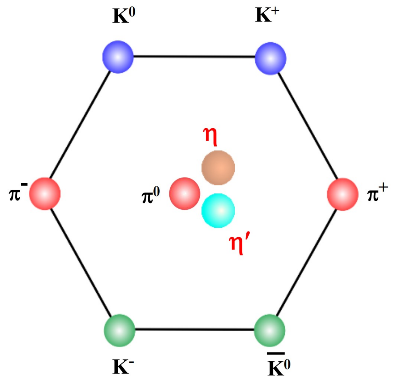
η/η' 物理研究进展



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超级陶粲装置研讨会，2024年7月7日-11日，兰州大学

η Physics

Standard Model Tests:
- Chiral symmetry and anomalies
- Extract $\eta - \eta'$ mixing angle and quark mass ratio
- Theory inputs to HLbL for $(g-2)_\mu$
- QCD scalar dynamics

Fundamental Symmetry Tests:
- C, CP violations
- P, CP violations
- Lepton flavor violations

BSM Physics in Dark Sector:
- Vector bosons (B boson, dark photon and X boson)
- Dark scalars
- Pseudoscalars (ALPs)
- BSM weak decays

Channel	Expt. branching ratio	Discussion
$\eta \to 2\gamma$	39.41(20)%	Chiral anomaly, η–η' mixing
$\eta \to 3\pi^0$	32.68(23)%	$m_u - m_d$
$\eta \to \pi^0\gamma\gamma$	$2.56(22) \times 10^{-4}$	χPT at $\mathcal{O}(p^6)$, leptophobic B boson, light Higgs scalars
$\eta \to \pi^0\pi^0\gamma\gamma$	$<1.2 \times 10^{-3}$	χPT, axion-like particles (ALPs)
$\eta \to 4\gamma$	$<2.8 \times 10^{-4}$	$<10^{-11}$[55]
$\eta \to \pi^+\pi^-\pi^0$	22.92(28)%	$m_u - m_d$, C/CP violation, light Higgs scalars
$\eta \to \pi^+\pi^-\gamma$	4.22(8)%	Chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation
$\eta \to \pi^+\pi^-\gamma\gamma$	$<2.1 \times 10^{-3}$	χPT, ALPs
$\eta \to e^+e^-\gamma$	$6.9(4) \times 10^{-3}$	Theory input for $(g-2)_\mu$, dark photon, protophobic X boson
$\eta \to \mu^+\mu^-\gamma$	$3.1(4) \times 10^{-4}$	Theory input for $(g-2)_\mu$, dark photon
$\eta \to e^+e^-$	$<7 \times 10^{-7}$	Theory input for $(g-2)_\mu$, BSM weak decays
$\eta \to \mu^+\mu^-$	$5.8(8) \times 10^{-6}$	Theory input for $(g-2)_\mu$, BSM weak decays, P/CP violation
$\eta \to \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs
$\eta \to \pi^+\pi^-e^+e^-$	$2.68(11) \times 10^{-4}$	Theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \to \pi^+\pi^-\mu^+\mu^-$	$<3.6 \times 10^{-4}$	Theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \to e^+e^-e^+e^-$	$2.40(22) \times 10^{-5}$	Theory input for $(g-2)_\mu$
$\eta \to e^+e^-\mu^+\mu^-$	$<1.6 \times 10^{-4}$	Theory input for $(g-2)_\mu$
$\eta \to \mu^+\mu^-\mu^+\mu^-$	$<3.6 \times 10^{-4}$	Theory input for $(g-2)_\mu$
$\eta \to \pi^+\pi^-\pi^0\gamma$	$<5 \times 10^{-4}$	Direct emission only
$\eta \to \pi^\pm e^\mp \nu_e$	$<1.7 \times 10^{-4}$	Second-class current
$\eta \to \pi^+\pi^-$	$<4.4 \times 10^{-6}$ [56]	P/CP violation
$\eta \to 2\pi^0$	$<3.5 \times 10^{-4}$	P/CP violation
$\eta \to 4\pi^0$	$<6.9 \times 10^{-7}$	P/CP violation

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Channel	Expt. branching ratio	Discussion
$\eta' \to \eta\pi^+\pi^-$	42.6(7)%	Large-N_c χPT, light Higgs scalars
$\eta' \to \pi^+\pi^-\gamma$	28.9(5)%	Chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation
$\eta' \to \eta\pi^0\pi^0$	22.8(8)%	Large-N_c χPT
$\eta' \to \omega\gamma$	2.489(76)% [58]	Theory input for singly-virtual TFF and $(g-2)_\mu$
$\eta' \to \omega e^+e^-$	$2.0(4) \times 10^{-4}$	Theory input for doubly-virtual TFF and $(g-2)_\mu$
$\eta' \to 2\gamma$	2.331(37)% [58]	Chiral anomaly, η–η' mixing
$\eta' \to 3\pi^0$	2.54(18)% (⋆)	$m_u - m_d$
$\eta' \to \mu^+\mu^-\gamma$	$1.09(27) \times 10^{-4}$	Theory input for $(g-2)_\mu$, dark photon
$\eta' \to e^+e^-\gamma$	$4.73(30) \times 10^{-4}$	Theory input for $(g-2)_\mu$, dark photon
$\eta' \to \pi^+\pi^-\mu^+\mu^-$	$<2.9 \times 10^{-5}$	Theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, dark photon, ALPs
$\eta' \to \pi^+\pi^-e^+e^-$	$2.4(^{+1.3}_{-1.0}) \times 10^{-3}$	Theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, dark photon, ALPs
$\eta' \to \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs
$\eta' \to \pi^+\pi^-\pi^0$	$3.61(17) \times 10^{-3}$	$m_u - m_d$, C/CP violation, light Higgs scalars
$\eta' \to 2(\pi^+\pi^-)$	$8.4(9) \times 10^{-5}$	Theory input for doubly-virtual TFF and $(g-2)_\mu$
$\eta' \to \pi^+\pi^-2\pi^0$	$1.8(4) \times 10^{-4}$	
$\eta' \to 2(\pi^+\pi^-)\pi^0$	$<1.8 \times 10^{-3}$	ALPs
$\eta' \to K^\pm\pi^\mp$	$<4 \times 10^{-5}$	Weak interactions
$\eta' \to \pi^\pm e^\mp\nu_e$	$<2.1 \times 10^{-4}$	Second-class current
$\eta' \to \pi^0\gamma\gamma$	$3.20(24) \times 10^{-3}$	Vector and scalar dynamics, B boson, light Higgs scalars
$\eta' \to \eta\gamma\gamma$	$8.3(3.5) \times 10^{-5}$ [59]	Vector and scalar dynamics, B boson, light Higgs scalars
$\eta' \to 4\pi^0$	$<4.94 \times 10^{-5}$ [60]	(S-wave) P/CP violation
$\eta' \to e^+e^-$	$<5.6 \times 10^{-9}$	Theory input for $(g-2)_\mu$, BSM weak decays
$\eta' \to \mu^+\mu^-$		Theory input for $(g-2)_\mu$, BSM weak decays
$\eta' \to \ell^+\ell^-\ell^+\ell^-$		Theory input for $(g-2)_\mu$
$\eta' \to \pi^+\pi^-\pi^0\gamma$		B boson
$\eta' \to \pi^+\pi^-$	$<1.8 \times 10^{-5}$	P/CP violation
$\eta' \to 2\pi^0$	$<4 \times 10^{-4}$	P/CP violation

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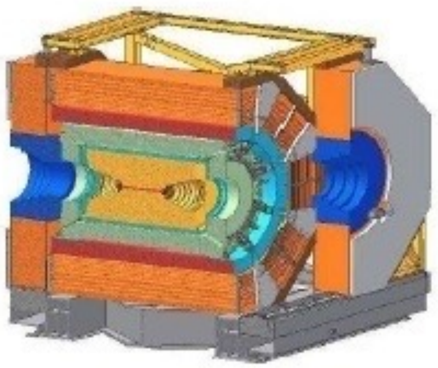
η/η' experiment

e⁺e⁻ Collider

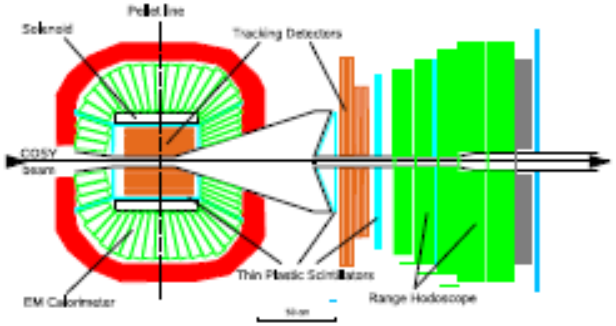
BESIII at BEPCII



KLOE-2

Fixed-target

WASA at COSY



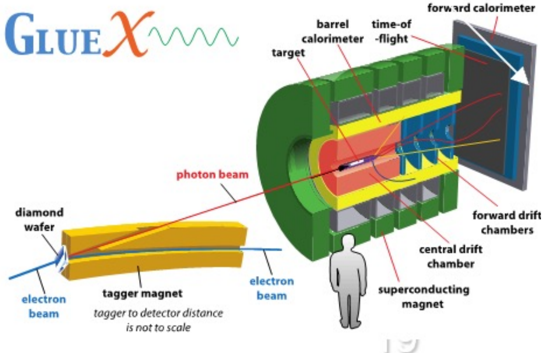
CEBAF Large Acceptance Spectrometer

CLAS(12)

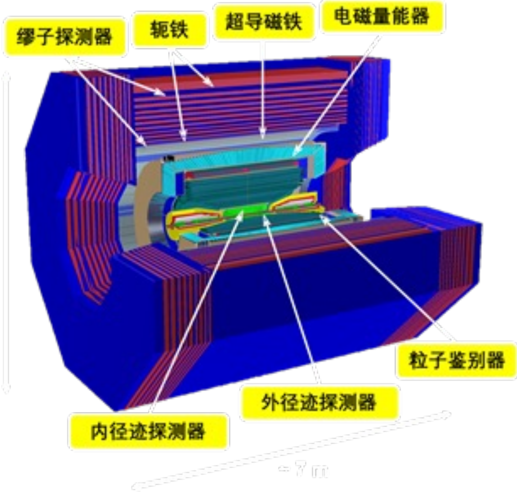
Collaboration

Crystal Ball

JEF at JLab



Future Exps



BESIII: an important role in η/η′ decays

PDG2022

η REFERENCES

ABLIKIM	21AM	PR D104 092004	M. Ablikim et al.	(BESIII Collab.)
BABUSCI	20A	JHEP 2010 047	D. Babusci et al.	(KLOE-2 Collab.)
ZHEVLAKOV	19	PR D99 031703	A.S. Zhevlakov et al.	(TMSK, MAINZ, TUBIN+)
ACHASOV	18B	PR D98 052007	M.N. Achasov et al.	(SND Collab.)
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PRAKHOV	18	PR C97 065203	S. Prakhov et al.	(A2 Collab. at MAMI)
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ARNALDI	16	PL B757 437	R. Arnaldi et al.	(NA60 Collab.)
ABLIKIM	15G	PR D92 012014	M. Ablikim et al.	(BESIII Collab.)
ADLARSON	14A	PR C90 045207	P. Adlarson et al.	(WASA-at-COSY Collab.)
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NEFKENS	14	PR C90 025206	B.M.K. Nefkens et al.	(A2 Collab. at MAMI)
NIKOLAEV	14	EPJ A50 58	A. Nikolaev et al.	(MAMI-B, MAINZ, BONN)
ABLIKIM	13	PR D87 012009	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	13G	PR D87 032006	M. Ablikim et al.	(BESIII Collab.)
BABUSCI	13	PL B718 910	D. Babusci et al.	(KLOE/KLOE-2 Collab.)
BABUSCI	13A	JHEP 1301 119	D. Babusci et al.	(KLOE-2 Collab.)
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GOSLAWSKI	12	PR D85 112011	P. Goslawski et al.	(COSY-ANKE Collab.)
ABLIKIM	11G	PR D84 032006	M. Ablikim et al.	(BESIII Collab.)

Few results on η decays at BESIII

η′(958) REFERENCES

ABLIKIM	21I	PR D103 072006	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	21J	PR D103 092005	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	20E	PR D101 032001	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	19AW	PR D100 052015	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	19T	PRL 122 142002	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	18	PR D97 012003	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	18C	PRL 120 242003	M. Ablikim et al.	(BESIII Collab.)
ADLARSON	18A	PR D98 012001	P. Adlarson et al.	(A2 Collab. at MAMI)
GONZALEZ-S...	18A	EPJ C78 758	S. Gonzalez-Solis, E. Passemar	(BEIJ, IND+)
AAIJ	17D	PL B764 233	R. Aaij et al.	(LHCb Collab.)
ABLIKIM	17	PRL 118 012001	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	17T	PR D96 012005	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	16M	PR D93 072008	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	15AD	PR D92 051101	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	15G	PR D92 012014	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	15O	PR D92 012001	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	15P	PR D92 012007	M. Ablikim et al.	(BESIII Collab.)
ACHASOV	15	PR D91 092010	M.N. Achasov et al.	(SND Collab.)
AKHMETSHIN	15	PL B740 273	R.R. Akhmetshin et al.	(CMD-3 Collab.)
PDG	15	RPP 2015 at pdg.lbl.gov		(PDG Collab.)
ABLIKIM	14M	PRL 112 251801	M. Ablikim et al.	(BESIII Collab.)
DONSKOV	14	MPL A29 1450213	S. Donskov et al.	(GAMS-4π Collab.)
PDG	14	CP C38 070001	K. Olive et al.	(PDG Collab.)
ABLIKIM	13	PR D87 012009	M. Ablikim et al.	(BESIII Collab.)
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ABLIKIM	13O	PR D87 092011	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	13U	PR D88 091502	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	12E	PRL 108 182001	M. Ablikim et al.	(BESIII Collab.)
PDG	12	PR D86 010001	J. Beringer et al.	(PDG Collab.)
ABLIKIM	11	PR D83 012003	M. Ablikim et al.	(BESIII Collab.)
ABLIKIM	11G	PR D84 032006	M. Ablikim et al.	(BESIII Collab.)

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- $J/\psi \to \gamma\eta \to 1.1\times10^{7}\ \eta$
- $J/\psi \to \gamma\eta' \to 5.2\times10^{7}\ \eta'$
- $J/\psi \to \gamma\eta',\ \eta' \to \pi^{+}\pi^{-}\eta \to 2.2\times10^{7}\ \eta$

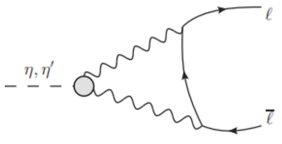
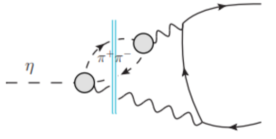
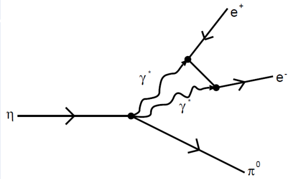
With η' constraint to suppress the background!

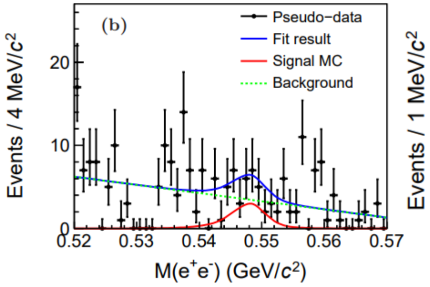
- With 1 trillion J/ψ hypothesis at STCF
- 5.2 billion η' inclusive decays (Pseudo-data) are simulated at STCF.

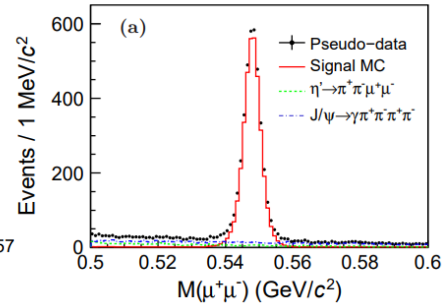
Novel approach to investigate η decays via $\eta' \to \pi\pi\eta$

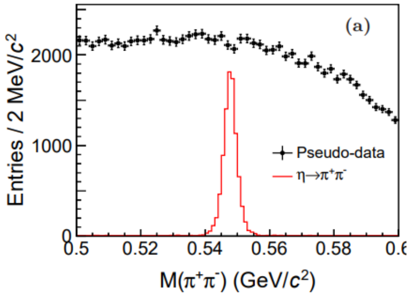
X. L. Kang, Y. Y. Ji, X. Q. Yuan, B. H. Xiang, et al, PRD 108 (2023) 014038

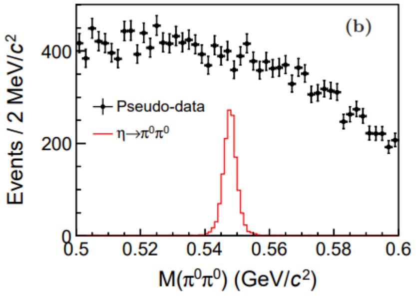
	SM Prediction	PDG	STCF Estimation	Comment
$\eta \to e^+e^-$	$10^{-9}\sim10^{-10}$	$< 7\times10^{-7}$ (SND)	$< 10^{-9}$	• Dominant by $\eta \to \gamma^*\gamma^* \to l^+l^-$ and intermediate hadronic states • fourth-order electromagnetic transition
$\eta \to \mu^+\mu^-$	$10^{-6}\sim10^{-7}$	$(5.8 \pm 0.8)\times10^{-6}$ SPECII	$(5.88 \pm 0.09)\times10^{-6}$	
$\eta \to \pi^+\pi^-$	$\sim10^{-16}$	$< 4.4\times10^{-6}$ KLOE-2	$< 7.8\times10^{-8}$	• P and CP violating decays • contribute to neutron EDM
$\eta \to \pi^0\pi^0$		$< 3.5\times10^{-4}$ GAM4	$< 6.9\times10^{-7}$	
$\eta \to \pi^0e^+e^-$	$10^{-11}\sim10^{-8}$	$< 7.5\times10^{-6}$ WASA	$< 2\times10^{-7}$	• $\eta \to \pi^0\gamma^* \to \pi^0l^+l^-$ is C-violated process • Dominant by C-conserving process $\eta \to \pi^0\gamma^*\gamma^* \to \pi^0l^+l^-$
$\eta \to \pi^0\mu^+\mu^-$		$< 5\times10^{-6}$ SPEC	$< 8.5\times10^{-8}$	

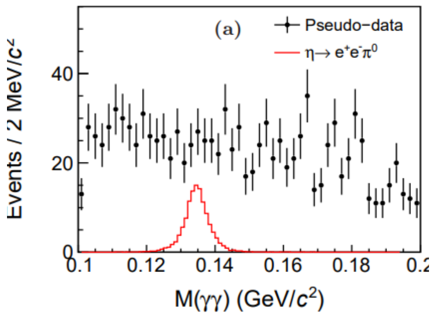




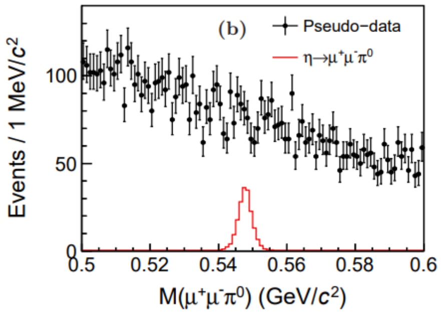










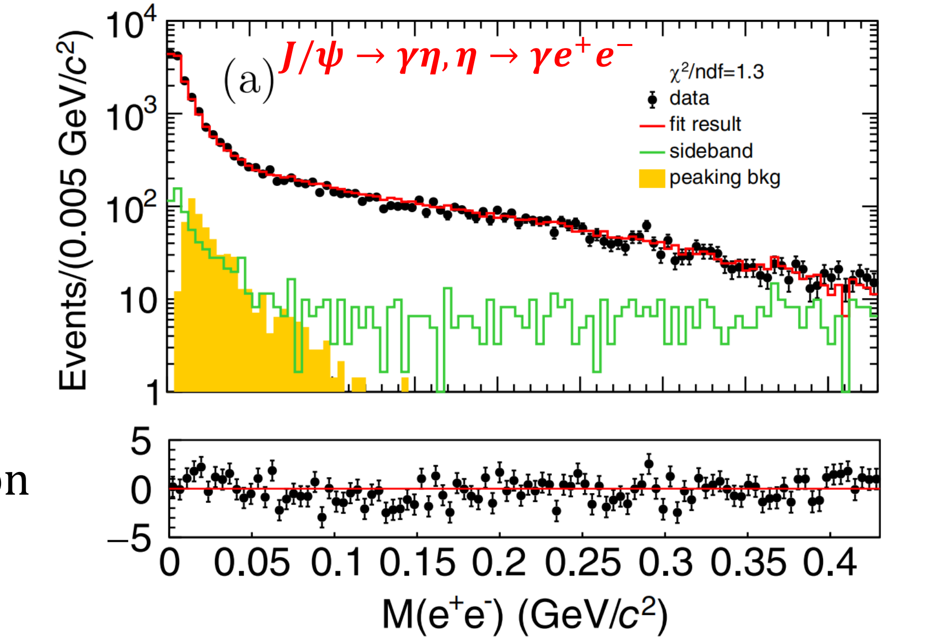


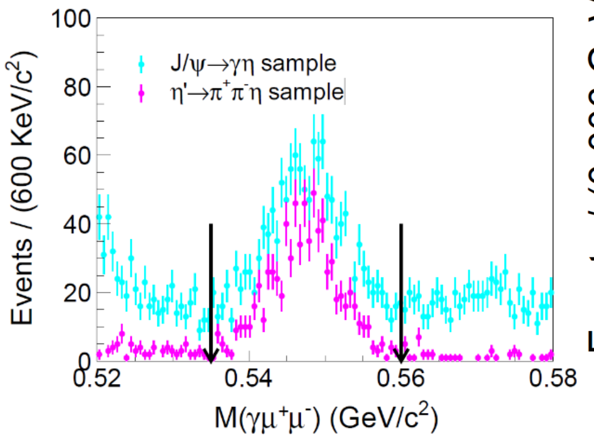
Transition form factor of $\eta \to \gamma l^+ l^-$

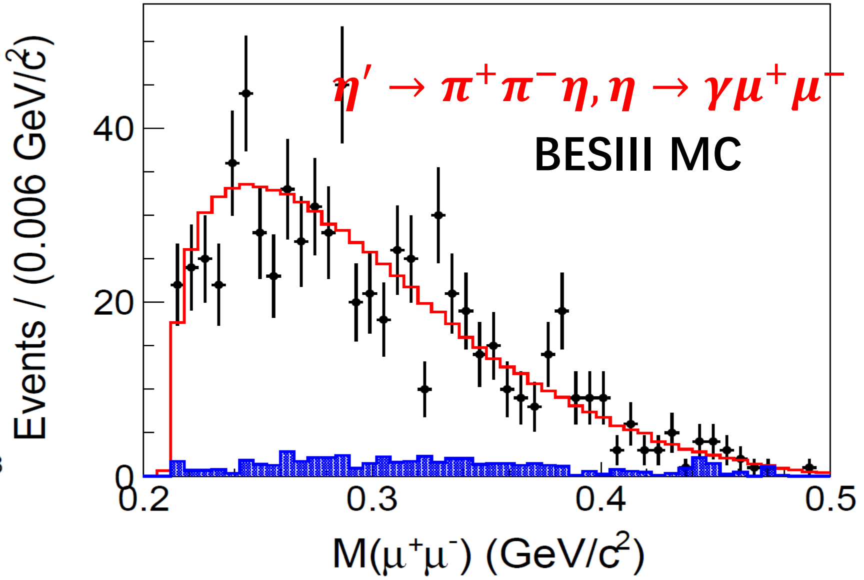
$\Lambda_\eta^{-2} = 1.781 \pm 0.125$

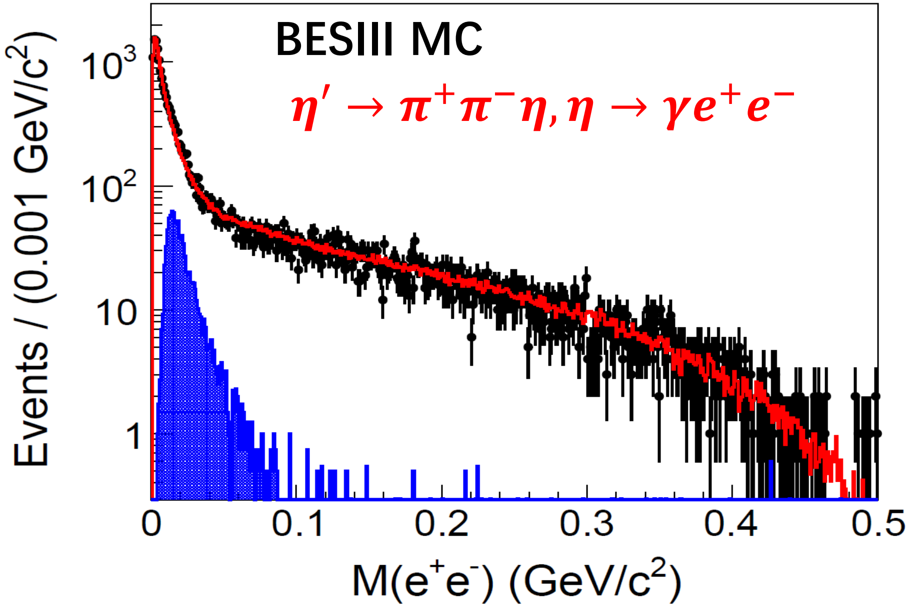
PRD 109 (2004) 072001

- Important input for HLbL contributions
- The latest TFFs are $\Lambda^{-2}(\eta \to \gamma e^+ e^-) = 1.97 \pm 0.11$ from A2 and $\Lambda^{-2}(\eta \to \gamma \mu^+ \mu^-) = 1.934 \pm 0.084$ from NA60
- For $J/\psi \to \gamma\eta, \eta \to \gamma e^+ e^-$, main backgrounds are and Bhabha, $J/\psi \to e^+ e^-$and γ conversion
- For $\eta' \to \pi^+\pi^-\eta, \eta \to \gamma e^+ e^-$, main background only remain γ conversion
- Main background are $\eta \to \gamma\pi^+\pi^-$ *for* $\eta' \to \pi^+\pi^-\eta, \eta \to \gamma\mu^+\mu^-$



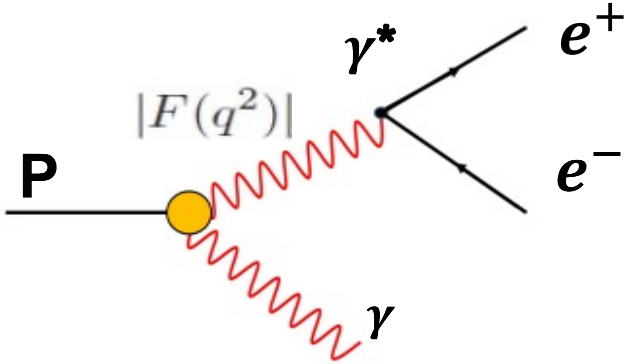






Transition form factor of $\eta \to \gamma l^+ l^-$

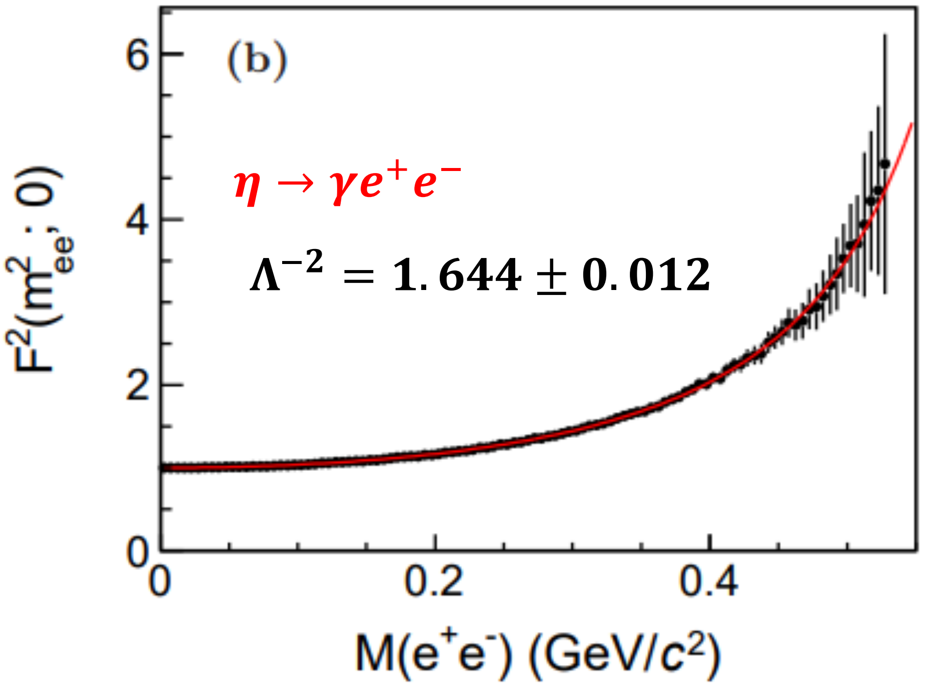
- With 5.2 billion Pseudo-data of η',
  - 0.2 million $\eta \to \gamma\mu^+\mu^-$ and 1.7 million $\eta \to \gamma e^+ e^-$.

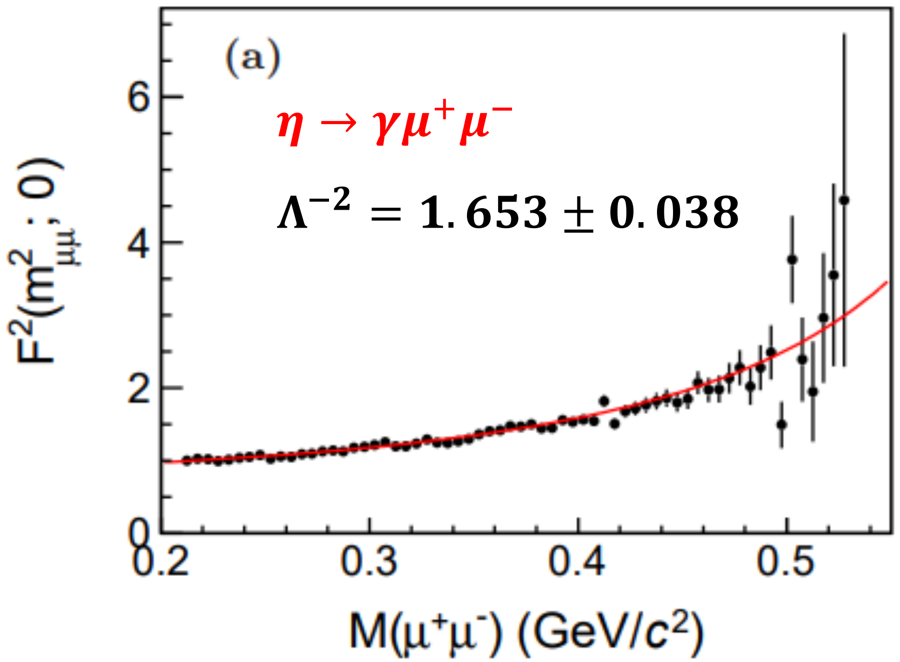


$$\frac{d\Gamma(P \to \gamma l^+ l^-)}{dq^2 \Gamma_{\gamma\gamma}} = \frac{2\alpha}{3\pi}\frac{1}{q^2}\sqrt{1-\frac{4m_l^2}{q^2}}\left(1+\frac{2m_l^2}{q^2}\right)\left(1-\frac{q^2}{M_P^2}\right)^3 \left|F_P(q^2,0)\right|^2$$

Single-pole model:

$$F(q^2) = \frac{1}{1 - q^2/\Lambda^2}$$





More works about η/η'

- The generators of η and η' have been organized and developed, which almost contains all decay modes of η and η'.
- More decays involving η and η' simulations are in progress.
- With STCF, more precise prediction results can be expected.

Contents

Hadronic decays $\eta/\eta' \to \pi\pi\pi$

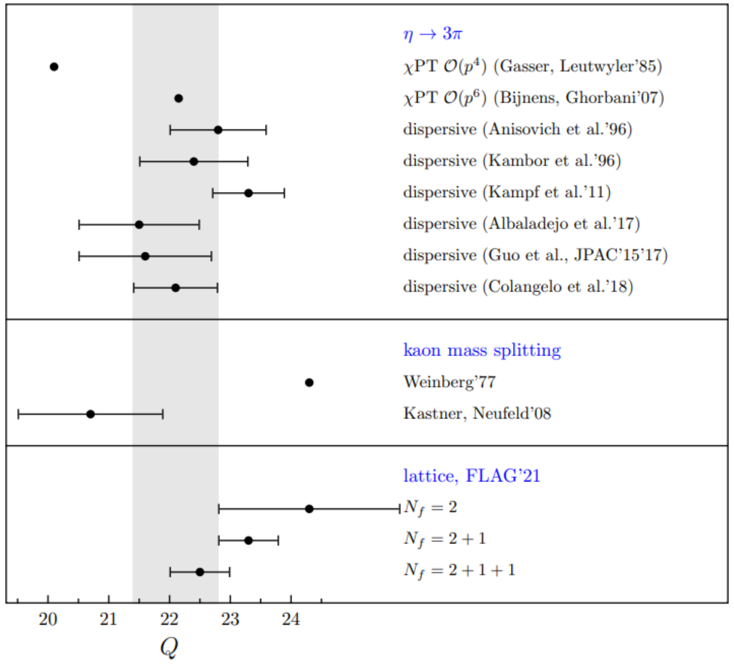
- Isospin violating process
- Electromagnetic effects are strongly suppressed due to Sutherland's theorem
- Almost exclusively caused by the light quark mass difference $m_u - m_d$

$$\Gamma(\eta \to \pi^+\pi^-\pi^0) = \frac{1}{Q^4}\frac{M_K^4(M_K^2 - M_\pi^2)^2}{6912\pi^3 M_\eta^3 M_\pi^4 F_\pi^4}\int_{s_{\min}}^{s_{\max}} ds \int_{u_-(s)}^{u_+(s)} du\, |M(s,t,u)|^2$$

$$Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} =$$

22.0 ± 0.7 G. Colangelo, S. Lanz, H. Leutwyler, E. Passemar, PRL 118, 022001 (2017)

21.6 ± 1.1 P. Guo, I. V. Danilkin, C. Fernández-Ramírez, V. Mathieu, A. P. Szczepaniak, PLB 771, 497 (2017)



- Investigation on final state interactions

Hadronic decays $\eta \to \pi\pi\pi$

$\eta \to \pi^+\pi^-\pi^0$ $\quad X = \frac{\sqrt{3}}{Q}(T_{\pi^+} - T_{\pi^-}), Y = \frac{3T_{\pi^0}}{Q} - 1,$

$$|A(X,Y)|^2 \propto 1 + aY + bY^2 + cX + dX^2 + eXY + fY^3 + gX^2Y + \cdots$$

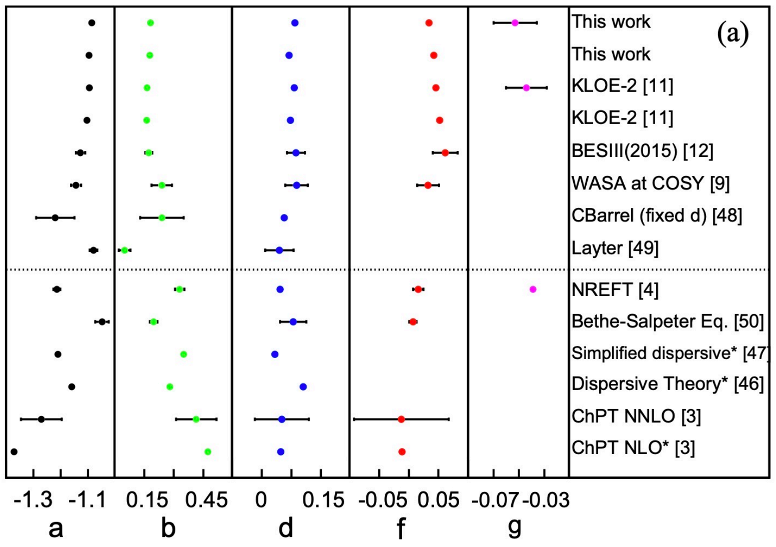
$\eta \to 3\pi^0$ $\quad Z = X^2 + Y^2 = \frac{2}{3}\sum_{i=1}^{3}(\frac{3T_i}{Q} - 1)^2$

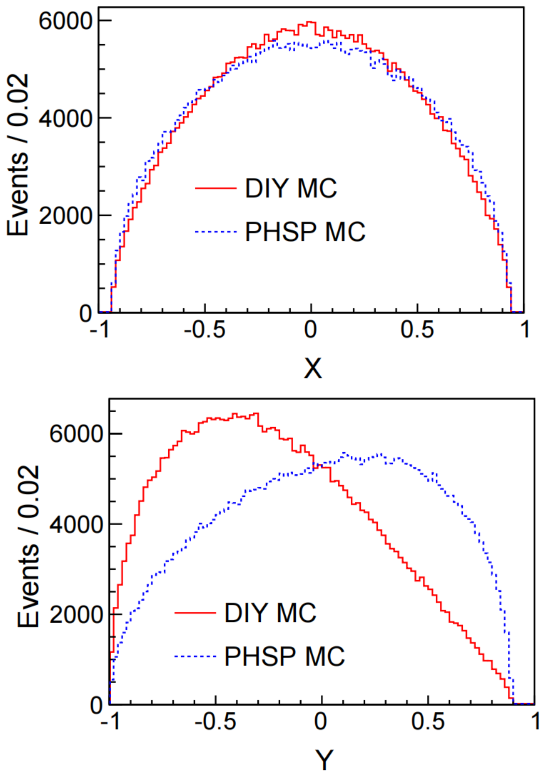
$$|A(X,Y)|^2 \propto 1 + 2\alpha Z + 2\beta(3X^2Y - Y^3) + 2\gamma Z^2 + \cdots$$

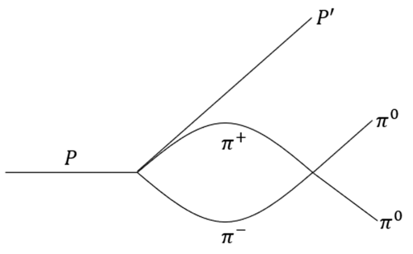
$$|A(X,Y)|^2 \propto 1 + 2\alpha Z + 2\delta \sum_{i=1}^{3} \Re\sqrt{1 - s_i/4m_{\pi^\pm}^2}$$

A sizeable cusp structure is expected in $\eta \to 3\pi^0$

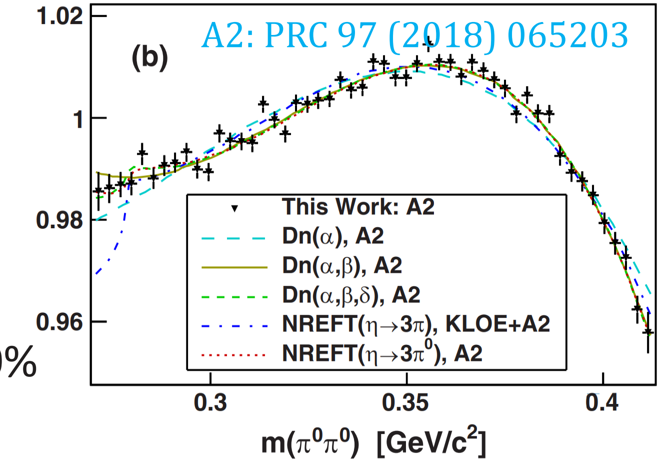
PRD 107 (2023) 092007

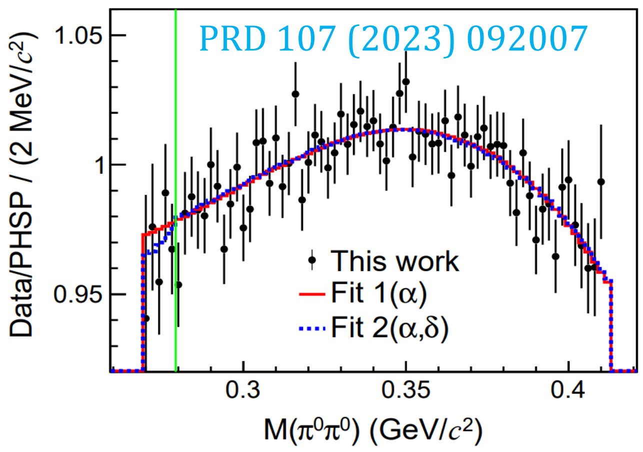


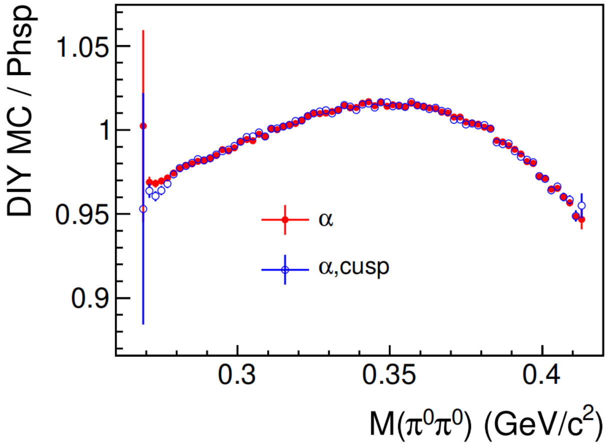




The cusp effect @ A2
with an uncertainty of ~50%



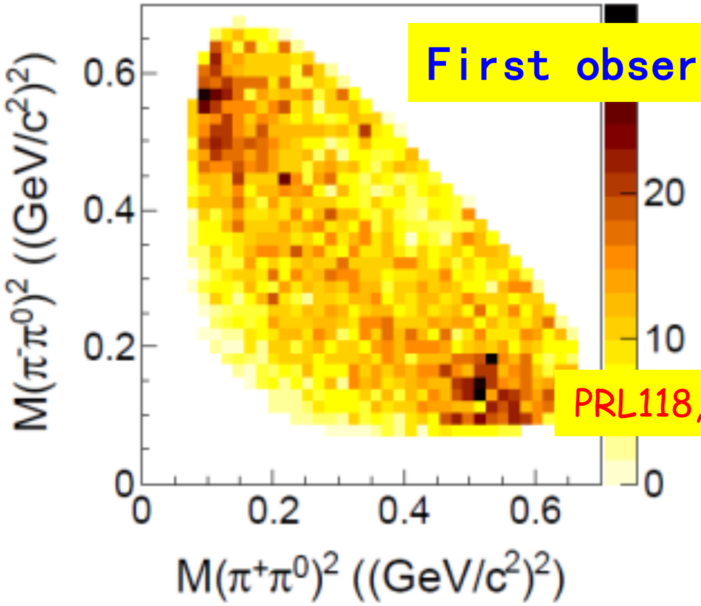


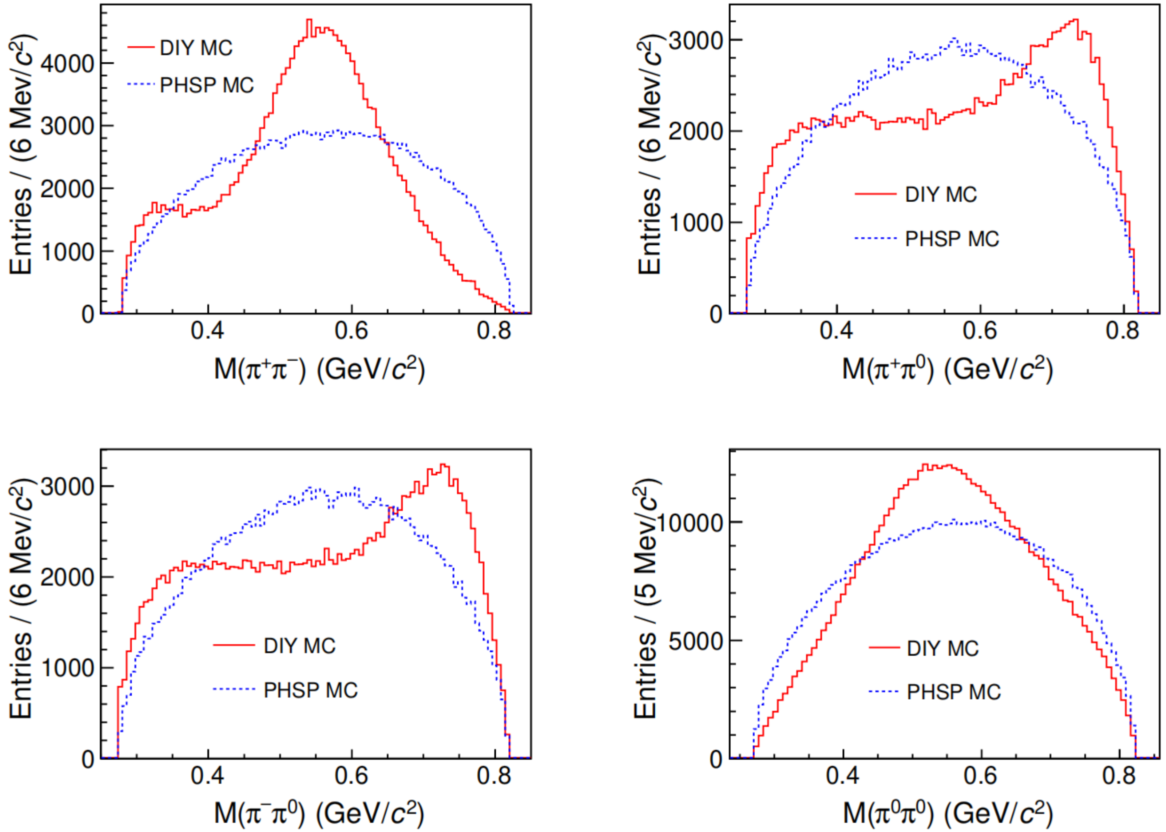


no obvious cusp contribution @ BESIII

Hadronic decays $\eta' \to \pi\pi\pi$

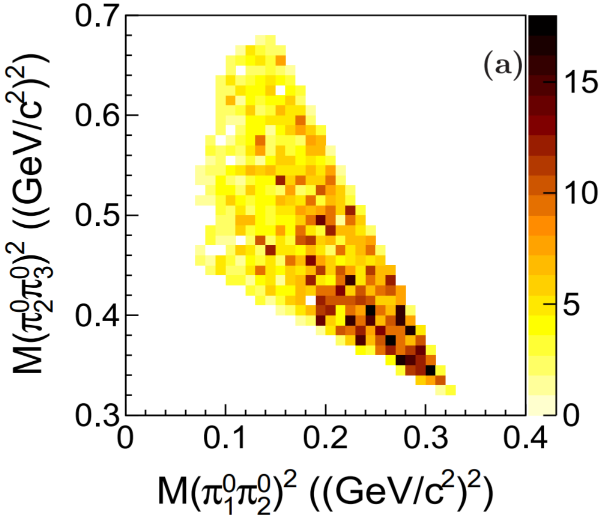
- Established by the CLEO collaboration in 2008





- $\pi - \pi$ scattering S and P (for $\eta' \to \pi^+\pi^-\pi^0$) wave to describe data

Decay mode	$\mathcal{B}$ (10^{-4})
$\pi^+\pi^-\pi^0$	$35.91 \pm 0.54 \pm 1.74$
$\pi^0\pi^0\pi^0$	$35.22 \pm 0.82 \pm 2.54$
$\rho^\pm\pi^\mp$	$7.44 \pm 0.60 \pm 1.26 \pm 1.84$
$(\pi^+\pi^-\pi^0)_S$	$37.63 \pm 0.77 \pm 2.22 \pm 4.48$



- The branching fraction for $\eta' \to 3\pi^0$
  - PDG average: $(3.57 \pm 0.26) \times 10^{-3}$
  - PDG fit result: $(2.50 \pm 0.17) \times 10^{-3}$
- Need to shuttled with more exp. data

Hadronic decays $\eta' \to \pi\pi\eta$

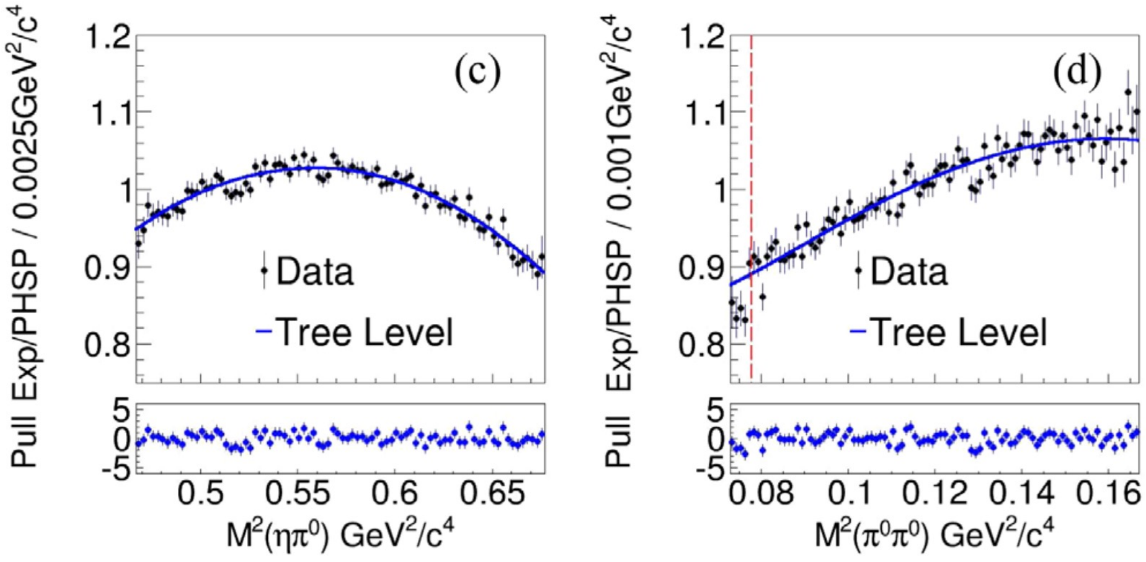
S. Gonzalez-Solls, E. Passemar EPJC78, 758 (2018)

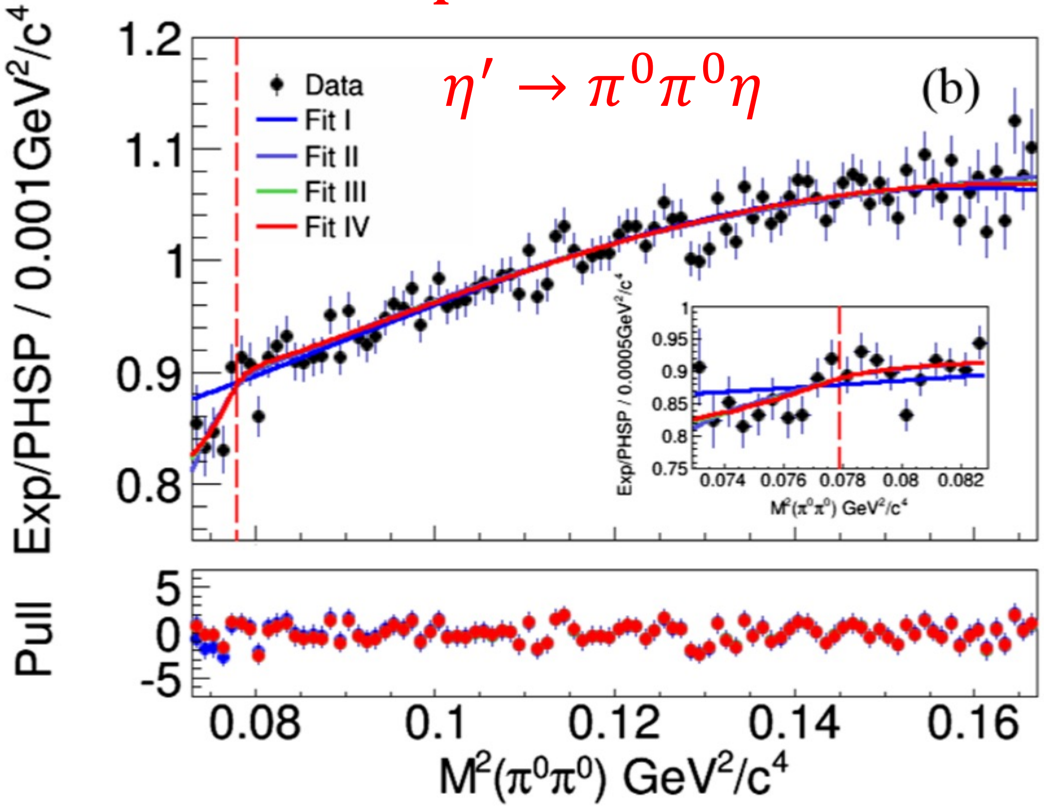
- S-wave charge-exchange rescattering: $\pi^+\pi^- \to \pi^0\pi^0 \to \pi^+\pi^-$
- The size of cusp effect is predicted to be about 6% in $\eta' \to \pi^0\pi^0\eta$ within NREFT

B. Kubis and S. P. Schneider, EPJC 62, 511 (2009)

Cusp effect with 3σ!

$$\mathcal{M}_{\eta'\to\eta\pi^0\pi^0} = \mathcal{M}_N^{tree} + \mathcal{M}_N^{one-loop} + \mathcal{M}_N^{two-loop} + \cdots$$
$$\mathcal{M}_{\eta'\to\eta\pi^+\pi^-} = \mathcal{M}_C^{tree} + \mathcal{M}_C^{one-loop} + \mathcal{M}_C^{two-loop} + \cdots$$





PRL 130 (2023) 081901

Hadronic decays $\eta' \to \pi\pi\eta$

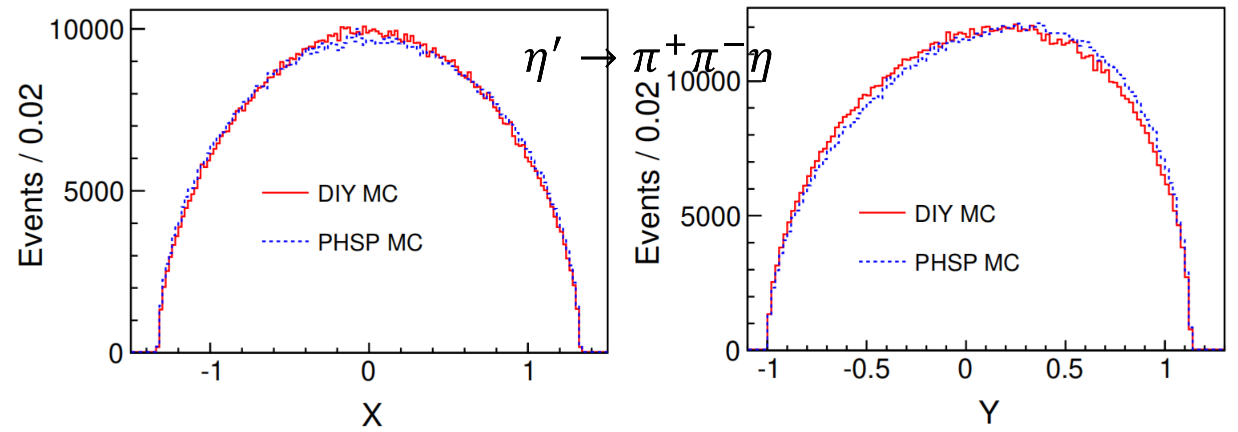
- The general presentation

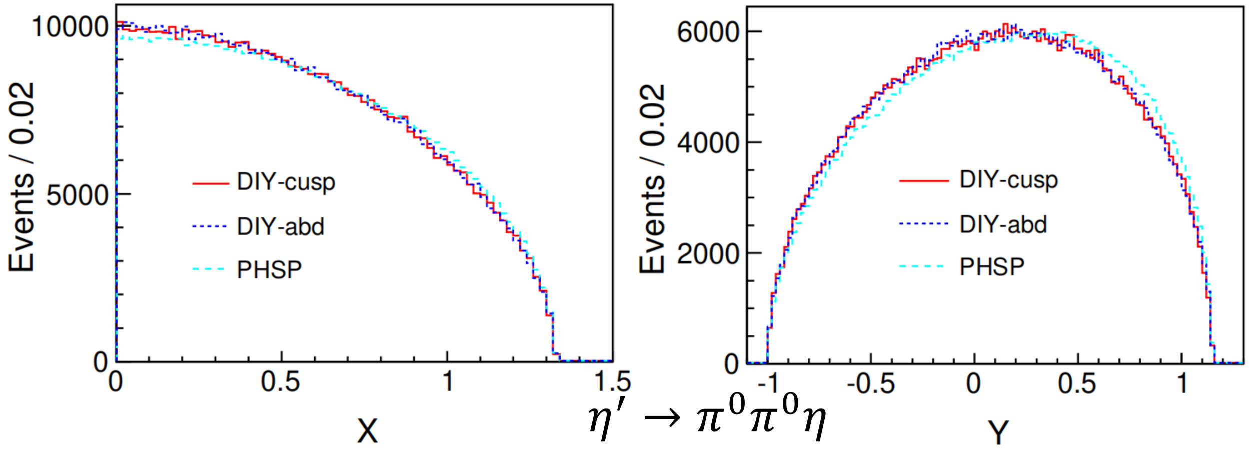
$$X = \frac{\sqrt{3}}{Q}(T_{\pi^+} - T_{\pi^-}), Y = \frac{m_\eta + 2m_\pi}{m_\pi}\frac{T_\eta}{Q} - 1$$

$$|M(X,Y)|^2 = N(1 + aY + bY^2 + cX + dX^2 + \dots)$$

- Tree level amplitude is equivalent to the general presentation

$\eta' \to \pi^+\pi^-\eta$

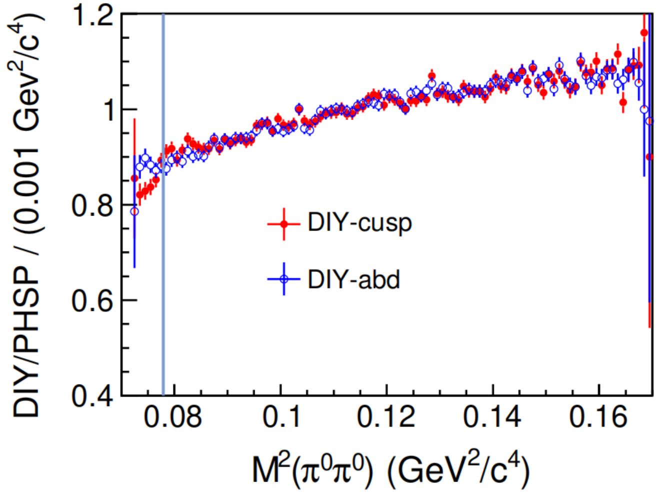




$\eta' \to \pi^0\pi^0\eta$

PRD 97 (2018) 012003

Para-meter	$\eta' \to \eta\pi^+\pi^-$					$\eta' \to \eta\pi^0\pi^0$		
	EFT [5]	Large N_C [7]	RChT [7]	VES [10]	This work	EFT [5]	GAMS-4π [12]	This work
a	−0.116(11)	−0.098(48)	(fixed)	−0.127(18)	−0.056(4)(2)	−0.127(9)	−0.067(16)	−0.087(9)(6)
b	−0.042(34)	−0.050(1)	−0.033(1)	−0.106(32)	−0.049(6)(6)	−0.049(36)	−0.064(29)	−0.073(14)(5)
c	...	...	...	+0.015(18)	0.0027(24)(18)	...	...	...
d	+0.010(19)	−0.092(8)	−0.072(1)	−0.082(19)	−0.063(4)(3)	+0.011(21)	−0.067(20)	−0.074(9)(4)



- The $\pi\pi$ and $\pi\eta$ interaction are included in the generator

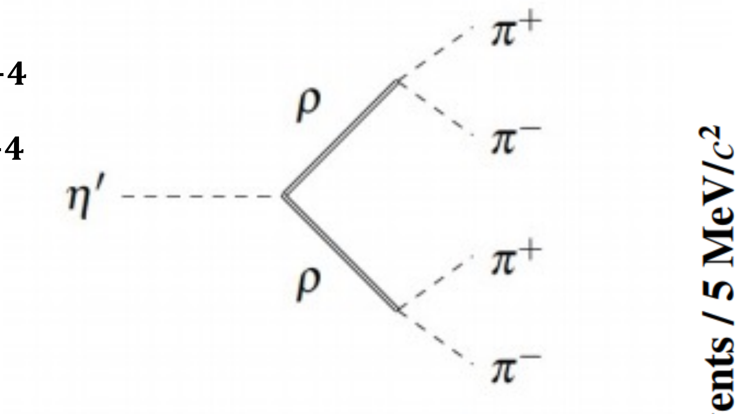
Hadronic decays $\eta' \to \pi^+\pi^-\pi^{+(0)}\pi^{-(0)}$

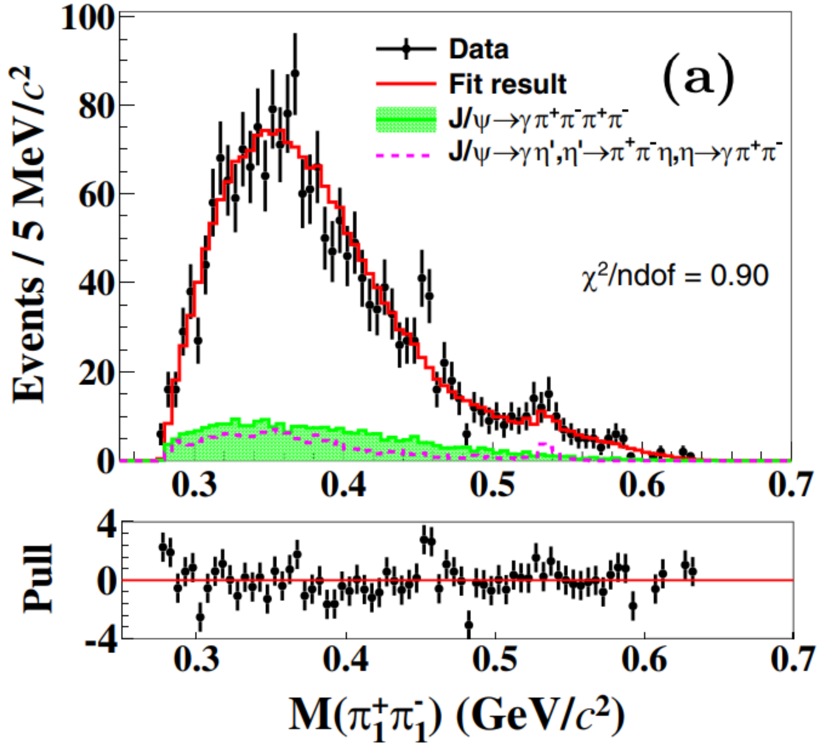
The decay amplitude based on the ChPT and VMD model

F. K. Guo et al, PRD 85,014014 (2012)

$$= \epsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu p_3^\alpha p_4^\beta \times \left\{ \left[\frac{s_{12}}{D_\rho(s_{12})} + \frac{s_{34}}{D_\rho(s_{34})} - \frac{s_{14}}{D_\rho(s_{14})} - \frac{s_{23}}{D_\rho(s_{23})} \right] + \alpha \left[\frac{M_\rho^2(s_{12}+s_{34})}{D_\rho(s_{12})D_\rho(s_{34})} - \frac{M_\rho^2(s_{14}+s_{23})}{D_\rho(s_{14})D_\rho(s_{23})} \right] \right\}$$

$$Br(\eta' \to \pi^+\pi^-\pi^+\pi^-) = (1.0 \pm 0.3)\times 10^{-4}$$
$$Br(\eta' \to \pi^+\pi^-\pi^0\pi^0) = (2.4 \pm 0.7)\times 10^{-4}$$





BESIII measured the doubly virtual isovector form factor

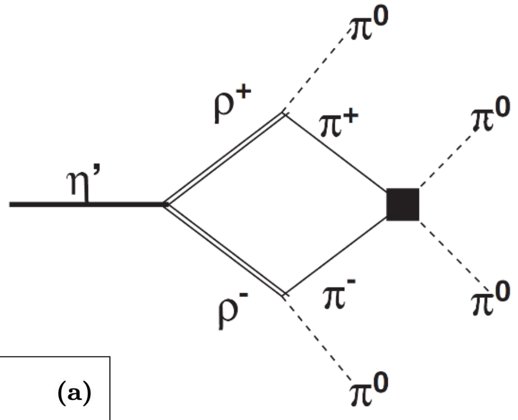
PRD 109, 032006 (2024)

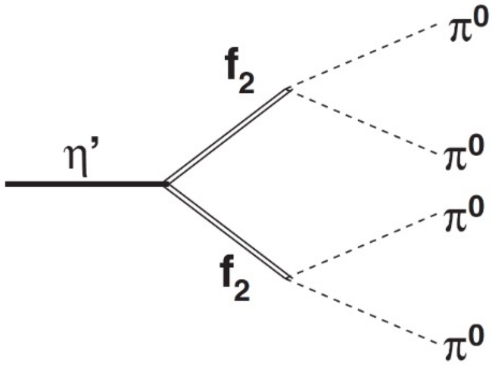
$$\alpha = 1.22 \pm 0.33 \pm 0.04$$
$$Br(\eta' \to \pi^+\pi^-\pi^+\pi^-) = (8.56 \pm 0.34)\times 10^{-5}$$
$$Br(\eta' \to \pi^+\pi^-\pi^0\pi^0) = (2.12 \pm 0.16)\times 10^{-4}$$

Rare decay $\eta' \to \pi^0\pi^0\pi^0\pi^0$

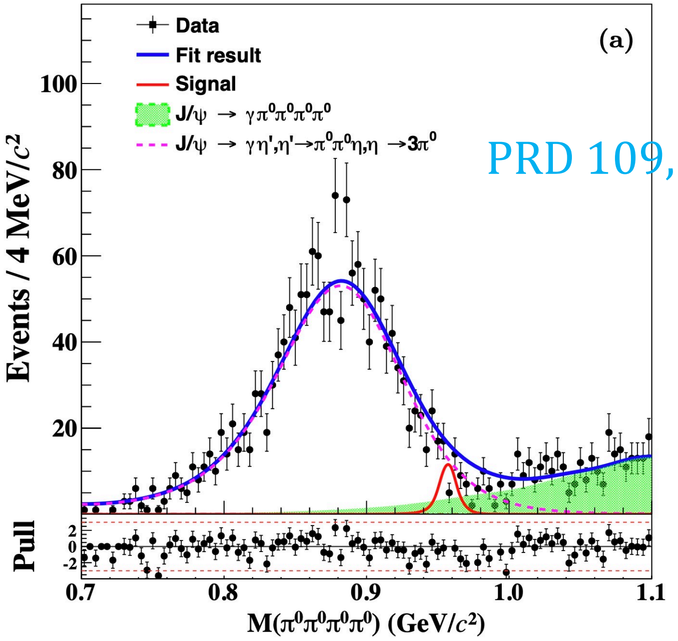
- CP-violation S-wave, induced by the QCD Lagrangian θ-term ⇒ $Br \sim 10^{-23}$
- CP-conserving higher order ⇒ $Br \sim 10^{-8}$ F. K. Guo et al, PRD 85,014014 (2012)

D-wave pion loop





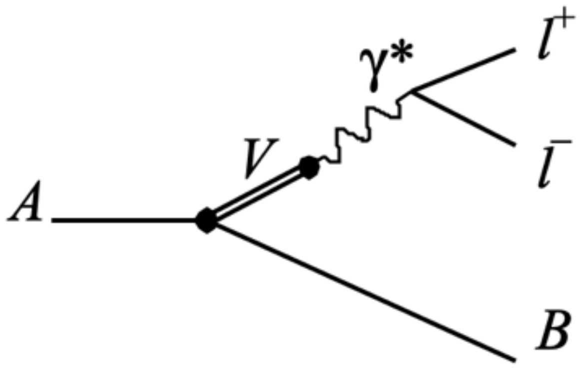
Small contribution from two f_2 tensor mesons



PRD 109, 032006 (2024)

- Using 10 billion J/ψ, the UL at 90% CL is set as 1.24×10^{-5}

Radiative decays $\eta/\eta' \to \gamma l^+ l^-$

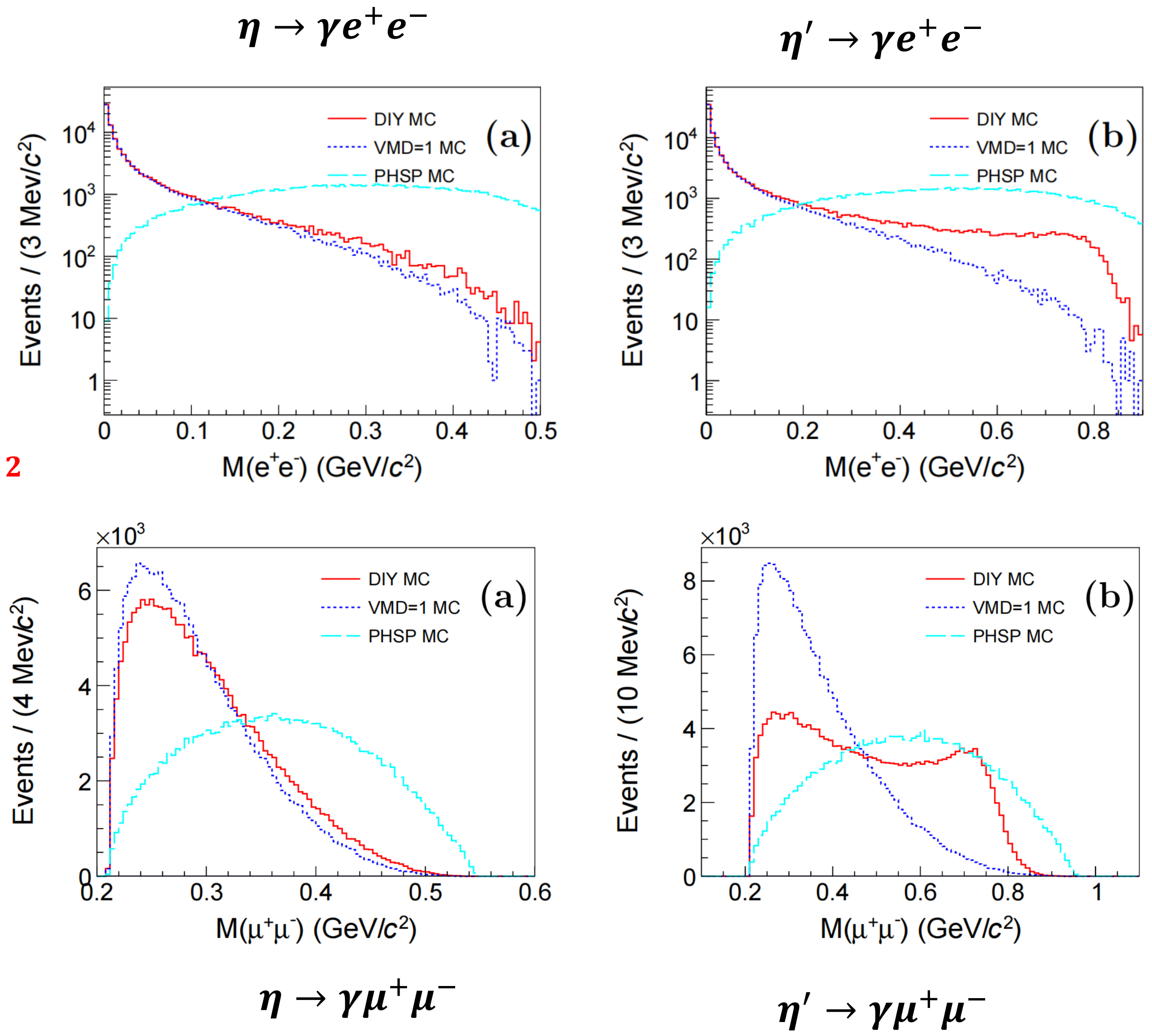


$$\frac{d\Gamma(P\to\gamma l^+l^-)}{dq^2\Gamma_{\gamma\gamma}} = \frac{2\alpha}{3\pi}\frac{1}{q^2}\sqrt{1-\frac{4m_l^2}{q^2}}\left(1+\frac{2m_l^2}{q^2}\right)\left(1-\frac{q^2}{M_P^2}\right)^3\left|F_P(q^2,0)\right|^2$$

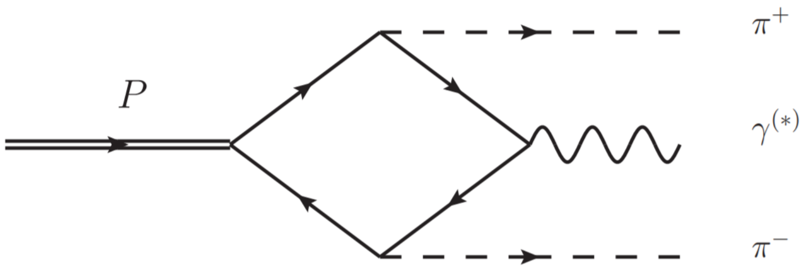
For $\eta \to \gamma l^+ l^-$ Single-pole model:

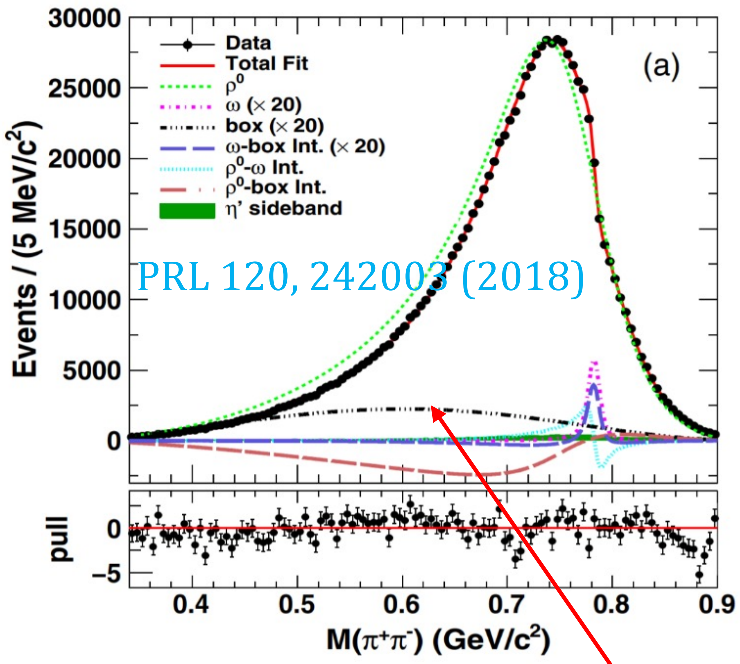
$$F(q^2) = \frac{1}{1-q^2/\Lambda^2}$$

For $\eta' \to \gamma l^+ l^-$ $$|F(q^2)|^2 = \frac{\Lambda^2(\Lambda^2+\gamma^2)}{(\Lambda^2-q^2)^2+\Lambda^2\gamma^2}$$



Radiative decays $\eta/\eta' \to \gamma\pi\pi$





box anomaly in $\eta' \to \pi^+\pi^-\gamma$

$$\sum_{\text{pol}=1}^{2} |\mathcal{A}_{P\to\pi^+\pi^-\gamma}|^2(s_{\pi\pi},\theta_\pi) = \frac{\lambda(m_P^2, s_{\pi\pi}, 0)\, s_{\pi\pi}\beta_\pi^2 \sin^2\theta_\pi}{16\, m_P^6}\left(|M_G|^2 + |E_G|^2\right)$$

CP conserving magnetic contribution

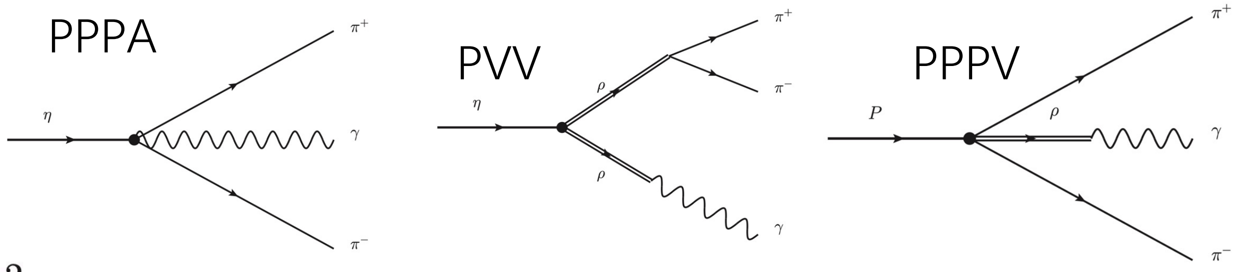
$$M_G(s_{\pi\pi}) = m_P^3 \mathcal{M} \times VMD(s_{\pi\pi})$$

CP violating electric contribution

T. Petri, arXiv:1010.2378

$$\mathcal{M} = \begin{cases} \frac{e}{8\pi^2 f_\pi^3} & \text{if } P = \pi^0; \\ \frac{e}{8\pi^2 f_\pi^3}\frac{1}{\sqrt{3}}\left(\frac{f_\pi}{f_8}\cos\theta_{mix} - 2\sqrt{2}\frac{f_\pi}{f_0}\sin\theta_{mix}\right) & \text{if } P = \eta; \\ \frac{e}{8\pi^2 f_\pi^3}\frac{1}{\sqrt{3}}\left(\frac{f_\pi}{f_8}\sin\theta_{mix} + 2\sqrt{2}\frac{f_\pi}{f_0}\cos\theta_{mix}\right) & \text{if } P = \eta'. \end{cases}$$

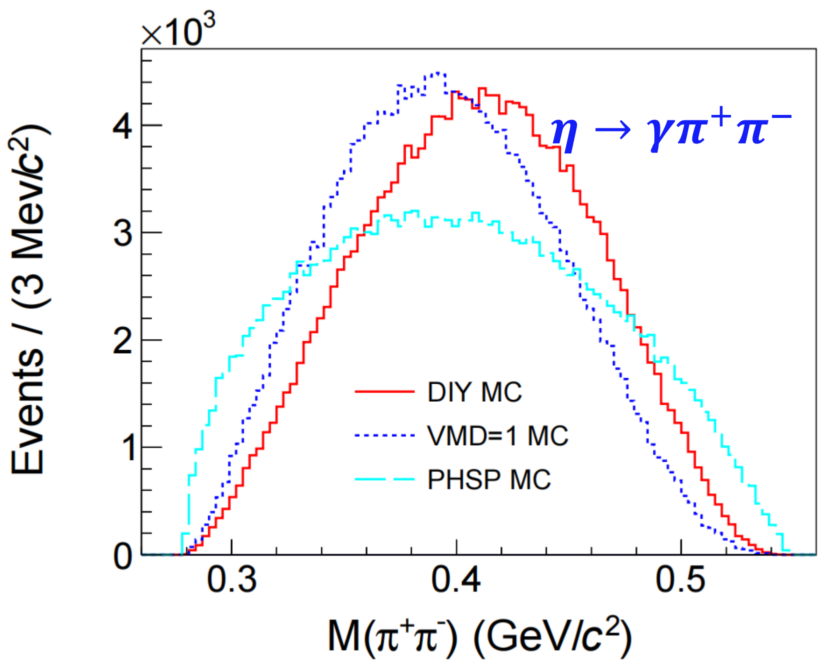
For $\eta \to \gamma\pi^+\pi^-$ VMD form factor

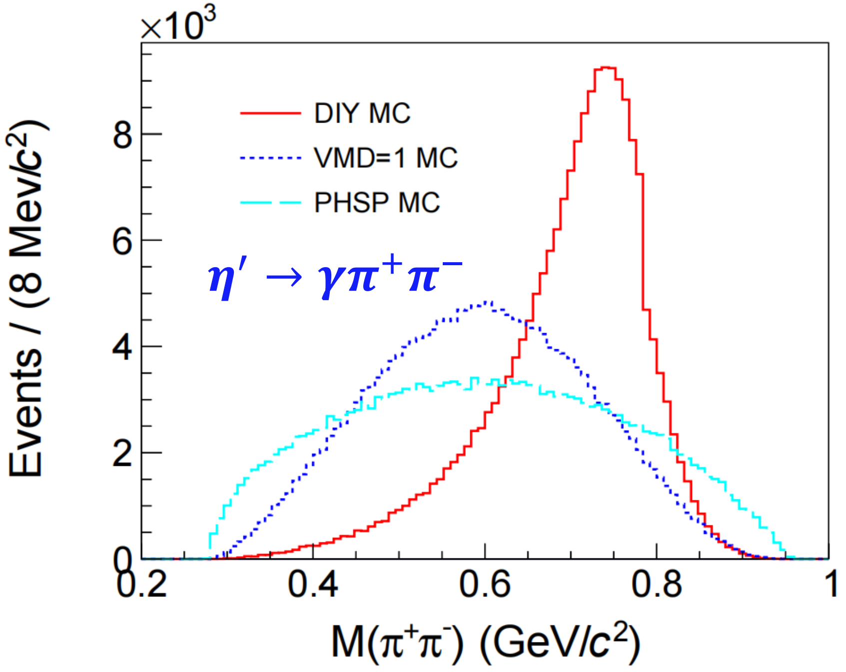


$$VMD_1(s_{\pi\pi}) = 1 - \frac{3}{2}c_3 + \frac{3}{2}c_3 \frac{m_V^2}{m_V^2 - s_{\pi\pi} - i m_V \Gamma(s_{\pi\pi})}.$$

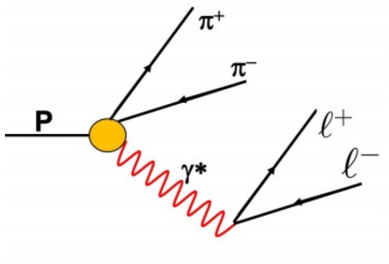
For $\eta' \to \gamma\pi^+\pi^-$, box anomaly + ρ + ω

$$\frac{d\Gamma}{dm} \propto k_\gamma^3 q_\pi^3(m) |\mathrm{BW}_\rho^{\mathrm{GS}}(1 + \delta \frac{m^2}{m_\rho^2} \mathrm{BW}_\omega) + \beta|^2$$

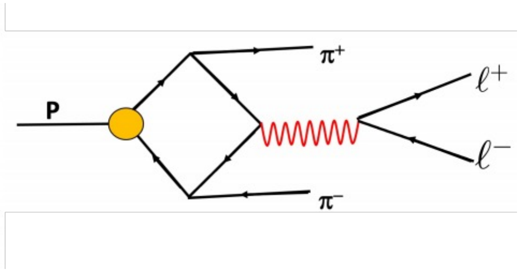




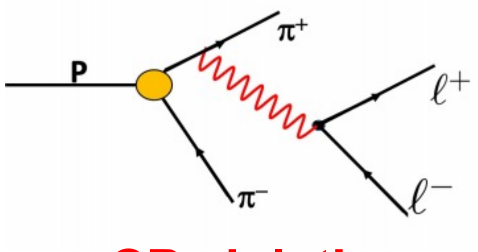
Electromagnetic dalitz decays $\eta/\eta' \to \pi\pi l^+ l^-$



VMD



Box-anomaly



CP violation

- Similar to η/η′→π⁺π⁻γ, replacing the γ with an off-shell one decays into a lepton pair

$$\overline{|\mathcal{A}_{\eta'\to\pi^+\pi^-l^+l^-}|}^2(s_{\pi\pi}, s_{ll}, \theta_\pi, \theta_1, \varphi) = \frac{e^2}{8k^2}|M(s_{\pi\pi}, s_{ll})|^2 \times \lambda\left(m_{\eta'}^2, s_{\pi\pi}, s_{ll}\right) \times [1 - \beta_1^2 \sin^2\theta_1 \sin^2\varphi] s_{\pi\pi}\beta_\pi^2 \sin^2\theta_\pi$$

$$M(s_{\pi\pi}, s_{ll}) = \frac{e}{8\pi^2 f_\pi^3}\frac{1}{\sqrt{3}}\left(\frac{f_\pi}{f_8}sin\theta_{mix} + 2\sqrt{2}\frac{f_\pi}{f_0}cos\theta_{mix}\right) \times VMD(s_{\pi\pi}, s_{ll})$$

constant

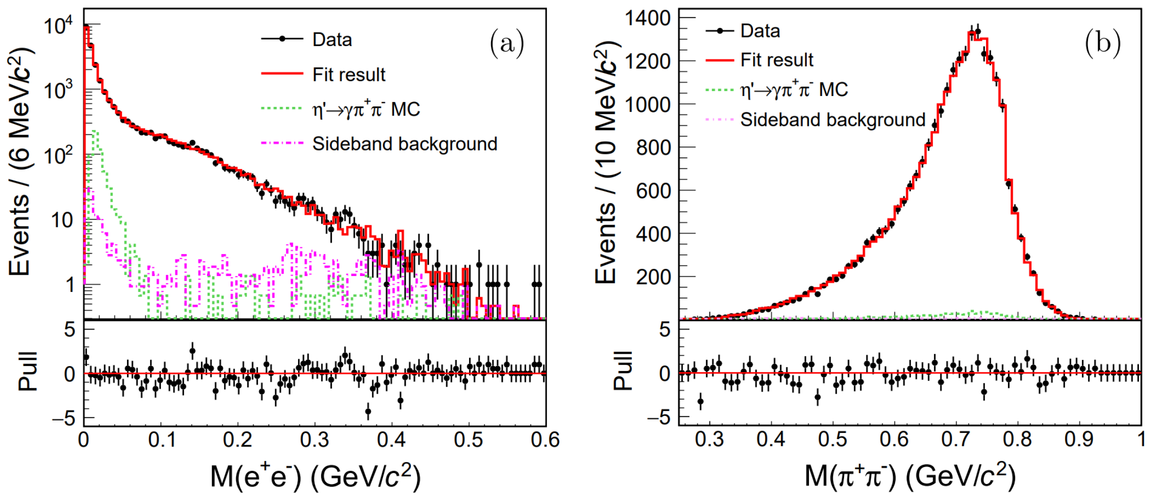
A. Faessler, C. Fuchs, and M. I. Krivoruchenko, PRC 61, 035206 (2000)

B. Borasoy and R. Nissler, EPJA 33, 95 (2007)

T. Petri, arXiv:1010.2378

$$VMD(s_{\pi\pi}, s_{ll}) = 1 - \frac{3}{4}(c_1 - c_2 + c_3) + \frac{3}{4}(c_1 - c_2 - c_3)\frac{m_V^2}{m_V^2 - s_{ll} - im_V\Gamma(s_{ll})} + \frac{3}{2}c_3\frac{m_V^2}{m_V^2 - s_{ll} - im_V\Gamma(s_{ll})}\frac{m_{V,\pi}^2}{m_{V,\pi}^2 - s_{\pi\pi} - im_{V,\pi}\Gamma(s_{\pi\pi})}$$

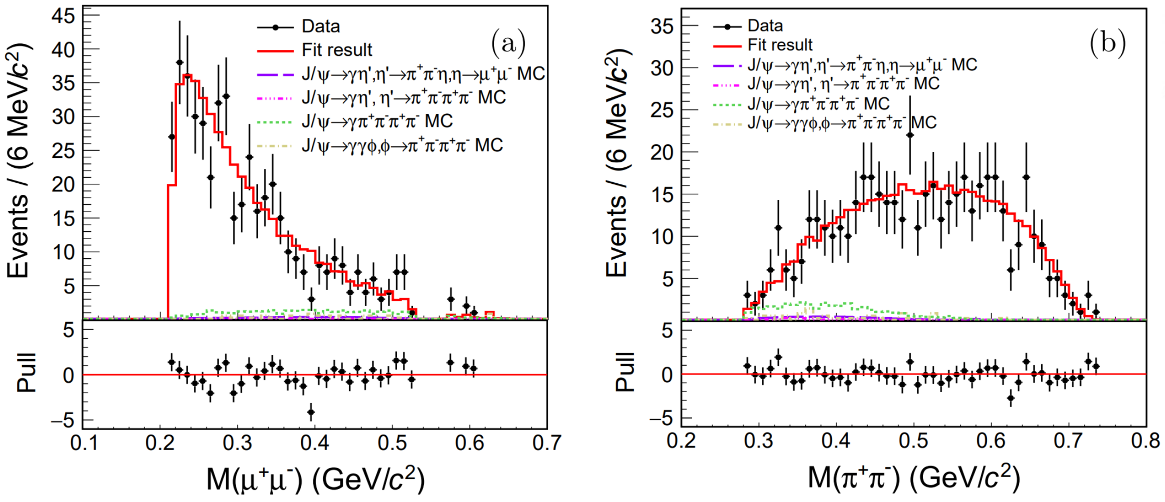
Box anomaly　　VMD contribution　　VMD contribution



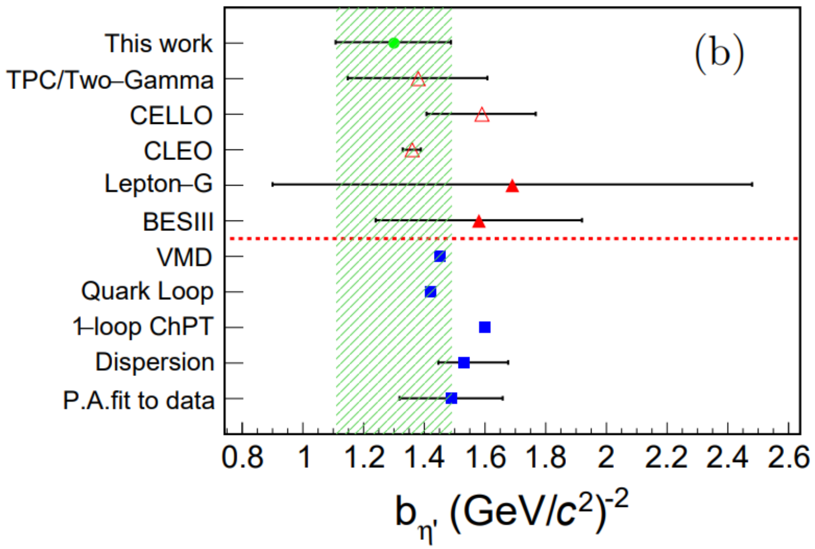
BESIII: arXiv:2402.01993v1

Hidden gauge **Full VMD** **Modified VMD**

$\eta' \to \pi^+\pi^- e^+ e^-$	Model I	Model II	Model III
	$c_1 - c_2 = c_3 = 1$	$c_1 - c_2 = 1/3, c_3 = 1$	$c_1 - c_2 \neq c_3$
$m_V\,(\mathrm{MeV}/c^2)$	$954.26 \pm 82.53 \pm 36.36$	857.37 ± 74.31	787.53 ± 137.90
$m_{V,\pi}\,(\mathrm{MeV}/c^2)$	$765.32 \pm 1.12 \pm 20.20$	765.35 ± 1.12	764.75 ± 1.25
$m_\omega\,(\mathrm{MeV}/c^2)$	$778.69 \pm 1.26 \pm 17.29$	778.70 ± 1.26	778.70 ± 1.36
$\beta(10^{-3})$	$8.53 \pm 1.40 \pm 0.71$	8.52 ± 1.40	8.11 ± 1.43
θ	$1.43 \pm 0.31 \pm 0.12$	1.43 ± 0.31	1.44 ± 0.35
$c_1 - c_2$	1	1/3	-0.03 ± 0.87
c_3	1	1	1.03 ± 0.02

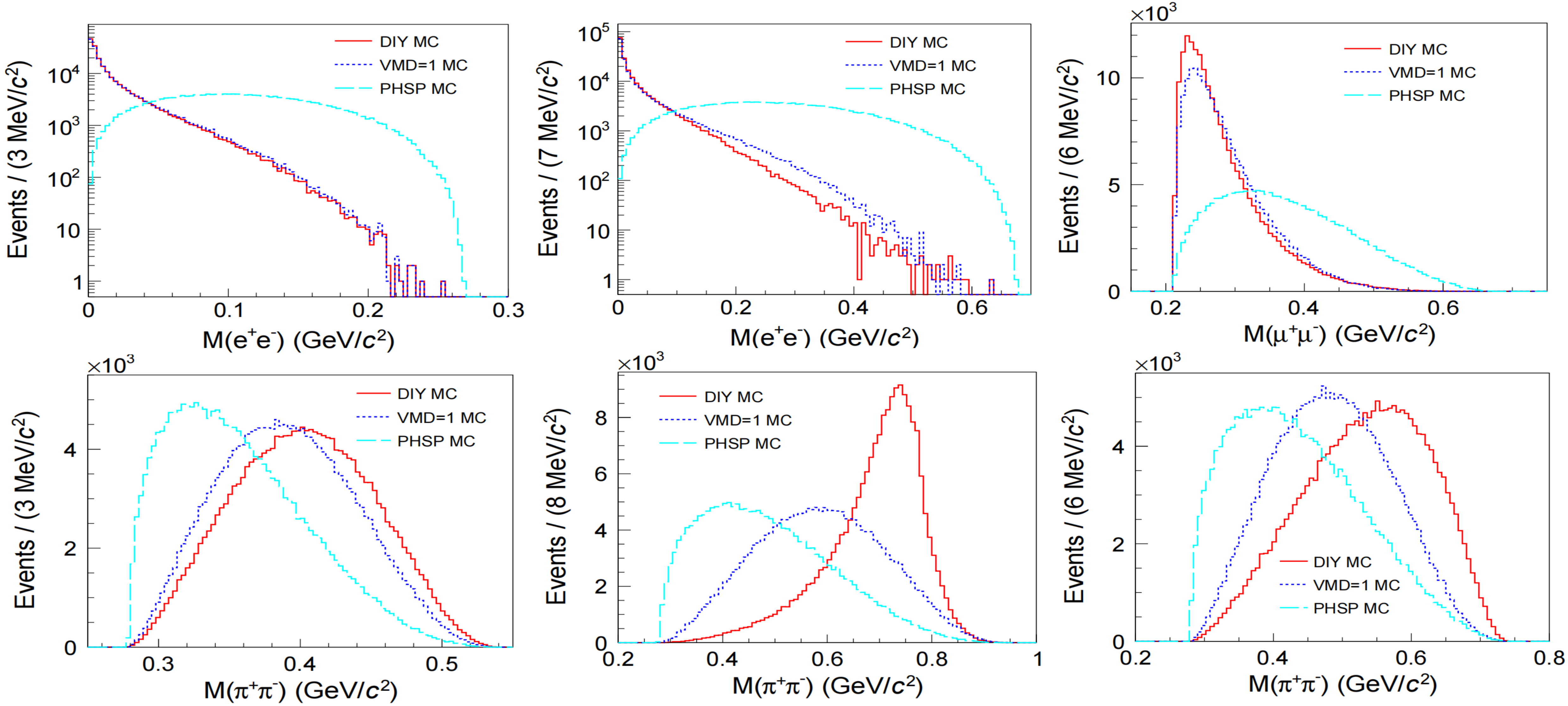


Also possible to access the TFF



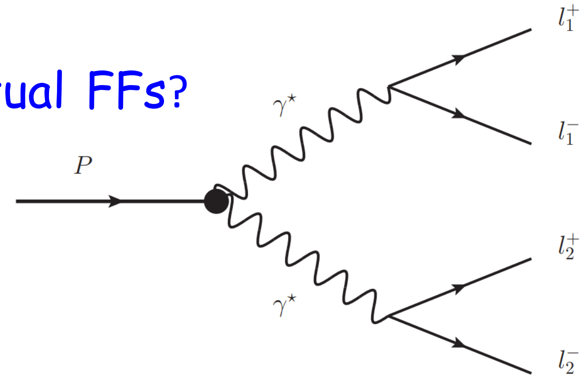
- *Box anomaly*
- ρ *and* ω *are necessary to describe data*

$\eta \to \pi\pi e^+e^-$ $\eta' \to \pi\pi e^+e^-$ $\eta' \to \pi\pi\mu^+\mu^-$

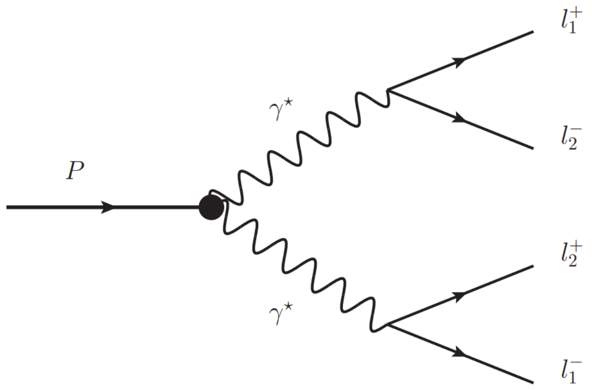


Electromagnetic dalitz decays $\eta/\eta' \to l^+l^-l^+l^-$

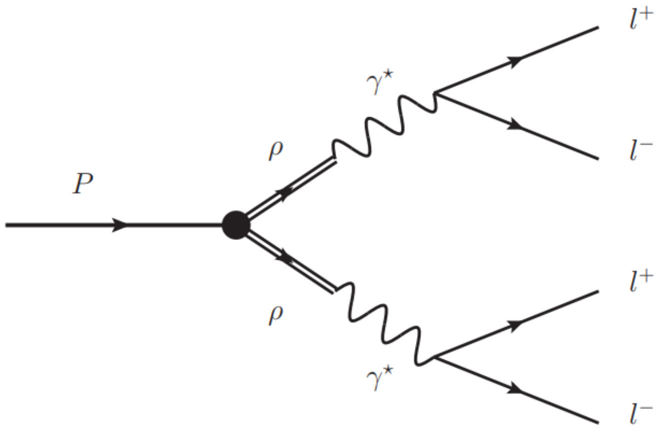
- Double virtual FFs?



Double Dalitz diagram for A1



Double Dalitz diagram for A2



Full VMD term LPVV

$$\overline{|\mathcal{A}_1^2(P \to e^+e^-\mu^+\mu^-)|} = \frac{e^4|M_1(s_{12}s_{34})|^2}{s_{12}^2 s_{34}^2}\lambda(m_P^2, s_{12}, s_{34})\left[2 - \beta_1^2 2\sin^2\theta_1 2 - \beta_{34}^2\sin^2\theta_3 4 + \beta_{12}^2\beta_{34}^2\sin^2\theta_1 2\sin^2\theta_3 4\sin^2\phi\right]$$

$$\overline{|\mathcal{A}_2^2(P \to l^+l^-l^+l^-)|} = \frac{e^4|\mathcal{M}(s_{14}s_{23})|^2}{s_{14}^2 s_{23}^2}\lambda(m_P^2, s_{14}, s_{23})\left[2 - \beta_1^2 4\sin^2\theta_1 4 - \beta_{23}^2\sin^2\theta_{23} + \beta_{14}^2\beta_{23}^2\sin^2\theta_{14}\sin^2\theta_{23}\sin^2\phi\right]$$

T. Petri, arXiv:1010.2378

$$M(s_{12}, s_{34}) = \mathcal{M} \times VMD_1(s_{12}, s_{34})$$

$$VMD_1(s_{12}, s_{34}) = 1 - c_3 + c_3 \times \frac{m_V^2}{m_V^2 - s_{l2} - im_V\Gamma(s_{l2})} \times \frac{m_V^2}{m_V^2 - s_{34} - im_V\Gamma(s_{34})}$$

- Statistics limited !

Rafel, Sergi, CPC 42 (2018) 023109

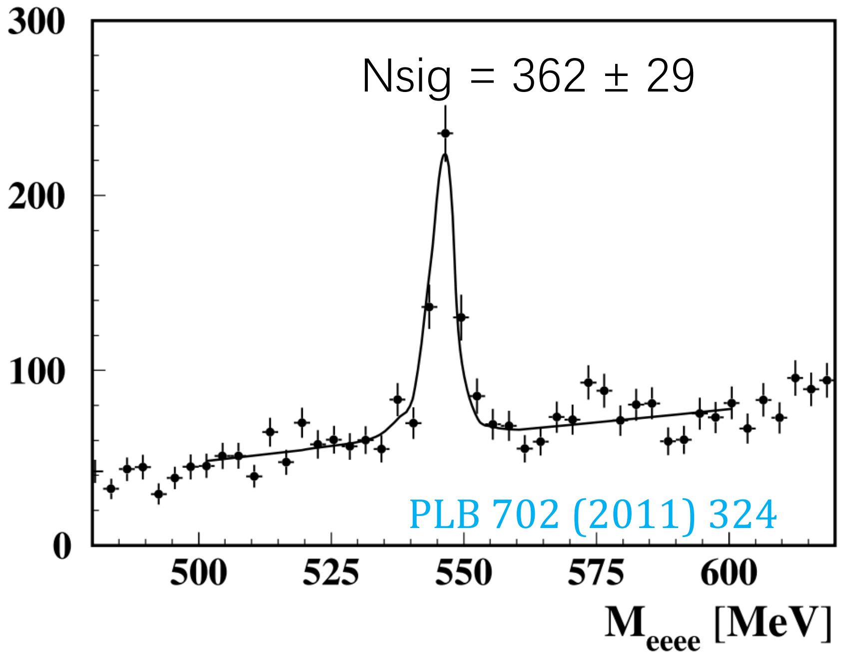
decay	this work	experimental value [1]	n_σ
$\eta \to e^+e^-e^+e^-$	$2.71(2)\times10^{-5}$	$2.4(2)(1)\times10^{-5}$	1.38
$\eta \to \mu^+\mu^-\mu^+\mu^-$	$3.98(15)\times10^{-9}$	$<3.6\times10^{-4}$	
$\eta \to e^+e^-\mu^+\mu^-$	$2.39(7)\times10^{-6}$	$<1.6\times10^{-4}$	
$\eta' \to e^+e^-e^+e^-$	$2.10(45)\times10^{-6}$	not seen	
$\eta' \to \mu^+\mu^-\mu^+\mu^-$	$1.69(36)\times10^{-8}$	not seen	
$\eta' \to e^+e^-\mu^+\mu^-$	$6.39(91)\times10^{-7}$	not seen	

$$B(\eta \to e^+e^-e^+e^-) = (2.40 \pm 0.22)\times10^{-5}$$

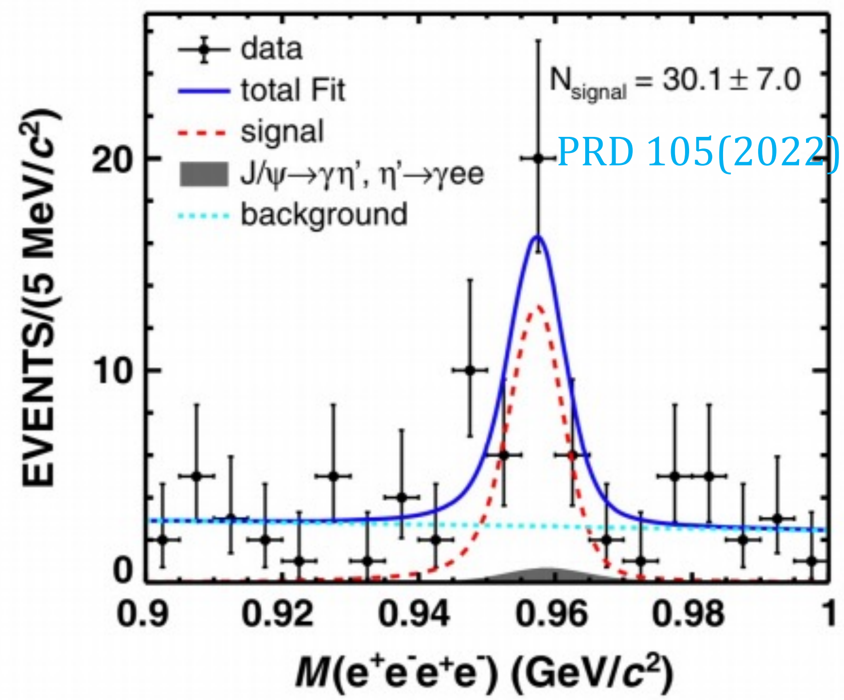
KLOE: PLB 702 (2011) 324

$$B(\eta' \to e^+e^-e^+e^-) = (4.5 \pm 1.0 \pm 0.5)\times10^{-6}$$

BESIII: PRD 105(2022)112010

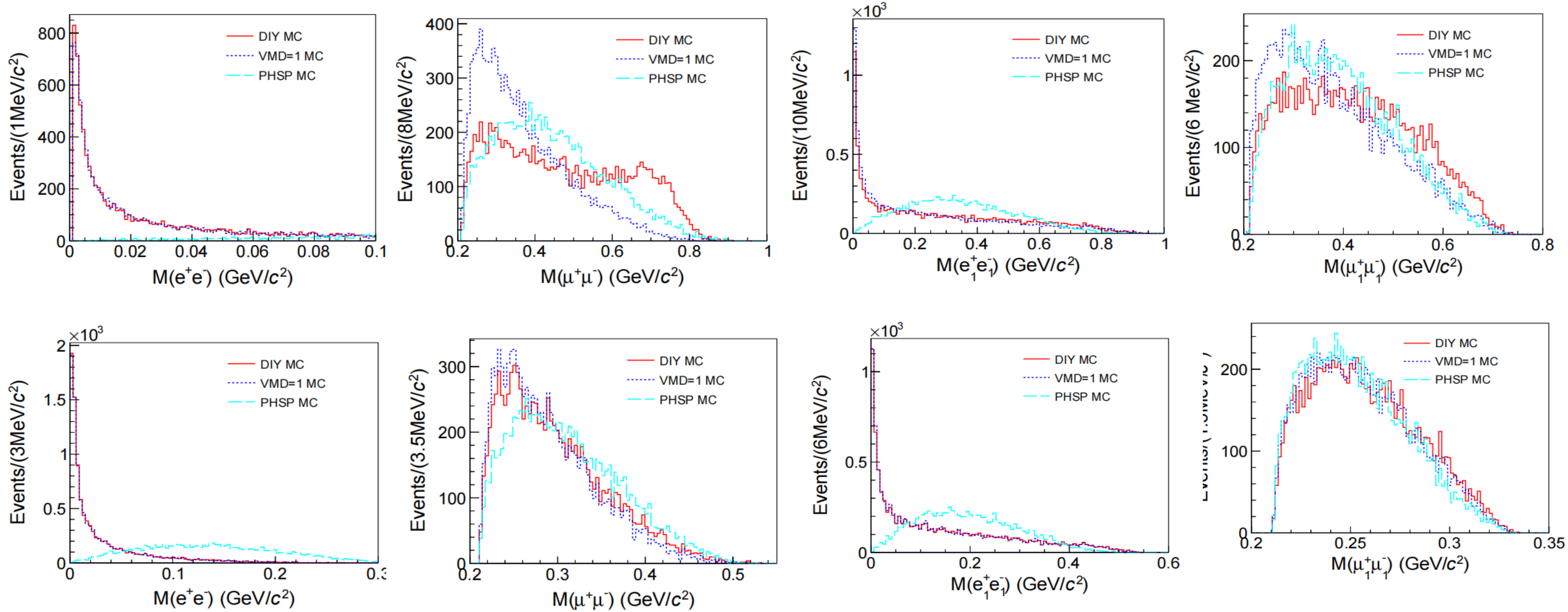


PLB 702 (2011) 324



PRD 105(2022)112010

$\eta' \to e^+e^-\mu^+\mu^-$ $\eta' \to e^+e^-e^+e^-$ $\eta' \to \mu^+\mu^-\mu^+\mu^-$



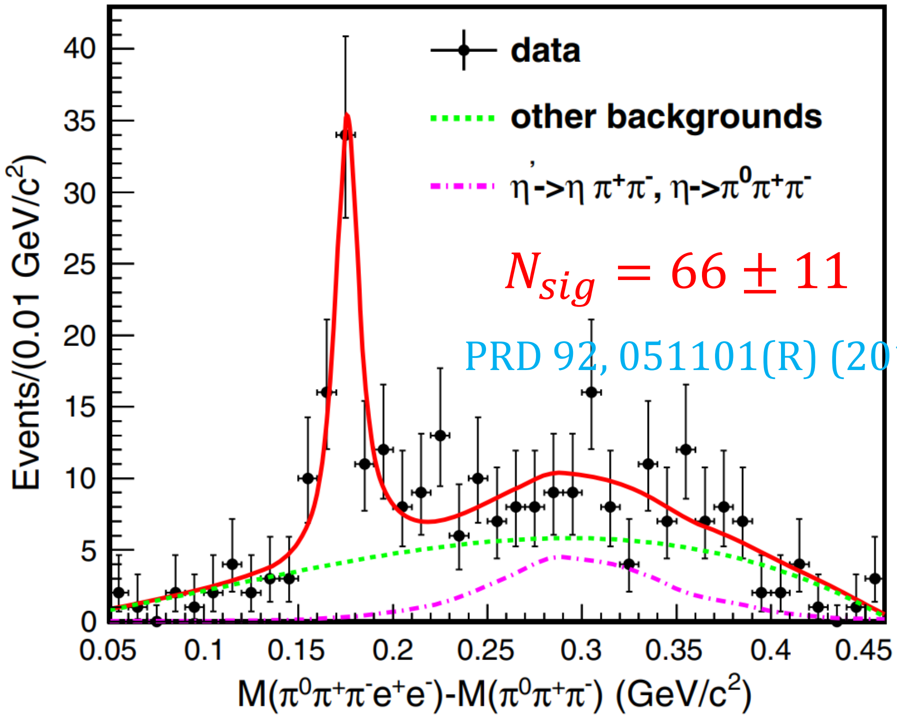
$\eta \to e^+e^-\mu^+\mu^-$ $\eta \to e^+e^-e^+e^-$ $\eta \to \mu^+\mu^-\mu^+\mu^-$

Electromagnetic dalitz decays $\eta' \to \omega e^+ e^-$

A. Faessler, C. Fuchs, M.I. Krivoruchenko,
PRC61 (2000) 035206

Within the framework of effective meson theory, decay amplitude

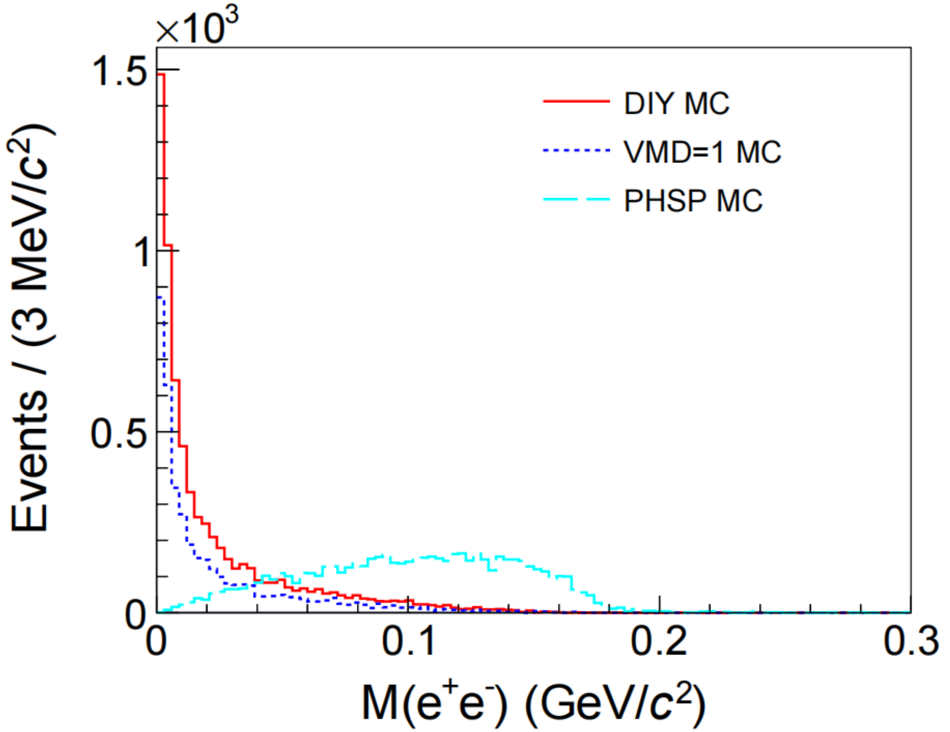
$$\overline{|\mathcal{A}_{P\to Ve^+e^-}|}^2 = \frac{2^6\pi^2 M_P^3 \alpha \Gamma_{P\to\gamma V}}{(M_P^2-M_V^2)^3}|VMD(p)^2|\frac{(M_P^2-p^2-M_V^2)^2-4M_V^2p^2}{p^2}(2-\beta_p^2\sin^2\theta_p),$$



$N_{sig} = 66 \pm 11$

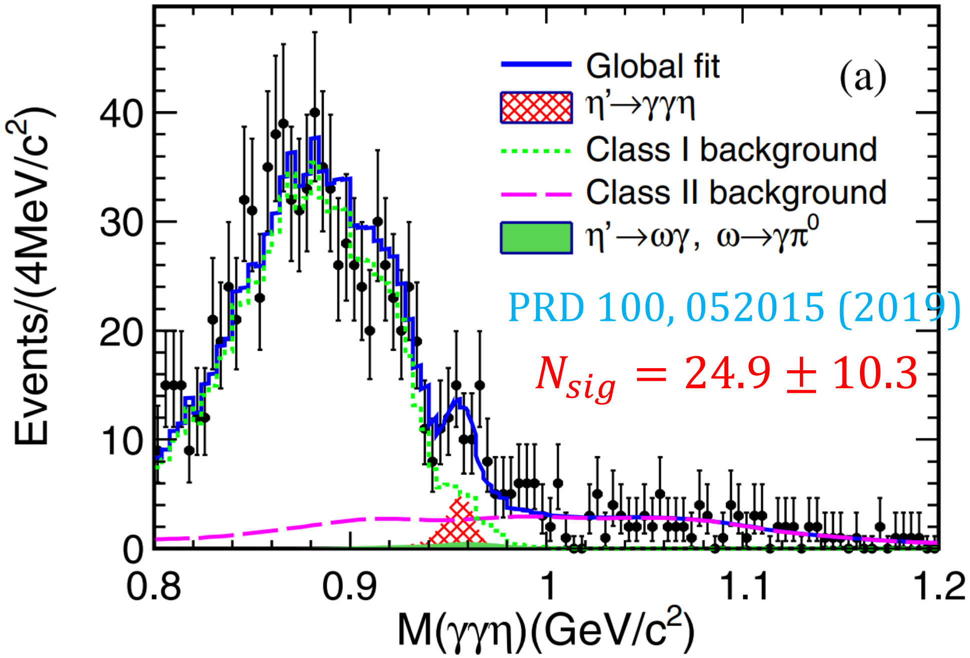
PRD 92, 051101(R) (2015)

$\mathrm{Br} = (1.97 \pm 0.34 \pm 0.17)\times 10^{-4}$

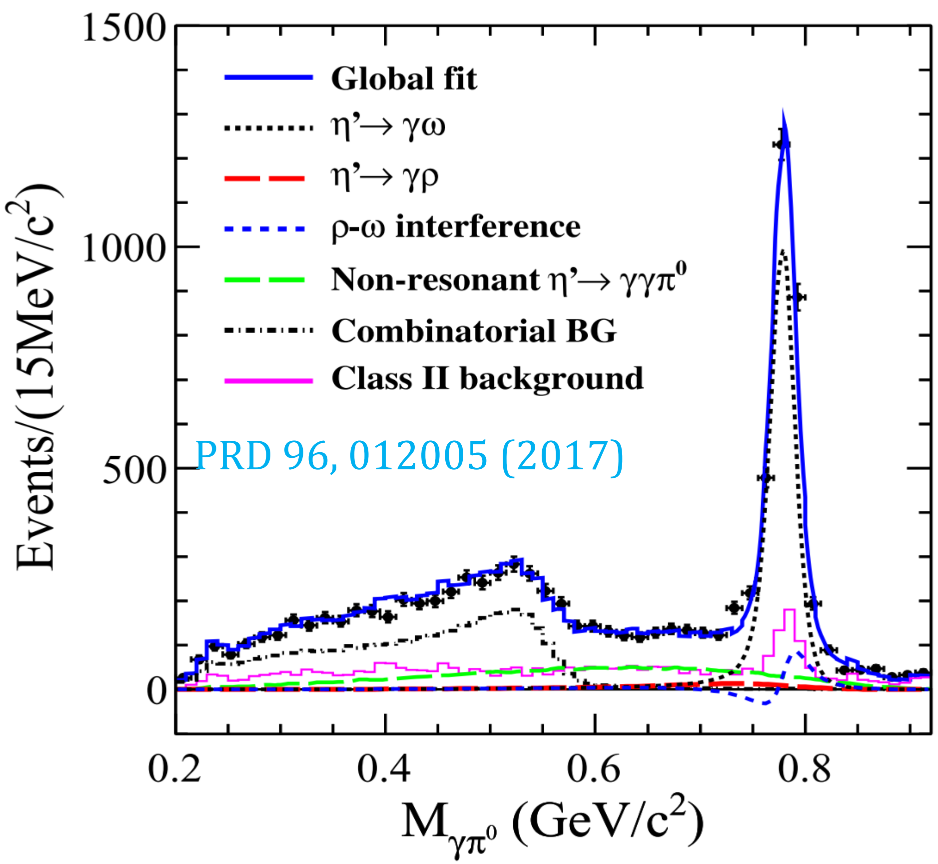


Double radiative decays $\eta' \to \gamma\gamma\pi^0/\eta$

- Within the frameworks of the LσM and VMD
  - ✓ $BF(\eta' \to \gamma\gamma\pi^0) = 2.91(21) \times 10^{-3}$
  - ✓ $BF(\eta' \to \gamma\gamma\eta) = 1.17(8) \times 10^{-4}$



$$Br(\eta' \to \gamma\gamma\eta) = (8.25 \pm 3.41 \pm 0.72) \times 10^{-5}$$

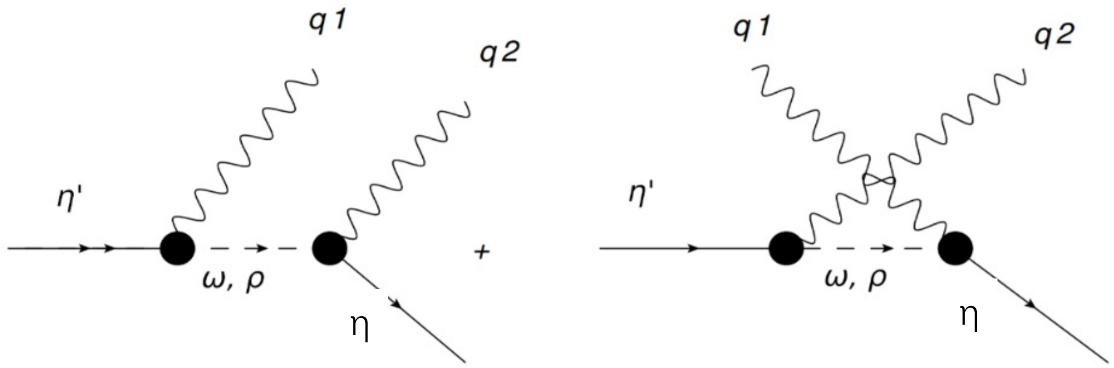


$$Br(\eta' \to \gamma\omega \to \gamma\gamma\pi^0) = (23.7 \pm 1.4 \pm 1.8) \times 10^{-4}$$

$BF(\eta' \to \gamma\gamma\pi^0)_{NR} = (6.16 \pm 0.64 \pm 0.67) \times 10^{-4}$

$\eta' \to \gamma\gamma\pi^0$ is dominant by ω resonance

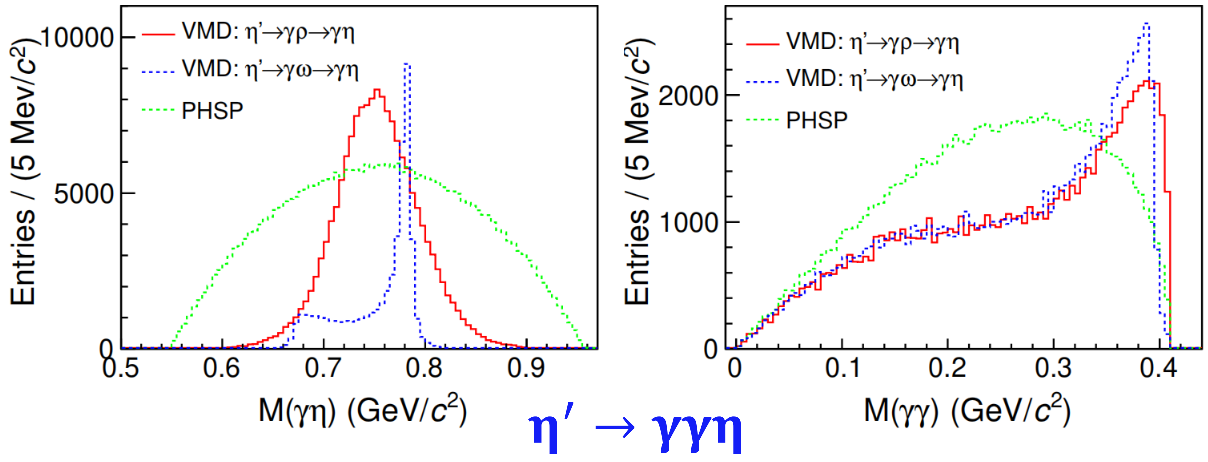
Double radiative decays $\eta' \to \gamma\gamma\pi^0/\eta$

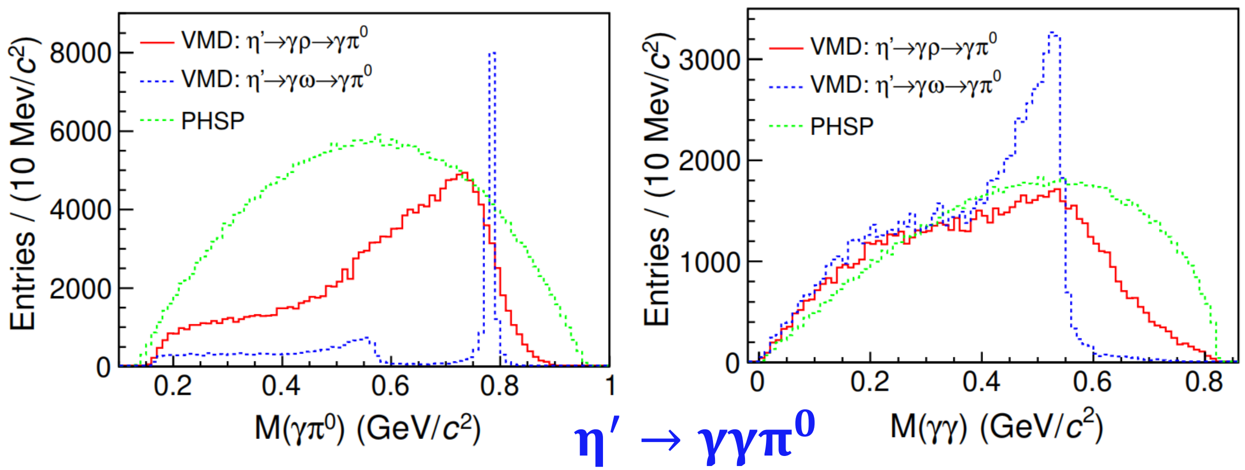


R. Escribano, S. Gonzalez-Solis, R. Jora, E. Royo, PRD 102 (2020) 034026

- Within the frameworks of the linear σ model (LσM) and VMD model

$$\mathcal{A}^{VMD}_{P\to\gamma\gamma P'} = \sum_{V=\rho^0,\omega,\phi} g_{VP\gamma}g_{VP'\gamma}\left[\frac{(P\cdot q_2 - m_P^2)\{a\} - \{b\}}{m_V^2 - t - im_V\Gamma_V} + \left\{\begin{array}{c} q_1 \leftrightarrow q_2 \\ t \leftrightarrow u \end{array}\right\}\right]$$

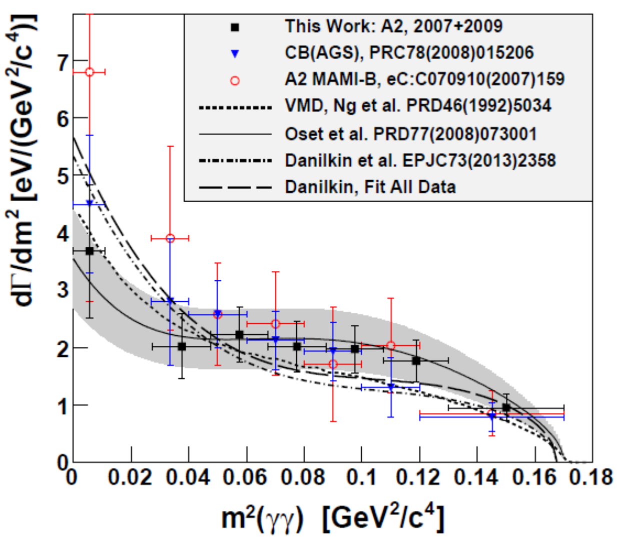




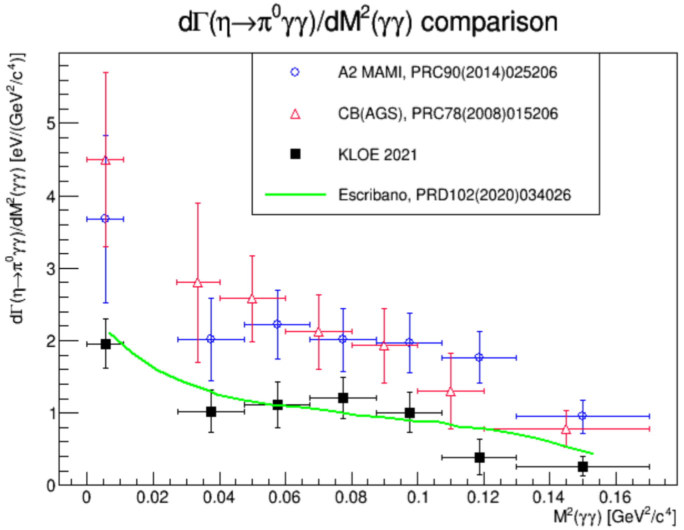
Double radiative decays $\eta \to \gamma\gamma\pi^0$

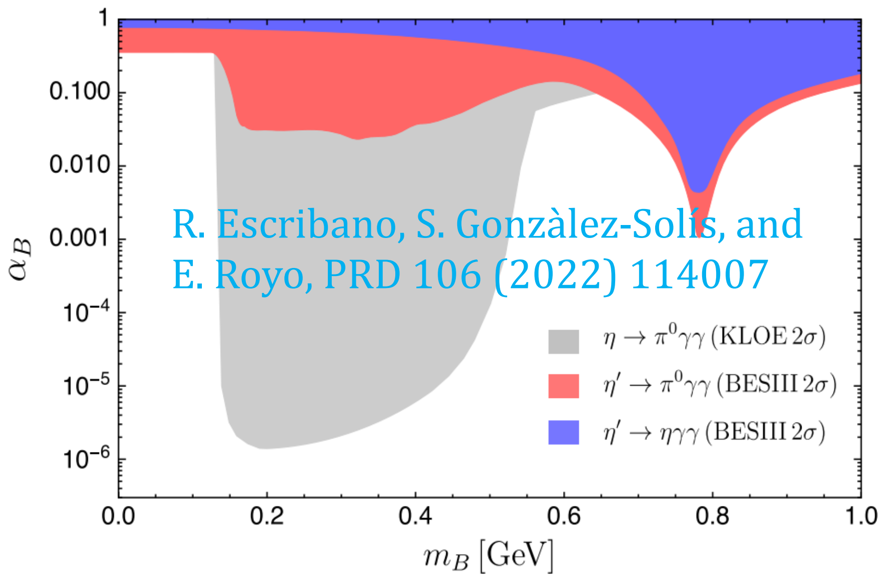
- ChPT "golden mode": O(p^2) null, O(p^4) suppressed, O(p^6) dominates [PLB 276(1) (1984) 185]
- Discrepancy between Exp. and The.
  - ✓ Br($\eta \to \pi^0\gamma\gamma$) = $(1.30\pm0.08)\times10^{-4}$ LσM+VMD
  - ✓ Br = $(2.21\pm0.24\pm0.47)\times10^{-4}$ CB@AGS (2008)
  - ✓ Br = $(2.52\pm0.25)\times10^{-4}$ CB@MAMI (2014)
  - ✓ Br = $(0.99\pm0.11\pm0.24)\times10^{-4}$ KLOE-2

A2: PRC 90 (2014) 025206

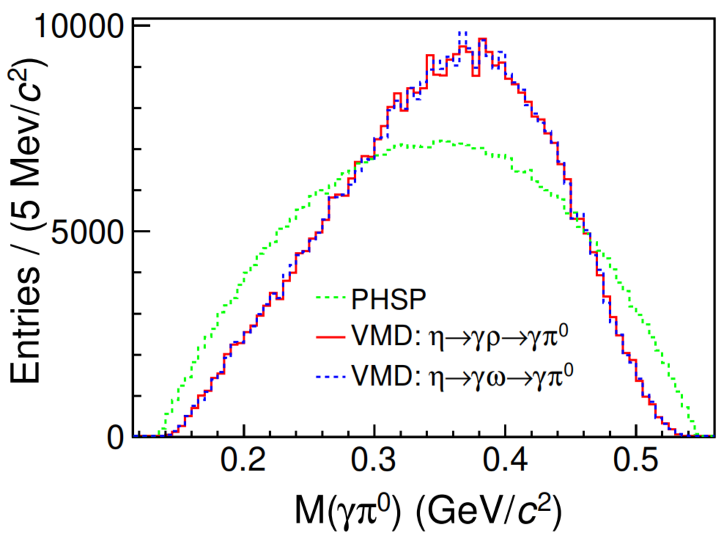


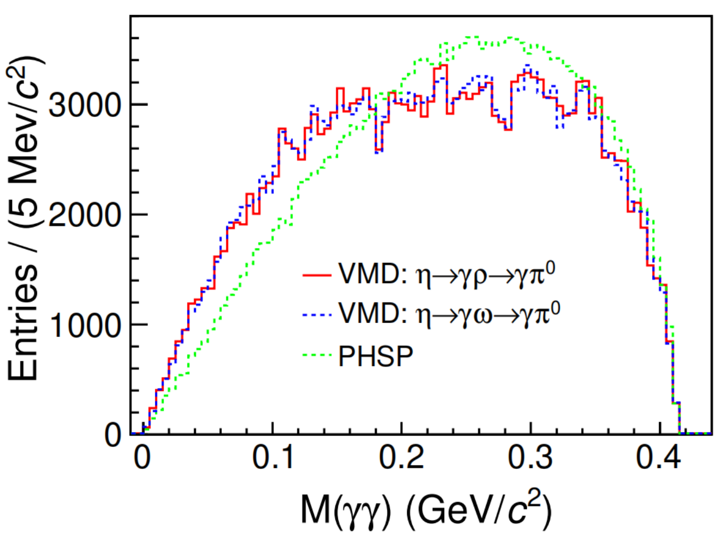
KLOE-2: EPS-HEP2023





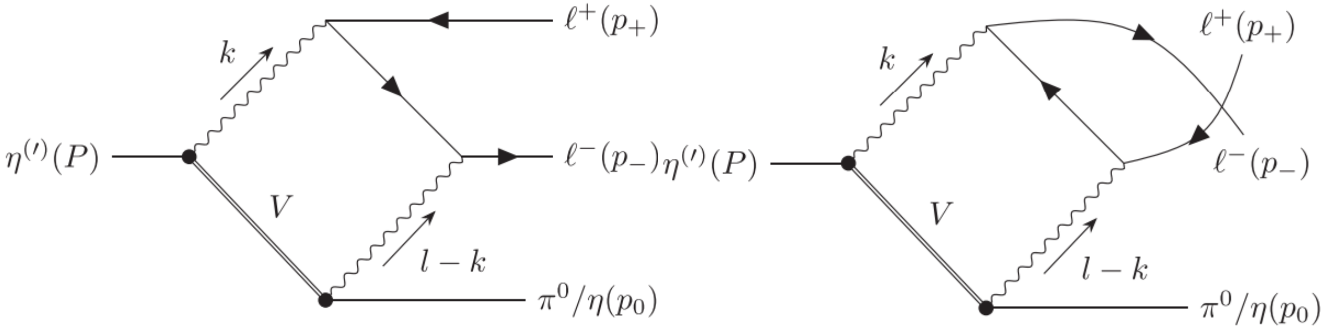
R. Escribano, S. Gonzàlez-Solís, and E. Royo, PRD 106 (2022) 114007





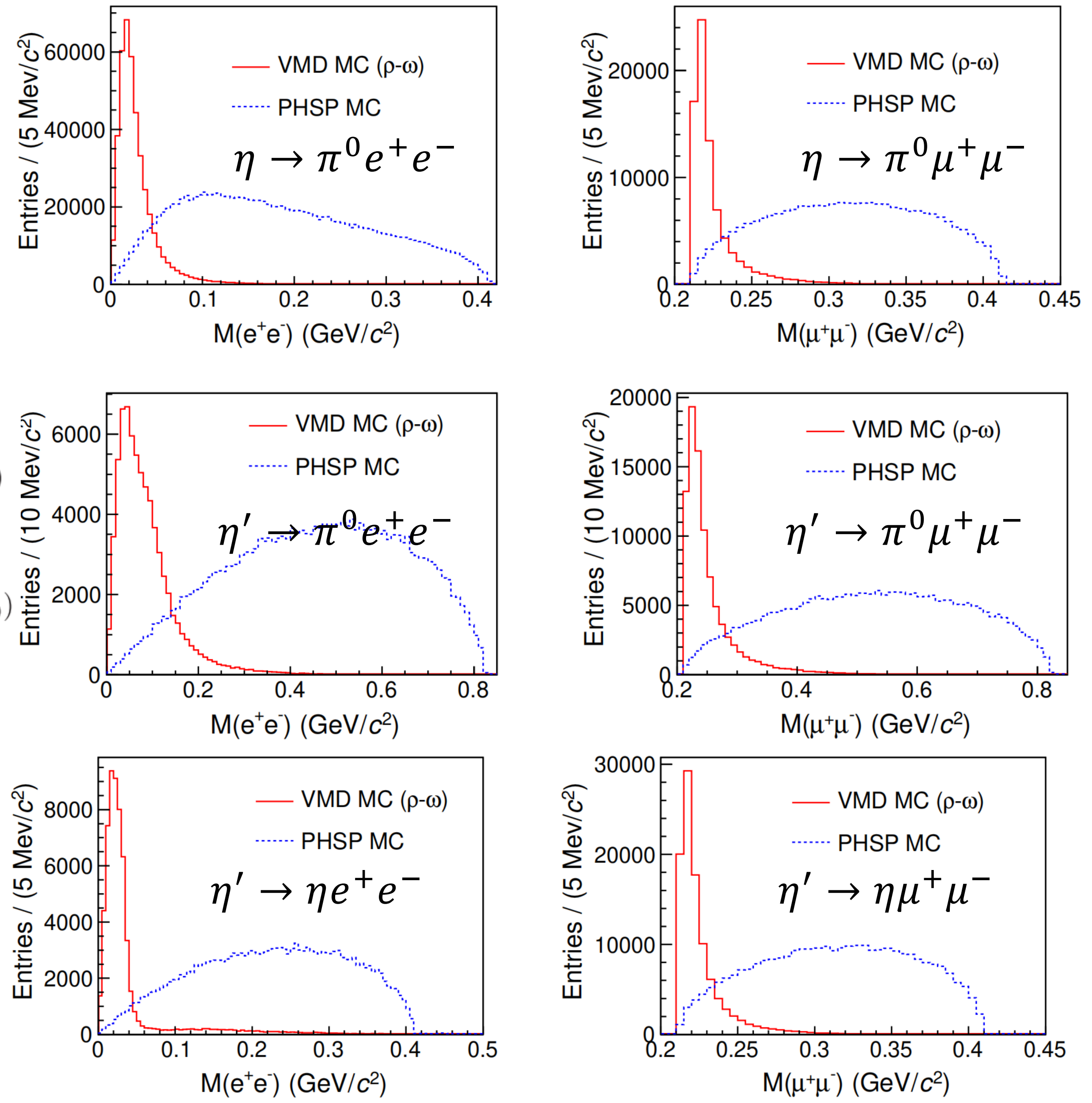
Semileptonic decays $\eta/\eta' \to \pi^0 l^+ l^-, \eta' \to \eta l^+ l^-$

- $\eta \to \pi^0 \gamma^* \to \pi^0 l^+ l^-$ is C-violated process
- Proceed via the C-conserving process of $\eta \to \pi^0 \gamma^* \gamma^* \to \pi^0 l^+ l^-$, the BR predicted in 10^{-11} to 10^{-8}



✓ **Decay amplitude** H. Schäfer, M. Zanke, Y. Korte, B. Kubis, PRD 108, 074025 (2023)

$$|\overline{\mathcal{M}}|^2 = 256\pi^4\alpha^4[C_\rho^2|\overline{\mathcal{M}}_{\rho,\rho}|^2 + C_\omega^2|\overline{\mathcal{M}}_{\omega,\omega}|^2 + C_\phi^2|\overline{\mathcal{M}}_{\phi,\phi}|^2 + C_\rho C_\omega|\overline{\mathcal{M}}_{\rho,\omega}|^2 + C_\rho C_\phi|\overline{\mathcal{M}}_{\rho,\phi}|^2 + C_\omega C_\phi|\overline{\mathcal{M}}_{\omega,\phi}|^2]$$



Summary

- **The decay of $\eta' \rightarrow \pi^+\pi^-\eta$ is found to be particularly attractive for studies of η rare decays**
- **MC generators for η/η' main decays are organized and developed**
- **BESIII: More results are expected to come soon**
- **STCF provide more opportunities to precisely studies η and η' decays**

Thanks for your attention!