\int_0^1 due^{-iup\cdot z}\left[\not{p}\gamma_5\phi(u) + m_0^\pi(\mu)\gamma_5\phi^p(u) + m_0^\pi(\mu)\gamma_5(1-\not{n}\not{\bar{n}})\phi^t(u)\right]_{ji}$$

$$\phi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^\pi(\mu)C_n^{3/2}(u)$$

$$\phi^\sigma(u) = 6u(1-u)\left[1+5\eta_{3\pi}C_2^{3/2}(u)\right]$$

$$\phi^p(u,\mu) = \left[1+30\eta_{3\pi}C_2^{1/2}(u) - 3\eta_{3\pi}\omega_{3\pi}C_4^{1/2}(u)\right]$$

- $\phi(x)$ and $\phi^{p,t}(u)$ are the twist two and twist three LCDAs
- $a_0^\pi = f_\pi$, $a_{n\geqslant 2}^\pi(\mu_0)$ and $m_0^\pi(\mu_0)$ are universal nonpertubative parameters
- μ dependences in a_n^π and others the integration over the **transversal dof**

  [Brodsky & Lepage1980, Balitsky & Braun1988]
- $C_n(u)$ are Gegenbauer polynomials $\sim$ Jacobi Polynomials $P_n^{j_1,j_2}\left(\frac{\overleftrightarrow{D}_+}{\overleftrightarrow{\partial}_+}\right)$ in the local collinear conformal expansion **longitudinal dof**

  [Lepage & Brodsky 1979, 80, Efremov & Radyushkin 1980, Braun & Filyanov 1990]

Pion LCDAs

$\phi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^{\pi}(\mu) C_n^{3/2}(u)$

- QCD definition $\qquad a_n^{\pi}(\mu) = \langle \pi | q(z)\bar{q}(z) + z_\rho \partial_\rho q(z)\bar{q}(z) + \cdots | 0 \rangle$
- **LQCD**: 0.334 ± 0.129[UKQCD 2010], 0.135 ± 0.032[RQCD 2019], $0.258^{+0.079}_{-0.052}$ [LPC 2022]

  △ default scale at 1 GeV $\qquad$ scale running

$$a_n(\mu) = a_n(\mu_0) \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{\frac{\gamma_n^{(0)} - \gamma_0^{(0)}}{2\beta_0}}, \quad \gamma_n^{\perp(\parallel),(0)} = 8C_F \left(\sum_{k=1}^{n+1} \frac{1}{k} - \frac{3}{4} - \frac{1}{2(n+1)(n+2)} \right)$$

  △ a_4^{π} is not available $\leftarrow$ the growing number of derivatives in $q\bar{q}$ operator

- **QCDSR**: 0.19 ± 0.06[Chernyak 1984], $0.26^{+0.21}_{-0.09}$[Khodjamirian 2004], $0.28^{+0.08}_{-0.08}$[Ball 2006]

  △ nonlocal vacuum condensate is introduced and modeled for $a_{n>2}^{\pi}$ [Bakulev 2001]

- Dispersion relation as an **Inverse problem** [Li 2020, Yu 2022]

  quark-hadron duality $\rightarrow$ Laguerre Polynomials to construct spectral density

$$\{a_2, a_4, a_6, a_8\} = \{0.249, 0.134, 0.106, 0.096\}$$

Pion LCDAs

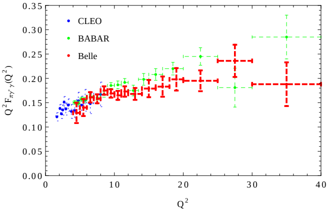
$\phi_\pi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^\pi(\mu) C_n^{3/2}(u)$

- **Data-driven** with QCD calculations for the π involved exclusive processes

△ $F_{B\to\pi}$: 0.19 ± 0.19 [Ball 05], 0.16 [Khodjamirian 11], large error from B meson

△ $F_{\pi\gamma\gamma^*}$: 0.14 [Agaev 2010] BABAR+CLEO, 0.10 [Agaev 2012] Belle+CLEO

large uncertainty of $a_{n>2}^\pi$, discrepancy data at large Q^2



[SC 1901.06071]

DiPion LCDAs

Method	a_2^π(2 GeV)	Refs.
LO QCDSR, CZ model	0.39	[30,31]
QCDSR	$0.18^{+0.15}_{-0.26}$	[32]
QCDSR	0.19 ± 0.06	[33]
QCDSR, NLC	0.13 ± 0.04	[34,35]
$F_{\pi\gamma\gamma^*}$, LCSRs	0.12 ± 0.04 (2.4 GeV)	[36]
$F_{\pi\gamma\gamma^*}$, LCSRs	0.21 (2.4 GeV)	[37]
$F_{\pi\gamma\gamma^*}$, LCSRs, R	0.19	[38]
$F_{\pi\gamma\gamma^*}$, LCSRs, R	0.31	[39]
$F_{\pi\gamma\gamma^*}$, LCSRs, NLO	0.096	[40]
$F_{\pi\gamma\gamma^*}$, LCSRs, NLO	0.068	[41]
F_π^{em}, LCSRs	$0.17 \pm 0.10 \pm 0.05$	[42]
F_π^{em}, LCSRs, R	0.14 ± 0.02	[43]
$F_{B\to\pi}$, LCSRs	0.13 ± 0.13	[44]
$F_{B\to\pi}$, LCSRs	0.11	[45,46]
LQCD, TWST, $N_f = 2$, CW	0.201 ± 0.114	[47]
LQCD, TWST, $N_f = 2+1$, DWF	0.233 ± 0.088	[48]
LQCD, MST, $N_f = 2$	0.136 ± 0.03	[27]
LQCD, MST, $N_f = 2+1$, CW	0.0762 ± 0.0127	[29]

△ F_π: 0.24 ± 0.17 [Bebek1978] Wilson Lab+NA7, 0.20 ± 0.03 [Agaev 2005] JLab

large uncertainty of $a_{n>2}^\pi$, available data only in small spacelike q^2

Pion LCDAs from F_π

- Spacelike data is available in the narrow region $q^2 \in [-2.5, 0]$ GeV^2
- Perturbative QCD calculations are valid in the intermediate/large $|q^2|$
  N^{2}LO calculation in collinear factorization $\sim$ NLO [Chen^2, Feng, Jia 2312.17228]
- The mismatch destroys the direct extracting programme from $F_\pi(q^2 < 0)$
- Timelike form factor $F_\pi(q^2 > 0)$ provides another opportunity
  - △ $e^+e^- \to \pi^+\pi^-(\gamma)$, $4m_\pi^2 \leqslant q^2 \lesssim 9$ GeV^2 [BABAR 2012]
  - △ $\tau \to \pi\pi\nu_\tau$, $4m_\pi^2 \leqslant q^2 \leqslant 3.125$ GeV^2 [Belle 2008]
  - △ $e^+e^-(\gamma) \to \pi^+\pi^-$, $0.6 \leqslant Q^2 \leqslant 0.9\,\mathrm{GeV}^2$ with ISR [BESIII 2016]
- TL measurement and SL predictions are related by dispersion relation
- The standard dispersion relation and The modulus representation

$$F_\pi(q^2 < s_0) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im} F_\pi(s)}{s - q^2 - i\epsilon}$$

⇓ [**SC**, Khodjamirian, Rosov 2007.05550]

$$F_\pi(q^2 < s_0) = \exp\left[\frac{q^2\sqrt{s_0 - q^2}}{2\pi}\int_{s_0}^{\infty} \frac{ds \ln|F_\pi(s)|^2}{s\sqrt{s - s_0}\,(s - q^2)}\right]$$

$$|\mathcal{F}_\pi(s)|^2 = \Theta(s_{\max} - s)\,|\mathcal{F}_{\pi,\mathrm{Inter.}}^{\mathrm{data}}(s)|^2 + \Theta(s - s_{\max})\,|\mathcal{F}_\pi^{\mathrm{pQCD}}(s)|^2$$

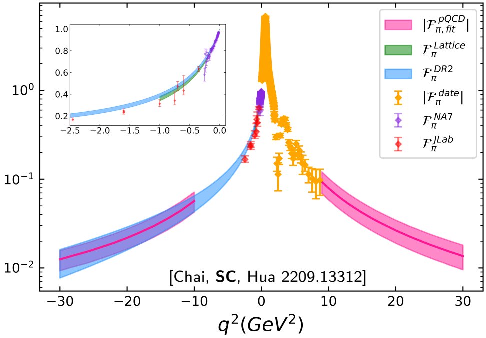
Pion LCDAs from F_π

- $a_2 = 0.275 \pm 0.055,\ a_4 = 0.185 \pm 0.065,\quad m_0^\pi = 1.37^{+0.29}_{-0.32}$ GeV

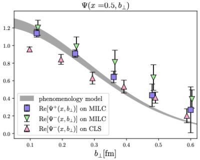
  △ Pion deviates from the purely asymptotic one △ a_2^π is not enough

  △ $0.258^{+0.079}_{-0.052}$ [LPC 2201.09173[hep-lat]], $0.249^{+0.005}_{-0.006}$ [Li 2205.06746]

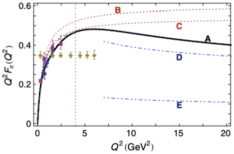
- a slight derivation in the small region



- intrinsic transverse momentum ?[LPC 2302.09961]



- dynamical chiral symmetry breaking ?[Chang et.al. 1307.0026]



Pion LCDAs from F_π

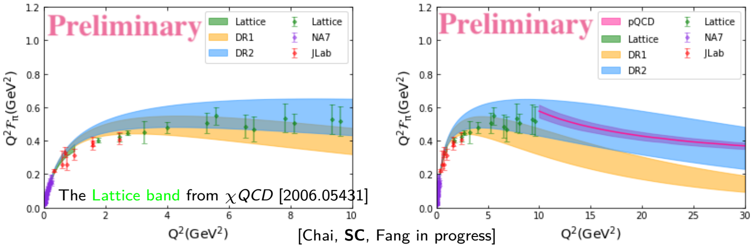
- Taking into account the contribution from the iTMD

$$\frac{f_\pi m_0^{\mathcal{P}}}{2\sqrt{6}}\phi^p(u,\mu) = \int \frac{d^2\vec{k}_T}{16\pi^3}\phi^p_{2p}(u,\vec{k}_T) + \int \frac{d^2\vec{k}_{T1}}{16\pi^3}\frac{d^2\vec{k}_{T2}}{4\pi^2}\phi^p_{3p}(u,\vec{k}_{T1},\vec{k}_{T2}).$$

$$\psi^p_{2p}(u,\vec{k}_T) = \frac{f_\pi m_0^{\mathcal{P}}}{2\sqrt{6}}\phi^p_{2p}(u,\mu)\Sigma(u,\vec{k}_T),$$

$$\psi^p_{3p}(u,\vec{k}_{1T},\vec{k}_{2T}) = \frac{f_\pi m_0^{\mathcal{P}}}{2\sqrt{6}}\eta_{3\pi}\phi^p_{3p}(u,\mu)\Sigma'(\alpha_i,\vec{k}_{1T},\vec{k}_{2T}).$$

- comparison with the impressive LQCD calculation [H.T Ding et.al, 2404.04412]

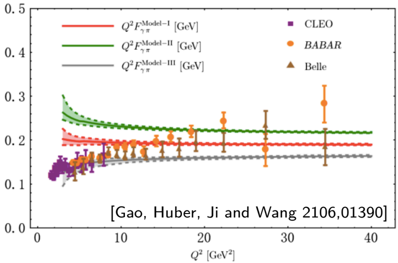


[Chai, **SC**, Fang in progress]

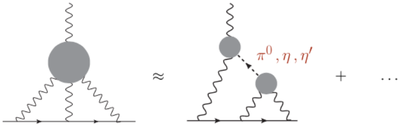
- the slight derivation is still there (not sensitive to iTMD)
- Form factors of K and $\eta^{(\prime)}$ mesons are in tuning

Pion LCDAs from $F_{\pi\gamma\gamma^*}$

- $F_{\pi\gamma\gamma^*}$ is the theoretically most clean observable $\propto a_n^\pi$
- Two-loop calculation of $F_{\pi\gamma\gamma^*}$ in hard-collinear factorization theorem
  $N^2LO \sim NLO$



Model-I [Brodsky, Teramond 0707.3859, RQCD 1903.08038]
Model-II [**SC**, Khodjamirian, Rosov 2007.05550]
Model-III [Mikhailov, Pimikov, Stefanis 1604.06391]



[Gérardin, Meyer, Nyffeler 1607.08174]
Hadronic light-by-light scattering (HLbL) contribution to $a_\mu^{HLbL;\pi^0}$

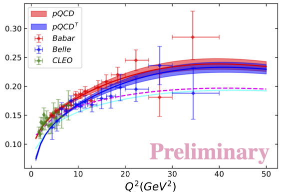
- pQCD calculation with taking into account the iTMD

† improve the pQCD power in the intermediate momentum transfers

† modification in the small and intermediate regions is significant

- more result of the $\eta^{(\prime)}$, η_q and η_s transition form factors

[Chai, **SC**, Fang in progress]

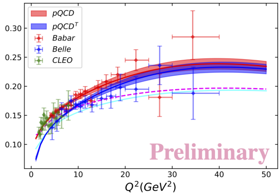


Pion LCDAs from $F_{\pi\gamma\gamma^*}$

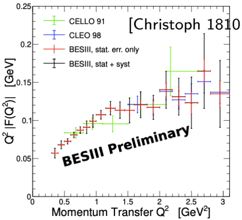
- pQCD calculation with taking into account the iTMD

† modification in the small and intermediate regions is significant (sensitive to the measurement)

[Chai, **SC**, Fang in progress]



- The measurement discrepancy starts from ~ 7 GeV^2
- BEPCII up to 5.6 GeV, Belle-II (4+7 GeV)
- STCF, 2-7 GeV, to settle down the "fat pion" issue



[Christoph 1810.00654[hep-ex]]

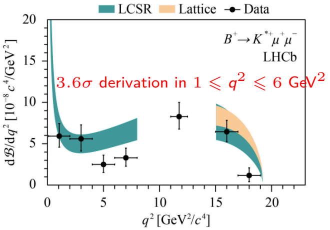
New physics hunter

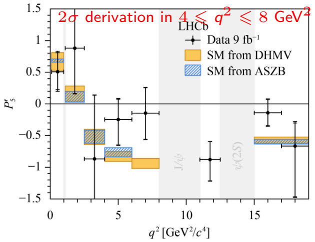
$D \to \pi l^+ l^-$

New physics hunter $D \to \pi l^+ l^-$

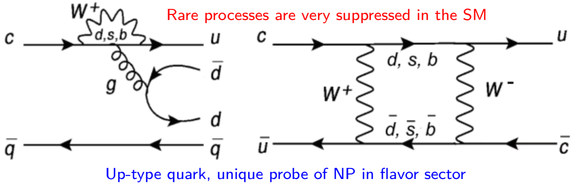
Anomalies in Flavor Physics

LHCb: a Ten-Year Review [2111.14360]





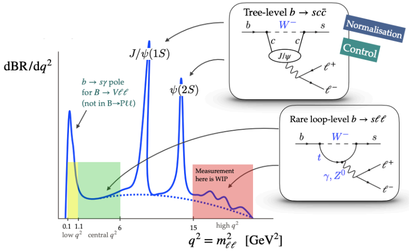
- $b \to s l^+ l^-$ anomalies are indeed a sign of NP ?
- if yes, a plausible effect in other FCNC processes like $c \to u$ transition



New physics hunter $D \to \pi l^+ l^-$

- relative difficulty to make theoretical prediction
- $\Uparrow$ reduced hierarchy $\mathcal{O}(\Lambda_{\rm QCD}/m_c)$

  $+$ the resonances effect a larger portion of the phase space



$b \to s l^+ l^-$

† the light resonants are negligible due to the large typical bin size, can be circumvented as the resulting effect in binned observables

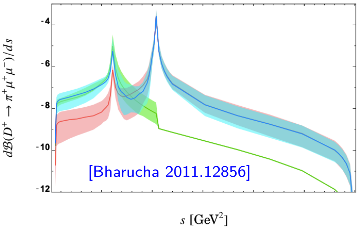
† polluting resonant effect from c-loop is much larger, so this kinematical regime is ignored and vetoed in the experimental analysis

- In D decays, the resonances effect a large of available phase space
- the hadronic resonances is more important than the non-resonant tails

New physics hunter $D \to \pi\mu^+\mu^-$

- In D decays, the resonances effect a large of available phase space
- Breit-Winger function + partonic result of quark vacuum polarisations
  [Hiller 1510. 00311, Kośnik 1510. 00965]
- Breit-Winger function + substracted dispersion relation [Feldmann 1705. 05891]
- Regge trajectories + asymptotically recovered [Bharucha 2011.12856]

  $D \to \pi$ form factor input of π LCDAs [**SC**, Khodjamirian, Rosov 2007.05550]



$$\mathcal{B}^{\rm SM}_{\rm lowq^2} = \left(8.1^{+5.9}_{-6.1}\right) \times 10^{-9}$$

$$\mathcal{B}^{\rm SM}_{\rm highq^2} = \left(2.7^{+4.0}_{-2.6}\right) \times 10^{-9}$$

current best-world limit

$$2.5 \times 10^{-8},\ 0.250^2 \leqslant q^2 \leqslant 0.525^2$$

$$2.9 \times 10^{-8},\ q^2 > 1.25^2$$

- the flat term $F_H(s)$ and the forward-backward asymmetry $A_{FB}(s)$
  $\sim 10\%$ sensitive to NP

New physics hunter $D \to \pi\mu^+\mu^-$

- Experimentail potentials

Experiment	Measurement	Sensitivity
LHCb	Angular observables	$\sim 0.2\%$ with $50\,\mathrm{fb}^{-1}$,
		$\sim 0.08\%$ with $300\,\mathrm{fb}^{-1}$
LHCb	Branching ratio	$\sim 10^{-8}$ with $50\,\mathrm{fb}^{-1}$,
		$\sim 3\times10^{-9}$ with $300\,\mathrm{fb}^{-1}$
Belle-II	Branching ratio	$\sim 10^{-8}$ (rescaling BaBar)

[talk at Towards the Ultimate Precision in Flavour Physics, Durham U.K. (2019)]

[BABAR Collaboration 1107.4465]

Run 4 ~ 2030
Run 5 ~ 2038

$N(D\bar{D}) \sim 10^9/\mathrm{ab}^{-1}$ angular observables $\sim 0.2\%$

- BESIII Collaboration in the electron channel [BESIII Collaboration 1802.09752]
  $\mathcal{B}(D \to \pi^+\pi^- e^+ e^-) < 0.7 \times 10^{-5}$ with $N(c\bar{c}) = 2 \times 10^7$ at 3.7 GeV

3.770	1	$D^0\bar{D}^0$	3.6	3.6×10^9	
		$D^+\bar{D}^-$	2.8	2.8×10^9	
		$D^0\bar{D}^0$		7.9×10^8	Single Tag
		$D^+\bar{D}^-$		5.5×10^8	Single Tag

STCF $N(D\bar{D}) \sim 8 \times 10^9$ Branching ratio $\sim 10^{-8}$

DiPion LCDAs

a comprehensive partial-wave analysis

DiPion LCDAs and B_{l4} decays

- **DiPion LCDAs are the most general object to describe the $\pi\pi$ mass spectral in diffractive production, provides a new nonperturbative objects to describe the transition from partons to hadrons**

- Comparison between $B \to \pi l \bar{\nu}$ and $B \to \rho l \bar{\nu}$ [Gao, Lü, Shen, Wang, Wei 1902.11092]

$$|V_{ub}| = \left(3.05^{+0.67}_{-0.52}\Big|_{\rm theo}\ {}^{+0.19}_{-0.20}\Big|_{\rm exp}\right) \times 10^{-3}, \quad \text{from B} \to \rho l \nu$$

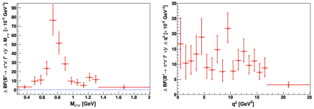
$$|V_{ub}|_{\rm PDG} = (3.70 \pm 0.12|_{\rm theo} \ \pm 0.10|_{\rm exp}) \times 10^{-3} \quad \text{from B} \to \pi l \nu$$

- Propose to measure the $B \to \pi^+ \pi^0 l^- \bar{\nu}$ decay with the $B \to \pi^+\pi^0$ form factor calculated from B meson LCSRs [**SC**, Khodjamirian, Virto 1701,01633]

- $B \to \pi\pi l \bar{\nu}_l$ has already been measured, mainly its resonant part $B \to \rho l \bar{\nu}_l$ $(1.58 \pm 0.11) \times 10^{-4}$ [CLEO 2000, BABAR 2011, Belle 2013]

- First measurement of the branching fraction of $B^+ \to \pi^+\pi^- l^+ \bar{\nu}_l$ $(2.3 \pm 0.4) \times 10^{-4}$ [Belle 2005.07766]

  More data on the way from Belle II



- First Lattice QCD study of the $B \to \pi\pi l \bar{\nu}$ transition amplitude in the region of large q^2 and $\pi\pi$ invariant mass near the ρ resonance [Leskovec et.al. 2212.08833[hep-lat]]

DiPion LCDAs and D_{l4} decays

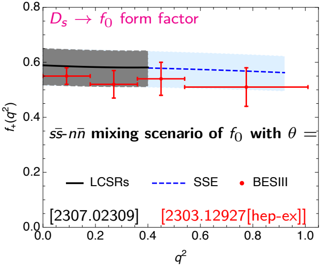
- $\mathcal{B}(D^0 \to \rho^- e^+ \nu) = \mathcal{B}(D^0 \to \pi^- \pi^0 e^+ \nu) = (1.45 \pm 0.08) \times 10^{-3}$ [BESIII 19]
- $\mathcal{B}(D^+ \to \rho^0 e^+ \nu) = (1.9 \pm 0.1) \times 10^{-3}$ [CLEO 13]
  $\mathcal{B}(D^+ \to f_0(500)[\to \pi^+ \pi^-] e^+ \nu) = (0.64 \pm 0.06) \times 10^{-3}$ [BESIII 19]
  $\mathcal{B}(D^+ \to \pi^+ \pi^- e^+ \nu) = (2.45 \pm 0.11) \times 10^{-3}$ [BESIII 19]
- $\mathcal{B}(D_s^+ \to f_0(980)[\to \pi^0 \pi^0] e^+ \nu) = (0.79 \pm 0.15) \times 10^{-3}$ [BESIII 22]
  $\mathcal{B}(D_s^+ \to f_0(980)[\to \pi^+ \pi^-] e^+ \nu) = (1.72 \pm 0.15) \times 10^{-3}$ [BESIII 23]
- DiPion LCDAs provides a solution to describe both the resonance contribution and nonresonant background in the heavy flavor decays
- Contributions at different partial wave are calculable in principle if the strong phase shifts are available from $\pi\pi$ scattering or heavy decays, interplay with the partial-wave analysis of the data samples
- Provides a supplement study to the scalar meson structure
- Improvement with the width effect ($\pi\pi$ invariant mass spectral)

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_S l^+ \nu)}{ds dq^2} = \frac{1}{\pi} \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1^2 \beta_\pi(s)}{|m_S^2 - s + i\left(g_1^2 \beta_\pi(s)) + g_2^2 \beta_K(s)\right)|^2}$$

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_S l^+ \nu)}{dk^2 dq^2} = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} \frac{\beta_{\pi\pi}(k^2)\sqrt{\lambda_{D_s}} q^2}{16\pi} \sum_{\ell=0}^{\infty} 2|F_0^{(\ell)}(q^2, k^2)|^2$$

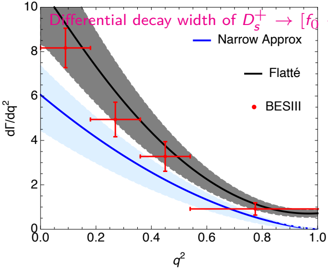
DiPion LCDAs and D_{l4} decays

$D_s \to f_0$ form factor

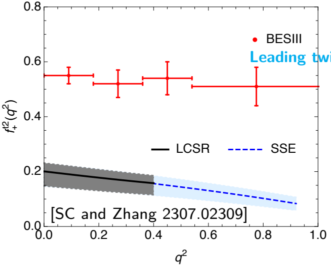


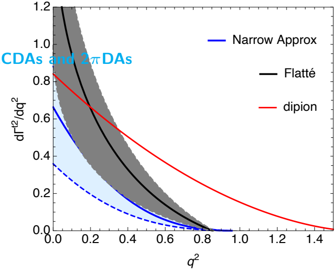
$s\bar{s}$-$n\bar{n}$ **mixing scenario of** f_0 **with** $\theta = 20^\circ \pm 10^\circ$

Differential decay width of $D_s^+ \to [f_0 \to] [\pi\pi]_S e^+ \nu_e$



Leading twist of f_0 **LCDAs and** 2π**DAs**





DiPion LCDAs and D_{l4} decays

- Twist-3 LCDAs give dominate contribution in $D_s \to f_0, [\pi\pi]_S$ transitions
- further measurements would help us to understand the DiPion system

4.180	1	$D_s^{+*}D_s^-$+c.c. $D_s^{+*}D_s^-$+c.c. $\tau^+\tau^-$	0.90 3.6	9.0×10^8 1.3×10^8 3.6×10^9	Single tag

BESIII $\mathcal{O}(10^6) D_s^+/D_s^{*+}$ production

Belle II $\mathcal{O}(10^9) D_s^+/D_s^{*+}$ production

Conclusion

- In the light-cone dominated processes, hadron structure is studied in terms of **LCDAs**
- $F_{\pi\gamma\gamma^*}$ to determine leading twist pion LCDAs, to check the LQCD evaluations, key input to further study of pion
- FCNC channel $D \to \pi l^+ l^-, \pi\pi l^+ l^-$ in charm decay to hunt NP
- Pure leptonic weak decay $D_s^* \to e^+ \nu$ to determine the total width, the electromagnetic coupling $g_{D_s^* D_s \gamma}$, a benchmark of different nonperturbative approaches
- **DiPion LCDAs** are introduced to describe the resonant contribution and the nonresonant background in heavy flavor decays   Two dimension measurement of $D_{(s)} \to \pi\pi e^+ \nu$

Thank you for your patience.