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# Axion search in eta decay

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# Introduction

## Strong CP problem

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \sum_q \bar{q}(i\not{D} - m_q e^{i\theta_q})q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \theta\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$q \to e^{i\gamma_5\alpha}q \quad \Longrightarrow \quad \theta_q \to \theta_q + 2\alpha\,, \qquad \theta \to \theta - 2\alpha$$

**implying the invariant quantity:** $\bar{\theta} = \theta + \theta_q$

$$\Rightarrow \quad \mathcal{L}_{\rm QCD}^{\rm axion} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \boxed{\bar{\theta}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

**Constraints from neutron electric dipole moment (nEDM)**

$$|\bar{\theta}| \lesssim 10^{-10}$$

- **Strong CP problem: why is $\bar{\theta}$ so unnaturally tiny ?**

# Axion: an elegant solution to strong CP

[Peccei,Quinn,PRL'77]
[Weinberg,PRL'78] [Wilzeck,PRL'78]

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \bar{\theta}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \cdots$$

**Spontaneous breaking of $U(1)_{PQ}$ symmetry & anomalous term: axion (pseudo-NGB)**

- **$f_a$: the axion decay constant. Invisible axion: $f_a >> v_{EW}$. Axion interactions with SM particles are accompanied with the factors of $(1/f_a)^n$.**

- **Stringent constraints for $m_{a,0} = 0$ (QCD axion)** $m_a^2 = m_{a,0}^2 + \frac{F_\pi^2 m_\pi^2}{4 f_a^2}$

  **[Di Luzio, et al., Phy.Rep'20] [Sikivie, RMP'21] [Irastorza, Redondo,PPNP'18] ... ...**

  **Cosmology, Astronomy, Collider, Quantum precision measurements , Cavity Haloscope, ... ...**

- **It is possible to introduce other model-dependent axion interactions, such as axion-quark, axion-photon and axion-lepton terms.**

**We will focus on the QCD-like axion: $m_{a,0}\,(\neq 0) \ll f_a$ with model-independent $aG\tilde{G}$ interaction, i.e., the MODEL INDEPENDENT QCD axion interactions.**

- **Axion-hadron interactions are relevant at low energies.**

# Axion chiral perturbation theory (A$\chi$PT)

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \boxed{\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

**Two ways to proceed:**

**(1) Remove the $aG\tilde{G}$ term via the quark axial transformation**

$$q \to e^{i\frac{a}{2f_a}\gamma_5 Q_a} q$$

$\mathrm{Tr}(Q_a) = 1$

$$-\frac{a\alpha_s}{8\pi f_a}G\tilde{G} - \frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma_5 Q_a q \qquad M_q \to M_q(a) = e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a}$$

**Mapping to $\chi$PT**

$$\mathcal{L}_2 = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi_a U^\dagger + U\chi_a^\dagger\rangle + \frac{\partial_\mu a}{2f_a}J_A^\mu|_{\rm LO}$$

$$\chi_a = 2B_0 e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \qquad J_A^\mu|_{\rm LO} = -i\frac{F^2}{2}\langle Q_a(\partial^\mu U U^\dagger + U^\dagger \partial^\mu U)\rangle$$

- **$Q_a = M_q^{-1}/\mathrm{Tr}(M_q^{-1})$ is usually taken to eliminate the LO mass mixing between axion and pion [Georgi,Kaplan,Randall, PLB'86], though any other hermitian $Q_a$ should lead to the same physical quantities.**

- **$J_A^\mu\,\partial_\mu a$ will cause the kinematical mixing. Should be consistently included. [Bauer, et al., PRL'21]**

## (2) Explicitly keep the $aG\tilde{G}$ term and match it to χPT

**Reminiscent:**

**QCD $U(1)_A$ anomaly that is caused by topological charge density** $\omega(x) = \alpha_s G_{\mu\nu}\tilde{G}^{\mu\nu}/(8\pi)$ **is responsible for the massive singlet $\eta_0$.**

**Axion could be similarly included as the $\eta_0$ via the U(3) χPT:**

$$\mathcal{L}^{\rm LO} = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \frac{F^2}{12}M_0^2 X^2$$

$$U = u^2 = e^{i\frac{\sqrt{2}\Phi}{F}}\,, \qquad \chi = 2B(s+ip)\,, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u\,,$$

$$u_\mu = i u^\dagger D_\mu U u^\dagger\,, \quad D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)$$

$$X = \log(\det U) + i\frac{a}{f_a} \qquad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \overline{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

- **$Q_a$ is not needed in U(3) χPT.**
- **$M_0^2 = 6\tau/F^2$, with τ the topological susceptibility. Note that $M_0^2 \sim O(1/N_c)$.**
- **δ expansion scheme: $\delta \sim O(p^2) \sim O(m_q) \sim O(1/N_c)$.**
- **Axion interactions enter via the axion-meson mixing terms.**

# $\pi$-$\eta$-$\eta'$-$a$ mixing in U(3) AχPT

[Gao,ZHG,Oller,Zhou,JHEP'23]

$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = M^{\rm LO+NLO} \begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix} \qquad M^{\rm LO+NLO} = \begin{pmatrix} 1+(0.015\pm0.001) & 0.017+(-0.010\pm0.001) & 0.009+(-0.007\pm0.001) & \frac{12.1+(0.48\pm0.08)}{f_a} \\ -0.019+(0.007\pm0.001) & 0.94+(0.21\pm0.01) & 0.33+(-0.22\pm0.03) & \frac{34.3+(0.9\pm0.2)}{f_a} \\ -0.003+(-0.003\pm0.000) & -0.33+(-0.18\pm0.03) & 0.94+(0.13\pm0.02) & \frac{25.9+(-0.5\pm0.1)}{f_a} \\ \frac{-12.1+(-0.20\pm0.03)}{f_a} & \frac{-23.8+(1.6^{+0.8}_{-0.8})}{f_a} & \frac{-35.7+(-5.7^{+1.6}_{-1.7})}{f_a} & 1+\frac{27.6\pm1.0}{f_a^2} \end{pmatrix}$$

## Two-photon couplings

$$\mathcal{L}_{\rm WZW}^{\rm LO} = -\frac{3\sqrt{2}e^2}{8\pi^2 F}\varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2 \Phi\rangle$$

$$\mathcal{L}_{\rm WZW}^{\rm NLO} = it_1 \varepsilon_{\mu\nu\rho\sigma}\langle f_+^{\mu\nu} f_+^{\rho\sigma}\chi_-\rangle + k_3 \varepsilon_{\mu\nu\rho\sigma}\langle f_+^{\mu\nu} f_+^{\rho\sigma}\rangle X$$

**Note: one needs the π-η-η'-*a* mixing as input to calculate $g_{a\gamma\gamma}$.**

$$F_{\pi^0\gamma\gamma}^{\rm Exp} = 0.274 \pm 0.002\,{\rm GeV}^{-1}\,,$$
$$F_{\eta\gamma\gamma}^{\rm Exp} = 0.274 \pm 0.006\,{\rm GeV}^{-1}\,,$$
$$F_{\eta'\gamma\gamma}^{\rm Exp} = 0.344 \pm 0.008\,{\rm GeV}^{-1}\,.$$

→

$$t_1 = -(4.4 \pm 2.3) \times 10^{-4}\,{\rm GeV}^{-2}\,,$$
$$k_3 = (1.25 \pm 0.23) \times 10^{-4}$$

→

$$g_{a\gamma\gamma} = 4\pi\alpha_{em} F_{a\gamma\gamma} = -\frac{\alpha_{em}}{2\pi f_a}(1.63 \pm 0.01)$$

**which can be compared to: 1.92±0.04** [Grilli de Cortona, et al., JHEP'16] **and 2.05±0.03** [Lu, et al., JHEP'20]

- **IB corrections could cause visible effects (working in progress).**

# Axion production from $\eta\to\pi\pi a$ decay in SU(3) $\chi$PT

## Why focus on axion in $\eta$ decay:

- ✓ Valuable channel to search axion @colliders: many available experiments with large data samples of $\eta/\eta'$ [BESIII, STCF, JLab, RETOP, ... ...]
- ✓ $\eta\to\pi\pi\pi$ (IB suppressed), $\eta\to\pi\pi a$ (no IB suppression)
- ✓ $\eta\to\pi\pi a$: theoretically easier to handel than $\eta'\to\pi\pi a$ (next step)

## Previous works:

- ❖ Most of them rely on leading-order $\chi$PT
- ❖ Possible issue: bulk contributions@LO $\chi$PT are constant terms, and potential large corrections from higher orders may result.
- ❖ Hadron resonance effects may lead to enhancements.

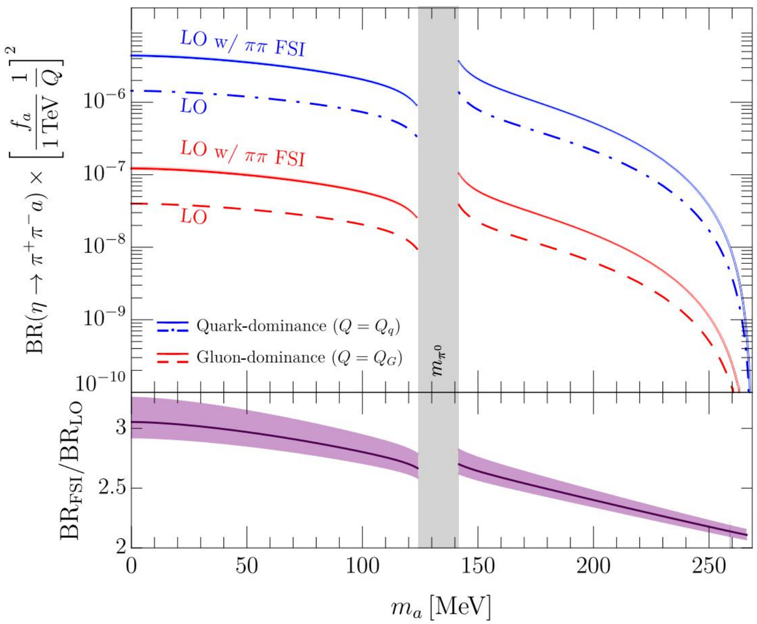
## Advances in our work :

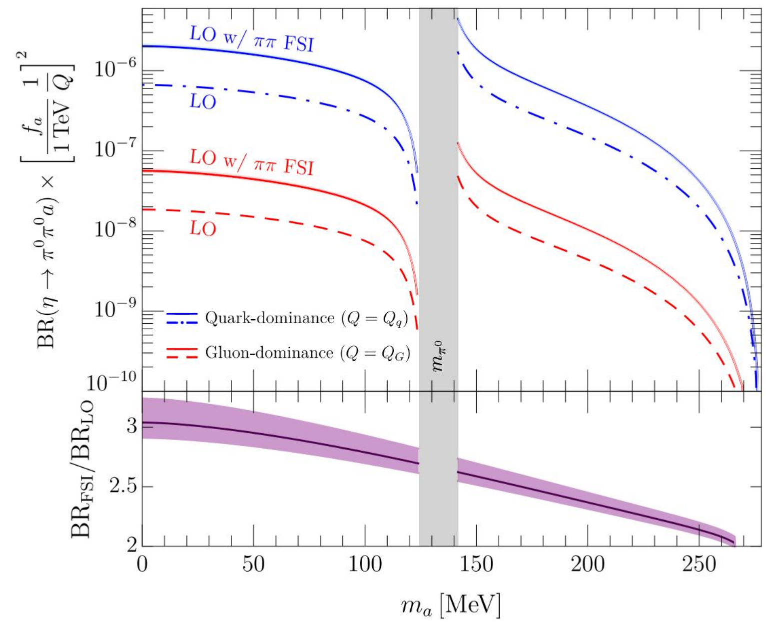
- ➢ Study of renormalization of $\eta\to\pi\pi a$ @1-loop level in SU(3) $\chi$PT
- ➢ To implement unitarization to the $\eta\to\pi\pi a$ $\chi$PT amplitude
- ➢ Uncertainty analyes in the phenomenological discussions

Alves and Sergi, arXiv:2402.02993 [hep-ph].

$$M_0(s) = P(s)\Omega_0^0(s)$$

**η→ππa LO amplitude** | **Omnes function: ππ FSI**




**Our improvements:**

- **NLO perturbative decay amplitude include $s$- and $t(u)$-channel interactions perturbatively.**
- **The unitarized decay amplitude will be constructed to account for the $s$-channel $\pi\pi$ final state interaction (FSI) effect that respect the chiral symmetry.**
- **Dalitz plots will be explored to decode the dynamics in η→ππa.**

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$q \to e^{i\frac{a}{2f_a}\gamma_5 Q_a} q$$

**Leading-order χPT Lagrangian**

$$\mathcal{L}_2 = \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + \chi_a U^\dagger + U\chi_a^\dagger \rangle + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\rm LO} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2$$

$$\chi_a = 2B_0 M(a) \qquad M(a) \equiv \exp\left(-i\frac{a}{2f_a}Q_a\right) M \exp\left(-i\frac{a}{2f_a}Q_a\right) \qquad J_A^\mu|_{\rm LO} = -i\frac{F^2}{2}\langle Q_a \left\{\partial^\mu U, U^\dagger\right\}\rangle$$

Note: we consider the octet part ($\overline{Q}_a$) of $Q_a$ in SU(3) χPT

**Next-to-leading order χPT Lagrangian**

$$\mathcal{L}_4 = L_1\langle \partial_\mu U \partial^\mu U^\dagger \rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \cdots + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\rm NLO},$$

$$J_A^\mu|_{\rm NLO} = -4iL_1 \langle \bar{Q}_a \{U^\dagger, \partial^\mu U\}\rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \cdots .$$

## Mixing between ALP and $\eta_8$

- **LO mixing**

$$\mathcal{L}_{\rm mix}^{\rm LO}=\frac{1}{2}\partial_\mu a\partial^\mu a+\frac{1}{2}\partial_\mu\eta_8\partial^\mu\eta_8+\frac{F}{f_a}C_k^{a\eta}\partial_\mu a\partial^\mu\eta_8-\frac{1}{2}\left(m_{a,0}^2+\frac{F^2}{f_a^2}C_m^a\right)a^2-\frac{1}{2}\bar{m}_{\eta_8}^2\eta_8^2\,,$$

$$C_k^{a\eta}=\frac{1}{2}\langle\bar{Q}_a\lambda_8\rangle=\frac{\bar{m}_{\eta_8}^2-\bar{m}_\pi^2}{2\sqrt{3}\bar{m}_{\eta_8}^2}\,,\quad C_m^a=\frac{B_0}{\langle M^{-1}\rangle}=\frac{\bar{m}_\pi^2(3\bar{m}_{\eta_8}^2-\bar{m}_\pi^2)}{12\bar{m}_{\eta_8}^2}\,,$$

LO contribution of pNGBs masses from light quarks masses $\bar{m}_\pi^2=2B_0\hat{m}\,,\quad \bar{m}_{\eta_8}^2=\frac{2B_0}{3}(\hat{m}+2m_s)$

- Fields redefinitions up to $O(1/f_a^3)$

$$a=\left[1+\frac{1}{2}(C_k^{a\eta})^2\frac{m_{a,0}^2(m_{a,0}^2-2\bar{m}_{\eta_8}^2)}{(m_{a,0}^2-\bar{m}_{\eta_8}^2)^2}\frac{F^2}{f_a^2}\right]\widetilde{a}-\frac{F}{f_a}C_k^{a\eta}\frac{\bar{m}_{\eta_8}^2}{\bar{m}_{\eta_8}^2-m_{a,0}^2}\widetilde{\eta}\,,$$

$$\eta_8=\left[1+\frac{1}{2}(C_k^{a\eta})^2\frac{\bar{m}_{\eta_8}^2(\bar{m}_{\eta_8}^2-2m_{a,0}^2)}{(m_{a,0}^2-\bar{m}_{\eta_8}^2)^2}\frac{F^2}{f_a^2}\right]\widetilde{\eta}-\frac{F}{f_a}C_k^{a\eta}\frac{m_{a,0}^2}{m_{a,0}^2-\bar{m}_{\eta_8}^2}\widetilde{a}\,.$$

$$\mathcal{L}_{\rm mix}^{\rm LO}=\frac{1}{2}\partial_\mu\widetilde{a}\partial^\mu\widetilde{a}-\frac{1}{2}\bar{m}_a^2\widetilde{a}^2+\frac{1}{2}\partial_\mu\widetilde{\eta}\partial^\mu\widetilde{\eta}-\frac{1}{2}\bar{m}_\eta^2\widetilde{\eta}^2+\mathcal{O}\left(\frac{F^3}{f_a^3}\right)\,,$$

$$\bar{m}_a^2=m_{a,0}^2+\frac{F^2}{f_a^2}\left(C_m^a+\frac{(C_k^{a\eta})^2m_{a,0}^4}{m_{a,0}^2-\bar{m}_{\eta_8}^2}\right)\,,\quad \bar{m}_\eta^2=\bar{m}_{\eta_8}^2+\frac{F^2}{f_a^2}\frac{(C_k^{a\eta})^2\bar{m}_{\eta_8}^4}{\bar{m}_{\eta_8}^2-m_{a,0}^2}\,.$$

- **NLO mixing and field redefinitions for ALP and η fields**

$$\mathcal{L}_{\rm mix}^{\rm NLO} = \frac{1}{2}(1-\Sigma_{aa}^{(4)\prime})\partial_\mu\widetilde{a}\partial^\mu\widetilde{a} + \frac{1}{2}(1-\Sigma_{\eta\eta}^{(4)\prime})\partial_\mu\widetilde{\eta}\partial^\mu\widetilde{\eta} - \Sigma_{a\eta}^{(4)\prime}\partial_\mu\widetilde{a}\partial^\mu\widetilde{\eta}$$
$$- \frac{1}{2}\left[\bar{m}_a^2 + \Sigma_{aa}^{(4)}(0)\right]\widetilde{a}^2 - \frac{1}{2}\left[\bar{m}_\eta^2 + \Sigma_{\eta\eta}^{(4)}(0)\right]\widetilde{\eta}^2 - \Sigma_{a\eta}^{(4)}(0)\widetilde{a}\widetilde{\eta}\,.$$

$\Sigma_{ij}^{(4)}(p^2)$ is the $O(p^4)$ two-point 1PI amplitude

$\Sigma_{aa}^{(4)} = O(p^4/f_a^2)\,,$ $\Sigma_{a\eta}^{(4)} = O(p^4/f_a)\,,$ $\Sigma_{\eta\eta}^{(4)} = O(p^4)\,.$

$\Sigma_{ij}^{(4)\prime}$ denotes $d\Sigma_{ij}^{(4)}(p^2)/dp^2$, which is momentum independent up to NLO.

- Fields redefinitions at NLO

$$\widetilde{a} = \left(1+\frac{1}{2}\Sigma_{aa}^{(4)\prime}\right)a_{\rm phy} + \frac{\Sigma_{a\eta}^{(4)}(m_\eta^2)}{m_\eta^2-m_a^2}\eta_{\rm phy}\,,\quad \widetilde{\eta} = \left(1+\frac{1}{2}\Sigma_{\eta\eta}^{(4)\prime}\right)\eta_{\rm phy} + \frac{\Sigma_{a\eta}^{(4)\prime}(m_a^2)}{m_a^2-m_\eta^2}a_{\rm phy}\,.$$

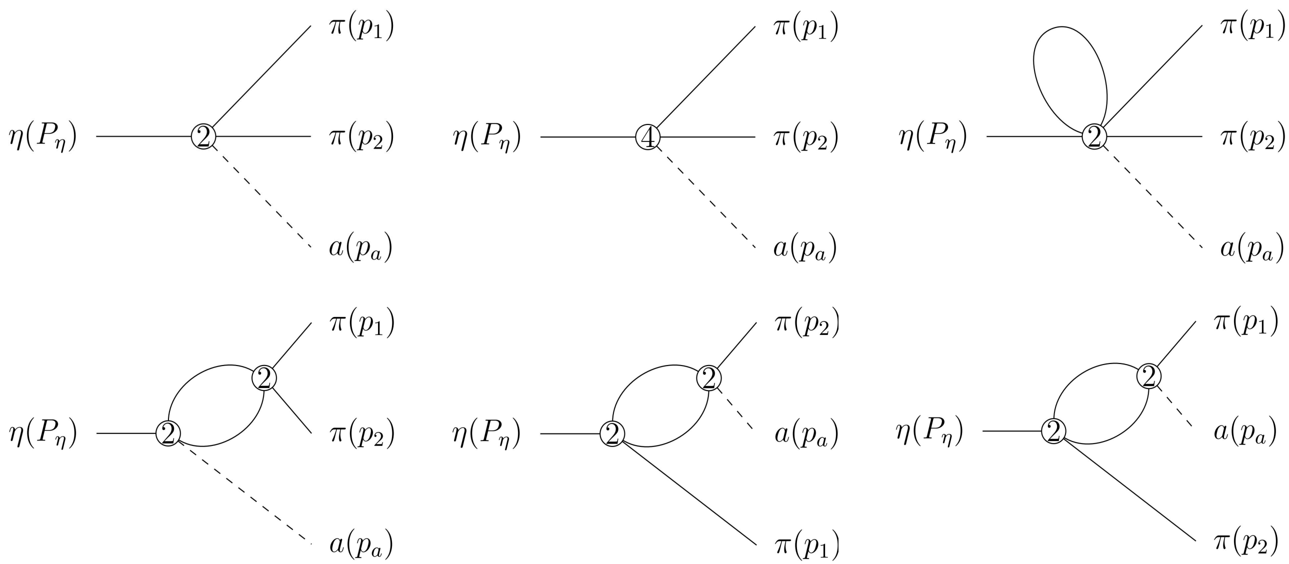
$$\mathcal{L}_{\rm mix}^{\rm NLO} = \frac{1}{2}\partial_\mu a_{\rm phy}\partial^\mu a_{\rm phy} - \frac{1}{2}m_a^2 a_{\rm phy}^2 + \frac{1}{2}\partial_\mu\eta_{\rm phy}\partial^\mu\eta_{\rm phy} - \frac{1}{2}m_\eta^2\eta_{\rm phy}^2$$

Perturbative calculation of the amplitude is performed by using field variables $a_{\rm phy}$ and $\eta_{\rm phy}$.

$$\langle a(p_a)\pi^i(p_1)\pi^j(p_2)|\hat{T}|\eta(P_\eta)\rangle = (2\pi)^4\delta^4(P_\eta - p_1 - p_2 - p_a)\mathcal{M}_{\eta;\pi^i\pi^j a}(s,t,u)\,,$$

$$s = (p_1 + p_2)^2 = m_{\pi\pi}^2,\ \ t = (p_a + p_2)^2 = m_{a\pi}^2,\ \ u = (p_a + p_1)^2$$

- **Feynman diagrams up to NLO**




- Parameters

| Masses and $F_\pi$ [MeV] | | | | LECs $L_i^r(\mu)$ at $\mu = 770$ MeV (in unit of $10^{-3}$) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $m_\pi$ | $m_K$ | $m_\eta$ | $F_\pi$ | $L_1^r$ | $L_2^r$ | $L_3^r$ | $L_4^r$ | $L_5^r$ | $L_6^r$ | $L_7^r$ | $L_8^r$ |
| 137 | 496 | 548 | 92.1 | 1.0(1) | 1.6(2) | −3.8(3) | 0.0(3) | 1.2(1) | 0.0(4) | −0.3(2) | 0.5(2) |

[J. Bijnens and G. Ecker, Ann. Rev. Nucl. Part. Sci. 64, 149 (2014)]

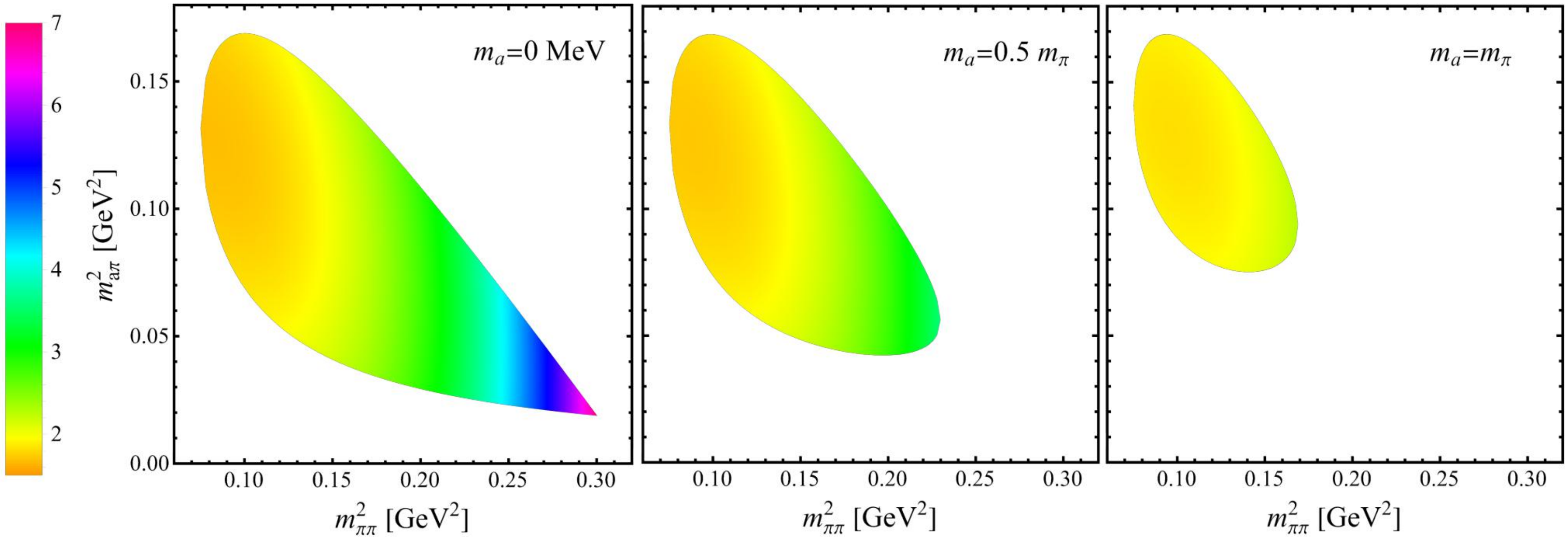
✓ **Renomarlization condition is verified to be consistent with conventional ChPT.**

# Observations:

- Strong isospin breaking effects enter the $\eta \to \pi\pi a$ amplitudes at the order of $(m_u - m_d)^2$
- In the isospin limit ($m_u = m_d$), the amplitudes with $\pi^+\pi^-$ and $\pi^0\pi^0$ in $\eta \to \pi\pi a$ processes are identical.

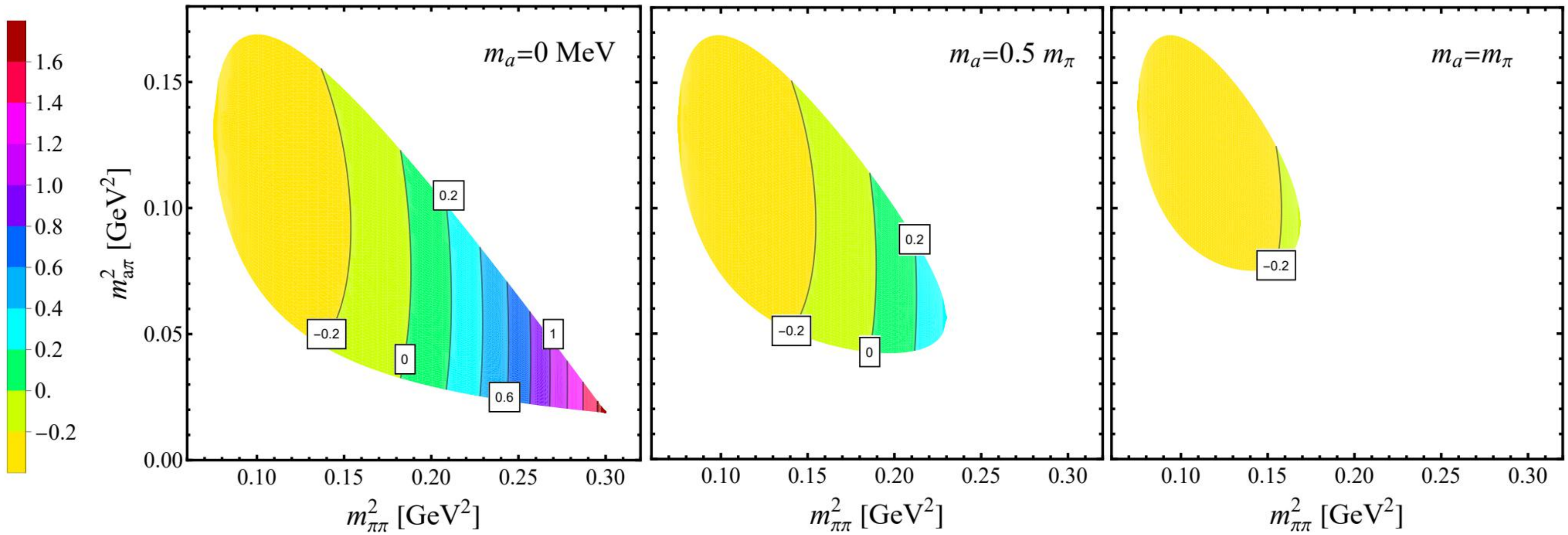
- **Dalitz plots**

$$10^6 f_a^2 \frac{d^2\Gamma_{\eta\to\pi^+\pi^- a}}{ds\,dt} = \frac{10^6 f_a^2}{32(2\pi)^3 m_\eta^3}\left(\left|\mathcal{M}^{(2)}_{\eta;\pi\pi a}\right|^2 + 2\mathcal{M}^{(2)}_{\eta;\pi\pi a}\mathrm{Re}\left(\mathcal{M}^{(4)}_{\eta;\pi\pi a}\right) + \left|\mathcal{M}^{(4)}_{\eta;\pi\pi a}\right|^2\right) \quad \text{in unit of GeV}^{-1}$$



● **Dalitz plots to show the NLO/LO convergence**

$$\left(2\mathcal{M}^{(2)}_{\eta;\pi\pi a}\mathrm{Re}\left(\mathcal{M}^{(4)}_{\eta;\pi\pi a}\right)+\left|\mathcal{M}^{(4)}_{\eta;\pi\pi a}\right|^2\right)/\left|\mathcal{M}^{(2)}_{\eta;\pi\pi a}\right|^2$$




**Important lessons:**

➢ **Non-perturbative effect in the ππ subsystem can be important.**

➢ **Perturbative treatment of the aπ subsystem is justified.**

## ● Unitarization of the partial-wave η→ππa amplitude

$$\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s) = \frac{\mathcal{M}^{00,\mathrm{L}}_{\eta;\pi\pi a}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)} \,,$$

$$G_{\pi\pi}(s) = -\frac{1}{(4\pi)^2}\left(\log\frac{m_\pi^2}{\mu^2} - \sigma_\pi(s)\log\frac{\sigma_\pi(s)-1}{\sigma_\pi(s)+1} - 1\right) \,,$$

$$\mathcal{M}^{00,\mathrm{L}}_{\eta;\pi\pi a}(s) = \mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s) + \mathcal{M}^{00,(4)}_{\eta;\pi\pi a}(s) - G_{\pi\pi}(s)\,\mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s)\,T^{00,(2)}_{\pi\pi\to\pi\pi}(s) \,.$$

**The unitarized amplitude satisfies the relation**

$$\mathrm{Im}\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s) = \rho_{\pi\pi}(s)\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s)\left(T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s)\right)^* \,, \qquad (2m_\pi < \sqrt{s} < 2m_K)$$

**with the unitarized PW ππ amplitude** $T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s) = \dfrac{T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}$

## ● Unitarized PW amplitude based on LO η→ππa amplitude

$$\mathcal{M}^{00,\mathrm{Uni-LO}}_{\eta;\pi\pi a}(s) = \frac{\mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)} \,.$$

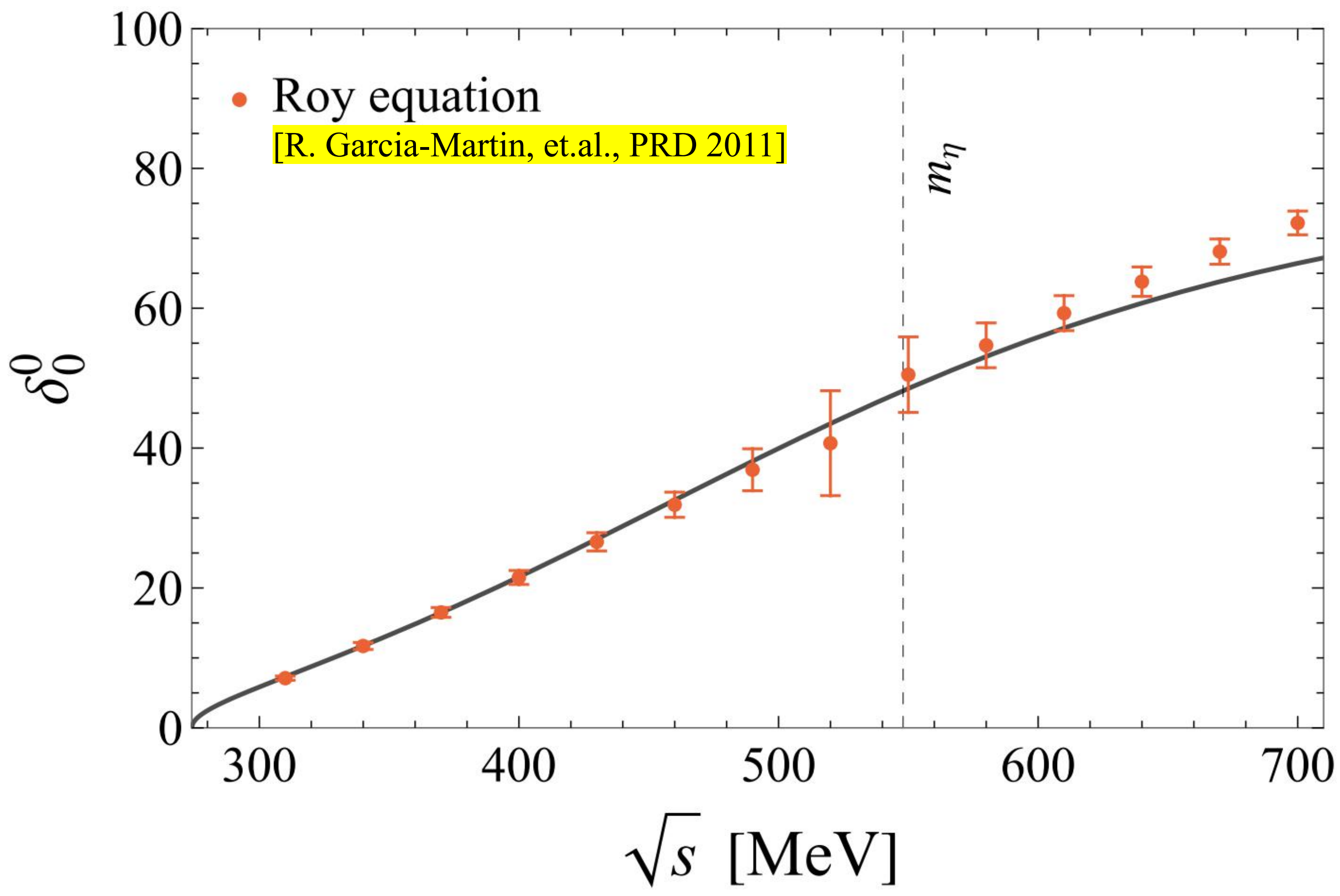
**Resemble the method:**

**Alves and Sergi, arXiv:2402.02993 [hep-ph].**

$$M_0(s) = P(s)\Omega_0^0(s)$$

# Phase shifts from the unitarized PW $\pi\pi$ amplitude

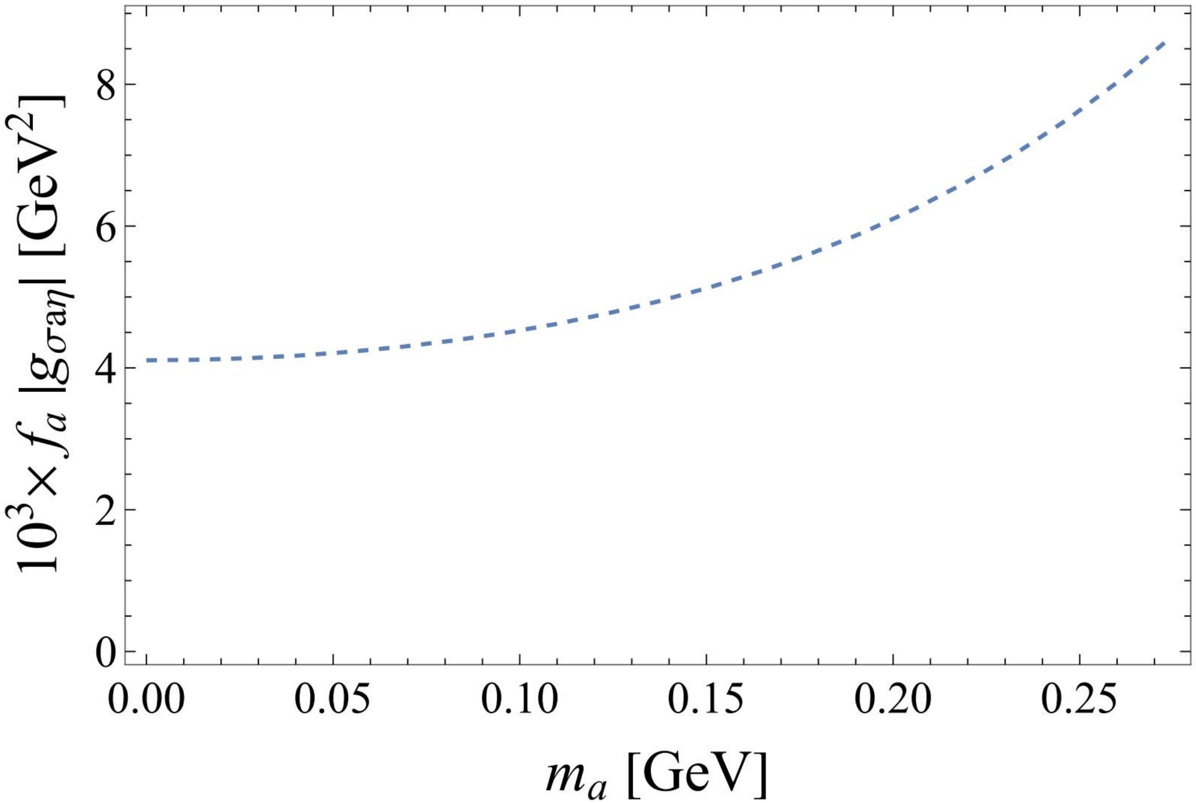
$$T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s) = \frac{T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}{1 - G_{\pi\pi}(s)T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}$$



- **Pole position of $f_0(500)/\sigma$ on the second Riemann sheet** $\sqrt{s_\sigma} = 457 \pm i251$ MeV

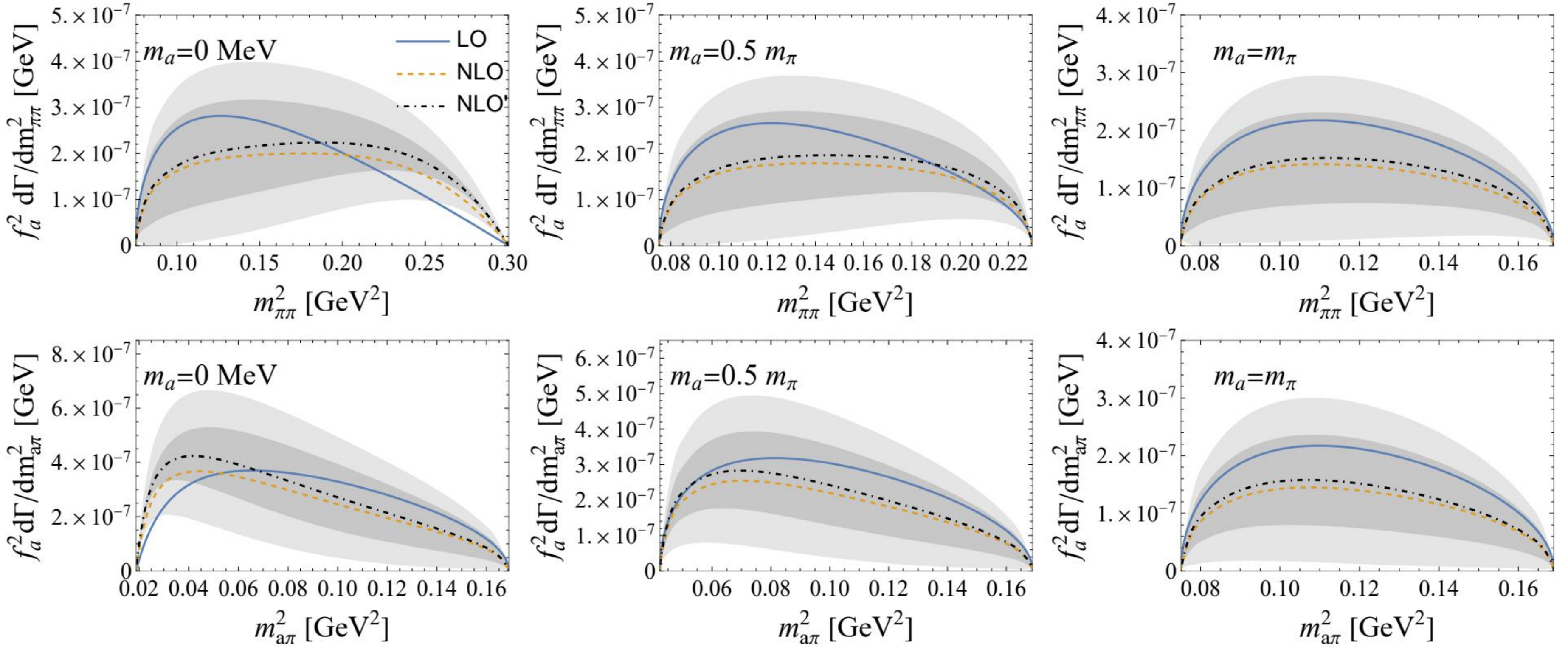
- $\sigma$-$a\eta$ coupling varying with $m_a$

$$\mathcal{M}^{00,\mathrm{Uni,II}}_{\eta;\pi\pi a}(s)\big|_{s\to s_\sigma} \sim -\frac{g_{\sigma\pi\pi}g_{\sigma a\eta}}{s-s_\sigma}$$



## Predictions of the $\pi\pi$ and $a\pi$ invariant-mass distributions at different axion masses

$$|\mathcal{M}|^2 = \left(\mathcal{M}^{(2)}\right)^2 + 2\mathcal{M}^{(2)}\mathrm{Re}\left(\mathcal{M}^{(4)}\right) + |\mathcal{M}^{(4)}|^2$$



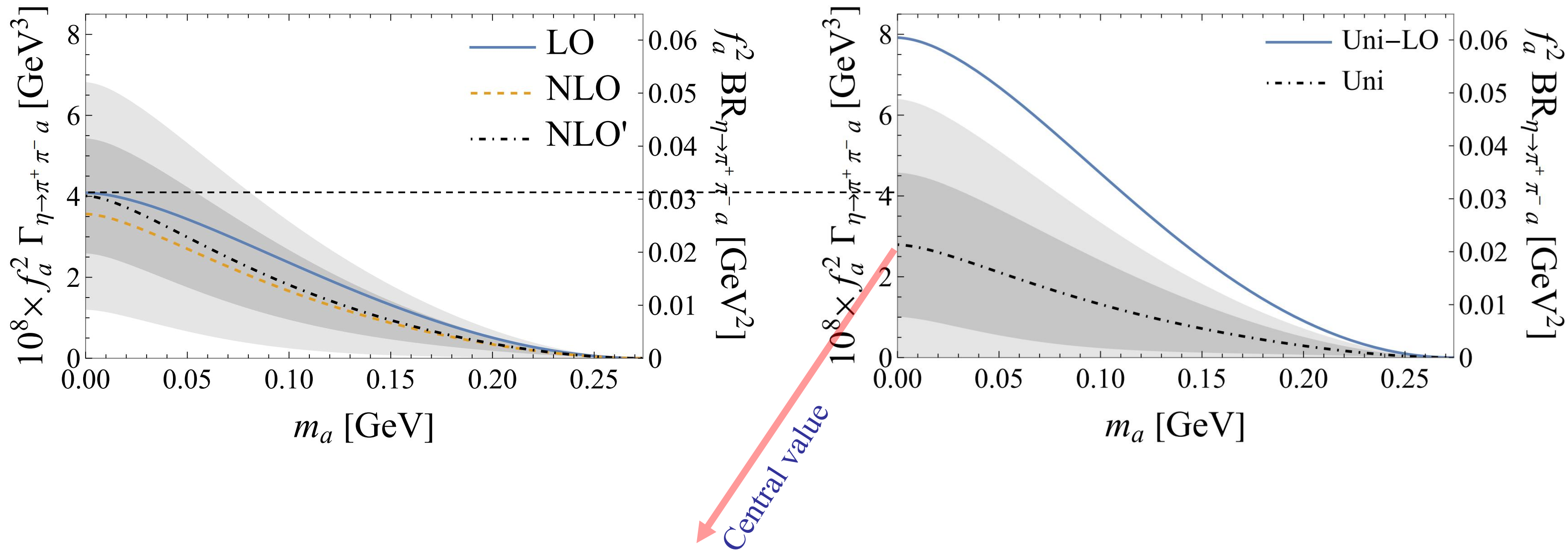

## Uncertainty bands:

- **Lighter regions:**

| $L_1^r$ | $L_2^r$ | $L_3^r$ | $L_4^r$ | $L_5^r$ | $L_6^r$ | $L_7^r$ | $L_8^r$ |
|---|---|---|---|---|---|---|---|
| 1.0(1) | 1.6(2) | −3.8(3) | 0.0(3) | 1.2(1) | 0.0(4) | −0.3(2) | 0.5(2) |

- **Darker regions: freeze the 1/Nc suppressed ones ($L_4$,$L_6$,$L_7$)**

# Predictions of the $\eta\to\pi\pi a$ branching ratios by varying $m_a$




$$\mathrm{BR}_{\eta\to\pi^+\pi^-a}\Big|_{m_a\to 0} = 2.1\times 10^{-2}\left(\frac{\mathrm{GeV}^2}{f_a^2}\right)$$

**Possible detection channels: $a\to\gamma\gamma$, $a\to e^+e^-$, $a\to\mu^+\mu^-$**

# Summary

- **Chiral perturbation theory provides a systematical and useful framework to study the axion-meson interactions .**

- **$\pi$-$\eta$-$\eta'$-*a* mixing and $g_{a\gamma\gamma}$ are predicted in U(3) A$\chi$PT by taking the various hadronic and lattice inputs.**

- **Axion production from the $\eta\to\pi\pi a$ decay is calculated. Large uncertainties from higher-order LECs are found.**

**谢谢！**